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Asset Price Volatility, Bubbles, and Process Switching

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ABSTRACT

Evidence of excess volatilities of asset prices compared with those of market fundamentals is often attributed to speculative bubbles. This study demonstrates that bubbles could in theory lead to excess volatility, but it shows that certain variance bounds tests preclude bubbles as an explanation. The evidence ought to be attributed to model misspecification or inappropriate statistical tests. One important misspecification occurs if a researcher incorrectly specifies the time series properties of market fundamentals. A bubble-free example economy characterized by a potential switch in government policies produces asset prices that would appear, to an unwary researcher, to contain bubbles.

THE INTRODUCTION AND USE of variance bounds tests by financial economists interested in asset pricing and market efficiency have generated considerable controversy. The first tests postulated a simple model in which market efficiency required assets to have a constant expected real rate of return. The rejection of this hypothesis in the work of Shiller [22, 23] and particularly Grossman and Shiller [12] was followed quickly by a number of different responses. The statistical properties of the tests in small samples and the time series assumptions of the data were criticized.[1] Substantial resources have also been devoted to complications of the model that allow time variation in discount rates and risk premiums, while remaining within the representative agent paradigm.[2] Others have taken the performance of the simple model and the excess volatility of asset prices described in the tests to be indicative of market inefficiency.

One particular type of market inefficiency that receives much attention in these discussions is that asset markets may be characterized by speculative bubbles. Representative of these statements is Ackley's [1, p. 13] discussion of Shiller's [22, 23] findings, in which he states, "But, surely, it is possible that speculative price bubbles, upward or downward, . . . supply part of the explanation." Similarly, in Fischer's [7, p. 500] discussion of Shiller [25], he states, "Backing up that empirical evidence was the development, by Shiller and others, of the theory of speculative bubbles, providing a reason that prices could fluctuate excessively without smart investors being able to profit from knowing they were living in a bubble." Similar statements have been made by others in discussions

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[1] See, in particular, Flavin [8], Kleidon [15, 16] and Marsh and Merton [20].

[2] Eichenbaum and Hansen [6] and Dunn and Singleton [5] explore alternatives that allow considerable variation in the representative agent's intertemporal marginal rate of substitution.

of stock price volatility and in discussions of the volatility of foreign exchange rates.[3]

In this paper we examine whether some of the variance bounds tests reported to date provide evidence for the hypothesis that asset prices contain speculative bubbles. The speculative bubbles discussed here are the type studied by Flood and Garber [9] and Blanchard and Watson [3]. The variance bounds tests we discuss are of the types that were conducted by Shiller [22, 23, 24, 25] and Mankiw, Romer, and Shapiro [19] and that were discussed by Grossman and Shiller [12]. We demonstrate the sense in which the existence of bubbles can in theory lead to excess volatility of asset prices relative to the volatility of market fundamentals, but we explain why certain variants of variance bounds tests preclude bubbles as a reason that asset price might violate such bounds. This result, without its formal demonstration, is mentioned by Mankiw, Romer, and Shapiro [19, p. 681], who state, "The inequalities ... hold even if there are bubbles." Since many researchers have mentioned bubbles as a possible reason for the failure of the simple rational expectations model in variance bounds tests, we thought it worthwhile to elaborate on the remark in Mankiw, Romer, and Shapiro [19].

The issue turns on how one measures the inherently unobservable construct that Shiller [22] denoted the ex post rational price. If one uses the sample's terminal market price to construct a measurable counterpart to the ex post rational price, as is done by Shiller [24, 25], Grossman and Shiller [12], and Mankiw, Romer, and Shapiro [19], failure of a variance bounds test cannot be attributed to the existence of speculative bubbles.[4] The reason is that use of the sample market price effectively builds bubbles into the null hypothesis. Rejection of the null must consequently be due to other sources. Potential explanations include general misspecification of the model, unknown small sample properties of the tests, and failure of the data to satisfy the ergodicity assumption implicit in the use of the statistics.

In bubble research one particularly important misspecification of the model occurs when the researcher incorrectly specifies agents' beliefs about the time series properties of market fundamentals. The second purpose of this paper is to explain, in terms of a simple model economy, how anticipated changes in market fundamentals may produce asset price paths that would appear, to an empirical researcher who is unaware of the potential change, to be characterized by bubbles, even though the economy is bubble free. The example economy is described by a potential change in government policies that we label a process switch.

Our presentation is in the next two sections. In Section I we describe a common asset pricing model and show how it responds to variance bounds tests when bubbles are present. In Section II we develop our example economy and explore possible process switches as explanations of bubble-type phenomena. Section III contains some concluding remarks.

[3] Dornbusch [4] argues that the flexible exchange rate system has not worked well and suggests that speculative bubbles may be one of the culprits. See Meese [21] for a test of speculative bubbles in the foreign exchange market.

[4] Marsh and Merton [20] follow Shiller [22, 23] and use the sample average price as the terminal price in constructing their counterexample to Shiller's derived variance bounds.

I. Variance Bounds Tests of an Asset Pricing Model

Most variance bounds tests examine present value relations that are derived from a representative consumer's optimization problem. If a_t is the real dividend of an asset at time t and z_t is the real price ex-dividend of the asset at time t, a typical first order condition of a representative agent requires

$$p_t = \rho E_t(p_{t+1} + d_{t+1}) \tag{1}$$

where $p_t \equiv U'(c_t)z_t$, $d_t \equiv U'(c_t)a_t$, $U'(c_t)$ is the marginal utility of consumption at time t, ρ is the fixed discount factor of the agent, and $E_t(\cdot)$ is the conditional expectation operator based on all time t information. The representative agent structure presumes homogeneous information across agents at each point in time. Equation (1) requires that the utility of the real value sacrificed by the individual in purchasing the asset be equal to the conditional expectation of the utility of the real value of the benefit from holding and selling the asset.

Equation (1) has the form of a linear difference equation that arises in many rational expectations models. Hence, a solution that depends only on *market fundamentals* can be written as

$$p_t^f = \textstyle\sum_{i=1}^{\infty} \lambda_i E_t(d_{t+i}), \tag{2}$$

and substitution of (2) into (1) with equality of p_t^f and p_t requires $\lambda_i = \rho^i$.

Notice that if (1) is postulated as the entire model, an additional arbitrary element, b_t, can be added to the market fundamentals solution to provide an alternative solution,

$$p_t = \textstyle\sum_{i=1}^{\infty} \rho^i E_t(d_{t+i}) + b_t. \tag{3}$$

The model requires only that the sequence of b_t's possesses the property that

$$E_t(b_{t+i}) = \rho^{-i} b_t, \quad i = 1, 2, \cdots, \tag{4}$$

since with this property the solution (3) satisfies (1). The time series $\{b_t\}$ is termed a rational bubble according to Flood and Garber [9], since it satisfies the Euler equation (1). Absence of bubbles requires that each element of the sequence is zero.[5] The time series property of bubbles described by (4) ensures that bubbles cannot be a reason for the Euler equation (1) to be deemed misspecified in an econometric investigation such as that of Hansen and Singleton [14].

Since we are interested in how various variance bounds tests perform in the presence of speculative bubbles, we take (3) as our representation of equilibrium asset price with no additional restrictions placed on the b_t sequence other than those imposed by (4).[6]

The basic insights of the variance bounds tests are that the variance of an actual variable must be greater than or equal to the variance of its conditional expectation and that this latter variance must be greater than or equal to the

[5] Satisfaction of a transversality condition such as $\lim_{h\to\infty} \rho^h E_t(p_{t+h}) = 0$ requires the absence of bubbles. Tests for speculative bubbles can consequently be thought of as tests of this model's transversality condition.

[6] The restrictions that (4) places on the b_t process are not very severe. The time series process can take many possible forms, including bubble innovations that are conditionally heteroscedastic.

variance of a forecast based on a subset of the information used by agents. In order to see how the existence of bubbles could lead in theory to a violation of variance bounds, consider the ex post rational price. The variance bounds literature defines the theoretical ex post rational price to be the price that would prevail if everyone knew the future market fundamentals with certainty and there were no bubbles. Therefore, the ex post rational price is

$$p_t^* = \sum_{i=1}^{\infty} \rho^i d_{t+i}. \tag{5}$$

The theoretical relation that is the foundation of many variance bounds tests is obtained by subtracting (3) from (5) and rearranging terms:

$$p_t^* = p_t + u_t - b_t, \tag{6}$$

where $u_t \equiv \sum_{i=1}^{\infty} \rho^i [d_{t+i} - E_t(d_{t+i})]$ is the deviation of the present value of dividends from its expected value based on time t information. By construction, u_t is uncorrelated with p_t and b_t, but p_t and b_t may be correlated with each other.

Notice in (4) that, since $\rho^{-1} > 1$, a rational stochastic bubble is nonstationary. Consequently, its unconditional moments are undefined. For this reason we address our arguments to variances and covariances of the innovations of processes, which are well defined.

Let the innovation in X_t from time $t - n$ be $[X_t - E_{t-n}(X_t)]$. Then the innovation variance and covariance operators are defined by

$$V_n(X_t) = E\{[X_t - E_{t-n}(X_t)]^2\}$$

and

$$C_n(X_t, Y_t) = E\{[X_t - E_{t-n}(X_t)][Y_t - E_{t-n}(Y_t)]\}$$

where $E(\cdot)$ denotes the unconditional mathematical expectation. In what follows we treat n as a finite positive integer.

Applying the innovation variance operator to both sides of (6) yields

$$V_n(p_t^*) = V_n(p_t) + V_n(u_t) + V_n(b_t) - 2C_n(p_t, b_t), \tag{7}$$

which follows from the conditional orthogonality of u_t to p_t and b_t.

Suppose that a researcher had errorless measurements of p_t^* and p_t over a long time series and could develop very good estimates of $V_n(p_t^*)$ and $V_n(p_t)$.[7] Assume also that the researcher knew that (1) was not rejected by the data. Since the innovation variances of u_t and b_t in (7) are strictly non-negative, a finding of $V_n(p_t) > V_n(p_t^*)$ could be rationalized, within the framework of the model, by $C_n(p_t, b_t) \geq 0$.

In typical presentations of variance bounds tests, the stochastic bubble is excluded from (6) because absence of bubbles is intended to be part of the joint null hypothesis. A theoretical variance bound derived in (7) in the absence of bubbles is $V_n(p_t^*) > V_n(p_t)$. It is easy to construct examples in which a sufficiently large innovation variance in b_t causes this variance bound to be violated.

[7] In order to simplify our argument we abstract from the sampling distribution of the sample statistics and regard them as precise estimates of their population counterparts.

Consider the situation in which the innovations in b_t are orthogonal to the innovations in market fundamentals. In this case, $C_n(p_t, b_t) = V_n(b_t)$. Therefore, the right-hand side of (7) reduces to $V_n(p_t) + V_n(u_t) - V_n(b_t)$, and a sufficiently large innovation variance in the bubble could cause $V(p_t^*) < V(p_t)$.[8]

We imagine that theoretical exercises similar to the above have spawned the popular argument that the failure of variance bounds tests can be due to speculative bubbles. Analogously, it has been argued that failure to reject the variance bounds inequalities is due to exclusion from the sample of time periods containing bubbles.[9] We now demonstrate that this theoretical intuition is not always correct. The practical implementation of many variance bounds tests precludes rational stochastic bubbles, per se, as the explanation for the failure of the test.

The difference between the theoretical exercise described above and its practical implementation arises, of course, in the construction of an observable counterpart to p_t^*. In practice it is impossible to measure the ex post rational price because it depends on the infinite future. Researchers therefore typically measure a related variable denoted $\hat{p}_t$. Since actual price and dividend data are available for a sample of observations on $t = 0, 1, \cdots, T$, researchers use

$$\hat{p}_t \equiv \textstyle\sum_{i=1}^{T-t} \rho^i d_{t+i} + \rho^{T-t} p_T, \quad t = 0, \cdots, T-1, \tag{8}$$

in place of p_t^*.[10] Notice from (5) and (6) that

$$\hat{p}_t = p_t^* - \rho^{T-t} p_T^* + \rho^{T-t} p_T, \tag{9a}$$

which implies from (6) that

$$\hat{p}_t = p_t^* + \rho^{T-t}(b_T - u_T). \tag{9b}$$

Since u_T is the innovation in the present value of dividends between time T and the infinite future, it is uncorrelated with all elements of the time T information set which includes time t information. Since b_T depends on the evolution of the stochastic bubble between t and T, it is not orthogonal to time t information.

Notice what happens when (9b) is solved for p_t^* and the result is substituted

[8] Geweke [11] notes that in linear environments a variance bounds test is not always powerful at detecting deviations from the theory. Since $p_t = p_t^f + b_t$ from (2) and (3), and because $p_t^* = p_t^f + u_t$ from (5) and the definition of u_t, $V_n(p_t^f) - V_n(p_t) = V_n(u_t) - V_n(b_t) - 2C_n(p_t^f, b_t)$. When $V_n(u_t) > V_n(b_t) + 2C_n(p_t^f, b_t)$, the variance bounds test is unable to detect bubbles even though they are present in the data. Geweke demonstrates that an alternative regression test is more powerful. Frankel and Stock [10] reach a similar conclusion but argue that variance bounds test may be more powerful against nonlinearities in the misspecification.

[9] Shiller [26, p. 689], in his discussion of Mankiw, Romer, and Shapiro [19], argues that one reason one of their tests does not find excess volatility is "that the major 'speculative bubbles' in this century of data, that of the 1920s and that of the 1950s, are given less weight."

[10] The variable $\hat{p}_t$ is used by Shiller [24], Grossman and Shiller [12], and Mankiw, Romer, and Shapiro [19]. Earlier, Shiller [22, 23] used the average market price over the sample period as the terminal price in (8). See note 4. Kleidon [15] and LeRoy [17] demonstrate that use of $\hat{p}_t$ produces a smooth series compared to p_t even in situations in which the data are constructed to satisfy (1). The series look smooth because, for small k, $V(p_t^* - p_{t-k}^*) < V(p_t - p_{t-k})$ with the results being quite dramatic for highly autocorrelated dividend series. Hence, the graphs in Grossman and Shiller [12] are quite misleading.

into (6). After slight rearrangement, one obtains

$$\hat{p}_t = p_t + w_t, \tag{10}$$

where

$$w_t \equiv (u_t - \rho^{T-t} u_T) + (\rho^{T-t} b_T - b_t). \tag{11}$$

Equation (10) is the empirical counterpart of (6) and forms the basis of the usual variance bounds tests. The only substantive difference between our version of (10) and that of previous researchers is that we have explicitly allowed for rational stochastic bubbles in our derivation.

Application of the innovation variance operator to (10) gives

$$V_n(\hat{p}_t) = V_n(p_t) + V_n(w_t) + 2C_n(p_t, w_t). \tag{12}$$

The important point concerning (12) is that the innovation covariance between p_t and w_t is zero. To understand why, consider the nature of the composite disturbance term, w_t. First, as noted above, both u_t and u_T are uncorrelated with p_t since p_t is in the time t information set, which is a subset of the time T information set. Second, the combined term $\rho^{T-t} b_T - b_t$ is uncorrelated with the time t information set even though each term separately is not orthogonal to time t information. This is true because $E_t(b_T) = \rho^{-(T-t)} b_t$, which follows from (4). Hence, $E_t(\rho^{T-t} b_T - b_t) = 0$, and $C_n(p_t, w_t) = 0$.

Therefore, (12) takes the form

$$V_n(\hat{p}_t) = V_n(p_t) + V_n(w_t), \tag{13}$$

from which it follows that

$$V_n(\hat{p}_t) \geq V_n(p_t), \tag{14}$$

by the non-negativity of $V_n(w_t)$. Condition (14) was derived in the presence of rational stochastic bubbles. If the variables $\hat{p}_t$ and p_t are actually used, as they have been in several tests, and the inequality in (14) is found to be violated, one cannot conclude that stochastic bubbles are an explanation of model misspecification.

Mankiw, Romer, and Shapiro [19] conduct unbiased variance bounds tests using a terminal market price to construct their counterpart to ex post rational price. They note that their variance bounds would not be rejected because of bubbles in asset prices. In his comments on their paper, Shiller [26] mentioned that their exclusion of certain periods that may contain bubbles could be a reason that they fail to reject excess volatility in certain tests. As they noted and as our analysis indicates, their tests should continue to satisfy the variance bounds inequalities even in the presence of bubbles.[11] Since the Mankiw, Romer, and Shapiro framework is slightly different than the one we have used, we have included an appendix showing that bubbles cannot cause violations of their variance bounds.

Research that does not use the terminal price p_T to construct $\hat{p}_t$ does not

[11] In fairness to Shiller, his statements in [25] clearly indicate that he rejects the notion of rational speculative bubbles discussed in this paper, although Fischer [7] argues that Shiller's fads and fashions may ultimately prove to be the same thing as speculative bubbles when quantified empirically.

discriminate among possible reasons for rejection of the null hypothesis. The research which does use the actual terminal market price therefore incorporates one of the alternative hypotheses into the null hypothesis. While this may make it more difficult to reject the null hypothesis, a reversal of the inequality in (14) cannot be attributed to rational stochastic bubbles.

II. A Cautionary Note in Tests for Bubbles

The discussion above assumes that bubbles could be present in the data and examines whether certain variance bounds tests could detect them. In this section we present a cautionary note to those who are interested in testing for bubbles. We develop an example economy that is bubble-free, but it is characterized by a potential switch in government policy that is known to agents but may not be known to the researcher. The asset price paths in the example economy could be taken to be bubbles by an unwary researcher. Consequently, we stress that all bubbles tests are conditional on the researcher having the correct specification of the model, including agents' beliefs about the nature of market fundamentals. Flood and Garber [9], Blanchard [2], and Tirole [27] all define the bubble as what is left over after market fundamentals have been removed from the price. Since neither bubbles nor market fundamentals are directly observable, one can never be sure that market fundamentals have been specified appropriately.[12] The example serves to illustrate the point in the context of the model of the previous section.

The Euler equation (1) may be rewritten as

$$U'(c_t)z_t = E_t[\rho U'(c_{t+1})(z_{t+1} + a_{t+1})]. \tag{15}$$

Consider an economy with a_{t+i} constant at $\bar{a}$, and for simplicity assume that aggregate output is not storable and is constant at $\bar{y}$. With population normalized to one, per capita consumption is simply $c_t = \bar{y}$ in equilibrium. We first solve the model without uncertainty, assuming the absence of bubbles. The solution for price is

$$z_t = \rho\bar{a}/(1 - \rho), \quad t = 0, 1, \cdots. \tag{16}$$

Now consider the solution if the agents know that a government will come into existence at T. Assume that the government will institute a tax system to finance its expenditures and that the service flow of government goods enters the utility function separably from the utility of private goods. Assume also that the government will take g units of the consumption goods each period, which lowers equilibrium consumption to $c = \bar{y} - g$ in each period from T onwards.

The advent of the government sector raises the marginal utility of private consumption in period T and thereafter. We parameterize this change by writing $U'(\bar{y} - g) = (1 + \alpha)U'(\bar{y})$. Suppose further that the government finance system taxes all income flows but not capital gains at the rate θ, so that $g = \theta\bar{y}$. After-tax income from owning an asset is therefore $(1 - \theta)\bar{a}$.

[12] This point was emphasized by Flood and Garber [9, pp. 749–50] and has been reiterated by Hamilton and Whiteman [13].

To determine the price of the asset in periods before T, consider first what price must hold after the advent of the government. The first order condition at T is

$$U'(\bar{y} - g)z_T = \rho U'(\bar{y} - g)[z_{T+1} + (1 - \theta)\bar{a}], \tag{17}$$

which has the no-bubbles solution

$$z_t = \rho(1 - \theta)\bar{a}/(1 - \rho), \quad t = T, T + 1, \cdots. \tag{18}$$

The price of the asset before T can now be determined by recognizing that agents know at $T - 1$ that dividends will be taxed and that the marginal utility of private consumption will rise at T. Price must obey

$$U'(\bar{y})z_{T-1} = \rho U'(\bar{y} - g)(z_T + (1 - \theta)\bar{a}), \tag{19}$$

which, from (18), implies $z_{T-1} = (1 + \alpha)z_T$. Price falls from $T - 1$ to T to offset the increase in marginal utility and the incidence of taxation. Treating z_{T-1} as a terminal price, we solve for asset prices in periods before time $T - 1$.

The solution for asset price is

$$z_t = A\rho^{-t} + \rho\bar{a}/(1 - \rho), \quad t = 0, 1, \cdots, T - 1, \tag{20}$$

where

$$A = [(1 - \theta)(1 + \alpha) - 1][\rho\bar{a}/(1 - \rho)]\rho^{(T-1)}. \tag{21}$$

The price path prior to $T - 1$ will rise, fall, or remain constant depending on the sign of A, which is governed by whether $(1 - \theta)(1 + \alpha)$ is greater than, less than, or equal to one.[13]

Now suppose agents are not sure that a government will be installed at T. Assume that their uncertainty can be represented by a time-invariant probability π that the government will begin operation at T, with corresponding probability $(1 - \pi)$ that the government will not begin operation then or at any other time in the future.

With this change the constant term (21) becomes

$$A' = [\pi + (1 - \pi)(1 + \alpha)(1 - \theta) - 1][\rho\bar{a}/(1 - \rho)]\rho^{(T-1)}. \tag{22}$$

If the transition probability is not constant but moves through time, a solution for the price path is

$$z_t = [\pi_t + (1 - \pi_t)(1 + \alpha)(1 - \theta) - 1][\rho\bar{a}/(1 - \rho)]\rho^{T-t-1} + \rho\bar{a}/(1 - \rho),$$
$$t = 0, 1, \cdots, T - 1, \tag{23}$$

which may vary stochastically through time in addition to its deterministic movements.

These simple examples illustrate situations in which an expected future event produces a price path which, if compared to the path of market fundamentals

[13] If $U(c_t) = c_t^{1-\beta}/(1 - \beta)$, $\beta > 0$, then $(1 + \alpha)(1 - \theta)^{\beta} = 1$ and the type of price path followed until date $T - 1$ depends on the relationship of β to unity. If $\beta > 1$, prices rise prior to T, whereas they fall if $\beta < 1$. For quadratic utility the relationship of $(1 + \alpha)(1 - \theta)$ to unity will depend on the scale of the economy and ratios of the utility function parameters.

prior to the future event, appears to be characterized by a bubble. Examination of (20) and (23) indicates that asset prices correspond to the no-bubbles system with constant fundamentals price in (18) plus something else. The additional element must fulfill the bubble property given in equation (4) since it is in the homogenous part of the solution. The homogenous part will be present in price either if there are bubbles or if the no-bubbles system needs to position itself in advance of a future switch in forcing processes. The last example as well as more complex versions would generate stochastic price paths that would appear to contain stochastic bubbles that satisfy (4) even though the examples are bubble-free.

The econometric problem arises because the investigator never knows precisely what information is used by economic agents. Consider a naive investigator who examined data from periods surrounding the possible institution of the above government policies in a situation in which the policies were not instituted. If A' in (22) is nonzero, the unwary researcher who treated data for a_t as market fundamentals would conclude that the price path prior to T contained a bubble that burst. The example, of course, was bubble-free. The problem is that a_t does not capture all of the market fundamentals. The potential taxation and government spending programs also are part of fundamentals. If agents receive periodic information about potential changes in policies as in (23), those sources of information are also part of market fundamentals. Given the complexity of political processes, it may be that such misspecification occurs frequently.

The point of this section was to provide a cautionary note to the interpretation of bubbles research. Empirical bubbles tests must be interpreted either conditionally, assuming an investigator has correctly modeled both market fundamentals and agents' beliefs about future market fundamentals, or as joint tests for bubbles and possibilities of misspecification of market fundamentals. Perhaps the latter interpretation is more attractive to some researchers. It is interesting, nevertheless, to inquire whether bubble-type processes characterize the data, and, if so, what might be the misspecification of market fundamentals behind such a finding.

III. Concluding Remarks

Some models imply that rational speculative bubbles should be absent from asset prices. For these models, well-designed tests for speculative bubbles are tests of the specification of the model. Other models are consistent with speculative bubbles inhabiting the data. Whether tests for rational speculative bubbles are viewed as specification tests or as tests for underlying indeterminacy of asset prices, the questions raised by considering rational bubbles are susceptible to scientific investigation.

The purpose of this paper was two fold. The first purpose was to demonstrate that failure of some variance bounds tests should not be taken as evidence of rational speculative bubbles, because design of the tests precludes bubbles as a reason for failure of the tests. The second purpose was to argue that bubble tests are hard to design since the path of a bubble in the data would look like some forms of incorrect modeling of agents' expectations.

The problem with interpreting some variance bounds tests as bubble tests lies in the construction of the observable benchmark counterpart to the unmeasurable theoretical concept of ex post rational price. In some tests researchers use a terminal market price in the construction. If bubble processes are present in the data, they will be in the terminal market price and will therefore be built into the observable benchmark. Since the bubble is both in market price and in the constructed benchmark, the bubble does not give a reason that the volatility of actual market price is greater than the volatility of the benchmark.

With bubbles precluded as a reason for failure of some variance bounds tests, alternative explanations of the results must be found. Recall that the typical variance bounds tests conducted to date are tests of a joint hypothesis with many parts, including the following: (i) rational expectations, (ii) identical risk-neutral asset holders, (iii) annual portfolio decisions, (iv) identical tax treatment of dividends and capital gains, and (v) ergodicity of the data. Although there are other parts of the joint hypothesis, a violation of any combination of the five listed above would result in rejection of the hypothesis.

The second point of the paper is to provide a cautionary note to those who would test for bubbles. Flood and Garber [9] noted the problem that arises in development of convincing bubble tests because bubble processes can look like an investigator's misspecification of agents' unobservable expectations. We reiterated this point in an optimizing example of the stock market. Bubble tests require that the investigator correctly identify the processes used by agents in forming their expectations. In this sense bubble tests are open to the same criticism as is any test conditioned on correct modeling of agents' beliefs—the researcher must in fact conduct a test of a joint hypothesis that correctly models expectations.[14]

In our view, recent empirical research on the volatility of asset prices suggests that relaxation of the strong assumptions listed above will be necessary to develop a positive theory of asset pricing. In this regard, research is being done on a wide variety of fronts to disentangle the joint hypothesis. For example, researchers are relaxing the assumption of risk neutrality, including taxes in the analysis, developing models of agents with heterogeneous information sets, and obtaining direct observations on agents' expectations.

Appendix

In this appendix we demontrate that bubbles cannot be a reason for violation of the variance bounds tests derived in Mankiw, Romer, and Shapiro [19]. Their tests were developed in response to Flavin [8] and Kleidon [16], who argued that estimation of the sample variance by subtraction of the sample mean rather than the population mean produces small sample bias in variance bounds tests. To develop an unbiased test, Mankiw, Romer, and Shapiro [19] consider a "naive

[14] The variance bounds test described by LeRoy and Porter [18] and the methodology for testing for bubbles proposed by West [28, 29] are also sensitive to the issue of modeling agents' expectations and the potential failure of the data to satisfy the ergodicity assumption.

forecast" of stock price defined by

$$p_t^0 = \sum_{i=1}^{\infty} \rho^i F_t(d_{t+i}) \tag{A1}$$

where $F_t(d_{t+i})$ is the naive forecast of dividends at time $t + i$ based on some information available at time t.

Consider the following identity:

$$p_t^* - p_t^0 = (p_t^* - p_t) + (p_t - p_t^0). \tag{A2}$$

In order to avoid sample means, Mankiw, Romer, and Shapiro [19] work with the conditional second moments of (A2). Substitute from (6) into (A2) and take conditional second moments to derive

$$E_{t-n}(p_t^* - p_t^0)^2 = E_{t-n}(p_t^* - p_t)^2 + E_{t-n}(p_t - p_t^0)^2 - 2E_{t-n}[b_t(p_t - p_t^0)]. \tag{A3}$$

The last term in (A3) appears because $p_t^* - p_t = u_t - b_t$, and, although u_t is orthogonal to $(p_t - p_t^0)$, b_t is not. Notice, therefore, that the two inequalities derived by Mankiw, Romer, and Shapiro [19] in the absence of bubbles,

$$E_{t-n}(p_t^* - p_t^0) \geq E_{t-n}(p_t^* - p_t)^2 \tag{A4a}$$

and

$$E_{t-n}(p_t^* - p_t^0)^2 \geq E_{t-n}(p_t - p_t^0)^2, \tag{A4b}$$

need not hold in theory in the presence of bubbles. From (A3), notice that, if $E_{t-n}[b_t(p_t - p_t^0)]^2 > 0$, bubbles would be one of the reasons that the theoretical construct p_t^* could fail a second moment test.

Now consider substitution for p_t^* in (A2) from (9b):

$$\hat{p}_t - p_t^0 = (\hat{p}_t - p_t) + (p_t - p_t^0). \tag{A5}$$

Since p_t^* appears on both sides of (A2), the term $\rho^{T-t}(b_T - u_T^*)$ in (9b) does not appear in (A5). From (10), notice that $\hat{p}_t - p_t$ on the right-hand side of (A5) is uncorrelated with information at time t. Therefore, since $p_t - p_t^0$ is in the time t information set,

$$E_{t-n}(\hat{p}_t - p_t^0)^2 = E_{t-n}(\hat{p}_t - p_t)^2 + E_{t-n}(p_t - p_t^0)^2, \tag{A6}$$

which provides two empirical counterparts to (A4a) and (A4b):

$$E_{t-n}(\hat{p}_t - p_t^0)^2 \geq E_{t-n}(\hat{p}_t - p_t)^2 \tag{A7a}$$

and

$$E_{t-n}(\hat{p}_t - p_t^0)^2 \geq E_{t-n}(p_t - p_t^0)^2. \tag{A7b}$$

Since (A7a) and (A7b) were derived under the hypothesis that rational stochastic bubbles are present in the data, rejection of either hypothesis cannot be attributed to the presence of stochastic bubbles.

REFERENCES

1. Gardner Ackley. "Commodities and Capital: Prices and Quantities." *American Economic Review* 73 (March 1983), 1–16.

2. Olivier Blanchard. "Speculative Bubbles, Crashes, and Rational Expectations." *Economic Letters* 3, 387–89.
3. Olivier Blanchard and Mark Watson. "Bubbles, Rational Expectations and Financial Markets." In Paul Wachtel (ed.), *Crisis in the Economic and Financial Structure.* Lexington, MA: Lexington Books, 1982.
4. Rudiger Dornbusch. "Equilibrium and Disequilibrium Exchange Rates." *Zeitschift fur Wirtschafts und Sozialwissenshaften* 102 (1982), 573–99.
5. Kenneth Dunn and Kenneth J. Singleton. "Modeling the Term Structure of Interest Rates Under Nonseparable Utility and Durability of Goods." Forthcoming, *Journal of Financial Economics.*
6. Martin Eichenbaum and Lars Peter Hansen. "Uncertainty, Aggregation, and the Dynamic Demand for Consumption Goods." Carnegie-Mellon University, manuscript, 1984.
7. Stanley Fischer. "Comments and Discussion." *Brookings Papers on Economic Activity* 2 (1984), 499–504.
8. Marjorie Flavin. "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence." *Journal of Political Economy* 91 (December 1983), 929–56.
9. Robert P. Flood and Peter M. Garber. "Market Fundamentals Versus Price Level Bubbles: The First Tests." *Journal of Political Economy* 88 (August 1980), 745–70.
10. Jeffrey Frankel and James Stock. "A Relationship Between Regression Tests and Volatility Tests of Market Efficiency." National Bureau of Economic Research, Working Paper No. 1105, 1983.
11. John Geweke. "A Note on Testable Implications of Expectation Models." University of Wisconsin, Social Science Research Institute, Working Paper No. 8024, November 1980.
12. Sanford Grossman and Robert J. Shiller. "The Determinants of the Variability of Stock Market Prices." *American Economic Review* 71 (May 1981), 222–27.
13. James Hamilton and Charles Whiteman. "The Observable Implications of Self-Fulfilling Expectations." *Journal of Monetary Economics* 16 (November 1985), 353–74.
14. Lars Peter Hansen and Kenneth J. Singleton. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50 (September 1982), 1269–86.
15. Allan W. Kleidon. "Variance Bounds Tests and Stock Price Valuation Models." Forthcoming, *Journal of Political Economy.*
16. ———. "Bias in Small Sample Tests of Stock Price Rationality." *Journal of Business* 59 (April 1986), 237–62.
17. Stephen F. LeRoy. "Efficiency and Variability of Asset Price." *American Economic Review* 74 (May 1984), 183–87.
18. Stephen F. LeRoy and Richard D. Porter. "The Present-Value Relation: Tests Based on Implied Variance Bounds." *Econometrica* 49 (May 1981), 555–74.
19. N. Gregory Mankiw, David Romer, and Matthew D. Shapiro. "An Unbiased Reexamination of Stock Market Volatility." *Journal of Finance* 40 (July 1985), 677–89.
20. Terry A. Marsh and Robert C. Merton. "Dividend Variability and Variance Bounds Tests for Rationality of Stock Market Prices." *American Economic Review* 76 (June 1986), 483–98.
21. Richard Meese. "Testing for Bubbles in Exchange Markets: The Case of Sparkling Rates." *Journal of Political Economy* 94 (April 1986), 345–73.
22. Robert J. Shiller. "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71 (June 1981), 421–36.
23. ———. "The Use of Volatility Measures in Assessing Market Efficiency." *Journal of Finance* 36 (June 1981), 291–304.
24. ———. "Consumption, Asset Markets and Macroeconomic Fluctuations." *Carnegie Rochester Conference Series on Public Policy* 17 (1982), 203–50.
25. ———. "Stock Prices and Social Dynamics." *Brookings Papers on Economic Activity* 2 (1984), 457–510.
26. ———. "Discussion." *Journal of Finance* 40 (July 1985), 688–89.
27. Jean Tirole. "Asset Bubbles and Overlapping Generations." *Econometrica* 53 (November 1985), 1499–1528.
28. Kenneth West. "Speculative Bubbles and Stock Price Volatility." Princeton University, Financial Research Center Memorandum No. 54, December 1984.
29. ———. "A Specification Test for Speculative Bubbles." Princeton University, Financial Research Center Memorandum No. 58, April 1985.