Rev Acc Stud (2007) 12:227–256
DOI 10.1007/s11142-007-9026-3

Using accounting information for consumption planning and equity valuation

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Published online: 8 March 2007

Abstract This article develops a consumption-based valuation model that treats earnings and cash flow as complementary information sources. The model integrates three ideas that do not appear in traditional valuation models: (i) earnings provide information about future shocks to cash flow; (ii) earnings contain indiscernible transient accruals; and (iii) investors use cash flow and earnings to make allocation and consumption decisions and set price. Accordingly, the quality of earnings affects production and consumption as well as price. Among other implications, the model reveals that a valuation coefficient is not just a capitalization factor; it is the product of a capitalization factor and a structural factor reflecting earnings quality and accounting bias.

JEL Classifications D51 · E21 · G12 · G14 · M21 · M41

Keywords DCF · Earnings quality · CAPM · Equity risk premium · Resource allocation

Introduction

According to the Financial Accounting Standards Board (1978, para 37–39), the primary objective of financial reporting is to provide information to help

This article was presented at the *2006 Review of Accounting Studies Conference* in Fontainebleau, France, the 2003 NYU-Columbia Conference, the 2005 AAA Meeting, and the 2006 FARS meeting. An earlier draft was titled "Using Cash Flow to Supplement Earnings Information in Equity Valuation."

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investors, creditors, and others assess the amount and timing of prospective cash flows. The FASB asserts that ‘‘earnings and its components measured by accrual accounting generally provide a better indication of enterprise performance than does information about current cash receipts and payments’’ (SFAC, 1978, para 44). Accruals contribute information not found in current cash flows because ‘‘accruals reflect management's expectation about future cash flows’’ (Beaver, 1998, para 6). Consistent with this view, numerous researchers have documented that current earnings dominate current cash flow as predictors of future cash flows (Dechow, Kothari, & Watts, 1998; Finger, 1994; Greenberg, Johnson, & Ramesh, 1986; Livnat & Zarowin, 1990; Lorek & Willinger 1996; Penman & Yehuda, 2003).

Nonetheless, the information content of earnings does not subsume that of cash flow. In large sample studies, Barth, Cram and Nelson (2001) show that current cash flow contributes incremental information that supplements accruals information for predicting future cash flows. Dechow and Dichev (2002) suggest that accruals contain estimates of future cash flows, and argue that the quality of accruals determines the relative predictive abilities of earnings and cash flow. Barth, Beaver, Hand, and Landsman (2004) find some reduction in mean prediction errors when earnings are disaggregated into cash flow and total accruals, and even more reduction when total accruals are further disaggregated. Observing that earnings are about three times more volatile than cash flows, Givoly and Hayn (2000) suggest that, even if earnings might be a superior information attribute, it is more difficult to forecast than cash flow. Hence, it is difficult to rely exclusively on earnings forecasts for equity valuation. In the same spirit, Lev, Li and Sougiannis (2005) argue that earnings contain enough subjectivity that cash flows make better valuation attributes if one relies on linear prediction models. Based on out-of-sample prediction tests, Lev et al. conclude that in linear prediction models ‘‘[a]ccruals do not improve the prediction of cash flows beyond that achieved by current cash flows.’’

This article develops a consumption-based equity valuation model that interpolates between discounted cash flow relying exclusively on cash flow information and accounting-based valuation using earnings information. In the interior regions of the model, cash flow and earnings act together as complementary information sources. The model is set in an infinite-period consumption economy with many risk-averse investors characterized by time-additive exponential utilities for consumption of cash. The investors hold a portfolio comprised of shares in one risky firm and a deposit of cash in risk-free constant-returns-to-scale production technology. The risky firm issues periodic earnings reports and pays dividends equal to its risky free cash flows. Upon receiving a periodic earnings announcement and a dividend payment, investors set a new price for the firm, consume, and allocate the remaining cash to constant-returns-to-scale production.[1] Therefore, investors not only

[1] Because the market return can be identified with the investors' aggregate consumption at each point in time, the finance literature generically refers to this type of economy as a consumption-CAPM economy (Breeden, 1979).

revise price in response to public earnings reports and dividend payments, they also revise their consumption and production plans. Accordingly, the quality of earnings affects the firm's price as well as investors' consumption and production plans.

This model extends Feltham and Ohlson (1996) to a consumption–production setting where earnings provide information that complements the information contained in contemporaneous cash flow. The model integrates three ideas that do not appear in traditional Feltham-Ohlson models: (i) earnings provide noisy information about future shocks to cash flow; (ii) earnings contain transient accruals that investors cannot discern; and (iii) investors use cash flow and earnings information to make allocation and consumption decisions and set price. Production choice plays a critical role in this setting. Absent an opportunity to allocate resources between the firm and at least one productive technology, investors' consumption is fixed by the firm's exogenous free cash flow, which means investors enjoy no consumption choice. Thus, even if earnings quality might affect price, it can have no affect on consumption absent production choice.[2]

This model contains two innovations pertaining to the representation of accounting information. First, it puts on the table that investors rationally use accounting earnings and cash flows as complementary information sources. As it is, cash flow has dual roles—as information and as a consumption good—-while earnings serve only one role as information. The second innovation is to articulate two reasons why earnings do not predict future cash flows with full certainty. First, earnings omit some information (usually by design). Second, earnings contain indiscernible transient accruals that noise up its information content. Accordingly, ***content quality*** refers to the amount of information that current earnings contain about future cash flows. On the other hand, ***precision quality*** refers to how precisely an earnings report communicates its information content in the presence of indiscernible transient accruals.

This article contributes to two streams of analytic research. First, it adds to the literature on using accounting information as valuation attributes for equity valuation (Feltham & Ohlson, 1996; Garman & Ohlson, 1980; Ohlson, 1990). Feltham and Ohlson build models of equity valuation where contemporaneous cash flow and contemporaneous earnings and book value are information substitutes. In their setting, one can represent price either by capitalized cash flow or as a weighted average of capitalized earnings and book value. Thus, earnings leave cash flow no role as an incremental information provider in these models except to indicate dividend

[2] Gerald Feltham deserves credit for pointing out the essential role of production in this model. Christensen and Feltham (2003, Chapters 6–7) describe the ''disappointing'' (p. 245) value of information in a multi-period exchange economy *sans* information-based production choice. In a multi-period exchange setting with time-additive utilities, homogeneous investors, and uninsurable endowments, investors do not strictly Pareto prefer an accounting system with higher earnings quality over one with worst earnings quality. In contrast, investors *might* prefer higher earnings quality if firms use accounting information to make price-maximizing production decisions (Christensen & Feltham, 1988).

displacement.[3] By treating cash flow and earnings as complementary information sources, the model in this article differs from existing literature.[4]

Second, this article articulates how earnings quality affects price and consumption and production in a parametrical model. That earnings quality affects consumption and production builds on Epstein and Turnbull (1980), Kunkel (1982), Christensen and Feltham (1988), and Yee (2006a). Epstein and Turnbull prove that the timing of uncertainty resolution affects price. In production and exchange settings, Kunkel and Christensen and Feltham delineate sufficient conditions that guarantee investors Pareto prefer more public information to less. Yee (2006a) shows that an indiscernible transient component to reported earnings increases the equity risk premium by delaying information about future dividends. This article articulates how earnings quality affects production and consumption smoothing over time when cash flow and earnings are complementary information sources of forecasting information as envisioned by FASB.

1 The model

This section sets forth the four assumptions A1–A4 that define the model. The first assumption characterizes the firm and implements FASB's assertion that "earnings and its components measured by accrual accounting generally provide a better indication of enterprise performance than does information about current cash receipts and payments" (SFAC, 1978, para 44).

1.1 Assumptions of the model

1.1.1 A1—the firm

There is one firm with free cash flows c_t and earnings x_t. For all $t \in \{0, 1, 2, \ldots\}$,

$$c_{t+1} = \gamma c_t + \theta_{t+1} + \varepsilon_{t+1}. \tag{1}$$

[3] There are two effects that potentially may enable cash flow to provide incremental information to earnings in the Feltham and Ohlson (1996) model. First, "other information" in their model could be contemporaneous cash flow, in which case cash flow does provide incremental information to earnings. Second, Clubb (1996) shows that unexpected cash flow has explanatory power for contemporaneous unexpected return even within the Feltham and Ohlson setting. Unexpected accruals are part of the total outlay invested in positive net-present-value projects. Therefore, while unexpected accruals deserve a higher capitalization factor than unexpected cash flow from operations that has not been reinvested, there is no reason for the capitalization factor of unexpected cash flow to vanish. The model presented here develops a complementary relationship between earnings and cash flow that differs from both the "other information" effect and the Clubb effect.

[4] While Reichelstein (2000) and Dutta and Reichelstein (2005) also consider cash flow and accruals as complementary information sources, they focus on performance evaluation and compensation design rather than valuation and consumption issues. Assuming risk-neutral pricing (i.e., exogenous state prices), their models do not admit consumption planning and do not formulate earnings quality as is done in this article.

$$x_t \equiv c_t + (\theta_{t+1} - \theta_t + \delta_t - \delta_{t-1})\lambda \tag{2}$$

where $\theta_0 = 0$, $\delta_{-1} = 0$, $\{\gamma, \lambda\}$ are strictly positive constants, $\{\theta_{t+1}, \varepsilon_{t+1}, \delta_t\}$ are independent, serially uncorrelated, mean-zero normally distributed shocks with strictly positive variances $\{\sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\delta^2\}$. At date $t \geq 1$, investors observe $\Omega_t = \{\{c_\tau\}_{\tau=0}^t, \{x_\tau\}_{\tau=0}^t, \{\theta_\tau\}_{\tau=1}^t\}$. Equations (1) and (2) and the values of $\{\lambda, \gamma, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\delta^2\}$ are common knowledge.

In A1, earnings are exclusively information attributes—they have no utility value in themselves and cannot be consumed. In contrast, cash flow serves dual roles as a payoff attribute and an information attribute.[5] Investors do not have to consume free cash flow immediately; they may deposit cash in the constant-returns-to-scale production technology (see A2 below) and consume the risk-free output over many subsequent periods.

In Eq. (1), future shocks to cash flow contain two components: θ_{t+1}, which is imperfectly forecasted by earnings, and ε_{t+1}, which is unpredictable. By making future cash flows uncertain, θ_{t+1} and ε_{t+1} create "fundamental risk" (i.e., uncertainty in future cash flows). The difference between θ_{t+1} and ε_{t+1} is that accountants can predict θ_{t+1} whereas ε_{t+1} is unpredictable at date t even with complete inside information. For example, θ_{t+1} may represent date $t+1$ revenue arising from a new contract signed at date t, whereas ε_{t+1} may represent a shock to cash flow arising from future changes to the market price of raw materials.

In accordance with the idea that accruals contribute information not found in current cash flows because "accruals reflect management's expectation about future cash flows" (Beaver, 1998, p. 6), earnings x_t in Eq. (2) communicates noisy information about θ_{t+1} to investors. Specifically, earnings aggregate two components: contemporaneous cash flow c_t and accrual

$$Accr_t \equiv \underbrace{(\theta_{t+1} + \delta_t - \delta_{t-1})}_{\substack{\text{noisy measure of} \\ \text{shock to forward} \\ \text{cash flow}}} \lambda - \underbrace{\lambda\theta_t}_{\substack{\lambda\theta_t \text{ had already} \\ \text{been recognized} \\ \text{at date } t-1}}.$$

$Accr_t$ is a linear combination of the cash flow realization and a noisy measure of θ_{t+1}, the shock to future cash flow. Because $\lambda\theta_t$ is recognized in $Accr_{t-1}$, it is deducted from $Accr_t$ to avoid its double counting. The structure of the accruals noise, $\delta_t - \delta_{t-1}$, accommodates the idea that abnormal accruals reverse in the subsequent period. δ_t is a new accrual shock originating at date t and $-\delta_{t-1}$ is the reversal of the accrual shock originating at date $t-1$. For example, if a firm abnormally accelerated the recognition of previously deferred revenues at date $t-1$ by δ_{t-1} dollars, then it has δ_{t-1} fewer dollars of deferred revenues to recognize at date t. Accordingly, date t earnings of this firm suffer an abnormal reduction of δ_{t-1} dollars. Dechow and Dichev (2002)

[5] Penman (2003) describes how to construct free cash flow from financial statements. Earnings correspond to what Penman calls "operating income".

consider a similar reversing accruals structure in their empirical research specification. Note that, in any given period, x_t may be positive even if contemporaneous cash flow is negative if the accruals component $\theta_{t+1} + \delta_t - \delta_{t-1}$ is sufficiently positive.[6]

A1 posits that, while date t investors observe $\{c_t, x_t, \theta_t\}$, they cannot observe δ_t, which represents unobservable accruals noise. As a result, x_t is a forecasting statistic that offers investors a noisy prediction of θ_{t+1}, one of the shocks to forward cash flow. The observability of realized θ_t (but not future θ_{t+1}) is a metaphor for the idea that investors can discern component shocks to realized cash flow based on line item and footnote information in financial statements.

λ in Eq. (2) is an accounting policy parameter that acts as an accounting-imposed discount parameter determining how θ_{t+1} is recognized in earnings x_t. If λ equals the risk-free discount rate,[7] then earnings would recognize θ_{t+1} on a "fair value" basis (i.e., $x_t = \lambda\theta_{t+1} + \cdots$). On the other hand, if λ is smaller than the risk-free discount rate, then earnings recognizes θ_{t+1} on an unconditionally "conservative" basis since earnings recognizes θ_{t+1} on a less than dollar-for-dollar basis. If $\lambda = 1$, earnings recognizes θ_{t+1} at nominal value without discounting.

The second assumption characterizes the constant-returns-to-scale production technology.

1.1.2 A2—risk-free constant-returns-to-scale ("CRS") production technology

There is a risk-free constant-returns-to-scale production technology that pays (demands) R dollars at the end of the period for each dollar deposited (borrowed) at the beginning of the period. The common-knowledge value of R, a constant, strictly exceeds $\max\{1, \gamma\}$, where γ is the cash flow growth parameter in A1.

The production technology acts as a metaphor for an alternative investment that investors can buy or short. Such an alternative investment could be the market portfolio (if one abstracts away market volatility) or an infinitely deep exogenously provided bank account that lends and borrows at a constant risk-free rate.[8] The production technology's significance in this setup stems not its risk-free or constant-returns-to-scale features, which are adopted merely for analytical convenience. Access to a constant-returns-to-scale investment gives investors an opportunity to affect the level of cash flow production in the economy by virtue of allocating wealth to it. Since Eq. (1) exogenously determines the firm's cash flow

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[6] Steve Penman deserves acknowledgement for pointing this out. Also note that the reversing structure of θ_{t+1} and δ_t in earnings implies that $\sum_{t=0}^{\infty} x_t = \sum_{t=0}^{\infty} c_t$,which means there is no double counting of the $\{\theta_t, \delta_{t-1}\}_{t=0}^{\infty}$ shocks, and that the average dividend payout ratio is $E\lfloor c_t \rfloor / E\lfloor x_t \rfloor = 1$.

[7] Calibrating to the risk-free (rather than risk-adjusted) discount rate makes sense here because the accountant observes θ_{t+1} for recognition purposes and, hence, the value of θ_{t+1} is not risky from the accountant's perspective.

[8] Institutionally, the closest approximation to risk-free CRS production technology is government-backed consol bonds indexed to the CPI index to adjust for inflation risk.

production independently of how much investors bid up its share price, investors cannot influence the firm's output. Absent an opportunity to allocate to an alternative production technology, investors' decisions would have no impact on cash flow production and consumption.

Let W_t^i denote the cumulative value of investor i's cum-dividend, pre-consumption wealth at date t. W_t^i encompasses the market value of the investor's holdings in the risky firm and the CRS production technology. Let q_t^i denote the (fractional) number of shares of the risky firm held by investor i at date t, and let P_t be the firm's per-share price. At the start of each period t, investor i starts with W_τ^i dollars, invests $P_\tau q_\tau^i$ in shares of the risky firm, consumes z_τ^i dollars, and deposits the remainder, $W_\tau^i - z_\tau^i - P_\tau q_\tau^i$, in the CRS production technology. Since there is no borrowing or lending constraint, the sign and magnitude of $W_\tau^i - z_\tau^i - P_\tau q_\tau^i$, is unconstrained. The investor's date $t+1$ pre-consumption wealth consists of the compounded value of her holdings in the CRS production technology and the risky firm, which can be written as

$$\begin{aligned} W_{\tau+1}^i &= \left(W_\tau^i - z_\tau^i - P_\tau q_\tau^i\right)R + (P_{\tau+1} + c_{\tau+1})q_\tau^i \\ &= \left(W_\tau^i - z_\tau^i\right)R + (P_{\tau+1} + c_{\tau+1} - RP_\tau)q_\tau^i. \end{aligned}$$

Each of the investors, labeled $i \in \{1,\ldots,M\}$, is endowed with initial wealth W_0^i and has time-additive CARA utility[9] $U_t^i = -E\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}e^{-\rho z_\tau^i}|\Omega_t\right]$ for cash consumption stream $\{z_\tau^i\}_{\tau=t}^{\infty}$. $\beta \in (0,1)$ is the investor's patience parameter and $\rho \in (0,\infty)$ is a strictly positive risk aversion parameter. While investors observe price P_t, the information in price is redundant to Ω_t because investors are homogeneously informed.

The third assumption defines the investors' portfolio-consumption choice problem.

1.1.3 A3—investors' problem

At each date $t > 0$, each investor i observes $\Omega_t = \left\{\{c_\tau\}_{\tau=0}^t, \{x_\tau\}_{\tau=0}^t, \{\theta_\tau\}_{\tau=1}^t\right\}$ and, conditional on Ω_t, chooses her equilibrium portfolio-consumption amounts $\{q_t^{i*}, z_t^{i*}\}$ to solve

$$\{q_t^{i*}, z_t^{i*}\} = \arg\max_{\{q_t^i, z_t^i\}} U_t^i$$

subject to[10]

[9] While the setting is tractable even if investors have heterogeneous risk-aversion, I assume investors are identical for simplicity. Asset pricing with exponential utility functions and incomplete or differential information has attracted enormous attention in both accounting and finance. He and Wang (1995), Christensen and Feltham (2003), and Yee (2006a, b) offer references. Christensen and Feltham (2005, Sect. 25.4.2) provide an introduction to the "LEN" framework, which uses exponential utilities in a non-market agency setting.

[10] Doubling strategies that may lead to unbounded amounts of borrowing are also forbidden. See Appendix.

(i) $W^i_{t+1} = \left(W^i_t - z^i_t\right)R + (P_{t+1} + c_{t+1} - RP_t)q^i_t$
(ii) $\lim_{s\to\infty} E\left[\frac{W^i_{t+s}}{R^s}|\Omega_t\right] = 0.$

The structure of the investors' problem, her initial wealth W^i_0, and her utility function parameters are common knowledge.

A3 says that each investor chooses her equilibrium portfolio-consumption amounts $\left\{q^{i*}_t, z^{i*}_t\right\}$ at each point in time to maximize her utility subject to her budget constraint and rational expectations about the future.

The final assumption is market clearance, which determines price.

1.1.4 A.4—market clearance: $\sum_{i=1}^{M} q^i_\tau = 1$

A4 normalizes the number of shares of the risky firm to unity without loss of generality. When aggregate demand exceeds the supply of shares (unity), price rises until aggregate demand falls to unity. When aggregate demand is below unity, price falls until aggregate demand equals unity.

1.2 A consumption equilibrium of the model

To characterize the equilibrium implied by A1–A4, introduce the following terminology. Investor i's ***portfolio plan*** $\left\{q^i_\tau\right\}^\infty_{\tau=t}$ at date t is i's share of the risky firm going forward. Analogously, the investor's ***consumption plan*** $\left\{z^i_\tau\right\}^\infty_{\tau=t}$ is the amounts she consumes going forward. Investor i's ***production plan*** $\left\{\Pi^i_\tau\right\}^\infty_{\tau=t}$ at date t is the amounts of cash that the investor receives from the firm and the CRS production technology going forward.

Portfolio and consumption choices determine production. Since the investor holds q^i_{t-1} shares of the risky firm and deposits $W^i_{t-1} - z^i_{t-1} - q^i_{t-1}P_{t-1}$ dollars in the CRS production technology, the investor's portfolio produces a gross cash return of

$$\Pi^i_t = \underbrace{q^i_{t-1}c_t}_{\substack{\text{payout from}\\ \text{risky firm}}} + \underbrace{\left(W^i_{t-1} - z^i_{t-1} - q^i_{t-1}P_{t-1}\right)R}_{\substack{\text{payout from}\\ \text{risk-free production technology}}}$$

dollars during period t. As c_t is exogenously given, the investor's portfolio and consumption choices $\left\{z^i_{t-1}, q^i_{t-1}\right\}$ determine the production of cash Π^i_t. Hence, portfolio and consumption choice is tantamount to production choice.

A ***consumption equilibrium*** at date t is a set of prices, and portfolio-consumption plans $\left\{P_\tau, \left\{q^{i*}_\tau, z^{i*}_\tau\right\}^M_{i=1}\right\}^\infty_{\tau=t}$ consistent with assumptions A1–A4. Lemma 1 characterizes a consumption equilibrium:

Lemma 1 *Let $\Re_t \equiv P_t + c_t - RP_{t-1}$ denote the firm's "return premium." In a consumption equilibrium, the price of the firm's share satisfies*

$$E[\Re_{t+1}|\Omega_t] = \frac{\phi^{-1}\rho}{M}\Sigma_P \quad t \geq 0, \tag{3}$$

where $\phi \equiv \frac{R}{R-1}$ *and*

$$\Sigma_P \equiv \text{var}[\Re_{t+1}|\Omega_t]. \tag{4}$$

In a consumption equilibrium, investor i's portfolio-consumption plans satisfy $q_t^{i*} = 1/M$ and

$$z_t^{i*} = \phi^{-1}\left\{W_t^{i*} - \frac{\mathrm{I}}{R-1} + \frac{CE\Re}{R-1}\right\}. \tag{5}$$

In Eq. (5) wealth evolves according to

$$W_t^{i*} = W_{t-1}^{i*} + I + q_{t-1}^{i*}{}'\Re_t - CE\Re, \tag{6}$$

where $I \equiv \frac{\ln(R\beta)}{\phi^{-1}}$ is production-driven savings and $CE\Re \equiv E\left[q_t^{i*}\Re_{t+1}|\Omega_t\right] - \frac{\phi^{-1}\rho}{2}\text{var}\left[q_t^{i*}\Re_{t+1}|\Omega_t\right]$ is the certainty equivalent of the return premium.

Proof All proofs are in the Appendix.

Equations (3) and (4) determine the equilibrium price P_t of the firm. Equation (3) is the "no arbitrage" equation that determines price when investors have CARA utility functions. In the limit of zero risk aversion (i.e., $\rho \to 0$), Eq. (3) reduces to the familiar no-arbitrage relation that is the starting premise of accounting-based equity valuation (e.g., see Ohlson, 1990).

Equations (5) and (6) characterize the investor's consumption plan. As is well known, CARA utility investors strive to smooth consumption (Breeden, 1979; Christensen & Feltham, 2005, Sect. 25.3). At each period, the investor starts with her cum-dividend, pre-consumption wealth W_t^{i*} and exempts I dollars of it from consumption in order to grow her deposit in the production technology. In addition, she anticipates receiving a stream of return premia from her holding in the risky firm, which increases the present value of her wealth. As a result, the investor calculates the wealth available for her consumption at date t as

$$\underbrace{W_t^{i*}}_{\substack{\text{cum-dividend}\\ \text{pre-consumption}\\ \text{wealth}}} + \underbrace{\left(\mathrm{I} - \mathrm{I} - \sum_{s=1}^{\infty}\frac{\mathrm{I}}{R^s}\right)}_{\substack{\text{current productive savings}\\ \text{minus the present value of}\\ \text{the current and future amounts}\\ \text{earmarked for productive savings}}} + \underbrace{\sum_{s=1}^{\infty}\frac{CE\Re}{R^s}}_{\substack{\text{present value of}\\ \text{certainty equivalent}\\ \text{of return premium}\\ \text{from investor's risky}\\ \text{firm holding}}}$$

$$= \underbrace{W_t^{i*} - \frac{\mathrm{I}}{R-1} + \frac{CE\Re}{R-1}}_{\substack{\text{wealth available for}\\ \text{consumption}}}.$$

To smooth consumption, the investor consumes a sustainable fraction[11] ϕ^{-1} of $\left\{W_t^{i*} - \frac{I}{R-1} + \frac{CE\Re}{R-1}\right\}$. The investor cannot smooth consumption completely because W_t^{i*} in Eq. (6) contains the full value of realization shocks to the return premium $\Re_t$.

According to Eq. (6), an investor sets aside a fixed amount of production-driven savings, I, each period. Earmarked for deposit in the CRS production technology, the value of I is determined by the return on the production technology and the investor's impatience for consumption. If $R\beta = 1$, then $I = 0$ because the investor is indifferent between consuming an annuity of $R - 1$ dollars in perpetuity and consuming a dollar immediately. If $R\beta > 1$, then $I > 0$ because the investor is patient enough that she prefers consuming an annuity of $R - 1$ dollars in all subsequent periods to consuming a dollar immediately. The investor would set aside the same amount of production-driven savings even if the economy has no risky firm.

In Eqs. (3)–(6), return premium $\Re_{t+1}$ and its certainty equivalent $CE\Re$ depend on earnings quality. As a result, earnings quality affects price and investors' consumption and production plans. The remainder of this article examines the impact of earnings quality on price and consumption and production.

2 Earnings quality: precision and content

Since Ω_t plays a central role in determining price and consumption in Eqs. (3)–(6), this section elaborates on the information structure implied by Assumptions A1–A4. To arrive at the consumption equilibrium, investors process $\left\{\{c_\tau\}_{\tau=0}^t, \{x_\tau\}_{\tau=0}^t, \{\theta_\tau\}_{\tau=1}^t\right\}$ in light of common knowledge about the structural equations, Eqs. (1) and (2), to forecast cash flows.

Investors do not observe and cannot infer exactly the values of $\{\theta_{t+1}, \varepsilon_{t+1}, \delta_t\}$ based on Ω_t. Investors have no information about ε_{t+1}; their best estimate is $\varepsilon_{t+1} \sim WN\left(0, \sigma_\varepsilon^2\right)$. In contrast, as explained in Appendix A.1, investors may infer the exact value of the trailing accrual shock δ_{t-1} and use it and other public information to construct an unbiased noisy estimator of θ_{t+1} at date t. Their estimator is $s_t \equiv \frac{1}{\lambda}\{x_t - c_t + \lambda\theta_t + \lambda\delta_{t-1}\}$. By Eq. (2),

$$s_t = \theta_{t+1} + \delta_t,$$

which implies $s_t \sim N\left(\theta_{t+1}, \sigma_\delta^2\right)$. Bayesian theory implies that $E[\theta_{t+1}|\Omega_t] = E[\theta_{t+1}|s_t] = Q_P s_t$ and $\text{var}[\theta_{t+1}|\Omega_t] = \text{var}[\theta_{t+1}|s_t] = (1 - Q_P)\sigma_\theta^2$, where

[11] If an investor starts with 1 dollar at the start of a period, consumes a fraction ϕ^{-1} of it and invests the remaining fraction in CRS production, then she has 1 dollar again at the end of the period. Hence, with access to CRS production, each dollar of wealth sustains period consumption of ϕ^{-1} dollars in perpetuity.

$$Q_P \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\delta^2}.$$

I identify Q_P with the precision quality of earnings. Q_P reflects how precisely earnings forecast θ_{t+1} in the presence of indiscernible accrual shocks. In the limit where the estimator s_t provides a perfect forecast of the value of θ_{t+1}, precision quality is perfect (i.e., $Q_P = 1$) and $\text{var}[\theta_{t+1}|\Omega_t] = 0$. When the estimator s_t provides no information about θ_{t+1}, precision quality vanishes (i.e., $Q_P = 0$) and $\text{var}[\theta_{t+1}|\Omega_t] = \sigma_\theta^2$. No matter how poor precision quality is, investors' uncertainty in the value of θ_{t+1} cannot exceed σ_θ^2; poor precision quality cannot create more cash flow risk than what exists without an earnings report.

Beyond precision quality, A1 manifests a second dimension of earnings quality that is content quality. Referring to the amount of information that earnings contain about future cash flow, content quality is the proportion of cash flow risk reduced by earnings information. According to Eq. (1), the total risk in next-period cash flow is $\sigma_c^2 \equiv \sigma_\theta^2 + \sigma_\varepsilon^2$. Absent accrual shocks, the fraction of σ_c^2 risk earnings could mitigate by revealing the value of θ_{t+1} to investors is

$$Q_C \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}.$$

Q_C is a summary measure of content quality. As the ratio of predictable cash flow risk to the total unconditional amount of cash flow risk, Q_C reflects the fraction of cash flow uncertainty mitigated by an earnings report that fully discloses the value of θ_{t+1} without accrual shocks. Absent accrual shocks, the posterior variance of c_{t+1} conditional on c_t and x_t is $\text{var}[c_{t+1}|\Omega_t] = (1 - Q_C)\sigma_c^2$. The limit of no accrual shocks and perfect content quality corresponds to $Q_C = 1$. In this limit, $\text{var}[c_{t+1}|\Omega_t] = 0$, which means earnings mitigate all forward cash flow uncertainty entirely. The opposite limit of non-existent content quality occurs when $Q_C = 0$. In this limit, $\text{var}[c_{t+1}|\Omega_t] = \sigma_c^2$, which means earnings provide no incremental information.

In summary, the useful part of the public information set at date t for forecasting cash flow and valuing equity is

$$\Omega_t = \{c_t, x_t, \delta_{t-1}\}.$$

From this information, investors construct $s_t \equiv \frac{1}{\lambda}\{x_t - c_t + \lambda\theta_t + \lambda\delta_{t-1}\}$ and forecast cash flows

$$E[c_{t+1}|\Omega_t] = \gamma c_t + Q_P s_t. \tag{7}$$

Hence, limited precision quality restricts information about θ_{t+1} while content quality and biasness parameter λ plays no role in forecasting cash flow. As a result, the two dimensions of earnings quality and accounting biasness differ in how they affect price and investors' consumption and production plans.

3 Production, consumption, and earnings quality

Lemma 1 tells the following story about price, production, and consumption. Earnings quality is valuable to investors because it enables them to implement a more preferred consumption sequence. According to Eq. (5), investors smooth lifetime consumption by consuming only a portion of their wealth each period. When earnings quality is higher, the firm's future cash flow stream appears less risky to investors and, so, its equity risk premium is smaller. As a result, the firm's share price is higher and investors, who each hold a fractional share $(q_t^{i*} = 1/M)$ of the firm, are wealthier by virtue of the higher share price. Consequently, higher earnings quality enables investors to shift consumption forward without surrendering consumption smoothness. Since this is what they prefer, investors have greater utility in a higher earnings quality regime. Interestingly, because investors consume more in earlier periods when earnings quality is higher, higher earnings quality results in less cash invested in CRS production and, so, less cash is produced for subsequent consumption. Nonetheless, investors have greater utility under higher earnings quality because this is their chosen consumption path given higher quality information.

Proposition 1 confirms this story:

Proposition 1 *In a consumption equilibrium, for all* $Q_j \in (0,1)$, $j \in \{P, C\}$, $i \in \{1, \ldots, M\}$, and $t \in \{1, 2, \ldots\}$:[12]

(a) $\frac{\partial}{\partial Q_j} E\left[q_t^{i*} \Re_{t+1} | \Omega_t\right] < 0$

(b) $\frac{\partial}{\partial Q_j} \mathrm{var}\left[z_{t+1}^{i*} - z_t^{i*} | \Omega_t\right] < 0$

(c) Let $\bar{s}$ be the largest integer strictly less than $\frac{1}{R-1}$. Then

$$\left.\frac{\partial}{\partial Q_j}\right|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}} E\left[z_{t+s}^{i*} | \Omega_t\right] \text{ is } \begin{cases} > 0 & \text{if } s \in \{0, 1, \ldots, \bar{s}\} \\ \leq 0 & \text{if } s \in \{\bar{s}, \bar{s}+1, \ldots\}. \end{cases}$$

(d) $$\left.\frac{\partial}{\partial Q_j}\right|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}} E\left[\Pi_{t+s}^{i*} | \Omega_t\right] \text{ is } \begin{cases} = 0 & \text{if } s = 0 \\ < 0 & \text{if } s \in \{1, 2, \ldots\} \end{cases}$$

(e) $$\left.\frac{\partial}{\partial Q_j}\right|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}} E\left[-\sum_{s=t}^{\infty} \beta^{s-t} e^{-\rho z_s^{i*}} | \Omega_t\right] > 0.$$

[12] When $s = 0$ in parts (c) and (d), $z_t^{i*} \equiv E\left[z_t^{i*} | \Omega_t\right]$ and $\Pi_t^{i*} \equiv E\left[\Pi_t^{i*} | \Omega_t\right]$. $\frac{\partial}{\partial Q_j}$ is the partial derivative holding all other model parameters, including $\sigma_c^2 \equiv \sigma_0^2 + \sigma_\varepsilon^2$ and $Q_{j'}$ for $j' \neq j$, fixed. $\left.\frac{\partial}{\partial Q_j}\right|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}}$ is the partial derivative holding $B_t^{i*} \equiv W_t^{i*} - (P_t + c_t) q_{t-1}$, investors' one-period-ahead cash flow expectation, and all other model parameters fixed. B_t^{i*} is the amount of pre-consumption, pre-dividend wealth investor i has deposited in the CRS production technology at the start of period t. Peter Christensen deserves credit for identifying B_t^{i*} as the reasonable variable to hold fix when computing these inequalities. That $E[c_{t+1}|\Omega_t]$ is held fixed highlights the higher earnings quality is guaranteed to benefit investors only on an *ex ante* basis.

Proposition 1 identifies the impact of earnings quality (precision or content) on investor planning. Parts (a)–(c) say that higher earnings quality reduces the firm's equity risk premium, enables investors to achieve a smoother consumption path, and induces greater contemporaneous consumption. Because investors consume more in the current period, they have less cash to deposit in CRS production. Thus, as part (d) indicates, higher earnings quality causes expected production to decrease. Since expected production is lower, investors expect[13] to have less cash to consume in the distant future in accordance with the $s \geq \bar{s}$ cases in Part (c). Nonetheless, because contemporaneous and near-term consumption is greater when earnings quality is higher, investors enjoy greater utilities when earnings quality is higher. Therefore, as indicated in part (e), investors prefer higher earnings quality.[14]

Proposition 1's main take-away is not necessarily that investors prefer higher earnings quality under assumptions A1–A4, which they do. The more important point is that, when investors use accounting information to make production and consumption decisions, they are better able to implement their consumption preferences when earnings quality is higher even though higher earnings quality does not lead to higher consumption levels at every point in the future. The implications of earnings quality for production and consumption are sensitive to the characteristics of the production opportunities available to investors. Under assumptions A1–A4, investors allocate between only two production technologies: the firm, whose cash flows are fixed independently of the price of its share, and a constant-returns-to-scale production technology. When earnings quality is higher, investors consume more up front, tie more cash in the firm (which has no effect on the firm's subsequent cash flows) and have less remaining cash to invest in the production technology. Accordingly, subsequent production and consumption decrease when earnings quality is higher. But given production opportunities with different characteristics, higher earnings quality could very well lead to higher production levels that sustain permanently higher levels of consumption.

4 Price and earnings quality

Equation (3) provides an equilibrium framework illustrating how one might extend traditional equity valuation theory to incorporate earnings quality. Rearranging Eq. (3) yields

[13] Part (c) might appear inconsistent with rational expectations in claiming that actual consumption z_t^{i*} increases with earnings quality for all $t \geq 1$ while expected consumption $E\left[z_{t+s}^{i*}|\Omega_t\right]$ decreases with earnings quality when $s \geq \bar{s}$. This apparent inconsistency resolves by noting that part (c) says that z_t^{i*} increases conditional on fixed B_t^{i*} and $E[c_{t+1}|\Omega_t]$ and that $E\left[z_{t+s}^{i*}|\Omega_t\right]$ decreases conditional on fixed B_t^{i*} and $E[c_{t+1}|\Omega_t]$ without any condition on B_{t+s}^{i*} and $E[c_{t+s+1}|\Omega_{t+s}]$. If B_{t+s}^{i*} and $E[c_{t+s+1}|\Omega_{t+s}]$ are held fixed, then $E\left[z_{t+s}^{i*}|\Omega_t\right]$ would also *increase* with increasing earnings quality.

[14] Part (e) supports results obtained by Kunkel (1982) and Christensen and Feltham (2003, Proposition 8.4) in an Arrow-Debreu setting. Because all investors act identically in the equilibrium of Proposition 1, it is tantamount to a situation where one investor plans consumption and production. Since this investor can choose to ignore any utility-decreasing information, higher earnings quality cannot reduce the investor's equilibrium utility.

$$P_\tau = \frac{1}{R}\left\{E[P_{\tau+1} + c_{\tau+1}|\Omega_\tau] - \frac{\rho}{\phi M}\Sigma_P\right\} \quad \forall\tau \geq 0.$$

Iterating this equation once and applying the law of iterated expectations yields

$$P_t = \frac{E[c_{t+1}|\Omega_t]}{R} + \frac{E[P_{t+2} + c_{t+2}|\Omega_t]}{R^2} - \frac{\rho}{\phi MR}\left\{1 + \frac{1}{R} + \frac{1}{R^2}\right\}\Sigma_P.$$

Repeating the iteration procedure *ad infinitem* yields

$$P_t = \sum_{\tau=1}^{\infty}\frac{E[c_{t+\tau}|\Omega_t]}{R^\tau} - \frac{\rho}{MR}\Sigma_P. \tag{8}$$

Therefore, price equals the present value of expected cash flows with a risk-neutral discount rate minus a risk adjustment. Following Eqs. (4) and Eq. (8), the risk adjustment is proportional to

$$\begin{aligned}\Sigma_P =& \mathrm{var}\left[\left(\frac{c_{t+1} + Q_P s_{t+1}}{R-\gamma}\right) + c_{t+1}|\Omega_t\right] \\ =& \left(\frac{R}{R-\gamma}\right)^2 \mathrm{var}[c_{t+1}|\Omega_t] + \left(\frac{Q_P}{R-\gamma}\right)^2 \mathrm{var}[s_{t+1}|\Omega_t]\end{aligned}$$

or

$$\Sigma_P = \left(\frac{R}{R-\gamma}\right)^2\left\{(1-Q_P)\sigma_\theta^2 + \sigma_\varepsilon^2\right\} + \left(\frac{Q_P}{R-\gamma}\right)^2\left\{\sigma_\theta^2 + \sigma_\delta^2\right\} \tag{9}$$

Solving the equations of Lemma 1 yields the equilibrium price as a function of current cash flow, earnings, and the accounting policy and earnings quality parameters.

Proposition 2 *Define* $V_t^c \equiv \frac{\gamma c_t}{R-\gamma}$ *and* $V_t^x \equiv \phi\hat{x}_t - c_t$, *where* $\hat{x}_t \equiv \{x_t + (\theta_t + \delta_{t-1})\lambda\}\left(\frac{R-1}{R-\gamma}\right)$ *is "adjusted earnings." In a consumption equilibrium*

$$P_t = (1-\kappa)\left\{V_t^c - \frac{\rho}{RM}\Sigma_P\right\} + \kappa\left\{V_t^x - \frac{\rho}{RM}\Sigma_P\right\}, \tag{10}$$

where $\phi \equiv \frac{R}{R-1}$, $\kappa \equiv \frac{Q_P}{R\lambda}$, *and*

$$\Sigma_P(Q_P, Q_C) \equiv (1 - Q_P Q_C)\left(\frac{R}{R-\gamma}\right)^2\sigma_c^2 + Q_P Q_C\left(\frac{1}{R-\gamma}\right)^2\sigma_c^2.$$

Equation (10) implies that equilibrium price is a weighted average of capitalized cash flows, V_t^c, and ex-dividend capitalized adjusted earnings, V_t^x, minus an equity risk premium $\frac{\rho}{RM}\Sigma_P$. By linking price to capitalized contemporaneous cash flow and earnings, Eq. (10) manifests FASB's idea that earnings provide incremental information to help investors forecast cash flow. Also, taking weighted averages is common in valuation practice because practitioners take weighted averages over different value estimates to try and improve accuracy. For example, according to the Delaware Block Method used by courts in appraisal proceedings, appraisal value is legally *defined* as a weighted average of liquidation value, capitalized earnings, and market price (Yee, 2004).

Equation (10) contrasts against the accounting-based valuation formulas studied by Feltham and Ohlson (1996). While Feltham and Ohlson also consider capitalized contemporaneous cash flow and dividend-adjusted capitalized earnings to be valuation attributes, they treat these two attributes as mutual substitutes rather than as complements. In their setting, the financial statement user observes every realized shock to cash flow and earnings, which means that earnings do not provide incremental information about future cash flow beyond what is already contained in contemporary cash flow. Consequently, it does not matter whether one uses ex-dividend capitalized earnings, capitalized cash flow, or some weighted average to forecast cash flow. In contrast, the weight κ is uniquely specified in Eq. (10) because earnings and cash flow act as information complements and both are necessary to obtain the most precise Bayesian cash flow forecast.

Proposition 2 implies that weight κ and the risk premium have different exposures to the two dimensions of earnings quality and accounting bias. While κ depends on precision quality and accounting policy parameter λ, the risk premium depends on precision and content quality but not on λ. Therefore, I discuss weight κ and the equity risk premium separately in subsects. 4.1 and 4.2.

4.1 Weight κ and earnings quality

A "steady-cash firm" is one whose future cash flow is not expected to suffer shocks that are substantial compared to current cash flow, that is, $\|E[\theta_{t+1}|\Omega_t]\| = Q_P\|s_t\|$ for a steady-cash firm. In contrast, a firm with a large anticipated θ_{t+1} shock is a "transient-cash firm" because its current cash flow is transient.

Conventional wisdom holds that capitalized contemporaneous cash flow is a sufficient valuation attribute for a steady-cash firm; that is, one expects weight $\kappa \approx 0$ in Eq. (10) in for a steady-cash firm. In contrast, conventional wisdom holds that current cash flow is a poor valuation attribute for a transient-cash firm since current cash flow (which might well be negative) is a poor indicator of future cash flows.[15] This is because the short-run cash flows of a transient-cash firm incur large shocks before settling into the long-run trajectory. Accordingly, one expects contemporaneous earnings to provide significant

[15] For example, Wal-Mart reported negative free cash flows that became more and more negative in the early 1990s as Wal-Mart expanded rapidly (Penman, 2003, Chapter 4). At the same time, Wal-Mart's share price grew along with its (positive) reported earnings.

incremental information to current cash flow for a transient-cash firm, which means $\kappa > 0$.

Equation (10) articulates why it makes sense to value steady-cash firms using capitalized cash flow and transient-cash firms using a combination of capitalized earnings and cash flow. Since a steady-cash firm's future cash flows are expected to grow smoothly, anticipated shocks to future cash flows must be small, which implies[16] $Q_P \approx 0$ and, hence, $\kappa \approx 0$. Hence, Eq. (10) implies that steady-cash firms are valued by capitalizing cash flow without reference to earnings. On the other hand, a transient-cash firm is characterized by small or negative c_t, positive earnings x_t, and a large anticipated shock $\|E[\theta_{t+1}|\Omega_t]\| = Q_P\|s_t\| >> \|\gamma c_t\|$, which implies Q_P and, thus, κ are non-zero. As a result, one needs both V_t^c and V_t^x to span the price of a transient-cash firm in accordance with Eq. (10). According to Eq. (10), even if capitalized cash flow V_t^c is negative, the value of a transient-cash firm can be positive when κ and its earnings value V_t^x are sufficiently positive.

Equation (10) implies that the cash flow valuation coefficient is $\left(1 - \frac{Q_P}{R\lambda}\right)\left(\frac{\gamma}{R-\gamma}\right)$ and the adjusted-earnings valuation coefficient is $\frac{Q_P}{R\lambda}\phi$. The point is that a valuation coefficient in a setting with biased and/or imperfect earnings quality is the product of *two* factors: an earnings quality factor and a capitalization factor. For cash flow, the earnings quality factor is $\left(1 - \frac{Q_P}{R\lambda}\right)$ and the capitalization factor is $\left(\frac{\gamma}{R-\gamma}\right)$. For adjusted earnings, the earnings quality factor is $\frac{Q_P}{R\lambda}$ and the capitalization factor is ϕ. This "two factor" picture of valuation coefficients extends the traditional view that equates valuation coefficients to capitalization factors. *In an imperfect earnings quality setting, valuation coefficients unavoidably contain an earnings quality factor that takes into account accounting bias and precision quality*.

This result has implications for the interpretation of coefficients in price-relevance studies. Suppose one regresses price on contemporaneous free cash flow and earnings and finds that the cash flow coefficient is small compared to the earnings coefficient. According to Eq. (10), this does not necessarily mean that contemporaneous cash flow is not value relevant. It only means that $\left(1 - \frac{Q_P}{R\lambda}\right)\left(\frac{\gamma}{R-\gamma}\right)$ is small. Since a small capitalization factor $\left(\frac{\gamma}{R-\gamma}\right)$ is enough to render $\left(1 - \frac{Q_P}{R\lambda}\right)\left(\frac{\gamma}{R-\gamma}\right)$ small, one cannot conclude from this finding that cash flow provides no incremental information to capitalized earnings. To establish that cash flow provides no incremental information, one needs to show that the earnings quality factor $\left(1 - \frac{Q_P}{R\lambda}\right)$ vanishes.

To establish conditions for the price relevancy of current cash flow and earnings, introduce the following terminology.[17] Refer to the value of $\frac{\partial P_t}{\partial V_t^c}$ as the "price relevancy of current cash flow." Current cash flow is more price-

[16] "≈" means approximately equal to.

[17] I emphasize that the model here is a one-firm model. Hence, care must be taken before one can use these results to interpret cross-sectional studies. Even if current cash flow provides little incremental information to earnings on average in a large sample study (as Penman & Yehuda, 2003 find), it does not imply that cash flow is irrelevant for every firm.

relevant when $\frac{\partial P_t}{\partial V_t^c}$ is larger, and it is price-irrelevant if $\frac{\partial P_t}{\partial V_t^c} = 0$.[18] Likewise, the "price relevancy of earnings" is $\frac{\partial P_t}{\partial V_t^x}$. Earnings are more price-relevant when $\frac{\partial P_t}{\partial V_t^x}$ is larger, and it is price-irrelevant if $\frac{\partial P_t}{\partial V_t^x} = 0$.

Equation (10) reveals that the weight κ determines the price relevancy of cash flow and earnings. When $Q_P \in (0, 1)$, weight κ increases with conservative bias (i.e., $-\frac{\partial \kappa}{\partial \lambda} > 0$) and precision quality (i.e., $\frac{\partial \kappa}{\partial Q_P} > 0$), and decreases with the return on CRS production (i.e., $\frac{\partial \kappa}{\partial R} < 0$). Hence, earnings are more (and cash flow is commensurately less) price-relevant when accounting is more conservative, has higher precision quality, or when the risk-free rate decreases.

Under what situation is cash flow price-irrelevant? Cash flow is price-irrelevant if and only if $\kappa = 1$ or, equivalently, $Q_P = R\lambda$. Recall from the discussion of assumption A1 that $\lambda = 1/R$ pertains to fair value accounting while $\lambda < 1/R$ pertains to conservative accounting that recognizes less than the present value of anticipated cash flow. Thus, under fair value accounting, cash flow is price-irrelevant if, and only if, precision quality is perfect ($Q_P = 1$). Under conservative accounting, cash flow is price-irrelevant if, and only if, precision quality is imperfect ($Q_P = R\lambda < 1$). This means that, even if precision quality is imperfect, cash flow may be price-irrelevant under conservative accounting. Corollary 1 summarizes these observations:

Corollary 1 *Equation* (10) *implies that* $\frac{\partial P_t}{\partial V_t^c} = 0$ *if, and only if,* $\frac{\partial P_t}{\partial V_t^x} = 1$ *and, in addition, one of the following statements holds:*

(i) *accounting is fair value* $(\lambda = 1/R)$ *and precision quality is perfect* $(Q_P = 1)$
(ii) *accounting is conservative* $(\lambda < 1/R)$ *and precision quality is imperfect by exactly the right amount* $(Q_P = R\lambda)$.

Another situation is when precision quality is imperfect $(Q_P < 1)$ and anticipated future cash flows are recognized at full value and not discounted. In this case, $\lambda = 1$ and recognition is "aggressive." Then $0 < \frac{Q_P}{R\lambda} < 1$. In this situation, $\frac{\partial P_t}{\partial V_t^c} = \left(1 - \frac{Q_P}{R\lambda}\right) > 0$ and $\frac{\partial P_t}{\partial V_t^x} = \frac{Q_P}{R\lambda} > 0$, which means both cash flow and earnings are price-relevant. Corollary 1 implies that cash flow irrelevancy is incompatible with $\lambda > 1/R$.

Content quality Q_C is notably absent from the valuation coefficients. Thus, while accounting bias and precision quality influence the valuation coefficients, content quality plays no role. The reason content quality does not matter is because it refers to the level of unpredictable information that is not provided by both cash flow and earnings. As such, content quality does not change the balance of predictive power between cash flow and earnings.

[18] $\frac{\partial P_t}{\partial V_t^c} = 0$ corresponds to cash flow irrelevancy because, if $\frac{\partial P_t}{\partial V_t^c} = 0$, then $P_t = V_t^x - \frac{\rho}{RM}\Sigma_P$, which implies cum-dividend price $P_t + c_t = \phi x_t - \frac{\rho}{RM}\Sigma_P$ does not depend on cash flow.

4.2 Equity risk premium and earnings quality

This section turns to how accounting bias and limited earnings quality affect the risk premium. The equity risk reflects how risky the present value of future cash flow appears to investors. Equations (1) and (2) imply that σ_c^2 is the risk that arises from cash flows while σ_δ^2 is the uncertainty that arises from reported earnings. It is clear that σ_c^2 and σ_δ^2 do not contribute to the conditional variance of returns, Σ_P, in Eq. (11) in a symmetric way.

The reason that σ_c^2 and σ_δ^2 contribute to Σ_P in unequal ways is because cash flow and earnings serve unequal roles as consumption and information attributes. As a consumable, cash flow serves a dual role as a payout and an information attribute. In contrast, earnings cannot be consumed and, thus, serves only an informational role. The accrual shock δ_t in Eq. (2) does not affect cash flow realizations; its role derives only from its ability to reduce public information about θ_{t+1} so that, conditional on public information, θ_{t+1} *appears* more uncertain. Thus, whether the δ_t shocks affect the equity risk premium hinges on the θ_{t+1} shocks. Absent θ_{t+1} shocks, δ_t shocks do not matter. Accordingly, the accrual-shock variance σ_δ^2 enters in the equity risk premium only as a factor that enhances or reduces the impact of σ_θ^2. In contrast, the cash flow shock variances σ_θ^2 and σ_ε^2 increase the equity risk premium directly.

According to Eq. (11), $\Sigma_P(Q_P, Q_C)$ is a weighted average of two limiting values. One limit occurs when earnings are completely uninformative (i.e., $Q_P = Q_C = 0$), in which case $\Sigma_P(Q_P, Q_C)$ assumes its maximum value $\Sigma_P(0,0) = \left(\frac{R}{R-\gamma}\right)^2 \sigma_c^2$. This is because when earnings are completely uninformative

$$\Sigma_P(0,0) = \mathrm{var}\left(\underbrace{c_{t+1} + \sum_{s=1}^{\infty} \frac{c_{t+1+s}}{R^s}}_{\substack{\text{cum} - \text{dividend discounted} \\ \text{cash flows } t+1}} |c_t \right) = \mathrm{var}\left(R \sum_{s=1}^{\infty} \frac{c_{t+s}}{R^s} | c_t \right)$$

$$= R^2 \mathrm{var}\left(\frac{\gamma c_t + \theta_{t+1} + \varepsilon_{t+1}}{R-\gamma} | c_t \right) = \left(\frac{R}{R-\gamma}\right)^2 \mathrm{var}(\theta_{t+1} + \varepsilon_{t+1} | c_t) = \left(\frac{R}{R-\gamma}\right)^2 \sigma_c^2.$$

The pre-factor $\left(\frac{R}{R-\gamma}\right)^2$ in $\Sigma_P(0,0)$ is a capitalization factor that captures the expected present value of the entire future stream of cash flow shocks. Thus, $\Sigma_P(0,0)$ equals a capitalization factor times the variance of the total shock to period-ahead cash flow. The other limit occurs when earnings quality is maximal (i.e., $Q_P = Q_C = 1$). In this setting, earnings enable investors to forecast c_{t+1} with certainty, which implies that

$$\begin{aligned}\Sigma_P(1,1) &= \mathrm{var}\left(c_{t+1} + \sum_{s=1}^{\infty} \tfrac{c_{t+1+s}}{R^s} | c_{t+1}\right) = \mathrm{var}\left(\sum_{s=1}^{\infty} \tfrac{c_{t+1+s}}{R^s} | c_{t+1}\right) \\ &= \mathrm{var}\left(\tfrac{\gamma c_{t+1} + \theta_{t+2} + \varepsilon_{t+2}}{R-\gamma} | c_{t+1}\right) \\ &= \left(\tfrac{1}{R-\gamma}\right)^2 \mathrm{var}(\theta_{t+2} + \varepsilon_{t+2} | c_t) = \left(\tfrac{1}{R-\gamma}\right)^2 \sigma_c^2.\end{aligned}$$

Therefore, $\Sigma_P(1,1) = \left(\frac{1}{R^2}\right)\Sigma_P(0,0)$. The reason $\Sigma_P(1,1)$ is a factor of $\frac{1}{R^2}$ smaller than $\Sigma_P(0,0)$ is because perfect quality earnings inform investors about the value of forward cash flows and, so, reduces uncertainty about future cash flows by one period. Thus, $\Sigma_P(1,1)$ is discounted relative to $\Sigma_P(0,0)$ by an extra factor of R^2. For intermediate values of Q_P and Q_C, Eq. (11) implies that the equity risk premium is proportional to a weighted average of $\Sigma_P(1,1)$ and $\Sigma_P(0,0)$:

$$\Sigma_P \equiv (1 - Q_P Q_C)\Sigma_P(0,0) + Q_P Q_C \Sigma_P(1,1). \tag{11}$$

Therefore, the equity risk premium strictly increases with increasing content quality and increasing precision quality:

Corollary 2 *When* $Q_P \in (0,1)$ *and* $Q_C \in (0,1)$, *Eq. (11) implies (i)* $\frac{\partial \Sigma_P}{\partial Q_P} < 0$, *(ii)* $\frac{\partial \Sigma_P}{\partial Q_C} < 0$, *and (iii)* $\frac{\partial^2 \Sigma_P}{\partial Q_C \partial Q_P} < 0$.

Inequalities (i) and (ii) in Corollary 2 imply, respectively, that increasing precision quality or increasing content quality strictly reduces the equity risk premium. Inequality (iii) implies that precision quality matters more when content quality is higher, and that content quality matters more when precision quality is higher.

4.2.1 Accounting bias and the equity risk premium

The equity risk premium does not depend on accounting bias parameter λ. Since investors know the value of λ, its value does not cause incremental investor uncertainty. Hence, λ does not contribute to the expression for Σ_P in Eq. (11). Fischer and Verrecchia (2000) consider a model in which investors are uncertain about the value of a parameter determining accounting bias and show that parameter uncertainty affects equilibrium price.

4.2.2 Growth and the equity risk premium

Conventional wisdom asserts that, all things equal, high-growth firms are more risky than low-growth firms. That is, if two firms are of equal current price, one expects a high-growth firm to have greater risk and return. The model here captures the idea in a simple way. According to Eq. (11), the equity risk premium is proportional to $\left(\frac{R}{R-\gamma}\right)^2$, where $\gamma \in [0, R)$ is one plus the per-period growth rate of future cash flows. This implies that the equity risk premium not only increases with growth, but the greater the growth rate, the faster it increases with growth (i.e., $\frac{\partial \Sigma_P}{\partial \gamma} > 0$ and $\frac{\partial^2 \Sigma_P}{\partial \gamma^2} > 0$). Since growth compounds shocks to cash flows, if two firms have the same present value of cash flows in expectation, then risk-averse investors would consider the firm that has the greater cash flow growth rate to be more risky.

4.2.3 Earnings uncertainty versus returns uncertainty

Empirical studies often use the volatility of reported earnings as a proxy for risk (Baginski & Whalen, 2003; Beaver, Kettler, & Scholes, 1970; Beaver & Manegold, 1975; Elgers, 1980). The model here implies that earnings volatility is, at best, a noisy (and perhaps biased) risk measure.

Uncertainty about future returns, Σ_P, is driven by uncertainty about future cash flows. Earnings affect Σ_P only indirectly through what it reveals about future cash flows. To examine the difference between earnings volatility and price volatility, define "earnings uncertainty" as the volatility of capitalized adjusted earnings conditional on public information, $\text{var}(\phi \hat{x}_{t+1} | \Omega_t)$. The capitalization factor ϕ has been inserted for convenience. Evaluating the conditional variance of the expression for $\hat{x}_{t+1}$ yields

$$\text{var}(\phi \hat{x}_{t+1} | \Omega_t) = \Sigma_P + \left(\frac{R}{R-\gamma}\right)^2 \left\{1 - \left(\frac{Q_P}{R\lambda}\right)^2\right\} \sigma_c^2. \tag{12}$$

Equation (12) implies that earnings uncertainty is less than returns uncertainty only if accounting is sufficiently conservatively biased (i.e., $\lambda < \frac{Q_P}{R} \leq \frac{1}{R}$). If accounting is only mildly conservative (i.e., $\frac{Q_P}{R} < \lambda \leq \frac{1}{R}$) or outright aggressive (i.e., $\lambda > \frac{1}{R} \geq \frac{Q_P}{R}$), then $\text{var}(\phi \hat{x}_{t+1} | \Omega_t) > \Sigma_P$. Earnings uncertainty equals returns uncertainty only if cash flow is certain (i.e., $\sigma_c^2 = 0$), accounting is conservatively biased by a special amount (i.e., $\lambda = \frac{Q_P}{R} \leq \frac{1}{R}$), in which case price is $P_t = V_t^x - \frac{\rho}{RM} \Sigma_P$.

5 Diversification and book value

This section briefly describes two extensions of assumptions A1–A4 to accommodate (a) multiple firms and (b) book value as a third information variable.

5.1 Multiple firms and diversification

The article focuses on an economy with a one risky firm. Introducing additional firms, endowing investors with financial reports from these firms (whose performance may be correlated), and permitting investors to diversify their portfolios across firms raise two issues: diversification and information spillover across firms. While working out the details is beyond the scope of this article, this section outlines how these issues might affect the main results.

In Eqs. (1) and (2), θ_{t+1}, ε_{t+1}, and δ_t are undiversifiable shocks because, in an economy with one risky firm and risk-free CRS production technology, investors have no way to hedge away these mutually uncorrelated shocks. In contrast, in an economy multiple correlated risky securities, investors may partially diversify away each risk factor. In a multi-firm economy, one may decompose each risk factor, for example θ_{t+1}, into the sum of a systematic (or undiversifiable) component θ^S_{t+1} and an idiosyncratic (or diversifiable) component θ^I_{t+1}:

$$\theta_{t+1} = \theta^S_{t+1} + \theta^I_{t+1}.$$

CAPM theory implies that θ^S_{t+1} determines expected returns. Idiosyncratic component θ^I_{t+1} does not affect expected returns and, so, it has no impact on Σ_P and the equity risk premium. Thus, if one cares only about expected returns and the equity risk premium, one could interpret all shocks introduced in Eqs. (1) and (2) as representing just the systematic components of the full shocks without loss of generality.

On the other hand, if one cares about price realizations, the unexpected component of returns, and investors' actual consumption amounts, one must explicitly consider the idiosyncratic shocks. This is because, as Yee (2006b) shows, idiosyncratic shocks affect equilibrium price realizations[19] and, hence, unexpected returns and investors' consumption paths. As a result, the idiosyncratic components of all shocks affect the price–earnings relationship, the weight κ, and equilibrium consumption even if they do not alter returns in expectation.

The way that the idiosyncratic shocks affect the price–earnings relationship depends on details of the cross-firm correlation between the shocks. In particular, suppose there are three risky firms with cash flows

$$c_{i\,t+1} = \gamma_i c_t + \theta_{i\,t+1} + \varepsilon_{i\,t+1} \quad i \in \{1, 2, 3\},$$

where $\operatorname{cov}(\theta^I_{1\,t+1}, \theta^I_{2\,t+1} | \Omega_t) \neq 0$ and $\operatorname{cov}(\theta^I_{j\,t+1}, \sum_{i=1}^3 \theta_{i\,t+1} | \Omega_t) = 0$ for $j \in \{1, 2\}$. This means that the idiosyncratic components of firms #1 and #2 are correlated even though they are uncorrelated with the market portfolio. In this situation, spillover about both systematic and idiosyncratic information occurs between firms #1 and #2. As a result, the expected cash flows and, hence, price of firm #1 depends on the contemporaneous earnings of firm #2 and vice versa. Hence, information spillover of *idiosyncratic* shocks causes price functions to depend on the earnings of other firms. Information spillover does not diversify

[19] For example, if somebody robs a bank of a million dollars, the bank's valuation reduces by a million dollars even if crime is a fully diversifiable risk factor. This is simply because the realization of gain or loss (even if idiosyncratic) affects *ex post* valuations.

away even if the spillover pertains exclusively to idiosyncratic shocks because investors' cash flow forecasts and, accordingly, price realizations depend on all publicly available information.

Yee (2006b) works out fundamental betas and price–earnings relations in the presence of idiosyncratic shocks. The main take-way is that idiosyncratic shocks do not necessarily diversify away in valuation coefficients and valuation weights even when they diversify away in the equity risk premium in accordance with CAPM. This means that one-firm valuation models, such as the one studied in this article, do not straightforwardly extend to a multi-firm setting unless one assumes away idiosyncratic shocks and information spillover. When idiosyncratic shocks or information spillover are important, one has to rethink accounting-based equity valuation.

5.2 Book value as a third complementary information variable

Does the integration of book value negate the incremental informativeness of capitalized cash flow? Does cash flow provide incremental information to both earnings and book value? This section provides a model that extends the weighted average formula, Eq. (10), to an accounting system that integrates book value as a valuation attribute.

Ohlson and Zhang (1998) consider accounting systems that express price as a weighted average of book value and ex-dividend capitalized earnings. These accounting systems differ from the accounting policy studied in the preceding sections of this article because they treat book value as a necessary information variable. To implement such an accounting system with imperfect quality, replace Eq. (2) with[20]

$$x_t \equiv -(1-d_0)oa_t + (1+d_1)c_t + d_2(\theta_{t+1} + \delta_t - \delta_{t-1}), \tag{13}$$

where oa_t is the book value of operating assets, $d_0 \equiv \frac{Rw}{R-1+w}$, $d_1 \equiv \frac{\left(\frac{R-1}{R-\gamma}\right)\gamma - w}{R-1+w}$, $d_2 \equiv \frac{R-1}{(R-\gamma)(R-1+w)}$, and $w \in [0,1]$ is an accounting policy parameter. The clean surplus relation, $oa_t = oa_{t-1} + x_t - c_t$, implies that the book value of operating assets must evolve according to

$$oa_t \equiv d_0 oa_{t-1} + d_1 c_t + d_2(\theta_{t+1} + \delta_t - \delta_{t-1}).$$

These constructions of x_t and oa_t and endow earnings with information about future cash flow shock θ_{t+1} and with indiscernible accrual shocks. While investors observe realizations $\Omega_t = \{c_t, oa_t, x_t, \theta_t\}$ at date t, they do not observe the accrual shock δ_t.

Under perfect precision quality, when $\delta_\tau = 0$ for all τ, Ohlson and Zhang's Corollary 2 implies that

[20] To keep the expressions shorter, I set accounting biasness parameter λ to unity in this subsection.

$$\frac{E[c_{t+1}|oa_t, x_t, c_t]}{R-\gamma} \overset{\delta_\tau=0}{\equiv} (1-w)oa_t + w[\phi x_t - c_t].$$

Hence, under perfect precision quality, price is a weighted average of book value and ex-dividend capitalized earnings. Cash flow has no role aside from adjusting capitalized earnings for displacement.

Now, suppose $\delta_\tau \neq 0$ so that precision quality is imperfect. In this case, financial statement analysis (analogous to the procedure described in Appendix A.1) allows investors to infer the value of the trailing accrual shock δ_{t-1} and construct $\hat{x}_t \equiv x_t + d_2\delta_{t-1}$ and $\hat{oa}_t \equiv oa_t + d_2\delta_{t-1}$. From this, they construct the estimator $s_t \equiv (R-\gamma)V_t^{(oa,x)} - \gamma c_t$, where

$$V_t^{(oa,x)} \equiv (1-w)\hat{oa}_t + w\lfloor \phi\hat{x}_t - c_t \rfloor.$$

Plugging in the definitions of $\hat{oa}_t$ and $\hat{x}_t$ reveals that $s_t = \theta_{t+1} + \delta_t$, which means s_t is an unbiased noisy estimator of θ_{t+1} with normal distribution: $s_t \sim N(\theta_{t+1}, \sigma_\delta^2)$. Bayesian theory then implies that $E[\theta_{t+1}|s_t] = Q_P s_t$. As a result,

$$\frac{E[c_{t+1}|oa_t, x_t, c_t]}{R-\gamma} = \frac{\gamma c_t + Q_P s_t}{R-\gamma} = (1-Q_P)V_t^c + Q_P V_t^{(oa,x)},$$

which means that price equates to $P_t = (1-Q_P)V_t^c + Q_P V_t^{(oa,x)} - \frac{\rho}{RM}\Sigma_P$. This price function extends Eq. (10) to accommodate book value. The weight κ in Eq. (10) reduces to Q_P here because Eq. (13) sets $\lambda = R$ for simplicity. Otherwise, price continues to be a weighted average of capitalized cash flow V_t^c and an earnings-based valuation estimate, which is now $V_t^{(oa,x)}$. Since $V_t^{(oa,x)}$ is itself a weighted average of $\hat{oa}_t$ and V_t^x, the cash flow dynamic Eq. (1) and the accounting rule Eq. (13) implies that

$$P_t = (1-K_1-K_2)\left[V_t^c - \frac{\rho}{RM}\Sigma_P\right] + K_1\left[\hat{oa}_t - \frac{\rho}{RM}\Sigma_P\right] + K_2\left[V_t^x - \frac{\rho}{RM}\Sigma_P\right]. \tag{14}$$

The weight parameters are $K_1 = (1-w)Q_P$ and $K_2 = wQ_P$, where Q_P is the precision quality parameter and $w \in [0,1]$ is an exogenously chosen accounting policy parameter. Eq. (14) implies that, under an Ohlson-Zhang accounting system with imperfect precision quality, price is a weighted average over valuation estimates based on book value, cash flow, and earnings. Eq. (10) is the special case of Eq. (14) where accounting policy parameter w equals unity, in which case $V_t^{(oa,x)} = V_t^x$. When $w > 0$, cash flow provides incremental information to both earnings and book value.

In conclusion, the presence of a book-value-based valuation estimate does not negate the information role of capitalized cash flow. Cash flow may offer incremental information to a valuation model that integrates both book value and earnings. This result is consistent with the empirical findings of Penman and Yehuda (2003), who show that cash investment (and, hence, free cash flow) provides incremental information to net operating assets and operating earnings.

6 Concluding remarks

This article studies a consumption-based valuation model that treats earnings and cash flow as complementary information sources. The model integrates three ideas that do not appear in traditional valuation models: (i) earnings provide noisy information about future shocks to cash flow; (ii) earnings contain indiscernible transient accruals shocks; and (iii) investors use cash flow and earnings information to make allocation and consumption decisions and set price. Accordingly, the quality of earnings affects production and consumption as well as price. Higher earnings quality increases price and enables investors to consume more up front and to smooth consumption better. Because investors consume more up front, they retain less capital to invest in CRS production. As a result, higher earnings quality reduces subsequent production and consumption. Nonetheless, because it enables investors to choose a more preferred consumption path, higher earnings quality increases investors' utilities. Thus, the model depicts a situation where higher earnings quality strictly benefits investors.

From the narrow lens of equity valuation, the most useful result of this article is Proposition 2. Proposition 2 identifies a setting where investors use accounting information to make consumption and production choices and equilibrium price equates to

$$P_t = (1-\kappa)\left(V_t^c - p_0\right) + \kappa\left(V_t^x - p_0\right), \tag{15}$$

a weighted average of a risk-adjusted cash flow-based estimate, $V_t^c - p_0$, and a risk-adjusted earnings-based estimate, $V_t^x - p_0$. In Eq. (15), p_0 is the equity risk premium. Weight κ depends on precision quality and an accounting conservatism parameter. If accounting biases the value of contemporaneous earnings and, hence, depresses the value of κ downwards, then κ is bigger to compensate. Indeed, κ exceeds 1 when earnings are sufficiently conservatively biased. Equation (15) also reveals that a valuation coefficient (i.e., the coefficient of c_t or x_t in the price-cash-flow-earnings relation) is not just a capitalization factor as in traditional valuation models; it is the product of a capitalization factor and a structural factor reflecting earnings quality and accounting bias. The variance of the shocks to prospective cash flow and the content and precision quality of earnings determine the equity risk premium p_0. Accounting bias, because it is common knowledge in the model, does not affect p_0 because what is known by definition does not exacerbate uncertainty.

The model rests on several assumptions that limit the generality of the results. The most important limitation is that only one firm is considered. As such, the model does not directly address the diversification issue or manifest the possibility that one firm's financial reports may affect the quality of available public information about another firm. Since assumption A1 posits that the firm's cash flows are unaffected by investors' capitalization of the firm, another open issue is how earnings quality affects the firm's production of cash flows. Extending Eq. (1) to allow earnings quality a role in the production function, perhaps analogous to how Dutta and Reichelstein (2005) model cash flows as a moral hazard process, would be interesting. Finally, I have assumed that the parameters and distributions governing the evolution of cash flow and earnings are common knowledge to investors. This assumption implies that investors fully unravel accounting conservatism and do not learn dynamically about firm fundamentals as time advances. If earnings help investors learn about the nature of a firm, then poor earnings quality hinders the learning process, an effect this model does not capture.

Despite these limitations, this study raises a rich set of issues that deserve more attention from both empiricists and theorists. Are there additional important dimensions to earnings quality besides content and precision quality? How does other financial reporting information supplement cash flow information beyond revealing θ_t Eq. (2)? How does earnings quality affect investors' allocation of wealth if firms have non-trivial returns to market capitalization?

Acknowledgements I am indebted to Gerald Feltham (editor), whose insight into the role of production enriched my understanding, and to Peter O. Christensen (RAST discussant) for many stimulating comments. Acknowledgement is also due an anonymous referee, James Ohlson, Steve Penman, Bruce Miller, Jack Hughes, Brett Trueman, Jing Liu, David Aboody, Dan Gode (NYU-Columbia Conference discussant), Mike Kirschenheiter (AAA discussant), Jennifer Francis (FARS discussant), and other participants of these three conferences for their feedback. Jennifer Hersch and research assistants Yong Gyu Lee and Jing Li proofread earlier versions of this article.

Appendix

Inferring the value of the trailing accrual shock

Investors infer the realization of ε_t by inverting Eq. (1): $\varepsilon_t = c_t - \gamma c_{t-1} - \theta_t$. On the other hand, investors do not observe and cannot infer the realizations of the contemporaneous accrual shock δ_t. The best they can do at date t is to infer the trailing accrual shock from public information $\{\delta_{t-2}, c_{t-1}, x_{t-1}, \theta_t\}$ by inverting Eq. (2):[21]

$$\delta_{t-1} = \delta_{t-2} + (x_{t-1} - c_{t-1})\lambda^{-1} - \theta_t + \theta_{t-1}.$$

[21] Starting with $\delta_{-1} \equiv 0$, investors roll this equation forward at each period to infer the value of δ_0 at date $t = 1$, the value of δ_1 at date $t = 2$, and so on.

This means that at any date $t \geq 1$, investors know the values of $\{\varepsilon_t, \delta_{t-1}\}$ by inference. Thus, the public information set at each date $t \geq 1$ is effectively $\left\{\{c_\tau\}_{\tau=0}^{t}, \{x_\tau\}_{\tau=0}^{t}, \{\theta_\tau\}_{\tau=1}^{t}, \{\varepsilon_\tau\}_{\tau=1}^{t}, \{\delta_\tau\}_{\tau=0}^{t-1}\right\}$.

Proofs of lemmas and propositions

Setup for remainder of proofs

The rest of the proofs build on the following setup. Each investor i solves the program given in assumption A3 for her optimal portfolio-consumption plan. When the variables of information set Ω_τ are normally distributed and P_t is linear in these variables, this dynamic optimization problem can be solved using standard methods (He & Wang, 1995), which I only sketch here. The optimization problem is equivalent to

$$J\left(W_\tau^i\right) = \max_{z_\tau^i, q_\tau^i}\left\{-e^{-\rho z_\tau^i} + \beta E\left[J\left(W_{\tau+1}^i\right)\middle|\Omega_\tau^i\right]\right\}$$

subject to two conditions:

$$\text{budget: } W_{\tau+1}^i = \left(W_\tau^i - z_\tau^i\right)R + (P_{\tau+1} + c_{\tau+1} - RP_\tau)q_\tau^i \quad \forall\tau \geq t$$
$$\text{transversality: } \lim_{\tau\to\infty} E\left[\beta^\tau e^{-\rho z_\tau^i}\middle|\Omega_t^i\right] = 0 \text{ and } \lim_{\tau\to\infty} E\left[\frac{W_{t+\tau}^i}{R^\tau}\middle|\Omega_t^i\right] = 0.$$

Market clearance,

$$\sum_{i=1}^{M} q_\tau^i = 1 \quad \forall\tau \geq t,$$

determines equilibrium market prices at each date. The second transversality condition prevents unlimited borrowing in perpetuity. In addition, I require (as is standard practice in solving these models) that the solution does not involve ''doubling down'' strategies that may entail going infinitely into debt, which is unrealistic (Harrison & Kreps, 1979). To solve for equilibrium prices and consumption paths:

(a) Guess a trial solution $J(W_\tau^i) = -e^{-\alpha W_\tau^i + \varsigma}$ for all $\tau \geq t$, where $\{\alpha, \varsigma\}$ are undetermined constants.

(b) Assuming this trial function, derive the two first order conditions for $\{z_\tau^i, q_\tau^i\}$ for each date $\tau \geq t$. Solving these two first order conditions for all $\tau \geq t$ yields $\{z_\tau^{i,*}, q_\tau^{i,*}\}$ in terms of the undetermined coefficients $\{\alpha, \varsigma\}$.

(c) Require $J(W_\tau^i) = -e^{-\rho z_\tau^{i,*}} + \beta E[J((W_\tau^i - z_\tau^{i,*})R + (P_{\tau+1} + c_{\tau+1} - RP_\tau)q_\tau^{i,*})|\Omega_\tau^i]$ to obtain two equations. These equations imply that $\alpha = \rho\phi^{-1}$ and

$$\varsigma = \ln\phi + \left(\frac{1}{R-1}\right)\left\{\ln(R\beta) - \rho\phi^{-1}\left(E\left[q_\tau^i \Re_{\tau+1}\middle|\Omega_\tau^i\right] - \frac{\rho\phi^{-1}}{2}\mathrm{var}\left[q_\tau^i \Re_{\tau+1}\middle|\Omega_\tau^i\right]\right)\right\},$$

where $\phi \equiv \frac{R}{R-1}$ and $\Re_{\tau+1} \equiv P_{\tau+1} + c_{\tau+1} - RP_\tau$.

(d) Plug these equilibrium values of $\{\alpha, \varsigma\}$ into the expressions for $\{z_\tau^{i,*}, q_\tau^{i,*}\}$ determined in Step b to obtain the expressions for the equilibrium portfolio-consumption plans stated in Lemma 1.

(e) Impose $\sum_{i=1}^M q_\tau^i = 1$ to obtain the fundamental pricing equation

$$E\left[P_{\tau+1} + c_{\tau+1} - RP_\tau\middle|\Omega_\tau^i\right] = \frac{\rho}{\phi M}\Sigma_P \quad \forall\tau \geq 0, \tag{A1}$$

where $\Sigma_P \equiv \mathrm{var}\left[\Re_{\tau+1}\middle|\Omega_\tau^i\right]$. While not used in this article, the stochastic discount factor in this economy is $\mu_{t+1}^i = \beta\exp\left(-\rho\left(z_{i+1}^{i*} - z_i^{i*}\right)\right)$, where z_i^{i*} is the equilibrium consumption of any investor i.

Proof of Lemma 1 Step d of "Setup for remainder of proofs" states the procedure for deriving q_t^{i*} and z_t^{i*}. Note that $q_t^{i*} = 1/M$ simply because each of the M identical investors owns an identical fraction of the firm. When homogeneous information, $\Omega_t^i = \Omega_t$ for all i and Eq. (A1) implies Eq. (3) with Σ_P as defined following Eq. (A1). □

Proof of Proposition 1 Lemma 1 implies that $\Sigma_P \equiv \mathrm{var}[\Re_{t+1}|\Omega_t]$, $CE\Re = \frac{\phi^{-1}\rho}{2M^2}\Sigma_P$, $E\left[q_t^{i*}\Re_{t+1}|\Omega_t\right] = \frac{\phi^{-1}\rho}{M^2}\Sigma_P$, and $z_{t+1}^{i*} - z_t^{i*} =\backslash$ $\phi^{-1}\{q_t^{i*}\Re_{t+1} + \mathrm{I} - CE\Re\}$, which implies $\mathrm{var}\left[z_{t+1}^{i*} - z_t^{i*}|\Omega_t\right] = (\frac{1}{\phi M})^2\Sigma_P$. Since $\frac{\partial\Sigma_P}{\partial Q_j} < 0$ for $j \in \{1,2\}$ by Corollary 2 (proved independently below), parts (a) and (b) follow. Parts (c) and (d) are proved by relating $E\left[z_{t+s}^{i*}|\Omega_t\right]$ and $E\left[\Pi_{t+s}^{i*}|\Omega_t\right]$ to Σ_P. The formulas in Lemma 1 imply $W_{t+s}^{i*} = W_t^{i*} + \mathrm{I}s + \sum_{u=1}^s \{q_{t+u-1}^{i*}\Re_{t+u} - CE\Re\}$,

$$E\left[z_{t+s}^{i*}|\Omega_t\right] = \phi^{-1}\left\{W_t^{i*} - \left(\frac{1}{R-1} - s\right)\mathrm{I} + \left(\frac{\rho\phi^{-1}}{2M^2}\right)\left(\frac{1}{R-1} + s\right)\Sigma_P\right\},$$

and

$$E\left[\Pi_{t+s}^{i*}|\Omega_t\right] = W_t^{i*} - s\mathrm{I} + \left\{\phi + \frac{1}{2}(s-2)\right\}\frac{\rho\phi^{-1}}{M^2}\Sigma_P - \frac{1}{M}\left(\frac{\gamma^s}{R-\gamma}\right)E[c_{t+1}|\Omega_t].$$

The latter two formulas are valid for all $t \geq 1$ and $s \geq 0$. Plugging in $W_t^{i*} = B_t^{i*} + (P_t + c_t)q_{t-1}$ and taking the partial derivatives holding B_t^{i*} and $E[c_{t+1}|\Omega_t]$ fixed yields parts (c) and (d). Finally, the discussion leading up to Eq. (A1) and the formulas in Lemma 1 imply that the value of investor i's equilibrium utility function (a.k.a. the "derived utility") $J\left(W_\tau^{i*}\right) = E[-\sum_{\tau=t}^\infty \beta^{\tau-t}e^{-\rho z_\tau^{i*}}|\Omega_t]$ equals

$$
\begin{aligned}
J\left(W_\tau^{i*}\right) &= -e^{-\rho\phi^{-1}W_\tau^{i*}+\ln\phi+\left(\frac{1}{R-1}\right)\left\{\ln(R\beta)-\left(E\left[q_\tau^i\Re_{\tau+1}\middle|\Omega_\tau^i\right]-\frac{\rho\phi^{-1}}{2}\mathrm{var}\left[\mathrm{q}_\tau^{\mathrm{i}}\Re_{\tau+1}\middle|\Omega_\tau^{\mathrm{i}}\right]\right)\right\}} \\
&= -\phi e^{-\rho\phi^{-1}\left\{W_\tau^{i*}-\left(\frac{\mathrm{I}-CE\Re}{R-1}\right)I\right\}} \\
&= -\phi e^{-\rho z_\tau^{i*}}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\frac{\partial}{\partial Q_j}\Bigg|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}} E\left[-\sum_{\tau=t}^{\infty}\beta^{\tau-t}e^{-\rho z_\tau^{i*}}|\Omega_t\right] \\
&\quad = -\phi\frac{\partial}{\partial Q_j}\Bigg|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}} e^{-\rho z_\tau^{i*}} = +\phi\rho e^{-\rho z_\tau^{i*}}\frac{\partial}{\partial Q_j}\Bigg|_{\substack{B_t^{i*} \\ E[c_{t+1}|\Omega_t]}} z_\tau^{i*} > 0,
\end{aligned}
$$

which proves part (e). I thank Peter Christensen for pointing out an error in an earlier statement of part (e). □

Proof of Proposition 2 Equation (1) implies $\sum_{\tau=1}^{\infty}\frac{E[c_{t+\tau}|\Omega_t]}{R^\tau} = \frac{E[c_{t+1}|\Omega_t]}{R-\gamma}$, which means evaluating Eq. (8) requires evaluating $E[c_{t+1}|\Omega_t]$. Plugging $s_t \equiv \frac{1}{\lambda}\{x_t - c_t + \lambda\theta_t + \lambda\delta_{t-1}\}$ into Eq. (7) and rearranging yields

$$
\begin{aligned}
E[c_{t+1}|\Omega_t] &= \gamma c_t + \tfrac{Q_P}{\lambda}\{x_t - c_t + \lambda\theta_t + \lambda\delta_{t-1}\} \\
&= \gamma c_t + \tfrac{Q_P}{\lambda}\left\{\left(\tfrac{R-\gamma}{R-1}\right)\hat{x}_t - c_t\right\} \\
&= (R-\gamma)\left\{\tfrac{\gamma c_t}{R-\gamma} + \tfrac{Q_P}{R\lambda}\left[\phi\hat{x}_t - \tfrac{R\,c_t}{R-\gamma}\right]\right\},
\end{aligned}
$$

where ϕ and $\hat{x}_t$ are as defined in the Proposition. Rearranging this further yields

$$
\frac{E[c_{t+1}|\Omega_t]}{R-\gamma} = \left(1-\frac{Q_P}{R\lambda}\right)\frac{\gamma c_t}{R-\gamma} + \frac{Q_P}{R\lambda}[\phi\hat{x}_t - c_t]. \tag{A2}
$$

Plugging Eq. (A2) into Eq. (8) yields Eq. (10). Next, we evaluate Σ_P starting from Eq. (9). To this end, recalling that $Q_P \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}$, $Q_c \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$, and $\sigma_c^2 \equiv \sigma_\theta^2 + \sigma_\varepsilon^2$ and rearranging Eq. (9) yields

$$
\begin{aligned}
\Sigma_P &= \left(\frac{R}{R-\gamma}\right)^2\left\{(1-Q_P)\sigma_\theta^2 + \sigma_\varepsilon^2\right\} + \left(\frac{Q_P}{R-\gamma}\right)^2\left\{\sigma_\theta^2 + \sigma_\delta^2\right\} \\
&= \left(\frac{R}{R-\gamma}\right)^2\left\{\left[(1-Q_P)\sigma_\theta^2 + \sigma_\varepsilon^2\right] + \left(\frac{1}{R}\right)^2 Q_P\sigma_\theta^2\right\} \\
&= \left(\frac{R}{R-\gamma}\right)^2\left\{[(1-Q_P)Q_C + (1-Q_C)] + \left(\frac{1}{R}\right)^2 Q_P Q_C\right\}\sigma_c^2.
\end{aligned}
$$

Finally, observing that $(1-Q_P)Q_C + (1-Q_C) = 1 - Q_P Q_C$ proves the formula for Σ_P. □

Proof of Corollary 1 $\frac{\partial P_t}{\partial V_t^c} = 0$ implies $P_t = V_t^x - \frac{\rho}{RM}\Sigma_P = \phi\hat{x}_t - c_t - \frac{\rho}{RM}\Sigma_P$. Hence, $\frac{\partial P_t}{\partial V_t^x} = 1$. The second part of the Proposition is proved in the main text preceding the Proposition. □

Proof of Corollary 2 Follows from taking the partial derivative of Eq. (11). □

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