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A significant application of conjoint analysis is in pricing decisions for new products. Conceptually, profit maximization is an important criterion for selecting the price of a product. However, the maximization of profit necessitates estimation of fixed and variable costs, which are difficult to estimate reliably for the large number of products available for evaluation in conjoint analysis. Consequently, users of conjoint models have begun to use a share simulation to screen a small set of attractive products. For each screened product, fixed and variable costs are estimated separately and used to simulate its profits at different price levels. The limitation of this approach is that the profit simulations are based on the assumption that the conjoint data, and hence the predicted profits, are error free. Also, though the purpose of examining alternative prices is to determine the best price at which to offer a new product, current conjoint simulators do not focus explicitly on optimal pricing decisions. The authors describe and illustrate a model for optimal pricing of screened products in conjoint analysis, incorporating the effect of measurement and estimation error on predicted profits.

A Reservation-Price Model for Optimal Pricing of Multiattribute Products in Conjoint Analysis

Since its introduction in the early 1970s, conjoint analysis has been widely accepted as a procedure for measuring customers' tradeoffs among multiattribute products and services. Wittink and Cattin (1989) esti-

mate that about 200 to 400 commercial applications of conjoint analysis per year were carried out during the early 1980s. Most of those applications were in new product concept identification, competitive analysis, pricing, segmentation, and product repositioning.

One significant application of conjoint analysis is in pricing decisions (Mahajan, Green, and Goldberg 1982; Page and Rosenbaum 1987). The objective in such applications is to determine the best price at which a new product should be offered to customers. The determination of the best price typically is achieved by using conjoint choice simulators, which predict the effect of alternative feasible prices on the share of a new product concept. Thus, choice simulators assess the sensitivity of a new product's share to changes in its price, and therefore are useful for selecting a price at which the new product is offered to consumers. The availability of choice simulators in computer packages such as Sawtooth and Bretton-Clark has contributed to the popularity of the conjoint approach for pricing new products.

Though useful for assessing the sensitivity of a prod-

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uct's share to changes in its price, current choice simulators do not extend to (1) assessing the profitability of a new product and (2) identifying a price that maximizes the profit from a new product. The principal reason is that a profit criterion necessitates estimates of fixed and variable costs for every feasible product. In many conjoint studies, estimating costs individually for each potential new product is not practical because of the large number of feasible products available for evaluation. Conceptually, multiattribute cost functions offer an alternative approach for estimating variable costs. Practically, as suggested by Green and Krieger (1989), multiattribute cost functions are difficult to estimate reliably. The difficulty of obtaining accurate cost estimates has been a major impediment in the development of conjoint models using a profit objective.

Faced with the difficulty of cost function estimation, users of conjoint models have begun to adopt a two-step approach to conjoint simulations. First, traditional share simulations (which require no cost information) are used to screen a small number of attractive product concepts. The fixed and variable costs for each screened product then are estimated individually and used to simulate its profits at different price levels (see, e.g., Page and Rosenbaum 1987 for an illustration of this approach at the Sunbeam Corporation, which manufactures blenders and choppers for household use). However, the profit simulations are based on the assumption that the conjoint data, and hence the predicted demand and profits, are error free. Also, though the purpose of examining alternative prices is to determine the best price at which to offer a new product, current conjoint simulators do not focus explicitly on optimal pricing decisions.

We propose a model for determining the price that maximizes the profit of a product that has been screened by using a share criterion and for which fixed and variable costs have been estimated separately. In contrast to current simulators in which conjoint data are assumed to be error free, the proposed approach incorporates the effect of error in demand estimates that can arise from measurement and estimation error in conjoint utility functions. The error in demand in turn affects the price that maximizes the profit of a new product.

The proposed approach also differs from conjoint-based models for optimal product and product-line design that have been proposed recently in the literature (see, e.g., Green and Krieger 1989). Those models assume error-free utility function estimates and formulate the problem of selecting an optimal product or product line as a mathematical programming problem. Though a mathematical programming approach can be used to formulate the pricing problem, we pursue the proposed approach because it explicitly models error in consumer preferences.

In the following section, we detail the development of the model. Then we use the model in an application to determine the optimal prices for new apartments. We conclude with a discussion of limitations and possible extensions of the proposed approach.

MODEL DEVELOPMENT

Conceptual Underpinnings

To explicate the conceptual underpinnings of the proposed model, let us consider the choice of apartments by a group of university students. Assume that the apartments are described in terms of the six attributes (including price) in Table 1A. Assume also that a student (or a segment of students) currently occupies the apartment described in Table 1B. Consider a real estate developer who has screened the four apartment concepts in Table 1C on the basis of a conjoint share simulation. Only one type of apartment is to be constructed, and there is no constraint on the number of units that can be built. Given the information on the currently occupied apartment, what should be the monthly rent that maximizes the (independent) profit of each screened apartment? To address the pricing question, we assume that each consumer has a maximum (i.e., reservation) price that he or she is willing to pay for a new product. Also, we assume that a consumer's reservation price for a new product is determined by his or her (estimated) utility for the product *in relation* to the price and utility for his or her most preferred product among all product offerings in his or her evoked set. For expository purposes, assume that each student's current apartment is his or her most preferred among available apartments. Thus, for the apartments example, we assume that the maximum monthly rent a student is willing to pay for a new apartment depends on its utility in relation to the utility and rent of the currently occupied apartment.

Estimates of consumer preference functions can contain error because (1) consumer ratings of product profiles can contain error and (2) missing attributes and misspecification of the preference function can contribute estimation error. A reservation price estimate therefore also can contain error because it reflects a consumer's utility for a product in relation to his or her utility for a status quo product. Hence, we assume that each consumer's reservation price estimate is an observation from an idiosyncratic reservation price distribution. Note that conjoint preference functions typically are estimated by regression (Wittink and Cattin 1989) and therefore assume that consumers have deterministic preferences (Green and Srinivasan 1978). Under that modeling assumption, a consumer's reservation price estimate is a realization of a random variable not because his or her preferences are stochastic, but because the measurement and estimation of his or her preferences contains error.

We further assume that each consumer's reservation price estimate is an observation from a distribution that has the same functional form, but different parameter values, across consumers. The parameters reflecting preference heterogeneity are assumed to have an appropriate conjugate distribution over the population. By specifying both the individual reservation price distribution and the across-individual heterogeneity distribution, we obtain the unconditional reservation price dis-

Table 1
APARTMENT ATTRIBUTES AND PROFILE DESCRIPTIONS

A. Attributes				
Attribute	Levels			
Rent	\$225, \$270, \$315, \$360, \$405, \$450, \$495, \$540			
Walking time to class	10, 15, 20, 30 minutes			
Noise level of apartment	Very quiet, average, noisier than average, very noisy			
Safety of apartment location	Very safe, average, less safe than average, very unsafe			
Condition of apartment	Newly renovated throughout, renovated kitchen only, fair condition, poor condition			
Size of living/dining area	24' × 30', 15' × 24', 12' × 15', 9' × 12'			
B. Profile Description of Status Quo Apartment				
Attribute	Levels			
Rent	\$360			
Walking time to class	20 minutes			
Noise level of apartment	Average			
Safety of apartment location	Average			
Condition of apartment	Renovated kitchen only			
Size of living/dining area	12' × 15'			
C. Profile Description of Four New Apartment Concepts				
Attribute	Concept 1	Concept 2	Concept 3	Concept 4
Walking time to class	10 minutes	30 minutes	10 minutes	30 minutes
Noise level of apartment	Average	Very quiet	Average	Average
Safety of apartment location	Average	Very quiet	Average	Average
Condition of apartment	Poor	Renovated kitchen	Renovated throughout	Renovated throughout
Size of living/dining area	9' × 12'	12' × 15'	9' × 12'	24' × 30'

tribution across consumers. This unconditional distribution is related to the demand function for a new product and is used to estimate the price at which the product's expected profit is maximized.

Analytical Underpinnings

To describe the analytical underpinnings of the proposed model, let us consider a product described in terms of K attributes, including price. Let $k = 1, \dots, K$ denote the attributes, and let $k = p$ specifically denote the price attribute. We assume that the conjoint data have been collected for all K attributes, and that the preference functions are estimated with price as a continuous attribute. As in most conjoint studies, consumers are assumed to provide ratings of product concepts (Wittink and Cattin 1989). Individual preference functions are estimated by a regression constrained to ensure that as price increases, a consumer's utility for a product linearly decreases.¹

¹The model represents the simplest situation in which the utility of a product is a linearly decreasing function of price. Alternative assumptions about the effect of price on the utility of product concepts are also possible (e.g., an inverted-U relationship between utility and price, and/or interactions between price and other attributes). As described subsequently, a linearly decreasing relationship between utility and price justifies the use of a normal distribution for individual reservation prices. For other utility-price relationships, the derivation of reservation price distributions is significantly more difficult and should be examined in future research.

Let u_i^* denote the highest estimated utility of any currently available product in consumer i 's evoked set. The value of u_i^* includes the utility contribution due to price. Let t denote the index of a new product for which the optimal price is to be assessed. Let $u_{i|t-p}$ denote the multiattribute utility of product t , where the notation emphasizes that the utility contribution due to price is *not included* in $u_{i|t-p}$. We assume that individual i prefers product t at price p to any product in his or her evoked set if

$$(1) \quad u_{i|t-p} + u_i(p) \geq u_i^* + \epsilon,$$

where $u_i(p)$ is the contribution of price p to individual i 's utility for a product and ϵ is an arbitrarily small positive number (we assume that a small value of ϵ is selected by the user in any application of the proposed model). Then p_{ii} is an estimate of individual i 's reservation price for product t if $p = p_{ii}$ is the price at which equation 1 is satisfied as an equality; that is, it is the price at which the utility of item t exceeds by ϵ the utility of the most preferred item in consumer i 's evoked set.²

²An alternative approach is to ask consumers directly to state the maximum (positive or negative) price they are willing to pay to switch to a new product from their status quo. This task may be difficult for respondents and can potentially affect the reliability (and hence the measurement error) of the direct evaluations. However, direct evaluations do not have the estimation error that is present in conjoint-based estimates of reservation prices. Future research should examine alternative approaches to reservation price estimation and identify conditions under which each approach is most useful.

As $u_i(p)$ is assumed to be a single-valued, decreasing function of price, a single reservation price p_{ii} is estimated for each consumer.

As the reservation price reflects the marginal value of product t over the most preferred offering in a consumer's evoked set, this distribution should accommodate both positive and negative reservation prices. For reasons discussed subsequently, we assume that a consumer's reservation price is described by a normal distribution $r(p_{ii}) = N(\mu_{ii}, \sigma_{ii}^2)$ with mean μ_{ii} and variance σ_{ii}^2 , $i = 1, 2, \dots, n$. Heterogeneity in individual reservation prices is modeled by assuming that the variance σ_{ii}^2 of the distribution of individual reservation prices is the same but the mean μ_{ii} of the individual reservation price distributions is normally distributed across the consumer population; that is, $\mu_{ii} \sim N(\mu_t, \sigma_t'^2)$, where $E(\mu_{ii}) = \mu_t$ is the expected value of μ_{ii} and $\text{var}(\mu_{ii}) = \sigma_t'^2$ is the variance of μ_{ii} across the population of consumers. Let $g(\mu_{ii}) = N(\mu_t, \sigma_t'^2)$ denote the density of μ_{ii} across the consumer population. Let $h(p_t)$ denote the *unconditional density* of reservation prices for product t across the population of consumers. The $h(p_t) = \int_{-\infty}^{\infty} r(p_{ii})g(\mu_{ii})d\mu_{ii} \sim N(\mu_t, \sigma_t^2)$, where $\sigma_t^2 = \sigma_{ii}^2 + \sigma_t'^2$ (Johnson and Kotz 1970, p. 87). Let $H(p_t)$ denote the cumulative density function of $h(p_t)$. Observe that $1 - H(p_t)$ denotes the probability that a randomly chosen individual from the consumer population will have a reservation price less than p_t , and therefore will prefer product t at price p_t over all products in his or her evoked set. Equivalently, $1 - H(p_t)$ is the proportion of consumers who select product t at price p_t over all items in their (idiosyncratic) evoked sets; that is, it is an estimate of the *share of choices* of product t at price p_t . If m denotes the total number of consumers in the target market, the number of consumers who will buy product t at price p_t is

$$(2) \quad D(p_t) = m[1 - H(p_t)].$$

If each consumer buys one unit of a product, equation 2 describes the demand function for product t . The expected profit from selling product t at price p_t is

$$(3) \quad \pi(p_t) = m[1 - H(p_t)](p_t - c_t) - f_t,$$

where f_t is the fixed cost of producing product t and c_t is its (constant) unit variable cost (in the concluding section, we discuss relaxing the assumption of constant variable cost). The firm's profit is maximized when $d\pi(p_t)/dp_t = 0$, which in turn implies that the optimal price of product t is

$$(4) \quad p_{t,\text{opt}} = \frac{1 - H(p_t)}{h(p_t)} + c_t.$$

Note that a dollar increase in the variable cost c_t increases the optimal price $p_{t,\text{opt}}$ by a dollar. Thus, if the cost is estimated to be in the range $c_t \pm \delta$, the optimal price is in the range $p_t \pm \delta$.

As $h(p_t)$ is a normal density function, an explicit for-

mula for $H(p_t)$, and hence a closed-form solution for $p_{t,\text{opt}}$, is not possible from equation 4. However, the optimal price can be obtained by numerically solving equation 4. Given estimates of μ_t and σ_t^2 , one can readily compute the value of the normal density $h(p_t)$ for different values of p_t . The corresponding value of $H(p_t)$ can be obtained from standard normal tables. The optimal price is given by the value of p_t at which $H(p_t)$ intersects $Z(p_t) = 1 - h(p_t)(p_t - c_t)$ and can be determined graphically from a simultaneous plot of $H(p_t)$ and $Z(p_t)$.

Our analytical development has its roots in the work by Gabor and Granger (1966). Those authors postulated and successfully tested several hypotheses about customer behavior in markets where a product's quality is imputed from its price. Gabor and Granger found that any given customer has a price range within which he or she will consider purchasing a product. Brands priced below that acceptance range will be rejected by the consumer as being too shoddy. Similarly, brands priced above the range will be rejected as being too expensive. The minimum and maximum acceptable prices differ from person to person. However, for a homogeneous group of customers, the minimum or the maximum price that a customer is willing to pay for the product is a lognormally distributed random variable (i.e., the logarithm of the price is normally distributed). Further, the standard deviations of the log distributions for the minimum and maximum prices are the same.

The concept of a reservation price suggested in our formulation corresponds to the maximum price suggested by Gabor and Granger. Unlike them, however, we assume that the reservation price for a customer follows a normal distribution. Our use of the normal distribution is based on the following three reasons.

1. Conjoint models are used most often with ratings data to estimate a linear preference model (Wittink and Cattin 1989). A normal error assumption is both common and robust in linear models parameterized with interval-scaled data. In turn, a normal error assumption in a linear model implies a normal distribution for the utilities u_{it-p} and u_i^* , and hence a normal distribution for the marginal utility $\Delta u_{ii} = u_{it-p} - u_i^*$ of product t over the most preferred item in consumer i 's evoked set. If, as is common in conjoint studies, we assume a linear relationship between utility and price, a normal distribution for Δu_{ii} implies a normal distribution for the individual reservation prices p_{ii} .
2. We define a consumer's reservation price p_{ii} to reflect the marginal value of product t in relation to the value of the most preferred item in his or her evoked set. Hence, both negative and positive reservation prices should be defined, which is possible if the reservation prices are normally distributed but not if they are lognormally distributed. The normal model retains this advantage over alternative reservation price models (e.g., exponentially distributed individual reservation prices with gamma heterogeneity) that are restricted to the positive price domain.
3. If the empirical distribution of (positive) reservation prices is skewed to the left, it can be approximated by both a

lognormal distribution and a (truncated) normal distribution. However, an empirical reservation price distribution that is skewed to the right can be approximated by a (truncated) normal distribution, but not by the lognormal distribution. Hence, a (truncated) normal distribution is likely to be more flexible than a lognormal distribution in approximating alternative reservation price distributions.

Estimation

The preceding description suggests that the unconditional distribution of reservation prices should be described by a normal density function $h(p_i)$ defined over the range $[-\infty, \infty]$. However, conjoint preference functions are estimated only over a restricted positive range $[p_{\min}, p_{\max}]$ of prices that consumers are likely to encounter in their actual purchases. Consequently, a consumer's reservation price can be estimated from conjoint data only if it is in the range $[p_{\min}, p_{\max}]$. Let $u_i(p_{\min})$ and $u_i(p_{\max})$ denote the utility contributions of prices p_{\min} and p_{\max} , respectively, in consumer i 's multiattribute preference function, $i = 1, \dots, n$. Then consumer i 's reservation price estimate is less than p_{\min} if $u_{i|p} + u_i(p_{\min}) < u_i^* + \epsilon$, and is higher than p_{\max} if $u_{i|p} + u_i(p_{\max}) > u_i^* + \epsilon$. In either case, a reservation price estimate for consumer i cannot be obtained because preference functions are not estimated below price p_{\min} or above price p_{\max} . However, the fraction q_{1i} of consumers whose reservation price estimates are below p_{\min} , and the fraction q_{2i} of consumers whose reservation price estimates are above p_{\max} , can be estimated.

Thus, the following data are available from conjoint analysis for the estimation of the normal density $h(p_i)$: (1) the estimates q_{1i} and q_{2i} of the fractions of consumers with reservation price estimates below p_{\min} and above p_{\max} , respectively, and (2) the estimates of reservation prices in the range $[p_{\min}, p_{\max}]$ for a fraction $1 - (q_{1i} + q_{2i})$ of the consumers. Procedures for estimating a normal distribution from truncated data have been developed for cases when q_{1i} and q_{2i} are not known (see, e.g., Johnson and Kotz 1970, p. 81-86). The proportions q_{1i} and q_{2i} are allowed to take any value in those estimation procedures. However, q_{1i} and q_{2i} are known in the present instance and should be used to constrain parameter estimates further. Such constraints use all available information about reservation prices and provide a stricter test of the assumption that the reservation prices have a normal distribution. A constrained maximum likelihood estimation procedure is described next to estimate the mean μ_i and variance σ_i^2 of the normal density $h(p_i)$ of the unconditional reservation prices.

Let $z_i = (p_i - \mu_i)/c_i$ denote the standardized value of p_i . Then z_i has the standard normal density $f(z_i) = (1/\sqrt{2\pi})^{1/2} \exp(-z_i^2/2)$. Let $F(z_i)$ denote the cumulative density function of z_i . Let $z_{i,\min}$ ($z_{i,\max}$) denote the value of z_i at $p = p_{\min}$ ($p = p_{\max}$). Let \bar{p}_i and s_i denote the average and standard deviation of the reservation price observations between p_{\max} and p_{\min} . If q_{1i} and q_{2i} are not known,

the maximum likelihood estimates of μ_i and σ_i^2 are given by (Johnson and Kotz 1970, p. 82-83)

$$\hat{\mu}_i = \bar{p}_i - R\hat{\sigma}_i^2$$

and

$$\hat{\sigma}_i^2 = \frac{s_i^2}{C}$$

respectively, where:

$$R = \frac{f(z_{i,\min}) - f(z_{i,\max})}{F(z_{i,\max}) - F(z_{i,\min})}$$

and

$$C = 1 + \frac{z_{i,\min}f(z_{i,\min}) - z_{i,\max}f(z_{i,\max})}{F(z_{i,\max}) - F(z_{i,\min})} - R^2$$

Note that R and C depend on the values of z_i , $f(z_i)$, and $F(z_i)$ at the truncation points $z = z_{\min}$ and $z = z_{\max}$. If those values are not known, iterative solution procedures that maximize the likelihood function are needed to estimate R and C (e.g., Harter and Moore 1966). However, q_{1i} and $1 - q_{2i}$, which are known in the present instance, are estimates of the cumulative proportions under the normal density curve $f(z_i)$ at $z_i = z_{i,\min}$ and $z_i = z_{i,\max}$, respectively; that is, $\hat{F}(z_{i,\min}) = q_{1i}$, $\hat{F}(z_{i,\max}) = 1 - q_{2i}$. The corresponding values of $z_{i,\max}$ and $z_{i,\min}$ can be determined from standard normal tables (e.g., $F(z_i) = .5$ implies $z_i = 0$). Once $z_{i,\max}$ and $z_{i,\min}$ are estimated, the corresponding values of the unit normal variate $f(z_i)$ can be computed. Thus the values of z_i , $f(z_i)$, and $F(z_i)$ can be estimated at $z_i = z_{i,\min}$ because q_{1i} is known, and can be estimated at $z_i = z_{i,\max}$ because q_{2i} is known. Given these values, one can readily estimate the expressions for R and C , and hence the expressions for μ_i and σ_i^2 .

To illustrate the estimation procedure, consider $q_{1i} = q_{2i} = .025$ (i.e., 2.5% of the sample consumers have reservation price estimates below p_{\min} and another 2.5% have them above p_{\max}). Then $\hat{F}(z_{i,\min}) = 1 - \hat{F}(z_{i,\max}) = .025$. From the standard normal tables, $F(z_i = -1.96) = .025$ and $F(z_i = 1.96) = .975$, which implies $z_{i,\min} = -z_{i,\max} = -1.96$. Thus,

$$f(z_{i,\min}) = f(z_{i,\max}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(-1.96)^2}{2}\right) = .0584$$

Hence,

$$R = \frac{.0584 - .0584}{.975 - .025} = 0$$

and

$$C = 1 + \frac{(-1.96)(.0584) - (1.96)(.0584)}{.975 - .025} - 0^2 = .7591$$

If \bar{p}_i and s_i^2 are the values of the mean and variance for the 95% reservation price estimates between p_{\min} and p_{\max} , the mean and variance estimates for the untruncated nor-

mal distribution of reservation prices are $\hat{\mu}_t = \bar{p}_t$ and $\hat{\sigma}_t^2 = (1/.7591)s_t^2 = 1.317s_t^2$, respectively.

AN APPLICATION

As an illustrative application of the proposed model, consider again the problem of determining independently the optimal pricing policy for each of the four apartments in Table 1C, assuming that only one of the apartments is to be offered by a real estate developer. The apartments described represent actual alternatives that students at an urban northeastern university are likely to encounter. The apartment described by concept 1 is small, close to campus, in poor condition, and average in terms of both the noise and safety attributes. The apartment described by concept 3 is similar to the apartment described by concept 1, but has been thoroughly renovated. Apartments described by concepts 2 and 4 are farther from campus and larger than the apartments described by concepts 1 and 3. However, in choosing between the apartments described by concepts 2 and 4, a student must make a tradeoff between the larger size and better condition of the latter and the lesser noise and greater safety of the former.

The conjoint data for our study were a replication of the data used by Green, Helsen, and Shandler (1988). Six of the seven original attributes (including price) were used in the replication. Each of 177 students, comprising both MBA students and business undergraduates, rated 32 apartment concepts, which were used to estimate idiosyncratic multiattribute preference functions. Each student's preferences for apartments were described by a main effects model in which price was used as a continuous variable. All other attributes in the model appeared at a discrete number of levels. The parameters for each individual's preference function were estimated by OLS regression. For expository purposes, all consumers were assumed to have the same status quo apartment, described in Table 1B.

An analysis of the data suggested that for apartments described by concepts 3 and 4, the number of reservation prices in the \$225 to \$375 range was small (≤ 5). However, the proportion of reservation price estimates below \$225 exceeded .20 for each of those apartments. Consequently, the lowest reservation price used in the following analysis is \$375, and the observations in the \$225 to \$375 range are reflected in the computation of q_t , for apartment $t = 1, \dots, 4$. Also, for the purpose of this illustration, the (linear) price term in the multiattribute preference functions is extrapolated to \$825. That is, we assume that the individual reservation price estimates could be obtained over the range $[p_{\min}, p_{\max}] = [\$375, \$825]$.³

³We extrapolate the upper truncation point beyond the maximum price used in the conjoint study for expository purposes. In commercial settings, the conjoint data collection should be designed to include a price range that is wide enough for a reasonable sample of reservation prices to be estimated.

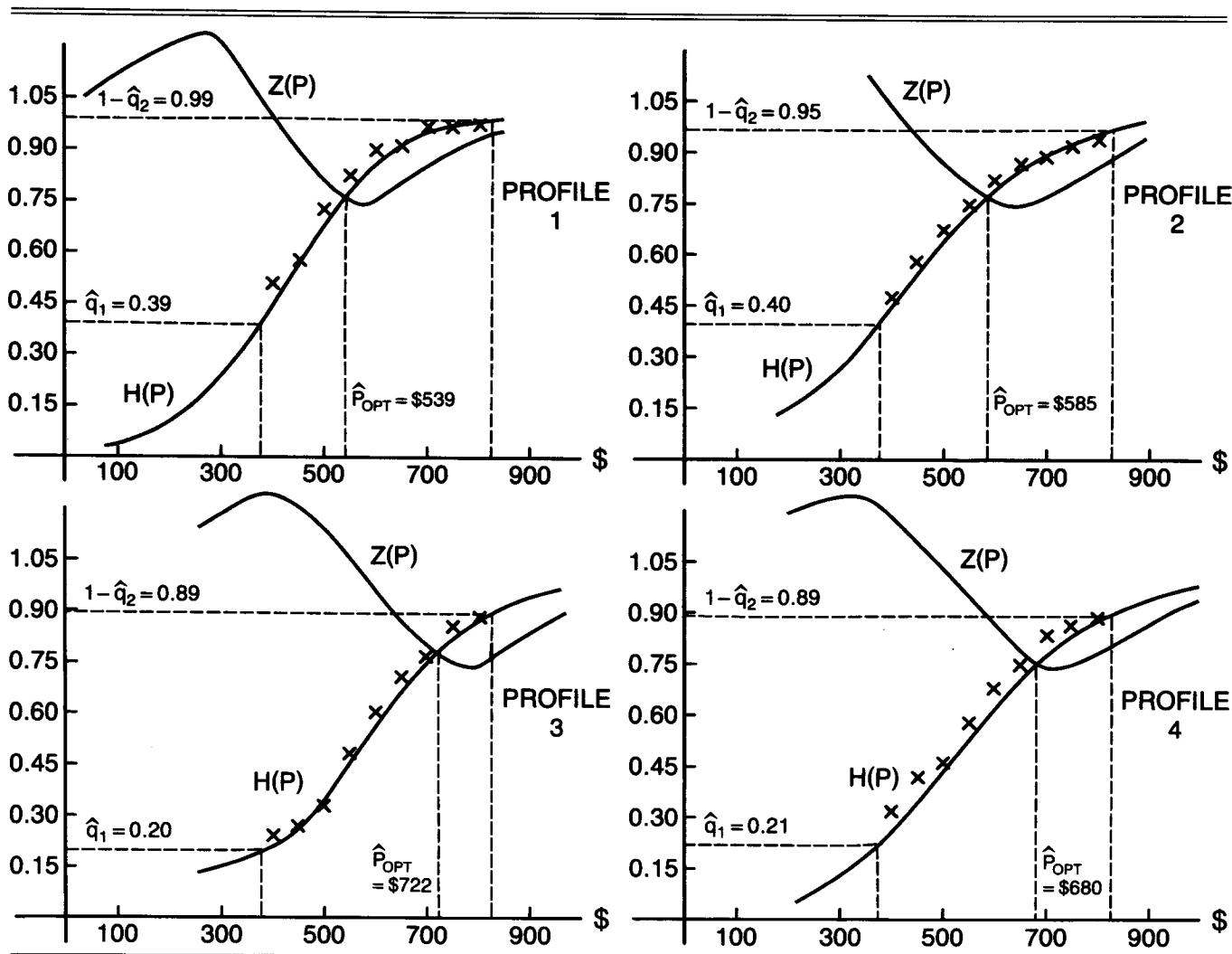
The parameters of the normal distribution $h(p_t)$ of unconditional reservation prices were estimated for the four apartments by the previously described estimation procedure. Figure 1 shows the estimated cumulative density function and the observed cumulative proportions of reservation prices at each of 11 equally spaced points (including the endpoints) in the range $[p_{\min}, p_{\max}] = [\$375, \$825]$. Let M_t denote maximum difference between the empirical cumulative distribution and the estimated cumulative normal distribution of reservation prices for apartment t . Let $prob_t$ denote the probability of obtaining no less than the observed value of M_t if we assume that the unconditional reservation prices have a normal distribution. If the normal distribution provides a good fit to the empirical reservation price distributions, the value of $prob_t$ should be high. The asymptotic value of $prob_t$ (which is valid for large sample sizes) was computed by using the Kolmogorov-Smirnov test (Owens 1962). As shown in the following table, the value of $prob_t$ was large for all four apartments, suggesting that the normal distribution provided a good fit of the empirical distribution to reservation prices.

Concept	q_1	q_2	$\hat{\mu}_t$	$\hat{\sigma}_t$	M_t	$Prob_t$
1	.39	.01	418.33	161.18	.04	.94
2	.40	.05	423.72	209.83	.02	.99
3	.20	.11	582.25	188.67	.05	.76
4	.21	.11	521.38	203.21	.05	.76

To determine the optimal prices, we used an arbitrary variable cost of $c_t = \$400$ per month for each new apartment (the variable cost for apartments can include items such as interest and insurance payments, utilities and maintenance costs, and employee salaries). Recall that increasing the variable cost by a dollar also increases the optimal price by a dollar. Hence, the optimal prices for variable costs different from \$400 can be computed readily by increasing (decreasing) the optimal price by the same amount as the increase (decrease) in the unit variable cost above (below) \$400.

The points of intersection between the estimated cumulative density $H(p_t)$ of the unconditional reservation prices and the function $Z(p_t) = 1 - h(p_t)(p_t - 400)$ are shown in Figure 1. The profit-maximizing monthly rent for the apartment described by concept t is identified by the price p_t at which $H(p_t)$ and $Z(p_t)$ intersect. Note that the only difference between the apartments described by concepts 1 and 3 is that the former apartment is in poor condition and the latter apartment has been thoroughly renovated. The estimated mean reservation price (rent) for the apartment described by concept 3 is approximately \$160 higher, and its estimated optimal monthly rent (assuming a constant variable cost $c_1 = c_3 = \$400$ per month) is more than \$180 higher, than that for the apartment described by concept 1. Also, note that though the apartments described by concepts 2 and 4 are both 30 minutes from campus, the former is safer and less noisy, but smaller and in poorer condition, than the latter. Our analysis suggests that the estimated mean res-

Figure 1
 NORMAL CUMULATIVE DENSITY FUNCTION ESTIMATE OF RESERVATION PRICES, AND OPTIMAL PRICES OF FOUR
 PRODUCT CONCEPTS ASSUMING A UNIT VARIABLE COST OF \$400
 (X = empirical cumulative density of reservation prices)



ervation price is about \$97 higher for the larger apartment in better condition (i.e., for the apartment described by concept 4), and that its profit-maximizing monthly rent (assuming $c_2 = c_4 = \$400$ per month) is higher by \$95, than that for the apartment described by concept 2. Given the market size m , and the values of f_i and c_i for apartment t , the corresponding monthly profit at the optimal rental price $p_{t,opt}$ can be obtained from equation 3. To illustrate, let $m = 1000$. Let the fixed-cost allocations per month be $f_1 = \$20,000$, $f_2 = \$30,000$, $f_3 = \$35,000$, and $f_4 = \$45,000$. Let $c_t = \$400$ per month for $t = 1, 2, 3, 4$. Then the monthly profit at the optimal rent is \$23,090 for apartment 1, \$25,500 for apartment 2, \$59,990 for apartment 3, and \$46,000 for apartment 4.

CONCLUSIONS

We propose a procedure to determine the optimal price of a product by using the data typically obtained by conjoint analysis. The conjoint data are used to infer individual reservation prices. Both an individual's reservation price and the mean reservation price over the target population are assumed to be normally distributed, implying a normal distribution for the unconditional reservation prices. The cumulative density for the unconditional reservation prices is related to the demand function for a product, which in turn is used to determine a price that maximizes the product's profit. As an illustrative

example, we consider the problem of pricing student apartments. Additional applications, particularly in commercial settings, would be useful for further validation of the proposed method.

In our analysis, we assume constant variable cost because (1) it keeps the analysis simple and (2) in most practical applications, it is easier for a user to estimate a range of costs (based on the share-of-choice simulations) than to estimate a cost function that reflects scale effects. Conceptually, however, it is possible to reflect scale effects by modeling cost as a function of demand; that is, $c_i = f(D(p_i))$. However, note that if (as here) the unconditional reservation price distribution is assumed to be normal, the demand, and hence the cost, depends on $H(p_i)$. A closed-form expression for unit cost cannot be obtained in this case. Numerical estimation procedures that can overcome the potential problem of local optima would be useful in extending the model to reflect scale effects.

Another assumption of the proposed model is that each consumer buys one unit of the product. One way to relax this assumption is to define as many "dummy" customers as the number of units purchased by a customer. Each dummy customer then can be assumed to purchase one unit of the product. However, recall that our model also assumes that the individual reservation prices are independent. The introduction of dummy consumers violates this independence assumption. It would be useful to consider alternative methods that incorporate purchase rate differences in the proposed model (e.g., by making the market size parameter m a random variable reflecting purchase rate heterogeneity).

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