# Commitment versus Flexibility with Costly Verification

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A principal faces an agent who is better informed but biased toward higher actions. She can verify the agent's information and specify his permissible actions. We show that if the verification cost is small enough, a threshold with an escape clause (TEC) is optimal: the agent either chooses an action below a threshold or requests verification and the efficient action above the threshold. For higher costs, however, the principal may require verification only for intermediate actions, dividing the delegation set. TEC is always optimal if the principal cannot commit to inefficient allocations following the verification decision and result.

# I. Introduction

Organizations establish capital budgeting procedures to ensure the right allocation of capital to projects. Budgeting decisions are complicated by

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the presence of information costs and misaligned incentives. In a survey of manufacturing firms, Ross (1986) observes that top corporate management is often too busy and preoccupied with other responsibilities to have the time and resources to evaluate every investment opportunity. At the same time, lower-level division heads who possess the required information for decision-making are prone to overstate their investment opportunities and spend excessively (e.g., Brealey and Myers 1981; Donaldson 1984).<sup>1</sup> In fact, the combination of these factors is seen as a main reason why top management often imposes capital rationing on the firm's divisions.<sup>2</sup>

Analogous problems of information and incentives arise in other applications. In the operation of fiscal policy, citizens do not have the capacity to evaluate every budget allocation that must be made, whereas elected officials who know the value of outlays are biased toward overspending because of political interests (e.g., Aguiar and Amador 2011; Halac and Yared 2014, 2019a). In the context of international trade, the World Trade Organization cannot assess the appropriateness of every tariff response to dumping, whereas the governments that understand the true circumstances are biased toward imposing high tariffs to protect their domestic industries (e.g., Amador and Bagwell 2013; Beshkar and Bond 2017).<sup>3</sup>

Starting with the work of Holmström (1977, 1984), decision-making in these environments has been formally analyzed as a delegation problem. The canonical setting consists of a principal who faces a better informed but biased agent and cannot rely on transfers.<sup>4</sup> Importantly, it is assumed that the principal's cost of verifying the agent's information is prohibitively high, so all the principal can do is specify a set of allowable actions from which the agent can select. The optimal delegation set is shaped by a fundamental trade-off between commitment and flexibility: a narrow set limits biased decisions by the agent, whereas a wide set lets the agent utilize his private information about the efficient action. A main insight from the literature is that under weak conditions, this trade-off is optimally resolved by threshold delegation. That is, the prescription is to

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<sup>&</sup>lt;sup>1</sup> Using survey data, Pruitt and Gitman (1987) find that senior managers are aware that junior managers overstate estimated project revenues. They also find that this overstatement is to a large extent considered intentional.

<sup>&</sup>lt;sup>2</sup> See Mukherjee and Rahahleh (2011). As the authors describe, a large literature finds evidence that firms operate under capital constraints and, moreover, that these constraints are imposed internally by senior managers rather than externally by the suppliers of funds.

<sup>&</sup>lt;sup>3</sup> As another example, in a retail setting, it is too costly for managers to scrutinize the best sales strategy for every client, whereas the sales representatives who are able to do so generally offer too many discounts (e.g., Lo et al. 2016).

<sup>&</sup>lt;sup>4</sup> In various applications, like those described above, (contingent) transfers may be ruled out because of institutional reasons or ethical considerations. In their study of capital budgeting practices, Mukherjee and Rahahleh (2011) report that directly rewarding employees for proposing good investments is uncommon in large firms.

set budget caps for managers in organizations, deficit limits in the context of fiscal policy, and tariff caps as part of trade agreements.

A key limitation of existing analyses of delegation concerns the use of verification. In practice, while it is costly for principals to verify agents' information, this cost is not as high as to rule out verification altogether. Realworld delegation rules typically feature a cap on allowable actions, as in the canonical model, together with review and approval procedures for requests that exceed the cap. In organizations, "[s]maller projects can typically be approved by division heads, and thus, within the budget limits, decision-making for these projects is completely decentralized. . . . Larger projects, by contrast, must be approved by a central investment committee or even the board of directors" (Taggart 1987, 18).<sup>5</sup> Because the committee in charge of approval must spend costly time gathering information, evaluating cash flow projections, and deliberating on the appropriate investment, only certain projects are verified: "Given their priorities, top management often copes with productivity improvement by allocating small fixed sums to divisions and plants. That leaves them the time to carefully analyze the large projects" (Ross 1986, 21). Similar procedures are used in the context of fiscal and trade policy, where rules specify deficit or tariff caps together with escape clause and dispute settlement provisions for breaching these caps under verified special conditions.<sup>6</sup>

Motivated by these applications, we study a general delegation framework in which verification is costly but feasible. We model costly verification as in the seminal work of Townsend (1979) and explore how it affects optimal delegation. How does the principal choose the delegation set to optimally resolve the trade-off between commitment and flexibility while at the same time minimizing verification costs? We find that optimal rules can take complicated forms, as verification effectively allows the principal to relax incentive constraints by dividing the delegation set into subsets. Yet we show that under certain conditions, an optimal rule takes the simple form of a *threshold with an escape clause (TEC)*. We define TEC as a rule in which the agent either selects an action below a threshold or requests verification and the efficient action above the threshold by triggering the escape clause. As noted above, rules of this form are commonly observed in applications. Our paper provides a theoretical foundation for the broad use of TEC and shows how its optimality depends on the principal's cost of verification and her commitment power.

Our model features an agent who is biased toward higher spending relative to the principal. The agent's private information, or *type*, concerns

<sup>&</sup>lt;sup>5</sup> See also Bower and Lesard (1973), Ross (1986), Mukherjee and Henderson (1987), Gitman and Vandenberg (2000), and Grinstein and Tolkowsky (2004), among others.

<sup>&</sup>lt;sup>6</sup> See, e.g., Schaechter et al. (2012), Lledó et al. (2017), and Coate and Milton (2019) on fiscal rules and Beshkar and Bond (2017) on trade agreements.

the value of spending; a higher type corresponds to a higher marginal value of spending for both the principal and the agent. Following the literature, we build upon a setting in which, absent verification, an optimal delegation rule would be a threshold, allowing the agent to choose any spending up to a maximum level. We depart from prior work by letting the principal verify and perfectly learn the agent's type. Verification entails an additive cost for the principal, which may also be partially born by the agent.

We begin our analysis by assuming that the principal can fully commit to a delegation rule. The problem can be viewed in three steps: first, the principal chooses a mapping from the agent's verification decision and result to a set of allowable spending; second, the agent decides whether to seek verification; third, the agent chooses a spending level from the allowable set. Formally, a delegation rule is a pair of schedules specifying, for each agent type, whether he is verified and his spending level. A delegation rule is optimal if it maximizes the principal's expected welfare subject to the incentive compatibility constraint that each agent type prefer his verification assignment and spending level to those of any other type.<sup>7</sup> In particular, each type must prefer his allocation to that of any other type who is not prescribed verification. Deviations to types who are verified can be trivially deterred as they get revealed by the principal's verification. As an implication, the use of verification can make local incentive compatibility constraints slack while nonlocal constraints bind; our analysis makes use of perturbation methods to address these issues.

Our first main result shows that TEC is optimal if the cost of verification is sufficiently small. Importantly, we also show that verifying all agent types is never optimal; hence, no matter how small the verification cost is, an optimal rule prescribes no verification for some types. The intuition why TEC is optimal is that verifying an upper region of agent types not only allows the principal to improve their spending allocation but also is an efficient means of imposing discipline on lower agent types who are not verified; these types select from a set of lower spending levels and cannot mimic higher types who are verified. To prove the result, we show that any rule with decreasing verification—prescribing verification for a set of agent types and no verification for a set of higher types—can be dominated. Decreasing verification is expensive for the principal because it requires incentivizing types in the verification region to seek verification rather than mimic a higher type in a no-verification region above them, and this in turn requires inducing significant overspending in the no-verification region. We show that when the verification cost is small

<sup>&</sup>lt;sup>7</sup> We restrict attention to deterministic rules (see sec. VI for a discussion) and prove that a revelation principle in terms of payoffs holds in our setting.

enough, a perturbation that verifies all types in the decreasing verification region increases the principal's welfare.

TEC, however, may not be optimal if the verification cost is relatively larger. Our second main result shows that decreasing verification can be strictly optimal in this case. For example, a rule that verifies only an intermediate set of types can yield the principal higher welfare relative to not verifying any type as well as relative to using a TEC rule. The main reason why verifying only intermediate types can dominate not verifying any type is that the verification region serves to discipline types in the noverification region below. The main reason why verifying only intermediate types can dominate TEC is that it allows the principal to save on verification costs. We show that these benefits can outweigh the cost of overspending that is needed to incentivize intermediate types to be verified. Thus, a rule that involves decreasing verification can be optimal when the verification cost is not small (and not large) enough.

The optimality of decreasing verification does not rely on any sort of asymmetry in the payoff or distribution functions. As noted, our setting is one in which threshold rules are always optimal absent verification, and in fact we prove the result by taking the widely studied case of quadratic preferences and a uniform distribution of types. An interior verification region can be beneficial because it allows the principal to divide the delegation set while keeping verification at a minimum. Observe that the agent wants to overspend relative to the principal but his preferred spending level depends on his type. Consequently, requiring verification for intermediate spending levels may suffice to limit the spending of relatively low types: these types are unable to justify increasing their spending to an intermediate level via verification, and increasing their spending further would not be attractive to them.

The above result raises the question of why rules with decreasing verification are rarely observed in reality. We provide an answer based on a practical consideration: implementing such a rule requires strong commitment power from the principal, stronger than what may be feasible in applications. Take the aforementioned rule in which the principal verifies only an intermediate set of types. Under this rule, the principal commits to an allocation that may be inefficient ex post, following the verification decision and result. In particular, the rule may assign an inefficient spending level after the agent's type is verified both in the case that the agent's seeking verification is on path as well as when this verification is part of a deviation. Moreover, the rule may induce an allocation after the agent decides not to seek verification that is inefficient conditional on no verification, that is, when ignoring the incentives of verified types.

What happens if the principal is unable to commit ex ante to these ex post inefficient allocations? In organizations, for example, even if top management specifies certain budgets and requirements ex ante, it is common for these to be changed ex post. Ross (1986) documents that in firms whose budgeting procedures resemble TEC, budget approvals do not conform to preannounced criteria but depend on the discretion of top management.<sup>8</sup> Investment committees decide the scope of projects that are brought up for verification and approval as well as the budget cap for projects that have not been submitted for review. In related work on chief executive officers and corporate boards, Grinstein and Tolkowsky (2004) find that corporate boards exert significant discretion in reviewing and approving annual budgets and large capital requests made by the chief executive officer.

Our third main result shows that if the principal's commitment power is limited, then TEC is optimal whenever verification is optimal. In terms of the three-step timing described previously, limited commitment power means that the principal now revises the agent's allowable spending set following the agent's verification decision and result. We prove that in this case, any incentive-compatible rule must have weakly increasing verification everywhere. The reason is that inducing decreasing verification requires incentivizing verified types not to deviate and choose a higher spending level in a no-verification region above them, and under limited commitment power it also requires incentivizing nonverified types not to seek a verification that guarantees them efficient spending. When unable to fully commit to a rule ex ante, the principal cannot implement the spending levels that would be needed to make these deviations unattractive, and thus decreasing verification is not feasible. Consequently, we obtain that under limited commitment power, any optimal rule featuring verification must be TEC.

Altogether, our results provide a theoretical justification for the prevalence of TEC in the real world. When verification costs are small enough, even a principal who can commit to any class of delegation rule will find it optimal to choose one with the simple form of TEC. When verification costs are larger, more complex rules may perform better, but TEC remains the principal's optimal rule if her commitment power is limited. An implication is that limitations to commitment power, as we have considered, appear to be prevalent in applications and an important reason behind the broad use of TEC rules.

*Related literature.*—Our paper is related to several literatures. First, we contribute to the literature on optimal delegation and self-control, starting with Holmström (1977, 1984). Main references include Melumad and Shibano (1991) and Alonso and Matouschek (2008) on delegation

<sup>&</sup>lt;sup>8</sup> See also Bower and Lesard (1973) and Taggart (1987, 18). The latter notes that "[i]f a division's worthwhile projects exceed its budget, top management may be wiling to renegotiate." Additionally, Mukherjee and Henderson (1987) observe that firms' investment criteria are sometimes unclear, as they depend on the class of project. This gives top management more discretion to make decisions ex post.

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under quadratic preferences; Amador, Werning, and Angeletos (2006) on consumption-savings problems with hyperbolic preferences; and Amador and Bagwell (2013), which considers a general framework that we take as our baseline.<sup>9</sup> As in this literature, we study a principal-agent environment with no transfers in which the agent is better informed about the efficient action but biased relative to the principal. In contrast to this literature, we allow the principal to verify the agent's information at a cost. By introducing this additional tool, we are able to explore how escape clauses are optimally used and how optimal delegation depends on the extent of the principal's commitment power.<sup>10</sup>

Second, we contribute to the literature on costly verification, starting with Townsend (1979). Both that paper and others that followed it-including Gale and Hellwig (1985), Border and Sobel (1987), and Mookherjee and Png (1989)-analyze settings with transfers, which we rule out. More recently, Ben-Porath, Dekel, and Lipman (2014) and Erlanson and Kleiner (2015) consider costly verification in one-good and collective allocation problems without transfers, and Glazer and Rubinstein (2004, 2006) and Mylovanov and Zapechelnyuk (2017) study related questions using different verification technologies.<sup>11</sup> Our main departure from this literature (in addition to other differences specific to each paper) is that we study a delegation setting in which we allow for different degrees of bias by the agent relative to the principal. This is also a main distinction with respect to Harris and Raviv (1996)-and the dynamic version in Malenko (2019)-who consider costly verification in a delegation model where the agent always benefits from higher actions.<sup>12</sup> Such an extreme bias assumption implies that granting the agent flexibility has no value to the principal. We instead build on a canonical delegation framework in which flexibility is valuable; that is, the agent's most preferred action

<sup>9</sup> See also Athey, Atkeson, and Kehoe (2005), Ambrus and Egorov (2013, 2017), Halac and Yared (2014, 2018, 2019a, 2019b), and Amador, Bagwell, and Frankel (2018). Auster and Pavoni (2017) study a delegation problem with limited awareness that gives rise to a noninterval delegation set.

<sup>10</sup> We study the effects of the principal not being able to commit to not changing the agent's allowable spending set following the verification decision and result. A different question that a literature on auditing has investigated concerns a principal's ability to commit to an audit strategy; see, e.g., Reinganum and Wilde (1986), Banks (1989), and Chatterjee, Morton, and Mukherji (2008). Work on delegation and self-control has also studied lack of commitment to rules; this includes Bernheim, Ray, and Yeltekin (2015), Dovis and Kirpalani (2017), and Halac and Yared (2019a).

<sup>11</sup> More broadly, there is a literature on mechanism design and implementation with evidence, including Green and Laffont (1986), Bull and Watson (2007), Deneckere and Severinov (2008), Ben-Porath and Lipman (2012), and Kartik and Tercieux (2012).

<sup>12</sup> The model in Harris and Raviv (1996) also differs from ours in other respects: there are only three agent types, the agent receives a noncontingent transfer from the principal, and the principal can choose to verify the agent with an interior probability. Harris and Raviv (1998) consider an extension in which capital is allocated across multiple projects. Malenko (2019) analyzes a dynamic version in which projects of independent and identically distributed quality arrive stochastically over time.

is higher than the principal's but not necessarily the highest possible action. Our paper provides the first study of optimal delegation and verification in a setting in which the principal faces a commitment versus flexibility trade-off. We show that this trade-off introduces new conceptual issues into our mechanism design problem and shapes the principal's optimal rule.

Finally, our paper is also related to a literature that studies policy rules with escape clauses in macroeconomic models. Building on the seminal work of Rogoff (1985) on commitment versus flexibility, Flood and Isard (1988) and Lohmann (1992) consider the use of escape clauses in monetary policy. Obstfeld (1997) discusses the merits of escape clauses in the context of fixed exchange rate systems, where member countries are allowed to realign in the face of severe shocks. Beshkar and Bond (2017) analyze trade agreements within the class of tariff caps with escape clauses, where the reliance on verification relative to tariff overhang optimally depends on the level of international externality. Coate and Milton (2019) consider the optimal design of fiscal limits for a politician who is allowed to override the limit and select his preferred action with the citizens' approval. We share with this literature our motivation of examining the role of escape clauses. Our main departure is that we use mechanism design to study optimal rules with verification without restricting their structure.

## II. Model

Our baseline model of delegation is the same general principal-agent environment of Amador and Bagwell (2013), where we focus on the case in which the agent's bias is toward higher actions. We expand this delegation model by allowing for costly state verification, following Townsend (1979).

#### A. Environment

There are a principal and an agent. The state is  $\gamma \in \Gamma \equiv [\underline{\gamma}, \overline{\gamma}]$  for  $\underline{\gamma} > 0$ , with continuous density  $f(\gamma) > 0$  for all  $\gamma$ . The corresponding distribution function is  $F(\gamma)$ . The level of spending is denoted by  $\pi \in [\underline{\pi}, \overline{\pi}]$ .

The principal's welfare is  $U_P(\gamma, \pi)$ , twice continuously differentiable with  $\partial^2 U_P(\gamma, \pi)/\partial \pi^2 < 0$ . We assume that the principal's optimum,  $\pi_P(\gamma) \equiv \operatorname{argmax}_{\pi} U_P(\gamma, \pi)$ , is interior, and we refer to it as the *efficient* level of spending. We impose the following single-crossing condition:

$$\frac{\partial^2 U_P(\gamma, \pi)}{\partial \gamma \partial \pi} > 0. \tag{1}$$

Thus, the efficient level of spending is increasing in the state:  $\pi'_{P}(\gamma) > 0$ .

The agent's welfare is  $U_A(\gamma, \pi) = \gamma \pi + b(\pi)$ , with  $b(\pi)$  twice continuously differentiable and  $b''(\pi) < 0$ . We assume that the agent's optimum,  $\pi_A(\gamma) \equiv \operatorname{argmax}_{\pi} U_A(\gamma, \pi)$ , is interior, and we refer to it as the *flexible* level of spending. Note that the agent's welfare satisfies the single-crossing condition  $\partial^2 U_A(\gamma, \pi)/\partial\gamma\partial\pi > 0$ .<sup>13</sup> We consider an agent who is biased toward higher spending relative to the principal. Specifically, we add the following assumption to the setting of Amador and Bagwell (2013):

$$\frac{\partial U_A(\boldsymbol{\gamma}, \boldsymbol{\pi})}{\partial \boldsymbol{\pi}} > \frac{\partial U_P(\boldsymbol{\gamma}, \boldsymbol{\pi})}{\partial \boldsymbol{\pi}}.$$
(2)

Condition (2) says that the agent not only benefits from increasing spending whenever the principal does but also benefits from any spending increase more than the principal. Note that since we took the efficient and flexible spending levels to be interior and the parties' utilities from spending to be strictly concave, an implication of this condition is that the flexible level of spending always exceeds the efficient level. That is, our assumptions yield

$$\frac{\partial U_{A}(\boldsymbol{\gamma},\boldsymbol{\pi})}{\partial \boldsymbol{\pi}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{\pi}_{P}(\boldsymbol{\gamma})} > \frac{\partial U_{P}(\boldsymbol{\gamma},\boldsymbol{\pi})}{\partial \boldsymbol{\pi}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{\pi}_{P}(\boldsymbol{\gamma})} = 0 = \frac{\partial U_{A}(\boldsymbol{\gamma},\boldsymbol{\pi})}{\partial \boldsymbol{\pi}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{\pi}_{A}(\boldsymbol{\gamma})},$$

which, given  $\partial^2 U_A(\gamma, \pi) / \partial \pi^2 < 0$ , implies  $\pi_A(\gamma) > \pi_P(\gamma)$  for all  $\gamma \in \Gamma$ .

The state  $\gamma$  is private information to the agent, that is, the agent's *type*. The principal can perfectly verify  $\gamma$  by paying an additive cost  $\phi > 0$ . The agent's cost of verification is  $\alpha \phi$  for  $\alpha \in [0, 1]$ . This formulation allows us to cover situations in which the agent pays no verification cost ( $\alpha = 0$ ) as well as situations in which he pays a cost no larger than the principal's ( $\alpha \in (0, 1]$ ). One could also allow for the agent to pay a higher cost than the principal's. Our results in section IV continue to hold under  $\alpha > 1$ , provided that the agent's bias is sufficiently large; our results in section V hold independently of the value of  $\alpha \ge 0$ .

By featuring both a bias and private information by the agent, our environment gives rise to a commitment versus flexibility trade-off. If the agent were not biased relative to the principal, the principal could implement the efficient level of spending by providing full flexibility to the agent (who would in this case choose  $\pi_A(\gamma) = \pi_P(\gamma)$ ). Similarly, if the state  $\gamma$  were not the agent's private information, the principal could implement the efficient level of spending by committing the agent to a fully contingent spending plan. In the presence of both a bias and private information, however, the principal cannot implement efficient spending  $\pi_P(\gamma)$  for all  $\gamma$  without verification, and she faces a nontrivial trade-off between commitment and flexibility.

<sup>&</sup>lt;sup>13</sup> For both the principal and the agent's preferences, we will refer to single crossing as the (stronger) supermodularity condition that we have assumed these preferences satisfy.

Special cases.—The model of delegation described above encompasses specific cases commonly studied in the literature. One example is the case of quadratic preferences with a constant bias (which we will refer to as simply quadratic preferences), examined by Melumad and Shibano (1991) and Alonso and Matouschek (2008) and used extensively in applied work. Under these preferences, the principal's welfare is  $-(\gamma - \pi)^2/2$  and the agent's welfare is  $-(\gamma + \beta - \pi)^2/2$  for some  $\beta > 0$  representing the agent's bias. This formulation is equivalent to letting  $U_P(\gamma, \pi) = \gamma \pi + b(\pi) - \beta \pi$  and  $U_A(\gamma, \pi) = \gamma \pi + b(\pi)$  for  $b(\pi) = \beta \pi - \pi^2/2$  and is therefore a special case of our model. We will use the quadratic preferences case to illustrate some of our results.

Another example is the model of consumption under hyperbolic preferences, analyzed by Amador, Werning, and Angeletos (2006) and Halac and Yared (2014, 2018, 2019a).<sup>14</sup> The principal's welfare in this case is  $\gamma u(c) + w(y - c)$  and the agent's welfare is  $\gamma u(c) + \beta w(y - c)$ , where uand w are utility functions; c and y represent consumption and exogenous income, respectively; and  $\beta \in (0, 1)$  captures the degree of present bias by the agent. This formulation is equivalent to letting  $U_P(\gamma, \pi) =$  $\gamma \pi + (1/\beta)b(\pi)$  and  $U_A(\gamma, \pi) = \gamma \pi + b(\pi)$ , with  $\pi = u(c)$  and  $b(\pi) =$  $\beta w(y - u^{-1}(\pi))$ , and is thus also encompassed by our model.

#### B. Timing

The order of events is as follows:

- 1. The principal sets a rule, which maps a verification decision and result into an allowable spending set  $\Pi$ .
- 2. The agent chooses whether to seek verification,  $a \in \{0, 1\}$ , and the principal perfectly verifies his type  $\gamma$  if a = 1.
- 3. The agent chooses a spending level  $\pi$  from the allowable set II.

The above timing assumes that the agent learns his type  $\gamma$  before the principal sets a rule in step 1. Our analysis is unchanged if instead the agent learns his type after the rule has been set, that is, at the beginning of step 2.

#### C. Delegation Rules

Given the game form described above, we can analyze the principal's problem as that of choosing a delegation rule *M* that consists of a pair

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<sup>&</sup>lt;sup>14</sup> Halac and Yared (2014, 2018, 2019a) use this model to study fiscal rules, where a government's deficit bias may emerge from the aggregation of heterogeneous, time-consistent citizens' preferences (Jackson and Yariv 2015, 2014) or from turnover in a political economy setting (Aguiar and Amador 2011; Alesina and Passalacqua 2016). See Yared (2019) for a broad discussion of this application.

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of schedules  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ , specifying a verification decision and spending level for each type  $\gamma$ . The principal chooses a rule M to maximize her expected welfare:

$$\max_{\{a(\gamma),\pi(\gamma)\}_{\gamma\in\Gamma}}\int_{\underline{\gamma}}^{\gamma} (U_P(\gamma,\pi(\gamma)) - a(\gamma)\phi)f(\gamma)d\gamma$$
(3)

subject to

$$U_{A}(\gamma, \pi(\gamma)) - a(\gamma)\alpha\phi \geq U_{A}(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\hat{\gamma}) = 0.$$
(4)

The objective (3) is the principal's expected welfare under a given rule, taking into account the additive verification costs. The constraint (4) is an incentive compatibility (or truth-telling) constraint: it guarantees that an agent of type  $\gamma$  prefers his assigned verification decision and spending level,  $a(\gamma)$  and  $\pi(\gamma)$ , to a different allocation  $a(\hat{\gamma})$  and  $\pi(\hat{\gamma})$  for some type  $\hat{\gamma}$  who is not verified (i.e., with  $a(\hat{\gamma}) = 0$ ). Note that it is sufficient to consider deviations to nonverified types: since a deviation in which an agent of type  $\gamma$  mimics a verified type  $\hat{\gamma}$  would be detected by the principal (as verification reveals the true type) and the principal can arbitrarily punish the agent (through the spending allocation) when she learns that he has deviated, we do not need to consider such a deviation.<sup>15</sup>

We also note that the formulation above does not rule out mixed strategies by the agent. If the agent were willing to mix over verification and no verification or over different spending levels, he would be indifferent over these allocations, and thus the principal can select one of these that maximizes her expected welfare.<sup>16</sup> In fact, building on this observation, we can show that our results are not limited to the game form in section II.B but continue to hold when allowing for any indirect mechanism specifying a message space for the agent and a deterministic allocation function to which the principal commits. Such a mechanism induces a game in which the agent sends a message, is either verified or not as a function of the message, and is assigned a spending level as a function of the message and verification result. We show in appendix B (available online) that a version of the revelation principle in terms of payoffs holds in our setting, implying that to study the optimal deterministic mechanism for the principal, it is without loss to restrict attention to deterministic direct

<sup>&</sup>lt;sup>15</sup> The principal can punish a deviation of a type  $\gamma$  in which he mimics a type  $\hat{\gamma} \neq \gamma$  with  $a(\hat{\gamma}) = 1$  by assigning following verification some spending level  $\pi(\hat{\gamma}, \gamma)$  such that  $U_A(\gamma, \pi(\hat{\gamma}, \gamma)) \leq U_A(\gamma, \pi(\gamma))$ . It is clear that such a spending level exists; in fact, setting  $\pi(\hat{\gamma}, \gamma) = \pi(\gamma)$  would be a sufficient punishment.

<sup>&</sup>lt;sup>16</sup> While this selection relaxes the principal's problem, it is not used under the optimal rule described in our main result in proposition 3, which induces a unique best response by the agent. Hence, the result does not rely on selection of equilibria of the game in sec. II.B.

mechanisms (i.e., where the message space coincides with the agent's type space) that induce truthful reporting by the agent, as considered in the program in (3) and (4) above.

Because there is a continuum of types, it is possible that the problem in (3) and (4) admits multiple solutions that are identical everywhere except for a measure zero set of types. As a means of selecting the optimum in such a situation, we say that a rule M is *optimal* if it solves (3) and (4) and there is no other solution  $\tilde{M}$ , with associated verification and spending schedules  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ , such that

$$U_P(\gamma, \tilde{\pi}(\gamma)) - \tilde{a}(\gamma)\phi \ge U_P(\gamma, \pi(\gamma)) - a(\gamma)\phi$$
(5)

for all  $\gamma$  and strictly for some  $\gamma \in \Gamma$ . Although multiple solutions can in principle continue to exist under this condition, this criterion turns out to be sufficient for our characterization.

# III. No Verification Benchmark

Before analyzing the optimal delegation rule with verification, we review the results of the literature by considering the optimal rule in the absence of verification. Consider the principal's problem in (3) and (4) subject to the additional constraint that  $a(\gamma) = 0$  for all  $\gamma$  (so that constraint (4) becomes  $U_A(\gamma, \pi(\gamma)) \ge U_A(\gamma, \pi(\hat{\gamma}))$  for all  $\gamma, \hat{\gamma}$ ). Amador and Bagwell (2013) study this problem. To solve it, they make the following assumption 1 on the distribution of  $\gamma$ ; we extend this assumption to any truncation from above, with support  $[\gamma, \overline{\gamma}']$  for  $\overline{\gamma}' \le \overline{\gamma}$ , density  $f(\gamma)/F(\overline{\gamma}')$ , and distribution function  $F(\gamma)/F(\overline{\gamma}')$ :

Assumption 1. Take the distribution of  $\gamma$  truncated from above by  $\bar{\gamma}' \leq \bar{\gamma}$ . For each such truncated distribution, there exists  $\gamma^*$  such that for  $\kappa \equiv \inf_{\{\gamma,\pi\}} \left( (\partial^2 U_P(\gamma,\pi)/\partial^2 \pi)/b''(\pi) \right)$ , i.

$$\kappa F(oldsymbol{\gamma}) = rac{\partial U_P(oldsymbol{\gamma}, oldsymbol{\pi}_A(oldsymbol{\gamma}))}{\partial oldsymbol{\pi}} f(oldsymbol{\gamma})$$

is nondecreasing for all  $\gamma \in [\underline{\gamma}, \gamma^*]$ ; and ii.

$$(\gamma - \gamma^*)\kappa \geq \int_{\gamma}^{\widetilde{\gamma}'} rac{\partial U_P(\widetilde{\gamma}, \pi_A(\gamma^*))}{\partial \pi} rac{f(\widetilde{\gamma})}{1 - F(\gamma)} d\widetilde{\gamma}$$

for all  $\gamma \in [\gamma^*, \overline{\gamma}']$ , with equality at  $\gamma^*$ .

One can verify that for the special cases typically studied in the literature, such as those with quadratic or hyperbolic preferences, assumption 1 is satisfied under commonly used distribution functions, including



FIG. 1.—Optimal rule under no possibility of verification. The figure is drawn for the quadratic preferences case (see sec. II.A), where we let  $\underline{\gamma} = 0.5$ ,  $\overline{\gamma} = 1.5$ ,  $\beta = 0.12$ , and  $F(\gamma)$  uniform.

exponential, lognormal, and any nondecreasing density.<sup>17</sup> Given assumption 1, the results in Amador and Bagwell (2013) yield the following:

PROPOSITION 1 (Optimal rule under no possibility of verification). Take the distribution of  $\gamma$  truncated from above by  $\bar{\gamma}' \leq \bar{\gamma}$ . If the principal is constrained to  $a(\gamma) = 0$  for all  $\gamma \in [\underline{\gamma}, \bar{\gamma}']$ , an optimal rule is a threshold  $\gamma^* < \bar{\gamma}'$  such that

$$\pi(\boldsymbol{\gamma}) = \min\{\pi_A(\boldsymbol{\gamma}), \pi_A(\boldsymbol{\gamma}^*)\}$$

for  $\gamma \in [\gamma, \overline{\gamma'}]$ .

Under no verification, an optimal rule is a threshold  $\gamma^*$  such that all types  $\gamma \leq \gamma^*$  spend at their flexible level and all types  $\gamma > \gamma^*$  are bunched at the flexible spending level of  $\gamma^*$ . The principal can implement this rule by setting a spending limit  $\pi^* = \pi_A(\gamma^*)$  and allowing the agent to choose any spending level up to this limit.

Figure 1 illustrates an optimal rule under no verification for the case of quadratic preferences. The level of spending is on the vertical axis and the agent's type on the horizontal axis. In this simple example, both efficient and flexible spending are increasing linear functions of the state  $\gamma$ , and flexible spending exceeds efficient spending by a constant amount

<sup>&</sup>lt;sup>17</sup> We also note that assumption 1 on the original distribution implies that the assumption is satisfied for all truncations from above if the conditions in proposition 2 of Amador and Bagwell (2013) hold.



FIG. 2.—Rule without verification with a hole  $[\pi_L, \pi_H]$ . Parameters are the same as in figure 1.

representing the agent's bias. The rule characterized in proposition 1 specifies a spending level that coincides with the agent's flexible level for  $\gamma \leq \gamma^*$  and equals  $\pi_A(\gamma^*)$  for  $\gamma > \gamma^*$ .

A key insight behind the result in proposition 1 is that holes are suboptimal. More precisely, the principal can always improve upon a rule as that depicted in figure 2, which does not allow the agent to choose spending  $\pi \in [\pi_L, \pi_H]$  for some interior  $\pi_L$  and  $\pi_H$  but does allow the agent to choose spending immediately below  $\pi_L$  and immediately above  $\pi_H$ . The hole  $[\pi_L, \pi_H]$  implies that an agent of type  $\gamma$  for whom  $\pi_A(\gamma) \in (\pi_L, \pi_H)$ is not allowed to spend at his flexible level. Such an agent spends at the lower limit of the hole  $\pi_L < \pi_A(\gamma)$  if his type is relatively low, but he spends at the upper limit of the hole  $\pi_H > \pi_A(\gamma)$  if his type is higher. The role of assumption 1 is to guarantee that if the principal removes the hole, the benefit of reducing overspending for the types that bunch at  $\pi_H$  would outweigh any potential costs of increasing spending for the types that bunch at  $\pi_L$ .

#### **IV.** Optimal Rule

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We now turn to the study of optimal delegation when costly verification is possible. The following class of rules will play an important role in our analysis:

DEFINITION 1. A rule is TEC if it consists of  $\{\gamma^*, \gamma^{**}\}$ , with  $\gamma^* < \gamma^{**}$  and  $\gamma < \gamma^{**} < \bar{\gamma}$  such that

- i. (threshold) if  $\gamma \leq \gamma^{**}$ ,  $a(\gamma) = 0$  and  $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$ ; and
- ii. (escape clause) if  $\gamma > \gamma^{**}$ ,  $a(\gamma) = 1$  and  $\pi(\gamma) = \pi_P(\gamma)$ .

Figure 3 illustrates a TEC rule using the quadratic preferences example. Under TEC, types  $\gamma \leq \gamma^*$  are not verified and spend at their flexible level, types  $\gamma \in (\gamma^*, \gamma^{**}]$  are not verified and are bunched at the flexible spending level of  $\gamma^*$ , and types  $\gamma > \gamma^{**}$  are verified and are assigned their efficient spending level. The principal can implement this rule by allowing the agent to either choose a spending level up to a limit  $\pi^* = \pi_A(\gamma^*)$  or request verification by triggering an escape clause. When the agent is verified, he is assigned his efficient spending level provided that it is above a specified level  $\pi^{**} = \pi_P(\gamma^{**})$  (and is otherwise punished).

An important feature of TEC is that the verification function  $a(\gamma)$  is weakly increasing; that is, there is no decreasing verification:

DEFINITION 2. A rule features decreasing verification at  $\gamma'$  if  $a(\gamma)$  jumps from 1 to 0 at  $\gamma'$ ; that is, either (i)  $a(\gamma') = 0$  and  $\limsup_{\gamma\uparrow\gamma'} a(\gamma) = 1$  or (ii)  $a(\gamma') = 1$  and  $\liminf_{\gamma\downarrow\gamma'} a(\gamma) = 0$ . A rule features weakly increasing verification at  $\gamma'$  if neither i nor ii holds.

Note that we will refer to decreasing/increasing verification in the strict sense, and we will clarify whenever we use decreasing/increasing verification in the weak sense. Figure 4 depicts an example of a rule with decreasing verification. This rule specifies verification only for types between two interior cutoffs,  $\gamma_L$  and  $\gamma_H > \gamma_L$ . Types above and below this



FIG. 3.—TEC rule. Parameters are the same as in figure 1, with  $\phi = 0.008$  and  $\alpha = 0$ . Solid line depicts the allocation of nonverified types; dashed line corresponds to verified types.



FIG. 4.—Rule with decreasing verification. Parameters are the same as in figure 3. Solid line depicts the allocation of nonverified types; dashed line corresponds to verified types.

region are not verified, and hence the rule features decreasing verification at  $\gamma_{H}$ . We will return to this example in section IV.C.

Another feature of TEC is that it specifies verification for some agent types but not for all. We begin by showing in section IV.A that inducing no verification for some types is in fact a property of any optimal rule. Furthermore, building on this result, we show that TEC is optimal whenever optimal verification is everywhere weakly increasing. We consider a simple extreme-bias case in section IV.B and provide an analysis for our general setting in section IV.C.

#### A. Preliminaries

The next lemma shows that verifying all agent types is never optimal for the principal:

LEMMA 1. A rule with  $a(\gamma) = 1$  for all  $\gamma \in \Gamma$  is not optimal.

The logic is simple. Suppose that a rule that verifies all types is optimal. Such a rule must trivially assign efficient spending to all types. Now consider a perturbation in which the principal allows the agent to choose  $\pi_P(\underline{\gamma})$  without verification. Under the perturbed rule, a set of types  $[\underline{\gamma}, \gamma']$  for  $\gamma' \geq \underline{\gamma}$  will prefer  $\pi_P(\underline{\gamma})$  over being verified and assigned efficient spending.<sup>18</sup> Moreover, since the agent is biased toward higher spending

<sup>&</sup>lt;sup>18</sup> Note that if  $\alpha = 0$ ,  $\gamma' = \gamma$ , yet our argument applies, given our optimality condition (5). In the appendix, we also provide an alternative proof for the case of  $\alpha = 0$  that does not rely on condition (5).

and pays a verification cost no larger than the principal's, it must be that the principal is strictly better off by not verifying these types. Hence, we find that incentivizing low types to not overspend is cheaper than verifying them, and thus verifying all types cannot be optimal.

Given lemma 1, we establish the following:

LEMMA 2. If an optimal rule features verification that is weakly increasing everywhere, then TEC is optimal.

Since verifying all agent types is suboptimal, an optimal rule with verification that is weakly increasing everywhere must feature a no-verification region followed by a verification region; that is, there must be a type  $\gamma^{**}$ such that  $a(\gamma) = 0$  for  $\gamma < \gamma^{**}$  and  $a(\gamma) = 1$  for  $\gamma > \gamma^{**}$ . Consider a rule that optimizes over each of these regions separately. Conditional on the agent's type being in the no-verification region, an optimal rule is a threshold  $\gamma^* < \gamma^{**}$  (by proposition 1). Conditional on the agent's type being in the verification region, an optimal rule assigns efficient spending to all types. To prove lemma 2, we show that the rule that results from optimizing over each region separately is incentive compatible—and therefore optimal—over the whole set of types.

Specifically, we establish that no type  $\gamma > \gamma^{**}$  who is prescribed verification under the proposed rule would have an incentive to deviate to the no-verification region.<sup>19</sup> Note that an optimal rule for the no-verification region sets a maximum allowable spending level  $\pi_A(\gamma^*) \le \pi_P(\gamma^{**})$ . Moreover, by optimality of  $\gamma^{**}$ , the principal prefers to pay the cost of verifying type  $\gamma > \gamma^{**}$  to assign him  $\pi_P(\gamma)$  rather than bunch him at  $\pi_A(\gamma^*)$ . Since the agent is biased toward higher spending and pays a verification cost no larger than the principal's, it follows that types  $\gamma > \gamma^{**}$  also prefer to be verified rather than deviate to  $\pi_A(\gamma^*)$ . This proves that the proposed rule is incentive compatible, which implies that it is also optimal, and by construction this rule is TEC.

#### B. Extreme Bias

Before turning to our main results, we consider a setting in which the agent's bias is extreme. Suppose  $b(\pi) = 0$  for all  $\pi \in [\underline{\pi}, \overline{\pi}]$ , so that the agent's welfare is simply  $U_A(\gamma, \pi) = \gamma \pi$ . The agent in this case always prefers higher levels of spending: his flexible spending level is  $\pi_A(\gamma) = \overline{\pi}$  for all  $\gamma \in \Gamma^{20}$  As mentioned in the introduction, such an extreme bias corresponds to what is assumed in other models of costly verification, including the seminal work of Townsend (1979), the delegation model

 $<sup>^{19}</sup>$  By proposition 1, no type  $\gamma \leq \gamma^{**}$  who is prescribed no verification would have an incentive to deviate either.

<sup>&</sup>lt;sup>20</sup> As assumed in sec. II.A, we are primarily interested in the case in which  $\pi_A(\gamma)$  is interior rather than a corner; however, we find it is instructive to study this corner case first.

of Harris and Raviv (1996, 1998), and more recent contributions, such as Ben-Porath, Dekel, and Lipman (2014).

An extreme bias implies that if the agent is not verified, he will choose the highest allowable level of spending, regardless of his type. Moreover, the agent will seek verification only if that allows him to spend more than under no verification. The analysis therefore is significantly simplified. The only incentive-compatible rule for an agent with an extreme bias involves bunching all nonverified types at one spending level; that is, flexibility has no value in this setting. Furthermore, any type that is verified must be assigned a higher spending level than that at which nonverified types are bunched. As a result, we have the following:<sup>21</sup>

PROPOSITION 2 (Optimal rule under extreme bias). Suppose  $b(\pi) = 0$  for all  $\pi \in [\underline{\pi}, \overline{\pi}]$ . Then if verification is optimal, TEC is optimal.

When the agent's bias is extreme and verifying some types is optimal, an optimal rule is TEC, with nonverified types  $\gamma \leq \gamma^{**}$  bunched and awarded no flexibility and verified types  $\gamma > \gamma^{**}$  spending at their efficient level. The optimality of TEC follows from the optimality of weakly increasing verification. Suppose by contradiction that an optimal rule featured decreasing verification. Take  $\gamma'$  to be a marginal nonverified type splitting a verification region and a higher no-verification region, so  $a(\gamma') = 0$  and  $a(\gamma' - \varepsilon) = 1$  for some  $\varepsilon > 0$  arbitrarily small. Let  $\pi_A(\gamma^*)$  be the level of spending at which nonverified types are bunched. The optimality of verifying  $\gamma' - \varepsilon$  implies

$$U_P(\gamma' - \varepsilon, \pi(\gamma' - \varepsilon)) - U_P(\gamma' - \varepsilon, \pi_A(\gamma^*)) \ge \phi, \tag{6}$$

where, as noted, incentive compatibility requires  $\pi(\gamma' - \varepsilon) \ge \pi_A(\gamma^*)$ , and since  $\phi > 0$ , (6) yields  $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$ . The optimality of not verifying  $\gamma'$  then implies

$$U_P(\gamma', \pi(\gamma' - \varepsilon)) - U_P(\gamma', \pi_A(\gamma^*)) \leq \phi.$$
(7)

However, (6) and (7) together with  $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$  violate the singlecrossing condition (1), yielding a contradiction. Intuitively, the principal can improve upon a rule with decreasing verification by verifying a higher agent type instead of a lower type, as the marginal benefit of letting the higher type spend more is higher. Note that such a perturbation is always incentive compatible for the agent because all nonverified types are bunched at the same spending level, which (by incentive compatibility) is lower than the spending level assigned to any verified type. This feature is of course due to the agent's bias being extreme.

<sup>&</sup>lt;sup>21</sup> Proposition 2, as well as propositions 3 and 5, describes an optimal rule when verification is optimal. Clearly, verification is optimal if and only if the verification  $\cot \phi$  is not too high.

#### C. Optimal Rule with Verification

We next study optimal delegation with verification in our general setting in which the agent's bias is not extreme. To this end, it is useful to consider a relaxed version of the problem in (3) and (4), in which we assume that the agent pays no verification cost ( $\alpha = 0$ ):

$$\max_{\{a(\gamma),\pi(\gamma)\}_{\gamma\in\Gamma}} \int_{\underline{\gamma}}^{\overline{\gamma}} (U_P(\gamma,\pi(\gamma)) - a(\gamma)\phi) f(\gamma) d\gamma$$
(8)

subject to

$$U_A(\gamma, \pi(\gamma)) \ge U_A(\gamma, \pi(\hat{\gamma}))$$
 for all  $\gamma, \hat{\gamma}$  for which  $a(\hat{\gamma}) = 0.$  (9)

Since the original incentive compatibility constraint (4) is tighter than the relaxed constraint (9), if a solution to (8) and (9) satisfies (4), then it is also a solution to the problem in (3) and (4).

Furthermore, we can show that if a solution to (8) and (9) is TEC, then it will indeed satisfy (4), implying the following:

LEMMA 3. If a TEC rule is a solution to (8) and (9), it is also a solution to (3) and (4).

To show that a TEC rule  $\{\gamma^*, \gamma^{**}\}$  that solves (8) and (9) satisfies the original constraint (4), we establish that any agent of type  $\gamma > \gamma^{**}$  would prefer to pay the verification cost  $\alpha\phi$  and spend at his efficient level  $\pi_P$  ( $\gamma$ ) rather than pay no verification cost and choose the threshold flexible spending level  $\pi_A(\gamma^*)$ . The logic is similar to that behind lemma 2, where we show that the optimality of verifying type  $\gamma > \gamma^{**}$  for the principal implies incentive compatibility of this verification for the agent. Hence, we obtain that to study whether TEC is optimal, it is without loss to focus on the relaxed problem in (8) and (9).<sup>22</sup> We analyze this problem for the remainder of this section.

The following two lemmas establish useful properties of any solution:

LEMMA 4. If a solution to (8) and (9) prescribes verification for type  $\gamma$ , it has  $\pi_P(\gamma) \leq \pi(\gamma) \leq \pi_A(\gamma)$ . If (9) does not bind for  $\gamma$ , then  $\pi(\gamma) = \pi_P(\gamma)$ .

LEMMA 5. In any solution to (8) and (9),  $\pi$  ( $\gamma$ ) is weakly increasing.

Lemma 4 states that if a type  $\gamma$  is verified, his assigned spending level is (weakly) between his efficient level and his flexible level. The argument is straightforward. If assigned spending for type  $\gamma$  is either below efficient or above flexible, then either increasing or decreasing this spending, respectively, makes the principal better off and is incentive compatible for the agent. Since the principal maximizes her expected welfare subject to incentive compatibility, if a verified type's incentive compatibility constraint is slack, the principal assigns this type efficient spending.

<sup>&</sup>lt;sup>22</sup> We maintain our optimality condition in (5) to select a solution.

Lemma 5 shows that the principal assigns a spending level that is weakly increasing in the agent's type  $\gamma$ . When comparing two agent types that are not verified, the result naturally follows from incentive compatibility: a type  $\gamma$  cannot be assigned higher spending than a higher type  $\gamma' > \gamma$ , as at least one of them would have an incentive to deviate, given that preferences satisfy single crossing. When comparing two agent types such that (at least) one of them is verified, the result follows from optimality: if a type  $\gamma$  is assigned higher spending than a higher type  $\gamma' > \gamma$ , the principal can improve welfare by swapping these types' spending levels and verification assignments, and if incentive compatibility was initially satisfied, it will continue to be satisfied after the swap, given single crossing.

By definition of TEC and lemma 2, whether a TEC rule is optimal depends on whether the principal can instead benefit from inducing decreasing verification, namely, a situation in which a set of types is verified and a set of higher types is not verified. Using lemmas 4 and 5, we next show that any rule featuring decreasing verification must induce significant overspending, limiting the welfare that such a rule can provide to the principal:

LEMMA 6. Suppose a solution to (8) and (9) features decreasing verification at  $\gamma' < \bar{\gamma}$ . Then the solution satisfies

$$\frac{\int_{\gamma'}^{\gamma} (U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))) f(\gamma) d\gamma}{1 - F(\gamma')} \ge \eta(\gamma')$$
(10)

for

$$\eta(\gamma') = \frac{\int_{\gamma'}^{\min\{\pi_P^{-1}(\pi_A(\gamma')),\bar{\gamma}\}} (U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi_A(\gamma'))) f(\gamma) d\gamma}{1 - F(\gamma')} > 0.$$
(11)

If an optimal rule features decreasing verification at an interior point, then the principal's expected welfare from types above this point is strictly bounded away from that under efficient spending. For intuition, consider first the example in figure 4, where the principal induces verification only for an interior set of types  $[\gamma_L, \gamma_H]$ . The principal must incentivize these types to seek verification rather than deviate and mimic a type in the no-verification region above  $\gamma_H$ . In the example, the principal achieves this by assigning types immediately above  $\gamma_H$  their flexible spending levels while assigning verified types immediately below  $\gamma_H$  the spending levels that make them indifferent over deviating to  $\pi_A(\gamma_H)$  under no verification. As a consequence, however, the principal induces overspending by a positive mass of types above  $\gamma_H$ . In fact, all types  $\gamma \in (\gamma_H, \pi_P^{-1}(\pi^*))$  spend above their efficient level in the example of figure 4.

More generally, for any optimal rule with decreasing verification at a point  $\gamma' < \bar{\gamma}$ , lemma 6 shows that the principal's expected welfare above

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 $\gamma'$  is lower than efficient welfare, with the difference being no smaller than  $\eta(\gamma')$  in (11). The bound  $\eta(\gamma')$  captures the minimum overspending above  $\gamma'$  that is needed to deter deviations by verified types below  $\gamma'$ . Specifically, let  $a(\gamma') = 1$  and thus  $a(\gamma' + \varepsilon) = 0$  for  $\varepsilon > 0$  arbitrarily small. By lemma 5, we know that all types above  $\gamma'$  spend more than those below, and by lemma 4, we know that verified types  $\gamma$  spend no more than their flexible amount  $\pi_A(\gamma)$ . Thus, for types in the verification region below  $\gamma'$  not to deviate to the no-verification region above  $\gamma'$ , it must be that  $\pi(\gamma' + \varepsilon) \ge \pi_A(\gamma')$ ; in fact, by optimality, this inequality must be strict.<sup>23</sup> Given that by lemma 5 all types  $\gamma \ge \gamma' + \varepsilon$  spend weakly above  $\pi(\gamma' + \varepsilon)$ , it follows that all types  $\gamma > \gamma'$  spend strictly above  $\pi_A(\gamma')$ , which exceeds efficient spending  $\pi_P(\gamma)$  for all  $\gamma \in (\gamma', \min\{\pi_P^{-1}(\pi_A(\gamma')), \bar{\gamma}\})$ . This yields the bound in (11).

Importantly, the bound identified in (11) is independent of the verification cost  $\phi$ . This allows us to establish our first main result. In what follows, let  $\eta(\bar{\gamma}) = \lim_{\gamma \uparrow \bar{\gamma}} \eta(\gamma)$ .

PROPOSITION 3 (Optimal rule under small verification cost). Let  $\bar{\phi} \equiv \min_{\gamma \in \Gamma} \eta(\gamma) > 0$ . If  $\phi < \bar{\phi}$  and verification is optimal, TEC is optimal.

The idea is as follows. By lemma 6, any optimal rule with decreasing verification implies a welfare loss due to overspending in the decreasing verification region. We show that if the principal's verification cost is small relative to the minimum such loss, then she can raise her welfare by verifying all types in the decreasing verification region and reducing their spending to the efficient level. It follows that an optimal rule must induce weakly increasing verification everywhere, and therefore TEC is optimal by lemma 2.

Formally, suppose by contradiction that an optimal rule induces decreasing verification at some point, and let  $\gamma^{**}$  be the lowest verified type under this rule. We consider a global perturbation: the principal verifies all types  $\gamma \ge \gamma^{**}$  and assigns them efficient spending  $\pi_P(\gamma)$  while solving for an optimal rule without verification for types  $\gamma < \gamma^{**}$ . By proposition 1, an optimal rule for the no-verification region is a threshold  $\gamma^* < \gamma^{**}$ , and since  $\pi_A(\gamma^*) \le \pi_P(\gamma^{**})$  (by optimality of  $\gamma^*$ ) and  $\alpha = 0$ , it is easy to verify that the perturbed rule is incentive compatible.

To show that the perturbation strictly raises the principal's welfare, note first that expected welfare conditional on  $\gamma < \gamma^{**}$  weakly increases because it is now maximized subject to fewer constraints: under the perturbed rule, types  $\gamma < \gamma^{**}$  cannot mimic a type  $\hat{\gamma} \ge \gamma^{**}$ . Thus, all we need to show is that expected welfare conditional on  $\gamma \ge \gamma^{**}$  increases strictly, namely, that the (allocative) benefit of verifying these types is strictly

<sup>&</sup>lt;sup>23</sup> If  $\pi(\gamma' + \varepsilon) = \pi_A(\gamma')$ , incentive compatibility requires  $\pi(\gamma') = \pi_A(\gamma')$ , but then the principal can improve upon the rule by setting  $a(\gamma') = 0$  while keeping everything else unchanged.

greater than the additional verification cost the principal incurs. Because verified types  $\gamma \geq \gamma^{**}$  are assigned efficient spending, the benefit of verifying them is weakly positive. Moreover, note that by the contradiction assumption, there exists a type above  $\gamma^{**}$  at which the original rule features decreasing verification. Thus, if  $\gamma' < \bar{\gamma}$  is the lowest such type, lemma 6 implies that the benefit of verifying types  $\gamma \geq \gamma^{**}$  is bounded from below by  $(1 - F(\gamma'))\eta(\gamma')$ , where  $\eta(\cdot)$  is defined in (11). The claim then follows in this case from the fact that, given  $\phi < \bar{\phi}$ , the additional cost of verifying types  $\gamma \geq \gamma^{**}$  is strictly smaller than  $(1 - F(\gamma'))\bar{\phi} = (1 - F(\gamma'))\min_{\gamma \in \Gamma} \eta(\gamma)$  and hence strictly smaller than the benefit of verifying these types. If the lowest type above  $\gamma^{**}$  at which the original rule features decreasing verification is  $\gamma' = \bar{\gamma}$ , an analogous argument applies, since in this case the original rule induces strict overspending by  $\bar{\gamma}$  and the benefit of verifying this type is no smaller than  $\bar{\phi}$ .

Figure 5 illustrates the result in proposition 3 in a setting with quadratic preferences (see sec. II.A) and a uniform distribution of types. In this setting, we obtain a closed-form expression for the cutoff  $\overline{\phi}$  and thus for the range of verification costs,  $\phi < \overline{\phi}$ , under which TEC is shown to be optimal under verification.<sup>24</sup> We find that  $\overline{\phi}$  is increasing in the agent's bias  $\beta$  and decreasing in the range of types  $\bar{\gamma} - \gamma$ . Intuitively, if the agent's bias toward higher spending is large, then inducing decreasing verification is very expensive for the principal, as she must allow high overspending above any interior verification region to deter deviations from verified types. In this case, the benefit of verifying all types above the verification region is large, and thus TEC is preferred even if the principal's verification cost  $\phi$  is relatively high. Similarly, if the range of types  $\bar{\gamma} - \gamma$  is small, then the mass of types above any interior verification region is also small, and therefore the cost of verifying all such types in a TEC rule is low even if  $\phi$  is relatively high. Figure 5 provides an illustration using the example of figure 3. The figure depicts values of  $\beta$  and  $\bar{\gamma} - \gamma$  under which the condition  $\phi < \overline{\phi}$  is satisfied (shaded areas) as well as the subset of those values under which verification-and thus TEC-is optimal (dark gray shaded area).25

<sup>24</sup> We derive this cutoff in app. B. We obtain

$$\bar{\phi} = \begin{cases} \frac{\beta^3}{6} \frac{1}{\bar{\gamma} - \underline{\gamma}} & \text{if } \beta \leq \bar{\gamma} - \underline{\gamma}, \\ \\ \frac{1}{6} \left[ \left( \bar{\gamma} - \underline{\gamma} \right)^2 - 3\beta \left( \bar{\gamma} - \underline{\gamma} \right) + 3\beta^2 \right] & \text{if } \beta > \bar{\gamma} - \underline{\gamma}. \end{cases}$$

<sup>25</sup> In this example, the optimal TEC rule bunches all nonverified types at one spending level, and as a result the optimality of verification given  $\phi < \overline{\phi}$  is independent of  $\beta$ . This, however, is not a general feature.



FIG. 5.—Optimality of TEC. Parameters are the same as in figure 3, with  $\phi = 0.008$ . Shaded areas indicate values of  $\beta$  and  $\bar{\gamma} - \gamma$  under which  $\phi < \bar{\phi}$ . Dark gray shaded area corresponds to values for which, in addition, verification is optimal.

The result in proposition 3 provides a justification for the broad use of TEC rules in applications.<sup>26</sup> As described in the introduction, capital budgeting studies (e.g., Ross 1986; Taggart 1987) report that TEC is common in organizations. Division managers are required to either abide a budgetary limit or provide project documentation to request a revision of their budgets. Schaechter et al. (2012) and Lledó et al. (2017) find that fiscal rules in many countries also take the form of TEC, namely, a spending or deficit limit with escape clause provisions that allow the government to break the limit under certain circumstances. Additionally, TEC rules are used in international trade agreements, in the form of a tariff cap with an escape clause (Beshkar and Bond 2017), and in price delegation in

<sup>&</sup>lt;sup>26</sup> Comparative statics are as one would expect. In particular, the lower is  $\phi < \overline{\phi}$ , other things equal, and the larger is the verification region (i.e., the smaller is  $\gamma^{**}$ ) in the optimal TEC rule.

firms, where sales people can unilaterally offer their customers discounts up to a certain percentage off the list price but must request a supervisor's approval for larger discounts (Lo et al. 2016).

Proposition 3 proves the optimality of TEC when the principal's cost of verification is small enough. What happens if the cost of verification is larger? Our next result shows that there exist environments and verification costs for which the principal induces verification but not in the form of TEC:

PROPOSITION 4 (Optimal rule under intermediate verification cost). There exist { $U_P$ , b, f,  $\phi$ ,  $\alpha$ } for which any optimal rule features decreasing verification.

To prove this result, we identify conditions on parameters under which verifying only an intermediate range of types  $[\gamma_L, \gamma_H]$  dominates both not verifying any type as well as using TEC.<sup>27</sup> The main reason why verifying only intermediate types can dominate not verifying any type is that an intermediate verification region imposes discipline on the no-verification region below. That is, even when the verification cost is large enough that the principal would not benefit from verifying types in  $[\gamma_L, \gamma_H]$  only to improve their allocation relative to flexible spending, she may benefit from verifying these types to discipline lower types: with the intermediate verification region, types  $\gamma < \gamma_L$  can no longer mimic types in  $[\gamma_L, \gamma_H]$ . The main reason why verifying only intermediate types can dominate using a TEC rule is that it allows the principal to save on verification costs. Specifically, with intermediate verification, the principal may be able to impose discipline on types  $\gamma < \gamma_L$  without prescribing verification for types  $\gamma > \gamma_H$  as she would under a TEC rule; this will be the case if  $\gamma_L$  has no incentive to deviate to mimic a type as high as  $\gamma_{H}$ . In such a situation, intermediate verification allows the principal to save on the cost of verifying types above  $\gamma_{H}$ .

These arguments yield that a rule with decreasing verification as that depicted in figure 4 can dominate any no-verification rule (as that in fig. 1) and any TEC rule (as that in fig. 3), provided that the cost of verification  $\phi$  is not small (or large) enough. We emphasize that proposition 4 does not rely on nonuniformity of the principal's objective across types or any other sort of asymmetry; we prove the result by taking the case of quadratic preferences and a uniform distribution of types, as depicted in our figures. We also note that while our construction implies the optimality of decreasing verification under some parameters with  $\phi > \overline{\phi}$ , the optimal rule in this case may not take the simple intermediate-verification structure that we consider to prove the result. In fact, we can show that even when restricting attention to quadratic preferences and a uniform distribution,

<sup>&</sup>lt;sup>27</sup> By lemma 2, any other rule with verification that is weakly increasing everywhere is thus also dominated. Hence, the claim in proposition 4 follows.

there exist parameters for which TEC, no verification, and intermediate verification are all dominated by a rule featuring multiple interior verification regions.<sup>28</sup> Intuitively, intercalating verification regions to further divide the delegation set can allow the principal to improve discipline while keeping verification costs at a minimum.<sup>29</sup>

The implications of proposition 4 for applications are immediate. For example, for organizations, this result tells us that it can be beneficial to define different categories of investment. Senior management could require division heads to either comply with a low budgetary limit meant for relatively small projects or choose from a higher range of investment levels meant for large projects; otherwise, documentation would be needed to have intermediate levels of investment approved. Such a verification requirement may suffice to discourage overinvestment by division managers with small projects: these managers lack proof to justify a small increase in their budget and would not want to increase their investment as much as for a large project.

Nevertheless, whereas delegation rules with decreasing verification can be optimal, they do not appear to be common in practice, and our analysis may help explain why. Our construction shows that implementing a rule with decreasing verification demands strong commitment power from the principal. Take, for example, the rule depicted in figure 4. The principal assigns spending strictly above the efficient level to some agent types  $\gamma \in [\gamma_L, \gamma_H]$  who are verified. By doing this, the principal incentivizes those types to be verified: if they were instead assigned efficient spending following verification, they would not seek verification in the first place. The principal must be committed to allowing this inefficient spending despite her learning the true type of the agent. Strong commitment power is also required to incentivize types  $\gamma < \gamma_L$  sufficiently close to  $\gamma_L$  to not seek verification. In the rule of figure 4, these types are punished if they seek verification, even though ex post, once verification took place, both the principal and the agent would strictly prefer efficient spending to punishment. Without the threat of punishment, the principal may not be able to prevent an agent of type  $\gamma < \gamma_L$  sufficiently close to  $\gamma_L$  from seeking verification, as an efficient allocation following verification would allow this agent to increase his spending toward his flexible level.

In practice, principals may not have sufficient commitment power to implement allocations that are inefficient ex post. We explore the implications of limited commitment power in section V.

<sup>&</sup>lt;sup>28</sup> In particular, the rule constructed in lemma 9 in the proof of proposition 4 in app. A is not optimal for some parameter values satisfying the assumptions of the lemma. For instance, taking the example of fig. 3, and consistent with our intuition behind fig. 5, we find that a rule with multiple interior verification regions becomes optimal if the range of types  $\bar{\gamma} - \gamma$  becomes large enough. <sup>29</sup> For this reason, when decreasing verification is optimal, the optimal rule is very sen-

<sup>&</sup>lt;sup>29</sup> For this reason, when decreasing verification is optimal, the optimal rule is very sensitive to parameters, such as the value of  $\bar{\gamma} - \gamma$ .

#### V. Limited Commitment

We study a setting in which the principal has limited commitment power. We modify the order of events in section II.B as follows:<sup>30</sup>

- 1. The principal sets a rule, which maps a verification decision and result into an allowable spending set  $\Pi$ .
- 2. The agent chooses whether to seek verification,  $a \in \{0, 1\}$ , and the principal perfectly verifies his type  $\gamma$  if a = 1.
- 3. The principal revises the allowable spending set  $\Pi$  to  $\Pi'$ .
- 4. The agent chooses a spending level  $\pi$  from the allowable set  $\Pi'$ .

The first two steps are the same as in our environment with full commitment power. What is new is step 3: after observing the agent's verification decision and the result if verification is chosen, the principal now revises the allowable spending set for the agent. This is a mild form of limited commitment. In particular, in step 2 we maintain the assumption that the principal is able to commit to a verification plan, so the agent's type is verified if and only if the agent requests verification.<sup>31</sup> Moreover, in step 4 we maintain the assumption that the principal is able to commit to allowing the agent to choose any spending level from the allowable spending set, so our problem is still one of delegation rather than cheap talk. The only assumption that we relax is about the principal's commitment to not changing the allowable spending set following the verification decision and result.

This form of limited commitment is relevant to applications of our model. For example, division managers in organizations may request a revision of their budgets for the next period. Can senior management commit to not changing their allocation ex post when no request is submitted? And in the case of a request, can senior management commit to an inefficient budget after verifying the benefits of the division's projects? As discussed in the introduction, the answer is often no. Senior management makes decisions on budget caps and the scope of projects brought up for review ex post, and these decisions do not always coincide with preannounced criteria (see Bower and Lesard 1973; Ross 1986; Taggart

<sup>&</sup>lt;sup>30</sup> We note that our results in this section are not limited to the exact game described below; analogous to our claims in sec. II.C, our findings can be extended to variations of this game that allow messages between the principal and the agent (while keeping our assumptions on the principal's limited commitment). Throughout this section, we maintain our optimality condition in (5).

<sup>&</sup>lt;sup>31</sup> As noted in n. 10, there is a literature that studies auditing when the principal cannot commit to an audit strategy. In many of the applications of our problem, however, we find that there are often institutions ensuring that principals cannot deny verification once it has been requested. In this sense, the agent can always choose to trigger verification. Lack of commitment by the principal in this respect would change the nature of our problem; we leave its analysis for future work.

1987). In fact, these criteria are sometimes left ambiguous, as they depend on the class of project, which may not be well specified (Mukherjee and Henderson 1987). This gives senior management more discretion to make budgetary decisions. Interestingly, in their study of fiscal rules across countries, Schaechter et al. (2012) also observe that escape clauses are sometimes not well specified: "in the past escape clause provisions have in several cases left too large a room for interpretation" (Schaechter et al. 2012, 20).

In our model, limited commitment on the side of the principal matters for two reasons. First, conditional on no verification, the principal must choose an allocation that is optimal for the nonverified types. That is, the principal assigns spending in this case taking into account the distribution of nonverified types and ignoring the incentives of verified types. Second, conditional on verification, the principal learns the agent's true type  $\gamma$  and must assign the agent the efficient spending level  $\pi_P(\gamma)$ . This is true both when the agent's seeking verification is on path as well as when this verification decision is part of a deviation. Hence, the agent can always choose to be verified to guarantee himself the efficient level of spending.

As a result, limited commitment implies certain conditions that any incentive-compatible rule must satisfy. In what follows, we restrict attention to strategies that specify piecewise continuous mappings  $\{a(\gamma), \pi(\gamma)\}$ .

LEMMA 7. Under limited commitment, any incentive-compatible rule satisfies the following:

i. If there is decreasing verification at  $\gamma_{H}$ , then

$$U_{A}(\boldsymbol{\gamma}_{H}, \boldsymbol{\pi}_{P}(\boldsymbol{\gamma}_{H})) - \boldsymbol{\alpha}\boldsymbol{\phi} = U_{A}(\boldsymbol{\gamma}_{H}, \boldsymbol{\pi}(\boldsymbol{\gamma}_{H})), \quad (12)$$

where  $\pi(\gamma_H) \equiv \lim_{\epsilon \downarrow 0} \pi(\gamma_H + \epsilon)$  if  $a(\gamma_H) = 1$ . Moreover,  $\pi(\gamma_H) > \pi_A(\gamma_H)$ . (13)

ii. If there is increasing verification at  $\gamma_L$ , then

$$U_A(\gamma_L, \pi_P(\gamma_L)) - \alpha \phi = U_A(\gamma_L, \pi(\gamma_L)), \qquad (14)$$

where  $\pi(\gamma_L) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon)$  if  $a(\gamma_L) = 1$ .

Part i shows that if  $\gamma_H$  splits a verification region from a higher noverification region, then  $\gamma_H$  must be indifferent between being verified and spending at the efficient level versus not being verified and spending at  $\pi(\gamma_H)$ , as allowed in the no-verification region above this type. Likewise, part ii shows that if  $\gamma_L$  splits a no-verification region from a higher verification region, then  $\gamma_L$  must be indifferent between being verified and spending at the efficient level versus not being verified and spending at  $\pi(\gamma_L)$ , as allowed in the no-verification region below this type. This result follows from the fact that a principal with limited commitment power assigns efficient spending whenever the agent seeks verification. Therefore, if there is a point at which a verification region either ends or starts, the marginal verified type at such point must weakly prefer verification with efficient spending to no verification, and the marginal nonverified type must weakly prefer no verification to verification with efficient spending. The marginal type must thus be indifferent.

Lemma 7 also shows that for type  $\gamma_H$  as defined in the lemma, an incentive-compatible rule must set  $\pi(\gamma_H) > \pi_A(\gamma_H)$ . This is required to make  $\gamma_H$  indifferent between verification and no verification: if this inequality is not satisfied, the marginal verified type would instead prefer to deviate and not seek verification.

For the remainder of our analysis, we require the following:

Assumption 2. If

$$R(m{\gamma}, m{\pi}_H) \equiv U_{\!\scriptscriptstyle A}(m{\gamma}, m{\pi}_P(m{\gamma})) - lpha m{\phi} - U_{\!\scriptscriptstyle A}(m{\gamma}, m{\pi}_H) \geq 0$$

for  $\pi_H > \pi_P(\gamma)$ , then

$$R(\gamma', \pi_H) > 0$$

for all  $\gamma' < \gamma$ .

This is a single-crossing property: we assume that if a type  $\gamma$  weakly prefers verification with efficient spending  $\pi_P(\gamma)$  to no verification with a higher spending level  $\pi_H > \pi_P(\gamma)$ , then any lower type  $\gamma' < \gamma$  strictly prefers verification with efficient spending  $\pi_P(\gamma')$  to no verification with the higher spending level  $\pi_H$ .<sup>32</sup> This property holds in the cases commonly studied in the literature, such as those with quadratic preferences or with hyperbolic preferences under common parameterizations.<sup>33</sup>

Given assumption 2, we obtain the following:

PROPOSITION 5 (Optimal rule under limited commitment). Under limited commitment, any incentive-compatible rule features weakly increasing verification everywhere. Moreover, if verification is optimal, TEC is optimal.

Under limited commitment, decreasing verification is not incentive compatible for the principal. As we discussed in section IV.C, decreasing verification requires that the principal commit to allowing the agent to spend at a level that is inefficient ex post, following the agent's verification decision and result. We prove that without this commitment, the

<sup>&</sup>lt;sup>32</sup> Our single-crossing conditions on preferences imply that if a type  $\gamma$  weakly prefers verification with efficient spending  $\pi_P(\gamma)$  to no verification with a lower spending level  $\pi_L < \pi_P(\gamma)$ , then any higher type  $\gamma' > \gamma$  strictly prefers verification with efficient spending  $\pi_P(\gamma')$  to no verification with the lower spending level  $\pi_L$ . Assumption 2 requires that this property be maintained in the opposite direction as well.

<sup>&</sup>lt;sup>33</sup> For example, in the hyperbolic preferences case (see sec. II.A), assumption 2 holds if the utility functions for present and future consumption are the same and either exponential or constant relative risk aversion with a coefficient weakly greater than 1.

principal cannot induce decreasing verification, and hence any incentivecompatible rule must feature weakly increasing verification at all types  $\gamma \in \Gamma$ . Analogous arguments to those behind lemmas 1 and 2 in our fullcommitment environment then imply that if verifying some agent types is optimal, a TEC rule is optimal.

A sketch of the proof of proposition 5 is as follows. Suppose by contradiction that there is an incentive-compatible rule that induces decreasing verification, with  $\gamma_H$  being a type splitting a verification region from a higher no-verification region. Given limited commitment, verified types immediately below  $\gamma_H$  are assigned efficient spending, and types  $\gamma$  immediately above  $\gamma_H$  spend at a level  $\pi_H > \pi_A(\gamma)$  that makes  $\gamma_H$  indifferent between verification and no verification (cf. lemma 7). This means that types immediately above  $\gamma_H$  must be strictly overspending, in fact spending above their flexible level. The heart of the proof is showing that the principal cannot commit to allowing such overspending.

It is clear that conditional on the agent not seeking verification, the principal would like to reduce the overspending by types immediately above  $\gamma_{H}$ . Reducing this overspending is ex post incentive compatible for these types: having chosen no verification, types  $\gamma > \gamma_H$  would prefer  $\pi_A(\gamma)$  to  $\pi_H > \pi_A(\gamma)$ . Hence, the only reason the principal would not reduce the overspending immediately above  $\gamma_H$  following no verification is if doing so would violate incentive compatibility for some other non-verified type. Such a nonverified type must be below  $\gamma_H$ ; specifically, there must exist a type  $\gamma_L < \gamma_H$  who is not verified and is exactly indifferent between his assigned spending level, call it  $\pi_L$ , and the spending level  $\pi_H > \pi_L$ . In fact, because of single crossing, this type must be the marginal type right below the verification region that ends at  $\gamma_H$ ; that is, the rule must induce verification for types  $\gamma \in [\gamma_L, \gamma_H]$  and no verification for types immediately below and above this set. An example is the rule depicted in figure 4.

Now if the principal induces such an interior verification region  $[\gamma_L, \gamma_H]$ , then by lemma 7, type  $\gamma_L$  must be indifferent between no verification with spending  $\pi_L$  and verification with efficient spending. Since we have defined  $\gamma_L$  as being indifferent between spending at  $\pi_L$  and spending at  $\pi_H$  under no verification, by transitivity, we obtain that  $\gamma_L$  must be indifferent between no verification with spending  $\pi_H$  and verification with efficient spending. However, recall that type  $\gamma_H$  is also indifferent between no verification with spending  $\pi_H$  and verification with efficient spending. However, recall that type  $\gamma_H$  is also indifferent spending. Hence, by assumption 2,  $\gamma_L < \gamma_H$  cannot hold, and we must have  $\gamma_L = \gamma_H$ .<sup>34</sup> This means that the principal verifies a single type at this

<sup>&</sup>lt;sup>34</sup> If  $\gamma_L < \gamma_H$ , the indifference of type  $\gamma_H$  between verification with efficient spending and no verification with spending  $\pi_H$  would imply that  $\gamma_L$  strictly prefers verification with efficient spending to no verification with spending  $\pi_H$ , a contradiction.

point who is indifferent between verification with efficient spending, no verification with higher spending at  $\pi_{H}$ , and no verification with lower spending at  $\pi_{L}$ . Conditional on no verification, this is thus an allocation in which the agent faces a hole  $[\pi_{L}, \pi_{H}]$ ; namely, he is not allowed to choose spending in this set but can choose spending immediately below and above it. But our analysis in section III shows that such a hole is suboptimal conditional on no verification; hence, following no verification, the principal would have a strict incentive to close the hole. This shows that a rule with decreasing verification cannot be incentive compatible when the principal has limited commitment power, allowing us to establish that TEC is optimal in this case.

Recall that in the full-commitment environment, TEC is optimal if the principal's cost of verification is small enough (as shown in proposition 3), but more complex rules may be optimal otherwise (as shown in proposition 4). In contrast, proposition 5 tells us that TEC is optimal under limited commitment for any verification cost for which verification is optimal.<sup>35</sup> Given the prevalence of TEC rules in the real world, these results suggest that limitations to commitment power are also prevalent. Moreover, these limitations may be an important reason behind the broad use of TEC in applications.

As a final remark, it is worth noting that while TEC is optimal both when the principal has full commitment power and a small verification cost as well as when she has limited commitment power, the specific details of an optimal TEC rule vary with each case. Under full commitment, an optimal TEC rule  $\{\gamma^*, \gamma^{**}\}$  is such that the principal prefers to verify types  $\gamma > \gamma^{**}$  to assign them efficient spending rather than bunch them at  $\pi_A(\gamma^*)$  without verification, whereas the opposite is true for types  $\gamma \in [\gamma^*, \gamma^{**}]$ . Hence, the principal is indifferent between verifying and not verifying the threshold type  $\gamma^{**}$ ; that is, the increase in assigned spending at  $\gamma^{**}$  exactly compensates the principal for the cost  $\phi$  of verifying this type. In contrast, under limited commitment, it is the agent who is indifferent at  $\gamma^{**}$ : as implied by lemma 7, type  $\gamma^{**}$  must be indifferent between being verified and assigned efficient spending versus not being verified and assigned  $\pi_A(\gamma^*)$ , and thus any increase in assigned spending at  $\gamma^{**}$  must exactly compensate this type for his verification cost  $\alpha\phi$ .

#### VI. Conclusion

This paper has studied the trade-off between commitment and flexibility in the presence of costly state verification. We have examined a general delegation problem in which a principal delegates decision-making to

 $<sup>^{</sup>_{35}}$  Additionally, as noted in sec. II.A, this result applies not only to  $\alpha \in [0,1]$  but also to any  $\alpha \geq 0.$ 

an agent who has superior information about the efficient action but is biased toward higher actions. A novel element of our framework is that the principal can verify the agent's private information. Because verification is costly, the principal wishes to use this technology selectively and in a way that supplements delegation and improves her commitment versus flexibility trade-off.

Our results provide insight into how the principal achieves this by designing an optimal delegation rule. We have shown that under full commitment power and a small enough verification cost, an optimal rule is a TEC, allowing the agent to freely select any action up to a threshold or to request verification and the efficient action above the threshold. When the verification cost is larger, the principal may instead prefer to require verification only for intermediate actions, still imposing some discipline on the agent but saving on verification costs. However, the optimality of TEC is restored under mild limitations to the principal's commitment power. Specifically, if the principal is unable to commit to not changing the agent's permissible action set following the verification decision and result, TEC is optimal for any verification cost for which verification is optimal.

As we have discussed, there are a variety of applications where delegation is central and rules make use of verification by specifying escape clauses. Our analysis sheds light on the optimal structure of escape clauses and provides a theoretical foundation for the common use of TEC rules. More broadly, our framework may help inform the empirical analysis of real-world rules. Data on delegation policies are increasingly available and offer an opportunity to explore the design of these rules in more detail. For instance, in the context of capital budgeting, it has been observed that the extent of capital rationing and the use of verification vary across firms (e.g., Ross 1986), and one could study how these differences relate to firm size, industry, and other factors that are likely to affect senior management's cost of verifying the quality of projects. In the context of fiscal policy, countries' fiscal rules vary in the use of escape clause provisions and their trigger events (Schaechter et al. 2012), and these may correlate with countries' institutional and macroeconomic conditions that affect the cost of auditing a government as well as the need for flexibility to respond to shocks.

Last, by uncovering a new set of issues that arise when verification is introduced to a setting in which both commitment and flexibility are valuable, our paper opens the door for further work that can help understand the optimal joint design of delegation and verification. We have focused on a simple model that emphasizes the main forces at play but abstracts from other potentially relevant aspects, for instance, associated with more complex verification technologies. We close by discussing some possible extensions and variations of our work. Random verification.—As in the seminal work of Townsend (1979), we have considered deterministic verification; namely, we assumed that the principal's rule assigns  $a(\gamma) \in \{0, 1\}$  to each agent type  $\gamma$ . More generally, one could allow for mechanisms in which the principal randomizes over the verification assignment, choosing a probability of verification for each type. The literature on financial contracting and tax collection finds that random verification can yield different results compared with deterministic verification; see Border and Sobel (1987) and Mookherjee and Png (1989).

Our focus on deterministic verification is motivated by the applications we study. Take capital budgeting. As captured by the game form that we have proposed, here the agent (division head) decides whether to request verification to obtain approval to choose actions that are not allowed by the principal (senior management) without verification. The principal commits to following the agent's request, and so it is the agent's choice whether to trigger the verification process. Unlike in other applications where verification/audit is used to determine fines for misbehavior (e.g., tax collection), random verification is not natural in these contexts. Using the timing of section II.B, random verification would mean that the agent chooses in step 2 not between verification. This is rarely observed in practice, possibly because committing to a nondegenerate lottery can be difficult for a principal.<sup>36</sup>

That said, in a setting in which there are no limitations to the principal's commitment power, the study of random verification could be an interesting extension of our work. As noted in the aforementioned literature, one issue is that an optimal randomized mechanism would depend on the extent to which the agent can be punished following verification, which in turn would depend on preference assumptions in our setting, given that punishments are imposed through the spending allocation only. Importantly, these punishments must be bounded; otherwise, the efficient allocation can be approached with a rule that verifies all agent types with very low probability and arbitrarily punishes the agent when verification reveals that he has deviated.<sup>37</sup> Such a possibility not only yields rather implausible predictions but also implies that an optimal rule in general will fail to exist unless a bound on punishments is imposed.

<sup>&</sup>lt;sup>36</sup> When the decision is simply over verification or no verification, commitment to the verification policy would in principle be facilitated by the fact that the principal's execution of the agent's request can be easily monitored. However, checking that the principal implements a specific lottery is harder, as it requires monitoring of the randomization itself rather than its outcome.

<sup>&</sup>lt;sup>37</sup> In our game form, a rule that approaches the efficient allocation would be implemented by inducing each agent type to choose a different lottery over verification.

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Imperfect verification.—Also following Townsend (1979), our analysis assumed that verification reveals the agent's type perfectly. An alternative would be to consider imperfect verification, namely, verification that provides only imperfect information about the agent's type. For example, in the context of capital budgeting in organizations, senior management may review information about the benefits of a project that a division manager advocates, but the available documentation may be incomplete and fail to reveal the full merits of the project.

A simple specification that may be possible to accommodate within our framework is when imperfect verification either reveals the agent's type perfectly or provides no information (i.e., when there are no false results). Provided that the principal can severely punish the agent (through the spending allocation), she would be able to prevent, at no cost, any deviation in which an agent type mimics another type who is verified, as is true in our problem with perfect verification. Yet a difference introduced by imperfect verification is that the principal may not observe the agent's type and thus may not be able to assign a type-dependent spending allocation for the case of verification and no information. Allowing for imperfect verification that may produce false results would naturally introduce further issues, as now punishing an agent type for mimicking another type who is verified would require imposing punishments on path.

How imperfect is imperfect verification? At one extreme, if verification is sufficiently accurate, we conjecture that our qualitative results would remain valid. At the other extreme, if verification is sufficiently inaccurate, it would become equivalent to money burning, and the results of the literature on when money burning is used in an optimal delegation rule would then apply (see Amador, Werning, and Angeletos 2006; Amador and Bagwell 2013; Ambrus and Egorov 2017). More generally, it would be of interest to explore the role of verification in delegation away from these two extremes.

*Verification costs.*—We have considered verification costs that are both type independent and exogenous. An extension of our problem could explore the effects of type-dependent verification costs: the principal's cost of verifying the agent's private information may be increasing in his type, for example, because more evidence is needed to verify larger project benefits, or one may take the view that verification costs are actually lower for extreme types, as these states are more visible. One possible difficulty is that monotonicity of the spending allocation (as shown in lemma 5) may fail to hold if verification cost increase very rapidly with the agent's type. But if the verification cost function is such that the principal would still prefer to swap the verification and spending allocations of two types  $\gamma$  and  $\gamma' > \gamma$  whenever type  $\gamma$  has higher spending than  $\gamma'$ ,

monotonicity will be satisfied and our analysis could be extended to allow for type-dependent verification costs.

Another variation would be to endogenize  $\alpha$ , so that the principal can affect the agent's cost of verification. Our results would continue to hold under this extension. Specifically, when the principal has full commitment power, we have derived conditions for the optimality of a TEC rule that are independent of the value of  $\alpha \in [0, 1]$ , and clearly the principal's welfare under this rule does not vary with  $\alpha$  either. Thus, the principal in this case would be indifferent over any  $\alpha \in [0, 1]$ , whereas  $\alpha = 0$  would be preferred by the agent. More generally, under full commitment it is always optimal for the principal to set  $\alpha = 0$ , as a zero verification cost for the agent maximally relaxes the agent's incentive compatibility constraint (4). Things are more interesting in the setting of section V, where the principal has limited commitment power. In this case, the principal may want to set a strictly positive verification cost for the agent, as that limits the set of agent types that may want to demand verification and efficient spending. In any case, for any given  $\alpha \ge 0$  that the principal would set, our analysis and the optimality of TEC apply without change.

*Transfers.*—Our focus has been on a canonical delegation problem in which transfers between the principal and the agent are not feasible. There are various ways in which transfers could be introduced in our framework and used to alter the feasibility and cost of inducing different allocations. Transfers could be contingent on the agent's verification decision and/or the verification result; moreover, the principal could offer different allowable spending sets for the agent to choose from and specify transfers associated with each set. These questions are beyond the scope of our paper, and so we leave them for future research.

#### Appendix A

#### Proofs

#### A1. Proof of Proposition 1

The claim follows from proposition 1 (part a) in Amador and Bagwell (2013, 1551).

#### A2. Proof of Lemma 1

Suppose by contradiction that a rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  with  $a(\gamma) = 1$  for all  $\gamma \in \Gamma$  is optimal. Since the incentive compatibility constraint (4) is trivially satisfied under this rule, it must be that  $\pi(\gamma) = \pi_P(\gamma)$  for all  $\gamma \in \Gamma$ . Define  $\gamma' \in \Gamma$  as the solution to

$$U_{A}(\gamma', \pi_{P}(\gamma')) - \alpha \phi = U_{A}\left(\gamma', \pi_{P}\left(\underline{\gamma}\right)\right)$$
(15)

if such a solution exists and  $\gamma' = \bar{\gamma}$  otherwise. Consider now a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ , with  $\tilde{a}(\gamma) = 0$  and  $\tilde{\pi}(\gamma) = \pi_P(\underline{\gamma})$  for  $\gamma \leq \gamma'$  and  $\tilde{a}(\gamma) = a(\gamma)$ ,  $\tilde{\pi}(\gamma) = \pi(\gamma)$  for  $\gamma > \gamma'$ . By single crossing and the definition of  $\gamma'$  in (15), the perturbed rule satisfies the incentive compatibility constraint (4). Conditional on  $\gamma > \gamma'$ , this rule yields the same expected welfare to the principal and the agent as the original rule. However, conditional on  $\gamma \leq \gamma'$ , the perturbed rule yields the agent a higher welfare than the original one, since by (15),

$$U_{A}(\gamma, \pi_{P}(\gamma)) - \alpha \phi \leq U_{A}\left(\gamma, \pi_{P}\left(\underline{\gamma}\right)\right)$$
(16)

for all  $\gamma \leq \gamma'$ . Moreover, note that (2) implies

$$U_{\!\scriptscriptstyle A}(\gamma, \pi_{\scriptscriptstyle P}(\gamma)) = U_{\!\scriptscriptstyle A}\!\left(\gamma, \pi_{\scriptscriptstyle P}\!\left(\underline{\gamma}
ight)
ight) \geq U_{\!\scriptscriptstyle P}(\gamma, \pi_{\scriptscriptstyle P}(\gamma)) = U_{\!\scriptscriptstyle P}\!\left(\gamma, \pi_{\scriptscriptstyle P}\!\left(\underline{\gamma}
ight)
ight)$$

for all  $\gamma > \gamma$ . Hence, using (16) and the fact that  $\alpha \in [0, 1]$  and  $\phi > 0$ ,

$$U_{\scriptscriptstyle P}(\gamma, \pi_{\scriptscriptstyle P}(\gamma)) - \phi < U_{\scriptscriptstyle P}\Bigl(\gamma, \pi_{\scriptscriptstyle P}\Bigl(\underline{\gamma}\Bigr)\Bigr)$$

for all  $\gamma \leq \gamma'$ . Conditional on  $\gamma \leq \gamma'$ , the principal is therefore strictly better off under the perturbed rule than under the original rule. It follows that the perturbed rule with no verification below  $\gamma'$  strictly dominates the original rule, contradicting the optimality of a rule that verifies all types.

REMARK. If  $\alpha = 0$ , then  $\gamma' = \underline{\gamma}$  and the perturbed rule we have constructed increases the principal's welfare from type  $\underline{\gamma}$  relative to verifying all types. The claim therefore follows in this case, given our optimality condition (5). Moreover, when  $\alpha = 0$ , we can also consider a different perturbation to prove the claim without relying on this condition. Specifically, take a perturbed rule that prescribes  $\tilde{a}(\gamma) = 0$  for all  $\gamma \in [\underline{\gamma}, \underline{\gamma} + \varepsilon]$  for  $\varepsilon > 0$  arbitrarily small, bunching all such  $\gamma$  at their average efficient spending level. This rule is incentive compatible and increases the principal's welfare relative to verifying all types: given  $\varepsilon$  small enough, the welfare loss from not assigning efficient spending to  $\gamma \in [\underline{\gamma}, \underline{\gamma} + \varepsilon]$ is second order, while the gain from saving on verification costs is first order.

#### A3. Proof of Lemma 2

Suppose that an optimal rule features verification that is weakly increasing everywhere. By lemma 1,  $a(\gamma) = 0$  for some  $\gamma \in \Gamma$ , and hence this rule must feature a no-verification region followed by a verification region. That is, the principal solves (3) and (4) by choosing a threshold  $\gamma^{**}$  such that  $a(\gamma) = 0$  for  $\gamma < \gamma^{**}$  and  $a(\gamma) = 1$  for  $\gamma > \gamma^{**}$  as well as a spending allocation  $\pi(\gamma)$  for each  $\gamma \in \Gamma$ .

Now consider a relaxed version of this problem in which the principal chooses an optimal allocation in the no-verification and verification regions separately, ignoring the incentives of types in one region to deviate to the other region. Taking the no-verification region to be  $[\underline{\gamma}, \gamma^{**}]$ , it follows from proposition 1 that an optimal allocation is a threshold  $\gamma^* < \gamma^{**}$  such that  $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$ for each  $\gamma \in [\underline{\gamma}, \gamma^{**}]$ . For the verification region  $(\gamma^{**}, \overline{\gamma}]$ , since incentive compatibility is trivially satisfied, an optimal allocation assigns  $\pi_P(\gamma)$  to each  $\gamma \in (\gamma^{**}, \overline{\gamma}]$ . Note that the resulting rule for the whole set  $\Gamma$  is TEC. Moreover, because this rule solves a relaxed problem, it is sufficient to show that it is incentive compatible over the whole set  $\Gamma$  to prove its optimality in the original problem.

To show incentive compatibility, note first that incentive compatibility within each region is guaranteed by construction. Furthermore, since, as explained in section II.C, no type would have incentives to deviate to mimic a different type that is verified, incentive compatibility is satisfied for all  $\gamma \in [\underline{\gamma}, \gamma^{**}]$ . All that is left to be shown is that no type  $\gamma \in (\gamma^{**}, \overline{\gamma}]$  has incentives to deviate to mimic a type  $\hat{\gamma} \in [\gamma, \gamma^{**}]$ :

$$U_{A}(\gamma, \pi_{\scriptscriptstyle P}(\gamma)) - lpha \phi \geq U_{A}(\gamma, \pi(\hat{\gamma})) ext{ for all } \gamma > \gamma^{**}, \hat{\gamma} \leq \gamma^{**}$$

The single-crossing condition in  $U_A$  implies that a sufficient condition for the above inequality to hold is

$$U_{A}(\gamma, \pi_{P}(\gamma)) - \alpha \phi \geq U_{A}(\gamma, \pi_{A}(\gamma^{*})) \text{ for all } \gamma > \gamma^{**}.$$
(17)

Now note that optimality of  $\gamma^{**}$  for the principal implies

$$U_{P}(\gamma, \pi_{P}(\gamma)) - \phi \geq U_{P}(\gamma, \pi_{A}(\gamma^{*})) \text{ for all } \gamma > \gamma^{**}.$$
(18)

Given the agent's bias (2) and  $\alpha \in [0, 1]$ , (18) implies (17) if  $\pi_P(\gamma) \ge \pi_A(\gamma^*)$  for all  $\gamma > \gamma^{**}$ , or, equivalently, since  $\pi'_P(\gamma) > 0$ , if

$$\pi_P(\boldsymbol{\gamma}^{**}) \ge \pi_A(\boldsymbol{\gamma}^*). \tag{19}$$

We prove that the TEC rule that we constructed satisfies (19). The optimal threshold  $\gamma^*$  in the no-verification region solves

$$\max_{\gamma^* \in [0,\gamma^{**})} \Bigg\{ \int_{\underline{\gamma}}^{\gamma^*} U_{\scriptscriptstyle P}(\gamma,\pi_{\scriptscriptstyle A}(\gamma)) f(\gamma) d\gamma + \int_{\gamma^*}^{\gamma^{**}} U_{\scriptscriptstyle P}(\gamma,\pi_{\scriptscriptstyle A}(\gamma^*)) f(\gamma) d\gamma \Bigg\}.$$

The first-order condition yields

$$\int_{\gamma^*}^{\gamma^{**}} \frac{\partial U_P(\gamma, \pi_A(\gamma^*))}{\partial \pi_A(\gamma^*)} \pi_A'(\gamma^*) f(\gamma) d\gamma = 0.$$

Note that  $\pi'_A(\gamma^*) > 0$ ,  $\partial U_P(\gamma, \pi_A(\gamma^*)) / \partial \pi_A(\gamma^*) < 0$  if  $\pi_P(\gamma) < \pi_A(\gamma^*)$ , and  $\partial U_P(\gamma, \pi_A(\gamma^*)) / \partial \pi_A(\gamma^*) > 0$  if  $\pi_P(\gamma) > \pi_A(\gamma^*)$ . Hence, the first-order condition requires  $\pi_P(\gamma) > \pi_A(\gamma^*)$  for some  $\gamma \in [\gamma^*, \gamma^{**}]$ , implying that (19) must hold.

#### A4. Proof of Proposition 2

Assume  $b(\pi) = 0$  for all  $\pi \in [\underline{\pi}, \overline{\pi}]$ . Suppose by contradiction that an optimal rule specifies  $a(\gamma) = 1$  for some  $\gamma \in \Gamma$  but TEC is not optimal. By lemma 2, this rule must feature decreasing verification. We proceed by showing that an optimal rule cannot feature decreasing verification at any  $\gamma' \in \Gamma$ .

Consider first decreasing verification at some  $\gamma' \in \Gamma$  with  $a(\gamma') = 0$ , so  $a(\gamma' - \varepsilon) = 1$  for some  $\varepsilon > 0$  arbitrarily small. As shown in the text, the optimality of verifying type  $\gamma' - \varepsilon$  implies (6) and  $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$ , whereas the optimality of not verifying  $\gamma'$  implies (7). However, the two equations together with  $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$  violate the single-crossing condition (1), a contradiction.

Consider next decreasing verification at some  $\gamma' \in \Gamma$  with  $a(\gamma') = 1$ , so  $a(\gamma' + \varepsilon) = 0$  for some  $\varepsilon > 0$  arbitrarily small. Analogous arguments to those above apply to this case and yield a contradiction.

#### A5. Proof of Lemma 3

Suppose that a TEC rule with cutoffs  $\gamma^*$  and  $\gamma^{**}$  is a solution to (8) and (9). Note that any rule satisfying constraint (4) will satisfy constraint (9). Hence, (8) and (9) are a relaxed version of (3) and (4), implying that any solution to (8) and (9) that satisfies (4) will also be a solution to (3) and (4). It follows that to prove the claim, all we need to show is that the TEC rule that solves (8) and (9) will satisfy constraint (4). It is immediate that for any  $\gamma$  with  $a(\gamma) = 0$ , (9) being satisfied implies that (4) will be satisfied. Now consider  $\gamma$  with  $a(\gamma) = 1$ . Optimality of verifying type  $\gamma$  under a TEC rule that solves (8) and (9) implies

$$U_P(\gamma, \pi_P(\gamma)) - \phi \ge U_P(\gamma, \pi_A(\gamma^*)), \tag{20}$$

since a perturbation that assigns no verification and spending level  $\pi_A(\gamma^*)$  to a type  $\gamma > \gamma^{**}$  is incentive compatible. Note that by the arguments in the proof of lemma 2, a TEC rule that solves (8) and (9) satisfies  $\pi_P(\gamma) \ge \pi_A(\gamma^*)$  for all  $\gamma > \gamma^{**}$ . Hence, combining (20) with (2) and the fact that  $\alpha \in [0, 1]$  implies

$$U_A(\gamma, \pi_P(\gamma)) - lpha \phi \geq U_A(\gamma, \pi_A(\gamma^*)).$$

It follows that (4) is satisfied for type  $\gamma$  with  $a(\gamma) = 1$ .

#### A6. Proof of Lemma 4

Suppose that a rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  solving (8) and (9) specifies  $a(\gamma') = 1$  for some  $\gamma' \in \Gamma$ .

To prove that the rule specifies  $\pi(\gamma') \leq \pi_A(\gamma')$ , suppose by contradiction that  $\pi(\gamma') > \pi_A(\gamma')$ . Consider a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma') = 1$  and  $\tilde{\pi}(\gamma') = \pi_A(\gamma')$  while keeping the allocation unchanged for all  $\gamma \neq \gamma'$ . This perturbation strictly increases the principal's welfare conditional on  $\gamma'$ , leaves the principal's welfare conditional on  $\gamma \neq \gamma'$  unchanged, and is incentive compatible for the agent.

Similarly, to prove that the rule specifies  $\pi(\gamma') \ge \pi_P(\gamma')$ , suppose by contradiction that  $\pi(\gamma') < \pi_P(\gamma')$ . Consider a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma') = 1$  and  $\tilde{\pi}(\gamma') = \pi_P(\gamma)$  while keeping the allocation unchanged for all  $\gamma \neq \gamma'$ . This perturbation strictly increases the principal's welfare conditional on  $\gamma'$ , leaves the principal's welfare conditional on  $\gamma \neq \gamma'$  unchanged, and is incentive compatible for the agent.

Finally, we prove that the rule must specify  $\pi(\gamma') = \pi_P(\gamma')$  if (9) does not bind for  $\gamma'$ . Suppose by contradiction that (9) does not bind for  $\gamma'$  and  $\pi(\gamma') \neq \pi_P(\gamma')$ . By the claim above,  $\pi(\gamma') \geq \pi_P(\gamma')$ , and thus the rule must set  $\pi(\gamma') > \pi_P(\gamma')$ . But then a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma') = 1$  and  $\tilde{\pi}(\gamma') = \pi(\gamma') - \varepsilon$ for  $\varepsilon > 0$  arbitrarily small while keeping the allocation unchanged for all  $\gamma \neq \gamma'$ strictly increases the principal's welfare conditional on  $\gamma'$ , leaves the principal's welfare conditional on  $\gamma \neq \gamma'$  unchanged, and is incentive compatible for the agent. 4560

#### A7. Proof of Lemma 5

Suppose by contradiction that a rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  that solves (8) and (9) specifies  $\pi(\gamma') > \pi(\gamma'')$  for some  $\gamma' < \gamma''$ . We consider four cases separately. *Case 1.*—Suppose  $a(\gamma') = a(\gamma'') = 0$ . Then (9) for  $\gamma'$  and  $\gamma''$  requires

$$egin{aligned} U_{\scriptscriptstyle\!A}(\gamma',\pi(\gamma')) &\geq U_{\scriptscriptstyle\!A}(\gamma',\pi(\gamma'')), \ U_{\scriptscriptstyle\!A}(\gamma'',\pi(\gamma'')) &\geq U_{\scriptscriptstyle\!A}(\gamma'',\pi(\gamma')), \end{aligned}$$

which together imply

$$U_A(\gamma', \pi(\gamma')) - U_A(\gamma', \pi(\gamma'')) \ge U_A(\gamma'', \pi(\gamma')) - U_A(\gamma'', \pi(\gamma'')).$$
(21)

However, given  $\gamma' < \gamma''$  and  $\pi(\gamma') > \pi(\gamma'')$ , (21) violates the single-crossing condition in  $U_{4}$ , a contradiction.

*Case* 2.—Suppose  $a(\gamma') = a(\gamma'') = 1$ . By lemma 4,  $\pi(\gamma'') \ge \pi_P(\gamma'')$ , and thus  $\pi(\gamma') > \pi(\gamma'')$  implies  $\pi(\gamma') > \pi_P(\gamma'') > \pi_P(\gamma'')$ . Using lemma 4 again, it then follows that (9) binds for  $\gamma'$ ; that is, there exists  $\hat{\gamma} \in \Gamma$  with  $a(\hat{\gamma}) = 0$  such that

$$U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\hat{\gamma})).$$
(22)

Furthermore, note that we must have  $\pi(\hat{\gamma}) \ge \pi(\gamma')$ , since  $\pi(\gamma') \le \pi_A(\gamma')$  and  $U_A$  is strictly concave. Incentive compatibility for  $\gamma''$  requires

$$U_{\scriptscriptstyle\!A}(\gamma'',\pi(\gamma''))\geq U_{\scriptscriptstyle\!A}(\gamma'',\pi(\hat{\gamma})),$$

which, combined with the observation that

$$\pi(\gamma'') < \pi(\gamma') \leq \pi_A(\gamma') < \pi_A(\gamma''), \tag{23}$$

implies

$$U_{A}(\gamma'',\pi(\gamma')) \geq U_{A}(\gamma'',\pi(\hat{\gamma})).$$
(24)

Combining (22) and (24) yields

$$U_{A}(\gamma',\pi(\hat{\gamma})) - U_{A}(\gamma',\pi(\gamma')) > U_{A}(\gamma'',\pi(\hat{\gamma})) - U_{A}(\gamma'',\pi(\gamma')).$$
(25)

However, given  $\gamma' < \gamma''$  and  $\pi(\hat{\gamma}) \ge \pi(\gamma')$ , (25) violates the single-crossing condition in  $U_{A}$ , a contradiction.

*Case 3.*—Suppose  $a(\gamma') = 1$  and  $a(\gamma'') = 0$ . Note that (23) must hold. Then consider a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma'') = 1$  and  $\tilde{\pi}(\gamma'') = \pi(\gamma')$ while leaving the allocation for types  $\gamma \neq \gamma''$  unchanged. Since incentive compatibility was initially satisfied and  $\gamma' < \gamma''$  while (23) holds, this perturbation is incentive compatible. Optimality of the original rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  therefore requires this perturbation to not strictly increase the principal's welfare, which requires

$$U_P(\gamma'',\pi(\gamma''))\geq U_P(\gamma'',\pi(\gamma'))-oldsymbol{\phi}.$$

The single-crossing condition in  $U_P$  then implies

$$U_P(\gamma', \pi(\gamma'')) > U_P(\gamma', \pi(\gamma')) - \phi.$$
(26)

Now consider a different perturbed rule  $\{\hat{a}(\gamma), \hat{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\hat{a}(\gamma') = 0$  and  $\hat{\pi}(\gamma') = \pi(\gamma'')$  while leaving the allocation for types  $\gamma \neq \gamma'$  unchanged. Equation (26) implies that this perturbation would strictly increase the principal's welfare. Hence, optimality of the original rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  requires that this perturbation violate incentive compatibility; that is, there must exist  $\hat{\gamma} \in \Gamma$  with  $a(\hat{\gamma}) = 0$  such that

$$U_{A}(\gamma', \pi(\hat{\gamma})) > U_{A}(\gamma', \pi(\gamma'')).$$
(27)

Note that since  $\pi(\gamma'') < \pi_A(\gamma')$ , we must have  $\pi(\hat{\gamma}) > \pi(\gamma'')$ . Moreover, by incentive compatibility being satisfied under the original rule, we have

$$U_A(\gamma'',\pi(\gamma''))\geq U_A(\gamma'',\pi(\hat{\gamma})).$$

Combining this equation with (27) yields

$$U_{A}(\gamma',\pi(\hat{\gamma})) - U_{A}(\gamma',\pi(\gamma'')) > U_{A}(\gamma'',\pi(\hat{\gamma})) - U_{A}(\gamma'',\pi(\gamma'')).$$
(28)

However, given  $\gamma' < \gamma''$  and  $\pi(\hat{\gamma}) > \pi(\gamma'')$ , (28) violates the single-crossing condition in  $U_A$ , a contradiction.

*Case 4.*—Suppose  $a(\gamma') = 0$  and  $a(\gamma'') = 1$ . By lemma 4,  $\pi(\gamma'') \le \pi_A(\gamma'')$ , and hence given  $\pi(\gamma') > \pi(\gamma'')$ , incentive compatibility for type  $\gamma''$  requires  $\pi(\gamma') > \pi_A(\gamma'')$ . Consider a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma') = 1$  and  $\tilde{\pi}(\gamma') = \pi(\gamma'')$  while leaving the allocation for types  $\gamma \neq \gamma'$  unchanged. Since the original rule satisfies incentive compatibility for  $\gamma''$ , single crossing implies that this perturbation is incentive compatible for  $\gamma'$ . Optimality of the original rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  then requires this perturbation to not strictly increase the principal's welfare, which requires

$$U_{\!\scriptscriptstyle P}(\gamma',\pi(\gamma')) \geq U_{\!\scriptscriptstyle P}(\gamma',\pi(\gamma'')) - oldsymbol{\phi}.$$

The single-crossing condition in  $U_P$  then implies

$$U_{P}(\gamma'', \pi(\gamma')) > U_{P}(\gamma'', \pi(\gamma'')) - \phi.$$
(29)

Now consider a different perturbed rule  $\{\hat{a}(\gamma), \hat{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\hat{a}(\gamma'') = 0$  and  $\hat{\pi}(\gamma'') = \pi(\gamma')$  while leaving the allocation for types  $\gamma \neq \gamma''$  unchanged. Equation (29) implies that such a perturbation would strictly increase the principal's welfare. Hence, optimality of the original rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  requires that this perturbation violate incentive compatibility; that is, there must exist  $\hat{\gamma} \in \Gamma$  with  $a(\hat{\gamma}) = 0$  such that

$$U_A(\gamma'', \pi(\hat{\gamma})) > U_A(\gamma'', \pi(\gamma')). \tag{30}$$

Note that since  $\pi(\gamma') > \pi_A(\gamma')$ , we must have  $\pi(\hat{\gamma}) < \pi(\gamma')$ . Moreover, by incentive compatibility being satisfied under the original rule, we have

$$U_A(\gamma',\pi(\gamma'))\geq U_A(\gamma',\pi(\hat{\gamma}))$$

Combining this equation with (30) yields

$$U_{\scriptscriptstyle A}(\gamma',\pi(\gamma')) - U_{\scriptscriptstyle A}(\gamma',\pi(\hat{\gamma})) > U_{\scriptscriptstyle A}(\gamma'',\pi(\gamma')) - U_{\scriptscriptstyle A}(\gamma'',\pi(\hat{\gamma})). \tag{31}$$

However, given  $\gamma' < \gamma''$  and  $\pi(\hat{\gamma}) < \pi(\gamma')$ , (31) violates the single-crossing condition in  $U_A$ , a contradiction.

#### A8. Proof of Lemma 6

Suppose that a rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  solves (8) and (9) and features decreasing verification at some  $\gamma' < \bar{\gamma}$ , with  $a(\gamma') = 1$ . Then  $a(\gamma' + \varepsilon) = 0$  for some  $\varepsilon > 0$  arbitrarily small. Suppose that it were the case that  $\pi(\gamma' + \varepsilon) = \pi(\gamma')$ . Then optimality of this rule would be violated, as a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma') = 0$  and  $\tilde{\pi}(\gamma') = \pi(\gamma')$  while keeping the allocation unchanged for  $\gamma \neq \gamma'$  would be incentive compatible and strictly increase the principal's welfare (recall  $\phi > 0$ ). It follows that  $\pi(\gamma' + \varepsilon) \neq \pi(\gamma')$ , and hence by lemma 5,  $\pi(\gamma' + \varepsilon) > \pi(\gamma')$ . Moreover, by lemma 4,  $\pi(\gamma') \leq \pi_A(\gamma')$ , and thus incentive compatibility for  $\gamma'$  would be violated if it were the case that  $\pi_A(\gamma') \geq \pi(\gamma' + \varepsilon) > \pi(\gamma')$ . It therefore follows that

$$\pi(\gamma' + \varepsilon) > \pi_A(\gamma') \tag{32}$$

for  $\varepsilon > 0$  arbitrarily small. Lemma 5 then implies  $\pi(\gamma) > \pi_A(\gamma')$  for all  $\gamma \in (\gamma', \gamma'')$ ,  $\gamma'' \equiv \min\{\pi_P^{-1}(\pi_A(\gamma')), \bar{\gamma}\}$ , which implies

$$\int_{\gamma'}^{\gamma'} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma < \int_{\gamma'}^{\gamma'} U_P(\gamma, \pi_A(\gamma')) f(\gamma) d\gamma.$$
(33)

Moreover, by definition,

$$\int_{\gamma'}^{\bar{\gamma}} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma \leq \int_{\gamma''}^{\bar{\gamma}} U_P(\gamma, \pi_P(\gamma)) f(\gamma) d\gamma.$$
(34)

Combining (33) and (34) and taking into account that  $1 - F(\gamma') > 0$  yields (10).

Suppose next that a rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  solves (8) and (9) and features decreasing verification at some  $\gamma' < \overline{\gamma}$ , with  $a(\gamma') = 0$ . Then  $a(\gamma' - \varepsilon) = 1$  for some  $\varepsilon > 0$  arbitrarily small, and arguments analogous to those above yield (10).

#### A9. Proof of Proposition 3

The arguments in the proofs of lemmas 1 and 2 apply to the relaxed problem, implying that if a solution to (8) and (9) involves verifying some type  $\gamma \in \Gamma$ , this solution is either a TEC rule or a rule that features decreasing verification at some  $\gamma' \in \Gamma$ . To prove the optimality of TEC for  $\phi < \overline{\phi}$ , we thus proceed by showing that for any such verification cost, a rule featuring decreasing verification cannot be a solution to (8) and (9).

Suppose that a rule  $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$  solves (8) and (9) and features decreasing verification. Denote by  $\gamma^{**}$  the infimum of the lowest verification region under this rule. Now consider a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that sets  $\tilde{a}(\gamma) = 0$  for  $\gamma < \gamma^{**}$ ,  $\tilde{a}(\gamma^{**}) = a(\gamma^{**})$ , and  $\tilde{a}(\gamma) = 1$  for  $\gamma > \gamma^{**}$ . If  $\tilde{a}(\gamma) = 0$ , let  $\tilde{\pi}(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$  for  $\gamma^*$  as defined in proposition 1 under  $\bar{\gamma}' = \gamma^{**}$ . If  $\tilde{a}(\gamma) = 1$ , let  $\tilde{\pi}(\gamma) = \pi_P(\gamma)$ . By the arguments in the proof of lemma 2, this rule is incentive compatible for types prescribed no verification and sets  $\pi_A(\gamma^*) \leq \pi_P(\gamma^{**})$ . Moreover, given this inequality and the fact that  $\alpha = 0$ , it follows that

the rule is also incentive compatible for types prescribed verification. We now show that this rule strictly increases the principal's expected welfare for  $\phi < \overline{\phi}$ , contradicting the optimality of the original rule. Denote by  $\gamma'$  the lowest type above  $\gamma^{**}$  featuring decreasing verification in the original rule. Then the change in the principal's expected welfare from using the perturbed rule instead of the original rule is

$$\int_{\underline{\gamma}}^{\gamma^{**}} (U_P(\gamma, \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}) - U_P(\gamma, \pi(\gamma))) f(\gamma) d\gamma + \int_{\gamma^{**}}^{\overline{\gamma}} (U_P(\gamma, \pi_P(\gamma))) (35) - U_P(\gamma, \pi(\gamma))) f(\gamma) d\gamma - \int_{\underline{\gamma}}^{\overline{\gamma}} [\phi(1 - a(\gamma))] f(\gamma) d\gamma.$$

Note that since all types above  $\gamma^{**}$  are verified, the principal's welfare conditional on the agent's type being in the no-verification region of the perturbed rule is optimized subject to fewer incentive compatibility constraints in this rule compared with the original rule. Hence, the first term in (35) is weakly positive.

To evaluate the second and third terms in (35), suppose first that  $\gamma' < \bar{\gamma}$ . Then by lemma 6, the second term in (35) satisfies

$$\int_{\gamma^{**}}^{\bar{\gamma}} (U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))) f(\gamma) d\gamma \geq \int_{\gamma'}^{\bar{\gamma}} (U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))) f(\gamma) d\gamma \\ \geq (1 - F(\gamma')) \eta(\gamma').$$
(36)

Moreover, the third term in (35) satisfies

$$-\int_{\gamma'}^{\bar{\gamma}} [\phi(1-a(\gamma))] f(\gamma) d\gamma > -(1-F(\gamma')) \bar{\phi}$$
  
=  $-(1-F(\gamma')) \min_{\gamma \in \Gamma} \eta(\gamma).$  (37)

Together, (36) and (37) imply that the perturbation strictly increases welfare.

Suppose next that  $\gamma' = \bar{\gamma}$ . Analogous arguments to those above imply that the perturbation makes the principal weakly better off conditional on  $\gamma < \bar{\gamma}$ . To evaluate the change in welfare conditional on  $\gamma = \bar{\gamma}$ , note that in this case we must have  $a(\bar{\gamma}) = 0$  and  $a(\bar{\gamma} - \varepsilon) = 1$  for  $\varepsilon > 0$  arbitrarily small. Analogous arguments to those in the proof of lemma 6 then imply  $\pi(\bar{\gamma}) \ge \pi_A(\bar{\gamma})$ . Moreover, by (11),

$$\eta(\bar{\gamma}) = \lim_{\gamma \uparrow \bar{\gamma}} \eta(\gamma) = U_P(\bar{\gamma}, \pi_P(\bar{\gamma})) - U_P(\bar{\gamma}, \pi_A(\bar{\gamma})) \ge \bar{\phi} > \phi,$$
(38)

where we have appealed to the definition of  $\bar{\phi}$ . It thus follows from (38) that the perturbation strictly increases the principal's welfare conditional on  $\gamma = \bar{\gamma}$ .

#### A10. Proof of Proposition 4

Consider the following quadratic-uniform setting: preferences satisfy  $U_P(\gamma, \pi) = \gamma \pi - \pi^2/2$  and  $U_A(\gamma, \pi) = (\gamma + \beta)\pi - \pi^2/2$  for  $\beta > 0$ , and  $f(\gamma) = 1$  for all  $\gamma \in \Gamma$ . In this setting, the efficient and flexible spending levels are given by

 $\pi_P(\gamma) = \gamma$  and  $\pi_A(\gamma) = \gamma + \beta$ , respectively. Let  $\alpha = 0$ , so that the agent pays no verification cost.

We first establish that in this setting, if the verification cost satisfies  $\phi > \beta^2/2$ , TEC is suboptimal, as it is dominated by a rule without verification.

LEMMA 8. Consider the quadratic-uniform setting with  $\alpha = 0$ . If  $\phi > \beta^2/2$ , then TEC is not optimal.

*Proof.* Take the quadratic-uniform setting with  $\alpha = 0$  and  $\phi > \beta^2/2$ . Consider the following problem:

$$\max_{\{\gamma^*,\gamma^{**}\}} \left\{ \begin{array}{l} \int_{\underline{\gamma}}^{\gamma^*} U_P(\gamma,\pi_A(\gamma)) f(\gamma) d\gamma + \int_{\gamma^*}^{\gamma^{**}} U_P(\gamma,\pi_A(\gamma^*)) f(\gamma) d\gamma \\ + \int_{\gamma^{**}}^{\tilde{\gamma}} (U_P(\gamma,\pi_P(\gamma)) - \phi) f(\gamma) d\gamma \end{array} \right\}.$$
(39)

Note that the solution to this program coincides with a rule without verification if it sets  $\gamma^{**} = \bar{\gamma}$ , and it coincides with a rule that verifies all types if it sets  $\gamma^{**} = \underline{\gamma}$ . By the definition of TEC, a necessary condition for a TEC rule to be optimal is that the solution to program (39) specify  $\underline{\gamma} < \gamma^{**} < \bar{\gamma}$ . We show that this cannot be satisfied when  $\phi > \beta^2/2$ .

The first-order condition for  $\gamma^*$ , given our assumptions on preferences and the distribution of  $\gamma$ , implies

$$\gamma^* = \max\left\{\frac{\underline{\gamma} + \gamma^{**}}{2} - \beta, \gamma^{**} - 2\beta\right\},\tag{40}$$

where we have taken into account the fact that  $\gamma^*$  may be lower than  $\underline{\gamma}$ . If the solution to (39) sets  $\gamma^{**}$  strictly interior, then the first-order condition for  $\overline{\gamma}^{**}$  implies

$$-\gamma^{**}(\gamma^*+\beta) + \frac{(\gamma^*+\beta)^2}{2} + \frac{\gamma^{**2}}{2} = \phi.$$

Substituting with (40) and rearranging terms yields

$$\frac{\left(\gamma^{**} - \max\left\{\left(\underline{\gamma} + \gamma^{**}\right)/2 - \beta, \gamma^{**} - 2\beta\right\} - \beta\right)^2}{2} = \phi.$$
(41)

Note that if  $\gamma^* \ge \underline{\gamma}$ , (41) implies  $\phi = \beta^2/2$ , contradicting the assumption that  $\phi > \beta^2/2$ . Therefore,

$$\gamma^* < \gamma, \tag{42}$$

and thus (41) implies

$$\gamma^{**} = \underline{\gamma} + 2\sqrt{2\phi}.$$

Substituting back into (40), we obtain

$$\gamma^* = \underline{\gamma} + \sqrt{2\phi} - \beta. \tag{43}$$

However, combined with (42), equation (43) implies  $\phi < \beta^2/2$ , contradicting the assumption that  $\phi > \beta^2/2$ . Therefore, the solution to (39) cannot set  $\gamma^{**}$  strictly interior when  $\phi > \beta^2/2$ . QED

We next show that there exists  $\phi > \beta^2/2$  under which a rule with verification is optimal.

LEMMA 9. Consider the quadratic-uniform setting with  $\alpha = 0$ . If  $\beta^2/2 < \phi < 2\beta^2/3$  and  $6\beta < \bar{\gamma} - \gamma$ , then a rule with verification is optimal.

*Proof.* Take the quadratic-uniform setting with  $\alpha = 0$ ,  $\beta^2/2 < \phi < 2\beta^2/3$ , and  $6\beta < \bar{\gamma} - \gamma$ . An optimal rule without verification sets  $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$ , where using (40) (with  $\gamma^{**} = \bar{\gamma}$ ) and the fact that  $\bar{\gamma} - 2\beta > \gamma + 4\beta > \gamma$ , we have

$$\gamma^* = \bar{\gamma} - 2\beta$$

We construct a perturbed rule  $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$  that features verification and yields the principal strictly higher expected welfare than this optimal rule without verification. For any given  $\gamma_H < \gamma^*$ , define  $\gamma_L$  as the solution to

$$U_{\!\scriptscriptstyle A}(\pmb{\gamma}_L,\pmb{\gamma}_L-\pmb{eta})\,=\,U_{\!\scriptscriptstyle A}(\pmb{\gamma}_L,\pmb{\pi}_A(\pmb{\gamma}_H)),$$

which after some algebra yields

$$\gamma_L = \gamma_H - 2\beta. \tag{44}$$

Take  $\gamma_H < \gamma^*$  sufficiently close to  $\gamma^*$  so that  $\gamma_L$  satisfies  $\gamma_L - 2\beta > \underline{\gamma}$  (note that the assumption that  $6\beta < \overline{\gamma} - \underline{\gamma}$  ensures that such a  $\gamma_H$  exists). Type  $\overline{\gamma}_L$  is defined so that he is indifferent between the flexible spending level of  $\gamma_H$  and the optimal spending limit under no verification for a distribution truncated at  $\gamma_L$  (which is given by  $\pi_A(\gamma_L - 2\beta) = \gamma_L - \beta$ ). Now construct the perturbed rule as follows: if  $\gamma < \gamma_L - 2\beta$  or  $\gamma > \gamma_H$ , then  $\tilde{a}(\gamma) = 0$  and  $\tilde{\pi}(\gamma) = \pi(\gamma)$ ; if  $\gamma \in [\gamma_L - 2\beta, \gamma_L)$ , then  $\tilde{a}(\gamma) = 0$  and  $\tilde{\pi}(\gamma) = \gamma_L - \beta$ ; and if  $\gamma \in [\gamma_L, \gamma_H]$ , then  $\tilde{a}(\gamma) = 1$  and  $\tilde{\pi}(\gamma)$  satisfies

$$U_A(\boldsymbol{\gamma}, \tilde{\boldsymbol{\pi}}(\boldsymbol{\gamma})) = U_A(\boldsymbol{\gamma}, \boldsymbol{\pi}_A(\boldsymbol{\gamma}_H)),$$

which after some algebra yields

$$\tilde{\pi}(\gamma) = 2\gamma - \gamma_H + \beta.$$

Note that given the definition of  $\gamma_L$ , this rule is incentive compatible. The perturbation changes the principal's welfare only for types  $\gamma \in [\gamma_L - 2\beta, \gamma_H]$ . The change in welfare is equal to

$$\int_{\gamma_{L-2eta}}^{\gamma_{L}} (U_{P}(\gamma,\gamma_{L}-eta)-U_{P}(\gamma,\gamma+eta))f(\gamma)d\gamma + \int_{\gamma_{L}}^{\gamma_{H}} (U_{P}(\gamma,2\gamma-\gamma_{H}+eta)-\phi -U_{P}(\gamma,\gamma+eta))f(\gamma)d\gamma.$$

After some algebra and substitution of (44), using our assumptions on preferences and the distribution of  $\gamma$ , this simplifies to

$$-\int_{\gamma_{H-4\delta}}^{\gamma_{H-4\delta}}\frac{\left(\gamma-\gamma_{H}+3\beta\right)^{2}}{2}\,d\gamma-\int_{\gamma_{H-2\delta}}^{\gamma_{H}}\frac{\left(\gamma_{H}-\gamma-\beta\right)^{2}}{2}\,d\gamma-\int_{\gamma_{H-2\delta}}^{\gamma_{H}}\phi\,d\gamma+\int_{\gamma_{H-4\delta}}^{\gamma_{H}}\frac{\beta^{2}}{2}\,d\gamma.$$

Simplifying further yields that the change in welfare is equal to

$$\frac{4}{3}\beta^3 - 2\beta\phi > 0,$$

where the inequality follows from the assumption that  $\phi < 2\beta^2/3$ . Therefore, the perturbed rule with verification strictly increases the principal's expected welfare relative to no verification. QED

It follows from lemmas 8 and 9 that in a quadratic-uniform setting with  $\alpha = 0$ ,  $\beta^2/2 < \phi < 2\beta^2/3$ , and  $6\beta < \bar{\gamma} - \gamma$ , verification is optimal but TEC is not. By lemma 2, any optimal rule must therefore feature decreasing verification.

#### A11. Proof of Lemma 7

*Part i.*—Suppose that an incentive-compatible rule induces decreasing verification at  $\gamma_{H}$ . Consider first the case in which  $a(\gamma_{H}) = 0$  and thus  $a(\gamma_{H} - \varepsilon) = 1$  for  $\varepsilon > 0$  arbitrarily small. Incentive compatibility for type  $\gamma_{H}$  requires

$$U_A(\gamma_H, \pi(\gamma_H)) \ge U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha \phi,$$
 (45)

since  $\gamma_H$  can choose to be verified and guarantee himself the efficient level of spending. Incentive compatibility for type  $\gamma_H - \varepsilon$  requires

$$U_{A}(\gamma_{H} - \varepsilon, \pi_{P}(\gamma_{H} - \varepsilon)) - \alpha \phi \geq U_{A}(\gamma_{H} - \varepsilon, \pi(\gamma_{H})), \tag{46}$$

since  $\gamma_H - \varepsilon$  can choose not to be verified and spend at  $\pi(\gamma_H)$ . Given the continuity of  $U_A$  and  $\pi_P$  in their respective arguments, we can take the limit of both sides of (46) as  $\varepsilon$  approaches 0 to obtain

$$U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha \phi \ge U_A(\gamma_H, \pi(\gamma_H)). \tag{47}$$

Combining (45) and (47) yields (12).

Consider next the case in which  $a(\gamma_H) = 1$  and thus  $a(\gamma_H + \varepsilon) = 0$  for  $\varepsilon > 0$  arbitrarily small. Analogous arguments to those above imply the following incentive compatibility constraints for  $\gamma_H$  and  $\gamma_H + \varepsilon$ , respectively:

$$U_{A}(\gamma_{H}, \pi_{P}(\gamma_{H})) - \alpha \phi \geq U_{A}(\gamma_{H}, \pi(\gamma_{H} + \varepsilon)), \qquad (48)$$

$$U_{A}(\gamma_{H} + \varepsilon, \pi(\gamma_{H} + \varepsilon)) \geq U_{A}(\gamma_{H} + \varepsilon, \pi_{P}(\gamma_{H} + \varepsilon)) - \alpha\phi.$$
(49)

Since the rule is piecewise continuous,  $\lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$  exists and can be defined as  $\pi(\gamma_H)$ . Taking the limit of both sides of (48) and (49) as  $\varepsilon$  goes to 0 yields (47) and (45), and combining these two inequalities yields (12).

To complete the proof of part i, we show that  $\pi(\gamma_H) > \pi_A(\gamma_H)$  must hold. Note that by (12), either  $\pi(\gamma_H) > \pi_A(\gamma_H)$  or  $\pi(\gamma_H) \le \pi_P(\gamma_H)$ . For the purpose of contradiction, suppose it were the case that  $\pi(\gamma_H) \le \pi_P(\gamma_H)$ . Consider the incentive compatibility constraint of type  $\gamma_H - \varepsilon$  for  $\varepsilon > 0$  arbitrarily small. Take first the case in which  $a(\gamma_H - \varepsilon) = 1$ . Then  $\gamma_H - \varepsilon$  must weakly prefer verification to no verification, which requires

$$U_{A}(\gamma_{H} - \varepsilon, \pi_{P}(\gamma_{H} - \varepsilon)) - \alpha \phi \geq U_{A}(\gamma_{H} - \varepsilon, \pi(\gamma_{H})).$$
(50)

Since  $\pi_P(\gamma_H - \varepsilon) < \pi_P(\gamma_H) < \pi_A(\gamma_H - \varepsilon)$ , (50) implies

$$U_{A}(\gamma_{H} - \varepsilon, \pi_{P}(\gamma_{H})) - \alpha \phi > U_{A}(\gamma_{H} - \varepsilon, \pi(\gamma_{H})).$$
(51)

Combining (12) and (51) yields

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$$U_{\scriptscriptstyle\!A}(\gamma_{\scriptscriptstyle H}-arepsilon, \pi_{\scriptscriptstyle P}(\gamma_{\scriptscriptstyle H})) = U_{\scriptscriptstyle\!A}(\gamma_{\scriptscriptstyle H}-arepsilon, \pi(\gamma_{\scriptscriptstyle H})) \geq U_{\scriptscriptstyle\!A}(\gamma_{\scriptscriptstyle H}, \pi_{\scriptscriptstyle P}(\gamma_{\scriptscriptstyle H})) = U_{\scriptscriptstyle\!A}(\gamma_{\scriptscriptstyle H}, \pi(\gamma_{\scriptscriptstyle H})).$$

Given  $\pi(\gamma_H) \leq \pi_P(\gamma_H)$ , this inequality violates the single-crossing condition in  $U_A$ , thus yielding a contradiction.

Consider next the case in which  $a(\gamma_H - \varepsilon) = 0$ . Given decreasing verification at  $\gamma_H$ , in this case we must have  $a(\gamma_H) = 1$  and  $a(\gamma_H + \varepsilon) = 0$  for  $\varepsilon > 0$  arbitrarily small. Moreover, given our definition of  $\pi(\gamma_H)$ ,  $\pi(\gamma_H) \leq \pi_P(\gamma_H)$  implies  $\lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon) \leq \pi_P(\gamma_H)$ . By incentive compatibility, type  $\gamma_H$  must weakly prefer verification to no verification, which requires

$$U_{A}(\gamma_{H}, \pi_{P}(\gamma_{H})) - \alpha \phi \geq U_{A}(\gamma_{H}, \pi(\gamma_{H} + \varepsilon)),$$
(52)

whereas type  $\gamma_H + \varepsilon$  must weakly prefer no verification to verification, which requires

$$U_{A}(\gamma_{H} + \varepsilon, \pi(\gamma_{H} + \varepsilon)) \geq U_{A}(\gamma_{H} + \varepsilon, \pi_{P}(\gamma_{H} + \varepsilon)) - \alpha\phi.$$
(53)

Combining (52) and (53) and using the fact that  $\pi_A(\gamma_H) > \pi_P(\gamma_H + \varepsilon) > \pi_P(\gamma_H)$  yields

$$U_{A}(\gamma_{H}, \pi_{P}(\gamma_{H} + \varepsilon)) - U_{A}(\gamma_{H}, \pi(\gamma_{H} + \varepsilon)) > U_{A}(\gamma_{H} + \varepsilon, \pi_{P}(\gamma_{H} + \varepsilon)) - U_{A}(\gamma_{H} + \varepsilon, \pi(\gamma_{H} + \varepsilon)).$$

Since  $\pi(\gamma_H + \varepsilon) \leq \pi_P(\gamma_H) \leq \pi_P(\gamma_H + \varepsilon)$  for  $\varepsilon$  approaching 0, this inequality violates the single-crossing condition in  $U_A$ , thus again yielding a contradiction.

Therefore, we obtain that  $\pi(\gamma_H) \leq \pi_P(\gamma_H)$  cannot hold, and we must thus have  $\pi(\gamma_H) > \pi_A(\gamma_H)$ .

*Part ii.*—Suppose an incentive-compatible rule induces increasing verification at  $\gamma_L$ . Then analogous arguments to those used to prove part i can be applied to show that (14) must hold at  $\gamma_L$ . Since the steps are analogous, we omit the details.

#### A12. Proof of Proposition 5

To prove this result, we first establish the following lemmas.

LEMMA 10. Under limited commitment, if an incentive-compatible rule features increasing verification at  $\gamma_L$ , then

$$\pi(\gamma_L) \leq \pi_P(\gamma_L), \tag{54}$$

where  $\pi(\gamma_L) \equiv \lim_{\epsilon \downarrow 0} \pi(\gamma_L - \epsilon)$  if  $a(\gamma_L) = 1$ .

*Proof.* Suppose an incentive-compatible rule features increasing verification at  $\gamma_L$ . By equation (14) in lemma 7, either  $\pi(\gamma_L) > \pi_A(\gamma_L)$  or  $\pi(\gamma_L) \leq \pi_P(\gamma_L)$ . For the purpose of contradiction, suppose  $\pi(\gamma_L) > \pi_A(\gamma_L)$  holds. Take first the case in which  $a(\gamma_L) = 1$ , so that  $a(\gamma_L - \varepsilon) = 0$  for  $\varepsilon > 0$  arbitrarily small and, given our definition of  $\pi(\gamma_L)$ ,  $\lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon) > \pi_A(\gamma_L)$ . By incentive compatibility, type  $\gamma_L - \varepsilon$  must weakly prefer no verification to verification, which requires

$$U_A(\gamma_L - \varepsilon, \pi(\gamma_L - \varepsilon)) \ge U_A(\gamma_L - \varepsilon, \pi_P(\gamma_L - \varepsilon)) - \alpha \phi.$$
 (55)

However, (14) and (55) together with the fact that  $\pi(\gamma_L) > \pi_A(\gamma_L)$  imply that assumption 2 is violated, yielding a contradiction.

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Consider next the case in which  $a(\gamma_L) = 0$ , so that  $a(\gamma_L + \varepsilon) = 1$  for  $\varepsilon > 0$  arbitrarily small. By incentive compatibility, type  $\gamma_L + \varepsilon$  must weakly prefer verification to no verification, which requires

$$U_{A}(\gamma_{L} + \varepsilon, \pi_{P}(\gamma_{L} + \varepsilon)) - \alpha \phi \geq U_{A}(\gamma_{L} + \varepsilon, \pi(\gamma_{L})).$$
(56)

Note that in this case,  $\pi(\gamma_L) > \pi_A(\gamma_L) > \pi_P(\gamma_L + \varepsilon)$  requires  $\pi(\gamma_L) > \pi_A(\gamma_L + \varepsilon)$ . However, (14) and (56) together with  $\pi(\gamma_L) > \pi_A(\gamma_L + \varepsilon)$  imply that assumption 2 is violated, yielding again a contradiction.

Therefore, we obtain that  $\pi(\gamma_L) > \pi_A(\gamma_L)$  cannot hold, and we must thus have  $\pi(\gamma_L) \leq \pi_P(\gamma_L)$ . QED

LEMMA 11. Under limited commitment, if an incentive-compatible rule features decreasing verification at  $\gamma_{H}$  then there exists  $\gamma' \leq \gamma_{H}$  satisfying

$$U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\gamma_H))$$
(57)

for  $\pi(\gamma') < \pi_A(\gamma')$ ,  $\pi(\gamma_H) \equiv \lim_{\epsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$  if  $a(\gamma_H) = 1$ , and either  $a(\gamma') = 0$ , or  $a(\gamma') = 1$ ,  $\lim_{\epsilon \downarrow 0} a(\gamma' - \varepsilon) = 0$ , and  $\pi(\gamma') \equiv \lim_{\epsilon \downarrow 0} \pi(\gamma' - \varepsilon)$ .

*Proof.* Suppose an incentive-compatible rule features decreasing verification at  $\gamma_{H}$ . By condition (13) in lemma 7,  $\pi(\gamma_{H}) > \pi_{A}(\gamma_{H})$ . Consider the problem of the principal after the verification decision  $a(\gamma)$  has been made and the verification result (in case of verification) has been obtained:

$$\max_{\{\pi(\gamma)\}_{\gamma\in\Gamma}} \int_{\underline{\gamma}}^{\bar{\gamma}} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma$$
(58)

subject to

$$\pi(\gamma) = \pi_P(\gamma) \text{ if } a(\gamma) = 1, \tag{59}$$

$$U_A(\gamma, \pi(\gamma)) \ge U_A(\gamma, \pi(\hat{\gamma}))$$
 for all  $\gamma, \hat{\gamma}$  for which  $a(\gamma) = a(\hat{\gamma}) = 0.$  (60)

This program takes into account that the principal will assign the efficient spending level to any agent type who chooses to be verified, and she will ignore the incentives of verified types when deciding the spending allocation of types who choose not to be verified. We now consider the optimal level of  $\pi(\gamma_H)$  given decreasing verification at  $\gamma_H$  and the conditions that are necessary for the principal to choose  $\pi(\gamma_H) > \pi_A(\gamma_H)$ .

Step 1.—Consider the spending allocation conditional on no verification. Note that analogous arguments to those used in the proof of lemma 5 imply that  $\pi(\gamma)$  must be weakly increasing for nonverified types  $\gamma$ . For each nonverified type  $\gamma$ , denote by  $\underline{\pi}(\gamma)$  the spending level closest to  $\pi_A(\gamma)$  from below in the allowable spending set for nonverified types (i.e., among all spending levels assigned to types who choose no verification). Analogously, denote by  $\overline{\pi}(\gamma)$  the closest spending level to  $\pi_A(\gamma)$  from above in the allowable spending set for nonverified types. Clearly, if  $\pi_A(\gamma)$  is in this allowable spending set, then  $\pi_A(\gamma) = \underline{\pi}(\gamma) = \overline{\pi}(\gamma)$ . The incentive compatibility constraint (60) together with the concavity of  $U_A$  requires that if  $a(\gamma) = 0$ , then

$$\pi(\gamma) = \arg \max_{\pi \in \{\underline{\pi}(\gamma), \overline{\pi}(\gamma)\}} U_A(\gamma, \pi).$$
(61)

*Step 2.*—As noted, given decreasing verification at  $\gamma_H$ , the rule must set  $\pi(\gamma_H) > \pi_A(\gamma_H)$ . We show that as a result, the rule must induce  $a(\gamma) = 0$  and  $\pi(\gamma) = \pi(\gamma_H)$ 

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for all types  $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$ . To see why, note first that by (61) and the single-crossing condition in  $U_A$ , any type  $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$  who is not verified necessarily chooses spending  $\pi(\gamma) = \pi(\gamma_H)$ . Therefore, it is sufficient to show that any type  $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$  must have  $a(\gamma) = 0$ . Suppose by contradiction that this were not the case. Then incentive compatibility for a type  $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$  with  $a(\gamma) = 1$  requires that this type weakly prefer verification to no verification, which requires

$$U_{A}(\gamma, \pi_{P}(\gamma)) - \alpha \phi \geq U_{A}(\gamma, \pi(\gamma_{H})).$$
(62)

However, (12) and (62) together with the fact that  $\gamma > \gamma_H$  and  $\pi(\gamma_H) > \pi_A(\gamma)$  violate assumption 2. The claim therefore follows.

Step 3.—We show that in an incentive-compatible rule, constraint (60) cannot be uniformly slack for all  $\gamma \leq \gamma_H$  and  $\hat{\gamma} = \gamma_H$ , where recall  $\pi(\gamma_H) > \pi_A(\gamma_H)$  by decreasing verification at  $\gamma_{H}$ . Suppose by contradiction that this is true. Note that from step 2, (60) is then uniformly slack for all  $\gamma \leq \gamma_H$  and  $\hat{\gamma} \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$ , where  $a(\hat{\gamma}) = 0$  for all such  $\hat{\gamma}$ . Now consider the following perturbation  $\{\tilde{\pi}(\gamma)\}_{\gamma\in\Gamma}$ : for  $\varepsilon > 0$  arbitrarily small and all  $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H) - \varepsilon)]$ , set  $\tilde{\pi}(\gamma) =$  $\pi(\gamma_H) - \varepsilon$ ; for all  $\gamma \in (\pi_A^{-1}(\pi(\gamma_H) - \varepsilon), \pi_A^{-1}(\pi(\gamma_H)))$ , set  $\tilde{\pi}(\gamma) = \pi_A(\gamma)$ ; and for all other types, leave the spending allocation unchanged. This perturbation strictly increases the principal's welfare as it reduces overspending by types  $\gamma \in$  $(\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$ . Moreover, since (by the contradiction assumption) (60) was uniformly slack before the perturbation for all  $\gamma \leq \gamma_H$ , it is still satisfied after the perturbation, and incentive compatibility for all types  $\gamma \geq \gamma_H$  is guaranteed as the perturbation satisfies (61). Therefore, we obtain that if (60) is uniformly slack for all  $\gamma \leq \gamma_H$  and  $\hat{\gamma} = \gamma_H$ , the principal can strictly improve upon the original rule by reducing  $\pi(\gamma_H)$  after the verification decision has been made, and hence the original rule violates incentive compatibility for the principal. The claim follows.

Step 4.—By step 3, in any incentive-compatible rule with decreasing verification at  $\gamma_H$ , there exists  $\gamma' \leq \gamma_H$  satisfying (57). Moreover, since decreasing verification at  $\gamma_H$  implies  $\pi(\gamma_H) > \pi_A(\gamma_H) \geq \pi_A(\gamma')$ , this requires  $\pi(\gamma') < \pi_A(\gamma')$ . This proves the lemma. QED

LEMMA 12. Under limited commitment, if an incentive-compatible rule features decreasing verification at  $\gamma_{H}$ , then there exists  $\gamma_L \leq \gamma_H$  at which the rule features increasing verification. Moreover,  $a(\gamma) = 1$  for all  $\gamma \in (\gamma_L, \gamma_H)$  and

$$U_A(\gamma_L, \pi(\gamma_L)) = U_A(\gamma_L, \pi(\gamma_H))$$
(63)

for  $\pi(\gamma_L) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon)$  if  $a(\gamma_L) = 1$  and  $\pi(\gamma_H) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$  if  $a(\gamma_H) = 1$ .

*Proof.* Suppose an incentive-compatible rule features decreasing verification at  $\gamma_{H}$ . By lemma 11, there exists a type  $\gamma' \leq \gamma_{H}$  satisfying (57) either with  $a(\gamma') = 0$  or at which there is increasing verification. We can establish that such a type is unique. Suppose by contradiction that there are two types,  $\gamma'' \leq \gamma_{H}$  and  $\gamma' < \gamma''$ , satisfying the condition in lemma 11. Then

$$U_{A}(\gamma'', \pi(\gamma'')) = U_{A}(\gamma'', \pi(\gamma_{H})), \qquad (64)$$

$$U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\gamma_H)).$$
(65)

Incentive compatibility requires

$$U_{A}(\gamma'', \pi(\gamma'')) \ge U_{A}(\gamma'', \pi(\gamma')), \tag{66}$$

$$U_{A}(\gamma', \pi(\gamma')) \geq U_{A}(\gamma', \pi(\gamma'')).$$
(67)

Combining (64)–(67) yields

$$U_A(\gamma',\pi(\gamma_H)) - U_A(\gamma',\pi(\gamma'')) \geq U_A(\gamma'',\pi(\gamma_H)) - U_A(\gamma'',\pi(\gamma'')).$$

Since  $\gamma' < \gamma''$  and  $\pi(\gamma_H) > \pi_A(\gamma_H) \ge \pi_A(\gamma'') > \pi(\gamma'')$  by decreasing verification at  $\gamma_H$  and lemma 11, this inequality violates the single-crossing condition in  $U_A$ , yielding a contradiction. Therefore, there exists a unique type below  $\gamma_H$  for which (57) holds, and denoting this type by  $\gamma_L$  yields (63).

Next, we show that  $a(\gamma) = 1$  for all  $\gamma \in (\gamma_L, \gamma_H)$ . Note first that a spending level  $\pi \in (\pi(\gamma_L), \pi(\gamma_H))$  cannot be allowed by the rule under no verification, since otherwise type  $\gamma_L$  would have a strict incentive to deviate to such a spending level. Consider the relevant case in which  $\gamma_L < \gamma_H$  and suppose by contradiction that  $a(\gamma) = 0$  for some type  $\gamma \in (\gamma_L, \gamma_H)$ . Let  $\gamma'$  denote the highest such type  $\gamma$ . Since, as noted, spending levels strictly between  $\pi(\gamma_L) < \pi_A(\gamma')$  and  $\pi(\gamma_H) > \pi_A(\gamma')$  are not allowed, it follows from (63) and  $\gamma' > \gamma_L$  that the rule must set  $\pi(\gamma') = \pi(\gamma_H)$ . Moreover, since by construction the rule features increasing verification at  $\gamma'$ , condition (14) in lemma 7 implies

$$U_A(\gamma', \pi_P(\gamma')) - \alpha \phi = U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\gamma_H)).$$
(68)

However, given (12) and (13), equation (68) violates assumption 2. It follows that  $a(\gamma) = 1$  for all  $\gamma \in (\gamma_L, \gamma_H)$ . QED

We can now prove the proposition. We begin by ruling out decreasing verification. Suppose by contradiction that an incentive-compatible rule features decreasing verification at some  $\gamma_H \in \Gamma$ . By lemma 12, there must exist a type  $\gamma_L \leq \gamma_H$ satisfying the conditions in the lemma. We proceed in two steps.

*Step 1.*—Suppose  $\gamma_L < \gamma_H$ . Then it follows from (14) and (63) that

$$U_A(\boldsymbol{\gamma}_L, \boldsymbol{\pi}_P(\boldsymbol{\gamma}_L)) - \boldsymbol{\alpha}\boldsymbol{\phi} = U_A(\boldsymbol{\gamma}_L, \boldsymbol{\pi}(\boldsymbol{\gamma}_H)). \tag{69}$$

However, (12) and (69) together with the fact that  $\gamma_L < \gamma_H$  and  $\pi(\gamma_H) > \pi_A(\gamma_H)$  (by [13]) imply that assumption 2 is violated, a contradiction.

Step 2.—By step 1, any incentive-compatible rule with decreasing verification must have  $\gamma_L = \gamma_H$  at each point  $\gamma_H$  at which there is decreasing verification. Now consider the principal's problem (58)–(60). Let  $\bar{\gamma}' \leq \bar{\gamma}$  be the highest non-verified type. Since the types with decreasing verification are atomistic and the rule is piecewise continuous, following a decision of no verification the principal solves

$$\max_{\{\pi(\gamma)\}_{\gamma^{dr}}} \int_{\underline{\gamma}}^{\underline{\gamma}} U_{P}(\gamma, \pi(\gamma)) f(\gamma) d\gamma$$
  
subject to

 $U_A(\gamma, \pi(\gamma)) \ge U_A(\gamma, \pi(\hat{\gamma}))$  for all  $\gamma, \hat{\gamma}$  for which  $a(\gamma) = a(\hat{\gamma}) = 0$ .

By proposition 1, the solution assigns  $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$  for  $\gamma \in [\underline{\gamma}, \overline{\gamma'}]$ and some  $\gamma^* < \overline{\gamma'}$ . However, in this case, conditions (13) and (54) (which require

 $\pi(\gamma_H) > \pi_A(\gamma_H)$  and  $\pi(\gamma_L) \le \pi_P(\gamma_L)$ , respectively) cannot be satisfied at a point  $\gamma_H \in [\underline{\gamma}, \overline{\gamma}']$  at which there is decreasing verification and thus  $\gamma_L = \gamma_H$ , a contradiction.

The claims above show that under limited commitment, any incentive-compatible rule features weakly increasing verification everywhere. Analogous arguments to those in the proofs of lemmas 1 and 2 can then be applied to show that a TEC rule is optimal if a rule with verification that is weakly increasing everywhere is optimal. Therefore, under limited commitment, if verification is optimal, TEC is optimal.

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