

# Biform Games\*

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## Abstract

Both noncooperative and cooperative game theory have been applied to business strategy. We propose a hybrid noncooperative-cooperative game model, which we call a biform game. This is designed to formalize the notion of business strategy as making moves to try to shape the competitive environment in a favorable way. (The noncooperative component of a biform game models the strategic moves. The cooperative component models the resulting competitive environment.) We give biform models of various well-known business strategies. We prove general results on when a business strategy, modelled as a biform game, will be efficient. We also suggest some connections to the area of corporate (multibusiness) strategy.

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# 1 Introduction

There have been a number of applications of game theory to the field of business strategy in recent years. Reflecting the two branches of game theory itself—the so-called noncooperative and cooperative branches—these applications have taken two forms. Noncooperative applications, which are the more numerous, use the more familiar language of game matrices and trees. A leading recent example of this approach is Ghemawat [11, 1997]. Cooperative applications use the less familiar characteristic-function language, and to date have been much fewer in number. For this approach, see Brandenburger and Stuart [7, 1996], Lippman and Rumelt [19, 2003b] (also their [18, 2003a]), MacDonald and Ryall [21, 2002], [22, 2003], Oster [27, 1999], Spulber [36, 2004], and Stuart [38, 2001].

The two approaches address different questions. The noncooperative model is useful for analyzing strategic moves in business—say the decision whether to enter a market, where to position a product, how much capacity to build, how much money to devote to R&D, etc. The cooperative model is useful for addressing the basic question of how much power the different players—firms, suppliers, customers, etc.—have in a given setting, and therefore, for saying how much value each player will capture.

Both models clearly have a role to play in understanding business strategy. Going back at least to the Five Forces framework (Porter [30, 1980]), the idea of talking about the power of the different players in the marketplace has been basic to the business-strategy field. A little more precisely, one can see the Five Forces (and other related frameworks) as a tool for assessing how much value is created in a certain environment, and how that ‘pie’ is divided up among the players. Cooperative game theory offers a theory of exactly this.<sup>1</sup>

But this analysis is only the starting point. The next step is to find ways to shape the environment in a favorable way. A good strategic move is one that brings about favorable economics to a player—one that enables the player to capture more value. This is where the noncooperative theory is important, since it gives us a formal language in which to write down such strategic moves (and countermoves).

In this paper, we put the two models together to create a hybrid noncooperative-cooperative

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<sup>1</sup>We remind the reader that, though standard, the terms “non-cooperative” and “cooperative” game theory are perhaps unfortunate. In particular, cooperative theory can indeed be used to analyze the implications of competition among the players. See also Section 4 below.

model which we call the **biform**<sup>2</sup> game model. A biform game is a two-stage game. The first stage is noncooperative and is designed to describe the strategic moves of the players (entry, position, capacity, R&D, etc. as above). But the consequences of these moves aren't payoffs (at least not directly). Instead, each profile of strategic choices at the first stage leads to a second-stage, cooperative game. This gives the competitive environment created by the choices that the players made in the first stage. Analysis of the second stage then tells us how much value each player will capture (i.e., gives us the payoffs of the players). In this way, the biform model is precisely a formalization of the idea that business strategies shape the competitive environment—and thereby the fortunes of the players.<sup>3</sup> See Figure 1.1 for a schematic of a biform game.<sup>4</sup>

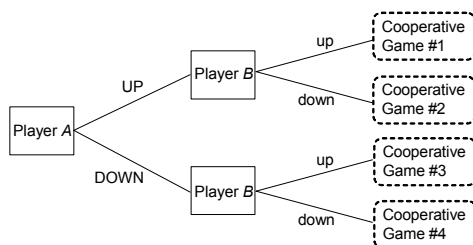


Figure 1.1

We note that at the colloquial level, phrases such as “changing the game” or “choosing the game” are often used when conveying the idea of business strategy. The biform model fits with this language: In the first stage, the players are each trying to choose the best game for themselves, where by “game” is meant the subsequent (second-stage) game of value. Equally, they are trying to change the game, if we define one of the second-stage games as the status quo.

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<sup>2</sup>Having, or partaking of, two distinct forms (OED, 2nd edition).

<sup>3</sup>Interestingly, Porter’s original article on the Five Forces (Porter [29, 1979]) is titled “How Competitive Forces Shape Strategy.” But there is no conflict with what we’re saying. In our model, it is the players’ anticipation of the resulting competitive environment that feeds back—in the usual game-theoretic way—to the strategic choices they make.

<sup>4</sup>Naturally, one can imagine more elaborate models, with a further stage of strategic choice after the second stage of our model, and so on. Our two-stage model is a first step in this direction.

## 2 Examples

We now give some examples of biform games. (More examples can be found in Section 6.)

**Example 2.1 (A Branded Ingredient Game)** *We start with a biform analysis of the idea of a ‘branded-ingredient’ strategy.<sup>5</sup> In the game, there are two firms each able to produce a single unit of a certain product. There is one supplier, which can supply the necessary input to at most one firm, at a cost of \$1. There are numerous buyers, each interested in buying a single unit of the product from one of the two firms. Every buyer has a willingness-to-pay of \$9 for firm 1’s product, and a willingness-to-pay of \$3 for firm 2’s product.*

*The supplier has the option of incurring an upfront cost of \$1 to increase the buyers’ willingness-to-pay for firm 2’s product to \$7. This is the branded-ingredient strategy, under which the supplier puts its logo on its customers’ products. (To make our point most clearly, we assume that this does not affect the buyers’ willingness-to-pay for firm 1, the stronger firm.)*

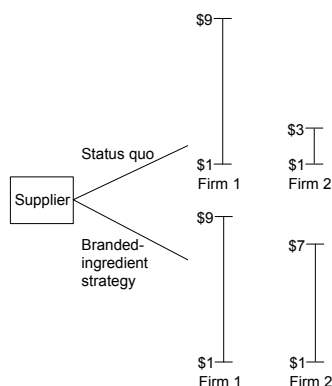


Figure 2.1

The game is depicted in Figure 2.1. Here, there is a simple game tree, in which only one player (the supplier) gets to move. The endpoints of the tree are the cooperative games induced by the supplier’s choices. The two vertical bars at the upper endpoint summarize the cooperative game that results if the supplier chooses the ‘status-quo’ strategy. The left bar shows the valued created if the supplier supplies firm 1 (and therefore not firm 2), and

<sup>5</sup>This example is taken from teaching materials written by Adam Brandenburger and Ken Corts. A well-known instance of a branded-ingredient strategy is the Intel Inside campaign. See Botticelli, Collis, and Pisano [5, 1997] for an account; also Footnote 7 below.

vice versa for the right bar. The interpretation of the bars at the lower endpoint is similar. (We don't show the upfront cost here, but it's included in the calculations below.)

How will this game be played? To see, we start by analyzing each of the cooperative games, and then work back to find the optimal strategy for the supplier. Formally, we'll analyze cooperative games using the Core, a solution concept that embodies competition among the players (see Section 5 below). It is straightforward to find the Cores in the current example. (See Appendix A for the calculations for this and the next example.)

The answer for the status-quo game is that  $\$9 - \$1 = \$8$  of value will be created. Firm 2 and the buyers won't capture any value. The supplier will receive between  $\$2$  and  $\$8$ , and firm 1 will receive between  $\$0$  and  $\$6$  (where the sum of what the supplier and firm 2 get must be  $\$8$ ). This is, of course, the intuitive answer: Competition among buyers ensures that the firms can get their full willingness-to-pay. Firm 2 can then bid up to  $\$3$  for the input from the supplier. Firm 1 has an advantage over firm 2 (since it commands the higher willingness-to-pay) and so will be the one to secure the input. But because of the presence of firm 2, it will have to pay at least  $\$3$  for it. Thus the supplier gets a minimum of  $\$3 - \$1 = \$2$  of value, and the remaining  $\$9 - \$3 = \$6$  is subject to negotiation between the supplier and firm 1, and this could be split in any way.

The analysis of the branded-ingredient game is very similar:  $\$8$  of value will be created gross of the  $\$1$  upfront cost, or  $\$7$  net. Again, firm 2 and the buyers won't capture any value. This time, the supplier is guaranteed  $\$5$ , and the remaining  $\$2$  of value will be split somehow between the supplier and firm 1.

We see that paying  $\$1$  to play the branded-ingredient strategy may well be worthwhile for the supplier. For example, if the supplier anticipates splitting the 'residual pies' equally with firm 1, then it anticipates getting  $\$2 + \$3 = \$5$  in the top game, and  $\$5 + \$1 = \$6$  in the bottom game.

The 'aha' of the strategy is that it wouldn't be worthwhile for firm 2 to pay  $\$1$  to increase willingness-to-pay for its product from  $\$3$  to  $\$7$ . It would still be at a competitive disadvantage. But it is worthwhile for the supplier to pay the  $\$1$  to increase this willingness-to-pay and thereby level the playing field. It gains by creating more 'equal' competition

among the firms.<sup>6</sup> This may be at least one effect of a branded-ingredient strategy in practice.<sup>7</sup>

The next example involves strategic moves by more than one player.

**Example 2.2 (An Innovation Game)** Consider the following game of innovation between two firms. Each firm has a capacity of two units, and (for simplicity) zero unit cost. There are three buyers, each interested in one unit of product. A buyer has a willingness-to-pay of \$4 for the current-generation product, and \$7 for the new-generation product. Each firm has to decide whether to spend \$5 to bring out the new product. The biform game is depicted in Figure 2.2. (Each vertical bar represents one unit. Again, upfront costs aren't shown.)

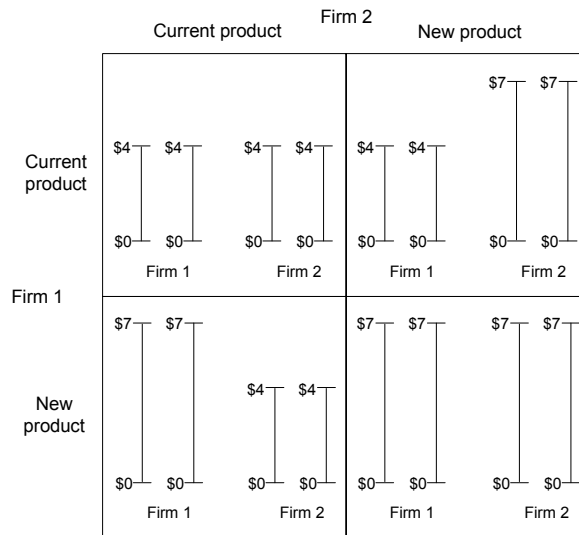


Figure 2.2

<sup>6</sup>In Porter terminology, we might say that the supplier's strategy has reduced buyer power. (Its buyers are the firms, of course.) The example is indeed one of strategy shaping the competitive environment.

<sup>7</sup>Some facts on Intel (taken from Botticelli, Collis, and Pisano [5, 1997]): In 1990, Intel created its Intel Inside campaign, which reimbursed PC makers for some portion of ad spending in return for their using the Intel Inside logo on PCs and in their ads. By 1993, Intel had spent \$500 million cumulatively on the campaign. In 1994, IBM and Compaq both opted out of Intel Inside. IBM said: "There is one brand, and it's IBM as far as IBM is concerned. We want to focus on what makes IBM computers different, not what makes them the same" (quoted from "IBM, Compaq Tire of the 'Intel Inside' Track," by B. Johnson, *Advertising Age*, 09/19/94, p.52). Compaq rejoined the campaign in 1996, IBM in 1997. (If we take IBM to be like firm 1 in our example, then we can interpret IBM's leaving the campaign as an attempt to undermine its effectiveness.)

The Cores of the four cooperative games are as follows: In the top-left game, each firm gets \$0 and each buyer gets \$4.<sup>8</sup> (This is intuitive, since supply exceeds demand.) In the bottom-left game, firm 1 gets \$6 gross (and \$1 net of its upfront cost), firm 2 gets \$0, and each buyer gets \$4. The same answer—with firms 1 and 2 reversed—holds in the upper-right game. These two cases are a bit more subtle. The Core effectively says that what the two firms offer in common is competed away by them to the buyers, but what firm 1 (resp. firm 2) uniquely offers is competed away to it by the buyers. Finally, in the bottom-right game, each firm gets \$0 gross (and thus  $-\$5$  net of its upfront cost) and each buyer gets \$7. (Supply again exceeds demand.)

We see that analysis of the second-stage, cooperative games yields an induced noncooperative game, which is the Battle of the Sexes (Figure 2.3). Here then, unlike Example 2.1, finding the best strategy reduces to a game problem, not a decision problem. If firm 1 thinks that firm 2 will innovate, then its best strategy is not to (and vice versa). Also, both firms may innovate, and lose overall, if each thinks the other won't. Buyers will win in this case.

|        |                 | Firm 2          |             |
|--------|-----------------|-----------------|-------------|
|        |                 | Current product | New product |
| Firm 1 | Current product | 0, 0            | 0, 1        |
|        | New product     | 1, 0            | -5, -5      |

Figure 2.3

### 3 Organization of the Paper

The preceding two examples of biform games illustrate some of the issues that we shall take up formally in the following sections. In Section 5 we give a general definition of a biform game, and also lay out the general analysis of a biform game. In doing so, we explain how we deal both with the case that the Core gives a unique payoff to each player, as in Example 2.2, and with the case that the Core gives a range of payoffs to a player, as in Example 2.1.

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<sup>8</sup>The structure of this cooperative game comes from Postlewaite and Rosenthal [31, 1974].

That is, we cover both determinate and indeterminate competition, since business strategies can give rise to either situation.

Note another difference between Examples 2.1 and 2.2. The efficient outcome in Example 2.2 is when one firm innovates and the other doesn't. The (net) value created is then  $\$7 + \$7 + \$4 - \$5 = \$13$ , as opposed to  $\$12$  when neither innovates and  $\$11$  when both innovate. The efficient outcome can therefore arise in this game, at least if we look at the two (pure-strategy) Nash equilibria. By contrast, in Example 2.1 we saw that the supplier will optimally choose the branded-ingredient strategy. This is inefficient—it costs  $\$1$  and does not increase the total (gross) value. In Section 6, we give a general result on the efficiency or inefficiency of strategies. We prove that three conditions on a biform game—Adding Up, No Externalities, and No Coordination—are sufficient for the outcome to be efficient. (In building up to this result, we also give some additional specific examples of biform games.) This analysis gives a general way of doing a kind of ‘audit’ of business strategies—if modelled as biform games—to investigate their efficiency properties. In Section 7, we apply the same analysis to the area of corporate (multibusiness) strategy, and use it to suggest a possible taxonomy of corporate strategies. Section 8 discusses some conceptual aspects of the biform model, and concludes. Additional technical material is contained in the appendices.

Before we begin the formal development of the biform model (in Section 5), we discuss some related models.

## 4 Related Models

The biform model is related to the two-stage models that are common in the game-theoretic industrial organization (IO) literature. These models are purely noncooperative. In a typical such model, firms first make strategic choices that define a resulting noncooperative subgame, in which they then set prices and thereby compete for customers. Shapiro [33, 1989] uses this set-up to give a taxonomy of a wide range of IO models. A similar formulation is also central to Sutton's [39, 1991] theory of market structure.

Clearly, the difference between these models and the biform model is in the treatment of the second stage. We have cooperative, rather than noncooperative, second-stage games, for two related reasons. First, as we said in the Introduction, the cooperative model ad-



dresses directly the question of the power of the different players (implied by the first-stage strategic choices). This is the central question of a number of frameworks in the business-strategy literature. (We’ve already mentioned Porter [30, 1980], and can add the Imitation-Substitution-Holdup-Slack framework of Ghemawat [10, 1991], and the framework in Spulber [36, 2004, Chapter 7], among others.) Second, the cooperative model embodies an ‘institution-free’ concept of competition. No player is given any price-setting power. No player can make a take-it-or-leave-it offer. Instead, every player is an active negotiator over price. In short, the cooperative model is one of buyers competing for sellers and sellers competing for buyers, without any specific ‘protocol.’<sup>9</sup> This is clear in Examples 2.1 and 2.2 above. In Example 2.1, we didn’t say whether the firms set prices that buyers must take or leave, or whether the buyers name prices. Nor did we specify how the supplier and the firms interact. The prices we mentioned were the consequences of free-form competition among the players. The same was true of Example 2.2—both the firms and the buyers were free to negotiate.

Of course, these two features of the cooperative approach—the direct analysis of power and the free-form modelling—are related. The free-form modelling ensures that the power of a player comes solely from the structure of the second-stage game, and not from any procedural assumptions. A good (first-stage) strategy is then one that creates a favorable (second-stage) structure for that player, in accordance with our conception of business strategy.

Finally, we believe that the cooperative model is empirically useful. In business-to-business relationships—such as supplier to firm, and firm to distributor—prices (and other terms) are often negotiated rather than posted by one or another party. This is not to deny that there are also posted-price settings. But the biform model makes negotiation the baseline case which, to repeat, does fit with the focus of a number of strategy frameworks.

Two-stage models are used in the economics of organizations; see, in particular, Grossman and Hart [13, 1986] and Hart and Moore [15, 1990]. Both these papers have a noncooperative first stage and a cooperative second stage, just as we have.<sup>10</sup> But then the approaches diverge, reflecting the different contexts being studied. Grossman-Hart (resp. Hart-Moore)

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<sup>9</sup>The term is from Kreps [17, 1990, pp.92-95]. Aumann [3, 1985, p.53] writes that “the core expresses the idea of unbridled competition.”

<sup>10</sup>To be quite precise, Grossman-Hart start with a noncooperative second-stage game, from which they then define an associated cooperative game.

use the Shapley Value (resp. the Nash Bargaining Solution) to say how players who jointly create some value might agree to divide it. We use the Core to capture free-form competition over the value created.<sup>11</sup>

Next, we mention briefly other work that applies cooperative game theory to business strategy.<sup>12</sup> In an earlier paper (Brandenburger and Stuart [7, 1996]), we proposed using cooperative theory—the Core in particular—to give foundations for business-strategy concepts. In effect, we proposed the second stage of the biform model developed in the current paper. Stuart [38, 2001] summarizes some applications different from the ones we present here. MacDonald and Ryall [21, 2002], [22, 2003] generate results based on the minimum and maximum values a player can get in the Core, and relate these results to business strategy. Lippman and Rumelt [19, 2003b] examine differences between cooperative game theory and general- or partial-equilibrium price theory, and discuss the benefits of the first as a basis for business-strategy research. (They consider several cooperative solution concepts apart from the Core—the Shapley Value, the Nash Bargaining Solution, and the Nucleolus.) Two recent textbooks on strategy (Oster [27, 1999] and Spulber [36, 2004]) use ideas from cooperative theory. Finally, we should mention work by Makowski and Ostroy ([23, 1994], [24, 1995]), which, while formally in the general-equilibrium setting, was a very important influence on this and our earlier paper ([7, 1996]). We say more about this connection in Section 6.

## 5 General Formulation

We now give the general definition of a biform game. Some notation: Given a set  $X$ , let  $\mathcal{P}(X)$  denote the power set of  $X$ , i.e., the set of all subsets of  $X$ . Also, write  $N = \{1, \dots, n\}$ .

**Definition 5.1** *An  $n$ -player biform game is a collection*

$$(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n),$$

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<sup>11</sup>The difference is more than a ‘technical’ one. In a separate note (Brandenburger and Stuart [8, 2004]), we consider a two-stage analysis of monopoly, in which the first stage is choice of capacity by the seller, and the second stage is a cooperative game between seller and buyers. We show that the Core and Shapley Value can give very different predictions in this game.

<sup>12</sup>For an application to the field of operations management, see Anupindi, Bassok, and Zemel [1, 2001], who employ a hybrid noncooperative-cooperative model similar to a biform game.

where:

- (a) for each  $i = 1, \dots, n$ ,  $S^i$  is a finite set;
- (b)  $V$  is a map from  $S^1 \times \dots \times S^n$  to the set of maps from  $\mathcal{P}(N)$  to the reals, with  $V(s^1, \dots, s^n)(\emptyset) = 0$  for every  $s^1, \dots, s^n \in S^1 \times \dots \times S^n$ ; and
- (c) for each  $i = 1, \dots, n$ ,  $0 \leq \alpha^i \leq 1$ .

The set  $N$  is the set of **players**. Each player  $i$  chooses a strategy  $s^i$  from **strategy set**  $S^i$ . The resulting profile of strategies  $s^1, \dots, s^n \in S^1 \times \dots \times S^n$  defines a **TU (transferable utility) cooperative game** with **characteristic function**  $V(s^1, \dots, s^n) : \mathcal{P}(N) \rightarrow \mathbb{R}$ . That is, for each  $A \subseteq N$ ,  $V(s^1, \dots, s^n)(A)$  is the value created by the subset  $A$  of players, given that the players chose the strategies  $s^1, \dots, s^n$ . (As usual, we require  $V(s^1, \dots, s^n)(\emptyset) = 0$ .) Finally, the number  $\alpha^i$  is player  $i$ 's **confidence index**. Roughly speaking, it indicates how well player  $i$  anticipates doing in the resulting cooperative games. The precise way the indices  $\alpha^i$  are used is explained below.

We note that the strategy sets  $S^1, \dots, S^n$  can, of course, come from a general extensive-form game, so that the definition of a biform game is certainly not restricted to a simultaneous-move first-stage game. (It covers the simple tree in Figure 2.1, and much more.) The analysis, too, can reflect a general extensive-form structure; see below, especially Footnote 16.

Write  $S = S^1 \times \dots \times S^n$ , with typical element  $s$ .

**Definition 5.2** *Call a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  **inessential** (resp. **super-additive**) if for each  $s \in S$ , the cooperative game  $V(s)$  is inessential (resp. superadditive).<sup>13</sup>*

The biform model is a strict generalization of both the strategic-form noncooperative, and TU cooperative, game models. Formally, write an  $n$ -player strategic-form noncooperative game as a collection  $(S^1, \dots, S^n; \pi^1, \dots, \pi^n)$ , where the sets  $S^i$  are as above and, for each  $i$ , player  $i$ 's payoff function  $\pi^i$  maps  $S$  to the reals. The following two remarks are then immediate:

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<sup>13</sup>See, e.g., Owen [28, 1995, pp.145-147] for definitions of inessential and superadditive games.

**Remark 5.1** Fix confidence indices  $\alpha^1, \dots, \alpha^n$ . There is a natural bijection between the subclass of  $n$ -player biform games  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  that are inessential and superadditive, and the class of  $n$ -player strategic-form noncooperative games  $(S^1, \dots, S^n; \pi^1, \dots, \pi^n)$ .

**Remark 5.2** Fix confidence indices  $\alpha^1, \dots, \alpha^n$ . There is a natural bijection between the subclass of  $n$ -player biform games  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  in which the sets  $S^i$  are singletons, and the class of  $n$ -player TU cooperative games.

We now turn to the analysis of a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$ , and adopt the following procedure:

(5.1) For every profile  $s \in S$  of strategic choices and resulting cooperative game  $V(s)$ ,

(5.1.1) compute the Core of  $V(s)$ ,<sup>14</sup>

and, for each player  $i = 1, \dots, n$ ,

(5.1.2) calculate the projection of the Core onto the  $i$ th coordinate axis,

and

(5.1.3) calculate the  $\alpha^i : (1 - \alpha^i)$  weighted average of the upper and lower endpoints of the projection.<sup>15</sup>

(5.2) For every profile  $s \in S$  of strategic choices, and each player  $i = 1, \dots, n$ ,

(5.2.1) assign to  $i$  a payoff equal to  $i$ 's weighted average as in (5.1.3) above,

and

(5.2.2) analyze the resulting strategic-form noncooperative game.

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<sup>14</sup>We assume that for each  $s \in S$ , the Core of  $V(s)$  is nonempty. See Section 8a.

<sup>15</sup>Note that the projection is a closed, bounded interval of the real line.

Given a profile of strategic choices  $s \in S$ , the first step is to restrict attention to Core allocations in the resulting cooperative game  $V(s)$  (Step 5.1.1). Second, we calculate the implied range of payoffs to each player  $i$  (Step 5.1.2). Third, each player  $i$  uses confidence index  $\alpha^i$  to evaluate the given cooperative game  $V(s)$  as a weighted average of the largest and smallest amounts of value that  $i$  can receive in the Core (Step 5.1.3). Use of the confidence indices reduces a biform game to a strategic-form noncooperative game (Step 5.2.1). This game may now be analyzed in standard fashion—say, by computing Nash equilibria, or by iteratively eliminating dominated strategies, or by some other method (Step 5.2.2).

Conceptually, this method of analysis starts by using the Core to calculate the effect of competition among the players at the second stage of the game—i.e., given the strategic choices made in the first stage. This determines how much value each player can capture. The Core might be a single point (as in Example 2.2). If so, competition fully determines the division of value. But there can also be a range of values in the Core, so that competition alone is not fully determinate, and (at least some) players face a ‘residual’ bargaining problem. (In Example 2.1, competition narrowed down the outcomes, but left a range of value to be negotiated between the supplier and firm 1. In the status-quo game, the range for the supplier was [\$2, \$8]; in the branded-ingredient game it was [\$5, \$7].) A confidence index  $\alpha^i$  close to 1 then indicates that player  $i$  anticipates capturing most of the value to be divided in the residual bargaining. If  $\alpha^i$  is close to 0, player  $i$  anticipates getting little of this residual value. We can say that player  $i$  has an optimistic or pessimistic view of the game according to whether  $\alpha^i$  is large or small. (In Example 2.1, the supplier would choose the branded-ingredient strategy if  $\alpha 7 + (1 - \alpha)5 > \alpha 8 + (1 - \alpha)2$ , or  $\alpha < 3/4$ . Unless the supplier is very optimistic, it pays it to follow this strategy.)

Use of the confidence indices gives us an induced noncooperative game among the players. Analysis of this game indicates which strategies the players will choose. Let us repeat that we don’t insist on a particular solution concept here. We can use Nash equilibrium, or some other concept.<sup>16</sup>

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<sup>16</sup>For example, if the underlying first-stage game is one of perfect information, we might want the backward-induction solution (BI), which we could get by performing iterated weak dominance (IWD) on the matrix. (See Brandenburger-Friedenberg [6, 2003] for an exact relationship between IWD and BI, and references to related results.)

A final comment on the formulation: Appendix B shows how the confidence index can be derived from axioms on a player’s preferences over intervals of outcome. The axioms are Order, Dominance, Continuity, and Positive Affinity. The first three axioms are standard. The fourth is what accounts for the specific weighted-average representation. But we argue in Appendix B that this axiom is immediately implied by our cooperative game context, so that the structure of a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  is a consistent whole.<sup>17</sup>

Section 8 has some additional comments on conceptual aspects of the biform model.

## 6 Efficiency and Business Strategy

We now give some more examples of biform games, beyond those in Section 2. The examples, like the earlier ones, show how to analyze business-strategy ideas using a biform game. They also lead to a general analysis of the efficiency and inefficiency of business strategies, as will be seen.

We start with some notation and a definition. Write  $N \setminus \{i\}$  for the set  $\{1, \dots, i-1, i+1, \dots, n\}$  of all players except  $i$ .

**Definition 6.1** *Fix a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$ , and a strategy profile  $s \in S$ . Then the number*

$$V(s)(N) - V(s)(N \setminus \{i\})$$

*is called player  $i$ ’s **added value** in the cooperative game  $V(s)$ .*

In words, player  $i$ ’s added value is the difference, in the given cooperative game, between the overall value created by all the players, and the overall value created by all the players except  $i$ . It is, in this sense, what player  $i$  adds to the value of the game. This is the standard cooperative concept of marginal contribution. Here we use the “added value” terminology from Brandenburger and Stuart [7, 1996].<sup>18</sup>

The following observation is immediate, and will be used below.

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<sup>17</sup>That said, note the assumption in our formulation that a player doesn’t distinguish between two Cores that yield the same projections for that player. One could imagine an alternative approach, where players have preferences over (entire) polytopes rather than intervals, that would allow this distinction. This could be an interesting extension.

<sup>18</sup>We do so for the same reason as there: One frequently sees the term “added value” in business-strategy writing, and the usage fits with the formal definition above.

**Proposition 6.1** Fix a biiform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$ , a strategy profile  $s \in S$ , and suppose that the Core of  $V(s)$  is nonempty. Then if

$$\sum_{i=1}^n [V(s)(N) - V(s)(N \setminus \{i\})] = V(s)(N),$$

the Core of  $V(s)$  consists of a single point, in which each player  $i$  receives  $V(s)(N) - V(s)(N \setminus \{i\})$ .

**Proof.** Given an allocation  $x \in \mathbb{R}^n$  and  $T \subseteq N$ , write  $x(T) = \sum_{j \in T} x^j$ . Fix some  $i$ , and note that the Core conditions  $x(N) = V(s)(N)$  and  $x(N \setminus \{i\}) \geq V(s)(N \setminus \{i\})$  imply that  $x^i \leq V(s)(N) - V(s)(N \setminus \{i\})$ . Summing over  $i$  yields  $x(N) \leq \sum_{i \in N} [V(s)(N) - V(s)(N \setminus \{i\})]$ , with strict inequality if  $x^i < V(s)(N) - V(s)(N \setminus \{i\})$  for some  $i$ . But the right side is equal to  $V(s)(N)$  by assumption. So strict inequality implies  $x(N) < V(s)(N)$ , a contradiction. ■

**Example 6.1 (A Negative-Advertising Game)** Here we look at a “generic” strategy (Porter [30, 1980]), with a difference. Consider the biiform game depicted in Figure 6.1.<sup>19</sup> There are three firms, each with one unit to sell at \$0 cost. There are two buyers, each interested in one unit of product from some firm. Firm 1 alone has a strategic choice, which is whether or not to engage in negative advertising. If it doesn’t, then each buyer has a willingness-to-pay of \$2 for each firm’s product. If it does, then willingness-to-pay for its own product is unchanged, but that for firm 2’s and 3’s products falls to \$1. (The negative advertising has hurt the image of firms 2 and 3 in the eyes of the buyers.)

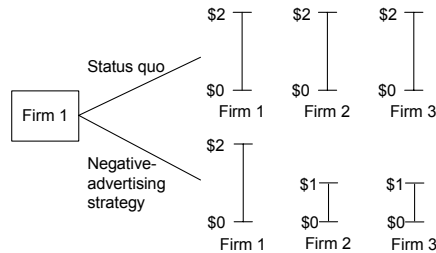


Figure 6.1

<sup>19</sup>Different formalisms aside, the example is essentially the same as Example 1 in Makowski-Ostroy [23, 1994], which they kindly say was prompted by discussions with one of us.

Along the status-quo branch, the overall value is \$4. Each firm's added value is \$0; each buyer's added value is \$2. (See Appendix A for the calculations for the examples in this section.) By Proposition 6.1, each firm will get \$0 in the Core, and each buyer will get \$2. (This is the intuitive answer, since the firms are identical, and supply exceeds demand.) Along the negative-advertising branch, the overall value is \$3. Firm 1's added value is \$1, firm 2's and 3's added values are \$0, and each buyer's added value is \$1. Again using Proposition 6.1, firms 2 and 3 will get \$0 in the Core, and firm 1 and each of the buyers will get \$1. (Similar to Example 2.2, the Core says that what the three firms offer in common is competed away to the buyers, while what firm 1 uniquely offers is competed away to it by the two buyers.)

Firm 1 will optimally choose the negative-advertising example, thereby capturing \$1 vs. \$0. This is the opposite of a differentiation strategy, where a firm creates added value for itself by raising willingness-to-pay for its product (perhaps at some cost to it). Here, firm 1 creates added value by lowering willingness-to-pay for its rivals' products. Notice that the strategy is inefficient: it shrinks the overall value created from \$4 to \$3.

**Example 6.2 (A Coordination Game)** There are three players, each with two strategies, labelled No and Yes. Player 1 chooses the row, player 2 chooses the column, and player 3 chooses the matrix. Figure 6.2 depicts the cooperative game associated with each strategy profile, where the value of all one-player subsets is taken to be 0.<sup>20</sup> This example can be thought of as a model of switching from an existing technology standard (the strategy No) to a new standard (the strategy Yes). The new technology costs \$1 more per player, and is worth \$2 more per player, provided at least two players adopt it.<sup>21</sup>

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<sup>20</sup>Properly, we should write  $V(\text{No}, \text{No}, \text{No})(N) = 6$ ,  $V(\text{No}, \text{No}, \text{No})({1,2}) = 4$ , etc. But the cell identifies the strategy profile, so we use the simpler notation in the matrix.

<sup>21</sup>In more detail, the characteristic functions shown can be built up from the following scenario. There are three firms, each with unit capacity, and numerous buyers, each interested in buying from one firm. (Only the firms have strategic choices, as shown.) Buyers value the current-generation technology at \$2, and the new generation at \$4. There is a 'network effect,' in that at least two buyers must purchase, for value to be created. The new technology costs a firm \$1 to operate, the current technology is costless. The choice that the majority of the firms make determines which technology is available to the buyers. Finally, the technology is backward, but not forward, compatible. Thus, if a firm is the only one choosing the new technology, it can sell to buyers, but not if it is the only one choosing the old technology. We can induce the three-player game in the text from this setup.



|     |   |   |     |   |   |
|-----|---|---|-----|---|---|
|     | No  | Yes   |     | No  | Yes   |
| No  | $v(N) = 6$<br>$v(1, 2) = 4$<br>$v(2, 3) = 4$<br>$v(3, 1) = 4$ | $v(N) = 5$<br>$v(1, 2) = 3$<br>$v(2, 3) = 3$<br>$v(3, 1) = 4$ | No  | $v(N) = 5$<br>$v(1, 2) = 4$<br>$v(2, 3) = 3$<br>$v(3, 1) = 3$ | $v(N) = 6$<br>$v(1, 2) = 3$<br>$v(2, 3) = 6$<br>$v(3, 1) = 3$ |
| Yes | $v(N) = 5$<br>$v(1, 2) = 3$<br>$v(2, 3) = 4$<br>$v(3, 1) = 3$ | $v(N) = 6$<br>$v(1, 2) = 6$<br>$v(2, 3) = 3$<br>$v(3, 1) = 3$ | Yes | $v(N) = 6$<br>$v(1, 2) = 3$<br>$v(2, 3) = 3$<br>$v(3, 1) = 6$ | $v(N) = 9$<br>$v(1, 2) = 6$<br>$v(2, 3) = 6$<br>$v(3, 1) = 6$ |
|     | No  |   |     | Yes   |   |

Figure 6.2

Using Proposition 6.1, it is easy to check that in each of the four cooperative games, the Core will give the players exactly their added values. We get the induced noncooperative game in Figure 6.3, which is a kind of coordination game. There are two (pure-strategy) Nash equilibria: (No, No, No) and (Yes, Yes, Yes). The first is inefficient (the total value is \$6), while the second is efficient (the total value is \$9).

|     |         |         |     |         |         |
|-----|---------|---------|-----|---------|---------|
|     | No      | Yes     |     | No      | Yes     |
| No  | 2, 2, 2 | 2, 1, 2 | No  | 2, 2, 1 | 0, 3, 3 |
| Yes | 1, 2, 2 | 3, 3, 0 | Yes | 3, 0, 3 | 3, 3, 3 |
|     | No      |         |     | Yes     |         |

Figure 6.3

We now show that Examples 6.1 and 6.2, together with Example 2.1, actually give a complete picture, within the biform model, of how strategies can cause inefficiencies. (Example 2.2 will also fit in, as we'll see.) We start with some definitions. As usual, we write  $S^{-i}$  for  $S^1 \times \dots \times S^{i-1} \times S^{i+1} \times \dots \times S^n$ .

**Definition 6.2** A biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  satisfies **Adding Up (AU)** if for each  $s \in S$ ,

$$\sum_{i=1}^n [V(s)(N) - V(s)(N \setminus \{i\})] = V(s)(N).$$

The game satisfies **No Externalities (NE)** if for each  $i = 1, \dots, n$ ;  $r^i, s^i \in S^i$ ; and  $s^{-i} \in S^{-i}$ ,

$$V(r^i, s^{-i})(N \setminus \{i\}) = V(s^i, s^{-i})(N \setminus \{i\}).$$

The game satisfies **No Coordination (NC)** if for each  $i = 1, \dots, n$ ;  $r^i, s^i \in S^i$ ; and  $r^{-i}, s^{-i} \in S^{-i}$ ,

$$V(r^i, r^{-i})(N) > V(s^i, r^{-i})(N) \text{ if and only if } V(r^i, s^{-i})(N) > V(s^i, s^{-i})(N).$$

The **Adding Up** condition says that in each second-stage cooperative game, the sum of the players' added values is equal to the overall value created in that game. (This is just the condition of Definition 6.1 applied to each second-stage game.) The **No Externalities** condition says that each player's strategic choice does not affect the value that the remaining players can create (without that player). The **No Coordination** condition says that when one player switches strategy, the sign of the effect on the overall value created is independent of the other players' strategic choices.

We need two additional (obvious) definitions: A profile of strategies  $s \in S$  in a biform game will be called a **(pure-strategy) Nash equilibrium** if it is a (pure-strategy) Nash equilibrium of the induced noncooperative game. (This is the noncooperative game induced as in Section 5. That is, given a profile  $r \in S$ , we assign each player  $i$  a payoff equal to the  $\alpha^i : (1 - \alpha^i)$  weighted average of the upper and lower endpoints of the projection onto the  $i$ th coordinate axis of the Core of the cooperative game  $V(r)$ .) A strategy profile  $s \in S$  will be called **efficient** if it solves  $\max_{r \in S} V(r)(N)$ .<sup>22</sup>

We can now state and prove two results on efficiency in biform games.

**Lemma 6.1** *Consider a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  satisfying AU and NE, and that for each  $r \in S$ , the game  $V(r)$  has a nonempty Core. Then a strategy profile  $s \in S$  is a Nash equilibrium if and only if*

$$V(s)(N) \geq V(r^i, s^{-i})(N) \tag{6.1}$$

for every  $r^i \in S^i$ .

**Proof.** By Proposition 6.1, AU implies that for each  $r \in S$ , the Core of  $V(r)$  consists of a single point, in which each player  $i$  receives  $V(r)(N) - V(r)(N \setminus \{i\})$ . Therefore, a profile  $s$  is a Nash equilibrium iff for each  $i$ ,

$$V(s)(N) - V(s)(N \setminus \{i\}) \geq V(r^i, s^{-i})(N) - V(r^i, s^{-i})(N \setminus \{i\}) \tag{6.2}$$

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<sup>22</sup> An efficient profile always exists, since we're assuming the strategy sets  $S^i$  are finite.

for every  $r^i \in S^i$ . But NE implies that

$$V(s)(N \setminus \{i\}) = V(r^i, s^{-i})(N \setminus \{i\})$$

for every  $r^i \in S^i$ . Thus, inequality (6.2) holds if and only if inequality (6.1) holds. ■

**Proposition 6.2** *Consider a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  satisfying AU, NE, and NC, and that for each  $r \in S$ , the game  $V(r)$  has a nonempty Core. Then if a strategy profile  $s \in S$  is a Nash equilibrium, it is efficient.*

**Proof.** Write

$$\begin{aligned} V(s)(N) - V(r)(N) = & \\ & V(s^1, r^2, \dots, r^n)(N) - V(r)(N) + \\ & V(s^1, s^2, r^3, \dots, r^n)(N) - V(s^1, r^2, r^3, \dots, r^n)(N) + \\ & \dots + \\ & V(s)(N) - V(s^1, \dots, s^{n-1}, r^n)(N). \end{aligned}$$

NC and inequality (6.1) in Lemma 6.1 together imply that each pair of terms on the right-hand side of this equation is non-negative, from which  $V(s)(N) \geq V(r)(N)$ . ■

**Proposition 6.3** *Consider a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  satisfying AU and NE, and that for each  $r \in S$ , the game  $V(r)$  has a nonempty Core. Then if a strategy profile  $s \in S$  is efficient, it is a Nash equilibrium.*

**Proof.** We are given that  $V(s)(N) \geq V(r)(N)$  for every  $r \in S$ , so certainly inequality (6.1) in Lemma 6.1 holds. Thus the profile  $s$  is a Nash equilibrium. ■

Propositions 6.2 and 6.3 provide, respectively, conditions for any Nash-equilibrium profile of strategies to be efficient and for any efficient profile of strategies to be a Nash equilibrium. They can thus be viewed as game-theoretic analogs to the First and Second Welfare Theorems of general-equilibrium theory. They are closely related to results in Makowski-Ostroy ([23, 1994], [24, 1995]), as we'll discuss below. First, though, we tie these general results back to our earlier examples of business strategies.

Table 6.1 summarizes which of the conditions—AU, NE, and NC—are satisfied in Examples 2.1, 6.1, and 6.3.

|                           | Adding Up | No Externalities | No Coordination |
|---------------------------|-----------|------------------|-----------------|
| Branded-ingredient game   | ✗         | ✓                | ✓               |
| Negative-advertising game | ✓         | ✗                | ✓               |
| Coordination game         | ✓         | ✓                | ✗               |

Table 6.1

Start with Example 2.1 (the branded-ingredient game). In the status-quo second-stage game, the supplier has an added value of \$8, and firm 1 has an added value of \$6 (firm 2 and the buyers have zero added value). The overall value is \$8, so AU fails. (We could equally have looked at the branded-ingredient second-stage game.) NE holds: Only the supplier has a strategic choice, and the value created without the supplier is constant—at \$0—regardless of which strategy the supplier follows. Finally, it is easy to see that NC is automatically satisfied in any game where only one player has a strategic choice.<sup>23</sup> We conclude that the likely inefficiency in this game (which we already noted in Section 3) comes from the failure of AU. In plain terms, there is a bargaining problem between the supplier and firm 1 over the \$6 of value that must be divided between them. To do better in this bargaining, the supplier may well adopt a strategy (the branded-ingredient strategy) that decreases the pie.

In Example 6.1 (the negative-advertising game), it is immediate from our earlier analysis that AU is satisfied. NC is satisfied for the same reason as in Example 2.1 (only firm 1 has a strategic choice). But NE fails: The value created by firm 2, firm 3, and the two buyers is \$4 when firm 1 chooses the status-quo strategy, but changes to \$2 when firm 1 chooses the negative-advertising strategy. Again, the outcome is inefficient.

Finally, in Example 6.2 (the coordination game), AU is satisfied. So is NE, as the reader can check from Figure 6.2. But NC fails: For example, when player 1 changes strategy

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<sup>23</sup>This is what we would want, of course. The possibility of inefficiency due to coordination issues should arise only when at least two players have (non-trivial) choices.

from *No* to *Yes*, the overall value falls from \$6 to \$5 when players 2 and 3 are both playing *No*, but rises from \$6 to \$9 when players 2 and 3 are both playing *Yes*. Notice that in this example, there is indeed an inefficient Nash equilibrium (*No*, *No*, *No*). But the efficient profile (*Yes*, *Yes*, *Yes*) is also a Nash equilibrium, as Proposition 6.3 says it must be.

We see how the general results enable us to identify the source of the inefficiency of the business strategy in each of our examples. We also know that if a biform model of a business strategy satisfies our three conditions, then we'll get efficiency. Of course, our examples are meant only to be suggestive of how one might model many other business strategies as biform games. But we now have a general way of doing a kind of 'audit' of any business strategy—if modelled as a biform game—to discover its efficiency properties.

Here is one more example, just to 'redress the balance,' since we've only given one example of efficiency so far (Example 2.2). This time, AU, NE, and NC will all hold.

**Example 6.3 (A Repositioning Game)** *Consider the biform game depicted in Figure 6.4. There are three firms, each with one unit to sell. There are two identical buyers, each interested in one unit of product from some firm. Under the status quo, the firms have the costs, and the buyers have the willingness-to-pay numbers, on the upper branch. Firm 2 has the possibility of repositioning as shown by its vertical bar on the lower branch. Specifically, it can spend \$1 to raise 'quality' (willingness-to-pay) and lower cost as shown.*

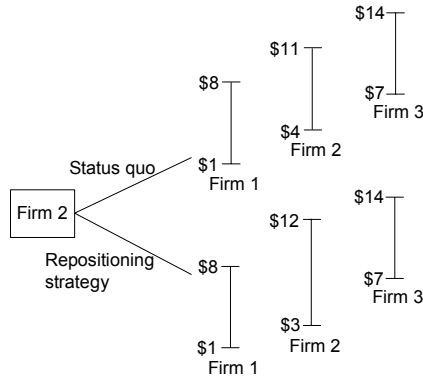


Figure 6.4

Along the status-quo branch, the overall value is \$14. Each firm's added value is \$0; each buyer's added value is \$7. Along the repositioning branch, the overall value is \$15 (\$16 minus the \$1 repositioning cost). Firm 2's added value is \$1; firm 1's and 3's added values

are \$0; each buyer's added value is \$7. *AU* is satisfied, as clearly are *NE* and *NC*. By Proposition 6.2, we know that firm 2 must optimally make the efficient choice of the lower branch, as indeed it will, to net \$1.

*This is a simple biform model of a (re)positioning strategy. Note that on the lower branch, firm 2 still doesn't have either the lowest cost or the highest quality among the three firms. But it does command the largest gap between quality and cost. This is the source of its added value.*

Apart from giving us information on the efficiency properties of a business strategy, the results of this section also give a way to frame some of the ideas in the corporate-strategy literature. We pick up on this in the next section, but, before that, we make some further comments on the results themselves.

First, note that Table 6.1 establishes the independence of the *AU*, *NE*, and *NC* conditions. It is possible for any two to hold, but not the third.<sup>24</sup>

Next, the efficiency conditions in Proposition 6.2 are sufficient, but not necessary. In Example 2.2, the (pure) Nash equilibria were efficient (as we noted in Section 3). But it is easily checked that *AU* and *NC* fail (*NE* holds).<sup>25</sup> This said, the conditions do seem 'tight.' None of them can be left out in Proposition 6.2, as our examples showed, and we do not see how they could be weakened in an interesting way.

Finally, we note the considerable debt we owe to Makowski-Ostroy ([23, 1994], [24, 1995]). Makowski-Ostroy (henceforth M-O) present welfare theorems to which our Propositions 6.2 and 6.3 are closely related. The M-O model is a reformulation of general-equilibrium theory, in which the agents make choices ("occupational" choices in their terminology) that define the subsequent economy in which they then produce and trade. It therefore has a two-stage structure, just as our purely game-theoretic model does. There are some differences. We define added value (marginal contribution) in the standard game-theoretic way, while M-O's definition effectively assumes that *NE* holds. Our *NC* condition is weaker than the corresponding condition in M-O (they call it No Complementarities) ruling out coordination issues.

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<sup>24</sup>It is easy to prove that if in a two-player game *AU* and *NE* are satisfied, then so is *NC*. This is why we needed three players to get a failure of *NC* alone (Example 6.2).

<sup>25</sup>The same example shows that it's also true that the conditions of Proposition 6.3 aren't necessary. Both efficient profiles are Nash equilibria, despite the failure of *AU*.

Appendix C contains more details on the relationship to M-O, and some other remarks on this section.

## 7 A Connection to Corporate Strategy

Corporate (multibusiness) strategy is the study of when ‘the whole is worth more than the sum of the parts.’ Alternatively put, corporate strategy is about the opportunities that arise because individual entities (divisions of a corporation, potential alliance partners, etc.) may each be maximizing the value they can capture, but appropriate joint action by the entities could capture more value in total.

The literature usually distinguishes two kinds of corporate strategy. The “market power” route involves joint action that leads to capturing a greater slice of the existing pie. A good example is when different divisions (or businesses) engage in joint purchasing of an input, and thereby negotiate a lower unit cost. The “efficiency” route involves joint action that leads to creating a larger pie, and capturing at least a part of the increase. The focus of the literature is on the second kind of corporate strategy, and we follow that here.

Our observation is that the previous section gives us a way to classify different kinds of efficiency-oriented corporate strategies. Think of a corporate strategy as a way of addressing an inefficiency. Then, by the previous section, we can then break this down into addressing a failure of Adding Up, or No Externalities, or No Coordination. (Of course, a given strategy could address more than one failure.) Here is a sketch of how this scheme works.

**a. No Adding Up** A big question in corporate strategy is that of the appropriate vertical scope of the firm. Within this topic, the Holdup problem has, of course, been central (e.g., Collis and Montgomery [9, 1998, pp.108-111]). Here is a simple biform model of Holdup. There are two players—call them the upstream firm  $U$  and the downstream firm  $D$ . If  $U$  makes an upfront investment of \$4, then  $U$  and  $D$  can transact and together create \$7 of (gross) value. Calculate added values: The overall pie is \$3,  $D$  creates \$0 on its own,  $U$  creates  $-\$4$  on its own.<sup>26</sup> The added value of  $D$  is thus \$7, the added value of  $U$  is \$3. In the Core, the \$3 pie will be divided so that  $D$  gets between \$0 and \$7, and  $U$  gets

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<sup>26</sup>As in earlier examples (see Appendix A), this is the correct treatment of  $U$ ’s upfront cost—viz., to subtract it from the value of any subset of players that includes  $U$ .

between  $-\$4$  and  $\$3$ . (Of course, these calculations simply reflect the fact that the game is a bilateral monopoly between  $U$  and  $D$ .) Will  $U$  make the efficient decision to invest? Not necessarily—which is the Holdup problem. (In our formalism, it will if and only if its confidence index is greater than  $4/7$ .)

Our analysis says that one way to understand the source of potential inefficiency in Holdup is as a failure of AU. (NE and NC are clearly satisfied.) Solutions to Holdup can be understood similarly. Vertical integration, bringing in a downstream competitor, etc. are ways of getting AU or closer to it. (For example, in its ideal form, vertical integration creates a one-player game, in which AU is automatically satisfied. A second downstream competitor would reduce the gap between the sum of the added values and the pie, ensuring the upstream firm more value.) But we are not suggesting any new solutions to Holdup, only pointing out that Holdup can be classified in our scheme.

Another area of corporate strategy where failures of AU come up is in horizontal relationships, e.g., in strategic alliances. Here, again, there may be inefficiency—usually, underinvestment—if a party isn’t assured its added value.

**b. Externalities** Typical corporate-strategy issues that fit in here are the problem of transferring knowledge across divisions of a corporation and managing a corporate brand. The externality is that one division could take an action to share its knowledge with another division that would increase the value created by the second division (together with its customers and suppliers). Or, it is that one division might make an investment in a common brand that is suitable for that division’s ‘image’ but unsuitable for another division’s image—i.e., lowers willingness-to-pay for the second division’s product, and so decreases the value created by that division, its customers, and its suppliers. In both cases, NE might fail, and inefficiency could result. Corporate-level knowledge and brand management are often discussed as appropriate strategies to overcome such potential inefficiencies.

**c. Coordination** Ideas around scale and scope would fit here. Consider two divisions where each has to decide which of two suppliers to source from. This could be analyzed as a (two-player) biform game, similar to Example 6.2, where the *No* and *Yes* labels correspond to choosing one or other supplier. The efficient profile of choices would be when both divisions choose the supplier corresponding to the *Yes* label. But both choosing the other supplier



(the *No* label) would also be a Nash equilibrium. We would again have a coordination game, and a failure of NC. The gains from coordination would come from the usual reasons of scale and scope—opportunities to share in defraying the fixed costs of the *Yes* supplier, to speed this supplier’s movement down the learning curve, etc. But with a failure of NC, there is no guarantee that the divisions, acting on their own, will make the efficient choice. Solutions to this kind of potential inefficiency are discussed in the literature under precisely the heading of coordination of the multibusiness firm (Collis-Montgomery [9, 1998, p.156]).

This is obviously not meant to be an exhaustive discussion of corporate strategy. Its purpose is simply to suggest that the efficiency framework of AU, NE, and NC may be a useful way to classify and unify some of the corporate-strategy literature. It does also suggest a ‘problem-driven’ approach to crafting corporate strategies: Try to spot inefficiencies in the organization (or across organizations) and then find ways of correcting them. Also, David Collis (private communication) has asked whether it might be possible to find a correspondence between the AU-NE-NC classification of inefficiencies and different organizational structures. Could it be argued that each type of inefficiency is best addressed via a particular organizational structure (hierarchy, incentive system, etc.)? We don’t know, but, again, perhaps this efficiency perspective on corporate strategy will prove fruitful.

## 8 Discussion of the Model

The goal of this paper was to combine the virtues of both branches of game theory—the noncooperative and the cooperative—into a hybrid model that could be used to analyze business strategies. We also made some connections to corporate strategy.

Here, we comment further on some conceptual aspects of the biform model we proposed.

**a. Emptiness of the Core** We’ve used the Core to analyze the various cooperative games that result from the players’ strategic choices. We did this because we wanted to capture the idea of free-form competition among the players. What if the Core is empty?

First, note that there are various important classes of TU cooperative games known to have nonempty Cores. Examples of such classes that are of economic interest include the “market games” and “assignment games” of Shapley and Shubik ([34, 1969], [35, 1971]). Of particular relevance to business-strategy applications, Stuart [37, 1997] proves nonemptiness

of the Core for a class of three-way assignment games (think of suppliers, firms, and buyers) that satisfy a local additivity condition.<sup>27</sup>

Next, we observe that a biform model can restore nonemptiness to at least one scenario that is known to produce an empty Core when modelled purely cooperatively. The point is essentially made by Example 2.2. Consider the following cooperative game: There are two firms, each with capacity of two units, and zero unit (marginal) cost. There are three buyers, each interested in one unit of product, and each with a willingness-to-pay of \$4. If a firm is ‘active,’ then it incurs a fixed cost of  $\$ \varepsilon$ , for some small  $\varepsilon > 0$ . (Formally, the value of any subset containing that firm and one or more buyers is reduced by  $\$ \varepsilon$ .) This game has an empty Core.<sup>28</sup> But there is a natural biform model of the scenario with nonempty Cores: Make paying the  $\$ \varepsilon$  cost a first-stage strategic choice for each firm, and write down the resulting second-stage cooperative games. (If a firm doesn’t pay  $\$ \varepsilon$  in the first stage, then it doesn’t increase the value of any subset at the second stage.) Very similar to Example 2.2, the Cores of the second-stage games will be singletons (in particular, nonempty!), and we’ll get an induced noncooperative game that is the Battle of the Sexes (Figure 8.1).

Arguably, finding an empty Core in a model is a ‘positive’ finding, telling us something important about the instability of the situation being studied. This seems true of the example of nonemptiness above, though we also showed a way to avoid emptiness. Interestingly, a kind of instability is still present in our biform resolution: Each firm will want to be the one to pay  $\$ \varepsilon$ , hoping the other won’t and thereby netting  $\$(8 - \varepsilon)$ . But both might end up losing overall.

Summing up, there are important classes of TU games that have nonempty Cores. The biform model may enable us to circumvent the emptiness of the Core in at least some other cases. (We just gave one example of this.) Finally, emptiness of the Core may be a valuable insight, in any case. This last possibility does raise the interesting question of how players, at the first-stage of a biform game, might evaluate a second-stage game with an empty Core. We don’t have an answer at present, and believe this question merits further study.

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<sup>27</sup>Example 2.1 is a three-way assignment game, and Examples 6.1 and 6.3 are two-way assignment games.

<sup>28</sup>Telser [40, 1994] contains other examples of empty Cores.

|        |                      |                      |                              |
|--------|----------------------|----------------------|------------------------------|
|        |                      | Firm 2               |                              |
|        |                      | Don't pay            | Pay $\$ \varepsilon$         |
| Firm 1 | Don't pay            | 0, 0                 | 0, $8 - \varepsilon$         |
|        | Pay $\$ \varepsilon$ | $8 - \varepsilon, 0$ | $-\varepsilon, -\varepsilon$ |

Figure 8.1

**b. Efficiency** An objection that is sometimes made of cooperative game theory is that it presumes efficiency. All potential value is created. This criticism is moot if the biform model is used. The biform model does incorporate what might be called conditional efficiency: Given a profile  $s \in S$  of strategic choices, use of the Core says that all of the value  $V(s)(N)$  that can then be created will, in fact, be created. But overall efficiency would require that the strategy profile  $s$  that the players actually choose maximize  $V(s)(N)$ , i.e., that the profile be efficient as defined in Section 6. We've seen several times in the paper that this need not be so. The biform model permits inefficiency.

**c. Externalities** Another long-standing issue in cooperative game theory is how to deal with situations where the value created by a subset  $A$  of players—call it  $v(A)$ —may depend on what players outside  $A$  do. Prima facie,  $v(A)$  does not depend on what the players outside  $A$  do, for the simple reason that a cooperative model has no concept of action or strategy in it. The approach in von Neumann and Morgenstern [41, 1944] was to start with a noncooperative game, which does have actions, of course, and to define an induced cooperative game from it. They then had to make a specific assumption about how the players outside  $A$  would behave. (They assumed minimax behavior.<sup>29</sup>) The biform model avoids this difficulty by simply positing that each strategy profile in the first-stage noncooperative game leads to a different, and independently specified, second-stage cooperative game. The quantity  $v(A)$  is then a function of the first-stage strategic choices of all the players—including, in particular, the choices of players outside  $A$ . But this dependence is without prejudice, so to speak.

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<sup>29</sup>Define  $v(A)$  to be the maximin payoff of  $A$  in the two-person zero-sum game in which the players are  $A$  and the complement of  $A$ , and the payoff to  $A$  from a pair of strategies is the sum of the payoffs to the players in  $A$  from those strategies. In effect, von Neumann-Morgenstern imagine that the players in  $A$  'assume the worst' about the players outside  $A$ .

There is no built-in assumption of minimax or any other kind of behavior.<sup>30</sup> Of course, the biform model also allows for the special case in which  $v(A)$  does not depend on what the players outside  $A$  do. This is precisely the situation of No Externalities, as in Definition 6.2.<sup>31</sup>

**d. Indeterminacy of the Core** The Core can be a single point (Examples 2.2, 6.1, 6.2, 6.3) or it can be indeterminate (Example 2.1). In the latter case, competition doesn't fully determine the division of value, and the Core reflects this. There is a residual bargaining problem—that depends presumably on ‘intangibles’ such as how skilled different players are at persuasion, bluffing, holding out, etc. (In Example 2.1, the supplier and firm 1 had to bargain over \$6 in the upper game of Figure 2.1, and \$2 in the lower game.) If one can make specific assumptions on how the residual bargaining proceeds, then it may be possible to reach a definite answer as to the division of value. But absent such specificity, the potential indeterminacy of the Core seems reasonable. Of course, we did assume that a player forms a definite view of a game's worth, even if the game involves residual bargaining. This was the role of the players' confidence indices, to be discussed next.

**e. Interpretation of the Confidence Index** The biform model makes no prediction about how the value created at the second stage will be divided, beyond saying it must be in accordance with the Core. It does say that players form views, as represented by their confidence indices, as to how much value they will get. We can think of these indices as giving the players' views on how good they think they are at dealing with the above ‘intangibles’ (persuasion, bluffing, holding out, etc.). Thus, returning again to Example 2.1, we noted (in Section 5) that the supplier would choose the branded-ingredient strategy if its confidence index  $\alpha < 3/4$ . A supplier who thought it could do very well ( $\alpha > 3/4$ ) in bargaining with firm 1 in the status-quo game would not spend the \$1 to play the branded-ingredient strategy.

The confidence indices are purely subjective. (They can be considered a representation

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<sup>30</sup>Zhao [42, 1992] and Ray and Vohra [32, 1997] have interesting alternative proposals for how to make what the players in subset  $A$  can achieve depend in a ‘neutral’ way on what the complementary players do. Unlike us, they derive cooperative games from noncooperative games. Also, their models are one-stage not two-stage.

<sup>31</sup>That definition was restricted to subsets  $A = N \setminus \{i\}$ , but generalizes in the obvious way to other subsets: Simply require that  $V(s)(A)$  be independent of any  $s^j$  for  $j \notin A$ .

of preference, as Appendix B shows.) The actual outcome of the game might be quite different from what a player anticipates. (In Example 2.1, the supplier might forego the branded-ingredient strategy, expecting to do very well in the bargaining with firm 1, and then even end up with less than \$5, the minimum it would get under the branded-ingredient strategy.)

We also stress that the confidence indices may or may not be mutually consistent, in the following sense. Fix a second-stage game. We assign to each player  $i$  an  $\alpha^i : (1 - \alpha^i)$  weighted average of the upper and lower endpoints of the corresponding Core projection. Of course, the resulting tuple of points may or may not itself lie in the Core. If it does for each second-stage game, we can say that the players' confidence indices are "mutually consistent." If not, they are "mutually inconsistent," and in at least one second-stage game, one or more players would definitely end up with an outcome different from that anticipated. We emphasize that there is no logical or conceptual difficulty with the inconsistent case, only a 'disagreement' among the players, which seems quite natural given the subjectivity of the confidence indices.<sup>32</sup>

**f. Epistemic Assumptions**<sup>33</sup> As in all game models, the question arises as to what we are assuming in this paper about the players' knowledge or beliefs about the model (including their knowledge or beliefs about one another's knowledge or beliefs, and so on). In particular, can we make the usual 'benchmark' assumption that the game in question is (informally) common knowledge among the players?

The answer is that the game can indeed be commonly known. True, the players' confidence indices might be mutually inconsistent (as above). But this is similar to a situation where the players in a game have different priors, and these priors are commonly known. (This is exactly what Harsanyi [14, 1967-8] called the "inconsistent" case.) There is no difficulty in such situations, just an 'agreement to disagree.' Or the game may not be commonly known; this is fine, too.

A related question is: How should we think about a Nash equilibrium of the game, given again that the  $\alpha^i$  are subjective, possibly even mutually inconsistent? The answer is that to

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<sup>32</sup>But there is also a natural question of whether mutual consistency is possible. See Appendix B.

<sup>33</sup>Pierpaolo Battigalli (private communication) posed the questions that we discuss in this section. We have borrowed freely from what he wrote to us.

get (pure-strategy) Nash equilibrium, it is sufficient that each player is rational and assigns probability one to the actual strategy choices of the other players.<sup>34</sup> So, in our set-up, a player doesn't actually need to be correct about the other players' confidence indices, just about the strategies they choose. There isn't any conceptual difficulty in talking about Nash equilibria.

We would like to give a formal treatment of the kinds of knowledge and belief ("epistemic") assumptions we are discussing here, but this isn't possible yet. Existing epistemic techniques apply only to noncooperative games, not to the hybrid noncooperative-cooperative structure of a biform game. We believe that these techniques can be adapted to the biform context, but this must wait for future work.

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<sup>34</sup>Aumann and Brandenburger [4, 1995, Preliminary Observation].

## Appendix A: Core Calculations

**Example 2.1.** It suffices to consider two buyers, so write the player set as  $N = \{s, f_1, f_2, b_1, b_2\}$ , where  $s$  is the supplier,  $f_1, f_2$  are the firms, and  $b_1, b_2$  are the buyers. Write the strategy set of the supplier as  $S = \{\sigma, \tau\}$ , where  $\sigma$  is the status-quo strategy and  $\tau$  is the branded-ingredient strategy. (We suppress the singleton strategy sets of the other players.)

We now build up the characteristic functions. Fix the indicator function  $\chi_{\{\tau\}}$  on  $S$  (i.e.  $\chi_{\{\tau\}}(\sigma) = 0$  and  $\chi_{\{\tau\}}(\tau) = 1$ ) and define, for  $\rho \in S$ ,

$$\begin{aligned} W(\rho)(\{s, f_1, b_1\}) &= W(\rho)(\{s, f_1, b_2\}) = 8, \\ W(\rho)(\{s, f_2, b_1\}) &= W(\rho)(\{s, f_2, b_2\}) = 2 + 4\chi_{\{\tau\}}(\rho). \end{aligned}$$

Next, for  $T \subseteq N$ , let  $\chi_T$  be the indicator function on  $N$ . Let  $M = \{\{s, f_1, b_1\}, \{s, f_1, b_2\}, \{s, f_2, b_1\}, \{s, f_2, b_2\}\}$ . Then for  $T \subseteq N$ , set

$$V(\rho)(T) = \begin{cases} -\chi_{\{\tau\}}(\rho)\chi_T(s) + \max_{\{m \in M : m \subseteq T\}} W(\rho)(m) & \text{if } \{m \in M : m \subseteq T\} \neq \emptyset, \\ -\chi_{\{\tau\}}(\rho)\chi_T(s) & \text{otherwise.} \end{cases}$$

Note that since  $8 > 2 + 4\chi_{\{\tau\}}(\rho)$  for all  $\rho$ , we have  $V(\rho)(N) = 8 - \chi_{\{\tau\}}(\rho)$ . Further,

$$\begin{aligned} V(\rho)(N) - V(\rho)(N \setminus \{s\}) &= 8 - \chi_{\{\tau\}}(\rho), \\ V(\rho)(N) - V(\rho)(N \setminus \{f_1\}) &= 6 - 4\chi_{\{\tau\}}(\rho), \\ V(\rho)(N) - V(\rho)(N \setminus \{f_2\}) &= 0, \\ V(\rho)(N) - V(\rho)(N \setminus \{b_1\}) &= 0, \\ V(\rho)(N) - V(\rho)(N \setminus \{b_2\}) &= 0. \end{aligned}$$

Write  $x(\rho) = (x^s(\rho), x^{f_1}(\rho), x^{f_2}(\rho), x^{b_1}(\rho), x^{b_2}(\rho))$  for a typical allocation, in the cooperative game corresponding to  $\rho$ . Consider the allocations  $x(\rho) = (z - \chi_{\{\tau\}}(\rho), 8 - z, 0, 0, 0)$ , where  $2 + 4\chi_{\{\tau\}}(\rho) \leq z \leq 8$ . Note that

$$x(\rho)(\{s, f_2, b_1\}) = x^s(\rho) \geq 2 + 3\chi_{\{\tau\}}(\rho) = V(\rho)(\{s, f_2, b_1\}).$$

It is then straightforward to show that these allocations satisfy  $x(\rho)(T) \geq V(\rho)(T)$  for all  $T \subseteq N$ , and also  $x(\rho)(N) = V(\rho)(N)$ . But player  $i$  cannot receive more than  $V(\rho)(N) -$

$V(\rho)(N \setminus \{i\})$  in the Core, so it follows that these are all the allocations in the Core, as required. ■

**Example 2.2.** Let  $N = \{f_1, f_2, b_1, b_2, b_3\}$ , where  $f_1, f_2$  are the firms and  $b_1, b_2, b_3$  are the buyers. Write the strategy sets of the firms as  $S^{f_1} = S^{f_2} = \{\sigma, \tau\}$ , where  $\sigma$  is the choice of the current product, and  $\tau$  is the choice of the new product. Set  $S = S^{f_1} \times S^{f_2}$ , with typical element  $\rho$ . (We suppress the singleton strategy sets of the buyers.)

Write  $\psi^{f_1}$  (resp.  $\psi^{f_2}$ ) for the indicator function  $\chi_{\{\tau\} \times S^{f_2}}$  (resp.  $\chi_{S^{f_1} \times \{\tau\}}$ ) on  $S$ . Also, for  $T \subseteq N$ , let  $r_T = \min\{2 \times |\{f_1, f_2\} \cap T|, |\{b_1, b_2, b_3\} \cap T|\}$ , where  $|X|$  denotes the cardinality of  $X$ . Then the characteristic functions are given by

$$V(\rho)(T) = 4r_T + 3 \min\{2[\psi^{f_1}(\rho)\chi_T(f_1) + \psi^{f_2}(\rho)\chi_T(f_2)], r_T\} - 5\psi^{f_1}(\rho)\chi_T(f_1) - 5\psi^{f_2}(\rho)\chi_T(f_2).$$

Note that

$$V(\rho)(N) = 12 + 3 \min\{2[\psi^{f_1}(\rho) + \psi^{f_2}(\rho)], 3\} - 5\psi^{f_1}(\rho) - 5\psi^{f_2}(\rho),$$

$$V(\rho)(\{f_1, b_1, b_2\}) = 8 + 6\psi^{f_1}(\rho) - 5\psi^{f_1}(\rho) = 8 + \psi^{f_1}(\rho), \quad (\text{A1})$$

$$V(\rho)(\{f_2, b_1, b_3\}) = 8 + \psi^{f_2}(\rho), \quad (\text{A2})$$

and, for  $i = b_1, b_2, b_3$ ,

$$V(\rho)(N \setminus \{i\}) = 8 + 6 \max\{\psi^{f_1}(\rho), \psi^{f_2}(\rho)\} - 5\psi^{f_1}(\rho) - 5\psi^{f_2}(\rho).$$

Also, if  $\rho \neq (\tau, \tau)$ , then

$$V(\rho)(N) = 12 + \psi^{f_1}(\rho) + \psi^{f_2}(\rho), \quad (\text{A3})$$

and, for  $i = b_1, b_2, b_3$ ,

$$V(\rho)(N) - V(\rho)(N \setminus \{i\}) = 4. \quad (\text{A4})$$

Consider the allocation  $x(\rho) = (\psi^{f_1}(\rho), \psi^{f_2}(\rho), 4, 4, 4)$ . It is straightforward to verify that  $x(\rho)(T) \geq V(\rho)(T)$  for all  $T \subseteq N$ . We now show that this is the only Core allocation. Adding A1 and A2 gives

$$\begin{aligned} x^{b_1}(\rho) + x(\rho)(N) &= x(\rho)(\{f_1, b_1, b_2\}) + x(\rho)(\{f_2, b_1, b_3\}) \geq \\ &V(\rho)(\{f_1, b_1, b_2\}) + V(\rho)(\{f_2, b_1, b_3\}) = 8 + \psi^{f_1}(\rho) + 8 + \psi^{f_2}(\rho) = 4 + V(\rho)(N), \end{aligned}$$



using A3. Thus  $x^{b_1}(\rho) \geq 4$ , so that  $x^{b_1}(\rho) = 4$ , using A4. A similar argument applies to  $x^{b_2}(\rho)$ . The condition  $x(\rho)(\{f_1, b_1, b_2\}) \geq V(\rho)(\{f_1, b_1, b_2\})$ , together with A1, then implies  $x^{f_1}(\rho) \geq \psi^{f_1}(\rho)$ . A similar argument yields  $x^{f_2}(\rho) \geq \psi^{f_2}(\rho)$ . Thus  $x^{f_1}(\rho) = \psi^{f_1}(\rho)$  and  $x^{f_2}(\rho) = \psi^{f_2}(\rho)$ , using A3 again.

The remaining case uses

$$V(\tau, \tau)(N) = 11, \quad (\text{A5})$$

and, for  $i = b_1, b_2, b_3$ ,

$$V(\tau, \tau)(N) - V(\tau, \tau)(N \setminus \{i\}) = 7. \quad (\text{A6})$$

Consider the allocation  $x(\tau, \tau) = (-5, -5, 7, 7, 7)$ . As before, it is straightforward to verify that  $x(\tau, \tau)(T) \geq V(\tau, \tau)(T)$  for all  $T \subseteq N$ . This is also the only Core allocation. Adding A1 and A2 gives

$$\begin{aligned} x^{b_1}(\tau, \tau) + x(\tau, \tau)(N) &= x(\tau, \tau)(\{f_1, b_1, b_2\}) + x(\tau, \tau)(\{f_2, b_1, b_3\}) \geq \\ &V(\tau, \tau)(\{f_1, b_1, b_2\}) + V(\tau, \tau)(\{f_2, b_1, b_3\}) = 18 = 7 + V(\tau, \tau)(N), \end{aligned}$$

using A5. Thus  $x^{b_1}(\tau, \tau) \geq 7$ , so that  $x^{b_1}(\tau, \tau) = 7$ , using A6. A similar argument applies to  $x^{b_2}(\tau, \tau)$ . The condition  $x(\tau, \tau)(\{f_1, b_1, b_2\}) \geq V(\tau, \tau)(\{f_1, b_1, b_2\})$ , together with A1, implies  $x^{f_1}(\tau, \tau) \geq -5$ . A similar argument yields  $x^{f_2}(\tau, \tau) \geq -5$ . Thus  $x^{f_1}(\tau, \tau) = -5$  and  $x^{f_2}(\tau, \tau) = -5$ , using A5 again. ■

**Example 6.1.** Let  $N = \{f_1, f_2, f_3, b_1, b_2\}$ , where  $f_1, f_2, f_3$  are the firms and  $b_1, b_2$  are the buyers. Write the strategy set of  $f_1$  as  $S = \{\sigma, \tau\}$ , where  $\sigma$  is the status-quo strategy and  $\tau$  is the negative-advertising strategy. (We suppress the singleton strategy sets of the other players.)

Fix the indicator function  $\chi_{\{\tau\}}$  on  $S$ . For  $T \subseteq N$ , let  $r_T = \min\{|\{f_1, f_2, f_3\} \cap T|, |\{b_1, b_2\} \cap T|\}$ . Then the characteristic functions are given by

$$V(\rho)(T) = \begin{cases} r_T(2 - \chi_{\{\tau\}}(\rho)) & \text{if } f_1 \notin T, \\ 2 + (r_T - 1)(2 - \chi_{\{\tau\}}(\rho)) & \text{if } f_1 \in T \text{ and } r_T \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Now

$$\begin{aligned} V(\rho)(N) &= 4 - \chi_{\{\tau\}}(\rho), \\ V(\rho)(N \setminus \{f_1\}) &= 4 - 2\chi_{\{\tau\}}(\rho), \end{aligned}$$

and, for  $i = f_2, f_3$ , and  $j = b_1, b_2$ ,

$$\begin{aligned} V(\rho)(N \setminus \{i\}) &= 4 - \chi_{\{\tau\}}(\rho), \\ V(\rho)(N \setminus \{j\}) &= 2. \end{aligned}$$

Thus

$$V(\rho)(N) - V(\rho)(N \setminus \{f_1\}) = \chi_{\{\tau\}}(\rho),$$

and, for  $i = f_2, f_3$ , and  $j = b_1, b_2$ ,

$$\begin{aligned} V(\rho)(N) - V(\rho)(N \setminus \{i\}) &= 0, \\ V(\rho)(N) - V(\rho)(N \setminus \{j\}) &= 2 - \chi_{\{\tau\}}(\rho), \end{aligned}$$

from which AU is satisfied. By Proposition 6.1, if the Core is nonempty, each player  $k \in N$  gets exactly  $x^k(\rho) = V(\rho)(N) - V(\rho)(N \setminus \{k\})$ . But it is straightforward to verify that  $x(\rho)(T) \geq V(\rho)(T)$  for all  $T \subseteq N$ . ■

**Example 6.2.** Let  $N = \{1, 2, 3\}$ , and let  $\rho$  be a strategy profile. AU is satisfied in each cooperative game. So, by Proposition 6.1, if the Core is nonempty, each player  $k \in N$  gets exactly  $x^k(\rho) = V(\rho)(N) - V(\rho)(N \setminus \{k\})$ . Now AU gives that for any  $i, j \in N$ , with  $i \neq j$ ,

$$V(\rho)(\{i, j\}) = [V(\rho)(N) - V(\rho)(N \setminus \{i\})] + [V(\rho)(N) - V(\rho)(N \setminus \{j\})].$$

It follows that  $x(\rho)(T) = V(\rho)(T)$  if  $|T| = 2$ . Also, since  $V(\rho)(\{k\}) = 0$ , we certainly have  $x(\rho)(T) > V(\rho)(T)$  if  $|T| = 1$ . Finally,  $x(\rho)(N) = V(\rho)(N)$ , by AU again. ■

**Example 6.3.** Let  $N = \{f_1, f_2, f_3, b_1, b_2\}$ , where  $f_1, f_2, f_3$  are the firms and  $b_1, b_2$  are the buyers. Write the strategy set of  $f_2$  as  $S = \{\sigma, \tau\}$ , where  $\sigma$  is the status-quo strategy and  $\tau$  is the repositioning strategy. (We suppress the singleton strategy sets of the other players.)

Fix the indicator function  $\chi_{\{\tau\}}$  on  $S$  and the indicator  $\chi_T$  on  $N$ . For  $T \subseteq N$ , let  $r_T = \min\{|\{f_1, f_2, f_3\} \cap T|, |\{b_1, b_2\} \cap T|\}$ . Then the characteristic functions are given by

$$V(\rho)(T) = \begin{cases} 7r_T & \text{if } f_2 \notin T, \\ 7r_T + \chi_{\{\tau\}}(\rho) & \text{if } f_2 \in T \text{ and } r_T \geq 1, \\ -\chi_{\{\tau\}}(\rho)\chi_T(f_2) & \text{otherwise.} \end{cases}$$

Now

$$\begin{aligned} V(\rho)(N) &= 14 + \chi_{\{\tau\}}(\rho), \\ V(\rho)(N \setminus \{f_2\}) &= 14, \end{aligned}$$

and, for  $i = f_1, f_3$ , and  $j = b_1, b_2$ ,

$$\begin{aligned} V(\rho)(N \setminus \{i\}) &= 14 + \chi_{\{\tau\}}(\rho), \\ V(\rho)(N \setminus \{j\}) &= 7 + \chi_{\{\tau\}}(\rho). \end{aligned}$$

Thus

$$V(\rho)(N) - V(\rho)(N \setminus \{f_2\}) = \chi_{\{\tau\}}(\rho),$$

and, for  $i = f_1, f_3$ , and  $j = b_1, b_2$ ,

$$\begin{aligned} V(\rho)(N) - V(\rho)(N \setminus \{i\}) &= 0, \\ V(\rho)(N) - V(\rho)(N \setminus \{j\}) &= 7, \end{aligned}$$

from which AU is satisfied. By Proposition 6.1, if the Core is nonempty, each player  $k \in N$  gets exactly  $x^k(\rho) = V(\rho)(N) - V(\rho)(N \setminus \{k\})$ . But it is straightforward to verify that  $x(\rho)(T) \geq V(\rho)(T)$  for all  $T \subseteq N$ . ■

## Appendix B: Axiomatization of the Confidence Index

Here we provide an axiomatic justification of the players' confidence indices. The axiomatization is closely related to early work by Hurwicz [16, 1951] and Milnor [25, 1954]. The main innovation may be in connecting this decision theory to cooperative game theory.

Let the choice set  $X$  consist of the closed bounded intervals of the real line, i.e.

$$X = \{[p, q] : p, q \in \mathbb{R} \text{ with } p \leq q\},$$

and let  $\succsim$  be a preference relation on  $X$ .

(For our application, fix a player  $i$ . The intervals are then the projections onto the  $i$ th coordinate axis of the Cores of cooperative games. The assumption is that player  $i$  evaluates these intervals according to the preference relation  $\succsim$ .)

Consider the following axioms on  $\succsim$ :

A1 (Order): The relation  $\succsim$  is complete and transitive.

A2 (Dominance): If  $p > s$ , then  $[p, q] \succ [r, s]$ .

A3 (Continuity): If  $[p_m, q_m] \succ [r_m, s_m]$  for all  $m$ , where  $[p_m, q_m] \rightarrow [p, q]$  and  $[r_m, s_m] \rightarrow [r, s]$ , then  $[p, q] \succsim [r, s]$ .

A4 (Positive affinity): If  $[p, q] \succ [r, s]$ , then  $[\lambda p + \mu, \lambda q + \mu] \succ [\lambda r + \mu, \lambda s + \mu]$  for any strictly positive number  $\lambda$  and any number  $\mu$ .

**Proposition B1** *A preference relation  $\succsim$  on  $X$  satisfies Axioms A1 through A4 if and only if there is a number  $\alpha$ , with  $0 \leq \alpha \leq 1$ , such that*

$$[p, q] \succsim [r, s] \text{ if and only if } \alpha q + (1 - \alpha)p \geq \alpha s + (1 - \alpha)r.$$

*Furthermore, the number  $\alpha$  is unique.*

**Proof.** Sufficiency and uniqueness are readily checked, so let us establish necessity.

Step 0: Let

$$A = \{\alpha' : \alpha' \in [0, 1] \text{ and } [\alpha', \alpha'] \succsim [0, 1]\}.$$

The set  $A$  is well-defined due to Order.

Step 1: The set  $A$  contains the point 0, and so is nonempty. To see this, note that Dominance implies that  $[0 - 1/n, 0 - 1/n] \prec [0, 1]$  for every integer  $n$ . Thus  $[0, 0] \precsim [0, 1]$  by Continuity.

Step 2: Set  $\alpha = \sup A$ . Then  $\alpha \in A$ . To prove this, it suffices to show that  $\alpha \leq 1$ . First, note that Dominance implies that  $[1 + 1/n, 1 + 1/n] \succ [0, 1]$  for all  $n$ . Thus by Continuity,

$$[1, 1] \succsim [0, 1]. \quad (\text{B1})$$

Second, note that by definition of  $\alpha$ ,

$$[\alpha - 1/n, \alpha - 1/n] \precsim [0, 1]. \quad (\text{B2})$$

Now suppose  $\alpha > 1$ . Then there is an  $n^*$  such that  $\alpha - 1/n > 1$  for  $n > n^*$ . Hence by Dominance,

$$[\alpha - 1/n, \alpha - 1/n] \succ [1, 1]. \quad (\text{B3})$$

Combining equations (B1), (B2), and (B3), and using Order, yields

$$[\alpha - 1/n, \alpha - 1/n] \succ [1, 1] \succsim [0, 1] \precsim [\alpha - 1/n, \alpha - 1/n]$$

for  $n > n^*$ , a contradiction. Thus  $\alpha \leq 1$ , as was to be shown.

Step 3: The number  $\alpha$  satisfies  $[\alpha, \alpha] \sim [0, 1]$ . First suppose that  $\alpha = 1$ . Then  $[1, 1] \precsim [0, 1]$  since  $\alpha \in A$ . Using equation (B1) and Order gives  $[1, 1] \precsim [0, 1] \precsim [1, 1]$ , from which  $[\alpha, \alpha] \sim [0, 1]$ . Next suppose that  $\alpha < 1$ . Note that  $[\alpha, \alpha] \precsim [0, 1]$  since  $\alpha \in A$ . Suppose, contra hypothesis, that  $[\alpha, \alpha] \prec [0, 1]$ . By the definition of  $\alpha$ , it must be that  $[\alpha + 1/n, \alpha + 1/n] \succ [0, 1]$  for all  $n$ . Using Continuity and Order then yields  $[\alpha, \alpha] \succsim [0, 1] \succ [\alpha, \alpha]$ , a contradiction.

Step 4: Using Positive Affinity,

$$[\alpha(q - p) + p, \alpha(q - p) + p] \sim [0(q - p) + p, 1(q - p) + p] = [p, q],$$

or

$$[p, q] \sim [\alpha q + (1 - \alpha)p, \alpha q + (1 - \alpha)p],$$

as required. ■

Some comments follow:

**i. Discussion of the Axioms** Axioms A1 through A3 are standard, and don't require an independent justification in the present context. Axiom A4 is crucial and accounts for the specific form that the representation of preferences takes. In fact, Axiom A4 is immediately implied by the context. Consider two cooperative games  $\Gamma_1$  and  $\Gamma_2$ . Fix a player  $i$ , and numbers  $\lambda > 0$  and  $\mu$ . Let  $\Gamma_3$  be derived from  $\Gamma_1$  by multiplying the value of every coalition in  $\Gamma_1$  by  $\lambda$  and, if the coalition contains player  $i$ , also adding  $\mu$ . (If you like, we change the 'currency' in which the game is played and give player  $i$  some money from outside the game.) Let  $\Gamma_4$  be derived from  $\Gamma_2$  in similar fashion. In cooperative game theory, the games  $\Gamma_1$  and  $\Gamma_3$  are considered strategically equivalent; likewise for the games  $\Gamma_2$  and  $\Gamma_4$ .<sup>35</sup> Now let player  $i$ 's Core projection in  $\Gamma_1$  be  $[p, q]$ , and that in  $\Gamma_2$  be  $[r, s]$ . Then player  $i$ 's Core projection in  $\Gamma_3$  will be  $[\lambda p + \mu, \lambda q + \mu]$ , and that in  $\Gamma_4$  will be  $[\lambda r + \mu, \lambda s + \mu]$ . If player  $i$  prefers the first interval to the second, then, using strategic equivalence, player  $i$  should prefer the third to the fourth. This is precisely Axiom A4.

**ii. Relationship to the Hurwicz Index** At a formal level, our axiomatization of the confidence index is closely related to Milnor's derivation of the Hurwicz optimism-pessimism index. (See Milnor [25, 1954]; also Hurwicz [16, 1951], Arrow [2, 1953].<sup>36</sup>) The contexts are, however, rather different. Milnor was concerned only with one-person decision problems and adopted a states-consequences formulation. Our context is multi-person and has no states; instead, there are just intervals of possible (monetary) consequences.

Luce and Raiffa [20, 1957, pp.282-298] list various criticisms of the Hurwicz decision criterion. On examination, however, it turns out that these criticisms have force only to the extent that the decision maker faces a problem with well-defined states. In our present, state-free context, they lose their bite. For example, the Hurwicz criterion cannot be made to satisfy admissibility without, at the same time, losing continuity (Milnor [25, 1954, p.55]).

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<sup>35</sup>See Owen [28, 1995, pp.215-216], where the (general) concept is called  $S$ -equivalence. Theorem X.3.4 there establishes that if two games are  $S$ -equivalent, then there is an isomorphism between their imputation sets that preserves the domination relation. This is the basis for treating the two games as equivalent.

<sup>36</sup>Ghirardato [12, 2001] obtains a similar representation. He starts with a state-space formulation, but then coarsens the decision maker's perception of the state space. Nehring and Puppe [26, 1996] present conditions under which preferences over sets depend on the maximal and minimal elements of the sets. The treatment in this paper is more specific than theirs in two ways. The choice set consists of closed bounded intervals of the real line, and the representation involves a convex combination of the maximal and minimal elements.

But in our set-up, admissibility and continuity do not conflict. To see this, consider the following extra axiom and proposition.

A5 (Admissibility): If  $p > r$ , then  $[p, q] \succ [r, q]$ ; if  $q > r$ , then  $[p, q] \succ [p, r]$ .

**Proposition B2** *A preference relation  $\succsim$  on  $X$  satisfies Axioms A1 through A5 if and only if there is a number  $\alpha$ , with  $0 < \alpha < 1$ , such that*

$$[p, q] \succsim [r, s] \text{ if and only if } \alpha q + (1 - \alpha)p \geq s + (1 - \alpha)r.$$

*Furthermore, the number  $\alpha$  is unique.*

**Proof.** Again, sufficiency and uniqueness are immediate, so we establish necessity. Using Proposition B.1, we have only to show that  $0 < \alpha < 1$ . We have  $[\alpha, \alpha] \sim [0, 1]$ . Admissibility implies  $[1, 1] \succ [0, 1]$  and  $[0, 1] \succ [0, 0]$ . Using Order, we find  $[1, 1] \succ [\alpha, \alpha] \succ [0, 0]$ . Dominance yields  $0 < \alpha < 1$ . ■

**iii. Application to Biform Games** To apply our axiomatization to a biform game, we have to decide whether to think of a player  $i$  as having one preference relation over all of the second-stage games, or a potentially different relation for each first-stage strategy profile  $s \in S$ . In the definition in the text (Definition 5.1), we assumed the former, to keep things simple. (The confidence indices played a relatively small role in the body of the paper. Except in Example 2.1, the Core was a singleton. Of course, in other applications, they could play a larger role.) In a more general case, a player might have different confidence indices for different strategy profiles  $s \in S$ .

This distinction is relevant to the issue of mutual consistency of the indices, discussed in Section 8e. Consider the biform game in Figure B1. There are three players, each with two strategies *No* and *Yes*. Player 1 chooses the row, player 2 the column, and player 3 the matrix. Figure B1 depicts the cooperative game associated with each strategy profile, where  $w > 0$  and the values of all subsets not shown are 0. First suppose that each player has a single confidence index for all the second-stage games—denote these  $\alpha^1$ ,  $\alpha^2$ , and  $\alpha^3$ . Then, considering the second-stage games following (*Yes*, *Yes*, *No*), (*No*, *Yes*, *Yes*), (*Yes*,

*No, Yes*), and (*Yes, Yes, Yes*) respectively, mutual consistency requires

$$\begin{aligned}\alpha^1 + \alpha^2 &= 1, \\ \alpha^2 + \alpha^3 &= 1, \\ \alpha^3 + \alpha^1 &= 1, \\ \alpha^1 + \alpha^2 + \alpha^3 &= 1,\end{aligned}$$

a contradiction. On the other hand, if we allow a player different confidence indices for different second-stage games, then mutual consistency can always be satisfied: For each second-stage game, take an arbitrary point in the Core, project it onto the players' axes, and treat each projected point as a weighted average of the upper and lower endpoints of the projection of the whole Core onto that axis.

We repeat what we said in Section 8e—that we don't see mutual consistency as conceptually necessary. But if it is wanted, we have now shown how it can be guaranteed.

|     |    |                                |     |                                |                                |
|-----|----|--------------------------------|-----|--------------------------------|--------------------------------|
|     | No | Yes                            |     | No                             | Yes                            |
| No  | —  | —                              | No  | —                              | $v(2, 3) =$<br>$v(N) =$<br>$w$ |
| Yes | —  | $v(1, 2) =$<br>$v(N) =$<br>$w$ | Yes | $v(1, 3) =$<br>$v(N) =$<br>$w$ | $v(N) = w$                     |
|     | No |                                |     | Yes                            |                                |

Figure B1



## Appendix C: Additional Comments on the Efficiency Results

**i. Efficiency and Rationality** We start by establishing a further implication of the AU, NE, and NC conditions.<sup>37</sup>

**Proposition C1** *Consider a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  satisfying AU, NE, and NC, and that for each  $s \in S$ , the game  $V(s)$  has a nonempty Core. Then each player has a (strongly) dominant strategy.*

**Proof.** Fix a player  $i$  and two strategies  $r^i, s^i \in S^i$ . Suppose that  $V(r^i, s^{-i})(N) > V(s^i, s^{-i})(N)$  for some  $s^{-i} \in S^{-i}$ . Then NC implies  $V(r^i, r^{-i})(N) > V(s^i, r^{-i})(N)$  for all  $r^{-i} \in S^{-i}$ . Applying NE yields

$$V(r^i, r^{-i})(N) - V(r^i, r^{-i})(N \setminus \{i\}) > V(s^i, r^{-i})(N) - V(s^i, r^{-i})(N \setminus \{i\})$$

for all  $r^{-i} \in S^{-i}$ . By AU, strategy  $r^i$  strongly dominates strategy  $s^i$ .

The remaining case is that  $V(r^i, s^{-i})(N) = V(s^i, s^{-i})(N)$  for all  $s^{-i} \in S^{-i}$ . But then NE yields

$$V(r^i, s^{-i})(N) - V(r^i, s^{-i})(N \setminus \{i\}) = V(s^i, s^{-i})(N) - V(s^i, s^{-i})(N \setminus \{i\})$$

for all  $s^{-i} \in S^{-i}$ . By AU, strategy  $r^i$  is payoff-equivalent to strategy  $s^i$  for player  $i$ . Finiteness of the strategy sets then implies that each player has a strongly dominant strategy (up to repetition of payoff-equivalent strategies). ■

Since a profile of dominant strategies is a Nash equilibrium, we know by Proposition 6.2 that it is efficient. Indeed, we can put Propositions 6.2, 6.3, and C1 together in the following way. In a noncooperative game, a player is **rational** if he chooses a strategy that maximizes his expected payoff, under some probability measure on (the product of) the strategy sets of the other players. In a biform game, each player still associates a single number with each strategy profile (equal to a weighted average of the Core projection), and so we can define the rational strategic choices of a player correspondingly. With this terminology, we have the following equivalence statement:

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<sup>37</sup>John Sutton prompted us to prove this result.

Assume AU, NE, and NC (and nonemptiness of the Core for each strategy profile). Then, a profile consists of rational strategies if and only if the profile is efficient.

The proof is immediate. By Proposition 7.1, under AU, NE, and NC, each player has a strongly dominant strategy (or strategies). This is then the only rational strategy (or strategies) for that player, and we just said that a profile of dominant strategies is efficient. Conversely, Proposition 6.3 says that under AU and NE (even without NC), an efficient profile is a Nash equilibrium. And a Nash-equilibrium strategy is rational—when the player assigns probability one to the other players’ strategies that make up the equilibrium.

Viewed this way, our results provide an answer to the question: “When is rationality equivalent to efficiency?” When are these two basic concepts, one from game theory and the other from welfare theory, equivalent? Our answer is that a sufficient condition for this is that AU, NE, and NC hold.

**ii. Weak No Coordination** Next, we note a possible weakening of the NC condition, and a corresponding counterpart to Proposition C1 (the proof mirrors the one above).

**Definition C1** A biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  is said to satisfy **Weak No Coordination (WNC)** if for each  $i = 1, \dots, n$ ;  $r^i, s^i \in S^i$ ; and  $r^{-i}, s^{-i} \in S^{-i}$ ,

$$V(r^i, r^{-i})(N) \geq V(s^i, r^{-i})(N) \text{ if and only if } V(r^i, s^{-i})(N) \geq V(s^i, s^{-i})(N).$$

**Proposition C2** Consider a biform game  $(S^1, \dots, S^n; V; \alpha^1, \dots, \alpha^n)$  satisfying AU, NE, and WNC, and that for each  $s \in S$ , the game  $V(s)$  has a nonempty Core. Then each player has a weakly dominant strategy.

**iii. Lack of Necessity Continued.** In Section 6, we already mentioned the lack of necessity of the conditions of Propositions 6.2 and 6.3. To see that the lack of necessity in Proposition C1, consider the Prisoner’s Dilemma (Figure C1), viewed as a biform game satisfying AU. (To see the game this way, note that AU in a two-player game implies that the game is inessential, so that the unique Core allocation gives the players their individually rational payoffs. These are the payoffs shown.) Here, NC holds but NE fails. Yet each player has a dominant strategy.

|      |      |
|------|------|
| 3, 3 | 0, 4 |
| 4, 0 | 1, 1 |

Figure C1

To see the lack of necessity in the equivalence statement in i. above, consider Matching Pennies (Figure C2), viewed as a biform game satisfying AU. Here, as in Figure C1, NC holds but NE fails. Yet each strategy of either player is rational and all profiles are efficient.

|       |       |
|-------|-------|
| 1, -1 | -1, 1 |
| -1, 1 | 1, -1 |

Figure C2

**iv. Relationship to Makowski-Ostroy Continued.** First, we explain the difference mentioned in Section 6 between our definition of added value (marginal contribution) and the definition in Makowski-Ostroy ([23, 1994], [24, 1995]). We talk about player  $i$ 's marginal contribution (defined in the standard game-theoretic way) in each of the cooperative games  $V(s)$  induced by different first-stage strategy profiles  $s \in S$ . Let us now cast the M-O definition in the biform model (recognizing that their model is different). M-O would select a particular strategy  $s_0^i \in S^i$  of player  $i$ , to be thought of as the strategy of “not participating in the game.” They would then say that player  $i$  receives his marginal contribution if his payoff from choosing any strategy  $s^i$ , when the other players choose strategies  $s^{-i}$ , is equal to  $V(s)(N) - V(s_0^i, s^{-i})(N \setminus \{i\})$ . That is, player  $i$  receives the difference between the value created when  $i$  is “in the game” (and playing  $s^i$ ) and the value created by the remaining players when  $i$  is “out of the game” (playing  $s_0^i$ ). By contrast, we say that  $i$  receives his marginal contribution if the payoff is  $V(s)(N) - V(s)(N \setminus \{i\})$ . Note that if NE is assumed, then  $V(s)(N \setminus \{i\}) = V(s_0^i, s^{-i})(N \setminus \{i\})$ , so that the two definitions then coincide. Thus, seen from the perspective of our formalism, M-O effectively assume NE from the beginning,

while we distinguish games that do or do not satisfy NE.<sup>38</sup>

Another difference between M-O and us is in the treatment of coordination. M-O use a condition (“No Complementarities”) which, in the formalism of the biform model, says: For each  $r, s \in S$ ,

$$V(s)(N) - V(r)(N) \leq \sum_{i=1}^n [V(s^i, r^{-i})(N) - V(r)(N)] .$$

In words, the change in the value of the game when the players switch from strategy profile  $r$  to strategy profile  $s$  is no more than the sum of the changes caused by the players’ switching one at a time. It can be shown that this condition holds if and only if we have: For each  $i = 1, \dots, n$ ;  $r^i, s^i \in S^i$ ; and  $r^{-i}, s^{-i} \in S^{-i}$ ,

$$V(r^i, r^{-i})(N) - V(s^i, r^{-i})(N) = V(r^i, s^{-i})(N) - V(s^i, s^{-i})(N).$$

By standard arguments, this latter condition holds if and only if the value of the game is additively separable as a function of the players’ strategy choices. Our No Coordination condition (Definition 6.2) is weaker than the M-O condition, and, in particular, does not impose additive separability.

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<sup>38</sup>As a consequence, M-O would classify some examples differently from us. Thus, we attributed the inefficiency in Example 6.1 to a failure of NE. M-O would attribute it to firm 1’s receiving \$1 from negative advertising, different from the marginal contribution of advertising, which M-O would calculate as  $-\$1$ . (Take the strategy of “not participating in the game” for firm 1 to be what we called the “status-quo” strategy.) We are grateful to Louis Makowski for much help on the difference between our formalisms.

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