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HEURISTICS FOR PRODUCT-LINE DESIGN USING CONJOINT ANALYSIS*

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Recently proposed methods for product-line selection use the total utilities of candidate items to construct product lines maximizing seller's return or buyers' welfare. For conjoint (hybrid conjoint) data, enumerating the utilities of candidate items can be computationally infeasible if the number of attributes and attribute levels is large and most multi-attribute alternatives are feasible. For such problems, constructing product lines directly from part-worths data is preferable. We propose such methods, extending Kohli and Krishnamurti's (1987) dynamic-programming heuristic for selecting a single item maximizing share to structure product lines maximizing share, seller's return, or buyers' (utilitarian) welfare. The computational performance of the heuristics and their approximation of product-line solutions is evaluated using simulated data. Across problem instances, the dynamic-programming heuristics identify solutions that are no worse, in terms of approximating optimal solutions, to the solutions of heuristics for the current two-step approaches to product-line design. An application using hybrid-conjoint data for a consumer-durable product is described.

(MARKETING—PRODUCT POLICY, CONJOINT ANALYSIS; PROGRAMMING—HEURISTICS)

1. Introduction

A number of researchers have recently proposed preference-based procedures for product line design. One approach, exemplified by the work of Sudharshan, May and Gruca (1988), considers product descriptions in terms of multiple, continuous attributes. Assuming an ideal-point model of consumer preferences, these researchers describe a procedure to select a share-maximizing set of multi-attribute product locations using idiosyncratic (or segment level) estimates of consumer preferences. The other approach, pursued by Dobson and Kalish (1988), Green and Krieger (1985), and McBride and Zufryden (1988), considers a finite set of candidate (reference set) items from which a product line is selected. Preference evaluations for each item are used to select a product line maximizing a return function for a seller (the "seller's problem"), or a utilitarian welfare function across buyers (the "buyers' problem"). If the number of candidate items is small, this approach can be implemented by obtaining direct preference evaluations of items from consumers. However, as the number of candidate items increase, direct measurement of preferences becomes difficult. Conjoint (hybrid conjoint) analysis using a finite number of discrete attributes is often used in these cases to estimate idiosyncratic part-worth preference functions from ordinal (combinations of ordinal and cardinal) evaluations of a subset of product profiles selected according to a fractional factorial experimental plan (Green and Srinivasan 1978; Green 1984). The utilities of candidate items, in turn, are estimated from the individual preference functions and are then used to evaluate product lines.

For conjoint data, an explicit enumeration of idiosyncratic utilities from part-worths functions is reasonable if the reference set contains a small number of preselected alternatives, either because product profiles are described using only a few attributes, or because

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a significant fraction of multi-attribute product profiles in larger problems is infeasible due to cost or technological reasons. For problems using a large number of attributes and attribute levels (e.g., problems using hybrid conjoint data), reference-set enumeration can become formidable if most multi-attribute items are feasible. For example, consider the problem of designing a product line for a nondurable product like cereal (or soup, frozen food) using data from a hybrid conjoint study involving 400 consumers and 12 attributes at 5 levels each. A complete enumeration of part-worths utilities for each item and each consumer takes over 250 hours on an FPS computer and about twice as long on a VAX-8600 computer. If a significant fraction of the alternatives are feasible, constructing product lines directly from part worths is preferable because it eliminates the intermediate step of enumerating utilities for reference-set items.

This paper presents a procedure which, similar to Sudharshan, May and Gruca's approach for continuous attributes, structures product lines directly from part-worth preference functions estimated using conjoint or hybrid conjoint analysis. The procedure builds on a previously proposed dynamic-programming heuristic which selects a single, multi-attribute item to maximize share of choices (Kohli and Krishnamurti 1987). We extend the method to (1) select product lines instead of a single item, and (2) to maximize any one of three objectives: share of choices, total utility obtained by buyers (the "buyers' problem"), and total return obtained by a seller (the "seller's problem").

§2 briefly reviews the literature on optimal product-line design. §3 describes the buyers', share-of-choices and seller's problems and formulates them as 0-1 integer programs. §4 presents the proposed heuristics for solving the problems. §5 assesses the computational performance of the heuristics and their approximation of the optimal solution via a Monte Carlo simulation. §6 presents an illustrative application using data from an actual hybrid conjoint study.

2. Background

The problem of optimal product design in a multi-attribute context was first formally considered by Shocker and Srinivasan (1974). These researchers considered the problem of maximizing share of choices in a multidimensional scaling (MDS) context and also discussed the problem of introducing a new product to maximize incremental profit. A number of MDS-based optimal product design procedures were subsequently developed (e.g., Albers and Brockhoff 1977; Eliashberg and Manrai 1989; Gavish, Horsky and Srikanth 1983; Hauser and Simmie 1981; Sudharshan, May and Shocker 1987). Albers (1976) examined the problem of product line design in MDS joint spaces. Recently, Sudharshan, May and Gruca (1988) proposed DIFFSTRAT, a procedure that constructs share maximizing product lines using deterministic or probabilistic choice rules in an ideal-point model. Both the repositioning of old items and the introduction of new ones is permitted. Simulation results suggest that DIFFSTRAT is a useful procedure for practical-sized problems.

Zufryden (1977, 1982) was the first to consider both product and product-line design problems in the context of conjoint analysis. Assuming a deterministic, first-choice model of preferences, he formulated the problems of identifying a share-maximizing product and product line as integer programs. As the problems are computationally intractable (i.e., NP-Hard), Kohli and Krishnamurti (1987) proposed a heuristic procedure for selecting a *single* multi-attribute item. Other recent contributors to the literature on product-line design are Green and Krieger (1985), McBride and Zufryden (1988) and Dobson and Kalish (1988). Like Zufryden, they assume a deterministic, first-choice model of consumer choice. Unlike Zufryden, who formulated the problem of selecting a product line directly from idiosyncratic part-worths data, these researchers assume that the composite utilities/returns of candidate, multi-attribute items are already known.

Green and Krieger first consider a buyers' problem in which a product line is selected to maximize total utility across consumers. They also consider a seller's problem in which a product line is selected to maximize the seller's return, constrained by each buyer's choice of his/her most preferred item from the product line. The problems are NP-Hard and thus they propose heuristic solution procedures. McBride and Zufryden (1988) formulate Green and Krieger's seller's problem as a mathematical program and use integer-programming code to obtain optimal solutions for a special case corresponding to the share-of-choices problem and for small instances of the general seller's problem. The authors note that as the problem is NP-Hard, solving the general problem with larger numbers of reference-set items may be difficult using general integer-programming code, a conclusion in accord with the experience of researchers in operations research (Fisher 1980).

Dobson and Kalish (1988) assume that each consumer's utility for a product is measured in dollars, and that each customer chooses from a set the product that maximizes the difference between the value of the product and its price. A buyer's problem, maximizing buyer welfare, and a seller's problem, maximizing the seller's profit subject to the choice constraint for each buyer, is formulated as a mathematical program. Dobson and Kalish explicitly consider fixed and variable costs in their formulations. Heuristics are described to solve the NP-Hard problems. Computational tests with small problems suggest that the heuristics closely approximate the optimal solution.

Finally, Moorthy (1984) describes a related line of research based on Pigou's theory of price discrimination. Like the conjoint and MDS approaches, Moorthy's model permits self selection by consumers and heterogeneity in consumer preferences. Unlike conjoint and MDS approaches, his model considers at most two (continuous) attributes and does not consider competitive offerings.

The present research departs from the work of Dobson and Kalish, Green and Krieger, and McBride and Zufryden by its focus on selecting product lines directly from attributes and attribute levels. We consider the buyers' and seller's problems, and also Zufryden's (1982) share-of-choices problem. Each problem is formulated as a 0-1 integer program. The problems are noted to be NP-Hard. Thus, like Green and Krieger and Dobson and Kalish, we present heuristic solution procedures. As noted above, the proposed heuristics extend Kohli and Krishnamurti's (1987) procedure for the optimal design of a single product in two ways: (1) from a single product to a product line, and (2) from maximizing share of choices to maximizing share, total utility to buyers, and the return to a seller.

3. Description of Problems

The proposed procedures for all three problems use data from conjoint or hybrid conjoint analysis, a buyer's preferences for hypothetical product profiles being scaled to estimate idiosyncratic, multi-attribute preference functions. For the seller's problem, a multi-attribute preference function is also estimated for the seller. That is, the seller evaluates a hypothetical set of product profiles in terms of the desirability of offering them to buyers. These preferences are used to estimate a seller's multi-attribute preference function. To reflect differences in the desirability of offering the same product to different buyers, the seller's preferences are allowed to vary by buyers (segments of buyers). We limit our attention to a part-worths model of individual preferences, a formulation that is most frequently used in applications of conjoint analysis.

Let $\Omega = \{1, 2, \dots, K\}$ denote the set of K attributes. Let $\Phi_k = \{1, 2, \dots, J_k\}$ denote the set of J_k levels of attribute $k \in \Omega$. Let $\Psi = \{1, 2, \dots, M\}$ denote the set of M items to be selected, where the multi-attribute description of each item is to be determined by solving the buyers', seller's or share-of-choices problem. Let $\Theta = \{1, 2, \dots, I\}$ denote

the set of I buyers. Let w_{ijk} denote the part worth of level $j \in \Phi_k$ of attribute $k \in \Omega$ for consumer $i \in \Theta$.

The *buyers' problem* is to select a product line that maximizes total utility across consumers, each consumer selecting that item in a product line with which he/she associates the highest utility. Let x_{ijkm} be a 0-1 variable which indicates whether (1) or not (0) level $j \in \Phi_k$ of attribute $k \in \Omega$ is assigned to product $m \in \Psi$ and individual $i \in \Theta$. The buyer's problem can be formulated as the following 0-1 integer program:

$$\max \sum_{i \in \Theta} \sum_{m \in \Psi} \sum_{k \in \Omega} \sum_{j \in \Phi_k} w_{ijk} x_{ijkm} \quad (1)$$

subject to

$$\sum_{j \in \Phi_k} \sum_{m \in \Psi} x_{ijkm} = 1, \quad i \in \Theta, \quad k \in \Omega, \quad (2)$$

$$\sum_{j \in \Phi_k} x_{ijkm} - \sum_{j \in \Phi_{k'}} x_{ijk'm} = 0, \quad k' > k, \quad k, k' \in \Omega, \quad i \in \Theta, \quad m \in \Psi, \quad (3)$$

$$x_{ijkm} + x_{i'j'km} \leq 1, \quad i' > i, \quad j' > j, \quad i, i' \in \Theta, \\ j, j' \in \Phi_k, \quad k \in \Omega, \quad m \in \Psi, \quad (4)$$

$$x_{ijkm} = 0, 1 \text{ integer}, \quad i \in \Theta, \quad j \in \Phi_k, \quad k \in \Omega, \quad m \in \Psi. \quad (5)$$

Constraint (2) requires that, across products, only one level of an attribute be associated with an individual. Constraint (3) requires that across attributes, the level assigned to an individual $i \in \Theta$ must correspond to the same product. Constraint (4) requires that the same level of an attribute must be specified for all individuals assigned to a product. Together, constraints (2), (3) and (4) require that each consumer be assigned one of the selected multi-attribute items. The objective function (1) selects the items to maximize the total utility across consumers, the maximization objective ensuring that each individual is assigned an item in the product line from which he/she obtains the highest utility.

The buyers' problem requires interpersonal utility comparisons, which are not reflected in interval-scaled conjoint data. Green and Krieger (1985) suggest treating normalized utilities *as if* they are interpersonally comparable. Gupta and Kohli (1990) compare via simulation the rankings of items in terms of their interpersonally comparable and normalized utilities. They report that the two sets of rankings are significantly correlated and that it may be suitable to implement the buyers' problem using normalized conjoint data.

In contrast to the buyers' problem, the share-of-choices and seller's problems consider a status-quo brand for each buyer. Let j_k^* denote the level of attribute $k \in \Omega$ that appears in the product profile of the status-quo brand for buyer $i \in \Theta$. Let $c_{ijk} = w_{ijk} - w_{ij_k^*k}$ denote the part worth of level j of attribute k relative to the part worth of level j_k^* of attribute k for consumer i , $i \in \Theta$, $j, j_k^* \in \Phi_k$, $k \in \Omega$. A buyer selects a product profile over status-quo only if its relative part worth utility is strictly positive.

As a number of sample consumers (i.e., the size of Θ) is fixed, the *share-of-choices problem* corresponds to maximizing the number of buyers in Θ who associate a higher utility with at least one item in the product line than with their status-quo alternative. Idiosyncratic importance weights (a_i) are associated with buyers to reflect differences in their purchase or consumption rates. The cannibalization of share from currently-offered brands is controlled by maximizing share of choices over the subset $\Theta' \subset \Theta$ of buyers whose status-quo brand is not offered by the seller (Kohli and Krishnamurti 1987).

Let each consumer's interval-scaled part worths be normalized to sum to 1. It follows that each product profile has a part-worths utility no larger than 1 and a relative part-worths utility that lies between -1 and 1 . Let x_{jkm} be a 0-1 variable which denotes

whether (1) or not (0) level $j \in \Phi_k$ of attribute $k \in \Omega$ appears in item $m \in \Psi$. The share-of-choices problem can be formulated as the following 0-1 integer program:

$$\min \sum_{i \in \Theta'} a_i x_i \quad (6)$$

subject to

$$\sum_{j \in \Phi_k} x_{jkm} = 1, \quad k \in \Omega, \quad m \in \Psi, \quad (7)$$

$$\sum_{k \in \Omega} \sum_{j \in \Phi_k} c_{ijk} x_{jkm} + x_{im} > 0, \quad i \in \Theta', \quad m \in \Psi, \quad (8)$$

$$x_i - \sum_{m \in \Psi} x_{im} \geq 1 - M, \quad i \in \Theta', \quad (9)$$

$$x_{jkm}, x_{im}, x_i = 0, 1 \text{ integer}, \quad i \in \Theta', \quad j \in \Phi_k, \quad k \in \Omega, \quad m \in \Psi. \quad (10)$$

Constraint (7) requires each item to be described by one level of each attribute. Observe that since the relative part-worth utility of an item cannot be less than -1 , $x_{im} = 1$ is sufficient to always satisfy constraint (8). Also, constraint (8) restricts x_{im} to be 1 only if item $m \in \Psi$ provides consumer $i \in \Theta'$ no higher utility than status quo. For each individual $i \in \Theta'$, constraint (9) restricts x_i to be 1 only if $x_{im} = 1$ for all $m \in \Psi$ (i.e., if none of the selected items has higher than status-quo utility for consumer i). The objective function (6) minimizes the number of instances (weighted by a_i) in which x_i is 1, and hence maximizes the (weighted) number of instances in which a consumer prefers at least one selected item to status quo.

The *seller's problem* maximizes the marginal utility the seller obtains by offering a product line of M items. The seller is permitted to have any current offerings on the market. Let v_{ijk} denote the part worth reflecting the seller's return if buyer $i \in \Theta$ purchases a product in which level $j \in \Phi_k$ of attribute $k \in \Omega$ appears. Let d_{ijk} denote the *marginal return* the seller associates with level $j \in \Phi_k$ of attribute $k \in \Omega$ for individual $i \in \Theta$, where $d_{ijk} = v_{ijk} - v_{ijk^*}$ if the i th buyer switches from a status-quo brand offered by the seller, and $d_{ijk} = v_{ijk} - v_{ijk^*}$ if the buyer switches from a status-quo brand offered by a competitor. The total marginal return to the seller is maximized assuming that each buyer selects that item, if any, from the product line which has the highest incremental utility over status quo. As in the formulation for the buyers' problem, let x_{ijkm} be a 0-1 variable which indicates whether (1) or not (0) level $j \in \Phi_k$ of attribute $k \in \Omega$ is assigned to product $m \in \Psi$ and individual $i \in \Theta$. The seller's problem can be formulated as the following 0-1 integer program:

$$\max \sum_{i \in \Theta} \sum_{m \in \Psi} \sum_{k \in \Omega} \sum_{j \in \Phi_k} d_{ijk} x_{ijkm} y_i \quad (11)$$

subject to

$$\sum_{m \in \Psi} \sum_{k \in \Omega} \sum_{j \in \Phi_k} w_{ijk} (x_{ijkm} - x_{i'jkm}) \geq 0, \quad i \neq i', \quad i \in \Theta, \quad (12)$$

$$y_i \sum_{m \in \Psi} \sum_{k \in \Omega} \sum_{j \in \Phi_k} w_{ijk} x_{ijkm} \geq y_i u_i^*, \quad i \in \Theta \quad (13)$$

$$y_i = 0, 1 \text{ integer and } (2), (3), (4), (5).$$

As for the buyer's problem, constraints (2)–(5) require that each individual be assigned one item, and that each item be described in terms of one level of each attribute. Constraint (12) requires that an individual be assigned to that selected item with which he/she associates the highest utility. Constraint (13) ensures that the seller obtains a return from consumer i only if the new product assigned to the consumer has higher utility than the utility u_i^* of his status quo item. The objective function (11) selects M items to maximize the seller's return, subject to the choice constraint for each buyer.

4. Solution Procedures

Each of the preceding problems is NP-Hard.¹ We therefore propose heuristics for their solution. Conceptually, the heuristics mimic a dynamic program, using attributes as stages and attribute levels as states. For each level of attribute $k \leq K$, step $(k - 1)$ of the heuristic constructs a line of MJ_{k-1} "partial" product profiles, the profiles being "partial" in the sense that they are described by the levels of only the k attributes considered by step $k - 1$. Each partial profile containing level j of attribute k is obtained by augmenting by level j the MJ_{k-1} partial profiles constructed at step $(k - 2)$ of the heuristic. Of these MJ_{k-1} candidate partial profiles, M are selected for level j of attribute k as follows. For the buyers' problem, a partial profile is selected if it maximizes the incremental sum of utilities across consumers, each consumer being assumed to select an available partial profile which gives him/her the highest utility. For the share-of-choices problem, a partial profile is selected to maximize the incremental number of consumers for whom at least one selected partial profile has positive relative-part-worths utility. For the seller's problem, a partial profile is selected if it maximizes the incremental partial return the seller obtains when each consumer picks an available partial profile with the highest, positive relative-part-worths utility.

Let $1, 2, \dots, K$ denote an arbitrary ordering of the attributes. Let $[0]$ denote a column vector of MJ_{k-1} zero elements. Let $[1]'$ denote all (conformable) row vectors of unit elements. We describe the heuristics more formally below.

The Buyers' Problem

Let $W(k)$ denote the individuals by attribute levels matrix of part worths for attribute $k \in \Omega$. Let $W_j(k)$ denote the j th column of $W(k)$.

Initialization: $S^*(1) = W(1)$, $W(K + 1) = [0]$.

Recursion:

$$S_j(k) = S^*(k - 1) + [W_j(k)][1]', \quad j \in \Phi_k,$$

$$S^*(k) = [S_1^*(k)S_2^*(k) \cdots S_{j_k}^*(k)],$$

where $S_j^*(k) = S_j(k)$ if $S_j(k)$ has no more than M columns. Otherwise, the p th column of $S_j^*(k)$, $1 \leq p \leq M$, is obtained as follows. For each row, identify the element with the largest value among all previously selected $p - 1$ columns and a candidate p th column. Select the candidate column for which the sum of these largest elements across the p columns is the greatest. The following criteria are used sequentially to resolve ties among columns: (1) select the column(s) with the largest sum of positive elements, (2) select the column(s) with the greatest number of positive elements, and (3) randomly select a column.

Termination: Stop if $k = K + 1$.

The im th element of $S^*(K + 1)$ corresponds to the i th buyer's part-worths utility for the m th item in the selected product line. Across the M items, each buyer selects an item with which he/she associates the highest utility.

The Share-of-Choices Problem

The preceding heuristic for the buyers' problem is altered in three ways to solve the share-of-choices problem. *First*, to account for the cannibalization of share of products currently offered by a firm, only the preferences of buyers $i \in \Theta'$ are considered. *Second*, the relative part worths, rather than the part worths, are used to evaluate the items. *Third*,

¹ The proof for the share-of-choices problem consists of showing that an NP-Hard problem called Maxsat (Kohli and Krishnamurti 1989) is transformed in a polynomial number of steps to the share-of-choices problem with $M = 1$. The (NP-Hard) K -Median problem is transformed in a polynomial number of steps to the buyer's problem with $K = 1$, and hence the buyers' problem is NP-Hard. Finally, the buyer's problem is transformed to the seller's problem with $d_{ijk} = c_{ijk}$ for all $i \in \Theta$, $j \in \Phi_k$, $k \in \Omega$. Hence the latter problem is also NP-Hard.

the $S^*(k)$ matrix is constructed by selecting columns with the largest incremental number of positive elements in $S_j(k)$.

Let $C(k)$ denote the buyers by attribute levels matrix of relative part worths c_{ijk} for attribute $k \in \Omega$. Let $C_j(k)$ denote the j th column of $C(k)$.

Initialization: $S^*(1) = C(1)$, $C(K+1) = [0]$.

Recursion:

$$S_j(k) = S^*(k-1) + [C_j(k)][1]', \quad j \in \Phi_k,$$

$$S^*(k) = [S_1^*(k)S_2^*(k) \cdots S_k^*(k)],$$

where $S_j^*(k) = S_j(k)$ if $S_j(k)$ has no more than M columns. Otherwise, select the p th column of $S_j^*(k)$ from $S_j(k)$ to maximize the (weighted by a_i) number of positive elements among the subset of rows with nonpositive elements in all previously selected columns, $1 \leq p \leq M$. The following criteria are used sequentially to resolve ties among columns: (1) select the column(s) with the highest (weighted) number of nonnegative elements, (2) select the column(s) with the highest (weighted) sum of positive elements, and (3) randomly select a column.

Termination: Stop if $k = K+1$.

The im th element of $S^*(K+1)$ corresponds to the i th buyer's part-worths utility for the m th item in the selected product line. If all M entries in row i of $S^*(K+1)$ are nonpositive, buyer i selects none of the M items over status quo. Otherwise, he/she selects the item with which he/she associates the highest utility among the M items identified by $S^*(K+1)$. The share of choices for a selected item is the fraction of positive elements in the corresponding column of $S^*(K-1)$. The total share for the selected product line is the fraction of rows of $S^*(K+1)$ that have at least one positive element.

The Seller's Problem

Let $D(k)$ denote the buyers by attribute levels matrix of the seller's marginal returns, the ij th entry denoting the marginal return to the seller from level j of attribute k . As before, let $C(k)$ denote the buyers by attribute levels matrix of relative part worths. The dynamic-programming heuristic for the seller's problem is described as follows.

Initialization: $S^*(1) = C(1)$, $T^*(1) = D(1)$, $S^*(K+1) = T^*(K+1) = [0]$.

Recursion:

$$S_j(k) = S^*(k-1) + [C_j(k)][1]',$$

$$T_j(k) = T^*(k-1) + [D_j(k)][1]', \quad j \in \Phi_k,$$

$$S^*(k) = [S_1^*(k)S_2^*(k) \cdots S_k^*(k)],$$

$$T^*(k) = [T_1^*(k)T_2^*(k) \cdots T_k^*(k)],$$

where $S_j^*(k) = S_j(k)$ and $T_j^*(k) = T_j(k)$ if $S_j(k)$ and $T_j(k)$ have no more than M columns. Otherwise, select the p th column of $S_j^*(k)$ and $T_j^*(k)$, $1 \leq p \leq M$, as follows. For each row i of $S_j(k)$, identify $s_{ij}(k)$, the largest value among the candidate p th column and all $p-1$ previously-selected columns. Let $t_{ij}(k)$ denote the element in $T_j(k)$ that corresponds to $s_{ij}(k)$. Let

$$h_{ij}(k) = \begin{cases} t_{ij}(k), & \text{if } s_{ij}(k) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Select that candidate column for which $\sum_{i \in \Theta} h_{ij}(k)$ is the largest. The following criteria are used sequentially to break ties among candidate columns: (1) select the column(s) with the largest sum of positive elements, (2) select the column(s) with the largest number of positive elements, (3) select the column(s) with the largest number of nonnegative elements, and (4) randomly select a column.

Termination: Stop if $k = K + 1$.

If all M elements in row i of $S^*(K + 1)$ are nonpositive, buyer i prefers his/her status-quo to all items in the product line. Otherwise, buyer i prefers the item corresponding to the column in which the largest positive value occurs. Let this column be denoted m . The seller's return from buyer i is $t_i^* = t_{im}^*(K + 1)$, the im th element of $T^*(K - 1)$, and the total return across buyers is $\sum_{i \in \Theta} t_i^*$.

Example

To illustrate the structure of the dynamic-programming heuristics, consider a small example for the buyers' problem with $I = 3$ consumers and $K = 3$ dichotomous attributes. Assume that a product line of $M = 2$ items is to be selected from the 8 possible product profiles. Let the individual by attribute levels matrices of part worths be

$$W(1) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad W(2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad W(3) = \begin{pmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

Initialization: $S^*(1) = W(1)$, $W(4) = (0 \ 0 \ 0 \ 0)'$.

Step 1.

$$S_1(2) = S^*(1) + W_1(2)[1] = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (1 \ 1) = \begin{pmatrix} 2 & 1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$S_2(2) = S^*(1) + W_2(2)[1] = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (1 \ 1) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Each element in the first (second) column of $S_j(2)$ corresponds to the sum of an individual's part worths for level 1 (level 2) of attribute 1 and level j of attribute 2, $j = 1, 2$. Because $M = 2$ and $S_j(2)$ contains two columns, $S_j^*(2)$ contains both columns of $S_j(2)$, $j = 1, 2$; i.e., $S_1^*(2) = S_1(2)$, $S_2^*(2) = S_2(2)$. Thus

$$S^*(2) = [S_1^*(2) S_2^*(2)] = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

Step 2.

$$S_1(3) = S^*(2) + W_1(3)[1]'$$

$$= \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} (1 \ 1 \ 1 \ 1) = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 2 & 2 & 3 & 3 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

$$S_2(3) = S^*(2) + W_2(3)[1]'$$

$$= \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} (1 \ 1 \ 1 \ 1) = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 2 & 3 & 2 & 3 \end{pmatrix}.$$

To construct $S_1^*(3)$, $M = 2$ columns are selected from $S_1(3)$. As each column of $S_1(3)$ has the same sum (=4), column 2 with the highest number of positive elements is first selected. Columns 1, 3 and 4 of $S_1(3)$ are candidate second columns for $S_1^*(3)$. For each row, the larger of the two elements in column 2 and a candidate column $j = 1, 3$, or 4 of $S_1(3)$ is

CANDIDATE COLUMN			
	$j = 1$	$j = 3$	$j = 4$
Row 1	2	1	1
Row 2	2	3	3
Row 3	1	1	1

The sum of these elements across rows is the same ($=5$) for each candidate column. Thus columns 1, 3 and 4 of $S_1(3)$ are tied as second candidate columns for $S_1^*(3)$. Each of the candidate columns also has 2 positive elements, so that the secondary criterion of selecting a column with the largest number of positive elements also does not resolve the tie. One of columns 1, 3 or 4 of $S_1(3)$ is therefore randomly selected. Assume that this is column 1. Then

$$S_1^*(3) = \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}.$$

To construct $S_2^*(3)$, $M = 2$ columns are selected from $S_2(3)$. As each column of $S_2(3)$ has the same sum ($=4$), column 3 with the highest number of positive elements is first selected. Columns 1, 2 and 4 of $S_2(3)$ are candidate second columns for $S_2^*(3)$. For each row, the larger of the two elements in column 3 and a candidate column $j = 1, 2$, or 4 of $S_1(3)$ is

CANDIDATE COLUMN			
	$j = 1$	$j = 2$	$j = 4$
Row 1	2	1	1
Row 2	1	1	1
Row 3	2	3	3

As the sum across rows of these elements is the same ($=5$) for each candidate column and as each candidate column has 2 positive elements, one of columns 1, 2 or 4 of $S_2(3)$ is randomly selected as the second column of $S_2^*(3)$. Assume that this is column 1. Then

$$S_2^*(3) = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 2 & 2 \end{pmatrix},$$

and

$$S^*(3) = [S_1^*(3)S_2^*(3)] = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 2 & 1 & 0 \\ 1 & 0 & 2 & 2 \end{pmatrix}.$$

Step 3. $S_1(4) = S^*(3) + W_j(4)[1]' = S^*(3) + [0][1]' = S^*(3)$. Hence $S^*(4)$ is constructed by selecting $M = 2$ columns from $S_1(4) = S^*(3)$. As each column of $S^*(3)$ has the same sum ($=4$), either column 1 or column 3 with the highest number of positive elements is first selected. Assume that column 1 is selected. Columns 2, 3 and 4 of $S^*(3)$ are candidate second columns for $S^*(4)$. Across column 1 and a candidate column $j = 2, 3$, or 4 of $S^*(3)$, the larger value in rows 1–3 is as follows:

CANDIDATE COLUMN			
	$j = 2$	$j = 3$	$j = 4$
Row 1	2	1	2
Row 2	2	2	2
Row 3	1	2	2

The sum across rows of these elements is the greatest ($=6$) for candidate column 4, which is selected as the second column of $S^*(4)$. Thus

$$S^*(4) = S_1^*(4) = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 2 \end{pmatrix}.$$

Each column of $S^*(4)$ corresponds to a product profile and each consumer obtains a utility of 2 from his/her more preferred of the two selected items. The total utility across consumers is 6. It can be verified that the optimal solution in this instance provides a total utility of 7 across consumers. The profiles corresponding to each column of $S^*(4)$ are identified by tracking the k levels associated with each column of $S_j(k)$, $k \leq K$. It can be verified that the item corresponding to the first column of $S^*(4)$ is identified by levels 2, 1 and 1 of attributes 1, 2 and 3, respectively. The second heuristic item corresponds to levels 1, 1 and 2 of attributes 1, 2 and 3, respectively. The optimal items can be verified to correspond to levels 2, 2, 2 and 1, 2, 1 of the first, second and third attributes, respectively.

Eliminating Infeasible Items

As described above, the heuristics can select product profiles that are infeasible from a technological or cost standpoint. Thus it is important to incorporate a mechanism in the heuristics to eliminate infeasible alternatives from the solution. We describe such a mechanism below.

Let N denote the number of infeasible product profiles. Let K denote the last attribute considered by the heuristic. Let level j of attribute K appear in N_{jK} infeasible product profiles. It can be verified that each column of $S^*(K)[T^*(K)]$ corresponds to a product profile, the j th set of M columns describing product profiles in which level j of attribute K appear. In turn, the M product profiles for each level of attribute K are selected from the MJ_{K-1} columns of $S_j(K)[T_j(K)]$. Hence if $MJ_{K-1} \geq N_{jK} + M$, $j \in \Phi_K$, then at least M columns in each $S_j(K)$ ($T_j(K)$) must correspond to feasible product profiles, permitting only those columns that correspond to feasible items to be selected for $S^*(K)$ ($T^*(K)$). This condition must be satisfied for each level of attribute K . Let l denote a (not necessarily unique) level of attribute K that, across all its levels, appears in the largest number of infeasible product profiles; i.e., $N_{lK} = \max \{N_{jK} : j \in \Phi_K\}$. To ensure a feasible product line, it is sufficient to satisfy $MJ_{K-1} \geq N_{lK} + M$. As a smaller value of N_{lK} , and a larger value of J_{K-1} , makes it more likely that the condition is satisfied, choose attribute K so that N_{lK} is the smallest across attributes, and choose attribute $K-1$ so that J_{K-1} is the largest among all attributes, except attribute K . If the condition is still not satisfied, construct a composite $(K-1)$ st attribute (Green and Srinivasan 1978), ensuring that it is.

5. Performance Evaluation

A Monte Carlo simulation is performed to assess the performance of the proposed heuristics. Problems are generated using a 3^4 experimental plan involving the number of attributes (4, 5, 6), levels per attribute (2, 3, 4), consumers (50, 100, 150) and items in the product line (2, 3, 4) as the design factors. Of the 81 possible problems, subsets of 51, 45 and 43 problems are solved for the share-of-choices, buyers' and seller's problem, respectively. The remaining problems are not solved because the computational time is exorbitant for enumeration, which is used to identify optimal solutions.

Four replicates are solved for each problem. The buyers' part worths and the seller's idiosyncratic returns are randomly generated from a uniform distribution on (0, 1), then normalized within respondent.² The normalized part worths are assumed interpersonally

² Examples of simulations using random data from a (continuous or discrete) uniform distribution to evaluate the performance of heuristics are Green and Kreiger (1985), Moily (1986) and Lee and Guignard (1988). The buyers' utility, share-of-choices and seller's return for an optimal product line follow the extreme-value distribution of a statistic with a fixed maxima (Gumbel 1958). This distribution is independent of the distribution giving rise to data, and hence does not depend upon whether a uniform, or some other, distribution is used to generate the part worth.

TABLE 1

Empirical Performance of Dynamic-Programming Solutions Relative to Optimal, Greedy and Random Solutions

	Average Performance-Ratio for		99% Confidence Range: Highest DP Solution Across Attribute Orderings	% Problems DP Solution Value No Smaller Than:		Ratio of DP Solution Value to Greedy Solution Value	
	Highest DP Solution Value Across Attribute Orderings	Lowest DP Solution Value Across Attribute Orderings		Greedy Solution Value	Random Solution Value	Smallest	Largest
	Buyers' Problem	0.987		0.962	0.986-0.989	91.67%	98.76%
Share-of-Choices Problem	0.989	0.921	0.987-0.991	94.61%	99.95%	0.95	1.09
Seller's Problem	0.987	0.849	0.983-0.990	64.5%	87.4%	0.90	2.40

comparable for the buyers' problem. Three currently available multi-attribute items are also randomly generated for each replicate. For the share-of-choices and seller's problems, a randomly selected item is specified as being currently offered by the seller. Each buyer's most preferred item among those currently available is specified as his/her status quo. All product profiles are assumed feasible. Equal importance weights are assigned to consumers for the share-of-choices problem.

For each problem, 24 distinct attribute orderings are specified. All $4! = 24$ possible attribute orderings are used for the 4 attribute problem. For each other problem, the orderings are randomly selected. The heuristic is implemented for each attribute ordering and the largest solution selected. Each problem is also solved using Green and Krieger's greedy heuristic, the individual utilities for items being enumerated from the part worths. The dynamic-programming solution is compared to the greedy solution, and to the optimal solution obtained by complete enumeration. Also, 100 random solutions are generated for each problem instance. The number of cases in which the dynamic-programming heuristic obtains a lower value than the random solutions is determined to assess if the proposed heuristics, if they perform well, do so only because *any* solution closely approximates the optimal.

Table 1 summarizes the results of the simulation. Across problem instances, the average value of the best dynamic-programming solution over attribute orderings exceeds 0.98 of the optimal for each of the buyers', share-of-choices and seller's problems. As the average value of the lowest dynamic-programming solution across the attribute orderings is as low as 0.849 of the optimal for the seller's problem, implementing the proposed heuristics across alternative attribute orderings does appear to improve their performance.³ Also, the comparison with the greedy solutions suggest that using the proposed heuristic to select product lines leads to solutions whose value is generally no worse than the solutions obtained by a two-step greedy. Finally, the dynamic-programming solutions seldom have a value less than the value of random solutions. This suggests that the good performance of the dynamic-programming heuristic is not a result of the data being such that random solutions reasonably approximate the optimal solution.

As the distribution of the performance ratio is not known, a bootstrap resampling procedure (Efron 1979) is used to estimate its confidence ranges for each problem. Briefly,

³ The ability of heuristics to identify near-optimal solutions is discussed by Fisher (1980). For examples of heuristics that exhibit similar empirical performance, see, e.g., Corneujols, Fisher and Nemhauser (1977), Green and Kreiger (1985), Dobson and Kalish (1988) and Lee and Guignard (1988).

the bootstrap uses the empirical distribution of the performance ratio as an approximation to its population distribution, and uses Monte Carlo simulation to make inferences regarding a statistic of the performance ratio. In particular, consider the bootstrap distribution of the mean of the 180 observations (4 replicates \times 45 problem types) of the sample performance ratio for the buyers' problem. The bootstrap resamples with replacement 180 times from the sample observations, computes the resampled mean performance ratio, and repeats the resampling a large number of B times. In the present instance, $B = 1000$. The reference distribution of the resampled means is then used to assess confidence ranges. A similar procedure is used for the share-of-choices and seller's problems. The 99 percent confidence range for the mean performance ratio is 0.986–0.989 for the buyers' problem, 0.987–0.991 for the share-of-choices problem and 0.983–0.990 for the seller's problem. These results suggest that the dynamic-programming heuristics perform well in approximating the optimal solution across problem instances. However, note that these results are valid only for problems described by the simulation parameters (i.e., the problem sizes and data generating mechanisms used).

A stepwise regression is performed to assess if the performance ratio for the heuristic systematically varies by (1) the type of problem (i.e., buyers', share of choices or seller's), (2) the number of product profiles, (3) the number of product in the line and (4) the number of individuals. The final equation retains only the number of profiles ($\beta = -0.0000239$, std. err. = 0.00000344) and the number of items in the product line ($\beta = -0.00234$, std. err. = 0.00234) as the significant factors. However, it explains little variation in the performance ratio ($R^2 = 0.0805$, adjusted $R^2 = 0.0772$). Thus while the number of product profiles and the number of items in the product line have statistically significant effects on the performance ratio, the amount by which the performance ratio decreases as these variables increase is small and these factors account for little variation in the performance ratio.⁴

The computational time for the heuristic can be shown to be proportional to $KJMR(JI + \log M)$, where K is the number of attributes, J the average number of levels per attribute, M the number of items in the product line, I the number of consumers and R the number of attribute orderings for which the heuristic is implemented. The computational time for enumeration can be shown to be proportional to $IJ^{KM}/(M - 1)! + M \log M$. Hence the ratio, t , of the heuristic to enumeration computational-time ratio is proportional to $IRJ^{KM}/M!JK(JI + \log M)$ for $J + I \gg \log M$.⁵ This implies that the computational time for the heuristic increases at a much slower rate than the computational time for enumeration. The above noted relationship between t and the problem parameters is verified using OLS regression in which $\log t$ is used as the dependent measure ($R^2 = 0.995$). Both the heuristic and enumeration solve the smallest buyers', share-of-choices and seller's problems in less than a second. For the largest problem instances, the heuristic solutions are obtained in less than 13 seconds. In contrast, the computational time for enumeration is 8.13 hours for the buyers' problem, 5.54 hours for the share-of-choices problem and 3.70 hours for the seller's problem. Thus for large problems, the heuristic significantly outperforms enumeration in terms of computational time.

⁴ Two alternative regression models are also tested. In the first, the number of profiles is replaced by the number of attributes and number of attribute levels as independent variables. In the second, the number of possible product lines replace the number of profiles and number of items in the product line as an independent variable. The R^2 for both analyses are lower than for the reported regression. The substantive conclusions remain unchanged.

⁵ Details of the computational time performance of the heuristic and complete enumeration can be obtained from the first author.

6. Application

As an illustrative example, we describe an application of the heuristics to design product lines of an actual consumer durable product. For expository purposes, we consider the product to be telephones. Sixteen attributes are used in the study, eight attributes at four levels each, seven attributes at three levels each and one attribute at two levels. Hybrid conjoint data are used, the idiosyncratic part worths being estimated for each of 187 sample consumers. A single return function is estimated for the seller across customers, the marginal return for each level of each attribute being estimated as the marginal profit of the attribute level over a base level.

Enumerating utilities for all $4^8 \times 3^7 \times 2$ product profiles requires an exorbitant amount of computational time, exceeding 600 hours on an FPS computer. A two-step approach, which first requires enumeration of the reference set, therefore is not possible for product line structuring in this case if all possible product profiles are of interest. In contrast, the dynamic-programming heuristic takes seconds to construct a product line of $M \leq 4$ items.

To assess how well the heuristic solutions compare to the optimal, we analyze a sub-problem involving 6 dichotomous attributes for which the optimal product line for $M \leq 4$ items can be obtained via enumeration on an FPS computer in approximately 5 hours. The attributes selected are: (1) Rotary Dial/Push Button, (2) Pulse/Tone, (3) No Memory/Memory, (4) No Redial/Redial, (5) Wall-mountable/Desk Telephone, and (6) Red/White Color. For each customer, a status-quo telephone, assumed to be offered by a firm other than the seller's, is described by the profile (1) rotary, (2) pulse, (3) no memory, (4) redial, (5) wall mountable and (6) white. For the largest product line of 4 items, the heuristic solutions across 24 attribute orderings are obtained in 40.8 seconds, 39.36 seconds and 67.2 seconds, for the buyers', share-of-choices and seller's problems, respectively. The corresponding computational times for enumeration are 3.8 hours, 2.5 hours and 5.17 hours. The performance ratio, r , of the heuristic solution to the optimal solution is as follows for each problem:

	$m = 1$	$m = 2$	$m = 3$	$m = 4$
Buyers'	1	0.98	1	1
Share-of-Choices	1	0.94	0.979	0.966
Seller's	1	0.998	0.996	0.989

For the smallest value of $r = 0.94$ ($m = 2$, share-of-choices problem), the optimal solution consists of (1) a white, rotary, wall mountable phone with pulse, no redial and no memory, and (2) a red, push button, desk phone with tone, redial and no memory. The heuristic selects a phone which differs from the first profile above in terms of having tone rather than pulse, and differs from the second also in terms of color (white instead of red), and mountability (wall mountable instead of desk). More than any other, the pulse/tone difference has significant implications for the technological skills required to design a telephone. In the present instance, the heuristic selects the tone feature. However, if the heuristic had selected the pulse feature, and if this technology was infeasible for the firm, alternative product lines with the next-highest share could be identified by selecting multiple, instead of a single, product line. Alternatively, the effect of substituting pulse for tone on the share of choices could be directly simulated, keeping the levels of each other attribute fixed in the product line.

7. Conclusion

In contrast to two-step procedures, which are best suited to problems in which the reference set of items is small, we propose one-step methods for structuring product lines

using conjoint (hybrid conjoint) data when the number of attributes and attribute levels used are large, and most multi-attribute alternatives are feasible. Because they are implemented in a computationally efficient manner and provide near-optimal solutions, the proposed heuristics also are useful for efficiently performing sensitivity analyses (e.g., assessing the robustness of solutions to perturbations in utility function estimates and across subsets of sample consumers). Also, like previous work, the proposed procedures assume a fixed set of competitive offerings. Models that explicitly consider actions and reactions by firms need to be considered in future research. Further, the present models on product-line selection assume that a buyer selects at most one of a set of substitutable items. Choices of multiple items reflecting demand complementarity also need to be considered. Finally, the various conjoint and MDS models for optimal product design assume that consumer preferences remain stable, even as the set of market offerings change. This is perhaps an appropriate assumption for mature markets. Alternative models that consider changes in preferences over time are also important to examine in future research.⁶

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