

# Discussion of “Centrality-Based Capital Allocations”\*

Alireza Tahbaz-Salehi

Columbia Business School, Columbia University

## 1. Introduction

Since the global financial crisis of 2007–9, a growing literature has focused on whether the complex web of interactions within the financial system can function as a mechanism for the propagation and amplification of shocks. This body of work, which for the most part models the interdependencies between different financial institutions by the means of a network, studies how stress at a few institutions can spread to others via interbank linkages, leading to system-wide crises.

Even though motivated by recent events, the financial networks literature has mostly focused on theoretical (and highly stylized) models of interbank contagion. In fact, aside from a few notable exceptions, there has been little work to document the detailed structures of real-world interbank networks and empirically assess the importance of different contagion channels emphasized by this literature.

The recent work by Alter, Craig, and Raupach (in this issue) belongs to the small set of papers that try to fill this important void. The paper relies on a rich data set of credit exposures in the German banking system to recover the detailed patterns of interbank linkages. More importantly, however, by building on a variant of the contagion model of Eisenberg and Noe (2001) and utilizing the interbank network information, it explores whether certain network-based capital reallocation policies—i.e., policies that determine each

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bank's capital requirements as a function of the bank's location in the network, while keeping the total capital in the system constant—can increase financial stability. The main results of the paper suggest that capital reallocation rules that are based on the financial institutions' so-called eigenvector centralities can reduce the extent of contagious defaults in a meaningful way and, hence, enhance the overall stability of the system.

## 2. Some Key Challenges

As mentioned above, the paper by Alter, Craig, and Raupach is part of the literature whose main aim is to provide a quantitative assessment of regulatory policies that are based on the intricate details of the financial network. In what follows, I will try to outline what I believe are some of the key challenges that such quantitative studies face in employing network-based tools in devising regulatory policies.

### 2.1 *Endogeneity of Financial Networks*

First and foremost, it is important to keep in mind that financial institutions enter into contracts with one another voluntarily. This means that financial interlinkages, and hence the structure of the resulting networks, are themselves equilibrium objects that are determined endogenously. As a result, network-based interventions that do not treat the network structure as part of the equilibrium are subject to the Lucas critique: the introduction of such policies may affect the banks' lending and borrowing incentives and, hence, alter the underlying network of financial interlinkages.

Thus, a proper assessment of the implications of network-based policies (such as the capital reallocation policies studies by Alter, Craig, and Raupach) needs to be coupled with a theory of network formation that takes the lending and borrowing incentives of the banks into account. Some recent papers, such as Zawadowski (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014), Babus (2014), Erol and Vohra (2014), and Farboodi (2014) have taken preliminary steps towards understanding how financial networks are formed. However, developing frameworks that can serve as the basis for quantitative evaluations of network-based regulatory policies requires further progress in this direction.

## 2.2 Network Centralities

The paper's analysis relies on two key ingredients: (i) a structural model of interbank contagion that determines how shocks to a given financial institution propagate over the network of financial liabilities; and (ii) a host of different centrality measures (such as eigenvector centrality, Opsahl centrality, etc.) that capture the relative importance of different banks in the financial network. These two ingredients, however, may be inconsistent with one another, in the sense that the structural model of interbank contagion may point to a notion of systemic importance that is different from the off-the-shelf measures of network centrality used in the paper. Such a possibility means that the choice of network centrality measures (and, hence, the policies that are based on such measures) should be informed by the underlying model of microeconomic spillovers between the banks.

To demonstrate the importance of micro-interactions in determining the relevant notion of systemic importance, it is useful to consider a simple model in which spillover effects between different institutions are linear. In particular, suppose that

$$L_i = \sum_{j=1}^n \pi_{ji} L_j + L_i^{\text{fund}}, \quad (1)$$

where, following the paper's notation,  $L_i^{\text{fund}}$  denotes bank  $i$ 's “fundamental” losses on loans to the real economy,  $L_i$  denotes its total losses, and  $\pi_{ji} \geq 0$  captures the extent to which losses to a bank  $j$  spill over to bank  $i$ . For simplicity, also assume that the column sums of the interaction matrix  $\pi = [\pi_{ji}]$  are equal to  $\alpha < 1$ . Denoting the total losses in the economy by  $L^{\text{agg}} = \sum_{i=1}^n L_i$ , one can show that

$$\frac{\partial L^{\text{agg}}}{\partial L_i^{\text{fund}}} = v_i,$$

where  $v_i$  is the *eigenvector centrality* of bank  $i$ , defined as the  $i$ -th element of the top (left) eigenvector of  $\pi' + \frac{(1-\alpha)}{n} \mathbf{1}\mathbf{1}'$ , with  $\mathbf{1}$  denoting the vector of all ones.<sup>1</sup> This simple result shows that when interbank

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<sup>1</sup>For a detailed derivation of this result, see Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015a).

interactions are linear, banks with higher eigenvector centralities are indeed the more systemically important institutions: negative shocks to the real assets of such banks would lead to larger increases in system-wide losses.

Such a result, however, may no longer hold if interbank spillover effects are non-linear. To see this, consider the following variant of the interaction model studied in the paper,

$$L_i = \min \left\{ l_i, \max \left\{ \sum_{j=1}^n \pi_{ji} L_j + L_i^{\text{fund}} - k_i, 0 \right\} \right\}, \quad (2)$$

where  $l_i$  denotes bank  $i$ 's total interbank liabilities and  $k_i$  is the capital held by the bank. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015b) show that if interbank spillovers are governed by (2), the proper notion of network centrality is indeed distinct from eigenvector centrality: shocks to banks with identical sizes and eigenvector centralities may still have dramatically different effects on the extent of contagion.

The stylized examples above highlight that network statistics that are meant to serve as the basis for network-based capital reallocation policies should not be divorced from the structural model of interbank contagion. Rather, the design of such policies should be informed by the underlying microeconomic interactions between banks.

### *2.3 Equilibrium Multiplicity*

My final comment concerns the fact that many models of contagion over financial networks exhibit multiple equilibria. For example, in the model used by Alter, Craig, and Raupach, the solution concept is a collection of mutually consistent repayments on interbank loans, called a clearing vector. The key observation here is that when bankruptcy costs are large enough, the clearing vector of a given financial network may not be unique. To address this issue, the paper selects the clearing vector for which the number of bank defaults is minimized. This is a perfectly reasonable exercise in equilibrium selection in the presence of multiplicity, as it would lead to the efficient outcome among all possible equilibria. Yet, at the same time, it also

presents a challenge for policies that are based on identifying systemically important financial institutions in the network: the set of systemically important financial institutions and, hence, the optimal capital reallocation rules, may change significantly depending on the equilibrium selected.

To see this more concretely in the context of the model used in the paper, consider a stylized financial network consisting of two fully connected components of sizes  $m$  and  $n$ , where  $m < n$ . Also suppose that banks in each component have no financial liabilities to banks in the other component. Given the complete symmetry between banks within a component, it is easy to see that, under the best equilibrium, banks in the smaller component are more systemically important: a relatively small shock to one such bank can lead to the default of all  $m$  banks in that component, whereas an equally sized shock can be absorbed by banks' excess capital if it hits a bank in the larger component. This result, however, is no longer true under the worst equilibrium, in which bank failures and contagion can happen simply due to coordination failures and self-fulfilling expectations. In particular, in the least efficient equilibrium, a shock to a bank in the larger component would lead to  $n > m$  defaults, implying that banks in the larger component are more systemically important.

This simple example highlights that quantitative evaluation of network-based regulatory policies (such as the ones advocated in the paper) should take the possibility of equilibrium multiplicity into account. It is not always the case that policies that are optimized for a certain equilibrium would perform equally well under the model's alternative equilibria.

### 3. Concluding Remarks

In summary, the paper by Alter, Craig, and Raupach in this issue provides a comparison of different network-based capital reallocation rules that are aimed at enhancing financial stability. It obtains a number of interesting results, in particular, on the quantitative relevance of capital reallocation rules based on financial institutions' eigenvector centralities. While, in my opinion, a thorough assessment of such policies requires a more comprehensive approach than

the one pursued in the current paper, I believe that the paper takes a first and important step in the right direction.

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