Coordinating Buyer-seller Transactions Across Multiple Products

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Joint ordering policies are examined as a method for reducing the transactions cost for multiple products sold by a seller to a homogeneous group of buyers. The problem of determining efficient joint ordering policies has the same structure as the previously-examined problem of determining the efficient ordering policy for a single product. Efficient joint lot-sizes are independent of prices, and are supported by a range of average-unit prices that permit every possible allocation of the transactions-cost saving between the buyer and the seller. Product bundling supports efficient joint orders across products, just as a quantity discount supports efficient transactions for a single product.
(*Marketing; Buyer-Seller Interaction; Product Bundling; Inventory Models; Transactions Cost*)

1. Introduction

A number of researchers have proposed models in which quantity discounts and two-part tariffs are used for achieving efficient transactions between a seller and a homogeneous group of buyers with constant demand (Dolan 1978, Lal and Staelin 1984, Dada and Srikanth 1987, Kohli and Park 1989). These models examine a single seller and a group of buyers with the same holding and ordering costs. The total transactions cost, which includes the holding costs for both parties, the ordering cost for the buyer, and the order-processing cost for the seller, is shown to be minimized if the lot size for a buyer exceeds his/her economic order quantity. A quantity discount or a two-part tariff acts as a mechanism that ensures efficient transactions and provides a division of the transactions-cost savings between the buyer and the seller.[1]

The analysis in these previous models assumes the transaction of only one product between a buyer and a seller. Many sellers, however, supply multiple products to buyers. In the automobile industry, joint ventures and ancillary operations can involve one firm producing multiple products for another firm. Chrysler Corporation has a contract to supply manual transmissions, transaxles and transfer cases to General Motors (Holusha 1989). General Motors has an agreement to supply engine parts and catalytic converters to Volga Auto Works (Levin 1990). In the electronics industry, some firms subcontract assembly work for their product de-

[1] The efficiency results of these models do not require heterogeneity in the holding and ordering costs of buyers, and are derived assuming that both the seller and the buyers incur holding and ordering/order-processing costs. In contrasts, the traditional motivation for a quantity discount is that it permits a seller to charge different prices to customers with heterogeneous ordering and holding costs. Gerstner and Hess' (1987) explanation for why firms charge different unit prices for different sizes of a product is driven by heterogeneity in the holding and ordering costs of buyers. Unlike the efficiency models, the seller's order-processing and holding costs are not considered by Gerstner and Hess. Models of promotions by Blattberg, Eppen and Lieberman (1981) and Eppen and Lieberman (1984) also focus on price discrimination between customers with high and low holding costs, a promotion being designed to induce purchases only from buyers with low holding costs. Ordering costs, which are central to the efficiency models, are not included in (and are not important for) the latter models. Finally, the models by Blattberg et al. and Eppen and Lieberman require the holding cost to be smaller for the deal-prone buyer than for the seller, whereas the transactions-efficiency models require the reverse relationship between the buyer's and seller's holding costs.

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signs (e.g., AT&T assembles PC configurations for IBM), others market product lines for original equipment manufacturers (e.g., Intel markets DRAMs for a Japanese manufacturer). In the machine tools and hardware industries, many firms produce multiple products, which they sell to wholesalers, who in turn sell product assortments to retailers.

For a single product, transactions efficiency can be increased only if a buyer's lot-size is larger than his economic order quantity. Multiple products offer the possibility of further reducing transactions costs by combining purchases across products. The reduction in the number of orders can simultaneously decrease the buyer's ordering cost and the seller's order-processing cost. On the other hand, departing from order quantities that minimize the independent transactions cost for each product can increase the total inventory holding cost incurred by the buyer and the seller. An efficient joint-ordering policy can therefore require trading off the decrease in the ordering and order-processing costs against a potential increase in the buyer's and seller's holding costs.

This paper extends the work described in Kohli and Park (1989) to examine efficient transactions across multiple products. It shows (1) that the problem of determining efficient joint ordering policies has the same structure as the problem of determining efficient ordering policies for a single product, and (2) that efficient joint orders can provide cost savings over efficient independent orders for the products. As it has the same structure as the single-product problem, the efficient lot size for joint orders is independent of prices, and is supported by a range of average unit prices that permit every possible allocation of the transactions-cost savings between the buyer and the seller. However, efficient joint orders differ from efficient orders for a single product in the following four ways:

(1) Unlike quantity discounts for a single product, a larger holding cost for the buyer than for the seller does not ensure the existence of efficient joint orders.

(2) Efficient joint orders can be feasible even if quantity discounts for individual products are not (i.e., if there are products for which the buyer's holding cost is smaller than the seller's).

(3) A price change is not always required for joint purchases, whereas it is required to support an efficient lot-size order for a single product.

(4) While a quantity discount can support efficient lot-sizes orders for a single product, product bundling is needed to support efficient joint orders.

2. The Model

Let n denote the number of products sold by the seller to the buyers. Let P_i denote the average unit price and D_i a buyer's demand for product i over a planning horizon. Let Q_i denote the lot-size order for product i if it is separately ordered. To compare the efficiency savings from joint orders to the efficiency savings from efficient independent orders, we assume that a quantity discount is feasible for each product. Equivalently, we assume that the efficient, independent lot size for product i is (Kohli and Park 1989)

$$Q_i = \sqrt{D_i(A_i + a_i)/H_i - h_i},$$

where A_i (a_i) is the ordering (order-processing) cost for the buyer (seller) and H_i (h_i) is the unit holding cost of product i for the buyer (seller), where $H_i > h_i$ for all i. Over the planning horizon, a buyer pays a purchase price of P_iD_i for product i, a holding cost (assuming constant usage rate) of $H_iQ_i/2$, and an ordering cost of A_iD_i/Q_i where D_i/Q_i is the number of orders for the product. Thus, the buyer's total cost from separately ordering the n products is

$$C_0 = \sum_{i=1}^{n} D_i P_i + H_i \frac{Q_i}{2} + A_i \frac{D_i}{Q_i}. \quad (1)$$

Assuming a fixed production schedule and a constant rate of shipments (see Lal and Staelin 1984), the corresponding profit for the seller is

$$\pi_0 = \sum_{i=1}^{n} D_i(P_i - V_i) + h_i \frac{Q_i}{2} - a_i \frac{D_i}{Q_i} - F_i, \quad (2)$$

where V_i (F_i) is the variable (fixed) cost for product i, h_i and a_i are the unit holding and the order-processing costs for product i, $h_iQ_i/2$ is the reduction in the holding cost for the seller upon transferring an average inventory of $Q_i/2$ of product i to a buyer, and a_iD_i/Q_i is the cost of processing D_i/Q_i orders of product i.

Let A denote a buyer's cost of placing a joint order for the n products. Let a denote the seller's cost of processing a joint order. Let m denote the number of joint orders. Let q_i denote the order quantity for product i in each joint order. Then $m = D_i/q_i$ for all i because the m orders must satisfy the annual demand for each product. Let b denote any arbitrary product. Then $m = D_b/q_b$ and $q_i = D_i q_b/D_b = k_i q_b$, where $k_i = D_i/D_b$ is the ratio of the annual demands for products i and b, respectively.

Let p_i denote the average unit price for product i under the joint ordering policy. A buyer's total cost from joint orders is

$$C = \left(\sum_{i=1}^{n} D_i p_i + H_i \frac{q_i}{2}\right) + A\frac{D_b}{q_b}, \qquad (3)$$

where $D_i p_i$ is the cost of purchasing D_i units of product i, $H_i q_i/2$ is the cost of holding an average inventory of $q_i/2$ units of product i for a year, and AD_b/q_b is the cost of placing D_b/q_b orders over the year. Similarly, the seller's profit from joint orders is

$$\pi = \left(\sum_{i=1}^{n} D_i(p_i - V_i) + h_i \frac{q_i}{2} - F_i\right) - a\frac{D_b}{q_b}, \qquad (4)$$

where $(p_i - V_i)$ is product i's unit contribution, $h_i q_i/2$ is the decrease in annual holding cost from transferring to the buyer an average inventory of $q_i/2$ units of product i, and aD_b/q_b is the annual cost of processing the D_b/q_b joint orders.

Let $\Delta C = C_0 - C$ be the decrease in the buyer's cost, and $\Delta\pi = \pi - \pi_0$ be the increase in the seller's profit due to joint orders. Let $P = \sum_{i=1}^{n} k_i P_i$ and $p = \sum_{i=1}^{n} k_i p_i$ denote the weighted prices for a collection of k_i units of product i, purchased at the independent and joint prices, respectively. Let $H = \sum_{i=1}^{n} k_i H_i$ denote the buyer's cost for holding k_i units of each product in inventory over the planning horizon. Let $h = \sum_{i=1}^{n} k_i h_i$ denote the corresponding cost for the seller. Equations (1) and (3), and equations (2) and (4), imply

$$p = P - \frac{\Delta C}{D_b} - \frac{1}{2D_b}\left(Hq_b - \sum_{i=1}^{n} H_i Q_i\right) - \left(\frac{A}{q_b} - \sum_{i=1}^{n} \frac{k_i A_i}{Q_i}\right), \qquad (5)$$

$$p = P + \frac{\Delta\pi}{D_b} - \frac{1}{2D_b}\left(hq_b - \sum_{i=1}^{n} h_i Q_i\right) + \left(\frac{a}{q_b} - \sum_{i=1}^{n} \frac{k_i a_i}{Q_i}\right). \qquad (6)$$

The above expressions describe the buyer's iso-cost and the seller's iso-profit curves, respectively, and are identical in structure to the corresponding equations for efficient ordering policies for a single product. It can be verified that joint orders of

$$q_i^* = k_i q_b^*, \quad i = 1, 2, \ldots, n,$$

$$q_b^* = \sqrt{\frac{2D_b(A + a)}{H - h}}, \qquad (7)$$

provide a greater cost saving than separate quantity discounts if $p_{\min} \le p_{\max}$, where $p_{\min}$ ($p_{\max}$) is the average unit price at which the seller (buyer) is indifferent between separate and joint purchases of the products. Setting $\Delta\pi = \Delta C = 0$ in equations (5) and (6) gives

$$p_{\min} = P - \frac{1}{2D_b}\left(hq_b^* - \sum_{i=1}^{n} h_i Q_i\right) + \left(\frac{a}{q_b^*} - \sum_{i=1}^{n} \frac{k_i a_i}{Q_i}\right), \qquad (8)$$

$$p_{\max} = P - \frac{1}{2D_b}\left(Hq_b^* - \sum_{i=1}^{n} H_i Q_i\right) - \left(\frac{A}{q_b^*} - \sum_{i=1}^{n} \frac{k_i A_i}{Q_i}\right). \qquad (9)$$

As $H_i > h_i$ for all i, $H > h$ in equation (7). The requirement $H > h$ for $q_i \ge 0$ generalizes to multiple products the condition $H_i > h_i$ for an efficiency gain via a quantity discount for product i. (Observe that although we assume that a quantity discount is feasible for each product (i.e., $H_i > h_i$ for all i), this assumption is not necessary because the condition $H > h$ can be satisfied even if $H_i \le h_i$ for one or more individual products.)

The condition $p_{\min} \le p_{\max}$ is equivalent to the requirement that joint orders reduce the transactions cost for the buyer-seller system. If each product is separately purchased, the system transactions cost is $\sum_{i=1}^{n} \sqrt{2D_i(A_i + a_i)(H_i - h_i)}$. Similarly, if the products are jointly purchased, the system transactions cost is $\sqrt{2D_b(A + a)(H - h)}$. It follows that joint orders are more efficient than separate orders if

$$A + a \le \left(\sum_{i=1}^{n} \sqrt{\frac{A_i + a_i}{1 + \sum_{j, j\ne i} \frac{D_j}{D_i}\frac{H_j - h_j}{H_i - h_i}}}\right)^2. \qquad (10)$$

Thus, joint orders are likely to yield a cost saving over separate orders if products that have the higher ordering and order-processing costs also have higher demands and a larger difference between the buyer's and seller's holding costs. Consider the following two cases.

Case 1: $A_i + a_i = A_j + a_j$ for all $i, j = 1, 2, \ldots, n$.

This case occurs when products do not differ much in their sizes, delivery modes, handling requirements, and lead times. Examples of such products include lines of comparably priced clothing sold by a garment wholesaler to retailers, canned goods sold to supermarkets by a food distributor, and stationary items sold to organizations by an office-supplies vendor.

Let $A + a = (ns)(A_i + a_i)$, where $s < 1$ (>1) if the joint cost of ordering and processing orders is less (more) than the sum of the independent ordering and order-processing costs for the n products. The value $s < 1$ will be observed if joint orders can reduce the costs of transportation (e.g., freight and insurance, which often decrease with increases in shipment volume), packing and unpacking, breakage and damage, inspection and goods handling, and the administration of ordering and processing activities (e.g., paperwork, computer processing, and tracking). The greater the savings due to these factors, the smaller the value of s. Thus, equation (10) can be rewritten as

$$s \leq \frac{\{\sum_{i=1}^{n} \sqrt{D_i(H_i - h_i)}\}^2}{n \sum_{i=1}^{n} D_i(H_i - h_i)}, \tag{11}$$

where $A_i + a_i = A_j + a_j$ for all $i, j = 1, 2, \ldots, n$. The right side of equation (11) obtains its maximum value of one if $D_i(H_i - h_i)$ is identical across products. Thus, joint orders reduce transactions costs if (1) the value of s is small (i.e., joint orders significantly reduce ordering and order-processing cost) and (2) the variability in $D_i(H_i - h_i)$ is small across products (i.e., if the demand for a product is negatively correlated with the difference in the buyer's and seller's holding costs).

Case 2: $H_i - h_i = H_j - h_j$ for all $i, j = 1, 2, \ldots, n$.

This case occurs when the products are similar in terms of their breakage, damage, and insurance costs, have comparable costs for the buyer and seller, and have similar storage and handling costs. Examples of products for which the condition may be satisfied are wines in comparable price ranges, frozen foods, shoes of different styles and sizes, and small electronic components. In this case, equation (10) simplifies to

$$\sqrt{A + a} \leq \sum_{i=1}^{n} \sqrt{\frac{A_i + a_i}{1 + \sum_{j, j \neq i} k_j}}. \tag{12}$$

Let $A_{i,\min} = \min_i A_i$, and $a_{i,\min} = \min_i a_i$. Then

$$\frac{A_{i,\min} + a_{i,\min}}{n} < \frac{A_i + a_i}{1 + \sum_{j, j \neq i} k_j}. \tag{13}$$

Thus, $A + a \leq n(A_{i,\min} + a_{i,\min})$ is a sufficient condition for efficient joint orders to provide a cost saving over efficient independent orders. Thus, if the cost of placing and processing joint orders is no more than n times the lowest cost of processing and placing any separate order, then joint orders are likely to yield a cost saving over separate orders. As noted in case (1) above, there are a variety of sources for these cost savings, including transportation, handling, packing, inspection, and the administration of ordering and processing activities.

One difference between previous efficiency models for a single product and the present model is that unlike a single product, efficient joint orders do not always require a change in the average unit prices. The conditions under which this occurs can be examined by setting $p_i = P_i$ in equations (5) and (6) and re-arranging terms to obtain the conditions $\sum_{i=1}^{n} h_i \Delta Q_i \leq 2\Delta a$ and $\sum_{i=1}^{n} H_i \Delta Q_i \geq 2\Delta A$ for the buyer and seller, respectively, where $\Delta Q_i = Q_i - q_i$, $\Delta a = (aD_b/q_b) - \sum_i a_i D_i/Q_i$, and $\Delta A = (AD_b/q_b) - \sum_i A_i D_i/Q_i$. Given values of ΔA and Δa, the two conditions are more likely to be satisfied if $H_i - h_i$ is large (small) for products with a large increase (decrease) in the lot-size order.

If the original prices support efficient joint orders, the buyer and the seller can potentially agree to not change prices. However, consider the case where the original prices are not feasible, and/or the buyer and the seller negotiate a price $p^* \in (p_{\min}, p_{\max})$ at which both the buyer and the seller prefer joint orders to separate orders (see Kohli and Park 1989 for a discussion of bargaining in the present context). The outcome of bargaining will either be a disagreement (i.e., no joint purchases) or an agreement under which the buyer will obtain a specified part of the cost saving from joint orders by purchasing $q_i^* = k_i q_b^*$ units of each product i at a total purchase price of $p^* q_b^*$. The joint purchases correspond to bundled purchases by the buyer, the cost saving being contingent upon the buyer switching to the joint ordering policy. Thus, in the present model, product bundling serves as a mechanism supporting efficient buyer-seller transactions. This contrasts with the usual explanation for bundling as a method for a seller to increase profits by exploiting the heterogeneity in consumer reservation prices across products (see, e.g., Adams and Yellen 1976, Schamalensee 1984, Hanson and Martin 1990). The present analysis suggests that there also is a transaction-efficiency rationale for product bundling that does not require customer heterogeneity. Adams and Yellen (1976, p. 476) note this as a possible cost-based explanation for bundling. Another cost-based motivation for bundling—economies of scope—is discussed by Hanson and Martin (1990, p. 164). In a similar spirit, Porter (1985) presents examples to illustrate how a firm can use product bundling to control manufacturing set-up costs.

Just-In-Time (JIT) inventory management offers another way for firms to reduce their transactions costs. In an EOQ context, a firm facing fixed, deterministic demand can invest in technology or process methods that reduce its order-processing (or set-up) costs, which in turn reduces the optimal lot-size order for a buyer. American Hospital Supplies' computerized ordering system is an example of such an investment. Porteus (1985) (see also Zangwill 1987) examines conditions under which such an investment reduces the total cost for a firm. These results directly apply to the present model, except that investments can be aimed at reducing either or both of a buyer's ordering cost and the seller's order-processing costs. These reductions, in turn, will decrease the buyer's optimal lot-size order.

An Example

To illustrate the preceding analysis, consider the following example involving two products ($i = 1, 2$). Let the buyer's annual demand be $D_1 = 1{,}000$ units for product 1 and $D_2 = 2{,}000$ units for product 2. Let the buyer incur an ordering cost of $A_1 = \$15/\text{order}$ and $A_2 = \$60/\text{order}$, and an inventory holding cost of $H_1 = \$3/\text{unit}/\text{year}$ and $H_2 = \$6/\text{unit}/\text{year}$ for products 1 and 2, respectively. Similarly, let the seller incur an order-processing cost of $a_1 = \$75/\text{order}$ and $a_2 = \$100/\text{order}$, and an inventory holding cost of $h_1 = \$1/\text{unit}/\text{year}$ and $h_2 = \$2/\text{unit}/\text{year}$ for products 1 and 2, respectively.

If the products are sold at (any) fixed unit prices, the buyer's EOQ is $l_1 = 100$ units for product 1 and $l_2 = 200$ units for product 2, where $l_i = \{2A_iD_i/H_i\}^{1/2}$. The corresponding total transactions cost incurred by the buyer and the seller is $\sum_{i=1}^{2} (D_i/l_i)(A_i + a_i) + (l_i/2)(H_i - h_i) = \$1{,}000 + \$2{,}000 = \$3{,}000$. If the seller offers a separate quantity discount for each product, the efficient lot size is $Q_1 = 200$ units for item 1 and $Q_2 = 300$ units for item 2, where $Q_i = \{[2(A_i + a_i)D_i]/(H_i - h_i)\}^{1/2}$ (Kohli and Park 1989). The corresponding total transactions cost incurred by the buyer and the seller, is $\sum_{i=1}^{2} (D_i/Q_i)(A_i + a_i) + (Q_i/2)(H_i - h_i) = \$600 + \$1{,}600 = \$2{,}200$.

Now consider joint orders for the two products. Let $A = \$70/\text{order}$ and $a = \$110/\text{order}$ be the ordering and order-processing costs for joint orders. Let product 1 represent the base product (i.e., $b = 1$). Then $k_1 = 1$, $k_2 = D_2/D_1 = 2000/1000 = 2$, $H = k_1H_1 + k_2H_2 = 15$ and $h = k_1h_1 + k_2h_2 = 5$. From equation (7), the optimal joint order quantities are $q_1^* = 60\sqrt{10}$ and $q_2^* = 120\sqrt{10}$. The corresponding total transactions cost incurred by the buyer and the seller is $(D_b/q_b^*)(A + a) + (q_b^*/2)(H - h) = \$600\sqrt{10} = \$1{,}897.37$. Thus, the independent quantity discounts for products reduce the total transactions costs by 26.67%, from \$3,000 to \$2,200. Joint orders further reduce the total transactions cost by 13.75%, from \$2,200 to \$1,897.37. Also, it can be verified that product bundling is necessary to achieve efficient transactions, which cannot be supported by the original prices of the products.

3. Conclusion

The above analysis considers joint purchases of all n products. However, it is possible that the products may be grouped so that a common order is placed for items in each group. If each product is restricted to appear in exactly one group, the problem of optimally partitioning products into subsets is equivalent to the set-partitioning problem, for which efficient approximate solution procedures are discussed by Balas and Padberg (1976). A more realistic problem is the optimal partitioning of products into subsets without the restriction that each item appears in only one subset. In this case, the buyer's annual demand may be met from purchases of one or more subsets in which it is included. This problem is significantly more complex than the set partitioning problem because the buyer's optimal purchasing policy need not be cyclical and periodic. Finally, the above analysis is restricted to joint orders for existing products. It may be useful to examine the problem of product-line design, taking into account economies of scope, transactions costs, demand interdependence, and heterogeneity among consumers.[2]

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