

The Price of Diversifiable Risk in Venture Capital and Private Equity

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ABSTRACT

This paper explores the private equity and venture capital (VC) markets and extends the standard principal-agent problem between the investors and venture capitalist to show how it alters the interaction between the venture capitalist and the entrepreneur. Since the investor-VC contract is set before the VC finds any investments, we show that it is the entrepreneur who must compensate the venture capitalist for any extra risk in the project even though it is the investor who requires the VC to hold the risk and even though the entrepreneur holds all of the market power in the model. Furthermore, although perfectly competitive investors expect zero alpha in equilibrium, the nature of the three way interaction results in a correlation between total risk and investor returns even net of fees. Thus, we show how and why diversifiable risk should be priced in VC deals even though investors are fully diversified. We then take our theory to a unique data set and show that while investors do earn zero alpha on average there is a strong correlation between realized risk and investor returns, exactly as predicted by the theory.

The Price of Diversifiable Risk in VC and Private Equity

Venture capitalists (often called VCs) are known to use high discount rates in assessing potential investments. This may just be a fudge factor that offsets optimistic entrepreneurial projections, but VCs claim to use high discount rates even in internal projections. Furthermore, Cochrane (2004) looks at individual VC projects and shows that they are surprisingly profitable, and earn large positive alphas. In general, VCs seem overly concerned with total risk, especially considering that fund investors are well diversified. Why?

This paper answers this question with a novel theory that links the principal-agent problem to asset prices, with empirical tests of the theory on newly available venture fund data. Our theory extends the principal-agent problem to interactions between the agent and a third party and demonstrates the impact of this extension in the venture capital arena. The principal-agent problem between an investor and a venture capitalist alters the interaction between the venture capitalist and the entrepreneur. We show that it is the *entrepreneur* who must compensate the venture capitalist for the extra risk that the *investor* requires the VCs to hold, even though the entrepreneur holds all of the market power in the model. Furthermore, although perfectly competitive investors expect zero alpha in equilibrium, the nature of the three-way interaction results in a correlation between total risk and average return even net of fees. Thus, we show that diversifiable risk can be priced in VC deals, even if the outside investors are fully diversified.

We then take our theory to a unique data set and show that while investors do earn zero alpha on average as expected, there is a strong correlation between realized risk and investor returns, exactly as predicted by the model: the quartile of VC funds with the greatest idiosyncratic risk has an alpha of 2.52% per quarter, while the lowest quartile has a per quarter alpha of -1.09%.

The trade-off between risk and incentives is a classic feature of contracts. Much of the work on this aspect of the principal-agent problem has focused on either the optimal contract (for example see Holmstrom and Weiss (1985) or Holmstrom and Milgrom (1987)) or on the attempt to see the resulting trade-offs empirically (see Prendergast (1999) for a recent survey). In this paper, our main contribution is to examine both theoretically and empirically the effect of the principal-agent problem on equilibrium asset prices. The VC market is the ideal arena

to examine the economic significance of the principal-agent problem. The investing principals are legally barred from actively monitoring their VC agents, so the principal-agent problem should be a central concern. Thus, if the principal-agent problem, a staple of economic theory, is fundamental it should have a large impact on the VC market.

To bring the principal-agent problem to asset prices we develop a simple yet novel model. The basic setting is that the private equity and venture capital markets are characterized by entrepreneurs with ideas, and outside investors who are well-diversified, but have little ability to screen and manage potential investments. Investors hire VCs who have considerable expertise in assessing and overseeing entrepreneurial ideas.¹ Typically VCs have little capital of their own, so they are in essence money managers, helping investors supervise their investments. Because of standard incentive problems, VCs receive an interest in the firms they fund. They are unable to monetize their holdings and are instead forced to hold a substantial amount of their wealth in the form of these contingent stakes. Furthermore, significant time is required to oversee an investment, which means that a VC can manage only a few investments. This means that VCs hold considerable idiosyncratic risk and must be compensated for bearing this risk.

The standard principal-agent problem ends here. If VCs were simply compensated by the principal for the services they provide and the risk they bear, then fund returns net of VC fees, earned by well-diversified investors, would be uncorrelated with idiosyncratic risk. Investors would compete away excess returns, resulting in zero net alphas.²

In this paper, we go one step further, studying the impact of the investor-VC problem on negotiations between the VC and the entrepreneur. The sequencing is key. VCs and investors strike their bargain first. The contract that exposes the VC to (and compensates the VC for) the risk of the portfolio is set before the VC actually locates any projects. Thus, the investor-VC contract is designed to compensate the VC for the expected risk in the portfolio, not the realized risk. It is not possible to contract on realized risk both because it is unverifiable but also because, at least in the U.S., laws governing the investor-VC relationship require the

¹For example, the average entrepreneurial idea may have a negative NPV if not guided correctly, but the VC eliminates projects with the least potential and manages the others to success, making the average VC project NPV positive.

²It is important to distinguish between gross returns on VC investments and net returns to investors. VC fees of any sort drive a wedge between these two. To get to zero average alphas on net returns, gross expected returns must exhibit positive alphas. Thus, Cochrane's (2001) finding that individual VC investments earn positive alphas should not be surprising and need not imply that outside investors earn excess returns.

investor to be passive and thus uninformed about the specific projects.³

The VC, with contract in hand, now negotiates with an entrepreneur. He does not care about the average risk, but only cares about the risk in the project being considered. All else equal, if this project has greater (less) than average risk then the VC requires a larger (smaller) return to invest. The contract with the investor is set, so the VC requires the entrepreneur to provide a price reduction on the deal that justifies his *personal* level of risk. However, this price reduction also benefits the outside investors. Thus, this ‘externality’ requires the entrepreneur to give up more than the amount necessary to just compensate the VC! As a result, higher than expected risk projects earn investors higher returns on average and lower than expected risk projects earn investors lower returns, even though on average investors earn zero alphas.

Thus, if our model is correct, fund returns net of fees should have zero alphas but should still be correlated with ex post total risk! We show formally that this result would not arise if there were no principal-agent problem. Even if, say, VCs must work harder to oversee higher risk firms (and thus demand higher fees), without a principal-agent problem investors’ net returns would not depend on risk. Thus, this surprising prediction can *only* be tested with VC investment fund returns *net* of fees, and this key prediction distinguishes our model from others.

We use newly available data on VC and buyout fund returns to test our model. These data cover a significant fraction of the US venture capital and buyout fund universe, so they provide some of the first systematic insights into the overall performance of this asset class. In contrast to Gompers and Lerner (1997), in our large sample we find that fund investors do not earn positive alphas on average. Buyout funds have a value-weighted IRR of 4.57%, and venture capital funds have a value-weighted IRR of 19.31%, but these are commensurate with the factor risks that these investors bear (i.e., the alphas are insignificantly different from zero). These insignificant alphas are consistent with our model (and any model with perfect competition between investors).

More importantly, we provide evidence that idiosyncratic risk is priced, even in net fund returns. Using proxies for the number of investments, we find that concentrated portfolios have higher net returns, consistent with our predictions. We also measure idiosyncratic risk

³In a typical fund the venture capitalists are the general partners (GPs) and the investors are the Limited Partners (LPs). To maintain their status as limited partners the LP must not monitor or participate in the investing process. Thus, the principal-agent problem in the VC market is enshrined in U.S. law.

directly. Funds with higher realized risk have higher average *net* returns, exactly as predicted by the model. Thus we find strong support for the theory. Furthermore, the calibration of our model suggests that the dead weight loss associated with the principal-agent problem could be 7% of the total amount invested, which was \$175 billion in 1999 alone! Thus the principal-agent problem could have considerable economic importance in the venture capital market.

The impact of idiosyncratic risk also introduces a different notion of competition in the VC market. Low risk aversion is one dimension of competitive advantage in the VC industry. A VC who is less risk-averse than the marginal VC could earn excess returns. Furthermore, in our model, competition between VCs is not about the ability to find ‘better’ (higher NPV) projects, because competition among investors ensures all excess return goes to the entrepreneur. Instead, a VC who could manage *more* projects than the marginal VC would hold less idiosyncratic risk. This would leave excess return for either the VC or investors.

The interaction between risk, incentives and returns might suggest that either the investor would like to encourage the VC to take on extra risk or that the VC himself may want to spend time altering the risk or makeup of his portfolio. However, we show this is not true. First, the investors only earn positive excess returns on projects with risk *above what is expected*. If VCs and investors agree to pursue higher risk projects (say, in biotech), the VC fee structure would reflect this and investors would still earn zero alphas on average. Second, we show that within our extended principal-agent model two odd features of the investor-VC contract ensure that the VC does not want to alter risk. The typical contract gives a VC an in-the-money option on the portfolio. This is odd both because an option would seem to encourage excess risk taking, and because we would expect the investor to require the VC to beat some positive benchmark, at least the risk-free rate, before participating in the upside. However, we show that the trade-offs faced by the VCs imply that only an in-the-money option creates the right incentives for the VC to focus on returns and not spend energy altering risk that does not affect diversified investors.

The focus of this paper is the pricing effect of the principal-agent problem. However, the principal-agent problem can also explain other features of the venture industry. For example, a principal-agent problem plus constant returns to scale implies that VC should not be a concentrated industry. Funds must be small enough to ensure that the agent’s compensation can be tied directly to the performance of his portfolio companies. As another example, if VC risk aversion is declining in wealth, then following a period of overall success, the VC industry

as a whole should be more willing to support riskier projects. The same principal-agent problem also affects pricing in other arenas. We argue that the same principal-agent problem is often present for investment decisions made inside a firm.⁴ This means that idiosyncratic risk is priced in internal capital markets. In fact, in any situation where agents that hold idiosyncratic risk set asset prices, those prices should be decreasing in the amount of risk.

Finally, it is worth noting that other aspects of the VC and private equity markets, such as illiquidity or a lack of competition, undoubtedly affect returns. However, these should affect average alphas rather than the cross-section of returns. In any case, we abstract away from these issues to focus on the asset pricing effects of the principal-agent problem.

Our paper is organized as follows. Section I develops the model and examines the effect of an equity contract between the investors and VC. Section II calibrates the model to determine the economic significance. Section III provides empirical support for the model using venture capital fund return data. Section IV examines the VC's incentive to choose more or fewer projects and provides an explanation for the standard option-like contract. Section V explores other implications and extensions, and Section VI concludes.

I The Principal-Agent Problem

The trade-off between risk and incentives is a classic issue between a principal and an agent. Because the principal does not have perfect information about the agent's types and/or complete information about the agent's actions, the principal-agent contract must leave the risk-averse agent with too much risk relative to the first-best solution. The standard problem is that an agent could be a bad type or that an agent must take an action that is costly and unverifiable, such as expending effort. To combat either problem, the principal commits to a contract where the agent's payoff depends on an observable output.⁵ If output is subject to shocks that are beyond the agent's control, then these contingent contracts impose risk onto the agent.

⁴Simultaneous work by Himmelberg, Hubbard, and Love (2002) argues that agency conflict between inside managers and outside owners of a firm leads managers to hold large positions, particularly in countries where investor protection is low. Thus, the cost-of-capital reflects idiosyncratic risk.

⁵Holmstrom and Ricart-I-Costa(1986) offer the idea that even if contracts do not explicitly depend on output, principals use the outcome of the agent's decision as a signal of the agent's quality. Since principals promote the high quality agents (they cannot commit to provide full insurance), agents hold risk and their incentives are distorted.

In our model, the venture capitalist is an agent who must be compensated for the opportunity cost of his time. Due to the investor's (principal's) lack of information about the type of the VC or the VC's actions, the VC's compensation must depend on the returns of his chosen projects. This matches reality, as the standard compensation scheme in private equity and VC is a fixed payment (typically near 2% of the fund per year) and a fraction of the return above some benchmark. Since the VC has limited wealth, a significant portion of his wealth is the present value of his portion of the project returns.^{6,7} The VC must invest significant time and effort (including board meetings, meeting with management/customers/suppliers, understanding the market, etc.) to help a project realize its value. Therefore, a VC can only manage a limited number of investments. Gompers and Lerner (1999) note that funds typically invest in at most two dozen firms over about three years. In addition, a VC's expertise may be limited to a particular sector or industry, which means that the VC may remain exposed to sector risk no matter how many projects he selects. Even if a VC can diversify across the entire VC industry, he may not be fully diversified because all VC projects may contain a correlated idiosyncratic risk component. For these reasons, the VC is exposed to significant idiosyncratic risk.⁸

Why not just combine a large number of VC investments into one much larger fund and compensate the VC based on total fund performance? The answer is that this would eliminate the link between a venture capitalist's compensation and his chosen projects. If the principal-agent problem were due to costly effort, VCs would exert too little of it. Said another way, the principal-agent problem remains regardless of how VC investments are aggregated into funds, so removing risk from the VC is not optimal. Furthermore, with aggregation VCs may still hold idiosyncratic risk because even in a larger fund the individual VC's career would depend

⁶In practice, when VCs have significant wealth, they are typically required to invest a large fraction of it (perhaps 30-70%) in the fund to show that they "believe" in what they are doing. In other words they must invest in the fund either as a costly signal that they are good or to ensure greater effort.

⁷Furthermore, the VC's ability to raise future funds depends on the success of his first fund (See Kaplan and Schoar and Gompers (1996). Chevalier and Ellison (1995) also look at the same issue in mutual funds). Therefore, the VC's future income stream depends on the success of the fund. This compounds the effect of any idiosyncratic risk held by the VC. This idea is similar to Holmstrom and Weiss (1985) who focus on future career concerns rather than specifically contingent contracts.

⁸Meulbroek (2001) expresses a related idea that managers of internet firms who receive stock cause a deadweight loss because outsiders can diversify. In our model, we assert that the relevant outsider is probably a venture capitalist who cannot diversify either. We agree that there is a deadweight loss relative to first-best, and we show that it manifests itself in venture capital asset prices.

on the projects he chooses rather than the overall portfolio.

Our theory says that prices should be low in VC and private equity even if there is intense competition among VCs for projects. Prices get bid up until the VC is just indifferent, but this price is still below the price implied by, say, the CAPM. Since the VC needs to be compensated, gross expected returns on the venture capital investment are higher than the factor risks would suggest.

Since VCs correctly use a higher discount rate to evaluate projects, some projects are not taken that are positive NPV based on factor risk alone. This is in line with earlier work, as the principal-agent problem has consistently been shown to distort investment. In papers such as Holmstrom and Ricart-I-Costa (1986), and Harris, Kriebel and Raviv (1982) principal-agent problems lead to underinvestment by the principal. In papers such as Lambert (1986) and Holmstrom and Weiss (1985) the agent's investment choice is distorted. However, none of these papers explicitly consider prices. In our model, the necessary pricing of idiosyncratic risk can result in the decision not to invest. The principal-agent problem leads to a contract that may cause a competitive VC and an entrepreneur to be unable to find a price to fund an otherwise profitable project.

Even though the VCs correctly use higher discount rates, the well-diversified investors who give money to the VCs do not earn excess returns in expectation. The contract between the investor and VC awards part of the fund's return to the VC. Since the investor can easily diversify, competition between investors means that they earn zero alphas on average.

Our extension of the principal-agent theory to the interaction between the VC and entrepreneur produces a novel prediction that is not easily explained by other potential theories. The VC and the investors negotiate their contract before the VC locates the eventual fund investments. Thus, the contract is negotiated based on the expected level of risk. However, once the VC identifies profitable projects, the actual amount of risk may differ from the expected amount. Since the realized level of risk cannot be verified by the investor, the contract cannot depend on the actual risk. When the actual risk is higher than the *ex ante* expectation, the VC demands more from the entrepreneur to compensate him for the additional risk. Therefore, although investors receive zero alphas on average, returns net of fees are positively correlated with *ex post* idiosyncratic risk.

If the principal-agent problem truly has economic significance it is here in the VC market that we should see its impact. The prediction from our model is not easily explained by other potential theories, and its presence in the data gives us some confidence that the principal-

agent problem does indeed affect prices and returns.

II The Model

The model has three participants: investors, venture capitalists and entrepreneurs. Investors are willing to invest a total of I dollars into a fund that invests in N projects. Each project receives I/N dollars. Entrepreneurs have project ideas but need some help and guidance to realize the value of their ideas. Their ideas produce random output of $(1 + R_i)I/N$ if they are overseen by a skilled, involved investor and zero if they are not. Even with guidance, the projects have both systematic and idiosyncratic risk and an uncertain return $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$, where R_m is the return on the market and ε_i is idiosyncratic risk. There is also a risk free asset with zero return, and the single-period CAPM holds for all traded assets. The projects may be positive or negative NPV, $\alpha_i \geq 0$. R_i and R_m are jointly normal, with $E[\varepsilon_i] = 0$, $E[\varepsilon_i R_m] = 0$, and $E[\varepsilon_i \varepsilon_j] = 0$, for all project pairs i and j .⁹ Therefore, the expected return on a project is $\mu_i = 1 + \alpha_i + \beta_i E[R_m]$ with variance $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$. We let the subscripts i and j represent particular projects from the space of all possible projects, Ω .

Entrepreneurs have no money. Investors have money, but do not have the skill to determine whether a project is positive or negative NPV, or to manage a project.¹⁰ The VCs, also with zero wealth¹¹, have the ability to locate and successfully manage projects, and determine the characteristics of those projects, α_i , β_i and σ_{ε_i} (the investor knows the distribution of these parameter values but cannot verify a particular project's characteristics). In order to successfully manage a project, a VC must exert effort with an opportunity cost of e_{vc} (as in Grossman and Hart (1981)). The effort of the VC is unverifiable. Therefore, the VC must be compensated in order to manage investments for the investors, and this compensation must

⁹ $E[\varepsilon_i \varepsilon_j] = 0$ is not required, but it simplifies the exposition of the results. The appendix drops this assumption.

¹⁰There are many more negative NPV projects than positive ones or, equivalently, the losses from the negative NPV projects are larger than the gains from the positive NPV projects. Therefore, if the investor invests in a randomly chosen project he loses money in expectation even if he could successfully manage the project.

¹¹This assumption ensures that the VCs compensation is a significant fraction of the VC's wealth and therefore its impact cannot be diversified away with outside wealth. In actuality VCs with prior significant wealth are often required to put between 30-70% of their total net worth in their fund. We argue that this signals their competence and ensures their effort by reducing their ability to diversify.

Limited wealth also ensures that the VC can only receive a positive payoff from the fund.

provide the VC with the incentive to provide effort.

Managing a project is a time consuming process. Therefore, initially we assume the VC is unable to successfully manage more than N projects.¹² However, in Section V we relax this assumption and examine the VC's incentives to choose more or fewer projects.¹³

Investors, venture capitalists, and entrepreneurs are all risk-averse and require compensation for the risk that they hold. However, investors have enough wealth outside the fund that they are well diversified and therefore only require returns for their undiversifiable or factor risk.

The timing of the model is a three-stage game. In the first stage, the investors and VC form a fund and agree on a contract to govern their relationship. In the second stage, the VC negotiates the payoff schedule that the entrepreneurs give up to get I/N dollars from the fund. The investments also occur in the second stage. In the final stage, project values are realized and payoffs are distributed.

We assume that there are enough investors competing for VCs that the VC's are able to maximize what they receive from the investors. Further, we assume that there are few enough positive NPV projects that entrepreneurs maximize what they negotiate from the fund for an investment. Since all rents accrue to the entrepreneur, this minimizes the chance of finding any positive alphas in equilibrium.

Due to the principal-agent problem between the investor and VC, the optimal contract between them depends on the output of the projects.¹⁴ Further, due to the need to share risk between the investor who can diversify and the entrepreneur who cannot, the optimal contract between the fund and the entrepreneur must depend on output.¹⁵ The results of this paper hold as long as the contracts depend on the output of the projects. However, in order

¹²In addition, the typical VC contract restricts the amount of money that can be invested in one investment. Therefore, since the size of the fund is given, the VC must invest in a minimum number of projects.

¹³The principal-agent problem that surrounds many aspects of project choice is an interesting problem studied by Dybvig, Farnsworth and Carpenter (2001), Kihlstrom (1988), Stoughton (1993) and Sung (1995). Here we focus only on the choice of risk.

¹⁴The use of a contract that depends on the performance of the fund can also be motivated with a signaling story. For example, the contract can be used to separate the VCs that are able to accurately screen projects from those who cannot. Those VCs that have no skill would be less willing to take a contract that rewards them only if the fund does well.

¹⁵An output-based contract between the fund and the entrepreneur could also be motivated by a principal-agent problem, or if the VC has more information about the success of the project than the entrepreneur, etc.

to achieve simple closed form solutions we assume that both contracts are equity contracts. Thus, the negotiations are first over ϕ , the fraction of the fund given to the VCs, and then over the fraction, θ_i , of the company I/N dollars will purchase.¹⁶

Holmstrom and Milgrom (1987) show that a linear sharing rule is optimal when effort choice and output are continuous, but monitoring by the principal is periodic. The idea is “that optimal rules in a rich environment must work well in a range of circumstances and therefore would not be complicated functions of the outcome” (Holmstrom and Milgrom (1987) p 325). Dybvig, Farnsworth and Carpenter (2001) show that an option-like contract may be optimal if the agent chooses both effort and a portfolio. Under different conditions different contracts are optimal. We do not want to focus on the actual effort choice (see Gompers and Lerner (1999), Gibbons and Murphy (1992) and, of course, Holmstrom and Milgrom (1987) for interesting work which focuses on the effort decision) and we are not interested in the trade-offs involved in a particular contract. Instead we simply wish to motivate the use of a sharing rule rather than fixed compensation. In our work we take the form of the contract as given and focus on its implications for asset prices; the implications are the same as long as the contracts depend on the output.

II.1 Benchmark: No Venture Capitalist Needed.

In order to provide a benchmark to compare to the more interesting results to follow, we first consider the pricing when the investors can profitably invest directly in the entrepreneurial projects and projects require no oversight. Thus, for a moment we remove the VC from the problem, but assume that the project is still positive NPV (specifically, we assume the investors can determine and expect to earn the α_i , β_i and σ_{ε_i} of project i).

Given this setup, mean-variance preferences plus perfect competition among investors ensures that investors are willing to fund the project as long as $\alpha_i \geq 0$. That is, investors are willing to fund positive NPV projects, where discount rates are determined using the CAPM. Perfect competition among well-diversified outside investors implies that the entrepreneur retains all the economic rents from the project. That is, outside investors are willing to fund the project on terms such that their expected return on investment just compensates them for the systematic risk they bear. Thus, we will show that outside investors earn a zero alpha in expectation.

¹⁶As already mentioned, it is possible that no θ exists that is acceptable to both the VC and entrepreneur. We examine this more thoroughly in a moment.

In the absence of a VC, investors fund a project directly. To begin we assume for simplicity that $N = 1$ (the appendix considers multiple projects). Investors put up I dollars and receive a fraction θ_i of the firm. The firm's random payoff is $(1 + R_i)I$, where, as described earlier, project returns follow the single factor model

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i. \quad (1)$$

Thus the investors receive $\theta_i(1 + R_i)I$. This implies that the beta of the investors' returns with respect to the market return is equal to $\theta_i\beta_i$.¹⁷ Given this setup, the entrepreneur minimizes θ_i subject to giving the investors a fair return:

$$\min \theta_i \text{ s.t. } \frac{\theta_i(1 + \alpha_i + \beta_i E[R_m])I}{1 + \theta_i\beta_i E[R_m]} = I, \quad 0 \leq \theta_i \leq 1. \quad (2)$$

The constraint is the expected payoff to the investors discounted at the appropriate CAPM rate, and generates an NPV of exactly zero.

There is only one θ_i that satisfies the constraint, and it is given by:

$$\theta_i = (1 + \alpha_i)^{-1}, \quad (3)$$

provided that $\alpha_i \geq 0$. Since the fraction θ_i of the firm is worth I dollars, the so-called *post-money* implied value of the whole firm is $I\theta_i^{-1}$ or simply $I(1 + \alpha_i)$, which reflects the investment plus the expected value added by taking on this positive NPV project. The so-called *pre-money* value of the firm is just the post-money value less the amount contributed by investors, or in this case $I\alpha_i$. Thus, all the rents accrue to the entrepreneur.¹⁸

II.2 The Venture Capitalist's Impact on Prices.

Now we address the more interesting case where there is a VC present. As before, the entrepreneur gives up a fraction of the firm θ_i to the investors, but now the investor gives the VC a fraction of the firm $\phi\theta_i$ and retains the fraction $(1 - \phi)\theta_i$.

¹⁷Although the concept of beta is scale-independent, a change in θ changes the fraction of the project owned by the investor even though he invests the same amount. Thus, his exposure to the risk of the project changes with the fraction he owns. For example, suppose the investor puts up all of the cash but receives only 75% of the project payoff. An additional 1% return on the project increases the investor's return by only 0.75%.

¹⁸A simple example will help clarify the terms post-money and pre-money value. If the entrepreneur convinces the investor that his firm/idea is worth \$2 million and the investor invests \$1 million at that valuation, then the value of the firm pre-money was \$2 million and the post-money value is \$3 million. These terms are used to make it clear that although the investment increases the value of the firm it does not change the price of the stock (there is more money but also more shares).

The VC is risk-averse with exponential utility over terminal wealth w :

$$u(w) = -\exp(-Aw), \quad (4)$$

where A is the VC's coefficient of absolute risk aversion. If terminal wealth is normally distributed, then maximizing expected utility is equivalent to

$$\max \mu_w - \frac{1}{2}A\sigma_w^2, \quad (5)$$

where μ_w and σ_w^2 are the mean and variance of wealth. This functional form for utility allows for a closed-form solution but does not drive the results. All that is necessary is a risk-averse VC. The appendix allows for a general risk averse utility function.

The VC has no other wealth but has outside opportunities that give him a certain payoff e_{vc} . Thus, in order to manage potential investments, the VC requires compensation that generates at least as much utility as e_{vc} , the opportunity cost of his effort.

We derive the solution using backward induction. Thus, we begin with the second stage negotiations. In the second stage the VC locates a suitable project, determines its characteristics, α_i , β_i and σ_{ε_i} , and negotiates with the entrepreneur. As in the benchmark case, the entrepreneur minimizes the fraction θ_i . However, this minimization is now subject to providing the VC with utility greater than the opportunity cost of his effort. Thus, the introduction of the VC alters the constraint faced by the entrepreneur from the benchmark case. Formally, the minimization problem is

$$\begin{aligned} \min \theta_i \\ \text{s.t. } \phi\theta_i I\mu_i - \frac{1}{2}AI^2\phi^2\theta_i^2\sigma_i^2 \geq e_{vc}, \end{aligned} \quad (6)$$

where ϕ is the predetermined contract between the VC and the investor, $\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_{\varepsilon_i}^2$ is the total variance of payoffs, and $\mu_i = 1 + \alpha_i + \beta_i E[R_m]$ is the expected return on the project.¹⁹

The entrepreneur wants to minimize the VC's and investor's take subject to the VC's constraint, and since the market is competitive, the offered contract provides a certainty equivalent of exactly e_{vc} . The binding constraint is quadratic in ϕ and yields the following expression for the share of the company offered to the VC in equilibrium:

$$\theta_i^* = \frac{\mu_i - (\mu_i^2 - 2Ae_{vc}\sigma_i^2)^{\frac{1}{2}}}{A\phi I\sigma_i^2} \quad (7)$$

¹⁹We assume that for $\theta \in [0, 1]$ the VC's utility increases with θ . Specifically this requires $\phi I\mu_i - AI^2\phi^2\sigma_i^2 > 0$. This eliminates the economically unreasonable (but mathematically possible) outcome that the VC's utility is improved by decreasing the fraction of the firm he receives.

Note that the model implicitly assumes that the VC has no access to the capital markets. If he could, the VC might want to hedge out his market risk by trading in the market portfolio. Based on our conversations with venture capitalists, we do not believe that such hedging is common. If the VC were able to eliminate all market risk, pricing would still be affected by idiosyncratic risk rather than total project risk.

Moving back one stage, to the first stage of the model, the VC and the investors must negotiate the equity contract that will govern their relationship. At this stage the VC has not yet located a suitable investment so the characteristics of the investment are not known. However, both the VC and investors can determine the distribution of the subset of projects that the VC would accept in the second stage, $\Omega_{vc} \subset \Omega$. As discussed above, we assume the VC's skill enables him to manage N projects from the set Ω_{vc} . Therefore, before an investment has been located investors' expectations are over the subset of projects Ω_{vc} .²⁰

In negotiating a contract, the VC attempts to maximize his expected utility subject to the investors receiving a fair return in expectation for the risk they hold:

$$\begin{aligned} & \max_{\phi} E_{\Omega_{vc}} [u(w(\phi))] \\ & \text{s.t. } E_{\Omega_{vc}} \left[\frac{\theta_i^*(1-\phi)\mu_i I}{1 + \theta_i^*(1-\phi)\beta_i E[R_m]} \right] \geq I, \end{aligned} \quad (8)$$

As in the benchmark case, the constraint binds because we have assumed perfect competition for VCs and investments. However, the constraint now includes an expectation because the investor does not know the project characteristics when he negotiates with the VC.

In order to get an easy-to-interpret closed-form solution we make the following simplifying assumption. We assume that there are only two equally likely projects within Ω_{vc} , and each project only differs in its level of idiosyncratic risk $\sigma_{\varepsilon_i}^2$. Thus, the VC receives a different fraction θ_i^* from each project. These assumptions do not drive any of our results and are dropped in the general formulation in the appendix; they simply make clear the effect of idiosyncratic risk. The binding constraint becomes,

$$\frac{\theta_1^*(1-\phi)\mu_1 I}{1 + \theta_1^*(1-\phi)\beta_1 E[R_m]} \frac{1}{2} + \frac{\theta_2^*(1-\phi)\mu_2 I}{1 + \theta_2^*(1-\phi)\beta_2 E[R_m]} \frac{1}{2} = I, \quad (9)$$

²⁰The expected returns are not normally distributed because a mixture of normals is not normal. However, the investors are assumed to have mean-variance preferences and the VC's expected wealth is still normal conditional on a chosen project. Therefore, the investor is willing to accept a contract as long as the expected present value of his return is greater than his investment.

where $\mu_1 = \mu_2 \equiv \mu$ and $\beta_1 = \beta_2 \equiv \beta$. Solving for ϕ yields²¹,

$$\phi^* = 1 - \frac{1}{\sqrt{(1 + \alpha)\theta_1^*\theta_2^*\beta E[R_m] + \frac{1}{16}(\theta_1^* + \theta_2^*)^2(1 + \alpha - \beta E[R_m])^2 + \frac{1}{4}(\theta_1^* + \theta_2^*)(1 + \alpha - \beta E[R_m])}}. \quad (10)$$

There is clearly a solution to Equations (7) and (10).²² However, as developed below there is not necessarily a solution with θ^* and ϕ^* between zero and one. Thus, a deal may be economically impossible.

II.2.1 What if there were no principal-agent problem?

A central insight of our model is that in pricing and in capital budgeting, the principal-agent problem introduces a wedge between gross returns on investment and net returns to investors. We will show that the size of this wedge varies with the level of idiosyncratic risk and affects expected returns to investors. However, even without a principal-agent problem, there would still be a wedge as long as there are VC fees paid out of the gross returns. Thus, it is important to distinguish which results are due simply to the wedge between gross and net returns, and which results are due specifically to the principal-agent problem. To do this, we develop a parallel model without the principal-agent problem. The formal development of this model can be found in the appendix, but we provide an overview here.

If there were no principal-agent problem, the VC would still locate projects and negotiate with the entrepreneur for a share of the project, but the investor would rely on the VC to take actions in the investor's best interest. VC compensation would not need to depend on fund performance. In this case, the VC's payment would depend on the project's expected returns (μ_i) rather than the realized returns. The VC's compensation must, of course, still be greater than or equal to the opportunity cost of his effort, e_{vc} , but the VC would hold no risk.

The effort required from the VC might also depend directly on the variance of the project. For example, if higher risk projects are more costly to evaluate or more costly to oversee, then the VC's effort can be written as $e_{vc}(\sigma_i^2)$, an increasing function of project variance. Note that even if VC effort depends on the risk, in the absence of the principal-agent problem, the

²¹We are only interested in the positive root since $\sqrt{\frac{1}{16}(\theta_1^* + \theta_2^*)^2(1 + \alpha - \beta E[R_m])^2 + \frac{1}{4}(\theta_1^* + \theta_2^*)(1 + \alpha - \beta E[R_m])}$ the negative root would yield $\phi > 1$, which would not make economic sense.

²²Proof: $\phi^*(\theta_1^*, \theta_2^*)$ is monotone and for any $\theta_2^* : \phi^*(0, \theta_2^*) < \infty$ and $\lim_{\theta_1^* \rightarrow \infty} \phi^*(\theta_1^*, \theta_2^*) = 1$. The inversion of equation (7) is monotone and $\lim_{\theta_1^* \rightarrow 0} \phi^*(\theta_1^*) \rightarrow \infty$ and $\lim_{\theta_1^* \rightarrow \infty} \phi^*(\theta_1^*) = 0$. Therefore, single crossing is satisfied and there must be an equilibrium for any θ_2^* (though not necessarily an economically reasonable one).

principal can simply agree to a contract that pays the VC more if the required effort is more. Thus, the VC still holds no risk but is compensated more for greater required effort.

We now turn to the implications of our model. Some results hold as long as there is a VC paid out of gross returns, and some are specific to the principal-agent problem. This will allow us to empirically distinguish the principal-agent problem from many other theories.

II.2.2 Theoretical Implications

In what follows, we present the main results in Theorems 1 through 4 and then, in the corresponding corollaries we consider the alternative in which there is no principal-agent problem. The corollaries show that Theorems 1 through 3, while interesting, cannot help us empirically distinguish our model. Thus, theorem 4 is the key implication of our model and the theorem we test.

Theorem 1 *Venture capital gross investment returns have positive alphas. Investor returns net of fees have zero alphas on average.*

Proof. See Appendix. ■

It appears that the investment has expected returns that are too high. On average, this isn't so. Nobody expects excess returns in this model. The returns above the benchmark case are just enough in expectation to compensate the VC for his services and for the idiosyncratic risk he holds. This is consistent with Cochrane (2004), who finds that individual VC investments earn large positive alphas.

It is interesting to note that relative to the benchmark case with no VC needed, the entrepreneur must give away too much of the firm in return for the investment. Thus, it is the entrepreneur who must compensate the VC for the additional risk that the investor requires the VC to hold even though the entrepreneur has all of the market power in the model. This is because the investor must on average earn a return commensurate with the factor risks or he will not participate in the VC market. Thus, in equilibrium all investment expenses must be borne by the entrepreneur.

Theorem 1 is an important implication of the principal-agent problem. However, this result is not too surprising and cannot help us empirically distinguish our model. As the following corollary shows, Theorem 1 holds in a model without the principal-agent problem.

Corollary 1 *Even if there is no principal-agent problem gross investment alphas may still be positive.*

Proof. See appendix. ■

Our theory predicts positive alphas before fees. However, as long as VC fees are nonzero, alphas before fees should be positive. Thus, we cannot use Theorem 1 to test our theory. For example, Cochrane’s (2004) empirical finding of large alphas on VC projects does not provide any evidence of a principal-agent problem. We need more.

Theorem 2 *Given any positive NPV project, if the total risk is large enough then the VC does not invest. Furthermore, if the ex ante expected α of the universe of projects that the VC would accept is positive but sufficiently small then the investor does not invest.*

Proof. See appendix. ■

Conditional on a contract with the investors, the VC would not be willing to accept some positive NPV projects that have high risk. Furthermore, although (for a given ϕ) the VC would be willing to accept some negative NPV projects²³ with low risk, the set Ω_{vc} must include projects with high enough α or the investor would be unwilling to provide the VC with a contract at all. Thus, in equilibrium a draw from the set Ω_{vc} (the only investments that will occur) must have a high expected α relative to its total risk.

When a paper claims that the NPV rule is no longer valid, it is important to ask which NPV rule it is and whether a suitably adjusted NPV calculation might restore order. In this context, it is useful to think of the VC’s share of the firm as consisting of two parts. One part is compensation to the VC for his effort, and the other part is compensation to the VC for the idiosyncratic risk that he must hold. The value of the VC’s effort could and perhaps should just be taken out of the net cash flows. This would go part of the way toward restoring the NPV rule. Compensation to the VC for risk is not quite the same, however. In principle, this too could simply come out of the net cash flows, and the NPV rule would be completely restored. But a higher hurdle rate that accounts for the VC’s idiosyncratic risk also makes intuitive sense, because more risk borne by the VC is associated with a higher implied hurdle

²³For the VC to choose to invest the VC’s constraint must be satisfied. Given ϕ^* the constraint, $\phi^*\theta_i I\mu_i - \frac{1}{2}AI^2\phi^{*2}\theta_i^2\sigma_i^2 \geq e_{vc}$, can be satisfied if the venture capitalist has sufficiently low risk aversion (small A) but $\phi^*\theta_i I\mu_i \geq e_{vc}$. Since the entrepreneur increases θ_i until the constraint is satisfied, and the largest $\theta_i = 1$, the constraint reduces to $\phi^* I\mu_i \geq e_{vc}$ or $1 + \alpha_i + \beta_i E[R_m] \geq e_{vc}/\phi^* I$. Therefore, a VC with sufficiently low risk aversion accepts a negative NPV project as long as the total return is large enough.

rate. Gross expected returns on the investment really do have to be higher because of the added risk.

Theorem 2, although interesting, has the same problem as Theorem 1. The following corollary shows that the same result would hold for a model without the principal-agent problem.

Corollary 2 *Even if there is no principal-agent problem some positive NPV projects may not get done.*

Proof. See appendix. ■

Since VCs use a higher discount rate to evaluate projects, some projects cannot get done that are positive NPV based on factor risk alone. However, it is also the case that compensation that does not depend on fund returns has the same qualitative effect. Therefore, as with Theorem 1, Theorem 2 cannot help us empirically determine if our theory is correct.

Theorem 3 *All else equal, the price the entrepreneur receives is decreasing in the amount of idiosyncratic risk. Therefore, gross returns are positively correlated with ex post idiosyncratic risk.*

Proof. See appendix. ■

If unavoidable principal-agent problems make diversification impossible, then idiosyncratic risk must be priced. Therefore, VC gross returns should be correlated with total risk, not just systematic risk. Projects with higher total risk should have higher returns.

It might seem that this theorem could help us empirically distinguish our model, since idiosyncratic risk is related to returns. However, the following corollary also shows that this prediction could also arise from other simple models. For example, if higher risk projects are more costly to evaluate or more costly to oversee, then even if there is no principal-agent problem the entrepreneur must pay more to get people to invest.

Corollary 3 *Even if there is no principal-agent problem the price the entrepreneur receives may still be decreasing in the amount of idiosyncratic risk.*

Proof. See appendix. ■

If high risk projects are more costly to oversee, then the VC must be paid more. The entrepreneur must give up more of his firm to get investors to invest, and gross returns would be higher for high risk projects. Therefore, Theorem 3 cannot help us distinguish the effect

of the principal-agent problem from other simple models. To do this we must look at net returns.

It might seem that, once we remove the VC's fees, the net return to investors should be unaffected by idiosyncratic risk, particularly since investors are perfectly competitive. However, the following theorem shows that the return to investors *is* affected by idiosyncratic risk.

Theorem 4 *On average, venture capital investment returns net of fees increase with the amount of ex post idiosyncratic risk, even though investors are well-diversified and face competitive market conditions.*

Proof. Assuming an economically reasonable equilibrium exists, the net fraction owned by the investors equals $\theta_i - \theta_i \phi^*$. Theorem 3 showed that the fraction given up by the entrepreneur, θ_i , varies with idiosyncratic risk, $d\theta_i/d\sigma_{\varepsilon_i}^2 > 0$. However, the share taken by the VC, ϕ^* , does not change with the realization of $\sigma_{\varepsilon_i}^2$ because it is determined ex ante before the project risk is known. Therefore, the fraction held by investors is a function of $\sigma_{\varepsilon_i}^2$. Thus, the net returns are correlated with realized idiosyncratic risk. Competitive conditions ensure that the expected alpha is zero, but realized alpha is positive on average for high idiosyncratic risk projects and negative for low idiosyncratic risk projects. ■

This is the most important and most surprising theorem in the paper. We later test this theorem and demonstrate that even though investors earn zero alphas in expectation, net returns are still correlated with ex post idiosyncratic risk. The idea is straightforward: the contract struck between the VC and investors is based on the expected level of idiosyncratic risk. When the realized risk is higher than expected the VC demands more from the entrepreneur to compensate him for the risk. And when the realized risk is lower than expected, competition between VCs means the VC demands less from the entrepreneur. The investor is affected by these demands. That is, the investor makes the required return on average, but earns a positive alpha sometimes and a negative alpha at other times.

This key prediction follows directly from the principal-agent problem. It is surprising on first blush because one would expect that the principal-agent problem would have no effect on returns to well-diversified investors. However, when we consider that the principal-agent problem arises because the investor cannot monitor the VC and must therefore negotiate a contract in advance, we begin to see how the principal-agent problem could affect investor returns. The contract is designed to compensate the VC for the expected risk of the portfolio, but since the VCs invest in so few projects realizations of high and low risk portfolios should

happen with some frequency. What happens if the VC locates a surprising number of high risk projects? He negotiates better terms from the entrepreneurs. The externality in this negotiation is that the investor also benefits from the VCs negotiation. This is not internalized by the investor-VC team because their contract is already set. Thus, to receive an investment from the VC the entrepreneur sets a lower price and the investor benefits.

This is the key prediction of our model because it is not easily generated without the principal-agent problem. If, for example, high risk investments had higher costs of investing, or monitoring high risk investments was more costly, then high risk projects would have higher gross returns, *but this would not show up in investors' returns*. The following corollary makes it clear that these results only occur when there is a principal-agent problem.

Corollary 4 *Without a principal-agent problem, even when the VC's compensation depends on idiosyncratic risk, the net returns are independent of the idiosyncratic risk.*

Proof. See appendix. ■

Without the principal-agent problem, any excess return required to compensate the VC goes to the VC. Therefore, the investors' return net of fees just compensates them for their beta risk. Theorem 4 allows us to distinguish between a positive alpha that is simply the result of an uncompetitive market or simply because VC's require compensation, and a positive alpha that is due to the pricing of idiosyncratic risk. If the principal-agent problem is of true significance in the VC arena, then the investors' returns should be correlated with risk even though the investors earn no excess returns on average.

III Calibration

In the model, it is clear that idiosyncratic risk is priced into venture capital financing. But is this effect economically large? In this section, we explore this question using recent empirical data on returns to venture capital investments. We find that the presence of idiosyncratic risk is quite costly. With realistic parameter values, venture capitalists can easily value their position at less than half of its value to a fully diversified investor.

To conduct the calibration exercise, we use summary statistics from Cochrane (2004), who studies all venture capital investments in the VentureOne database from 1987 through June 2000. After correcting for selection bias, he estimates an arithmetic average annual return of 59% and a CAPM alpha of 32%, along with an arithmetic annualized return standard

deviation of 107%.

Alternatively, we could have used data from Gompers and Lerner (1997) who measure returns for a single private equity group from 1972-1997 and find much lower returns, with CAPM and three-factor alphas of 8% per year.²⁴ Using 8% in place of 59% would drastically increase our estimate of the impact of idiosyncratic risk. Thus, we use the higher number to be conservative.

Suppose that a venture capitalist holds all of his wealth in a single project with this representative return distribution. Under our mean-variance assumptions, the VC would value this project using a Sharpe ratio equal to the Sharpe ratio for public equity. Over the sample period, and assuming a riskless rate averaging 5% (the calculations are insensitive to the choice of a riskless rate), the venture capital investment Sharpe ratio is $(59\% - 5\%)/107\% = 0.50$, compared to an annualized Sharpe ratio of $11.08\%/15.07\% = 0.73$ on public equity over the same interval (using data from Ken French's website).

Based on these return statistics, this representative VC investment has so much idiosyncratic risk that no investor would take it unless he were well-diversified. The good news is that venture capital fund investors are generally well-diversified. However, the VC is compensated via an equity interest, and he is forced to hold a lot of idiosyncratic risk. So we are still interested in the VC's private valuation of his holdings compared to the value of that interest in the hands of a well-diversified investor. That difference is a measure of the deadweight loss due to lack of diversification. Continuing the earlier example, in order to get a Sharpe ratio that matches the Sharpe ratio on public equity, the venture capitalist needs an expected return of $5\% + (0.73/0.50)(59\% - 5\%) = 84\%$.

Now suppose a venture capital investment of \$1 million and assume a horizon date of three years. The expected exit value is $\$1 \text{ million} * (1 + 59\%)^3 = \4.02 million . The venture capitalist applies a discount rate of 84% to this expected future cash flow for a present value of $\$4.02 \text{ million} * (1 + 84\%)^{-3} = \0.65 million . Thus, the value to the venture capitalist is only 65% of the value to a diversified investor. This 35% haircut is substantial, and indicates that these kinds of concentrated risks can sharply reduce value.

In our model, this valuation haircut only applies to the VC's interest, because other investors are well-diversified. If for simplicity we assume that the VC gets a pure 20% of the exit value of the fund (rather than the 2% of assets per year and 20% of the upside that is a common

²⁴Note that both studies report gross returns on the amount invested, not the net returns after fees and carried interest paid to the venture capitalist.

payment structure), then the deadweight loss from lack of diversification is $35\% * 20\% = 7\%$ of the total amount invested. This is economically significant considering that Lerner (2000) estimates that private equity funds managed \$175 billion in 1999.

Investors still earn fair returns for the systematic risk they bear, so they won't bear this deadweight loss. The entrepreneur bears this cost. Given these numbers, it is thus no surprise that entrepreneurs complain bitterly about the valuations that VCs apply to their businesses. There is a discount due to idiosyncratic risk. There is also a discount applied because the VC provides valuable services in return for stock rather than cash. We do not have the data to characterize the magnitude of this second effect, but the example demonstrates that the discount due to idiosyncratic risk alone can be substantial.

Why does the entrepreneur use venture capital at all? Perhaps he has no alternative if he has no wealth of his own, as we've assumed in the model. Even if he were wealthy enough to fund the project himself, he may still choose venture capital, because the same analysis of idiosyncratic risk also applies to the entrepreneur. If the entrepreneur has most of his financial and human capital tied up in this single source of risk, he too should apply the same discounts in arriving at his private valuation. In fact, the entrepreneur should prefer to fund via VC, all else equal, because external financing transfers (some of) the idiosyncratic risk to those better able to bear it.

IV Empirical Tests

The data have been obtained from Thomson Venture Economics. Venture Economics collects data on a substantial fraction of the venture capital and private equity funds formed in the United States. The dataset contains funds formed from 1969 to 2002, though there were very few such funds prior to the 1980's. Given the rapid growth of this industry in the 1990's, it is not surprising that most of the funds in the sample were formed in the last ten years. The dataset is not widely available, though it has been used by other authors, notably Kaplan and Schoar (2002).

When funds are formed, investors commit a specified amount of capital. Fund managers then call on these commitments as investment opportunities arise. These fund inflows are sometimes called takedowns. When investments are successful, the fund makes distributions to investors. These can be cash distributions if the investment is liquidated or acquired for cash, or they may be distributions of publicly traded shares if the investment comes public

or is acquired for stock by a public firm. Our database records the amount and date of each takedown and the value and timing of all distributions.

In addition, there are quarterly data on fund net asset values. We use these fund NAVs along with distribution and takedown amounts in the quarter to calculate a quarterly return for each fund. Naturally, it is difficult to value such privately-held and illiquid investments accurately, and such valuations are necessarily somewhat subjective. Different funds may also calculate asset values in different ways. For example, some funds may be conservative and delay writing up an investment's value until, say, another entity invests at a higher valuation. However, we adjust for these delays in our empirical work.

The database includes venture capital funds, buyout or private equity funds, mezzanine funds, and funds of funds. We exclude the mezzanine and fund-of-funds categories, and we limit the sample to funds formed in 1980 or later with at least \$5 million of committed capital. Funds with obvious data errors are also excluded.²⁵ Finally, we exclude funds that are formed after 1999. Return data are available through June 30, 2002, so funds formed after 1999 have at most 10 quarters of returns, which is insufficient to accurately measure either risk or return.

These filters leave us with a sample of 1,245 funds. Table 1 reports some summary statistics about the sample. About 70% of funds in the sample are venture capital funds. The rest are buyout funds. Venture capital funds tend to be much smaller than buyout funds, with a median committed capital of \$64.7 million vs. \$385.0 million for the typical buyout fund.

We have size and cash flow information about each fund, but we do not know the identity of the individual funds in our sample. Thus, we have no way of identifying a fund's individual investments. Instead, we must use available data to try to piece together a picture of the fund's characteristics. For example, the number of takedowns may be a useful proxy for the number of investments made by the fund. Conversations with venture capitalists and fund managers indicate that funds tend to make a separate call for capital for each investment made by the fund. The median VC fund has 10 takedowns, while the much larger median buyout fund calls for capital 21 times.

Similar statistics are available on distributions. The median VC fund makes 15 separate distributions, while the median buyout fund makes 14. When venture-capitalized firms go public, the VC fund typically cashes out of the investment in stages, so that a huge number of shares do not get sold in the market at the same time. Thus, VC funds make relatively

²⁵For example, two funds were written down to zero value and later made distributions. It is impossible to calculate a return in such a case. Another fund reported negative fund values.

more distributions to their investors. Distributions are less useful as a proxy for the number of investments, because only successful investments generate distributions.

We also report the average annualized IRR earned by a venture capital or buyout fund based on all its cash flows from inception to liquidation or June 30, 2002, whichever comes first. Over this sample period, the equal-weighted mean across all VC funds is 19.25%. For buyout firms the average is 9.67%. Weighted by committed capital amounts, these average IRR's are even lower. For example, the value-weighted buyout average return is only 4.57%. This is rather low, given that average stock market excess returns over this period are strongly positive. This IRR is also well below the comparable IRR of 19% in Kaplan and Schoar (2003). Our number is lower because we include more recent funds that have not been wound up, and these have done much worse recently. For instance, some of the very largest buyout funds, such as some of those run by KKR, have had well-publicized difficulties and mediocre recent returns. For comparison, we apply a filter to our buyout sample that is similar to Kaplan and Schoar and calculate average IRRs for funds that have either been liquidated or whose residual value is less than 10% of the original investment. The resulting equal-weighted mean buyout IRR is 18.3%, quite close to their results. We use funds formed up to 1999 because they provide additional data points, improving our statistical power, and because such a sample better reflects fund performance in both up and down stock markets.

Fund returns are extremely volatile. The cross-sectional standard deviation of annualized IRR's is over 51% for VC funds, and over 31% for buyout funds. IRR's themselves are an average log return over time, and if returns are independent over time, then the IRR standard deviation is the standard deviation of one-year log returns scaled down by $T^{1/2}$, where T is the duration of the investment in years. The average duration is about five years, so the cross-sectional standard deviation of annual returns is more than double the IRR standard deviation figures. Investments in these funds are volatile indeed.

Panel B of Table 1 demonstrates that funds mark to market with a substantial lag. All funds in an asset class are aggregated into a single NAV-weighted portfolio, and its quarterly returns are projected on contemporaneous and lagged market returns, measured using the CRSP value-weighted index of all NYSE, AMEX, and Nasdaq stocks:

$$r_{it} = \alpha_i + \beta_{0i}r_{mt} + \beta_{1i}r_{m,t-1} + \dots + \beta_{4i}r_{m,t-4} + \varepsilon_{it}. \quad (11)$$

All returns are excess returns over the T-bill rate. Betas in this time-series regression are

generally strongly positive out to lag 4.²⁶ For example, the contemporaneous beta for the portfolio of venture capital funds is estimated at 0.63, and the lagged beta estimates range from 0.25 to 0.34.

If the only problem with NAVs is lagged adjustment, due to accounting conservatism or any other source, then betas can be consistently estimated by simply summing up the betas on current and lagged factor returns. This method is also robust to various other simple measurement errors. For example, if all marks were biased upward by a certain fixed amount, all alphas would be biased upward, but this would not affect estimates of factor loadings or any of the cross-sectional relationships that we identify. Similarly, if individual fund marks are noisy, these should wash out in aggregating over many funds to form value-weighted portfolio returns. Finally, no matter how much the NAVs bias the quarterly returns, average NAV returns must eventually converge to cash flow IRRs over the life of the fund.

Summing up the current and lagged betas, the portfolio of VC funds has an estimated “long-run” beta of 1.80. It is also worth noting that buyout funds have much lower betas, with betas that sum to 0.65. The buyout fund betas are surprisingly small, especially since these are equity betas, and most buyout investments are significantly levered. However, these results are consistent with the conventional wisdom that buyout funds typically pursue companies with steady cash flows that are relatively insensitive to changes in aggregate conditions.

In the private equity industry and in some academic papers, such as Kaplan and Schoar (2003), a common performance measure is the public market equivalent, or PME. The PME measures the net present value of all cash flows to an investor in a fund net of fees, where discounting uses the ex post total return on the S&P500 or some other broad stock market index. Using PME as a performance measure is equivalent to assuming that all private equity and venture capital investments have a CAPM beta of one. The results in Table 1 Panel B show that betas are not equal to one, though betas for both VC and buyout funds are lower than our priors.

VC funds tend to make investments in small firms with strong growth prospects. Thus it seems appropriate to compare their performance to that of similar publicly-traded companies. To do this, we look at fund performance measured against the Fama-French (1993) three-factor model. Again, we aggregate funds into a value-weighted portfolio and run time-series regressions of quarterly fund returns on current and lagged factor returns. Specifically, we

²⁶We also estimated regressions with additional lags, but found insignificant covariances beyond a year.

estimate

$$r_{it} = \alpha_i + \sum_{j=0}^4 \beta_{ji}^{RMRF} RMRF_{t-j} + \sum_{j=0}^4 \beta_{ji}^{SMB} SMB_{t-j} + \sum_{j=0}^4 \beta_{ji}^{HML} HML_{t-j} + \varepsilon_{it}. \quad (12)$$

There are very few funds at the start of the sample, and many such alternative funds in the late 1990's. This implies that aggregated portfolio returns are likely to be much noisier in the early part of the sample. To adjust for this heteroskedasticity, all the time-series regressions in the paper are weighted by the number of funds included in a particular calendar quarter.

The results are in Table 2. The aggregate VC portfolio loads on the market factor and the value factor, while the aggregate buyout portfolio loads reliably only on the market factor. The buyout beta is quite low, with a point estimate of 0.81, but it is not very precisely measured, as the 95% confidence interval extends from about 0.50 to 1.13.

What stands out even more in Table 2 is that the aggregate value-weighted venture capital and buyout portfolios have alphas that are indistinguishable from zero. This is completely consistent with our model. However, this evidence is somewhat surprising, because it contrasts strongly with the high returns reported by Gompers and Lerner (1997) and Cochrane (2001). There are two likely explanations. First, our returns are net of all fees, while Cochrane's returns are gross returns on invested dollars. Second, our data extend through June 2002, and the stock market decline over the last two years of the sample sharply reduces overall average returns to these asset classes.

In Table 2, we also partition the sample by size, measured by the amount of capital committed to the fund. These results are initially in the spirit of exploratory data analysis, since our model does not make any direct predictions about the performance of small vs. large firms. In terms of factor loadings, buyout funds do not seem to load on either the small-firm or value factor. Venture capital funds have strong negative loadings on the book-to-market factor, consistent with their investments in small, high-growth opportunities. Interestingly, the smallest venture capital funds load positively on the small-firm factor, but the remaining VC funds do not.

Again, the biggest surprises are in performance measures. There is surprisingly little variation in performance between small funds and large funds. There is suggestive evidence that the largest quartile of venture capital funds outperforms, with an estimated alpha of 1.56% per quarter, but even this figure is not statistically distinguishable from zero. This might seem somewhat surprising at first glance, since the evidence in Kaplan and Schoar (2003) indicates that successful venture capitalists are able to raise larger follow-on funds,

and those successor funds tend to do well. However, the evidence is consistent with our assumption that the market for raising funds from investors is perfectly competitive, so that investors' alphas are always expected to equal zero.

Our other main empirical prediction is that average returns are increasing in the amount of idiosyncratic risk. As an initial investigation, we use the number of takedowns as a proxy for the number of investments made by the fund. All else equal, the number of investments is inversely related to the idiosyncratic risk borne by the fund managers. If the theory is correct and the number of takedowns is inversely correlated with idiosyncratic risk, then more takedowns should be associated with lower returns on average.

We focus on buyout fund takedowns, because VC takedowns are probably strongly correlated with success. Successful venture capital investments typically involve multiple rounds of funding. If a venture capitalist makes only one round of investment in a firm, it is usually because the VC evaluates the follow-on round and determines that the firm isn't doing well and isn't worth continued funding.²⁷ Thus, on average, a small number of takedowns probably implies fewer successes rather than fewer firms staked. Buyout funds do not suffer the same selection bias, because most buyouts do not typically require multiple stages of financing. Most are profitable, established businesses that are cash-flow positive. For buyout funds, takedowns are probably a good proxy for the number of firms that the fund invests in.

Because individual fund returns are so variable, we want to work with portfolios throughout. We sort funds into quartiles based on the number of takedowns, and run a time-series regression of portfolio returns on contemporaneous and lagged factor realizations. As before, we use the contemporaneous value and four lags of the three Fama-French factors.

The results are summarized in Table 3. Among buyout funds, the most concentrated funds do best. For those in the first quartile, with an average of four takedowns and at most eight cash draws, the average alpha was 2.36% per quarter. All other quartiles are negative, and the first quartile alpha is statistically different from the fourth quartile alpha ($p = 0.008$). Thus, buyout funds with few takedowns do better. While there is a positive correlation between takedowns and fund size, size is not driving these results, since Table 2 shows that there is no relationship between size and alpha.

As expected, the evidence runs the other way for VC funds. Funds with only a few takedowns strongly underperform funds with more. For example, funds in the smallest takedown

²⁷Late-stage venture capital funds could be an exception to this general rule, since they often hope to provide the last round of private financing before the firm goes public or is acquired.

quartile, with at most three takedowns, have negative (though insignificant) alphas, while those in the top quartile, with at least 14 separate cash calls, have an alpha of 2.62% per quarter. These differences are statistically significant.

Overall, the buyout evidence is consistent with the theory that idiosyncratic risk is priced. However, the evidence is only suggestive. We now turn to the non-trivial task of measuring idiosyncratic risk directly using the quarterly mark-to-market returns.

For publicly traded stocks with a complete price history, it is straightforward to estimate idiosyncratic risk. In a single factor model, for example, idiosyncratic risk is the variance or standard deviation of the residual in the market model regression:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}, \quad (13)$$

where r_{it} and r_{mt} are the excess returns over the risk free rate. This equation can be estimated consistently using a time-series of returns. In the present case, funds themselves report the value of their investments each quarter. If these estimates of value simply have i.i.d. measurement errors, then these errors bias upward the estimates of idiosyncratic risk. The measured residuals would also be negatively autocorrelated, though that is not particularly relevant here. The situation is analogous to bid-ask bounce, and the volatility effects of that particular measurement error are addressed by Roll (1984).

Measurement error is not a problem if these measurement errors are uncorrelated with returns or the true amount of idiosyncratic risk, because we do not care much about the actual level of idiosyncratic risk. As long as measured idiosyncratic risk remains informative about the true amount of idiosyncratic risk, then we can test the theory by testing for a positive association between measured idiosyncratic risk and returns.

A bigger problem is that funds tend to adjust their net asset values slowly. In fact, the National Venture Capital Association provides mark-to-market guidelines that explicitly encourage such conservatism. For example, according to the guidelines, a startup's value should only be written up if there is a subsequent financing round involving a third party at a higher valuation.

If a fund adjusts its net asset value in a consistent, time-stationary way with at most a one-period lag, and the measured market return itself has no such lags, then the problem is identical to the nonsynchronous trading problem studied by Scholes and Williams (1977), Lo and MacKinlay (1990), and Boudoukh, Richardson and Whitelaw (1994), among others.

Systematic risk can be measured by projecting on current and lagged market returns:

$$r_{it} = \alpha_i + \beta_{0i}r_{mt} + \beta_{1i}r_{m,t-1} + \varepsilon_{it}. \quad (14)$$

and then summing up the estimated slope coefficients to obtain an estimator for beta. The variance of the residual in this regression is a monotonic function of the true (unobserved) idiosyncratic risk. More lags can be added to this regression if the fund adjusts its returns with a longer lag.

Once the factor model is specified, there are a couple of different approaches to testing whether average returns are related to idiosyncratic risk. For example, Malkiel and Xu (2001) follow the general approach of Fama and MacBeth (1973) by estimating the risk premia γ in cross-sectional regressions of the general form:

$$r_{it} = \gamma_0 + \gamma_1\beta_i + \gamma_2\sigma_i + \nu_{it}, \quad (15)$$

where the set of test assets consists of a small number of portfolios and estimated betas on the right-hand side are either scalars in a single factor model or vectors in a multi-factor model. The time series independence of coefficient estimates is used to conduct inference on average risk premia. Lehmann (1990) provides an alternative cross-sectional approach.

A cross-sectional approach is not ideal for our fund return data. The cross-sectional regression approach is subject to well-known errors-in-variables bias because noisy estimates of factor loadings appear on the right-hand side. Even if we were to form portfolios of funds, each fund's return history is fairly short, typically ten years or less.²⁸ This, combined with the huge amount of idiosyncratic risk along with lagged marking to market, results in very noisy estimates of betas. The biases can be corrected (see, for example, Shanken (1992)), but the test does not have much power. In any case, we are interested in the risk premium on idiosyncratic risk, not the risk premium associated with beta or other factor loadings.

Rather than attempt a cross-sectional regression in the presence of these difficulties, we continue to use the time-series approach. This time, we sort on idiosyncratic risk directly. That is, for each of the 1,242 funds with a return history of at least six quarters, we regress

²⁸Forming portfolios is particularly challenging in this case. For maximum statistical power, it is important to form portfolios with the greatest possible cross-sectional variation in the RHS variables. The short return histories make it very difficult to calculate a pre-ranking period beta, and we have almost no data on characteristics of the funds other than their returns, making it impossible to sort on characteristics that might be correlated with factor loadings.

quarterly excess returns over the T-bill rate on the contemporaneous excess market return and four lags:

$$r_{it} = \alpha_i + \beta_{0i}r_{mt} + \beta_{1i}r_{m,t-1} + \dots + \beta_{4i}r_{m,t-4} + \varepsilon_{it}. \quad (16)$$

The variance of residuals is then a standard estimate of a fund’s idiosyncratic risk.²⁹

Funds are then sorted into quartiles based on their idiosyncratic risk, and value-weighted portfolios are formed. As in earlier tables, these portfolio returns are regressed on contemporaneous and lagged values of the three Fama-French factors:

$$r_{it} = \alpha_i + \sum_{j=0}^4 \beta_{ji}^{RMRF} RMRF_{t-j} + \sum_{j=0}^4 \beta_{ji}^{SMB} SMB_{t-j} + \sum_{j=0}^4 \beta_{ji}^{HML} HML_{t-j} + \varepsilon_{it}. \quad (17)$$

As before, this last time-series regression is weighted by the number of funds present in that quarter, and inference is conducted using robust Huber-White standard errors. As in Gibbons, Ross and Shanken (1989), we test the hypothesis that $\alpha_i = 0$.

The results are in Table 4. Consistent with the theory, venture capital and buyout funds with more idiosyncratic risk exhibit higher returns. For venture capital funds, the quartile with the lowest idiosyncratic risk has a quarterly alpha of -1.09% , the highest an alpha of 2.52% per quarter. These two quartiles have alphas that are statistically different at the 0.001 level. The evidence is qualitatively similar for buyout funds, though the magnitudes are smaller. For buyout funds, the quartile with the lowest idiosyncratic risk has a reliably negative quarterly alpha of -1.00% ; other quartiles are indistinguishable from zero. Alphas for the two extreme quartiles differ by 1.35% per quarter ($p = 0.065$).

While these results are consistent with the theory introduced here, there are other possible explanations for these results. For example, our tests hinge on the validity of the assumed linear factor model. If the linear factor model is wrong, then residual risk is correlated with expected returns, because at least some of the relevant covariance is not captured by the factor or factors and thus is lumped into the residual with the true, unpriced idiosyncratic risk.

To investigate this, we also redo the empirical tests using alternative factors. The results are nearly identical if we instead use just Nasdaq stocks to construct the market return, and the results are unchanged if we use a single market factor rather than the three Fama-French

²⁹Note that we do not include the Fama-French factors in this first-pass regression because they would remove too many degrees of freedom. If four lags of each factor were included, there would be 15 explanatory variables in total, so that any fund with a return history shorter than 16 quarters would have to be discarded. This might introduce significant selection bias. For robustness, we tried using only one lag of each factor (six RHS variables in total), as well as forcing a geometric decay structure onto the lagged factors (three contemporaneous factor loadings plus a single autoregressive decay parameter). The results were unchanged.

factors. We also check for parameter stationarity throughout the sample. When we split the sample, we find no evidence that the linear factor model differs across the two halves, though this test probably has little power given the short sample period. Finally, the evidence in Table 2 demonstrates that fund size is not behind these results.

We also examine whether these results depend on the use of reported fund values. In Table 5, we redo much of the analysis without using the marks. To accomplish this, we limit the sample to 759 funds formed between 1980 and 1994, and we report average IRRs based on actual cash flows. These funds are much more likely to have completed the investing and realization cycle by the end of our sample in 2002. This sample also more closely matches the sample in Kaplan and Schoar (2003). In the left half of Table 5, we sort on the number of takedowns and report the size-weighted average IRR. The results match those in Table 3. Again, we focus only on buyout funds and find that fewer takedowns are associated with higher IRRs. For example, the first quartile with the fewest takedowns has an average IRR of 29.32%, while the quartile with the most takedowns has an average IRR of 16.77%. We do not conduct formal statistical inference in this table, because we are not sure how to do this accurately, since there is only a single cross-section, and we would expect substantial cross-sectional correlation due to the overlap in the IRR calculation periods. However, the results are quite consistent with those in Table 3.

Similarly, we redo Table 4 using cash flow IRRs for funds formed between 1980 and 1994. We still need to form portfolios based on the amount of idiosyncratic risk, and thus we cannot avoid the marks completely. That is, we still do time-series regressions of individual fund returns on factor returns to get residual variance, and we sort on residual variance to form portfolios. But we calculate average IRRs for each portfolio based on actual cash flows for these funds. The results are the same as in Table 4: more idiosyncratic risk is associated with higher IRRs. For example, VC funds in the lowest risk quartile have a size-weighted average IRR of 7.98%, while the highest risk quartile averages an IRR of 32.51%. Overall, the evidence is consistent with the model's main implication that idiosyncratic risk in these particular asset classes is priced.

It is also worth noting that all of the empirical work in this section is limited by the confidential nature of the data. We have controlled for fund size, but we have essentially no other additional information about the funds beyond their cash flows. It would be useful, for example, to know how many entities a fund has invested in. It would also be useful to know industry focus and concentration for each fund. Both of these pieces of information

could also proxy for idiosyncratic risk. This is an advantage of the dataset used by Ljungqvist and Richardson (2003), who have extensive fund and individual investment data for funds managed by a single general partner. The advantage of the Venture Economics data is that it covers a much larger portion of the asset class.

V Extensions: The Risk of the Portfolio

All of the results in the paper so far hold with any form of contract between the investor and VC, as long as the compensation depends on the payoff of the portfolio. However, we have predicated these results on the assumption that the VC cannot alter the risk of the portfolio. This section considers the validity of that assumption by allowing the VC to maximize his utility by altering portfolio risk. We show that properly structured option contracts (similar to real VC compensation contracts) eliminate the VCs desire to alter the portfolio risk. Thus, relaxing the assumption that the VC cannot alter the portfolio risk would not invalidate our results. Therefore, our theory is robust to an extension that allows the VC to alter the risk of the portfolio.

This section also explains why the standard VC contract pays the VC a fraction of all positive returns on the portfolio. This makes the VC's payoff like a call option. Since increasing volatility increases the expected payoff from the option, it might seem that this type of contract would encourage excessive risk-taking by the VC. Such incentives could be removed by simply giving the VC an equity contract. Thus, why does the standard contract contain an option? Furthermore, why is the return benchmark above which the VC shares in the profits, usually equal to zero? It would seem that the VC should have to at least beat the risk-free rate (if not some higher benchmark) before sharing in the upside. This section shows that the answer to both questions is the same; the VC contract must neither encourage nor discourage the VC to spend time to alter the risk of the portfolio. The diversified investor does not care about the risk in any small part of his portfolio, so the investor wants the VC to increase the mean, not change the amount of idiosyncratic risk. We show that an equity contract encourages diversification, and an out-of-the-(expected)-money option contract encourages excessive risk taking. Thus, an option with a low strike price encourages effort while providing little benefit to changes in risk.

To consider the question of risk we examine a very general formulation of our model with $N > 1$ and VC utility function over wealth, $u(w)$. We assume that the VC is risk-averse but

likes wealth so that $u'(w) > 0$ and $u''(w) < 0$ and that the portfolio is sufficiently good that more of the portfolio is better for the VC.³⁰ Let R_p represent the return of the portfolio, μ_p represent the expected return, and σ_p^2 represent the portfolio variance. We assume that an agreement between the VC and investors exists. We then ask the following question. Given the equilibrium contract, does the VC wish to alter the risk of his portfolio? If he does, then it is not an equilibrium. If he does not want to alter risk, then allowing the VC to change the risk would not change the results.

Throughout the paper, equilibrium pricing is a result of a competitive environment, and therefore all the negotiating power was held by the entrepreneur. We wish to now consider the benefits and costs associated with a VC who has the ability to find one more project than the marginal VC (i.e., alter the risk on the margin). We assume that the price this VC receives stays the same (since he is competing against the marginal VC) but he has the ability to alter his surplus. Another way to phrase this is that we are assuming the contract between the VC and investors and entrepreneur is set, and we want to look at the VC's incentive to alter the portfolio risk. Thus, given an equilibrium we examine the VC's attempt to maximize utility.

$$\max_{\sigma_p^2} E[u(w) \mid \sigma_p^2] \tag{18}$$

First, consider the simple case of an equity contract between the VC and investors. It is straightforward to see that with an equity contract, if the VC can choose $N \in [\underline{N}, \bar{N}]$ then the VC will choose $N = \bar{N}$, or if the VC can decrease σ_p^2 then he will do so, all else equal. This follows directly from the fact that the VC is risk-averse. Therefore, if the VC takes on one more project than expected, then the VC's utility will improve because the variance of the portfolio decreases.

We now show that moving to an option-like contract reduces the incentive to take on too many projects by providing a benefit to volatility. However, if the effective strike price on the option is too high, the incentive flips and the VC wishes to increase risk. We assume that the VC-investor agreement gives the VC a fixed payment y and shares in a portion of the returns above some fixed benchmark, R_b .³¹ ω is sometimes referred to as the VC's carried interest.

³⁰We also assume continuity.

³¹While this could be a benchmark portfolio, the standard contract has a fixed benchmark, typically zero.

Therefore, the VC's payoff is of the form³²

$$\begin{aligned} X_{vc} &= y + \max[\omega I(R_p - R_b), 0] \text{ if } R_b \geq 0, \\ X_{vc} &= y + \omega IR_b + \max[\omega I(R_p - R_b), 0] \text{ if } R_b < 0. \end{aligned} \tag{19}$$

Given this form for the payoff we can show how the VC's incentives to change risk are affected by the choice of benchmark, R_b . The following theorem looks at how the VC's attempt to maximize his utility (equation 18) is affected by R_b .

Theorem 5 *If the VC receives an option payoff of the form in Equation (19), then for an appropriately chosen benchmark, R_b , which is below the expected return of the portfolio, μ_p , the VC has no incentive to change N or σ_p^2 .*

Proof. See appendix. ■

Thus, as long as R_b is chosen appropriately the VC has no incentive to change risk. Therefore, the VC does not spend time or effort trying to alter the risk of the portfolio. This ensures that our results are robust to the relaxation of the assumption that the VC does not alter the number of projects or the risk of the portfolio.

It is very interesting to note that the appropriate benchmark return, R_b , is not the expected return on the investments, R_p . Instead, the optimal benchmark makes the option in-the-money (in expectation) at inception. This provides the risk-averse VC with the least incentives to spend time altering the risk characteristics of his portfolio instead of focusing on returns. This is, of course, optimal for the well-diversified investor in the VC fund. Although most work on option based compensation suggests that options cause managers to increase risk Carpenter (2000) also shows that options may not always cause managers to increase risk. Here our point is to show that a properly structured in-the-money option eliminates the desire of the VC to increase or decrease risk and thus our model results are robust.

The intuition comes from thinking about the contract in the following way. Lowering R_b below R_p makes the contract like equity with a put. Increasing variance increases the value of the put but decreases the value of the equity (to a risk averse VC). At the appropriate strike price, these two effects offset, and the VC does not wish to alter risk. However, if R_b is too low then the contract is too much like equity and the VC wishes to reduce risk. If R_b is set at R_p (or above), then the contract is just a call option and the VC wishes to increase risk.

³²This equation differs from a standard option payoff because in theory we could require the VC to share in some of the negative returns.

Thus, a properly set, in-the-money option contract eliminates the VC's desire to alter the risk of the portfolio.

If the principal-agent problem and idiosyncratic risk are important in the VC industry then an in-the-money option-like contract is not odd at all. The appropriate option contract encourages the VC to focus on managing investments, not altering risk. Thus, we should be able to find our empirical predictions in real world data in which the VCs have option contracts.

VI Other Implications

The focus of this paper is the pricing effect of the principal-agent problem. However, the principal-agent problem also has other implications about the venture industry. The two we discuss below are implications about fund size and industry risk capacity. Furthermore, the same principal-agent problem also affects pricing in other arenas. Below we consider the pricing in internal capital markets, but generally our ideas should apply to any situation where agents hold risk and set prices.

VI.1 The Size of the Fund

The principal-agent problem requires VCs to be compensated based on fund results. Since VCs can manage only a limited number of investments, this has implications for scalability and the boundaries of the VC fund. Suppose, for example, that a large fund is considering hiring enough VCs to do a large number of small investments. The principal-agent problem rules this out, because an interest in the fund would not tie VC compensation sufficiently closely to that VC's effort, leading to shirking and free-riding. If there are no economies of scale, these diseconomies imply that small funds have a comparative advantage doing small investments, and large funds are comparatively better at large investments. In short, if a single team of VCs invests $\$X$ per project and can manage only Y projects, then the fund size should be $\$X*Y$. Larger funds with more VCs would worsen the principal-agent problem. Smaller funds would hold excessive idiosyncratic risk and be unable to compete effectively for projects.

Note that VC capacity constraints alone do not imply that small funds should make small investments and large funds should make large investments. Capacity constraints imply that if large funds wanted to make small investments, they would employ many VCs. In fact, we

might expect some funds to specialize in small investments and raise huge amounts of money to do so. This is not what we see. Instead, as fund size grows funds tend to make larger investments. Adding the principal-agent problem establishes a link between fund size and investment size.

Based on this, we should expect to find that smaller funds invest in early stage (smaller) deals, while larger funds invest in later stages. Furthermore, casual observation suggests that as fund sizes have grown in recent years, venture capital funds have looked to do larger and larger investments. Entrepreneurs talk of VCs who pushed them to take millions more than they set out to raise.

VI.2 Industry Concentration and Risk Capacity

When the venture industry began, a fraction of the early movers presumably got rich. With wealth comes the ability to diversify outside the fund. A more diversified VC can take higher risk projects because he does not need to be compensated as highly for the idiosyncratic risk. Overall, this implies that as time passes and successes are realized, the VC industry as a whole should be willing to support riskier projects. Furthermore, countries hoping to develop a VC industry may find that significant time and wealth is required before its venture capitalists become able to bear large amounts of idiosyncratic risk.

VI.3 Inside a Firm

In the case of the firm, we can think of the stockholders as investors, the executive team as the VCs, and the managers in charge of different projects as the entrepreneurs. A competitive stock market ensures that the investors only receive the required return. However, the future income of the CEO depends (implicitly or explicitly) on the success of the projects he chooses to fund. This dependence is required in order to increase unverifiable effort and/or to screen for good executives. Even if not required the dependence will arise because the market cannot commit to not base future salary on performance. Executives have a limited ability to manage projects. Thus, even in equilibrium executives hold idiosyncratic risk and so must be compensated for holding it. This results in lower prices for corporate projects than if a diversified pool of investors could somehow access and manage the projects directly. Thus, our model would predict the common observation that the internal cost of capital is above the market cost (see Poterba and Summers (1995)).

The variance of the projects chosen inside a firm probably has less impact on a CEO than projects that are done outside the firm have on the VC. Furthermore, greater oversight may exist over a firm (as this is encouraged rather than discouraged by the law), reducing the moral hazard. Therefore, as Prendergast (2000) suggests, a CEO's income can be far less dependent on the project than the VC's. As a result, the CEO can hold less risk, which implies that he needs lower returns than a VC.³³ Thus, although many positive NPV projects cannot get financing from the private equity market, some of these projects may be completed inside a firm where the impact of idiosyncratic risk is smaller.

This theory also helps explain why a VC is much more interested in an IPO than a private sale. He can expect the fully diversified (beta-based) price from a public sale, but a private buyer is likely to require compensation for some amount of idiosyncratic risk. Thus, as Black and Gilson (1999) argue, VC and a good stock market go together. We suggest that VCs don't want to sell to a private market, because the buyer faces the same principal-agent problem. The need to get the company to a diversified market drives everyone's desire to IPO. Furthermore, this theory is consistent with the anecdotal evidence that financial buyers are usually not willing to pay as much as a so-called strategic buyer if the strategic buyer faces less of a principal-agent problem in overseeing the acquisition (even if no other synergies exist).

VII Conclusion

Unavoidable principal-agent problems in the private equity and venture capital markets combined with the need for investment oversight result in idiosyncratic risk that must be priced. This extension of the principal-agent theory results in investor returns that are correlated with total risk, not just systematic risk. This leads us to expect: (1) excess gross returns for private equity and VC investments (as found by Cochrane (2004)), (2) zero alphas on average for funds net of fees, and surprisingly (3) higher alphas, even net of fees, for funds with higher total realized risk. We confirm the second and third predictions using VC and buyout fund return data. It is this third prediction that gives us confidence that we are finding evidence of the principal-agent problem, as it is not an obvious prediction from any of the typical

³³This seems to suggest that all projects should get done inside firms or at least in partnership with firms. However, Rhodes-Kropf and Santos (2001) suggest that the possibility that ideas can be stolen leads to some projects being funded by investors who have a lesser ability to steal.

influences in venture capital.

None of these effects require any excess returns: investors receive only required returns, and VCs receive an amount that includes their ‘salary’ plus compensation for bearing risk. It is this second component combined with a pre-set contract that causes the correlation between idiosyncratic risk and investor returns.

The ideas that we present lead logically to some general implications about the venture industry. First, VCs should enter the market if they are less risk-averse than the marginal VC, as they could accept less compensation or earn excess return. Thus, overall we should expect this field to attract smart individuals with a high risk tolerance. Second, if there are no economies of scale, VC should not be a concentrated industry. Funds must be small enough to ensure that the agent’s compensation can be tied directly to the performance of his portfolio companies. Also, if VC risk aversion is decreasing in wealth, then following a period of overall success we should expect the venture industry to become more and more willing to take on higher risk projects.

The standard contract between investors and VCs is essentially an option on the portfolio of chosen projects. The form of this contract suggests that idiosyncratic risk is very important in the VC and private equity industry. If VCs could properly diversify, then the option would only increase in value with increased volatility. This would mean that the standard contract was ensuring excessive risk taking. However, once we understand how idiosyncratic risk affects the VCs, we see that an option contract is in place to prevent over-diversification.

This paper also has broader implications for any situation in which agents compete. We briefly explore managers inside firms. If all private buyers must hold idiosyncratic risk, this might explain the appeal of an IPO, since public ownership maximizes risk-sharing by broadly distributing the idiosyncratic risk.

The venture arena is the perfect place to examine the impact of the principal-agent problem on asset prices. Investors hire VCs and are then legally barred from monitoring the VCs’ projects. Thus, the principal-agent problem should have a significant impact. We estimate that the dead weight loss from the need to employ the second best solution is on the order of tens of billions of dollars a year. Overall, we demonstrate that our extension of the principal-agent problem to pricing is a central issue in the venture market and has considerable economic importance.

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Appendix

The Model Without a Principal-Agent Problem: To parallel the main model, we assume that the VC must still negotiate with the investor to get a fraction, ϕ , but now this is a fraction of what the investor *expects* to receive from the investment, $\theta_i I \mu_i$. This is possible because the investor and VC can write a contract on μ_i and can rely on the VC to take actions in the investor's best interest. The VC's compensation must, of course, still be greater than or equal to the opportunity cost of his effort, e_{vc} . However, the VC's effort may depend directly on the variance of the project. In this case $e_{vc}(\sigma_i^2)$, with $e'_{vc}(\sigma_i^2) \geq 0$.³⁴ Note that even if VC effort depends on the risk, in the absence of the principal-agent problem the principal can simply agree to a contract that pays the VC more if the required effort is more. Thus, the VC still holds no risk but will be compensated more if her required effort is more.

With this setup and no principal-agent problem, the constraint in the entrepreneur's problem, Equation (6), would be *s.t.* $\phi \theta_i I \mu_i \geq e_{vc}(\sigma_i^2)$. Therefore, the fraction offered becomes $\theta'_i = \frac{e_{vc}(\sigma_i^2)}{\mu_i \phi I}$, where the superscript ' delineates no principal-agent problem. The constraint in the investor's problem, Equation (10), no longer requires an expectation over the type of project located because without the principal-agent problem the contract can depend directly on the realized parameters, α_i , β_i and σ_{ε_i} . Thus, the share received by the VC becomes

$$\phi' = 1 - \frac{1}{\theta'_i (\mu_i - \beta_i E[R_m])}. \quad (\text{A1})$$

Proof of Theorem 1: Assuming a deal is economically possible³⁵, the investors' constraint in Equation (8) is binding, and on average the investors earn zero alphas net of fees. However, the present value of the gross returns is $E_{\Omega_{vc}} \left[\frac{\theta_i^* \mu_i I}{1 + \theta_i^* \beta_i E[R_m]} \right]$ which is greater than I since $(1 - \phi) < 1$. Therefore, the gross returns are positive NPV.

Proof of Corollary 1: Without the principal-agent problem the constraint in the entrepreneur's problem, Equation (6), would be *s.t.* $\theta_i \phi I \mu_i \geq e_{vc}(\sigma_i^2)$. Therefore, the fraction offered to the VC becomes $\theta'_i = \frac{e_{vc}(\sigma_i^2)}{\mu_i \phi I}$. The constraint in the investor's problem, Equation (10), is unchanged. We assume a deal is economically possible.³⁶ Since $(1 - \phi') < 1$ Theorem

³⁴If $e'_{vc}(\sigma_i^2) = 0$ for all σ_i^2 then we are back in the world where the VC's effort does not depend on the level of risk.

³⁵Equations (7) and (10) must cross at a point where θ^* and ϕ^* are between zero and one, which always occurs for appropriately chosen parameters.

³⁶An economically possible solution with the principal-agent problem requires Equations (7) and (10) to cross at a point where θ^* and ϕ^* are between zero and one. Without the principal-agent problem an economically possible solution requires Equations (10) and $\theta'_i(\phi) = \frac{e_{vc}(\sigma_i^2)}{\mu_i \phi I}$ to cross at a point where θ' and ϕ' are between zero and one. This always occurs for appropriately chosen parameters.

1 ensures that alphas are still positive.

Proof of Theorem 2: Part 1: Given a function ϕ^* (based on projects the VC will accept, Ω_{vc}) negotiated between the VC and investors, for the VC to accept a particular project, the utility from accepting must exceed the opportunity cost of the time he must spend on the project. The resulting fraction of the firm that the entrepreneur must yield equals Equation (7). This share exceeds 1 if

$$\sigma_i^2 > \frac{2\mu_i\phi^*I - 2e_{vc}}{A\phi^{*2}I^2}, \quad (\text{A2})$$

which is possible even if α (inside μ_i) is positive. Therefore, the VC would not accept projects with large total risk even if they were NPV positive. Note also that if $\mu_i I < e_{vc}$ then the project does not produce enough to compensate the VC for his time regardless of the variance.

Part 2:

If there are two possible projects in Ω_{vc} , then for small enough α

$$1 \geq \sqrt{(1 + \alpha)\theta_1^*\theta_2^*\beta E[R_m] + \frac{1}{16}(\theta_1^* + \theta_2^*)^2(1 + \alpha - \beta E[R_m])^2} + \frac{1}{4}(\theta_1^* + \theta_2^*)(1 + \alpha - \beta E[R_m]). \quad (\text{A3})$$

Therefore, for small enough α , the ϕ negotiated between the investors and VC must be less than or equal to zero to satisfy the investor's constraint, and the VC would not accept this contract. Equation (A3) is easily shown to hold since the largest that θ_1^* and θ_2^* could be is 1. Even in this case, as $\alpha \rightarrow 0$ equation (A3) approaches

$$1 \geq \sqrt{\beta E[R_m] + \frac{1}{4}(1 - \beta E[R_m])^2} + \frac{1}{2}(1 - \beta E[R_m]), \quad (\text{A4})$$

$$1 \geq \sqrt{\frac{1}{4} + \frac{1}{2}\beta E[R_m] + \frac{1}{4}\beta E[R_m]^2} + \frac{1}{2}(1 - \beta E[R_m]) = 1. \quad (\text{A5})$$

Therefore, both the VC and the entrepreneur would have to give up everything as $\alpha \rightarrow 0$. Since $e_{vc} > 0$, the VC would be unwilling to do so. Expected α must be large enough to provide for all.

Q.E.D.

Proof of Corollary 2:

Part two of Theorem 2 shows that as long as $e_{vc}(\sigma_i^2) > 0$ then for small enough α the returns from the project would not be large enough to pay the VC and give the investor their required return.

Proof of Theorem 3: Assuming an economically reasonable equilibrium exists, then the constraint on the VC binds,

$$\theta_i\phi^*I\mu_i - \frac{1}{2}AI^2(\theta_i\phi^*)^2\sigma_i^2 = e_{vc}. \quad (\text{A6})$$

Taking the total derivative with respect to the variance σ_i^2 and solving for $d\theta_i/d\sigma_i^2$ yields

$$\frac{d\theta_i}{d\sigma_i^2} = \frac{\frac{1}{2}AI^2\phi^{*2}\theta_i^2}{[\phi^*I\mu_i - AI^2\phi^{*2}\theta_i\sigma_i^2]}. \quad (\text{A7})$$

The derivative is positive as long as

$$\phi^*I\mu_i - AI^2\phi^{*2}\theta_i\sigma_i^2 > 0. \quad (\text{A8})$$

Earlier we assumed that increasing the fraction offered to the VC always improved his utility therefore $\phi^*I\mu_i - AI^2\phi^{*2}\theta_i\sigma_i^2 > 0$. Thus, as long as θ_i is economically reasonable (between zero and one) θ_i increases with variance, and variance increases with idiosyncratic risk, $\sigma_{\varepsilon_i}^2$. A greater fraction given up by the entrepreneur is equivalent to receiving a lower implicit value for the firm. Therefore, total gross returns are higher. Thus, high gross returns are correlated with high idiosyncratic risk.

Proof of Corollary 3: Assuming there is no principal-agent problem and an economically reasonable equilibrium exists, then the constraint on the VC binds,

$$\theta_i\phi'I\mu_i = e_{vc}(\sigma_i^2). \quad (\text{A9})$$

Taking the total derivative with respect to the variance σ_i^2 and solving for $d\theta_i/d\sigma_i^2$ yields

$$\frac{d\theta_i}{d\sigma_i^2} = \frac{e'_{vc}(\sigma_i^2)}{\phi'I\mu_i} \quad (\text{A10})$$

The derivative is positive as long as $e'_{vc}(\sigma_i^2) > 0$. Thus, θ_i increases with variance, and variance increases with idiosyncratic risk, $\sigma_{\varepsilon_i}^2$. A greater fraction given up by the entrepreneur is equivalent to receiving a lower implicit value for the firm. Therefore, total gross returns are higher, and high gross returns are correlated with high idiosyncratic risk.

Proof of Corollary 4: Assuming an economically reasonable equilibrium exists, the net fraction owned by the investors equals $\theta'_i - \theta'_i\phi'$. Without the principal-agent problem we know

$$\phi = 1 - \frac{1}{\theta'_i(\mu_i - \beta_i E[R_m])}, \quad (\text{A11})$$

and

$$\phi' = \frac{e_{vc}(\sigma_i^2)}{\theta'_i I\mu_i}. \quad (\text{A12})$$

Substitution reveals that

$$\theta'_i - \theta'_i\phi' = \frac{1}{(1 + \alpha_i)}. \quad (\text{A13})$$

Therefore, the fraction held by investors is NOT a function of $\sigma_{\varepsilon_i}^2$. Thus, in the model without a principal-agent problem, net returns are not correlated with realized idiosyncratic risk.

The General Problem: This section shows that for some set of utility functions and exogenous model parameters (α 's, β 's, e_{vc} , *etc.*) a solution exists. We assume that the VC is risk-averse but likes wealth so that $u'(w) > 0$ and $u''(w) < 0$. We also assume that the utility function is continuous in all parameters.

Let ϕ_p represent the contract negotiated with the investors and let θ_p represent the vector of contracts negotiated with the entrepreneurs. Since the VC's wealth, w , depends on the fraction of the portfolio he receives, the VC's utility is a function of the negotiated fractions: $u(w(\phi_p, \theta_p))$. Furthermore, the fraction the VC receives is $\phi_p \theta_p$, therefore the VC's utility as a function of his fraction of the portfolio collapses to $u(\phi_p \theta_p)$. The last assumption we make is that the portfolio is sufficiently good that more of the portfolio is better: $\frac{d}{d\phi_p \theta_p} u(\phi_p \theta_p) > 0$.

We assume for simplicity that the VC negotiates with all entrepreneurs at once. This is unnecessary but eliminates the need to take expectations over future project parameters. Since the VC's are competitive the θ_i must all be set so that the VC's utility constraint binds:

$$u(\phi_p \theta_p) = e_{vc}, \quad (\text{A14})$$

There is only one constraint and N degrees of freedom in θ_p , and $\frac{d}{d\phi_p \theta_p} u(\phi_p \theta_p) > 0$, therefore, given a $\phi_p > 0$, there are an infinite number of solutions to this constraint. Since we are only trying to show that a solution exists we focus on a symmetric solution where $\theta_i = \theta_j$. If a symmetric solution exists, it is almost certain but irrelevant that other solutions also exist. Let $\theta_p^* > 0$ represent the symmetric fraction that is the solution to the VC's constraint ($\frac{d}{d\phi_p \theta_p} u(\phi_p \theta_p) > 0$ insures there is only one and that it is positive). Since each element of a solution vector would be the same there is no longer a need for vector representation and θ_p^* is simply a number.

Given the symmetric solution θ_p^* , and the competition among investors, ϕ_p must be set such that

$$E_{\Omega_{vc}} \left[\frac{\theta_p^*(1 - \phi_p)\mu_p}{1 + \theta_p^*(1 - \phi_p)\beta_p E[R_m]} \right] = 1, \quad (\text{A15})$$

where β_p is the beta of the portfolio. For any positive fraction θ_p^* , $E_{\Omega_{vc}} \left[\frac{\theta_p^*(1 - \phi)\mu_p}{1 + \theta_p^*(1 - \phi)\beta_p E[R_m]} \right]$ is a strictly decreasing function of ϕ . Therefore, a solution to the constraint always exists for some parameters μ_p , β_p , and $E[R_m]$ unless $\theta_p^* = 0$ and we showed above that $\theta_p^* > 0$. Let ϕ_p^* represent the solution to the investor's constraint.

To show that a solution to the overall problem exists we must show that the constraints cross in (θ_p, ϕ_p) space. In the limit as $\theta_p \rightarrow 0$, then for $u(\theta_p \phi_p) = e_{vc}$, $\phi_p \rightarrow \infty$. Furthermore, as $\theta_p \rightarrow \infty$, then for $u(\theta_p \phi_p) = e_{vc}$, $\phi_p \rightarrow 0$. Therefore, continuity of the utility function ensures iso-utility curves in the positive quadrant of (θ_p, ϕ_p) space are convex parabolas that approach the x-axis and y-axis in the limit. Looking at the second constraint, Equation (A15), we see that as $\theta_p \rightarrow 0$, $\phi_p \rightarrow -\infty$ and as $\theta_p \rightarrow \infty$, $\phi_p \rightarrow 1$. Therefore, iso-present value curves are increasing and concave, and we have achieved single crossing in the positive quadrant. Thus, we have proved that there is always only one symmetric solution. Furthermore, increasing β_p shifts the iso-present value line up and decreasing e_{vc} shifts the iso-utility curve down. Therefore, for some choice of parameters the single crossing occurs such that $0 < \theta_p^* < 1$ and $0 < \phi_p^* < 1$.

Proof of Theorem 5: The VC's payoff can be rewritten as

$$\begin{aligned} X_{vc} &= y + \sigma_p \omega I \max\left[\frac{R_p - \mu_p - (R_b - \mu_p)}{\sigma_p}, 0\right] \text{ if } R_b \geq 0, \\ X_{vc} &= y + \omega I R_b + \sigma_p \omega I \max\left[\frac{R_p - \mu_p - (R_b - \mu_p)}{\sigma_p}, 0\right] \text{ if } R_b < 0. \end{aligned} \quad (\text{A16})$$

Or,

$$\begin{aligned} X_{vc} &= y + \sigma_p \omega I \max\left[z - \frac{R_b - \mu_p}{\sigma_p}, 0\right] \text{ if } R_b \geq 0, \\ X_{vc} &= y + \omega I R_b + \sigma_p \omega I \max\left[z - \frac{R_b - \mu_p}{\sigma_p}, 0\right] \text{ if } R_b < 0, \end{aligned} \quad (\text{A17})$$

where $z \sim N(0, 1)$. Therefore, the VC's expected utility is

$$E[u(w)] = \int_{-\infty}^{\frac{R_b - \mu_p}{\sigma_p}} u(y) f(z) dz + \int_{\frac{R_b - \mu_p}{\sigma_p}}^{\infty} u(y + \sigma_p \omega I z - \omega I (R_b - \mu_p)) f(z) dz \text{ if } R_b \geq 0, \quad (\text{A18})$$

$$E[u(w)] = \int_{-\infty}^{\frac{R_b - \mu_p}{\sigma_p}} u(y + \omega I R_b) f(z) dz + \int_{\frac{R_b - \mu_p}{\sigma_p}}^{\infty} u(y + \sigma_p \omega I z + \omega I \mu_p) f(z) dz \text{ if } R_b < 0.$$

The derivative of each expected utility with respect to variance of the portfolio is

$$\begin{aligned} \frac{d}{d\sigma_p} E[u(w)] &= \int_{\frac{R_b - \mu_p}{\sigma_p}}^{\infty} u'(y + \sigma_p \omega I z - \omega I (R_b - \mu_p)) f(z) dz \text{ if } R_b \geq 0, \\ \frac{d}{d\sigma_p} E[u(w)] &= \int_{\frac{R_b - \mu_p}{\sigma_p}}^{\infty} u'(y + \sigma_p \omega I z + \omega I \mu_p) f(z) dz \text{ if } R_b < 0. \end{aligned} \quad (\text{A19})$$

Thus, if $R_b - \mu_p \geq 0$ then $\frac{d}{d\sigma_p}E[u(w)] > 0$. Furthermore, if $R_b - \mu_p < 0$ then³⁷

$$\lim_{R_b \rightarrow -\infty} \frac{d}{d\sigma_p}E[u(w)] = E[u'(y + \sigma_p \omega I z + \omega I \mu_p)] < 0. \quad (\text{A20})$$

Therefore, continuity implies that $\frac{d}{d\sigma_p}E[u(X_{vc})] = 0$ for some R_b such that $-\infty < R_b < \mu_p$.

Q.E.D.

³⁷The proof that $E[u'(y + \sigma_p \omega I z + \omega I \mu_p)] < 0$ is as follows.

Let $x \sim N(0, 1)$ then if $x > 0$ then $u'(a + x) < u'(a) \forall a \Rightarrow u'(a + x)x < u'(a)x \forall a$. If $x < 0$ then $u'(a + x) > u'(a) \forall a \Rightarrow u'(a + x)x < u'(a)x \forall a$ (remember $x < 0$ so inequality switches). Therefore, $u'(a + x)x < u'(a)x \forall a, x \Rightarrow E[u'(a + x)] < u'(a)E[x] = 0$.

Table 1
Summary statistics

The sample consists of 1,245 venture capital and private equity funds in the Thomson Venture Economics database that were formed 1980-1999. Panel A displays fund characteristics, and standard deviations across funds are in parentheses. An annualized IRR is calculated for each fund in the database, and cross-sectional averages are reported. Value weights are each fund's committed capital. Panel B displays coefficients of a quarterly time series regression of aggregate asset class returns on the contemporaneous and lagged value-weighted return on all CRSP stocks, where asset class returns are based on the quarter's inflows, outflows, and reported quarterly changes in net asset values. Standard errors are in parentheses.

| Panel A: Fund Characteristics | | | | | | |
|--------------------------------|-------------------|---------------|-------------------|---------------|---------------------|---------------|
| | <u>All Funds</u> | | <u>VC Funds</u> | | <u>Buyout Funds</u> | |
| | <u>Mean</u> | <u>Median</u> | <u>Mean</u> | <u>Median</u> | <u>Mean</u> | <u>Median</u> |
| Committed capital | 318.7 (607.1) | 104.0 | 147.4 (302.0) | 64.7 | 686.0 (875.8) | 385.0 |
| Number of takedowns | 17.4 (43.9) | 12.5 | 13.6 (50.8) | 10.0 | 25.4 (20.7) | 21.0 |
| Number of distributions | 23.2 (98.9) | 15.0 | 23.2 (118.4) | 15.0 | 23.3 (28.3) | 14.0 |
| Annualized equal-wtd. fund IRR | 16.38% (46.48) | 8.93% | 19.25% (51.37) | 9.13% | 9.67% (31.30) | 8.12% |
| Annualized value-wtd. fund IRR | 9.18% (38.99) | | 19.31% (58.79) | | 4.57% (23.91) | |
| Number of funds | 1,245 | | 866 | | 379 | |

| Panel B: Betas of the value-weighted portfolio of all funds (vs. $R_{m,t-j}$) | | | |
|--|-------------------------|-------------------|---------------------|
| | <u>All Funds</u> | <u>VC Funds</u> | <u>Buyout Funds</u> |
| | Contemporaneous (j = 0) | 0.36*** (0.05) | 0.63*** (0.10) |
| Lag 1 quarter (j = 1) | 0.19*** (0.05) | 0.27** (0.11) | 0.19*** (0.05) |
| Lag 2 quarters (j = 2) | 0.18*** (0.05) | 0.34*** (0.11) | 0.11** (0.05) |
| Lag 3 quarters (j = 3) | 0.19*** (0.05) | 0.25** (0.11) | 0.13** (0.05) |
| Lag 4 quarters (j = 4) | 0.13** (0.06) | 0.30*** (0.11) | 0.01 (0.05) |

* rejects two-sided null of zero at 10% level; ** rejects at 5%; *** rejects at 1%.

Table 2
Funds sorted by size (committed capital)

The sample consists of 1,245 venture capital and buyout funds in the Venture Economics database, sorted into quartiles each quarter based on committed capital in millions of dollars. Funds formed between 1980 and 1999 are in the sample. Time-series regressions on quarterly value-weighted returns weighted by the number of funds, including four quarterly lags of each factor return. Alphas are expressed in percent per quarter. Robust standard errors are in parentheses.

| Panel A: Venture Capital Funds | | | | | |
|--------------------------------|-------------------|-----------------|-----------------------|----------------------|----------------------|
| Quartile | 1994Q1 Size Range | α | $\Sigma \beta_{RMRF}$ | $\Sigma \beta_{SMB}$ | $\Sigma \beta_{HML}$ |
| 1 (smallest) | \$5.0 – 19.8 | 0.76 (0.50) | 0.28** (0.14) | 0.51*** (0.16) | -0.92*** (0.17) |
| 2 | \$20.0 – 36.0 | -0.13 (0.47) | 0.62*** (0.11) | 0.13 (0.15) | -0.42*** (0.12) |
| 3 | \$36.1 – 64.7 | 0.84 (0.92) | 1.05*** (0.21) | 0.47 (0.35) | -1.34*** (0.33) |
| 4 (largest) | \$65 – 1,775 | 1.56* (0.90) | 1.24*** (0.20) | 0.23 (0.35) | -1.32*** (0.28) |
| All | | 1.17 (0.83) | 1.11*** (0.18) | 0.28 (0.32) | -1.26*** (0.27) |
| Panel B: Buyout Funds | | | | | |
| Quartile | 1994Q1 Size Range | α | $\Sigma \beta_{RMRF}$ | $\Sigma \beta_{SMB}$ | $\Sigma \beta_{HML}$ |
| 1 (smallest) | \$5.0 – 60.3 | 1.25 (1.09) | 0.42 (0.27) | 0.11 (0.45) | 0.12 (0.17) |
| 2 | \$62.4 – 160.0 | 1.50 (2.03) | 0.70** (0.28) | -0.09 (0.43) | 0.14 (0.20) |
| 3 | \$163.6 – 400.0 | -0.48 (0.76) | 0.88*** (0.12) | 0.10 (0.16) | -0.01 (0.11) |
| 4 (largest) | \$410 – 5,600 | 0.23 (0.60) | 0.79*** (0.20) | 0.11 (0.16) | 0.14 (0.12) |
| All | | 0.18 (0.55) | 0.81*** (0.16) | 0.11 (0.14) | 0.11 (0.10) |

* rejects two-sided null of zero at 10% level; ** rejects at 5%; *** rejects at 1%.

Table 3
Funds sorted by number of takedowns

The sample consists of 1,245 venture capital and buyout funds formed between 1980 and 1999. Funds are sorted into quartiles based on the total number of takedowns over the life of the fund. Size is committed capital, in millions of dollars. Time-series regressions on quarterly value-weighted returns weighted by the number of funds, including four quarterly lags of each factor return. Alphas and RMSE's are expressed in percent per quarter. Robust standard errors are in parentheses.

| Panel A: Venture Capital Funds | | | | | | | |
|--------------------------------|------------------|------------------|------------------|------------------------------|-----------------------------|-----------------------------|------|
| <u>Pfl</u> | <u>Takedowns</u> | <u>Avg. Size</u> | α | $\Sigma \beta_{\text{RMRF}}$ | $\Sigma \beta_{\text{SMB}}$ | $\Sigma \beta_{\text{HML}}$ | RMSE |
| 1 | 1 – 3 | \$41.6 | -0.26 (0.40) | 0.42*** (0.11) | 0.27** (0.12) | -0.28*** (0.10) | 0.26 |
| 2 | 4 – 8 | 70.9 | 0.31 (0.60) | 0.80*** (0.14) | 0.17 (0.24) | -0.70*** (0.16) | 0.53 |
| 3 | 9 – 13 | 132.2 | 1.07 (0.73) | 1.25*** (0.17) | 0.09 (0.24) | -1.17*** (0.20) | 0.47 |
| 4 | 14 or more | 211.1 | 2.62** (1.10) | 1.31*** (0.31) | 0.21 (0.44) | -1.61*** (0.32) | 0.71 |
| Panel B: Buyout Funds | | | | | | | |
| <u>Pfl</u> | <u>Takedowns</u> | <u>Avg. Size</u> | α | $\Sigma \beta_{\text{RMRF}}$ | $\Sigma \beta_{\text{SMB}}$ | $\Sigma \beta_{\text{HML}}$ | RMSE |
| 1 | 1 – 8 | \$249.3 | 2.36** (0.99) | 0.16 (0.25) | 0.07 (0.31) | -0.03 (0.20) | 0.45 |
| 2 | 9 – 17 | 444.1 | -0.45 (0.83) | 1.12*** (0.18) | 0.33 (0.18) | 0.05 (0.14) | 0.33 |
| 3 | 18 – 29 | 688.2 | -0.06 (0.84) | 0.87*** (0.22) | 0.11 (0.14) | 0.09 (0.16) | 0.30 |
| 4 | 30 or more | 996.6 | -0.50 (0.67) | 0.95*** (0.24) | 0.12 (0.16) | 0.18 (0.16) | 0.27 |

* rejects two-sided null of zero at 10% level; ** rejects at 5%; *** rejects at 1%.

Table 4
Funds sorted by idiosyncratic risk

The sample consists of 1,242 venture capital and buyout funds formed between 1980 and 1999. Funds are sorted into quartiles based on idiosyncratic risk, which is measured using the following fund-level time-series regression of quarterly returns over the T-bill rate:

$$R_{it} = \alpha_i + \beta_{0i} R_{mt} + \beta_{1i} R_{m,t-1} + \dots + \beta_{4i} R_{m,t-4} + \varepsilon_{it}$$

The table reports the results of a quarterly time-series regression of value-weighted portfolio returns on contemporaneous and four quarterly lags of standard factor returns. Alphas and average RMSE's are expressed in percent per quarter. Robust standard errors are in parentheses.

| Panel A: Venture Capital Funds | | | | | |
|--------------------------------|-----------------------|--------------------|------------------------------|-----------------------------|-----------------------------|
| Quartile | <u>Avg. Fund RMSE</u> | α | $\Sigma \beta_{\text{RMRF}}$ | $\Sigma \beta_{\text{SMB}}$ | $\Sigma \beta_{\text{HML}}$ |
| 1 | 5.08 | -1.09*** (0.17) | 0.19*** (0.04) | 0.12*** (0.04) | -0.01 (0.04) |
| 2 | 10.76 | -0.81*** (0.30) | 0.56*** (0.08) | 0.20*** (0.07) | -0.18*** (0.06) |
| 3 | 16.11 | 0.38 (0.52) | 0.88*** (0.15) | 0.30** (0.13) | -0.51*** (0.11) |
| 4 | 47.45 | 2.52** (1.11) | 1.46*** (0.28) | 0.03 (0.42) | -1.71*** (0.30) |
| Panel B: Buyout Funds | | | | | |
| Quartile | <u>Avg. Fund RMSE</u> | α | $\Sigma \beta_{\text{RMRF}}$ | $\Sigma \beta_{\text{SMB}}$ | $\Sigma \beta_{\text{HML}}$ |
| 1 | 3.05 | -1.00*** (0.33) | 0.19*** (0.06) | 0.09* (0.05) | 0.12*** (0.04) |
| 2 | 8.08 | -0.45 (0.30) | 0.49*** (0.10) | 0.09 (0.06) | 0.08 (0.07) |
| 3 | 14.23 | 0.66 (0.56) | 0.71*** (0.13) | -0.03 (0.09) | 0.27*** (0.10) |
| 4 | 37.69 | 0.35 (0.83) | 0.98*** (0.24) | 0.12 (0.23) | 0.08 (0.15) |

* rejects two-sided null of zero at 10% level; ** rejects at 5%; *** rejects at 1%.

Table 5
Cash Flow IRRs for Takedown and Risk Portfolios

759 venture capital and buyout funds formed between 1980 and 1994. Funds are sorted into quartiles as in Tables 3 and 4 based on either the number of takedowns or the amount of idiosyncratic risk. Annual IRRs are calculated for each fund based on all its cash flows. Reported cross-sectional averages are weighted by size, which is defined as capital committed to the fund in millions of dollars. RMSEs are expressed in percent per quarter.

| Panel A: Venture Capital Funds | | | | | |
|--------------------------------------|------------------|------------|-------------------------------------|-------------|------------|
| <u>Sorted by number of takedowns</u> | | | <u>Sorted by idiosyncratic risk</u> | | |
| <u>Pfl</u> | <u>Takedowns</u> | <u>IRR</u> | <u>Pfl</u> | <u>RMSE</u> | <u>IRR</u> |
| 1 | 1 – 3 | 5.70% | 1 | 6.01 | 7.98% |
| 2 | 4 – 6 | 11.41% | 2 | 10.96 | 12.01% |
| 3 | 7 – 11 | 16.23% | 3 | 15.72 | 19.65% |
| 4 | 12 or more | 25.68% | 4 | 45.58 | 32.51% |

| Panel B: Buyout Funds | | | | | |
|--------------------------------------|------------------|------------|-------------------------------------|-------------|------------|
| <u>Sorted by number of takedowns</u> | | | <u>Sorted by idiosyncratic risk</u> | | |
| <u>Pfl</u> | <u>Takedowns</u> | <u>IRR</u> | <u>Pfl</u> | <u>RMSE</u> | <u>IRR</u> |
| 1 | 1 – 4 | 29.32% | 1 | 5.34 | 10.88% |
| 2 | 5 – 11 | 18.99% | 2 | 12.30 | 16.63% |
| 3 | 12 – 24 | 19.25% | 3 | 18.49 | 22.10% |
| 4 | 25 or more | 16.77% | 4 | 52.94 | 27.74% |