

**RISK, UNCERTAINTY, AND EXCHANGE RATES\***

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This paper is motivated by two facts: failure of log-linear empirical exchange rate models of the 1970's and the observed variability of risk premiums in the forward market. Rational maximizing models predict that changes in conditional variances of monetary policies, government spendings, and income growths affect risk premiums and induce conditional volatility of exchange rates. I examine theoretically how changes in these exogenous conditional variances affect the level of the current exchange rate and attempt to quantify the extent that this channel explains exchange rate volatility using autoregressive conditional heteroscedastic models.

**1. Introduction**

Most existing empirical models of exchange rates were designed to address the influence of the first moments of exogenous processes on exchange rates. The models are usually linear in natural logarithms, and their solutions express logarithms of exchange rates as the discounted expected values of the logarithms of the future driving processes with constant rates of discount. Many of the models assume that risk neutrality provides a good approximation of the preferences of actual economic agents.

Since only first moments of exogenous processes matter for behavior in these models, they cannot answer questions such as how does the exchange rate respond to an increase in the uncertainty of government spending, monetary policy, or the rate of technological change. Explicit nonlinear

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models based on the maximizing behavior of risk-averse agents are able to address these questions. The goal of this paper is to examine these issues.

There are several additional motivations for the first sections of this paper. The first comes from the work of Meese and Rogoff (1983), who explore the out-of-sample predictive ability of the log-linear models of the 1970's that were the first rational expectations models to study exchange rates as an asset market equilibrium. Their striking finding is the general failure of these models to beat a random walk prediction for the exchange rate, even when the models are given *ex post* values of the right-hand-side variables. One aspect of the economy that is ignored in constructing linear models is the nature of risk. Meese and Rogoff (1983) suggest that time-varying risk premiums could be an important determinant of their findings although they express skepticism about the likelihood of this being the complete explanation.

A second motivation is the finding of Fama (1984) who investigated regressions of *ex post* rates of currency depreciation on the forward premium, which is defined to be the expected rate of depreciation plus a risk premium. One interpretation of his results is that risk premiums in the forward market are more variable than expected rates of depreciation. This interpretation is valid under the hypothesis of rational expectations and the assumptions that the sample statistics are converging to the true moments of the population with correct asymptotic standard errors. Presumably, the variables that cause time-varying risk premiums in the forward market are potentially important in the determination of spot exchange rates and the levels of other asset prices.<sup>1</sup>

A third motivation is the partial equilibrium exercise that Frankel and Meese (1987) conducted to examine how a change in the conditional variance of the future spot exchange rate affects the level of the current exchange rate. Their surprising calculations indicate that plausible changes in the conditional variance of the exchange rate can have substantial effects on the level of the spot rate. Frankel and Meese (1987) acknowledge that their exercise is partial equilibrium, and they suggest that a two-period mean-variance model is unlikely to be appropriate in an environment in which conditional variances are moving. Intertemporal general equilibrium models are required.

Finally, the fourth motivation comes from the theoretical exercises that Abel (1986) and Giovannini (1987) performed. They examined how changes in the conditional variance of an exogenous aggregate real dividend process affect the level of stock prices in general equilibrium. The effect depends on the degree of relative risk aversion or the rate of intertemporal substitution of a representative agent, but their models predict opposite directions of the effect. Abel (1986) worked with the Lucas (1978) model, which is a real barter model, while Giovannini (1987) worked with the Svensson (1985b) model, which modified the timing of transactions in the monetary model of Lucas (1982).

<sup>1</sup>The empirical literature on risk premiums and the efficiency of forward and futures foreign exchange markets is critically reviewed in Hodrick (1987).

The analysis is conducted in the next four sections. Section 2 specifies the preferences and budget constraints of the countries and defines an equilibrium. Section 3 provides closed form solutions for some of the key variables of the model under assumptions on the time series properties of the exogenous processes. Section 4 contains the empirical analysis associated with the model. Some concluding remarks are contained in section 5.

## 2. A modified Svensson model

In this section I explore a version of the cash-in-advance model presented in Svensson (1985a, b) and discussed in Stockman and Svensson (1987). The model is a modification of the monetary model of Lucas (1982). I add a discussion of exogenous fiscal policy and examine time-varying conditional variances of the exogenous processes. These extensions allow consideration of the issues outlined above.

### 2.1. Countries and endowments

There are two goods that are the endowments of the two countries denoted 1 and 2. The endowments are exogenous, nonstorable, and have realizations denoted  $Y_{1t}$  and  $Y_{2t}$ . The timing of the model follows Svensson (1985b) with goods markets open in the beginning and asset markets open at the end of each period. The endowments are elements of the exogenous state at time  $t$  that is denoted  $x_t$ , and it will be demonstrated that the state follows a first-order Markov process with transition density given by  $F(x_{t+1}|x_t)$ .

### 2.2. Government sectors

Each government buys some of that country's goods in the competitive markets. The exogenous purchase is denoted  $G_{it}$ ,  $i=1,2$ . The government budget constraints require balance between purchases of goods and taxes collected net of securities issued and redeemed. I consider only real head taxes, which are denoted  $\tau_{it}$ , for  $i=1,2$ . Taxes are paid at the asset market. The governments also issue state-contingent claims to nominal money, where  $B_{it}(x_t)$  is the amount of currency  $i$  that the government of country  $i$  promises at time  $t-1$  to pay at time  $t$  contingent on the state of the world being  $x_t$ . The money stocks are exogenous and are given by  $M_{it}$ ,  $i=1,2$ , for the outstanding monies at the end of period  $t-1$ . Country 1 money is the 'dollar', and country 2 money is the 'pound'.

The governments' flow budget constraints are therefore

$$G_{it} = \tau_{it} + \left[ \int n_i(x_{t+1}, x_t) B_{it+1}(x_{t+1}) dx_{t+1} - B_{it}(x_t) \right] / P_{it} \\ + (M_{it+1} - M_{it}) / P_{it}, \quad i=1,2. \quad (1)$$

In (1)  $n_i(x_{t+1}, x_t)$  is the endogenous nominal pricing function associated with money  $i$ . It provides money  $i$  values at time  $t$  in state  $x_t$  of promises to amounts of money  $i$  at time  $t + 1$  given state  $x_{t+1}$ . The dollar price of good 1 is  $P_{1t}$  and the pound price of good 2 is  $P_{2t}$ .

The governments are subject to cash-in-advance constraints on purchases of goods, although with their access to the printing press they are not limited in their nominal spending by their previous accumulation of money. If  $M_{it}^g$  is the amount of money that government  $i$  acquired in the asset market at time  $t - 1$ , then the cash-in-advance constraints are

$$P_{it}G_{it} \leq M_{it}^g + (M_{it+1} - M_{it}), \quad i = 1, 2. \quad (2)$$

The time series of government spending, taxation, and money creation are exogenous, and the government is assumed to issue debt to be consistent with its budget constraint. The exogenous gross rate of monetary growth of country  $i$  in period  $t$  is  $\Omega_{it} \equiv M_{it+1}/M_{it}$ ,  $i = 1, 2$ .

### 2.3. Preferences and budget constraints

The preferences of the representative agent in each country are identical and homothetic. The agents are assumed to have identical initial wealth levels and are taxed equally by the two countries as in Sargent (1987). These assumptions facilitate the discussion of an equilibrium, since they lead to the perfectly pooled equilibrium of Lucas (1982).

The objective function of the representative consumer is to maximize expected lifetime utility as in

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_{1t}, C_{2t}) \right\}, \quad 0 < \beta < 1, \quad (3)$$

by choice of consumption of the good of country 1,  $C_{1t}$ , and of the good of country 2,  $C_{2t}$ . The period utility function is sufficiently concave that the Inada conditions are satisfied and an internal equilibrium is guaranteed.<sup>2</sup>

Information relevant to the decisions for the period is obtained at the beginning of the period when the representative consumer faces two cash-in-advance constraints dictating the quantities that can be consumed. In the period  $t - 1$  asset market the representative agent acquires  $M_{it}^p$  of currency  $i$ . In period  $t$  the purchasing power of the dollar in terms of good 1 is

<sup>2</sup>The Inada conditions require that the ratio of the marginal utility of good 1 to that of good 2 goes to zero when the consumption of good 1 goes to infinity, holding the consumption of good 2 constant, and the same ratio goes to infinity when the consumption of good 2 goes to infinity, holding the consumption of good 1 constant.

$\Pi_{1t} \equiv 1/P_{1t}$ , and the purchasing power of the pound in terms of good 1 is  $\Pi_{2t} \equiv S_t/P_{1t}$ , where  $S_t$  is the exchange rate of dollars per pound. The cash-in-advance constraints are

$$C_{1t} \leq M_{1t}^p \Pi_{1t}, \quad (4a)$$

$$\Theta_t C_{2t} \leq M_{2t}^p \Pi_{2t}, \quad (4b)$$

and the relative price of good 2 in terms of good 1, which describes a real terms of trade, is  $\Theta_t \equiv S_t P_{2t}/P_{1t}$ . Although this defines a terms of trade, it is not possible to trade goods for goods within a period.

The consumer's budget constraint requires that purchases of assets in the asset market be less than or equal to wealth at that time. Agents trade titles to the endowment processes of the two countries, and the number of shares to the two endowments purchased at time  $t$  is denoted  $Z_{it+1}$  with dollar prices of the shares denoted  $Q_{it}$ , for  $i = 1, 2$ . The total number of shares is normalized to unity for each of the two shares to endowments. The consumers can purchase state-contingent monies, where  $B_{it}^p(x_t)$  is the amount of money  $i$  purchased at time  $t - 1$  for delivery at the time  $t$  asset market conditional on the state being  $x_t$ . The agent's resources are any unspent monies from the goods markets, the payoffs plus resale values of their shares, and the state-contingent payoffs of monies, but minus the tax liabilities. The budget constraint in period  $t$  is

$$\begin{aligned} & \Pi_{1t} M_{1t+1}^p + \Pi_{2t} M_{2t+1}^p + \Pi_{1t} \int n_1(x_{t+1}, x_t) B_{1t+1}^p(x_{t+1}) dx_{t+1} \\ & + \Pi_{2t} \int n_2(x_{t+1}, x_t) B_{2t+1}^p(x_{t+1}) dx_{t+1} + \Psi_{1t} Z_{1t+1} + \Psi_{2t} Z_{2t+1} \\ & \leq (\Pi_{1t} M_{1t}^p - C_{1t}) + (\Pi_{2t} M_{2t}^p - \Theta_t C_{2t}) + \Pi_{1t} B_{1t}^p(x_t) + \Pi_{2t} B_{2t}^p(x_t) \\ & + (\Psi_{1t} + Y_{1t}) Z_{1t} + (\Psi_{2t} + \Theta_t Y_{2t}) Z_{2t} - \frac{1}{2}(\tau_{1t} + \Theta_t \tau_{2t}). \end{aligned} \quad (5)$$

In (5) the good 1 real price of a share of the endowment in country  $i$  is  $\Psi_{it} \equiv Q_{it}/P_{1t}$ ,  $i = 1, 2$ .

By adding the real value of current consumption and the real tax liabilities to both sides of (5), the right-hand side of the modified (5) is defined to be real wealth, which is denoted

$$\begin{aligned} W_t & \equiv \Pi_{1t} M_{1t}^p + \Pi_{2t} M_{2t}^p + \Pi_{1t} B_{1t}^p(x_t) + \Pi_{2t} B_{2t}^p(x_t) \\ & + (\Psi_{1t} + Y_{1t}) Z_{1t} + (\Psi_{2t} + \Theta_t Y_{2t}) Z_{2t}. \end{aligned} \quad (6)$$

#### 2.4. Solution of the agent's problem

Consider the value function of the agent's problem. The agent must choose consumption and portfolio allocations given current real wealth and real stocks of money and the nature of uncertainty about the future that is characterized by the probability distribution of future states of the world. Hence, the value function is

$$\begin{aligned}
 & V(W_t, \Pi_{1t}M_{1t}^p, \Pi_{2t}M_{2t}^p, x_t) \\
 &= \max \left\{ U(C_{1t}, C_{2t}) + \beta \int V(W_{t+1}, \Pi_{1t+1}M_{1t+1}^p, \Pi_{2t+1}M_{2t+1}^p, x_{t+1}) \right. \\
 & \quad \left. \times F(x_{t+1}|x_t) dx_{t+1} \right\}, \tag{7}
 \end{aligned}$$

where the choices of consumption goods and portfolio allocations are subject to the constraints in (4) and (5). The conditional expectation of the agent in (7) is rational because it is taken with respect to the true transition probability of the future state.

If  $\lambda_t$  is the multiplier for the period  $t$  budget constraint (5),  $\nu_{1t}$  is the multiplier for the period  $t$  dollar-good cash-in-advance constraint (4a), and  $\nu_{2t}$  is the multiplier for the period  $t$  pound-good cash-in-advance constraint (4b), the first-order conditions are

$$U_{1t} = \lambda_t + \nu_{1t}, \tag{8a}$$

$$U_{2t} = (\lambda_t + \nu_{2t})\Theta_t, \tag{8b}$$

$$\lambda_t \Pi_{1t} = \beta E_t[(\lambda_{t+1} + \nu_{1t+1})\Pi_{1t+1}], \tag{8c}$$

$$\lambda_t \Pi_{2t} = \beta E_t[(\lambda_{t+1} + \nu_{2t+1})\Pi_{2t+1}], \tag{8d}$$

$$\lambda_t \Psi_{1t} = \beta E_t[(\Psi_{1t+1} + Y_{1t+1})\lambda_{t+1}], \tag{8e}$$

$$\lambda_t \Psi_{2t} = \beta E_t[(\Psi_{2t+1} + \Theta_{t+1}Y_{2t+1})\lambda_{t+1}], \tag{8f}$$

$$\lambda_t \Pi_{1t} n_1(x_{t+1}, x_t) = \beta \lambda_{t+1} \Pi_{1t+1} F(x_{t+1}|x_t), \quad \forall x_{t+1}, \tag{8g}$$

$$\lambda_t \Pi_{2t} n_2(x_{t+1}, x_t) = \beta \lambda_{t+1} \Pi_{2t+1} F(x_{t+1}|x_t), \quad \forall x_{t+1}. \tag{8h}$$

In (8a)–(8b) the partial derivative of the utility function with respect to its  $i$ th argument is denoted  $U_{i,t}$ . Each cash-in-advance constraint in (4) also holds with equality when its associated multiplier is strictly greater than zero, and if the multiplier equals zero, the constraint is not binding. All expectations in (8c)–(8f) are with respect to the density function of  $x_{t+1}$  given  $x_t$ .

The interpretation of (8a)–(8h) is straightforward. Eq. (8a) relates the marginal utility of consumption of good 1 to the marginal value of real wealth in units of good 1 plus the marginal value of the real dollar balances of the agent. Similarly, (8b) relates the marginal utility of good 2 to the marginal value of wealth plus the marginal value of the real pound money balances held by the agent where both multipliers are multiplied by the relative price of good 2 in terms of good 1 because they are in units of good 1. An important aspect of these two expressions is that the current marginal utility of consumption is not equated to the marginal value of wealth unless the cash-in-advance constraint associated with that good is slack.<sup>3</sup>

Eqs. (8c)–(8h) are the Euler equations for the investment decisions of the agent. Eqs. (8c)–(8d) are associated with the decisions to increase money balances in period  $t$ , which involves a tradeoff of the product of the current real value of the money in terms of good 1 and the current marginal value of wealth against the expected utility value of the money in the next period's goods market which is its real purchasing power in terms of good 1 times the marginal value of wealth plus the marginal value of money at that time. Eqs. (8e)–(8f) are associated with the purchases of shares in the endowments. Investment at time  $t$  in a title to future output requires a utility sacrifice given by the product of the current real price of the asset and the current marginal value of wealth. Since all assets, other than monies, pay off and can be resold only in the next period's asset market, which is after consumption in that period, the utility gain to purchasing an asset is the expectation of the product of the real resources available from holding the asset with the marginal value of wealth at time  $t + 1$ . Eqs. (8g)–(8h) involve the purchase of state-contingent monies for delivery in the next asset market. If a unit of money  $i$  for delivery in a particular state  $x_{t+1}$  is purchased today at a nominal price of  $n_i(x_{t+1}, x_t)$ , the agent sacrifices real value given by the current purchasing power of that money times the marginal value of wealth. The value received in return is the real value of the unit of money conditional on the realization of the particular state times the marginal value of wealth in that state times the probability of that state being realized. These equations must hold for all possible future states.

<sup>3</sup>Townsend (1987) argues that disparities between the marginal utility of consumption and the marginal utility of wealth in models with explicit monetary technologies may help to resolve asset pricing anomalies. Other formulations of cash-in-advance constraints have been explored by Lucas (1984) and Lucas and Stokey (1983, 1987).

### 2.5. Definition of an equilibrium

An equilibrium is a set of initial conditions  $\{M_{i0} > 0, B_{i0}(x_0), i = 1, 2\}$  and stochastic processes for the exogenous variables  $\{Y_{it}, G_{it}, \tau_{it}, M_{it+1}^g, M_{it+1}, i = 1, 2\}_{t=0}^{\infty}$ , the endogenous choice variables  $\{C_{it}, M_{it+1}^p, B_{it+1}^p(x_{t+1}), Z_{it-1}, i = 1, 2\}_{t=0}^{\infty}$ , the prices of goods and assets  $\{\Pi_{it}, \Theta_t, \Psi_{it}, i = 1, 2\}_{t=0}^{\infty}$ , that are functions of the current state of the economy, and the pricing functions  $n_i(x_{t+1}, x_t), i = 1, 2$ , such that the following conditions are satisfied: (i) The two government budget constraints in (1) are balanced for all  $t \geq 0$ , and the cash-in-advance constraints (2) are satisfied with equality. (ii) Given the pricing functions for contingent money purchases, the real share prices, and the stochastic processes for  $\{\tau_{it}, \Pi_{it}, Y_{it}, i = 1, 2\}$  and the initial conditions, the choices of the households for consumption goods, money holdings, contingent claim purchases, and share purchases solve the agent's constrained maximization problem. (iii) There is market clearing in the competitive markets for goods, shares, and contingent claims on monies for all periods  $t \geq 0$ , where market clearing is given by the following:

$$2C_{it} + G_{it} = Y_{it}, \quad i = 1, 2, \quad (9a)$$

$$Z_{it+1} = \frac{1}{2}, \quad i = 1, 2, \quad (9b)$$

$$M_{it+1} = M_{it+1}^g + 2M_{it+1}^p, \quad i = 1, 2, \quad (9c)$$

$$B_{it+1}(x_{t+1}) = 2B_{it+1}^p(x_{t+1}), \quad i = 1, 2, \quad \forall x_{t+1}. \quad (9d)$$

One equilibrium is the perfectly pooled equilibrium of Lucas (1982). Agents equally share the endowments, net of government consumption, of the two goods.

### 3. Closed-form equilibrium solutions

In developing explicit solutions I work with particular time series properties for the exogenous variables. The processes on endowments and gross rates of growth of money supplies are assumed to be conditionally log normal. If lower-case letters indicate natural logarithms of upper-case counterparts, then the processes are

$$y_{it+1} = \rho_i y_{it} + (1 - \rho_i) y_i + \varepsilon_{it+1}, \quad 0 \leq |\rho_i| \leq 1, \quad i = 1, 2, \quad (10a)$$

$$\omega_{it+1} = \rho_{i+2} \omega_{it} + (1 - \rho_{i+2}) \omega_i + \varepsilon_{i+2t+1}, \quad 0 \leq |\rho_{i+2}| \leq 1, \quad i = 1, 2. \quad (10b)$$



In (10a) the  $y_i$ ,  $i = 1, 2$ , are the unconditional values of the logarithms of the endowments of the two countries, and in (10b) the  $\omega_i$ ,  $i = 1, 2$ , are the logarithms of the two unconditional gross rates of nominal monetary growth. Each  $\varepsilon_{it+1}$ ,  $i = 1, 4$ , is assumed to be normally distributed with conditional mean equal to zero and conditional variance given by  $h_{it}$ ,  $i = 1, 4$ . The series are assumed to be conditionally uncorrelated for simplicity.

I specify the distributions of the shares of government spending by letting the fraction of good  $i$  that the government buys be  $\xi_{it} \equiv G_{it}/Y_{it}$  and assuming the following processes:

$$\xi_{it+1} = \rho_{i+4}\xi_{it} + (1 - \rho_{i+4})\xi_i + \varepsilon_{i+4t+1}, \quad 0 \leq |\rho_{i+4}| < 1, \quad i = 1, 2, \quad (11)$$

where  $\varepsilon_{5t+1}$  is distributed uniformly on the interval  $[-h_{5t}, h_{5t}]$  and  $\varepsilon_{6t+1}$  is distributed uniformly on the interval  $[-h_{6t}, h_{6t}]$ .

The parameters characterizing the conditional variances of the six exogenous processes follow simple autoregressions such that

$$E_t(h_{it+1}) = \phi_i h_{it} + (1 - \phi_i)h_i, \quad i = 1, 6, \quad (12)$$

where  $h_i$  is the unconditional variance of the process for  $i = 1, 4$ , and  $\frac{1}{3}(h_i)^3$  is the unconditional variance for  $i = 5, 6$ .

The state of the economy is  $x_t = \{y_{it}, m_{it+1}, \omega_{it}, \xi_{it}, \tau_{it}, i = 1, 2, h_{jt}, j = 1, 6\}$  and, with the assumption that the taxation policies are Markov processes, the  $x_t$  vector is a Markov process as was assumed in the beginning. For closed-form solutions, I choose the period utility function to be

$$U(C_{1t}, C_{2t}) = [1/(1 - \gamma)] C_{1t}^{1-\gamma} + [1/(1 - \delta)] C_{2t}^{1-\delta}, \quad (13)$$

the constant relative risk aversion utility function that Abel (1986) and Giovannini (1987) use. In dynamic applications under uncertainty these utility functions have the unfortunate attribute of specifying the agent's risk aversion with the same parameter that characterizes the agent's preferences for intertemporal substitution. In this case the equilibrium marginal utilities of consumption are

$$U_{1t} = [(Y_{1t} - G_{1t})/2]^{-\gamma} \quad \text{and} \quad U_{2t} = [(Y_{2t} - G_{2t})/2]^{-\delta}. \quad (14)$$

From the definition of the shares of government spending in the economy, it follows that  $U_{1t} = 2^\gamma(1 - \xi_{1t})^{-\gamma} Y_{1t}^{-\gamma}$  and  $U_{2t} = 2^\delta(1 - \xi_{2t})^{-\delta} Y_{2t}^{-\delta}$ .

I also follow Giovannini (1987) and investigate explicitly only the case in which the parameters of the model result in an equilibrium in which the

cash-in-advance constraints hold as equalities.<sup>4</sup> With the assumption that the governments' cash-in-advance constraints hold as equalities, the goods-market clearing conditions and the money-market clearing conditions can be used to find expressions for the real purchasing powers of the two monies which are

$$\Pi_{1t} = Y_{1t}/M_{1t+1} \quad \text{and} \quad \Pi_{2t} = \Theta_t Y_{2t}/M_{2t+1}. \quad (15)$$

In (15) the dependence of  $\Pi_{2t}$  on the relative price  $\Theta_t$  indicates that this is not a final expression since the relative price is an endogenous variable.

The solution for the marginal utility of wealth is

$$\lambda_t = \beta 2^\gamma E_t \left[ (1 - \xi_{1t+1})^{-\gamma} Y_{1t+1}^{1-\gamma} / Y_{1t} \Omega_{1t+1} \right]. \quad (16)$$

The complete solution requires substitution from the specification of the time series processes on the exogenous variables. Similarly, a solution for the terms of trade is

$$\Theta_t = \beta 2^\delta E_t \left[ (1 - \xi_{2t+1})^{-\delta} Y_{2t+1}^{1-\delta} / Y_{2t} \Omega_{2t+1} \right] / \lambda_t. \quad (17)$$

### 3.1. Solution for the exchange rate

The exchange rate is  $S_t = \Pi_{2t}/\Pi_{1t} = \Theta_t (Y_{2t}/M_{2t+1}) / (Y_{1t}/M_{1t+1})$ . Taking its natural logarithm and solving (17) from the exogenous processes gives

$$\begin{aligned} s_t = & \alpha_{s0} + \alpha_{s1} m_{1t+1} - \alpha_{s2} m_{2t+1} - \alpha_{s3} \Xi_{1t} + \alpha_{s4} \Xi_{2t} - \alpha_{s5} y_{1t} + \alpha_{s6} y_{2t} \\ & + \alpha_{s7} \omega_{1t} - \alpha_{s8} \omega_{2t} - \alpha_{s9} h_{1t} + \alpha_{s10} h_{2t} - \alpha_{s11} h_{3t} + \alpha_{s12} h_{4t}. \end{aligned} \quad (18)$$

In (18)  $\Xi_{1t} \equiv \ln\{E_t[(1 - \xi_{1t+1})^{-\gamma}]\}$  and  $\Xi_{2t} \equiv \ln\{E_t[(1 - \xi_{2t+1})^{-\delta}]\}$ , which are given by the following:

$$\Xi_{1t} = \ln \left\{ \left[ -(1 - \mu_{1t} - h_{5t})^{1-\gamma} + (1 - \mu_{1t} + h_{5t})^{1-\gamma} \right] / (1 - \gamma) 2 h_{5t} \right\}, \quad (19a)$$

$$\Xi_{2t} = \ln \left\{ \left[ -(1 - \mu_{2t} - h_{6t})^{1-\delta} + (1 - \mu_{2t} + h_{6t})^{1-\delta} \right] / (1 - \delta) 2 h_{6t} \right\}. \quad (19b)$$

In (19a)  $\mu_{1t} \equiv \rho_5 \xi_{1t} + (1 - \rho_5) \xi_{11}$  and in (19b)  $\mu_{2t} \equiv \rho_6 \xi_{2t} + (1 - \rho_6) \xi_{21}$ . In (18) all of the  $\alpha_s$  parameters are defined to be positive when there is positive

<sup>4</sup>Svensson (1985b) studies the solution only for independently and identically distributed exogenous processes. He characterizes the equilibrium that in general involves times when the value of the multiplier is zero.

persistence of endowment processes and intertemporal substitution is high ( $\gamma < 1$  and  $\delta < 1$ ). Their values are  $\alpha_{s1} = \alpha_{s2} = \alpha_{s3} = \alpha_{s4} = 1$ ,  $\alpha_{s5} = (1 - \gamma)\rho_1$ ,  $\alpha_{s6} = (1 - \delta)\rho_2$ ,  $\alpha_{s7} = \rho_3$ ,  $\alpha_{s8} = \rho_4$ ,  $\alpha_{s9} = \frac{1}{2}(1 - \gamma)^2$ ,  $\alpha_{s10} = \frac{1}{2}(1 - \delta)^2$ , and  $\alpha_{s11} = \alpha_{s12} = \frac{1}{2}$ .

An increase in the money stock of country 1 or its rate of growth depreciates the dollar relative to the pound. The results for the level of a country's endowment depend on the intertemporal elasticity of substitution. Higher (lower) levels of output in country 1 (2) lead to an appreciation of the dollar relative to the pound when intertemporal substitution is high ( $\gamma < 1$  and  $\delta < 1$ ). The results are reversed if intertemporal substitution is low.

Notice that an increase in the expected share of country 1's output that the government will take in the next period appreciates the dollar relative to the pound. Similarly, if less of country 2's output is expected to be available next period, the pound appreciates relative to the dollar. These effects arise because of the influence of future government spending on the expected marginal utility of the respective goods. If less of country 1's endowment is expected to be available for consumption next period, the current relative price of the country 2 good in terms of the country 1 good,  $\Theta_t$ , must fall. Since the purchasing powers of the dollar in terms of the country 1 good and the pound in terms of the country 2 good are determined strictly by the outstanding quantities of monies and the currently available endowments, the entire change in the relative price of the 2 goods is accomplished through the exchange rate.

An increase in the conditional variance of the country 1 money growth rate or the country 1 endowment process causes an appreciation of the dollar. Increases in either conditional variance increase the expected purchasing power of the dollar. An increase in the conditional variance of the share of government spending in good 1 (2) causes an increase in  $\Xi_{1t}$  ( $\Xi_{2t}$ ) which also appreciates (depreciates) the dollar relative to the pound. These effects arise because an increase in the variance of the share of government spending increases the expected marginal utility of that good since agents are risk-averse.

### 3.2. Solutions for nominal interest rates

Let  $i_{1t}$  be the risk-free nominal interest rate of country 1 on a continuously compounded basis making  $\exp(-i_{1t})$  the dollars that one must sacrifice at time  $t$  for a dollar delivered unconditionally at the time  $t + 1$  asset market. Let  $i_{2t}$  be the similarly defined pound nominal interest rate. From the definitions of the nominal pricing kernels and (8g)–(8h),

$$\begin{aligned} \exp(-i_{jt}) &= \int n_j(x_{t+1}, x_t) dx_{t+1} \\ &= \beta E_t \left[ \Pi_{j,t+1} \lambda_{t+1} / \Pi_{jt} \lambda_t \right], \quad j = 1, 2. \end{aligned} \quad (20)$$

Taking natural logarithms of both sides of (20) and exploiting the assumed time series processes of the exogenous variables gives a solutions for  $i_{1t}$  and an analogous unrepresented solution for  $i_{2t}$ :

$$i_{1t} = \alpha_{i10} + \alpha_{i11}\bar{\xi}_{1t} - \alpha_{i12} \ln\{E_t[(1 - \xi_{1t+2})^{-\gamma}]\} + \alpha_{i13}(y_{1t} - y_{1t}) \\ + \alpha_{i14}(\omega_{1t} - \omega_1) + \alpha_{i15}h_{1t} + \alpha_{i16}h_{3t}. \quad (21)$$

In (21) all of the  $\alpha$  coefficients cannot be signed because they depend on the degrees of intertemporal substitution. When intertemporal substitution for good 1 is high and with positive persistence of real endowments, all of the  $\alpha_{i1}$  coefficients are positive, and their values are  $\alpha_{i10} = -\ln\beta + \omega_1 - \frac{1}{2}(1 - \phi_1)(1 - \gamma)^2h_1 - \frac{1}{2}(1 - \phi_3)h_3$ ,  $\alpha_{i11} = \alpha_{i12} = 1$ ,  $\alpha_{i13} = \rho_1(1 - \gamma)(1 - \rho_1)$ ,  $\alpha_{i14} = \rho_3^2$ ,  $\alpha_{i15} = \frac{1}{2}(1 - \phi_1 - \rho_1^2)(1 - \gamma)^2$ , and  $\alpha_{i16} = \frac{1}{2}[(1 - \phi_3) - (1 + \rho_3)^2]$ . Higher than average rates of monetary growth increase nominal interest rates because they increase the expected rate of inflation. If intertemporal substitution is strong and with positive persistence of real endowments, higher than average endowments cause high nominal interest rates because they increase the purchasing power of money and create expected inflation as the future purchasing power of money is expected to fall. This effect outweighs the real interest rate effect which would decrease nominal interest rates.

### 3.3. Risk premiums in the forward foreign exchange market

Although no explicit forward exchange market was introduced above, arbitrage allows the pricing of forward contracts. To prevent an arbitrage opportunity, the return from investing a dollar in a risk-free nominal dollar asset has to be identical to the return from converting the dollar into pounds, investing the pounds in a risk-free nominal pound asset, and making a forward contract to sell the pound proceeds for dollars. Hence, interest rate parity implies that

$$\exp(i_{1t}) = (1/S_t)\exp(i_{2t})F_t, \quad (22)$$

where  $F_t$  is the contract price of dollars per pound in the time  $t$  forward market for delivery and payment at time  $t + 1$ .

The logarithmic expression of the risk premium in the forward market is  $E_t(s_{t+1}) - f_t$  or  $E_t(s_{t+1} - s_t) - (i_{1t} - i_{2t})$ . Evaluating this gives

$$E_t(s_{t+1} - f_t) = \alpha_{r1}h_{1t} - \alpha_{r2}h_{2t} + \alpha_{r3}h_{3t} - \alpha_{r4}h_{4t} \\ - \{E_t(\bar{\xi}_{1t+1}) - \ln[E_t(1 - \xi_{1t+2})^{-\gamma}]\} \\ + \{E_t(\bar{\xi}_{2t+1}) - \ln[E_t(1 - \xi_{2t+2})^{-\delta}]\}, \quad (23)$$

where the  $\alpha_r$  parameters are defined to be positive when intertemporal substitution is higher for both goods and the values of the parameters are given by  $\alpha_{r1} = \frac{1}{2}\rho_1^2(1 - \gamma)$ ,  $\alpha_{r2} = \frac{1}{2}\rho_2^2(1 - \delta)$ ,  $\alpha_{r3} = \frac{1}{2}(1 + \rho_3)^2$ , and  $\alpha_{r4} = \frac{1}{2}(1 + \rho_4)^2$ . If risk premiums are highly variable as indicated in the analysis of Fama (1984) and if the model is true, the variability is produced by time variation in the conditional variances of the exogenous monetary growth rates and of the endowment processes. The variances of the shares of the endowments that the governments will take also affect the risk premium since  $E_t(\Xi_{1,t+1})$  is not in general equal to the logarithm of the expected value at time  $t$  of  $(1 - \xi_{1,t-2})^{-\gamma}$  with a similar condition for the country 2 expression.

#### 4. An empirical investigation

The model has a number of strong testable implications. In the remainder of the paper I test a limited number of these new ideas. The model indicates how conditional variances of exogenous processes become additional exogenous processes that influence the economy. Changes in uncertainty interact with the risk aversion of economic agents to cause movements in asset prices such as interest rates and exchange rates. Because true conditional variances are not observable, empirical work requires estimation of conditional variances.

In this section I model the conditional variances of economic processes with univariate autoregressive conditional heteroscedasticity (ARCH) or its generalized counterpart (GARCH)<sup>5</sup>.

A typical time series  $x_t$  is modelled as an ARIMA process with potential GARCH innovations. The innovation in  $x_t$  conditional on its past history is  $\varepsilon_t$  which has the property that  $E(\varepsilon_t | x_{t-1}, x_{t-2}, \dots) = 0$ . In a GARCH model the conditional variance of  $\varepsilon_t$  is  $V_{t-1}(\varepsilon_t) \equiv h_t$ , and it is modelled as

$$h_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad (24)$$

where  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_i \geq 0$ , for all  $i$ . The unconditional variance of  $\varepsilon_t$  is  $\sigma^2 = \omega[1 - \alpha(1) - \beta(1)]^{-1}$ , where  $\alpha(L) \equiv \sum_{i=1}^q \alpha_i L^i$  and  $\beta(L) \equiv \sum_{i=1}^p \beta_i L^i$  and  $\alpha(1) + \beta(1) < 1$  is required.

Several interesting aspects of GARCH models are noteworthy. First, although the innovations in a series are serially uncorrelated, they are not independent because of the dependence across time of the conditional second moments. Second, large innovations in the process cause an increase in the

<sup>5</sup>Engle and Bollerslev (1986) and the associated comments provide an introduction to the econometric and empirical literature associated with models of GARCH errors. I am grateful to Tim Bollerslev for sharing his computer program that was used in the identification and estimation of the GARCH models.

conditions of the future conditional variances damp down the value. Such a property is desirable in exchange rate movements. Exchange rate markets are characterized by tranquil and turbulent property that is desirable for asset prices in general. A particular property is that the fourth unconditional moment of  $\epsilon_t$  exceeds  $3\sigma^4$ . Hence, the unconditional distribution of  $\epsilon_t$  is leptokurtic relative to the normal distribution.

One unattractive feature of GARCH models is the assumption that the conditional variance is an exact function of the current information set of the econometrician. Just as it is possible that agents have more information about conditional means than the information set of the econometrician, it is possible that the true conditional variance is different from the GARCH specification. Nevertheless, the GARCH model provides an estimate of conditional variances, imposes strong testable restrictions and is a logical place to start inference about the model.

#### 4.1. Estimation of univariate models with monthly data

An empirical investigation of the model requires series that coincide with the theoretical constructs and a definition of a period. Given the availability of data, I examined monthly data for four countries, the United States, the United Kingdom, Japan, and West Germany, for the flexible exchange rate era that began in March 1973. The data are the money supplies, as measured by M1, the industrial production indexes, the consumer price indexes, and the exchange rates of the currencies relative to the U.S. dollar.<sup>6</sup>

The first step in the identification and estimation of univariate models was to determine the appropriate degree of differencing for the series. The autocorrelations and partial autocorrelations of the levels and first differences of the natural logarithms of the series were examined and in all cases first differencing was appropriate to induce stationarity since the autocorrelations of the levels of the series failed to damp significantly.

This reasoning was supported by examination of Dickey–Fuller tests of the null hypotheses that the level of the series contains a unit root and that there is a unit root in the first differences of the series. The results of the tests are presented in table 1. The test statistics are constructed by performing ordinary least squares regressions on the following equation either in the presence of a trend or without a trend:

$$\Delta z_t = \alpha_0 + \alpha_1 t + \alpha_2 z_{t-1} + \sum_{i=1}^3 \alpha_{i+2} \Delta z_{t-i} + \epsilon_t, \quad (25)$$

<sup>6</sup>The data are described in detail in the data appendix.

Table 1  
Dickey-Fuller unit root tests.<sup>a</sup>

$$\Delta z_t = \alpha_0 + \alpha_1 t + \alpha_2 z_{t-1} + \sum_{i=1}^3 \alpha_{i+2} \Delta z_{t-i} + \varepsilon_t$$

Series	No. of observations	$\tau_\mu(z)$	$\tau_\tau(z)$	$\tau_\mu(\Delta z)$	$\tau_\tau(\Delta z)$
JY	168	-0.337	-3.227	-3.955*	-4.082*
JP	168	-5.721*	-5.075*	-4.373*	-5.948*
JM	167	-1.870	-2.772	-10.882*	-11.100*
JS	169	-0.255	-1.555	-5.030*	-5.184*
GY	168	-1.120	-2.464	-6.889*	-6.897*
GP	169	-3.125**	0.034	-4.361*	-5.476*
GM	167	-1.276	-2.205	-11.031*	-11.072*
GS	169	-1.296	-1.232	-5.844*	-5.867*
UKY	168	-1.138	-2.280	-6.614*	-6.676*
UKP	169	-4.130*	-0.970	-4.370*	-6.351*
UKM	167	-1.667	-1.117	-7.449*	-7.712*
UKS	169	-1.524	-1.562	-5.411*	-5.439*
USY	169	-0.893	-2.816	-4.615*	-4.618*
USP	168	-1.882	-0.050	-3.243**	-3.719**
USM	169	-1.368	1.887	-10.360*	-9.759*

<sup>a</sup>The first one or two letters of each series denotes the country (J = Japan, G = Germany, UK = United Kingdom, US = United States) and the last letter denotes the economic aggregate (Y = industrial production, P = consumer prices, M = money supply, S = spot exchange rate). The Dickey-Fuller statistics are the ratios of the estimated  $\alpha_2$  to its standard error in the presence of a trend for  $\tau_\tau$  and without trend for  $\tau_\mu$ . The critical values for  $\tau_\tau$  are -3.41 (5%) and -3.96 (1%) and for  $\tau_\mu$  they are -2.86 (5%) and -3.43 (1%). Rejection of the null hypothesis of the presence of a unit root in the level of the natural logarithm of the series ( $z$ ) or its first difference ( $\Delta z$ ) is indicated by an asterisk (\*) at the 1% marginal level of significance or by two asterisks (\*\*) at the 5% marginal level of significance. The first observation is March 1973 and the last is either January 1987 (166 observations), February 1987 (167 observations), or March 1987 (169 observations).

and  $\Delta$  is the first difference operator. If  $z_t$  contains a unit root, a test of the null hypothesis that the coefficient  $\alpha_2$  in (25) is zero will not be rejected. The  $\tau_\mu(z)$  and the  $\tau_\tau(z)$  statistics are the 't-statistics' for the null hypothesis that  $\alpha_2$  is zero either without a trend in the regression or with a trend, respectively.<sup>7</sup> Notice that there is only marginal evidence against the hypothesis of a unit root in any of the series and only in the case of the price levels, and strong evidence against the hypothesis that the first differences of the series contain a second unit root. These tests are the  $\tau_\mu(\Delta z)$  and  $\tau_\tau(\Delta z)$  statistics. I therefore worked with the first differences.

<sup>7</sup>The marginal levels of significance of the statistics are from the tables reported in Fuller (1976). The third-order autoregression was chosen a priori under the hypothesis that this would remove most autocorrelation.

Table 2a  
Univariate models.<sup>a</sup>

Series	No. of observations	Model				
		Const. 1	Coeff. 1	Coeff. 2	Coeff. 3	Const. 2
JY	167	0.0003	-0.1867	0.1890	0.3133	0.00013
MA(3)		0.0013	0.0899	0.0844	0.0681	0.00002
ARCH(1)		0.8175	0.0378	0.0251	< 0.0001	< 0.0001
JP	151	-0.0013	0.2296	0.0291	0.2157	0.00002
AR(3)		0.0008	0.0945	0.0572	0.0507	0.000005
ARCH(1)		0.1042	0.0151	0.6110	< 0.0001	< 0.0001
JM	166	-0.00003	-0.4517	-0.3580		0.00033
AR(2)		0.00072	0.0760	0.0825		0.00003
		0.9667	< 0.0001	< 0.0001		< 0.0001
JS	168	-0.0035				0.00092
RW		0.0026				0.00011
ARCH(1)		0.1782				< 0.0001
GY	167	0.00002	-0.2354			0.00024
MA(1)		0.00061	0.0817			0.00003
ARCH(1)		0.9738	0.004			< 0.0001
GP	168		0.5321			0.000005
AR(1)			0.0683			0.0000004
			< 0.0001			< 0.0001
GM	166	0.0005	-0.1998	0.0631	0.3864	0.00007
MA(3)		0.0009	0.0847	0.0741	0.0611	0.00001
ARCH(1)		0.5785	0.0188	0.3944	< 0.0001	< 0.0001
GS	168	-0.0028				0.00123
RW		0.0026				0.00011
		0.2815				< 0.0001

0.0042  
0.1927  
0.0034

0.1356  
0.0786  
0.0845  
0.2840  
0.1268  
0.0251

0.3932  
0.1482  
0.008



UKY						0.00015	0.6651
RW	167	0.0023				0.00002	0.1648
ARCH(1)		0.0010				< 0.0001	< 0.0001
UKP		0.0214					
AR(1)	168	-0.0013	0.6350			0.00002	0.3651
ARCH(1)		0.0013	0.0849			0.000002	0.0833
		0.3173	< 0.0001			< 0.0001	< 0.0001
UKM		-0.00003	-0.1881			0.00024	
AR(1)	166	0.00083	0.0767			0.00002	
		0.9712	0.0142			< 0.0001	
UKS		0.0030				0.00098	
RW	168	0.0026				0.00009	
		0.2486				< 0.0001	
USY		0.0036	0.5055			0.00005	0.3089
AR(1)	168	0.0013	0.0674			0.000005	0.1261
ARCH(1)		0.0056	< 0.0001			< 0.0001	0.0143
USP		0.0057	0.5510			0.000006	0.3015
AR(2)	167	0.0006	0.0870		0.1653	0.000001	0.2044
ARCH(1)		< 0.0001	< 0.0001		0.0830	< 0.0001	0.1402
					0.0464		
USM		-0.0001	0.2456			0.00003	0.2705
AR(1)	168	0.0009	0.0908			0.000004	0.1354
ARCH(1)		0.9115	0.0068			< 0.0001	0.0457

<sup>a</sup> See table 1. 'Const. 1' refers to the estimated constant in the univariate ARMA model and 'Const. 2' refers to the estimated constant in the ARCH model. 'Coef. 1-3' are the estimated AR or MA coefficients and 'Coef. 4' is the estimated ARCH coefficient. The first number listed in each cell under the 'Model' section is the coefficient estimate and the second is the associated standard error. The third is the MLS for H<sub>0</sub>: coefficient = 0; \*, < 0.0001. implies that the MLS is less than 0.0001. A blank cell implies that the corresponding term does not enter into the series specification.

Table 2b  
Residual diagnostics of the univariate models.<sup>a</sup>

Series	$B_1$	$B_2$	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$Q(10)$	$Q(20)$
JY	-0.065	3.333	-0.003 -0.021	0.047 -0.107	0.057 0.077	-0.073 -0.049	13.828 7.353	39.294 15.090
JP	0.229	2.645	-0.047 -0.044	0.019 0.038	0.078 0.059	0.004 0.049	9.537 18.275	21.817 30.338
JM	0.405	3.860	0.038 0.008	0.008 -0.048	0.066 -0.012	0.004 -0.074	26.179 3.102	33.022 10.581
JS	-0.437	3.763	0.067 0.023	0.067 0.055	0.138 0.047	0.092 0.132	9.671 13.010	19.933 21.872
GY	-1.016	6.993	-0.080 0.055	-0.059 -0.042	0.197 0.014	-0.041 -0.026	15.064 2.755	26.736 10.141
GP	0.784	4.228	-0.069 0.063	-0.004 -0.086	0.142 0.003	0.069 0.008	14.041 3.511	34.106 9.850
GM	0.140	3.849	0.014 -0.038	-0.032 -0.073	0.028 0.058	-0.002 -0.079	11.552 10.273	35.333 17.079
GS	-0.171	4.027	-0.006 0.145	0.137 -0.026	0.004 -0.047	0.008 -0.032	8.188 5.179	17.689 13.867
UKY	-0.513	6.944	-0.046 -0.013	-0.098 0.108	0.005 0.050	0.047 -0.068	5.938 5.684	17.981 8.247
UKP	1.268	6.204	-0.050 0.010	-0.037 -0.020	0.230 0.121	-0.010 -0.060	26.881 14.815	44.392 18.032
UKM	0.358	3.947	0.007 0.017	0.048 -0.080	0.067 0.043	0.009 0.192	10.632 11.564	16.804 16.897
UKS	-0.619	4.541	0.036 -0.005	0.108 -0.041	-0.034 -0.038	0.076 0.284	6.149 14.951	21.120 18.312
USY	0.025	4.473	-0.013 0.039	0.050 -0.113	0.087 -0.027	0.082 0.066	5.444 5.088	14.778 21.472
USP	0.275	4.970	-0.033 -0.006	-0.078 -0.052	0.054 -0.003	0.074 -0.055	12.603 4.270	40.723 5.795
USM	0.080	3.146	0.004 0.024	-0.105 -0.150	0.151 0.054	-0.121 0.134	24.500 15.423	41.851 30.670

<sup>a</sup>See table 1. The statistic  $B_1 = m_3/(m_2)^{3/2}$  is a test of skewness (where  $m_i$  denotes the  $i$ th moment of the sampled population), and the statistic  $B_2 = m_4/(m_2)^2$  is a test of kurtosis for  $\varepsilon_i/\sqrt{h_i}$ . Under the null hypothesis of a normal distribution for the population, the 5% critical value for  $H_0$ : no skewness, is 0.299, and the 5% critical value for  $H_0$ : normal kurtosis, is 3.63 [see Pearson and Hartley (1966)]. The first four estimated autocorrelation coefficients are denoted  $\rho_1, \dots, \rho_4$  for  $\varepsilon_i/\sqrt{h_i}$  (first line) and for  $\varepsilon_i^2/h_i$  (second line). The 5% critical value for  $H_0$ :  $\rho_i = 0$  is 0.154.  $Q(10)$  and  $Q(20)$  are the corresponding Ljung-Box statistics; 5% critical values for  $H_0$ : no autocorrelation at 10 or 20 lags are 18.307 and 31.410, respectively.

Residual diagnostics were examined to determine if autocorrelation remained in the transformed series, and additional models were estimated where necessary. The autocorrelations and partial autocorrelations of the squared residuals of the series were examined to identify a possible GARCH model. This procedure follows the suggestions of Bollerslev (1986). The resulting models are presented in table 2a with residual diagnostics presented in table 2b. Of the 120 reported autocorrelation coefficients only four are significantly different from zero using the asymptotic  $1/\sqrt{T}$  test. For a few series the  $Q$ -statistics for 10 and 20 autocorrelations do indicate that higher-order autocorrelations may be significantly different from zero, but these effects appear in most cases to be due to seasonality.

The results for the United States indicate that the rates of growth of the money supply and the index of industrial production are well modelled by AR(1) processes with ARCH(1) innovations. The U.S. rate of inflation is an AR(2) with ARCH(1) innovations. For the United Kingdom, the results indicate a random walk with ARCH(1) innovations for the rate of growth of industrial production and an AR(1) for the rate of growth of the money supply. There is essentially no support for ARCH in the rate of growth of the U.K. money supply. The U.K. inflation rate was an AR(1) with ARCH(1) innovations, although higher-order autocorrelations do appear to be statistically different from zero. The pound-dollar exchange rate was identified to be a random walk, and the LR test in this case had a value of essentially zero. For Germany, the results indicate an MA(1) with ARCH(1) innovations for the rate of growth of industrial production, and an MA(3) with ARCH(1) innovations for the rate of growth of the money supply. The German rate of inflation appears to be an AR(1), and the LR test indicated no first-order ARCH. The deutsche mark-dollar exchange rate was a random walk, and there was some evidence in support of ARCH innovations since the LR test had a value of 2.774 with an MLS of 0.096. For Japan, the rate of growth of the money supply was modelled as an AR(2). The rate of growth of industrial production was estimated to be an MA(3) with ARCH(1) innovations. The Japanese rate of inflation was found to be an AR(3) with ARCH(1) innovations. The yen-dollar exchange rate was a random walk with some evidence in support of ARCH(1) innovations since the value of the LR test was 3.256 with an MLS of 0.071.

Maximum likelihood estimation maintains an assumption that the innovations in the series are conditionally normal. This is testable since the estimated innovations divided by their estimated standard deviations should be a unit normal. Table 2b reports two test statistics labelled  $B_1$  and  $B_2$ . The statistic  $B_1$  is a test of skewness, the ratio of the third sample moment around the sample mean to the second sample moment raised to the  $\frac{3}{2}$  power. The statistic  $B_2$  is a test of kurtosis, the ratio of the fourth sample moment to the squared second sample moment. The assumption of normality of several of the series

appears questionable given the large values of the tests of skewness and kurtosis. The German and U.K. industrial production series appear to be particularly bad. The exchange rates are also poorly behaved showing both signs of excess kurtosis and of negative skewness for the yen and the pound.

Two results about exchange rates are striking in table 2a. First, each rate of depreciation appears to be serially uncorrelated relative to its past history. This is a common finding, but the lack of strong evidence of ARCH in the monthly logarithmic changes in exchange rates is in strong contrast to the intuition described earlier and to the findings of conditional heteroscedasticity in studies of risk premiums with monthly data as described in Hodrick (1987). Perhaps it is an indication that the ARCH process is not a good economic model of the conditional heteroscedasticity apparently present in the data in other studies.

The finding of no or limited ARCH in the exchange rate data sampled at a monthly interval is also surprising in light of the strong evidence of ARCH in the data sampled at a weekly interval reported in Engle and Bollerslev (1986) and in the daily data reported in Baillie and Bollerslev (1987). The next section investigates a time series model of exchange rates with weekly sampling of the data to demonstrate that GARCH is present at that sampling interval.

#### 4.2. An exchange rate model with weekly data

Table 3a provides an investigation of data for seven currencies versus the U.S. dollar that are sampled on each Wednesday from June 13, 1973 to January 23, 1985. The only data employed in the model consist of the spot and one month forward exchange rates. The estimated time series model of the rate of depreciation of a currency relative to the U.S. dollar is the following:

$$s_{t+1} - s_t = b_0 + b_1(f_t - s_t) + (1 - \rho_1 L - \rho_2 L^2)^{-1} \varepsilon_{t+1}, \quad (26a)$$

$$\varepsilon_{t+1} | \Phi_t \sim N(0, h_{t+1}), \quad (26b)$$

$$h_{t+1} = \omega + \alpha \varepsilon_t^2 + \beta h_t + \delta (f_t - s_t)^2. \quad (26c)$$

The presence of the forward premium in the conditional mean of the rate of depreciation has a long history, and the presence of several negative estimated coefficients on the forward premium, the  $b_1$ 's, is consistent with the results of Fama (1984) and others. Whether this is valid evidence of variation in risk premiums that is greater than variation in expected rates of depreciation is a matter of considerable debate. Five of the currencies also show slight evidence of residual serial correlation in the conditional mean.

Table 3a  
Models of exchange rates with weekly observations.<sup>a</sup>

$$s_{t+1} - s_t = b_0 + b_1(f_t - s_t) + (1 - \rho_1 L - \rho_2 L^2)^{-1} \varepsilon_{t+1}$$

$$\varepsilon_{t+1} | \Phi_t \sim N(0, h_{t+1})$$

$$h_{t+1} = \omega + \alpha \varepsilon_t^2 + \beta h_t + \delta (f_t - s_t)^2$$

	Deutsche mark	Swiss franc	French franc	Japanese yen	Canadian dollar	British pound	Italian lira
$b_0$	0.113 (0.082)	0.165 (0.095)	0.029 (0.068)	0.023 (0.062)	-0.059 (0.017)	-0.224 (0.060)	-0.028 (0.038)
$b_1$	-0.518 (0.278)	-0.431 (0.203)	0.267 (0.141)	0.118 (0.106)	-0.394 (0.098)	-0.377 (0.169)	0.033 (0.038)
$\rho_1$	0.119 (0.047)	0.080 (0.043)	0.105 (0.051)	0.109 (0.044)		0.086 (0.047)	
$\rho_2$	0.083 (0.041)		0.151 (0.047)	0.085 (0.042)			
$\omega$	0.150 (0.053)	0.088 (0.034)	0.114 (0.029)	0.010 (0.003)	0.040 (0.011)	0.126 (0.035)	0.051 (0.007)
$\alpha$	0.176 (0.042)	0.162 (0.033)	0.257 (0.048)	0.056 (0.007)	0.269 (0.033)	0.112 (0.029)	0.420 (0.053)
$\beta$	0.646 (0.077)	0.763 (0.037)	0.679 (0.040)	0.935 (0.006)	0.605 (0.054)	0.806 (0.047)	0.652 (0.026)
$\delta$	153.805 (52.997)	49.508 (13.236)	10.627 (4.679)	1.569 (0.486)	0.2126 (10.208)		
$L$	-986.624	-1088.711	-967.155	-939.327	-421.169	-956.489	-894.116
$Q_1$	6.653	15.836	18.395	14.529	26.373	20.876	29.053
$Q_2$	11.874	7.796	12.848	7.194	14.308	10.227	55.445
$B_1$	0.013	0.412	-0.129	0.459	-0.296	-0.331	-0.789
$B_2$	3.886	4.910	4.421	6.604	5.260	5.701	7.928

<sup>a</sup>See table 2. The data are sampled weekly on Wednesdays for June 13, 1973 to January 23, 1985 for 607 observations. The logarithmic differences are multiplied by 100. The log likelihood function is  $L$ .  $Q_1$  is the chi-square statistic for testing the significance of the first fifteen autocorrelations of  $\varepsilon_t$ .  $Q_2$  is the analogous statistic for  $\varepsilon_t^2/h_t$ . The 5% critical values are 0.099 for  $B_1$  and 3.199 for  $B_2$ .

The comparatively new aspect of table 3a is the presence of the squared forward premium in the conditional variance. The tests of conditional heteroscedasticity conducted by Cumby and Obstfeld (1984) indicated that such a variable ought to be present, and it enters significantly in four of the seven currencies.

Table 3b reports some LR tests of the model with their associated marginal levels of significance in parentheses. These tests are not all independent, and consequently, care ought to be taken in considering their results. There is exceedingly strong evidence of GARCH as evidenced by the test of  $\alpha = 0$  and  $\beta = 0$  in row 3. The evidence for the importance of the squared forward premium in the conditional variance (row 2) is not as striking, but it seems

Table 3b  
Likelihood ratio tests of models in table 3a.

$H_0^a$	Test statistics						
	Deutsche mark	Swiss franc	French franc	Japanese yen	Canadian dollar	British pound	Italian lira
1.	98.734 ( $< 0.001$ )	86.976 ( $< 0.001$ )	89.344 ( $< 0.001$ )	112.656 ( $< 0.001$ )	90.414 ( $< 0.001$ )	N.A.	N.A.
2.	19.788 ( $< 0.001$ )	11.768 (0.001)	5.518 (0.019)	8.830 (0.003)	0.000 (0.999)	N.A.	N.A.
3.	69.282 ( $< 0.001$ )	63.468 ( $< 0.001$ )	67.052 ( $< 0.001$ )	92.226 ( $< 0.001$ )	56.516 ( $< 0.001$ )	34.092 ( $< 0.001$ )	230.242 ( $< 0.001$ )
4.	18.832 (0.001)	7.952 (0.047)	19.222 (0.001)	13.580 (0.009)	N.A.	16.862 (0.001)	N.A.
5.	18.808 ( $< 0.001$ )	7.946 (0.019)	17.402 (0.001)	13.064 (0.005)	N.A.	10.912 (0.004)	N.A.
6.	12.222 (0.002)	3.138 (0.076)	17.060 ( $< 0.001$ )	11.536 (0.003)	N.A.	N.A.	N.A.
7.	2.808 (0.246)	3.700 (0.157)	3.942 (0.139)	2.176 (0.337)	18.784 ( $< 0.001$ )	10.862 (0.004)	3.738 (0.154)

<sup>a</sup>The null hypotheses are the following: 1. ( $\alpha = \beta = \delta = 0$ ); 2. ( $\delta = 0$ ); 3. ( $\alpha = \beta = 0$ ); 4. ( $b_0 = b_1 = \rho_1 = \rho_2 = 0$ ); 5. ( $b_1 = \rho_1 = \rho_2 = 0$ ); 6. ( $\rho_1 = \rho_2 = 0$ ); 7. ( $b_0 = b_1 = 0$ ).

safe to reject the hypothesis of no effect. The null hypothesis in row 5 postulates no time variation in the conditional mean of the rate of depreciation. The LR tests are chi-square statistics with three degrees of freedom in this case, and it appears safe to reject the hypothesis for the Deutsche mark, 18.808 ( $< 0.0001$ ), the Swiss franc, 7.946 (0.019), the French franc, 17.402 (0.001), the Japanese yen, 13.064 (0.005), and the British pound, 10.912 (0.004).

#### 4.3. Tests of the theory

The previous sections established the presence of movements in the conditional variances of some of the exogenous processes of the model, and it remains to examine whether changes in these conditional variances induce changes in the exchange rate. The best test would be maximum likelihood estimation of the equations for the exchange rate and other asset prices subject to the restrictions of the theory while simultaneously estimating the laws of motion for the driving processes including specifications of the conditional covariances of the exogenous processes. While this is feasible, it is quite complicated. Instead, I conduct a preliminary investigation, under a set of restrictive assumptions.

I first estimate the conditional variances with the ARCH procedure discussed above, and second, I estimate an exchange rate equation with ordinary least squares (OLS) using the presumed exogenous data on monies and industrial productions and their estimated conditional variances from the first stage. Pagan (1984, theorem 12) examines the consistency and the asymptotic distribution of such a strategy and demonstrates that, if the first stage produces consistent estimates of the true conditional variances, the procedure produces consistent estimates of the parameters of interest. The ARCH estimates are consistent estimates of the true conditional variances if the true process is a univariate ARCH model, and agents are rational. If agents actually use more information than the econometrician to forecast conditional means or conditional variances, the ARCH estimates are not consistent. Pagan also demonstrates that, if the estimated conditional variances are consistent, the OLS estimates of the standard errors of the second-stage parameters will understate the true standard errors. Hence, failure to reject the hypothesis of no influence of the explanatory variables cannot be reversed by calculation of appropriate standard errors. Therefore, the two-stage procedure is a simple yet possibly appropriate first step in determining the validity of the model.

Since there is strong evidence that the levels of the natural logarithms of exchange rates contain a unit root, I first differenced eq. (18), and examined the following specification with ordinary least squares:

$$\begin{aligned} \Delta s_t = & \beta_0 + \beta_1 \Delta h_{1t} + \beta_2 \Delta h_{2t} + \beta_3 \Delta h_{3t} + \beta_4 h_{4t} + \beta_5 \Delta m_{1t} \\ & + \beta_6 \Delta m_{2t} + \beta_7 \Delta y_{1t} + \beta_8 \Delta y_{2t} + \beta_9 \Delta \omega_{1t} + \beta_{10} \Delta \omega_{2t} + \varepsilon_t. \end{aligned} \quad (27)$$

The specification (27) requires an explanation of the error term. Under a tight interpretation of the theory, the error term in (27) is the first difference in the expected shares of government spendings, which were assumed to be exogenous and independent of the right-hand-side variables included in (27). Hence, ordinary least squares is appropriate. Such a tight interpretation of the theory is no doubt inappropriate since the assumed time series processes of the exogenous variables that led to the specification of (18) as the solution of the model were in most cases not supported in the empirical investigation. This is particularly true of the industrial production series that were treated in the theory section as stationary in levels, while in the empirical section they were found to contain unit roots. The effect of solving the model with the estimated rather than the assumed processes would be to add the first differences of the rates of growth of industrial production to the list of explanatory variables and to increase the lag length of the included variable in cases where moving average processes or higher-order AR processes were identified.

The results in table 4 indicate that the data provide little support for the theory. The right-hand-side variables are essentially unrelated to changes in

Table 4  
Tests of the model.<sup>a</sup>

$\Delta s_t =$		$\beta_0 h_{mt} + \beta_4 \Delta h_{m^*t} + \beta_5 \Delta m_t + \beta_6 \Delta m_t^* + \beta_7 \Delta y_t + \beta_9 \Delta \omega_t + \beta_{10} \Delta \omega_t^* + \varepsilon_t$		
Variable	Coeff.	Deutsche mark	Japanese yen	British pound
Constant	$\beta_0$	0.003 (0.003) 0.295	0.003 (0.003) 0.311	-0.002 (0.002) 0.367
$\Delta h_y$	$\beta_1$	-79.881 (66.951) 0.235	-10.347 (60.989) 0.866	-6.430 (56.422) 0.320
$\Delta h_{y^*}$	$\beta_2$	6.188 (9.564) 0.519	21.728 (50.771) 0.669	-0.082 (3.788) 0.983
$\Delta h_m$	$\beta_3$	238.440 (167.465) 0.157	1.026 (160.353) 0.995	-23.303 (147.799) 0.873
$\Delta h_{m^*}$	$\beta_4$	-6.879 (31.610) 0.828		
$\Delta m$	$\beta_5$	0.585 (0.630) 0.355	0.293 (0.588) 0.619	1.021 (0.561) 0.071
$\Delta m^*$	$\beta_6$	-0.464 (0.409) 0.258	-0.172 (0.229) 0.453	-0.448 (0.251) 0.076
$\Delta y$	$\beta_7$	-0.414 (0.291) 0.157	0.149 (0.282) 0.597	-0.626 (0.261) 0.012
$\Delta y^*$	$\beta_8$	0.016 (0.147) 0.913	-0.238 (0.226) 0.294	0.234 (0.156) 0.136
$\Delta \omega$	$\beta_9$	-0.592 (0.499) 0.238	-0.267 (0.462) 0.565	-0.517 (0.439) 0.241
$\Delta \omega^*$	$\beta_{10}$	0.065 (0.267) 0.807	-0.009 (0.140) 0.946	0.131 (0.162) 0.419

<sup>a</sup> The dependent variable is the rate of depreciation of the dollar relative to the foreign currency. The sample is April 1973 to January 1987 for 166 observations. Variables without an asterisk are U.S. values, and variables with an asterisk are foreign variables. The conditional variances from the models of table 2 are denoted with an  $h$ . The money supply is denoted  $m$ , the industrial production index is denoted  $y$ , and money growth is denoted  $\omega$ .  $F$ -statistics (with marginal levels of significance in parentheses) and the adjusted  $R^2$ 's for the equations are the following: Deutsche mark,  $H_0$ : all  $\beta_i = 0$ , 0.735 (0.691),  $H_0$ :  $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ , 0.820 (0.514),  $R^2 = -0.017$ ; Japanese yen,  $H_0$ : all  $\beta_i = 0$ , 0.735 (0.691),  $H_0$ :  $\beta_1 = \beta_2 = \beta_3 = 0$ , 0.080 (0.966),  $R^2 = -0.042$ ; British pound,  $H_0$ : all  $\beta_i = 0$ , 1.296 (0.243),  $H_0$ :  $\beta_1 = \beta_2 = \beta_3 = 0$ , 0.017 (0.993),  $R^2 = 0.016$ .



exchange rates. Studies such as Meese and Rogoff (1983) have conditioned our response to the failure of money and industrial production to explain exchange rates making this finding not particularly surprising. Unfortunately, the conditional variances of the exogenous processes as measured by the GARCH models, also are not capable of explaining changes in exchange rates.

## 5. Conclusions

The purpose of this paper was to develop a model of exchange rate determination that provides some new directions for empirical work in the area by focusing on the way changes in the uncertainties in the economic environment interact with the risk aversion of economic agents to produce changes in asset prices. While the initial empirical investigation of the theory has not been very supportive of the model, there are some additional avenues of investigation that ought to be tried before the model is discarded. In this section I discuss some of the directions that could be taken, and I offer some additional ideas about the development of theoretical models that could allow them to achieve more consistency with the data.

We know that changes in nominal exchange rates are highly correlated with changes in real exchange rates, and that these changes in real exchange rates are highly persistent. One of the roles of the government spending variables in the theory was to provide policy variables that were potentially responsible for persistent changes in real exchange rates. I have not attempted to test this implication, and developing such tests would be useful.

Another challenging area for new research is the development and estimation of alternative models of the conditional variances of monies, incomes, and other variables that I have treated as exogenous. Although GARCH models may be good summaries of the serial dependence in a given data series, two problems are apparent. First, the estimates may be quite poor estimates of the true conditional variances. The resulting errors-in-variables problems that arise in the estimation make it difficult to derive consistent estimators of the influence of the true conditional variances. Second, as economists we want to understand the causes of changes in the conditional variances. Since univariate ARIMA models have proven useful in developing a theoretical forecasts of economic time series, we can expect similar success for GARCH models of conditional second moments. Nevertheless, the Lucas (1976) critique serves as a warning that we should look deeper into the economy than the capabilities of such time series models if we are going to be concerned about the policy implications of our models or about our ability to forecast when there are changes in policy regimes.

The theoretical model has implications for many asset prices other than exchange rates, and testing these restrictions in several asset markets simultaneously is desirable. The nominal interest rates and risk premiums in forward markets and in stock markets might be examined with alternative empirical

models of conditional variances. One serious problem in conducting these investigations that ought to be kept in mind is the peso problem. In Hodrick (1987) I examine many models of risk premiums in the forward and futures foreign exchange markets. An alternative interpretation of the apparent variability of risk premiums is the existence of peso problems. If these plague the forward market, they also plague the spot foreign exchange markets and the other asset markets of the world.

A third area of research on the model that may be warranted is the possible influence of time aggregation. Two problems are noteworthy in this area. The first is what is the appropriate time interval to identify as a period in a cash-in-advance model. The second area of concern is the influence of additional sources of information about the relevant exogenous variables. There are many sources of information in an economy about the monthly innovations in monies, incomes, and other economic aggregates and their future values that cause exchange rates to move and are not in the model.

One potential flaw in the theory that deserves investigation is the assumption of complete asset markets. Understanding exchange rates may require relaxation of this assumption in a sensible way. By sensible, I do not mean arbitrarily closing asset markets or prohibiting intertemporal trade just to government bonds, but I mean determining what assets are traded, in what amounts, why countries periodically prohibit intertemporal trade, and how many claims countries choose to accumulate against each other.

### Data appendix

All monthly data are from two tapes of the *International Financial Statistics* of the International Monetary Fund supplied to Northwestern University by the Inter-University Consortium for Political and Social Research. All of the data except the Japanese price index are from tape number ICPSR 7629. The series begin in March 1973 and end either in January, February, or March 1987. There are between 166 and 168 observations per series. The observations on the Japanese price index are from a previous ICPSR tape because I discovered a problem with the data on tape 7629.

The industrial production index is series 66.c, 'Industrial Production, Seasonally Adjusted'; these indexes are compiled from reported versions of national indexes.

The price index is series 64, 'Consumer Prices'; these indexes are compiled in the same way as are the industrial production indexes.

The money aggregate is series 34, 'Money'; this is the sum of currency outside banks and private sector demand deposits, plus (where applicable) private sector demand deposits with the postal checking system and the Treasury. This is an end of month series.

The exchange rate is series ae, 'Market Rate/Par or Central Rate'; this is the foreign currency unit value of the U.S. dollar which was quoted on the last trading day of each month.

Seasonal dummy variables are used with the four money supply series and the price indexes other than the U.S. series.

The data in table 3 are from Data Resources, Inc. The data are bid prices for spot and one month forward exchange rates.

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