CONTROL PREMIUMS, MINORITY DISCOUNTS, AND OPTIMAL JUDICIAL VALUATION*

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Abstract

This article characterizes controlling and minority share prices and optimal appraisal remedy valuation rules in a rational expectations equilibrium. The model identifies a set of optimal judicial valuation policies that would motivate shareholders to make optimal investment and freeze-out choices and identifies consequences of judicial valuation error. The model suggests that the share appraisal rule, under which minority shares are assessed a minority discount in freeze-out appraisals, would be more optimal than Delaware's pro rata doctrine, which forbids discounting of minority shares. In accordance with the share appraisal rule and in contradiction with their own pro rata doctrine, Delaware courts frequently allow implicit discounts to minority shares in freeze-out appraisals. In summary, the model articulates why courts should not equate minority share value with market price. Consistent with this implication, Delaware courts have always uniformly rejected the notion that market price should be the sole determinant of appraisal value.

I. Introduction

In most states of incorporation, including Delaware, a controlling shareholder dictates the freeze-out decision. Subject to mild constraints such as fiduciary duty and, perhaps, a vote of minority shareholders, the board determines if, when, and at what price to freeze out minority shareholders. Once freeze-out commences, dissenting minority shareholders must either accept the freeze-out price or challenge it in court. Depending on the corporate charter or state of incorporation, challengers may have two options. First, they can seek judicial appraisal of their shares, in which case the court determines what they get for their shares in lieu of the freeze-out price. If they do not qualify for appraisal, the remaining recourse is judicial review under the entire-fairness doctrine. If the court finds either a lack of fair dealing on the part of the control group or that the freeze-out price is inadequate, it may award damages. While an appraisal proceeding differs in many legal

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respects from an entire-fairness review, the aim of both is to determine the amount of compensation for the dissenting minority shares.[1]

The idea underlying the rational expectations model in the paper is as follows. One incentive for a controlling shareholder to freeze out minorities is that he wants to unlock hidden value that only a 100 percent owner can harvest.[2] Since shareholders anticipate the outcome of an appraisal proceeding and that minority shareholders will seek appraisal whenever it is advantageous for them to do so, an optimal judicial appraisal policy must adequately compensate the controlling shareholder to induce him to investigate the benefits of freeze-out and, then, to choose freeze-out only when it increases total shareholder wealth. The control premium represents the surplus the court offers the controlling shareholder to induce him to exert optimal ex ante effort and make the correct freeze-out choice. An optimal control premium is the payoff value of the controlling shareholder's real option to investigate and choose freeze-out supplemented with a correction for anticipated litigation fees. The minority discount is the commensurate deduction that should be taken from minority shareholders to reflect that they are short the option of control. Since dissenting minority shareholders' only decision right in a freeze-out process is the option to request judicial appraisal, an optimal judicial valuation policy should not grant minority shareholders a portion of hidden value.

The extant literature on the appraisal remedy focuses on two sets of issues. The first set concerns the merits of the appraisal remedy and what judicial valuation policy should be in appraisal proceedings.[3] Complicating the conversation is that Delaware courts do not seem to treat the control premium and minority discount coherently. For instance, they may allow a control premium without recognizing a commensurate minority discount or adopt

[1] John C. Coates IV, "Fair Value" as an Avoidable Rule of Corporate Law: Minority Discounts in Conflict Transactions, 147 U. Pa. L. Rev. 1251 (1999).

[2] Bernard Black & Reinier Kraakman, Delaware's Takeover Law: The Uncertain Search for Hidden Value, 96 Nw. U. L. Rev. 521 (2002). Other incentives include private benefits of control discussed in the agency cost literature. See Frank H. Easterbrook & Daniel R. Fischel, The Economic Structure of Corporate Law (1991).

[3] See Lucian Arye Bebchuk, Efficient and Inefficient Sales of Corporate Control, 109 Q. J. Econ. 957 (1994); Coates, *supra* note 1; John C. Coffee, Jr., Transfers of Control and the Quest for Efficiency, 21 Del. J. Corp. L. 359 (1996); Harry DeAngelo, Linda DeAngelo, & Edward M. Rice, Going Private: Minority Freezeouts and Shareholder Wealth, 27 J. Law & Econ. 367 (1984); Easterbrook & Fischel, *supra* note 2; Sanford Grossman & Oliver Hart, Disclosure Laws and Takeover Bids, 35 J. Finance 323 (1980); Benjamin Hermalin & Alan Schwartz, Buyouts in Large Companies, 25 J. Legal Stud. 351 (1996); Louis Lowenstein, Management Freeze-Outs, 85 Colum. L. Rev. 730 (1985); Lynn A. Stout, Are Takeover Premiums Really Premiums? Market Price, Fair Value, and Corporate Law, 99 Yale L. J. 1235 (1990); Robert B. Thompson, Exit, Liquidity, and Majority Rule: Appraisal's Role in Corporate Law, 84 Geo. L. J. 1 (1995); Barry M. Wertheimer, The Shareholders' Appraisal Remedy and How Courts Determine Fair Value, 47 Duke L. J. 613 (1998).

valuation procedures that build in implicit but doctrinally impermissible discounts.[4]

The second set of issues concerns the merits of the various judicial appraisal rules when an appraisal proceeding occurs. The appraisal remedy gives minority shareholders a real exit option and defines their payoff values in the event of appraisal. Since preappraisal transaction participants rationally anticipate their legal entitlements whether those entitlements make financial sense or not, appraisal policy influences pre-freeze-out shareholder behavior, transactions prices, and shareholder value. This observation raises questions about how shareholders bargain in the shadow of appraisal during reorganization,[5] why the remedy is exercised so infrequently,[6] and what impact the remedy has on premerger share prices.[7] Joel Seligman's survey found that of 16,479 mergers of U.S. companies between 1972 and 1981, only 20 resulted in judicial appraisal hearings nationwide.[8] Paul Mahoney and Mark Weinstein examined 1,350 mergers of publicly held firms across the United States and compared the premerger value of shares with an appraisal right against those without the right. After controlling for confounding effects, they concluded that "given the existence of legally enforceable fiduciary duties, appraisal does not benefit public company shareholders" compared to public shareholders in corporations not subject to an appraisal remedy.[9] Mahoney and Weinstein's result suggests that other safeguards, such as fiduciary duty standards, protect minority shareholders as much as the appraisal remedy. Yet when market prices are available for comparison, a reading of Delaware opinions suggests that judicial appraisals systematically exceed last-traded market prices. This seems paradoxical in view of Mahoney and Weinstein's findings because an appraisal premium would indicate that minority shareholders obtain windfall returns when an appraisal occurs.[10]

Although Sanford Grossman and Oliver Hart[11] and Benjamin Hermalin and Alan Schwartz[12] offer insightful models that address some of these issues, there is surprisingly little effort to develop an integrated framework that

[4] Coates, *supra* note 1.

[5] Lucian Arye Bebchuk & Howard F. Chang, Bargaining and the Division of Value in Corporate Reorganization, 8 J. L. Econ. & Org. 253 (1992); Thompson, *supra* note 3.

[6] Joel Seligman, Reappraising the Appraisal Remedy, 52 Geo. Wash. L. Rev. 829 (1984).

[7] Paul Mahoney & Mark Weinstein, The Appraisal Remedy and Merger Premiums, 1 Am. L. & Econ. Rev. 239 (1999).

[8] Seligman, *supra* note 6.

[9] Mahoney & Weinstein, *supra* note 7, at 273–74.

[10] The model in this paper supports the idea that, under normal circumstances, appraisal operates as a shadow remedy, so appraisal proceedings, when they actually occur, are rare extraordinary events (consistent with Joel Seligman's 1984 empirical survey, *supra* note 6). In the sense that appraisal proceedings are idiosyncratic events that do not represent the parties' ex ante expectations, the paradox is less weighty.

[11] Grossman & Hart, *supra* note 3.

[12] Hermalin & Schwartz, *supra* note 3.

establishes control premium, minority discount, and judicial valuation policy as endogenous values within a rational expectations equilibrium. This paper fills this void and presents a model that characterizes how judicial valuation policy affects the value of minority discounts in the shadow of the appraisal remedy. New features of the model presented here (and not in the extant literature) include the concepts of pre- and postinvestment control premiums and pre- and postinvestment minority discounts. Since shareholders anticipate the appraisal outcome, judicial appraisal and collateral litigation costs determine what price the controlling shareholder offers and what price minority shareholders are willing to accept in a freeze-out merger. The analysis clarifies how courts should invoke going concern value, control premium, minority discount, merger surplus, and prejudgment interest to motivate a controlling shareholder to maximize shareholder value. The result shows that Delaware law's pro rata doctrine is clearly not optimal. Further analysis of the model characterizes the harm caused by different types of judicial valuation error, including reliance on market price when the market is inefficient.

The analysis highlights a distinction between the preinvestment and postinvestment control premium. The preinvestment control premium, denoted δ_C, is the largest premium a controlling shareholder would be willing to pay for his control block. The postinvestment control premium, δ'_C, is what a controlling shareholder can command for his block should he decide, after investigating the possibility of freeze-out, to sell his shares rather than proceed. As I will explain, the preinvestment minority discount, δ_M, is proportional to δ_C and the postinvestment minority discount, δ'_M, is proportional to δ'_C. Because of the lemons effect,[13] liquidity issues, and other frictions, one expects that $\delta_C > \delta'_C$ or, equivalently, $\delta_M > \delta'_M$. The inequality $\delta_C > \delta'_C$ is also necessary to avoid unstable or nonsensical situations in which a controlling shareholder can amass arbitrage profits by simply buying and then reselling the same control block repeatedly.

The main analytical result, proposition 1, says that to motivate the controlling shareholder to make the right freeze-out choice, judges must discount minority shares by the postinvestment minority discount, δ'_M, rather than the preinvestment discount, δ_M. Ignoring prejudgment interest and other litigation costs, I find that the unique optimal linear valuation policy is to appraise minority shares as being worth continuing value (on a whole-firm basis) minus δ'_M. This means that appraisal optimally should exact a smaller minority discount than what the market had exacted when the controlling position was originally established. In other words, judges should award minority shareholders an apparent premium of size $\delta_M - \delta'_M$ over the earlier market price. This premium provides a possible explanation for the empirically observed premium over market price of judicial appraisals. Proposition 1 also tells us

[13] Lucian Arye Bebchuk & Marcel Kahan, The "Lemons Effect" in Corporate Freeze-Outs (Working Paper No. 6938, Nat'l Bur. Econ. Res. 1999).

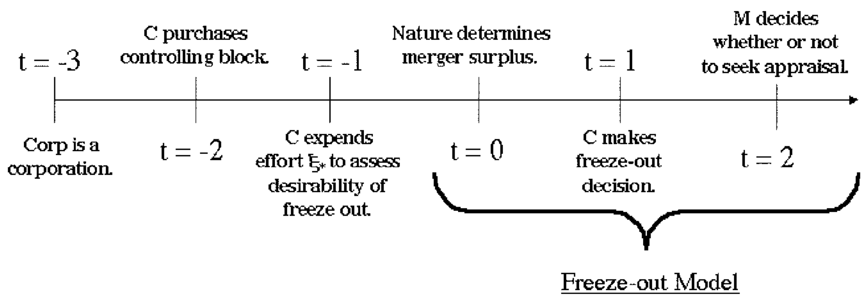

FIGURE 1.—Model time line

that, contrary to conventional wisdom, it is possible to design (nonlinear) valuation policies that award some of the controlling shareholder's ex post merger surplus to minority shareholders without inducing the controlling shareholder to invest suboptimally.

The analysis also clarifies the incentive effects of reallocating litigation costs. While the appraisal remedy should reimburse minority shareholders' prejudgment interest and litigation costs, the analysis suggests that the courts should not adjust for the litigation cost of the controlling shareholder. This is because there is no reason for minority shareholders to actually seek appraisal if the controlling shareholder acts rationally. Indeed, consistent with the observed paucity of appraisal cases, the model suggests that appraisal does not occur when all shareholders act rationally.

Section II characterizes the fundamental value elements and introduces the freeze-out model. Section III characterizes optimal judicial valuation policies—valuation rules that motivate a controlling shareholder to maximize total shareholder value. Section IV describes the equilibrium preinvestment control premium, which characterizes how market price responds in a regime with an appraisal remedy and an optimal judicial valuation policy. Section V considers what happens if judicial valuation policy is suboptimal owing to either systematic bias in favor of one side or the other or unbiased random judicial error. Section VI evaluates several judicial valuation doctrines in view of the normative implications of my analysis, and Section VII concludes.

II. FRAMEWORK FOR ANALYSIS

This section introduces the basic value concepts and a model of how judicial valuation affects the preappraisal choices of controlling and minority shareholders. Figure 1 indicates the key events leading up to a potential freeze-out situation at $t = 0$. Figure 2 provides additional details about the events and payoffs subsequent to $t = 0$.

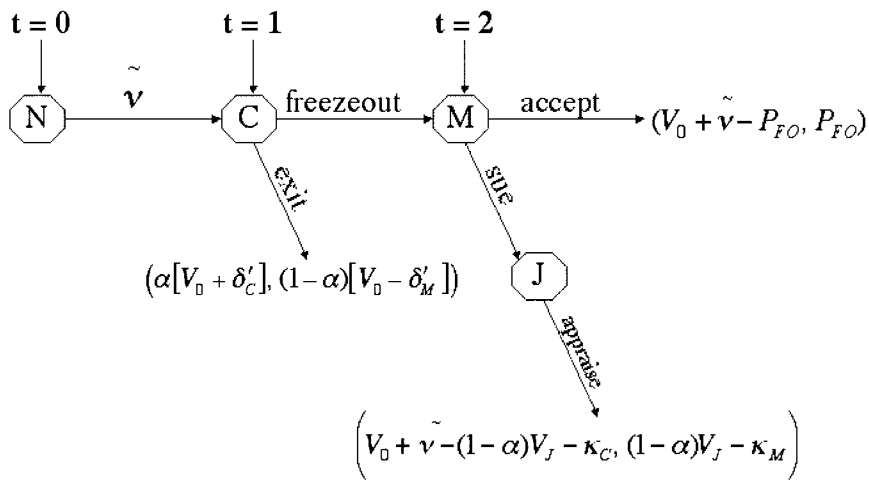

FIGURE 2.—Freeze-out model decision tree

A. Primitive Value Concepts

Consider a freeze-out candidate, a corporation called Corp. A similar discussion applies whether Corp is publicly traded or closely held. As indicated in Figure 1, at $t = -2$ a controlling shareholder C purchases a controlling fraction α of Corp's shares; suppose the remaining fraction $1 - \alpha$ is held by a representative minority shareholder M. While an effective controlling block may comprise less than half of the outstanding shares if the remaining shareholder group is diffuse, to keep the model simple I will assume that α is greater than 50 percent and a single minority shareholder holds the remaining shares.

I will use the modifier "preinvestment" to refer to features that are determined prior to $t = -1$, when C makes an irreversible investment of effort to investigate the desirability of freezing out M. The preinvestment value concepts are the following:

V_0 = the continuing or preinvestment value of the equity; that is, Corp's market value as a stand-alone entity committed to continuing existing operations;

δ_C = the preinvestment control premium in C's shares, which is the value of C's option to spend effort and explore the possibility of a freeze-out; the value of C's control block is $\alpha(V_0 + \delta_C)$; and

δ_M = the preinvestment minority discount the market exacts from M's shares; M's minority shares would be worth $(1 - \alpha)(V_0 - \delta_M)$ to another minority shareholder.

The values of δ_C and δ_M are endogenously determined by the rational expectations of C and M. In particular, how judicial valuation policy treats minority shareholders determines the difference between the value of controlling and minority shares.

The total value V_0 of outstanding Corp shares at $t = -3$ in Figure 1 equals the discounted present value of prospective dividends. In turn, prospective dividends are determined by future profitability and the amount of cash flow reinvestment required to sustain it.[14] The nature of the market, including its liquidity, efficiency, and aversion to systemic risk, determines the relationship between V_0 and expected dividends.[15]

At $t = -2$ in Figure 1, C pays $\alpha(V_0 + \delta_C)$ dollars to obtain a controlling fraction α of Corp shares. The preinvestment control premium δ_C is an extra amount that C must pay to obtain the status of controlling shareholder and its associated privileges.[16] I will think of δ_C as the value of C's option to explore the possibility of freezing out M at a later date. Thus, the model does not directly apply to situations in which a controlling shareholder precommits to going through with a freeze-out even if a freeze-out proves undesirable upon subsequent investigation.[17] Kevin Nathan and Terrence O'Keefe report that control purchasers paid, on average, a control premium δ_C/V_0 of between 41 and 75 percent for successful cash tender offers in the years from 1963 to 1985.[18]

At $t = -2$, a minority shareholder M holds the remaining $1 - \alpha$ fraction of outstanding Corp shares, which are worth $(1 - \alpha)(V_0 - \delta_M)$ dollars. Mi-

[14] Merton H. Miller & Franco Modigliani, Dividend Policy, Growth and the Valuation of Shares, 34 J. Bus. 411 (1961).

[15] In general, continuing value V_0 differs from Corp's net asset or liquidation value; however, the difference between V_0 and net asset value has many causes, such as extant market microstructure, tax effects, and traditional agency frictions, which are unrelated to freeze-out. As such, the difference between V_0 and net asset value does not generically refer to the control premium or minority discount. The potential for a substantial difference is dramatically highlighted by the widely known deviation of closed-end mutual fund share prices from their net asset values (NAVs). If one ignores liquidation costs (which may be substantial if there is market illiquidity or shareholder-level taxes), a fund's NAV is essentially its instant liquidation value. As is well known, closed-end funds invariably trade at a sizeable deviation from their NAVs. The difference does not seem to be completely attributable to liquidation costs, taxes, or other traditional market frictions. See Charles M. C. Lee, Andrei Shleifer, & Richard H. Thaler, Investor Sentiment and the Closed-End Fund Puzzle, 46 J. Fin. 75 (1991). When share prices deviate from NAV, this paper defines V_0 as the value of outstanding shares and not NAV.

[16] In this paper, I model the benefit of control simply as the option to explore freeze-out and harvest hidden value as 100 percent owner. This option does not include other benefits of control identified in the traditional agency literature, including the prerogative to expropriate (properly or improperly) resources of the firm or to give one's son-in-law a plush office. See Easterbrook & Fischel, *supra* note 2.

[17] Without some exogenous frictions—such as labor contracts or debt covenants—precommitting at $t = -2$ to a course of action is either irrational or not credible because restricting the future opportunity set reduces the total value of Corp.

[18] Kevin S. Nathan & Terrence B. O'Keefe, The Rise in Takeover Premiums: An Exploratory Study, 23 J. Fin. Econ. 101 (1989).

nority discount δ_M is the flip side of the control premium. If C's benefits of control occur at the expense of M, then C's control premium is a reallocation of wealth away from M to C. Accordingly, the value of M's shares in the presence of a controlling shareholder must be less than $(1 - \alpha)V_0$. The minority discount quantifies how much less.

Let $\theta \equiv (1 - \alpha)(V_0 - \delta_M) + \alpha(V_0 + \delta_C) - V_0$ denote the difference between Corp's equity value with and without a controlling shareholder. If the presence of a controlling shareholder does not matter, θ equals zero, which means the control premium comes exclusively out of the minority discount. If θ is greater than zero, the presence of C is good news, either because C's decision to purchase a control block signals private information about Corp's true value or because C creates value by his presence (for example, by monitoring the actions of management). If θ is less than zero, C's presence reduces value, which could occur, for instance, if C installs his own inefficient management and thwarts the threat of efficiency-enhancing takeovers.[19] The empirical evidence concerning the value of θ is indirect and inconclusive. Clifford Holderness and Dennis Sheehan analyzed 114 listed firms with values of α between 50 and 100 percent. Compared with firms with diffuse ownership structures, they found no statistical difference in investment expenditures, accounting rates of return, or Tobin's *q*. On the other hand, they also found that in a sample of 31 majority-block trades between 1978 and 1984, a firm's market value increased in the 30-day period surrounding a change of controlling shareholder by an abnormal 12.8 percent on average.[20]

Since the empirical evidence for nonzero θ is mixed, I will focus on the simplest case: when θ equals zero. In this case, firm value is shareholder invariant; that is, the continuing value of Corp is unaffected by its ownership structure. Analogous to Franco Modigliani and Merton Miller's financial policy invariance,[21] shareholder invariance means value cannot be created by a change of share ownership allocation, assuming that there is no collateral

[19] Rene Stulz presents a model in which low levels of management ownership ($\alpha \approx 5$ percent) align incentives and increase θ (Rene M. Stulz, Managerial Control of Voting Rights: Financing Policies and the Market for Corporate Control, 20 J. Fin. Econ. 25 (1988)). However, at higher levels of ownership, θ is less than zero because C's management becomes entrenched in office and has less incentive to operate efficiently because C can block takeovers.

[20] Clifford G. Holderness & Dennis P. Sheehan, The Role of Majority Shareholders in Publicly Held Corporations: An Exploratory Analysis, 20 J. Fin. Econ. 317 (1988).

[21] Franco Modigliani and Merton Miller's financial policy invariance stipulates that financing does not affect V_0 if it does not affect operations (Franco Modigliani & Merton H. Miller, The Cost of Capital, Corporation Finance, and the Theory of Investment, 48 Am. Econ. Rev. 261 (1958)). Shareholder invariance applies Modigliani and Miller's idea by recognizing that equity ownership is a form of financing. In other words, shareholder invariance says that the demographics of shareholders do not change V_0 as long as shareholders do not actively alter operations. When the controlling shareholder C actively contemplates altering operations at $t = -1$, shareholder invariance does not apply, and it will not be maintained in the model. Shareholder invariance is maintained only at $t = -2$ (and later at $t = 1$) when shareholders do not affect (or actively spend money to contemplate affecting) operations.

(or imminent) reorganization of operations. Since C has not made any efforts at $t = -2$ to exploit his control position, shareholder invariance applies and requires $(1 - \alpha)(V_0 - \delta_M) + \alpha(V_0 + \delta_C) = V_0$, which means that

$$\alpha\delta_C - (1 - \alpha)\delta_M = 0.$$

I emphasize that shareholder invariance does not imply or require that subsequent efforts by C to pursue a freeze-out or that an actual freeze-out does not increase Corp's value. Shareholder invariance says only that Corp's shares do not change in value at $t = -2$, when C is a passive owner and prior to his decision to explore the possibility of a freeze-out.

Once C has obtained control, hidden-value theory, according to which a firm's true value is visible to corporate directors but not to shareholders,[22] applies. At $t = -1$, C (who controls the board and may actually sit on the board) exercises his option to explore a freeze-out at cost ξ_*. Cost ξ_* represents the expense of time and resources, including the collection of private information and the hiring of financial analysts, consultants, and lawyers, to evaluate the benefits of a freeze-out. The expenses are unobservable and cannot be credibly communicated. C sets his effort level ξ_* to maximize his own expected net return.[23] Since C's return depends on judicial valuation policy in the event that M seeks appraisal, ξ_* also depends on judicial valuation policy. In general, the value of ξ_* that maximizes C's return is not the same as that which maximizes total shareholder value. Only special "optimal" judicial valuation policies align these two values. Suboptimal valuation policies cause C to choose a value of ξ_* that fails to maximize total shareholder value.

For any valuation policy, suppose that C has rationally chosen to exert ξ_* dollars of effort. C would not rationally invest ξ_* unless the expected marginal gain from his investment equals or exceeds his investment of effort. Thus, it must be the case that $P_C \geq \alpha(V_0 + \delta_C) + \xi_*$. Now suppose that $P_C > \alpha(V_0 + \delta_C) + \xi_*$; that is, the inequality is strict. This would mean that C achieves risk-free arbitrage profits: by exerting ξ_* dollars of effort, C increases the value of his position by strictly more than ξ_* dollars. But this would mean that another bidder, anticipating the possibility of earning arbitrage profits for himself, would have been willing to pay more than $\alpha(V_0 + \delta_C)$ for the control shares at $t = -2$. Such a competitor would outbid C at $t = -2$. Therefore, if the market for controlling shares is perfectly competitive, C can be the winning bidder for $\alpha(V_0 + \delta_C)$ dollars at $t = -2$ if, and only if, the postinvestment value of C's shares at $t = -1$ is worth

$$P_C = \alpha(V_0 + \delta_C) + \xi_*.$$

[22] Black & Kraakman, *supra* note 2.

[23] Hermalin & Schwartz, *supra* note 3.

This equality rests on the premise that the market for controlling share is efficient, an assumption that is consistent with the empirical evidence. In particular, target shareholders on average earn large positive abnormal returns from tender offers, while bidders' abnormal returns are, on average, close to zero.[24]

C's decision to exercise his option to explore a freeze-out induces a collateral change in the value of M's shares. I will denote the total value of M's shares at $t = -1$ as

$$P_{\mathrm{M}} = (1 - \alpha)(V_0 - \delta_{\mathrm{M}}) + \Delta.$$

The value of Δ, which will be endogenously determined in the freeze-out model, depends on judicial valuation policy.

Upon C's decision to invest effort, the total value of Corp's shares increases by $\xi_* + \Delta$ dollars, where $\xi_* + \Delta$ is the posttakeover and pre-freeze-out change in total shareholder value. This change reflects the ex post realization of (*a*) C's effort investment and (*b*) the news to M that C is actively contemplating a freeze-out. As such, $\xi_* + \Delta$ is the anticipated increase in expectation of Corp's total post-freeze-out value. It should be emphasized that the values of ξ_* and Δ (and, hence, $\xi_* + \Delta$) are not imposed on the model. Rather, they are determined endogenously by the rational expectations equilibrium.[25]

B. The Freeze-out Model

The freeze-out model traces the steps a controlling and a minority shareholder goes through in a potential freeze-out situation. Because C anticipates how the court will appraise the minority shares if appraisal occurs, judicial valuation policy affects whether C decides to freeze out and the freeze-out price he offers M should he choose to do so.

Figure 2 indicates the chain of events. In addition to the controlling (C) and minority (M) shareholders, two participants contribute to setting the freeze-out price P_{FO} of Corp: Nature (N) and judge (J). At date $t = 0$, Nature determines the values of the following variables:

$\tilde{\nu}$ = merger surplus, the incremental value Corp realizes if C gains 100 percent ownership; C's private valuation of Corp as a merged entity is

[24] Michael Bradley, Interfirm Tender Offers and the Market for Corporate Control, 53 J. Bus. 345 (1980).

[25] Empirical studies of tender offers suggest that $\xi_* + \Delta$ is positive. For example, Michael Bradley, Anand Desai, & E. Han Kim, Synergistic Gains From Corporate Acquisitions and Their Division between the Stockholders of Target and Acquiring Firms, 21 J. Fin. Econ. 3 (1988), finds that the joint market value of bidder and target tends to increase on average after a bid. That study focuses on tender offers in general rather than freeze-out mergers, and so it does not apply directly to $\xi_* + \Delta$.

$V_0 + \tilde{\nu}$; Nature draws $\tilde{\nu} \in [\underline{\nu}, \overline{\nu}]$ from a distribution density[26] $\phi(\tilde{\nu}, \xi_*)$ and discloses its value only to C; I assume that $\underline{\nu} < 0 < \overline{\nu}$;

δ'_C = expected postinvestment control premium C can obtain for his shares if he decides to sell rather than freeze out;

δ'_M = expected value of the postinvestment minority discount if C should forgo a freeze-out; the shareholder invariance principle implies $\alpha\delta'_C - (1 - \alpha)\delta'_M = 0$; and

κ_C and κ_M = respective litigation costs of C and M if there is an appraisal hearing; κ_M includes M's prejudgment interest, while κ_C incorporates any collateral harm the merged firm may suffer if it undergoes appraisal litigation;[27] in the analysis, I shall focus exclusively on the case when $\kappa_C > \kappa_M$.[28]

Unlike their preinvestment counterparts δ_C and δ_M, the values of the postinvestment premium and discount δ'_C and δ'_M are exogenously given in this model. One expects the postinvestment control premium δ'_C to be strictly positive because a new controlling shareholder who buys C's control block obtains an option to investigate the possibility of a freeze-out merger. Since the new controlling shareholder may be able to uncover hidden value even if C has failed to do so, the market price for C's control block commands a control premium even after C has decided not to freeze out. The expected postinvestment control premium δ'_C must be strictly less than the preinvestment control premium δ_C. This is because $\delta'_C < \delta_C$ is necessary to prevent arbitrage and sustain a rational expectations equilibrium. The cost of a round-trip for C, if he decides to avoid freezing out, is $\alpha(\delta_C - \delta'_C)$. This cost must be positive because a negative-cost round-trip would enable C to make unbounded arbitrage profits by perpetual repetition.

If one does not like the no-arbitrage argument, an alternative argument is as follows. Let π be the probability C's shares can be resold as a control block at a price reflecting the original control premium and $1 - \pi$ be the probability the block must be broken up and sold as noncontrol shares, in which case the control premium is zero. Hence, the postinvestment control

[26] The term $\phi(\tilde{\nu}, \xi_*)$ is a joint density function. To insure that C's choice of effort always has a unique optimum, define $\Psi(\xi) \equiv \int_0^{\overline{\nu}} d\tilde{\nu}\, \tilde{\nu}\, \varphi(\tilde{\nu}, \xi)$ and assume that $\phi(\tilde{\nu}, \xi_*)$ is such that $\partial\Psi/\partial\xi$ is a strictly decreasing function of ξ that crosses one at some unique value of ξ. This mathematical condition on $\phi(\tilde{\nu}, \xi_*)$ implements the assumption that there is always a first-best effort level that a controlling shareholder should exert.

[27] Collateral harm may include the disclosure of trade secrets during litigation that may help the merged firm's competitors, reduction of the merged firm's effectiveness if its officers and directors are distracted by shareholder litigation, and damage to its reputation for treating shareholders fairly.

[28] The case $\kappa_C > \kappa_M$ is more consistent with the empirical evidence indicating that appraisal is extremely rare. See Seligman, *supra* note 6. When $\kappa_C > \kappa_M$, controlling shareholders avoid appraisal proceedings at equilibrium. In contrast, when $\kappa_M \geq \kappa_C$, controlling shareholders are either indifferent to or prefer appraisal proceedings at equilibrium.

premium in expectation is $\delta'_C = (1 - \pi)0 + \pi\,\delta_C = \pi\,\delta_C$. Since $0 < \pi < 1$, this implies that $0 < \delta'_C < \delta_C$.[29]

C's investment of ξ_* effort dollars at $t = -1$, as indicated in Figure 1, determines the merger surplus distribution $\phi(\tilde{\nu},\ \xi_*)$ from which Nature draws the realization of $\tilde{\nu}$. The values of V_0, α, δ_C, δ_M, δ'_C, δ'_M, κ_C, and κ_M are common knowledge to all participants. However, because C's effort is unobservable and cannot be credibly communicated, $\tilde{\nu}$ is privately known only to C. The judge, the market, and M learn the realized value of $\tilde{\nu}$ only if and when there is an appraisal hearing.

The C, M, and J nodes in Figure 2 correspond to decision nodes of C, M, and J after C has already expended ξ_*. As depicted, upon learning the realization of $\tilde{\nu}$, C decides whether to freeze out M's shares or to exit. If C chooses to exit, he sells his shares for a terminal payoff of $\alpha(V_0 + \delta'_C)$ dollars and M is left with his existing shares worth $(1 - \alpha)(V_0 - \delta'_M)$, which implies that total shareholder value is $\alpha(V_0 + \delta'_C) + (1 - \alpha)(V_0 - \delta'_M) = V_0$, Corp's continuing value. The difference between Corp's value at $t = -1$ and the value if C exits, V_0, is $\xi_* + \Delta$, which is positive for any reasonable judicial valuation policy. Hence, this framework predicts that Corp's value decreases if its controlling shareholder chooses to exit rather than pursue a freeze-out merger.

If C chooses to freeze out, he has several procedural options, including share repurchase, share exchange, or a reverse triangular merger.[30] These procedures are significant only in how they affect M's chances of obtaining judicial appraisal.[31] In the freeze-out model, I assume that M has an appraisal right and C tenders a nonnegotiable offer of P_{FO} dollars for all of M's shares. If M accepts C's offer, M walks away with P_{FO} dollars and C has a net payoff of $V_0 + \tilde{\nu} - P_{FO}$. However, M has a right to refuse C's offer and seek judicial appraisal of his shares. If M seeks appraisal, C must disclose his private information (merger surplus $\tilde{\nu}$) to the judge. The judge appraises Corp according to a preset and publicly known valuation policy $V_J = F(V_0,\ \tilde{\nu},\ \delta_C,\ \delta_M,\ \delta'_C,\ \delta'_M,\ \kappa_C,\ \kappa_M)$, which allows the judge to take into account in her appraisal not only Corp's continuing value but also C's merger surplus, the control premium, the minority discount, prejudgment interests, and attorneys fees. Judicial valuation policy V_J cannot take into account C's effort expenditure

[29] The premium δ'_C would be further reduced by any transaction costs involved in reselling C's shares.

[30] In a reverse triangular merger, C creates a subsidiary S to which he transfers all of his Corp shares plus P_{FO} dollars. C, as S's exclusive shareholder, then directs S to pay M P_{FO} dollars for all of M's Corp shares. As a result, C is the sole owner of S, which holds 100 percent of Corp's shares.

[31] In an effort to justify fair dealing and fair price, C will usually appoint a subcommittee of disinterested directors, representing M, to approve the take-out price and obtain a fairness opinion from an investment banker. Fairness opinions are famous for being ambiguous with respect to price ranges. See William J. Carney, Fairness Opinions: How Fair Are They and Why We Should Do Nothing about It, 70 Wash. U. L. Q. 523 (1992).

ξ_* because, following a standard supposition in the contracting literature,[32] effort cannot be credibly observed by outsiders such as judges. As indicated in Figure 2, after appraisal, C walks away with the value of Corp net the appraised value of M's shares and litigation costs and M walks away with the appraised value of his shares net his litigation cost.

Given a known judicial valuation policy V_J, the equilibrium behavior of C and M can be determined by backward induction. Table 1 lists the resulting equilibrium market values of the control and minority positions at each period. The backward-induction argument used to obtain the values listed in Table 1 proceeds as follows (further details are in the proofs of propositions 1 and 2 in the Appendix):

a) If C chooses to exit at $t = 1$, according to Figure 2 the value of the control and minority positions are, respectively, $P_C = \alpha(V_0 + \delta'_C)$ and $P_M = (1 - \alpha)(V_0 - \delta'_M)$.

b) If C chooses to freeze out at $t = 1$, he sets the freeze-out offer P_{FO} as low as possible so that M will accept without requesting appraisal (in which case C would incur incremental litigation cost). Hence, $P_{FO} = (1 - \alpha)V_J - \kappa_M$, which gives M exactly what M would obtain in appraisal. As a result, the values of C's and M's positions at $t = 1$ in this case equal $P_C = V_0 + \tilde{\nu} - (1 - \alpha)V_J + \kappa_M$ and $P_M = (1 - \alpha)V_J - \kappa_M$.

c) C chooses to freeze out at $t = 1$ if, and only if, his position upon freeze-out exceeds the value of his position upon exit, that is, if $V_0 + \tilde{\nu} - (1 - \alpha)V_J(\tilde{\nu}) + \kappa_M > \alpha(V_0 + \delta'_C)$. Let ν_* denote the minimum threshold value of merger surplus $\tilde{\nu}$ that motivates C to choose freeze-out. (Clearly, the value of ν_* depends on the judicial valuation policy $V_J(\tilde{\nu})$.) At $t = 0$, even before C learns the value of the merger surplus $\tilde{\nu}$, both C and M anticipate C's $t = 1$ decision rule and the value of ν_* because judicial valuation policy is common knowledge. Accordingly, the value of C's position at $t = 0$ is the expected value of his $t = 1$ position conditional on his anticipated decision rule:

$$
\begin{aligned}
P_C &= E[\max\{V_0 + \tilde{\nu} - [(1 - \alpha)V_J(\tilde{\nu}) - \kappa_M],\ \alpha(V_0 + \delta'_C)\}] \\
&= \alpha(V_0 + \delta'_C)\Phi(\nu_*,\ \xi_*) + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*)\{V_0 + \tilde{\nu} - [(1 - \alpha)V_J(\tilde{\nu}) - \kappa_M]\}.
\end{aligned}
$$

Likewise, the value of M's position at $t = 0$ is the expected value of his $t = 1$ position conditional on C's anticipated decision rule:

$$
P_M = (1 - \alpha)(V_0 - \delta'_M)\Phi(\nu_*,\ \xi_*) + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*)[(1 - \alpha)V_J(\tilde{\nu}) - \kappa_M].
$$

[32] See Hermalin & Schwartz, *supra* note 3.

TABLE 1

EVOLUTION OF THE VALUES OF THE CONTROL AND MINORITY POSITIONS

t	Value of P_C at C's Position	Value of P_M at M's Position	$P_C + P_M$
-3	αV_0	$(1-\alpha)V_0$	V_0
-2	$\alpha(V_0 + \delta_C)$	$(1-\alpha)(V_0 - \delta_M)$	V_0
-1	$\alpha(V_0 + \delta_C) + \xi_*$	$(1-\alpha)(V_0 - \delta_M) + \Delta$	$V_0 + \xi_* + \Delta$
0	$\alpha(V_0 + \delta'_C)$ $+ \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi_*)\{V_0 + \tilde{\nu} - [(1-\alpha)V_J(\tilde{\nu}) - \kappa_M]\}$	$(1-\alpha)(V_0 + \delta'_M)\phi(\tilde{\nu}, \xi_*)$ $+ \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi_*)[(1-\alpha)V_J(\tilde{\nu}) - \kappa_M]$	$V_0 + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\{\phi(\tilde{\nu}, \xi_*) \times \tilde{\nu}\}$
1, if freeze out	$V_0 + \tilde{\nu} - (1-\alpha)V_J + \kappa_M$	$(1-\alpha)V_J - \kappa_M$	$V_0 + \tilde{\nu}$
1, if no freeze out	$\alpha(V_0 + \delta'_C)$	$(1-\alpha)(V_0 - \delta'_M)$	V_0

NOTE.—Shareholder invariance: $\alpha\delta_C - (1-\alpha)\delta_M = 0$ and $\alpha\delta'_C - (1-\alpha)\delta'_M = 0$; distribution of merger surplus $\tilde{\nu}$ conditional on C's effort: $\phi(\tilde{\nu}, \xi_*)$; C's equilibrium effort, which maximizes his own expected profit: ξ_*; minimum threshold value of $\tilde{\nu}$ that C requires to consummate freeze-out: ν_*; probability that $\tilde{\nu} > \nu_*$ given equilibrium effort ξ_*: $\Phi(\nu_*, \xi_*) \equiv \int_{\underline{\nu}}^{\nu_*} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi_*)$.

d) At $t = -1$, C chooses his effort level ξ_* to maximize his profit

$$\pi_C(\xi) = \underbrace{P_C(\xi)}_{\text{gain}} - \underbrace{\xi}_{\text{effort}},$$

where

$$P_C(\xi) = \alpha[V_0 + \delta'_C]\Phi(\nu_*, \xi) + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi)\{V_0 + \tilde{\nu} - [(1-\alpha)V_J(\tilde{\nu}) - \kappa_M]\}. \quad (1)$$

e) After an investment of ξ_* effort dollars at $t = -1$, C has committed a total of $\alpha(V_0 + \delta_C) + \xi_*$ dollars to establish his position. If the preinvestment control premium is priced efficiently, C's expected payoff—$P_C(\xi)$ in equation (1)—must equal his total investment. This implies that $P_C(\xi_*) = \alpha(V_0 + \delta_C) + \xi_*$, a condition that determines the endogenous value of the preinvestment risk premium δ_C. (Note that $P_C(\xi)$ depends on δ_C.)

According to the backward-induction argument, C's equilibrium effort ξ_*, his equilibrium freeze-out threshold ν_*, and the equilibrium value of the control premium δ_C depend on judicial valuation policy $V_J(\tilde{\nu})$. Sections III and IV define an optimal judicial valuation policy and derive expressions for the endogenous values of ξ_*, ν_*, and δ_C under such a policy.

III. Optimal Valuation Policy

Which value elements (continuing value, control premium, minority discount, litigation costs) should courts recognize and with what weight? This section characterizes judicial valuation policies V_J that maximize shareholder value.

I will assume that C discloses the true value of his merger surplus $\tilde{\nu}$ to the court in litigation. Of course, if C can get away with credibly understating the value of $\tilde{\nu}$, he will do so to bias the court's valuation in his favor. Discovery procedures and the benefit of hindsight concerning events subsequent to a freeze-out mitigate the ability of C to successfully mislead the court about the true value of $\tilde{\nu}$. However, one cannot rule out the possibility that C successfully hides the true value of $\tilde{\nu}$ some of the time and causes judicial valuation error. (Section IV will discuss the implications of judicial valuation error.)

Assuming that the judge learns the exact value of $\tilde{\nu}$, judicial valuation policy, denoted $V_J(\tilde{\nu})$, may depend on $\tilde{\nu}$ as well as any of the common knowledge parameters. I will call a judicial valuation policy $V_J(\tilde{\nu})$ optimal if it motivates the controlling shareholder to maximize total shareholder value, $P_T \equiv P_C + P_M$, at all dates $t \geq -1$. To maximize P_T, C must exert first-best effort ξ_* at $t = -1$ and, subsequently, make the first-best freeze-out decision (based on the first-best value of the freeze-out threshold ν_*).

The first proposition identifies what optimal judicial valuation policies look like:

PROPOSITION 1. Any optimal judicial valuation policy must be of the form

$$V_{\mathrm{J}} = V_0 + \left(\frac{\kappa_{\mathrm{M}} - \alpha\delta'_{\mathrm{C}}}{1-\alpha}\right) w(\tilde{\nu}),$$

where $w(\cdot)$ satisfies the following conditions: (i) $w(0) = 0$; (ii) $w(\tilde{\nu}) < \tilde{\nu}/(1-\alpha)\ \forall\ \tilde{\nu} \in [\underline{\nu}, \overline{\nu}]$; (iii) $\int_0^{\overline{\nu}} d\tilde{\nu}(\partial\phi/\partial\xi)(\tilde{\nu}, \xi_*)w(\tilde{\nu}) = 0$; and (iv) $\int_0^{\overline{\nu}} d\tilde{\nu}(\partial^2\phi/\partial\xi^2)(\tilde{\nu}, \xi_*)\{\tilde{\nu} - (1-\alpha)w(\tilde{\nu})\} < 0$.

Proof. All proofs are in the Appendix.

Conditions i and ii motivate C to chose the threshold merger surplus value ν_* that maximizes total shareholder value. As shown in the Appendix, this optimal threshold value is $\nu_* = 0$. Accordingly, an optimal valuation policy must motivate C to freeze out when $\tilde{\nu} > \nu_* = 0$, be indifferent when $\tilde{\nu} = 0$, and sell off when $\tilde{\nu} < 0$. Referring to C's $t = 1$ payoffs in the first column of Table 1, one sees that C's incentives are compatible with these goals if, and only if,

$$V_0 + \nu - [(1-\alpha)V_{\mathrm{J}}(\tilde{\nu}) - \kappa_{\mathrm{M}}]\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\alpha[V_0 + \delta'_{\mathrm{C}}] \quad \text{when } \tilde{\nu}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}0.$$

These three inequalities are satisfied if, and only if, conditions i and ii hold. Similarly, conditions iii and iv guarantee that judicial valuation policy motivates C to exert effort ξ_* that maximizes total shareholder value. The Appendix provides details.

Proposition 1 reveals several basic facts. Not all judicial valuation policies are optimal. The simplest plausible policy one can imagine, $V_{\mathrm{J}}(\tilde{\nu}) = V_0$, is not optimal. All optimal policies must adjust continuing value for the minority shareholder's litigation cost κ_{M} and the postinvestment control premium δ'_{C}.

The controlling shareholder's litigation cost κ_{C} is conspicuously absent from the formulas in proposition 1.[33] According to proposition 1, an optimal valuation policy does not need to consider C's litigation cost. Only the minority shareholder's litigation cost plays a role. This is because what the minority shareholder, M, anticipates as his potential litigation cost influences the freeze-out price he is willing to accept. Therefore, C rationally adjusts the freeze-out price he offers for M's potential litigation expense. On the other hand, C's anticipated litigation cost is irrelevant because M rationally

[33] The minority discount δ'_{M} is also absent, but it is not an independent parameter because, by shareholder invariance, $\delta'_{\mathrm{M}} = [\alpha/(1-\alpha)]\delta'_{\mathrm{C}}$.

accepts C's offer and avoids litigation, which means that C does not incur any litigation costs. In addition, the optimal valuation policy depends on the postinvestment control premium δ'_C rather than the preinvestment premium δ_C. This is because δ_C is a sunk cost once C's control is established at $t = -2$ and plays no role in C's decision making after that. On the other hand, δ'_C reflects the value of C's exit option and influences whether C seeks freezeout or not.

A simple way to meet the conditions of proposition 1 and achieve optimality is with a linear judicial valuation policy. Indeed, the optimal linear policy is unique and cannot depend on $\tilde{\nu}$.

COROLLARY 1. The unique optimal linear judicial valuation policy $V_J(\tilde{\nu})$ is $V_J = V_0 + [(\kappa_M - \alpha\delta'_C)/(1-\alpha)]$, which, by shareholder invariance, can also be written as $V_J = V_0 - \delta'_M + [\kappa_M/(1-\alpha)]$.

Corollary 1 says that judges should award minority shareholders their litigation cost κ_M and exact a minority discount δ'_M. However, this new minority discount is smaller than the preinvestment minority discount that the market accorded minority shares at $t < 0$. Thus, in addition to litigation expense, minority shareholders should get a premium over the preinvestment value of their shares. Specifically, the preinvestment value of minority shares on a whole-firm basis is, by definition, $V_0 - \delta_M$. Corollary 1 implies that judges optimally should grant minorities a difference equal to $\delta_M - \delta'_M$ over the preinvestment market price. Proposition 2 will relate this difference to the optimal effort level of C and show that $\delta_M - \delta'_M$ is a positive number.

According to proposition 1, optimal valuation policies are not unique and may depend on the merger surplus via a nonlinear function $w(\tilde{\nu})$. An example of a nonlinear optimal policy is when $\phi(\tilde{\nu}, \xi) = e^{-\tilde{\nu}^2/2\xi}/\sqrt{2\pi\xi}$ with $\nu \in (-\infty, \infty)$, the first-best effort level is $\xi_* = 1/8\pi$. Any valuation function of the form $V_J = V_0 + [(\kappa_M - \alpha\delta'_C)/(1-\alpha)] + \tilde{\nu}(1-2\tilde{\nu})/\lambda(1-\alpha)$ for any λ greater than one is optimal and satisfies conditions i–iv in proposition 1. These valuation policies build merger surplus into the appraisal remedy award in a nonlinear way. They do not motivate C to deviate from first-best effort because V_J is marginally independent of effort in ex ante expectation.

Under an optimal valuation policy, it is easy to show (simplifying the expressions in Table 1) that the value of C and M's shares at $t = 0$ are

$$P_C = \alpha(V_0 + \delta'_C) + \int_0^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi_*)\tilde{\nu} - (1-\alpha)\Psi_w(\xi_*)$$

and

$$P_M = (1-\alpha)(V_0 - \delta'_M) + (1-\alpha)\Psi_w(\xi_*),$$

where $\Psi_w(\xi) \equiv \int_0^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi)w(\tilde{\nu})$ is the amount of merger surplus, in expectation, reallocated from C to M by an optimal (nonlinear) valuation policy.

The first term of P_C is the value of C's shares if he does not go through with the freeze-out, the second term is the expected merger surplus in an optimal valuation policy regime, and the final term is the fraction of the merger surplus the optimal valuation policy reallocates from C to M. Total shareholder value, $P_T = P_C + P_M = V_0 + \int_0^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu}, \xi_*)\tilde{\nu}$, does not depend on $\Psi_w(\xi)$.

The literature often states that an optimal valuation policy cannot allocate merger surplus away from the acquirer without causing him to deviate from first-best effort.[34] As the preceding discussion shows, this is false: the optimal valuation policies in proposition 1, which motivate C to exert first-best effort, do not require merger surplus $\Psi_w(\xi_*)$ to be zero. Condition iii in proposition 1 says that C will deviate from first-best effort as long as $(\partial \Psi_w / \partial \xi)_{|\xi_* = 0} = 0$, which does not necessarily imply that $\Psi_w(\xi_*) = 0$. Thus, with a proper choice of $w(\tilde{\nu})$ (the nonlinear part of the optimal judicial valuation policy), it is possible to reallocate some of the merger surplus from C to M without distorting C's effort. In other words,

COROLLARY 2. An optimal judicial valuation policy may reallocate expected merger surplus from C to M provided that the amount reallocated is marginally independent of C's effort level in expectation.

The reason why corollary 2 is true is because C exerts his effort on an ex ante basis. Hence, he responds to ex ante incentives rather than ex post realizations. Therefore, optimal judicial valuation policy may reallocate merger surplus from C to M on an ex post basis, but only in a nonlinear way that does not reallocate the expected value of merger surplus conditional on C's effort level.

The Delaware appraisal remedy statute advises chancellors to appraise all shares "exclusive of any element of value arising from the accomplishment or expectation of the merger or consolidation."[35] In the sense that Delaware General Corporations Law, section 262, advises chancellors to appraise minority shares at their premerger value, this rule is optimal. However, Delaware courts try to follow the pro rata doctrine laid down in *Cavalier Oil,* which formally forbids discounts in freeze-out appraisals.[36] On the other hand, while pursuing pro rata appraisal, the courts frequently permit implicit discounts even though they do not recognize that they are doing so.[37] Hence, appraisal outcomes in Delaware may be more consistent with proposition 1 and more optimal than what one might expect on the basis of formal Delaware law as enunciated in *Cavalier Oil.*

[34] For example, proposition 2 in Hermalin & Schwartz, *supra* note 3, which follows because they consider a more restricted space of valuation policies.

[35] Del. Code Ann. tit. 8, § 262(h) (2004).

[36] See Cavalier Oil Corp. v. Harnett, 564 A.2d 1137, 1144 (Del. 1989).

[37] Coates, *supra* note 1.

IV. Preinvestment Control Premium

The preinvestment control premium δ_C is the value of C's option to invest effort to create and realize a merger surplus. As such, δ_C incorporates the expected payoff of control, which in turn depends on judicial valuation policy. Proposition 2 gives an expression for δ_C in the unique optimal linear judicial valuation regime.

Proposition 2. In an optimal valuation regime where the judicial valuation policy is $V_J(\tilde{\nu}) = V_0 + [(\kappa_M - \alpha\delta'_C)/(1-\alpha)]$, the preinvestment control premium is

$$\delta_C = \delta'_C + \frac{1}{\alpha}\left[\int_0^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\,\xi_*)\tilde{\nu} - \xi_*\right],$$

where the effort level ξ_* (which equals first best) is determined by

$$1 = \int_0^{\bar{\nu}} d\tilde{\nu}\frac{\partial\phi}{\partial\xi}(\tilde{\nu},\,\xi_*)\tilde{\nu}.$$

The commensurate preinvestment minority discount is $\delta_M = [\alpha/(1-\alpha)]\delta_C$.

Proposition 2 reveals that, assuming the optimal linear valuation policy, the preinvestment control premium has three components: δ'_C, the postinvestment control premium; the merger surplus in expectation, given optimal effort, grossed up by C's ownership fraction; and the optimal effort, also grossed up by C's ownership fraction. Since $\int_0^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\,\xi_*)\tilde{\nu} - \xi_*$ is C's expected net gain from his effort investment, it must be positive—otherwise, C would not rationally make the investment. Therefore,

Corollary 3. The preinvestment control premium exceeds the postinvestment control premium; that is, $\delta_C > \delta'_C$.

The control premium spread, $\alpha(\delta_C - \delta'_C)$, is the expected cost of C's roundtrip if he gains control and then resells without exploring a merger. If this spread were negative, C could make arbitrage profits by repeating the roundtrip ad infinitum. In the example where $\phi(\tilde{\nu},\,\xi) = e^{-\tilde{\nu}^2/2\xi}/\sqrt{2\pi\xi}$, $\xi_* = 1/8\pi$, which equals the first-best effort level, and $\alpha(\delta_C - \delta'_C) = +1/8\pi$.

Corollary 3 provides a possible explanation for why judicial valuations in practice tend to exceed last-traded prices in Delaware. The spread between what an optimal valuation policy recognizes and the market value of M's minority shares at $t = -2$ is $(1-\alpha)(\delta_M - \delta'_M)$. Since shareholder invariance implies $(1-\alpha)(\delta_M - \delta'_M) = \alpha(\delta_C - \delta'_C)$, and since corollary 3 says that $\alpha(\delta_C - \delta'_C) > 0$, this spread is positive. In other words, under an optimal valuation policy, the court should appraise minority shares at a higher value than what they traded for, $(1-\alpha)(V_0 - \delta_M)$, when C originally assumed control.

V. Consequences of Misvaluation

The practice of valuation is an inexact art, not a precise science.[38] Moreover, if a case goes to trial, the practical complexities applied to a large corporation should not be underestimated. *Technicolor* began with the original merger transactions in 1982 and has dragged on for the last 2 decades.[39] The Delaware Supreme Court invalidated the original appraisal because it did not take into account value-enhancing activities taken by the acquirer between the date of the merger agreement and the merger date.[40] Given the imprecision of valuation estimates and the duration and complexity of appraisal proceedings, what are the effects of judicial valuation error?

According to corollary 1, a linear valuation policy is optimal if, and only if, it looks like $V_J = V_0 + [(\kappa_M - \alpha\delta_C')/(1-\alpha)]$. What happens if courts deviate from an optimal policy and either systematically over- or underestimate continuing value, improperly award merger surplus, or make random, unpredictable valuation mistakes? Do judicial valuation errors hurt minority more than controlling shareholders? What does bias do to the control premium?

Consider the following suboptimal valuation policy:

$$V_J(\tilde{\nu}) = V_0 + \frac{\kappa_M - \alpha\delta_C'}{1-\alpha} + \eta\tilde{\nu} + e + \tilde{u},$$

where η and e are always common-knowledge constants and $\tilde{u} \sim N(0, \sigma)$ is a mean-zero, standard deviation σ, normally distributed random variable that realizes a value only on litigation. For technical reasons, I will confine η and e to the intervals $\eta \in (-\infty, 1/(1-\alpha))$ and $e \in [1-(1-\alpha)\eta](1-\alpha) \times [\underline{\nu}, \overline{\nu}]$. I will say judicial valuation policy has a type η, type e, or type $\tilde{u}$ error whenever, respectively, $\eta \neq 0$, $e \neq 0$, or $\sigma \neq 0$. Type η error reflects judicial reallocation of the ex post merger surplus from C to M. Type e error represents systematic bias in the appraisal of continuing value, prejudgment interest, litigation cost, control premium, or some ex ante measure of the expected merger surplus. Type $\tilde{u}$ error represents the unpredictable chance component of litigation caused by judicial mistakes and other factors outside the parties' control.

[38] For example, see Steven N. Kaplan & Richard S. Ruback, The Valuation of Cash Flow Forecasts: An Empirical Analysis, 50 J. Fin. 1059 (1995); and Kenton K. Yee, Perspectives: Combining Value Estimates to Increase Accuracy, Fin. Analysts J., July–August 2004, at 23.

[39] Cede & Co. v. Technicolor, 875 A.2d 602 (Del. 2005).

[40] Chancellor Chandler, the highly respected Delaware trial judge handling this case, describes the situation aptly: "The long history of the dispute between these parties is well known, not only to the parties, but also to all those who are familiar with Delaware corporate law. . . . As these parties launch their final campaign, the conflict between them not only appears to sustain both combatants, but has in part come to define them." Cede & Co. v. Technicolor, Inc., Civil Action No. 7129, 2001 Del. Ch. LEXIS 62, at *2 (Del. Ch. 2001).

Since $\tilde{u}$ is unpredictable, the market, C, and M would make their prelitigation choices on the basis of its expectation value, which is zero.[41] Hence, when C and M are risk neutral (or sufficiently diversified), type $\tilde{u}$ errors do not matter and can be dropped. The irrelevance of type $\tilde{u}$ adjudication errors when parties are risk neutral has been noted by many authors in differing contexts.[42]

On the other hand, types η and e errors cause suboptimal behavior that reduces shareholder value. The value V_J is suboptimal because $w(\tilde{\nu}) = \eta\tilde{\nu} + e$ does not generally satisfy conditions iii and iv in proposition 1 when either $\eta \neq 0$ or $e \neq 0$. Moreover, it turns out that type e and type η bias affect C's behavior along different dimensions and so cannot counteract each other. The complete proposition says,

PROPOSITION 3. Type $\tilde{u}$ errors do not matter. Absent type e error, type η error distorts C's freeze-out decision but not his effort level. On the other hand, type e error distorts both C's freeze-out decision and his effort level whether or not there is type η error.

An example illustrates proposition 3. Suppose that $\phi(\tilde{\nu}, \xi) = e^{-\tilde{\nu}^2/2\xi}/\sqrt{2\pi\xi}$ with $\nu \in (-\infty, \infty)$. Recall that in this example the first-best merger surplus threshold is $\nu_* = 0$, the first-best effort level is $\xi_* = 1/8\pi$, and the control premium is $\delta_C = \delta_C' + 1/8\pi\alpha$. Following the expression in the proof of proposition 3, judicial valuation bias shifts ν_* and ξ_* away from their first-best values so

$$\nu_*(e, \eta) = \frac{(1-\alpha)e}{1-(1-\alpha)\eta},$$

and $\xi_*(e, \eta)$ is the unique solution of $e^{-\nu_*^2/\xi_*(e,\eta)} = 8\pi[1-(1-\alpha)\eta]^2\xi_*(e, \eta)$,[43] which implies

$$\xi_*(e, \eta) = \frac{e^{-\nu_*^2/\xi_*(e,\eta)}}{[1-(1-\alpha)\eta]^2} \times \xi_{FB}.$$

Since the prefactor of ξ_{FB} may be either greater than or less than one, valuation bias may either increase or decrease C's investment of effort from first best.

[41] I am assuming that all shareholders are sufficiently diversified to behave as if they are risk neutral. If M were risk averse, risk bearing would reduce his utility for V_J. In this case, the error parameter e may be interpreted as having a risk-bearing cost component since a risk-averse shareholder's utility for the expected judicial award when $\sigma \neq 0$ equals its certainty equivalent, which is tantamount to V_J plus a shift in overall constant e.

[42] A. Mitchell Polinsky & Steven Shavell, Legal Error, Litigation, and the Incentive to Obey the Law, 5 J. L. Econ. & Org. 99 (1989).

[43] One can see that the solution is unique as follows. The left-hand side of the equation increases monotonically as a function of ξ_* from zero to one as ξ_* increases from zero to infinity. The right-hand side increases linearly from zero to infinity as ξ_* increases from zero to infinity. Because both sides are monotonic, the left-hand side equals the right-hand side at a unique value of ξ_*.

As long as $\eta < 1/(1 - \alpha)$, the sign of e determines whether ν_* exceeds or falls below the first-best threshold value of $\nu_{FB} = 0$. The control premium equals

$$\delta_C = \delta'_C + \frac{1}{\alpha}\left[\int_{\nu(e,\eta)}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*(e,\ \eta))\tilde{\nu} - \xi_*(e,\ \eta)\right].$$

This example shows that if e equals zero, ν_* achieves the first-best value $\nu_* = 0$ regardless of type η error. Type e error affects shareholder value in two ways. First, it shifts the threshold merger surplus, ν_*, away from the first-best level of zero. Positive or negative bias e induces a corresponding positive or negative value of ν_*, which tends to reduce the merger surplus and, accordingly, total shareholder value and the preinvestment control premium. Second, type e error causes C to choose a non-first-best effort level ξ_*. Examination of the defining equation for $\xi_*(e, \eta)$ shows that e bias alone or η bias alone is enough to cause deviations from ξ_{FB}. This is because both e and η error reallocate expected merger surplus from C to M.

VI. Appraisal in Delaware

As reviewed by John Coates, Delaware case law has evolved a complex web of valuation doctrines.[44] This section describes some of these doctrines and whether they meet the standards of optimality under proposition 1.

A. Market Inefficiency

A fundamental premise of judicial valuation is that markets are subject to inefficiencies and, accordingly, judges are justified if they choose to readjust or ignore market price in determining V_J.[45] While academic finance has been slow to accept the possibility of inefficient markets, this belief traces back in the courts to an influential Court of Chancery opinion issued (not surprisingly) during the Great Depression. The chancellor explained: "Markets are known to gyrate in a single day. . . . Even when conditions are normal and no economic forces at work unduly to exalt or depress the financial hopes of man, market quotations are not safe to accept as unerring expressions of value."[46] A related theme running through appraisal opinions is that valuation is an art rather than a science, and so trial judges should retain discretion rather than be locked into any mechanical valuation formula. Commenting on how judges who use the Delaware Block valuation method should weigh market price, net asset value, and capitalized earnings, the Delaware Supreme Court decided that "no rule of thumb is applicable to weighting;

[44] Coates, *supra* note 1.

[45] Yee, *supra* note 38, at 23.

[46] Chicago Corp. v. Munds, 172 A. 452, 455 (Del. Ch. 1934).

rather, the rule becomes one of entire fairness and sound reasoning . . . to the particular facts of each case."[47]

If the market price of minority shares is inefficient, should judicial valuation policy seek to correct the inefficiency? The analysis is subtle. Replacing market price with an independent valuation estimate not only gives rise to undesirable type η and type e errors; the procedure provides desirable incentives for shareholders to predict what that independent estimate will be. By definition, $V_0 - \delta'_M$ in corollary 1 equals the last-traded market price of M's shares before freeze-out, whether market price is efficient or not. Suppose the firm's continuing value from independent fundamental analysis is $V^* \neq V_0$, so the market is pricing minority shares incorrectly (or on the basis of inferior information relative to C's inside information). An otherwise optimal valuation policy that replaces V_0 with V^* deviates from the optimal policy in corollary 1 by the amount $V^* - V_0$. Depending on the nature of market inefficiency, this difference is tantamount to the type η and type e errors described in proposition 3. In this sense, valuation policy that ignores market price and assesses V^* independently is suboptimal. On the other hand, replacing an inefficient market price with a truer estimate provides incentives for all shareholders to figure out what V^* is, which has the fringe benefit of ultimately making market prices more efficient. Judicial valuation policy that highlights fundamental analysis and ignores bubble prices will encourage market prices to be more efficient. In deciding whether to use V^* or V_0, the court must balance the benefits of enticing market participants to improve market efficiency against the harm of motivating suboptimal freeze-outs whose only purpose is to allow C to capture market-mispricing surplus.

B. Share Appraisal Rule versus Pro Rata Doctrine

Delaware General Corporations Law advises, "[T]he Court shall appraise the shares, determining their fair value exclusive of any element of value arising from the accomplishment or expectation of the merger or consolidation."[48] The word "share" was originally incorporated in 1899, when Delaware General Corporations Law was originally adopted. The share appraisal rule suggests that the court should appraise the value of minority shares, that is, recognize a minority discount.

If the share appraisal rule is supplemented with a collateral award of prejudgment interest and attorneys fees, which it frequently is in Delaware, this rule is consistent with proposition 1. Share appraisal stipulates that $V_J = V_0 - \delta'_M$. This means that M's award under share appraisal is $(1 - \alpha)(V_0 - \delta'_M) = (1 - \alpha)\{V_0 - [\alpha/(1 - \alpha)]\delta'_C\}$, which is consistent with what

[47] Bell v. Kirby Lumber Corp., 413 A.2d 137, 143 (Del. 1980).

[48] Del. Code Ann. tit. 8, § 262(h).

M gets under the linear valuation policy in corollary 1 (ignoring prejudgment interest and attorneys fees).

Case law in Delaware has frequently deviated from share appraisal in favor of the pro rata doctrine. Under the pro rata doctrine, the value of every share, minority or controlling, is the continuing value of the firm divided by the number of outstanding shares. Hence, a minority share is worth exactly as much as a controlling share. In *Tri-Continental*, the Delaware Supreme Court pronounced that the aim of appraisal is to assess continuing value, "that which has been taken from [the shareholder], viz., his proportionate interest in a going concern." [49] In *Cavalier Oil*, the court reiterated that the corporation should be valued "as an entity" and that the objective of appraisal is "to value the corporation itself, as distinguished from a specific fraction of its shares as they may exist in the hands of a particular shareholder."[50] Accordingly, the court held that the dissenter shares were worth a "proportionate interest" in the going concern and that no minority discount should be applied.[51] The court explained that minority discounts unfairly penalize minority shareholders, provide incentives for a controlling shareholder to seek unfair windfalls by cashing out minority shareholders, and inject undue speculation in the appraisal process.[52]

According to proposition 1, pro rata valuation is suboptimal policy because it does not adjust for the preinvestment control premium or litigation cost. However, because implementation of the pro rata doctrine is complicated by estimation issues that may allow discounts and premiums in via the back door, it is not clear if Delaware's appraisals incorporate implicit discounts and premiums even under pro rata valuation. First, since going concern value incorporates corporate-level discounts and premiums (for example, differences between liquidation value and continuing value), the pro rata doctrine allows for corporate-level discounts. So, in practice, a Delaware court will admit a discount if a party can successfully label it as being corporate rather than shareholder level.[53] Second, any valuation methodology that relies on the market prices or rates of returns of comparable firms, such as the method of comparables, or CAPM, implicitly builds in minority discounts through market price, which disproportionately reflects the trading of minority shares. Pro rata estimation requires the judge to proactively take out the minority discount from market price by imposing a legal premium, which judges sometimes neglect to do.

Share appraisal has the added attraction of being more straightforward to implement than pro rata valuation. Since market prices of other minority

[49] Tri-Continental Corp. v. Battye, 74 A.2d 71, 72 (Del. 1950)

[50] Cavalier Oil Corp., 564 A.2d 1144.

[51] *Id.*

[52] *Id.* at 1144–45.

[53] Coates, *supra* note 1.

shares, by definition, incorporate the market value of any minority discount, the method of comparables automatically incorporates the required discount, and no additional adjustments are required. On the other hand, when suitable comparables are unavailable (as may be the case when one is valuing a closely held corporation) or when the method of choice is discounted cash flow, a court may need to actively impose the suitable discount or control premium. In this case, share appraisal is not straightforward since the methods for assessing the suitable adjustment is ad hoc and highly controversial.

C. Merger Surplus Exclusion Rule

Delaware Code section 262(h) advises that appraisal shall determine fair value "exclusive of any element of value arising from the accomplishment or expectation of the merger or consolidation."[54] In the context of proposition 1, this means that Delaware Code requires that $w(\tilde{v}) = 0$, which is compatible with optimal valuation by corollary 1. Hence, section 262 is consistent with the spirit of optimal linear valuation.

However, by redefining the line between merger surpluses and continuing values, Delaware courts have not acted to forbid merger surplus reallocations. In *Technicolor,* the Delaware Supreme Court overturned an appraisal result because the chancellor did not incorporate the value of the surplus-enhancing actions that the controlling shareholder took after contemplating freeze-out but before consummation of the process ($t = -1$ in Figure 1).[55] In particular, the controlling shareholder developed a strategy to liquidate the target firm and undertook preliminary negotiations with potential buyers.[56] The court held that these actions altered not the merger surplus but the basic nature of the target firm and, therefore, its continuing value V_0, which must be appraised.[57]

Technicolor seems to have restricted the scope of section 262 by interpreting "expectation of the merger" narrowly. In particular, the court held that plans and strategies the acquirer developed and began implementing in the 2 months between the merger agreement and the freeze-out date constituted realizations and not expectations.[58] Accordingly, the merger surplus exclusion rule did not apply, and the value impact of the acquirer's plans and strategies should be incorporated into V_J via V_0. Since delaying value-enhancing improvements to win a greater allocation of a fixed pie in court is never optimal, *Technicolor*'s reclassification is suboptimal because it pro-

[54] Del. Code Ann. tit. 8, § 262(h).
[55] Cede & Co v. Technicolor, Inc., 684 A.2d 289, 290 (Del. 1996)
[56] *Id.* at 293–94.
[57] *Id.* at 299.
[58] *Id.*

vides perverse incentives for the controlling shareholder to delay implementation of his freeze-out plan to get a more favorable valuation in court.

VII. Conclusion

This paper characterizes the rational expectations equilibrium in which controlling and minority share prices and the court's appraisal remedy valuation policy are endogenously determined in an integrated framework. The model identifies a set of optimal judicial valuation policies that would motivate shareholders to make optimal investment and freeze-out choices. The paper then discusses the extent to which Delaware appraisal law is optimal and characterizes the ways in which nonoptimal policies distort the behavior of controlling shareholders.

Proposition 1, the main analytical result, indicates that to motivate the controlling shareholder to make the right freeze-out choice, judges should discount minority shares by the smaller postinvestment (δ'_M) rather than preinvestment (δ_M) minority discount. Aside from prejudgment interest and other litigation costs, the unique optimal linear valuation policy is to appraise minority shares as being worth continuing value (on a whole-firm basis) minus δ'_M. Moreover, it is possible to design (nonlinear) valuation policies that award some of the controlling shareholder's ex post merger surplus to minority shareholders without inducing the controlling shareholder to invest suboptimally.

The analysis also clarifies the incentive effects of reallocating litigation costs. While the appraisal remedy should reimburse minority shareholders' prejudgment interest and litigation costs, the analysis suggests that the courts should not adjust for the litigation cost of the controlling shareholder since there is no reason for minority shareholders to seek appraisal if the controlling shareholder acts rationally. Indeed, consistent with the observed paucity of appraisal cases, the model suggests that appraisal never occurs when all shareholders act rationally.

These results are largely compatible, if not with existing judicial valuation policy, at least with ideas in the evolving institutional literature. The share appraisal rule and the merger surplus exclusion rule are consistent with optimal judicial valuation, while Delaware law's pro rata doctrine is not. The bottom line is that courts should not equate the value of minority shares with market price, because minority shares should be discounted to motivate controlling shareholders to exert optimal effort. Consistent with the implications of this model, Delaware courts have always uniformly rejected the notion that market price should be the sole determinant of appraisal value and have long struggled to find the appropriate level of control premiums and minority discounts to award shareholders.

APPENDIX

Proof of Proposition 1

This proof employs backward induction. At the final stage, whatever V_J may be, if C decides to freeze out M's shares, his optimal offer to M is an amount P_{FO} that exactly reproduces what M could obtain in appraisal. This is because, since $\kappa_C > \kappa_M$ by assumption, it is favorable for C to avoid litigation. Hence,

$$P_{FO} = (1 - \alpha)V_J - \kappa_M.$$

M would reject any amount less than P_{FO} in favor of appraisal,[59] in which case M would obtain P_{FO} and C would incur the additional expense of litigation. Of course, C would not rationally want to offer any more than P_{FO}.

C would choose to freeze out Corp if, and only if, what he gains from merger exceeds what he gains from selling his shares to the market (see Figure 2):

$$V_0 + \tilde{\nu} - [(1 - \alpha)V_J - \kappa_M] > \alpha(V_0 + \delta'_C).$$

If what he gains from merger equals what he gains from divesting, C is indifferent and flips a coin; if what he gains from merger falls below that of divestiture, C sells off. Hence, the value of C's Corp shares after he learns the value of his merger surplus $\tilde{\nu}$ is $\max\{V_0 + \tilde{\nu} - [(1 - \alpha)V_J - \kappa_M], \alpha(V_0 + \delta'_C)\}$. Accordingly, if V_J depends on $\tilde{\nu}$, the value of C's shares at $t = 0$—before he learns the value of $\tilde{\nu}$—is

$$\begin{aligned} P_C &= E[\max\{V_0 + \tilde{\nu} - [(1 - \alpha)V_J(\tilde{\nu}) - \kappa_M],\ \alpha(V_0 + \delta'_C)\}] \\ &= \alpha[V_0 + \delta'_C]\Phi(\nu_*,\ \xi_*) + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi_*)\{V_0 + \tilde{\nu} - [(1 - \alpha)V_J(\tilde{\nu}) - \kappa_M]\}, \end{aligned}$$

where ν_*, the cutoff value of $\tilde{\nu}$ below which C sells off and above which C buys out, is determined by solving

$$V_0 + \nu_* - [(1 - \alpha)V_J(\nu_*) - \kappa_M] - \alpha[V_0 + \delta'_C] = 0,$$

and $\Phi(z, \xi_*) \equiv \int_{\underline{\nu}}^{z} d\tilde{\nu}\,\phi(\tilde{\nu}, \xi_*)$ is the probability that C sells off.

Likewise, M's payoff is

$$\begin{cases} (1 - \alpha)V_J - \kappa_M & \text{if Cseeks buyout,} \\ (1 - \alpha)(V_0 - \delta'_M) & \text{if C sells off.} \end{cases}$$

Therefore, the $t = 0$ value of M's shares is

$$P_M = (1 - \alpha)(V_0 - \delta'_M)\Phi(\nu_*,\ \xi_*) + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi_*)[(1 - \alpha)V_J(\tilde{\nu}) - \kappa_M].$$

[59] I assume that M knows the value of $\tilde{\nu}$ when he gets C's offer for the following reasons. If Corp is publicly traded, Securities and Exchange Commission Rule 13e-3 and Schedule 13E-3 require C to file public documents explaining why he is seeking a freeze-out and disclosing his estimate of $\tilde{\nu}$. In any case, if C has verifiable information about $\tilde{\nu}$ that he would be required to disclose in an appraisal hearing, he has no choice but to verifiably disclose it to M when he makes his offer, since M infers larger values of $\tilde{\nu}$ if C remains silent. See Paul Milgrom & John Roberts, Relying on the Information of Interested Parties, 17 RAND J. Econ. 18 (1986).

Applying shareholder invariance, $\alpha\,\delta'_{\mathrm{C}} - (1-\alpha)\,\delta'_{\mathrm{M}} = 0$, total shareholder value, $P_{\mathrm{T}} = P_{\mathrm{C}} + P_{\mathrm{M}}$, is seen to equal

$$P_{\mathrm{T}} = V_0 + \int_{\nu_*}^{\bar{\nu}} d\tilde{\nu}\,\{\phi(\tilde{\nu},\ \xi_*) \times \tilde{\nu}\}.$$

Given this expression for P_{T}, it is easy to see that the value of ν_* that maximizes total shareholder value is zero. When $\nu_* < 0$, the P_{T} integrand becomes negative, which decreases P_{T}. On the other hand, when $\nu_* > 0$, the P_{T} integrand is always positive. Hence, the maximum value of the integral occurs when $\nu_* = 0$: since the P_{T} integrand is always positive when $\nu_* > 0$, increasing ν_* above zero reduces the range of the integral, which reduces the value of the integral and P_{T}. Since $\nu_* = 0$ maximizes total shareholder value, an optimal valuation policy must motivate C to freeze out when $\tilde{\nu} > \nu_* = 0$, be indifferent when $\tilde{\nu} = 0$, and sell off when $\tilde{\nu} < 0$. C's payoffs are compatible with these goals if, and only if,

$$V_0 + \nu - [(1-\alpha)V_{\mathrm{J}}(\tilde{\nu}) - \kappa_{\mathrm{M}}]\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\alpha[V_0 + \delta'_{\mathrm{C}}] \quad \text{when } \tilde{\nu}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}0.$$

These equalities are satisfied by any function of the form $V_{\mathrm{J}} = V_0 + [(\kappa_{\mathrm{M}} - \alpha\delta'_{\mathrm{C}})/(1-\alpha)] + w(\tilde{\nu})$, where $w(0) = 0$ and $w(\tilde{\nu}) < \tilde{\nu}/(1-\alpha)\ \forall\ \tilde{\nu} \in [\underline{\nu},\ \bar{\nu}]$. Hence, any optimal valuation policy must be of this form.

An optimal valuation policy must also motivate C to exert the optimal amount of effort. This first-best effort level, denoted ξ_{FB}, strikes the optimal balance between shareholder gain and the cost of effort. Accordingly,

$$\xi_{\mathrm{FB}} = \mathrm{argmax}_{\xi}\left\{\underbrace{V_0 + \int_0^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi)\,\tilde{\nu}}_{\text{shareholder value at optimal policy}} - \underbrace{\xi}_{\text{cost of effort}}\right\}.$$

C is generally not motivated to exert first-best effort since he cares only about his own gains. At $t = -1$, C chooses his effort level to maximize his expected return on investment,

$$\pi_{\mathrm{C}}(\xi) = \underbrace{P_{\mathrm{C}}(\xi)}_{\text{gain}} - \underbrace{\xi}_{\text{effort}},$$

where

$$P_{\mathrm{C}}(\xi) = \alpha[V_0 + \delta'_{\mathrm{C}}]\Phi(0,\ \xi) + \int_0^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi)\{V_0 + \tilde{\nu} - [(1-\alpha)V_{\mathrm{J}}(\tilde{\nu}) - \kappa_{\mathrm{M}}]\}$$

and

$$V_{\mathrm{J}} = V_0 + \left(\frac{\kappa_{\mathrm{M}} - \alpha\delta'_{\mathrm{C}}}{1-\alpha}\right) + w(\tilde{\nu}).$$

Hence, the effort level C actually exerts is

$$\xi_* = \mathrm{argmax}_{\xi}\{\pi_{\mathrm{C}}(\xi)\}.$$

Solving these optimization programs for ξ_* and ξ_{FB} yields that ξ_* equals ξ_{FB} if, and only if, $w(\tilde{\nu})$ satisfies conditions i–iv stated in proposition 1. Q.E.D.

Proof of Corollary 1

If V_J is linear, then $w(\tilde{\nu}) = \eta\tilde{\nu}$ for some constant η. Given an optimal judicial valuation policy, C's optimization program for his best effort yields $1 = \int_0^{\bar{\nu}} d\tilde{\nu}(\partial\phi/\partial\xi)(\tilde{\nu},\ \xi_*)\tilde{\nu}$, which defines ξ_*, and so is identically true. But this identity conflicts with condition iii in proposition 1 if $w(\tilde{\nu}) = \eta\tilde{\nu}$ unless η equals zero. Q.E.D.

Proof of Proposition 2

The optimal effort level is, by definition, $\xi_* = \xi_{FB}$, where the first-best effort is characterized in the proof of proposition 1. After an investment of ξ_* effort dollars, C has committed a total of $\alpha(V_0 + \delta_C) + \xi_*$ dollars into his Corp shares. C expects a net payoff from this investment of $\alpha(V_0 + \delta'_C) + \int_0^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi_*)\tilde{\nu}$ dollars. If the preinvestment control premium is priced efficiently, C's expected net payoff must equal his original investment, which means

$$\alpha(V_0 + \delta_C) + \xi_* = \alpha(V_0 + \delta'_C) + \int_0^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi_*)\tilde{\nu}.$$

Rearranging yields the expression for δ_C in proposition 2. Q.E.D.

Proof of Proposition 3

Following the proof of proposition 1, solving $V_0 + \nu_* - [(1-\alpha)V_J(\nu_*) - \kappa_M] - \alpha[V_0 + \delta'_C] = 0$ with the biased judicial valuation policy yields the merger surplus cutoff

$$\nu_*(e,\ \eta) = \frac{(1-\alpha)e}{1-(1-\alpha)\eta}.$$

C's effort level $\xi_*(e,\ \eta)$ is the solution of[60]

$$\frac{1}{1-(1-\alpha)\eta} = \int_{[(1-\alpha)e]/[1-(1-\alpha)\eta]}^{\bar{\nu}} d\tilde{\nu}\frac{\partial\phi}{\partial\xi}(\tilde{\nu},\ \xi_*(e,\ \eta))\left\{\tilde{\nu} - \frac{(1-\alpha)e}{1-(1-\alpha)\eta}\right\}.$$

Following the same logic as the proof of proposition 1, given an effort level ξ, the shareholders' expected payoffs are

$$P_C(\xi;\ e,\ \eta) = \alpha[V_0 + \delta'_C] - (1-\alpha)[1 - \Phi(\nu_*, \xi_*(e,\ \eta))]e$$
$$+\ [1-(1-\alpha)\eta]\int_{\nu_*(e,\eta)}^{\bar{\nu}} d\tilde{\nu}\,\phi(\tilde{\nu},\ \xi_*(e,\ \eta))\tilde{\nu}$$

[60] To guarantee that ξ_* solves C's optimization program, the following technical constraint is also required:

$$\int_{[(1-\alpha)e]/[1-(1-\alpha)\eta]}^{\bar{\nu}} d\tilde{\nu}\frac{\partial^2\phi}{\partial\xi^2}(\tilde{\nu},\ \xi_*(e,\ \eta))\left\{\tilde{\nu} - \frac{(1-\alpha)e}{1-(1-\alpha)\eta}\right\} < 0.$$

and

$$P_{\mathrm{M}}(\xi;\ e,\ \eta) = (1-\alpha)[V_0 - \delta'_{\mathrm{M}}] + (1-\alpha)[1 - \Phi(\nu_*,\ \xi_*(e,\ \eta))]e$$

$$+\ (1-\alpha)\eta \int_{\nu_*(e,\eta)}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*(e,\ \eta))\tilde{\nu},$$

where $\Phi(\nu_*,\ \xi_*) \equiv \int_{\underline{\nu}}^{\nu_*} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*)$ is the probability that C sells off. The expected payout, P_{C}, motivates C to invest an effort of $\xi_*(e,\ \eta) = \mathrm{argmax}_{\xi}\{P_{\mathrm{C}}(\xi;\ e,\ \eta) - \xi\}$. Total shareholder value at this effort level equals

$$P_T(e,\ \eta) = P_{\mathrm{C}}(\xi_*;\ e,\ \eta) + P_{\mathrm{M}}(\xi_*;\ e,\ \eta) = V_0 + \int_{\nu_*(e,\eta)}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*(e,\ \eta))\, \tilde{\nu}.$$

Following the proof of proposition 2, the preinvestment control premium under the biased valuation policy solves

$$\alpha(V_0 + \delta_{\mathrm{C}}) + \xi_*(e,\ \eta) = \alpha(V_0 + \delta'_{\mathrm{C}}) + \int_{\nu_*(e,\eta)}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*(e,\ \eta))\tilde{\nu},$$

which implies

$$\delta_{\mathrm{C}} = \delta'_{\mathrm{C}} + \frac{1}{\alpha}\left[\int_{\nu(e,\eta)}^{\bar{\nu}} d\tilde{\nu}\, \phi(\tilde{\nu},\ \xi_*(e,\ \eta))\tilde{\nu} - \xi_*(e,\ \eta)\right].$$

Examination of these expressions yields the following facts: $\nu_*(0,\eta) = 0$, which means that type η bias alone does not distort C's freeze-out decision. In contrast, type e bias alone can distort C's freeze-out decision since $\nu_*(e,\ 0) \neq 0$. On the other hand, an examination of the equation defining $\xi_*(e,\ \eta)$ shows that either type e or type η alone or both together distort C's effort level away from first best. Q.E.D.

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