A contingent claim approach to performance evaluation

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Final version accepted July 1993

Abstract

We show that valuing performance is equivalent to valuing a particular contingent claim on an index portfolio. In general the form of the contingent claim is not known and must be estimated. We suggest approximating the contingent claim by a series of options. We illustrate the use of our method by evaluating the performance of 130 mutual funds during the period 1968–82. We find that the relative performance rank of a fund is rather insensitive to the choice of the index, even though the actual value of the services of the portfolio manager depends on the choice of the index

1. Introduction

Evaluating the performance of portfolio managers has received wide attention in the financial economics literature, presumably due to the fact that a substantial part of the savings of investors is managed by professionals. The principle behind performance evaluation is rather straightforward. All we have to do is to assign the correct value to the cashflow (net of management fees) the manager generates from the amount entrusted to him by the investor. The difference between the assigned value and the amount entrusted to the manager is the

value of the services provided by the manager. If this difference is positive then we designate the manager as providing "valuable service."

There are several difficulties in implementing this rather simple principle. One of the more serious difficulties is due to the fact that financial economists have yet to come up with a satisfactory valuation model that consistently values arbitrary streams of cashflows sufficiently close to their market price. Every capital asset pricing model that has been suggested has performed poorly at least with respect to one subset of the assets examined. Hence, an analyst who uses a particular valuation model has to be aware of the collection of assets for which the model performs satisfactorily.

For example, linear beta pricing models like the Capital Asset Pricing Model (CAPM) that are commonly used in valuing financial assets assign negative prices to some states of nature, and hence will assign implausible values to some options, even when they assign the correct value to the primitive set of assets on which the options are written. This was first pointed out by Dybvig and Ingersoll (1982) who constructed an option on the market portfolio (with an associated positive cashflow) to which the CAPM assigned a negative price. This observation cannot be ignored by arguing that few portfolio managers use traded options in their portfolios. As Merton (1981) and Dybvig and Ross (1985) point out, portfolios managed using superior information will exhibit option-like features, even when the portfolio manager does not explicitly trade in options.

Grinblatt and Titman (1989) stress the relevance of these results for portfolio performance evaluation by pointing out the need to use valuation models with positive state price densities, since a manager selling a call option on the index will be incorrectly classified as a superior performer by an investor using Jensen's alpha [or Treynor–Black's (1972) appraisal ratio] to evaluate performance. While these observations provide important insights into the issues involved, they do not provide operationally useful guidelines. This is because not every model that uses a positive state price density and values the set of primitive securities correctly will assign the same value to options on those securities. Also, the numerous candidate state price densities (or period weighting measures, to use the Grinblatt–Titman terminology) that have performed poorly in empirical studies of Intertemporal Capital Asset Pricing Models are all strictly positive. Even the empirical state price density associated with the CAPM and the Arbitrage Pricing Theory (APT) very rarely take negative values [see Hansen and Jagannathan (1991b)]. Hence an analyst who chooses a positive state price density to avoid assigning a negative value to a contingent claim that pays a positive amount may still not assign the correct value.[2]

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[2]See Harrison and Kreps (1979) and Green and Srivastava (1985) who show that in every arbitrage free economy there will in general exist an infinite collection of positive state price densities which assign the same (correct) value to the primitive securities for which traded prices are observed. Hansen and Jagannathan (1991a) show how to construct a positive price density using data on asset prices and payoffs

In this paper we address the difficulty that arises from managed portfolios having option-like features in a different way than Grinblatt and Titman. We assume that options on certain stock index portfolios are either traded or can be valued using arbitrage methods. We suggest approximating the payoff on the managed portfolio using payoffs on a limited number of options on a suitably chosen index portfolio. We can then arrive at an approximate value for the managed portfolio by finding the value of the options used in approximating its payoff. We provide some guidelines for choosing the index portfolio and establish conditions under which our procedure will work reasonably well using theoretical arguments as well as simulation. We also illustrate its use by empirical examples.

Our approach can be viewed as the nonlinear analogue of the linear beta pricing model of Connor (1984). We assume, as Connor does, that the equilibrium marginal rate of substitution of at least one individual who has frictionless access to financial markets is a function of only a finite number of "factors," and that the factors are payoffs on portfolios of traded securities. However, unlike Connor, we do not assume that only that part of asset payoffs that is in the linear span of the factors is priced. This is because, as already pointed out, while such an assumption may be reasonable for the primitive set of securities, it will in general not be appropriate for derivative claims on the primitive assets. Instead, for implementation purposes, our approach can be viewed as assuming that only that part of an asset's payoff that is in the linear span of the factors and certain limited number of options on the factors is priced, where we allow payoffs to resemble options on the primitive assets. We provide asymptotic justifications for our approach. These justifications are in the same spirit as those provided by Ross (1976) and Chamberlain and Rothschild (1983) for linear factor pricing models.

Notice that when the analyst uses only one specific option on an index portfolio (along with the risk-less payoff) to approximate the managed portfolio payoff, our method resembles the Henriksson–Merton (1981) method. However, it is important to stress that the motivation for the two methods are entirely different. Henriksson and Merton were primarily interested in classifying the abilities of the portfolio manager into two dichotomous parts: (i) market timing ability and (ii) ability to select undervalued securities. We are interested in assigning a value to the portfolio management services provided by the manager without imposing such a dichotomy.

This is because models that try to classify abilities into two dichotomous parts have to reckon with the following difficulty. As Admati, Bhattacharya, Pfleiderer, and Ross (1986) point out, even in theory, it is rather difficult to arrive at rigorous and consistent definitions of timing and selectivity abilities. This distinction is even more difficult in practice for the following reason. As Jagannathan and Korajczyk (1986) show, a portfolio manager can show superior timing ability by following some fairly simple portfolio strategies. For

example, a manager who has no abilities of any sort and who writes covered calls will show inferior market timing ability and superior selectivity when evaluated by the Henriksson–Merton procedures. These criticisms do not apply to our method. In fact, the Henriksson–Merton method is also approximately valid when used to measure the total value of a portfolio manager's abilities, even when the manager does not behave as assumed by Merton in developing his performance evaluation framework.

The rest of the paper is organized as follows: In Section 2 we develop the theoretical framework for performance evaluation. In Section 3 we examine the ability of the approximations we suggest to capture the true underlying value of a portfolio strategy. We demonstrate that, in general, we will need more than one option on the index for our approximations to work reasonably well.[3] In Section 4 we demonstrate the use of our method by evaluating the performance of 130 mutual funds. The paper ends with a summary of our results and suggestions for further research.

2. Theoretical framework

2.1. Economic environment

We suppose the following scenario. A portfolio manager announces that he can provide R_p dollars at time T for each dollar invested now (time t), net of any management fee. Without doubting the veracity of the portfolio manager, the question is, should an investor invest in such a fund. The answer is yes if the gross value, at the margin, of R_p is greater than the required investment of one dollar; that is, if the net present value is positive.

The net value per period per dollar invested, at the margin, V^j, of a payoff R_p for an individual with nominal marginal rate of substitution M^j is given by:

$$V^j = E[M^j R_p | F^j] - 1 \tag{1}$$

where F^j denotes the date t information set of the investor j. Note that V^j is in the information set F^j. This expression will arise from a quite general intertemporal utility maximization problem. Furthermore, the net value at the margin of any traded return, R, is zero, and hence:

$$1 = E[M^j R | F^j] . \tag{2}$$

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[3]If the analyst knows how the manager takes his/her decisions, it may be possible to derive a priori restrictions on the functional form of the contingent claim This will improve the precision with which the manager's abilities can be valued.

Substituting into equation (1), we get

$$V^j = E[M^j(R_p - R)|F^j] .\qquad (3)$$

A convenient choice of R is the nominally risk-free return R_f. Defining $X_p = R_p - R_f$, we get

$$V^j = E[M^j X_p|F^j].\qquad (4)$$

Notice that the value, V^j, of the payoff X_p, will in general depend on the information set, F^j, as well as the preferences of agent j through the marginal rate of substitution M^j. Since preferences are not typically observable, equation (2) is not empirically implementable. However, as long as a sufficient number of securities are traded in the market, all agents whose marginal rates of substitution M^j depend only on the payoffs from traded claims will agree on the expectation of the value, $E[V^j]$, of the managed portfolio. Furthermore this value is the value of a traded contingent claim. Assumption 1, below, is the formal condition that a sufficient number of securities be traded. Assumption 2, below, allows us to conclude that all marginal rates of substitution that are functions of the payoffs on traded securities are equal. Assumption 2 can be derived as a theorem from Assumption 1 and the assumption of no arbitrage opportunities [see Hansen and Richard (1987)] but in the interest of simplicity we skip the intervening steps.

Assumption 1. Let F_T denote the information set generated by the time T payoffs from allowable trading strategies, and F the corresponding common knowledge information set at time t. Assume that if D is in F_T, that is, D is a function of the payoffs of allowable trading strategies, then D itself is the outcome of some allowable trading strategy.

Assumption 2. There is a unique Z in F_T, which is strictly positive with probability one, such that the price of a payoff D in F_T is $E[ZD|F]$.

Assumption 1 is similar to, but weaker than, the assumption that markets are complete. To illustrate Assumption 1, consider the case of one risk-free security with gross return R_f and one risky security with random date T gross return of Y which is generated by a lognormal diffusion process. The information represented by F_T is the specific realization of Y. Hence $D = \max(Y - k, 0)$ is in F_T—that is, the realization of D is known once Y is known. However, when limited to trading only once in the risk-free and the risky assets, it is not possible to create a portfolio with a date T payoff of D, and Assumption 1 will not be satisfied in this economy. When continuous trading in the risk-free and the risky

assets is possible between dates t and T, and when the price process of the risky security follows, for example, a lognormal diffusion, then D as well as any function of Y can be attained as the payoff of a self-financing portfolio strategy. Thus, Assumption 1 can be viewed as an assumption about either the availability of traded assets or the continuous arrival of information and the ability to trade continuously without friction.

2.2. Valuation methodology

We first consider the valuation by individuals with the common knowledge information set F, whose marginal rates of substitution are spanned by traded securities and securities created by allowable dynamic trading strategies. All such individuals will have the same marginal rate of substitution. This can be seen by noting that if a marginal rate of substitution is spanned by traded securities, then it is in F_T and furthermore satisfies $1 = E[MR|F]$ for all gross returns R on traded securities. But by Assumption 2, there is a unique Z in F_T such that $E[ZR|F] = 1$ and hence the marginal rate of substitution M is equal to Z. Thus, the value, at the margin, of the portfolio payoff, X_p, to any investor whose marginal rate of substitution is spanned by traded securities, based on the common information set F, is given by:

$$V = E[MX_p|F] = E[ZX_p|F] . \tag{5}$$

Note that X_p can show positive (or negative) value since the managed portfolio is not a traded asset. This is true even if the payoff X_p is in F_T. For example, suppose a portfolio manager has perfect information about the return on some index R_I, and suppose he invests in the index if R_I exceeds the risk-free return, R_f, and otherwise invests in the risk-free security. Then, R_p will be given by $R_p = \max(R_I, R_f)$ and the excess return will be given by $X_p = \max(R_I - R_f, 0)$. This return is in F_T, but as long as the manager's information is not completely reflected in prices it will not have a zero value. Put another way, the above excess return can be generated by any trader (by Assumption 1), but it will require an investment equal to the value of a call option on the index with exercise price equal to the gross risk-free rate, R_f. The portfolio manager can supply it with a zero investment (borrow at R_f and invest in the managed portfolio).

The relation in (5) could form the basis of a performance evaluation procedure. Notice that V is the value at the margin of a borrowed dollar invested in the managed portfolio conditional on the information set F generated by prices of financial claims alone, at date t. The common information set F may be complicated; hence it is easier to estimate the average value

$v = E[V]$,[4] given by:

$$v = E[MX_p] = E[ZX_p] \,.$$

The average value is appropriate in the following scenario. The manager accepts a dollar from the investor and returns R_p dollars after one period. This is repeated for several periods. The evaluator observes only the time series of returns, R_p, on the managed portfolio, along with returns on some index portfolios. We assume that even if the manager's abilities change over time, it does so in some systematic stochastic fashion such that average ability is well defined.

Determining Z could be very difficult in general, as it involves finding the marginal utility of an individual and evaluating it at his or her optimal consumption, or equivalently, observing all contingent claim prices. Proposition 1 suggests an evaluation procedure when there is an individual whose marginal rate of substitution is a function only of the return on some index portfolio.

Proposition 1. Suppose there is an individual whose marginal rate of substitution, M^j, is a function solely of the vector of returns, R_I, on some index portfolios. Then the average value of the portfolio, v, is the average price of the traded security with payoff $e(R_I) = E[X_p|R_I] = E[R_p - R_f|R_I]$.

Proof.

$$E(V^j) = E[E(M^jX_p|F^j)] = E[M^jX_p] = E[M^jE[X_p|R_I]] = E[M^je(R_I)]$$
$$= e[Ze(R_I)]$$

where Z is the unique element of F_T satisfying $E[ZR] = 1$. $Q.E.D.$

Intuitively, X_p can be decomposed into two parts: a payoff that is related to the marginal rate of substitution which is a function of the vector of returns on some indices, R_I, and a payoff that is uncorrelated with the marginal rate of substitution. This latter payoff has zero mean and a zero average price. Valuing the managed portfolio then consists of finding the average price of the contingent claim, $e(R_I)$. To simplify the analysis, henceforth, we will assume that R_I is a scalar, that is, there is only one index.

We should note that in general every individual's marginal rate of substitution will not be spanned by traded securities. An individual's marginal rate of substitution is a function of real consumption. In an intertemporal setting, real consumption is a function of nominal income from financial investments as well

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[4]Note that we use the term average value to mean expected value.

as investments in nontraded assets, the proportion of nominal wealth consumed and of the price level.

According to Proposition 1, we need not find the equilibrium marginal rate of substitution, Z. All we have to do is find an individual (who actively maximizes) who holds some portfolio with return R_I and whose consumption is determined by the return R_I. For example, as Rubinstein (1976) has pointed out, if all investors have the same information set and logarithmic utility (or power utility and the growth rate in consumption is i.i.d.) then R_I will be the return on the market index portfolio. Epstein and Zin (1991) show that when all investors have logarithmic utility, then the result will obtain even when they are not expected utility maximizers. The assumption that all investors have the same information set implicitly assumes that the actively managed portfolio is small relative to the size of the economy.

The empirical problem, then, consists of determining the index (which could be multi-dimensional), estimating the relation between the portfolio excess return, X_p, and the index return, R_I, and applying contingent claim valuation techniques to arrive at the average value of $e(R_I)$.

The assumption that the marginal rate of substitution depends only on the return, R_I, suggests an alternative approach. One could use the time series of returns on all or some subset of traded assets to estimate the marginal rate of substitution, $M = f(R_I)$. Since the function $f(.)$ is not known, it must be estimated. One approach would be to use splines or polynomials to approximate the function $f(.)$. If a sufficiently long time series of data is available one may also estimate the functional form $f(.)$ by nonparametric methods [see Gallant and Tauchen (1989)].[5] Estimating the value then consists of finding the average of the product of the estimated marginal rate of substitution and the portfolio excess returns. The advantage of this method is that it concentrates on the unobservable part of the valuation relation and this is an important direction for future research in this area. The advantage of our approach is that it provides the characteristics of the managed portfolio that are useful for valuing it. An investor may agree with our characterization of the attributes of the payoff generated by the manager but disagree with the value we assign to it. Our approach will provide useful information even to such an investor.

Our approach also allows the use of prior information regarding the trading strategy of the portfolio manager. Specifically, such information could be used to choose the functional form of the conditional expectation. Furthermore, the shape of the contingent claim (the conditional expectation) may be of independent interest to investors. Also, as we show later on in the paper, our procedure is applicable even when the assumption of Proposition 1 is not satisfied.

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[5]Bansal and Viswanathan (1991) follow this approach in their empirical examination of asset prices

In general, the value of X_p to an individual whose marginal rate of substitution is not spanned by traded securities will depend upon the covariance of that part of his marginal rate of substitution that is due to nontraded assets with the part of the managed return that is nontraded. However, the value to such an investor will be given by Proposition 1 if: (1) the investor can duplicate the payoff from the managed portfolio by trading in marketed securities (possibly at a higher cost than the manager) or (2) the correlation between the part of the individual's marginal rate of substitution due to nontraded assets and the part of the return due to nontraded assets is zero. Further, if the marginal rate of substitution is close to Z in mean square, then the values will be close. These results are stated formally and proven in Proposition 2.

Proposition 2. Let $v = E[Ze(Z)]$ *denote the average value of the payoff,* X_p, *to all investors whose marginal rate of substitution depend only on the returns on traded claims, where* $e(Z) = E[X_p|Z]$. *Let* M *be an individual's marginal rate of substitution (not necessarily measurable with respect to* F_T). *The average value of the payoff* X_p *to this individual is given by* $E(MX_p)$. *Then, we have the following characterization of* $|E(MX_p) - v|$.

 a. If X_p is in F_T, then $|E[MX_p] - v| = 0$.

 b. If $COV(X_p - e(Z), M - Z) = 0$, then $|E[MX_p] - v| = 0$.

 c. If $\{E[(M - Z)^2]\}^{0.5} < \varepsilon/SD(X_p - e(Z))$, then $|E[MX_p] - v| < \varepsilon$.

Proof. First note that for all returns, R, on traded securities,

$$1 = E[MR] = E[RE[M|F_T]].$$

Since $E[M|F_T]$ is in F_T and since Z is unique, $E[M|F_T] = Z$.

 a. If X_p is in F_T, then $E[MX_p] = E[X_pE[M|F_T]] = E[ZX_p]$

   $= E[Ze(Z)] = v$.

 b. $E[MX_p] = E[(M - Z + Z)X_p] = v + E[(M - Z)X_p]$

   $= v + E[(M - Z)(X_p - e(Z) + e(Z))]$

   $= v + COV(M - Z, X_p - e(Z))$.

 c. $|E[MX_p] - v| = |E[(M - Z)X_p]| = |E[(M - Z)(X_p - e(Z))]|$

   $\leqslant E[|(M - Z)(X_p - e(Z))|] \leqslant \{E[(M - Z)^2]\}^{0.5}$

   $\times SD(X_p - e(Z))$.

by the Cauchy–Schwarz inequality, where SD indicates standard deviation. Under the conditions of the proposition, the last expression is less than ε. $\square$

Before considering issues associated with implementing the procedure, it is appropriate to consider the generality of our model. Assumption 1 is the crucial assumption and is the one most likely to be violated. Allowing the possibility of informed trading by the portfolio manager may restrict the ability of traders to replicate payoffs. For example, in the presence of informed traders, one would expect there to be a bid–ask spread, and hence trading will not be frictionless. We are therefore implicitly assuming that the portfolio manager is "small" relative to the market in order to guarantee the consistency of our methodology. The assumption that the portfolio manager's abilities change over time (if at all they do) in a systematic stochastic fashion is likely to be very restrictive.[6] We need this assumption to enable the use of time-series methods to estimate and value the manager's abilities. This assumption can be relaxed if the analyst has access to substantial additional information regarding how the manager takes decisions (in addition to time series data on the return on the managed portfolio).

Given these limitations, the technique described in Proposition 1 is relatively robust and immune to manipulation by the portfolio manager. Given the assumption that the marginal rate of substitution of at least one investor is a function of some identifiable index return, and given a reasonably accurate specification of the form of the function $e(R_I) = E[X_p|R_I]$, a portfolio manager will not be able to show spurious value through continuous trading or creative use of options and other contingent claims. Use of the wrong functional form for $e(.)$ or use of the wrong index return, however, could lead to erroneous valuation of the portfolio manager's abilities.

2.3. Choosing the functional form

Several parametric as well as nonparametric methods are available for estimating the function $e(.)$, when the functional form of $e(.)$ is not known. When one takes the parametric approach, one can choose either polynomials or splines, since it is well known that any function can be arbitrarily closely approximated by a collection of spline functions or polynomial functions. Alternatively, one could use a semi parametric method to estimate the joint density of X_P and R_I and then compute the conditional expectation function, $e(.)$. Each has its strengths and weaknesses and we discuss these below.

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[6]For example, consider a portfolio manager who specializes by watching what happens in the Middle East. Such a person might have been able to predict the war in the Middle East and moved the money in and out of oil stocks at appropriate times during the recent Iraq—Kuwait crisis and earned "abnormal" returns for his clients. It is not clear when the manager will be able to repeat such a performance in the future.

2.3.1. Parametric estimation

If one were to choose the spline approach, then a continuous, piecewise linear fit would appear to be the easiest to value. Note that a piecewise linear fit will be of the form:

$$X_P = \beta_0 + \beta_1 R_I + \sum \delta_1 \max(R_I - t_i, 0).$$

Notice that $\max(R_I - t_i, 0)$ is the payoff, at expiration, on an index call option with exercise price t_i, when the current value of the index is one. The value of a dollar for sure, received in one period is $1/R_f$. The value of R_I received in one period is 1. Thus, given estimates $\hat{\beta}_i$ and $\hat{\delta}_i$, the estimated value is

$$\hat{v} = (\hat{\beta}_0/R_f) + \hat{\beta}_1 + \sum \hat{\delta}_i C_i,$$

where C_i is the average value of a call option with one period to expiration and exercise price t_i on the index with current value equal to one. The strength of this procedure is the ease and intuitive appeal of the valuation phase of the exercise. To implement such a procedure, one must specify the number and location of the knots (the t_i's), run the regression and then make some assumption about the distribution of the return R_I. A reasonable bench mark to start with is to assume that R_I is lognormally distributed, so the C_i are Black–Scholes prices. The major weakness lies in the specification of the location and number of knots. With a large sample, one could let least squares minimization choose the location of a given number of knots. Such a large sample size would not typically be available. Thus, one is forced to ad hoc specifications of the location of the knots.

An alternative procedure is to fit a polynomial relation:

$$X_p = \sum \gamma_i R_I^i.$$

In this case, one need only specify the order of the polynomial and run the regression to obtain estimates $\hat{\gamma}_i$. The value estimate is then given by:

$$\hat{v} = \sum \hat{\gamma}_i E[ZR_I^i].$$

As in the case of splines, a reasonable bench mark to start with is to assume that R_I is lognormally distributed in identifying Z and evaluating $E[ZR_I^i]$.[7] The strength of the polynomial approach, relative to the spline approach is that no

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[7]See Dybvig (1988) for examples showing how to compute Z under alternative stochastic process assumptions.

ad hoc placement of the knots is required. The polynomial approach has two weaknesses. First, in small samples, the estimated coefficients can be very sensitive to outliers. Second, while R_I^* is, in theory, a perfectly reasonable contingent claim, it is not one that is observed. Consequently, the evaluator may have little confidence in the pricing of this claim. We discuss the polynomial approximation so that we can point out the connection to the well known Treynor–Mazuy (1966) and Admati et al. (1986) methods.

Many of the measures of performance evaluation that have appeared in the literature *resemble* special cases of this general parametric methodology. Remember however that the theoretical support for our approach is different than the ones on which these approaches were originally developed. For example, suppose that one specifies R_I to be a broad portfolio of stocks, and the risk-free rate is a constant, r. In this case, instead of working with $E(X_p|R_I)$, we can work with $E(X_p|X_I) = e(X_I)$, where $X_I = R_I - r$. If there are a priori reasons to specify $e(X_I) = \alpha + \beta X_I$, then the estimate of value, $E[ZX_p]$ is given by α/R_f, a scaling of Jensen's alpha. Admati et al. (1986) suggest estimating $e(X_I) = \alpha_0 + \alpha_1 X_I + \alpha_2 X_I^2$ and testing the significance of α_2 for market timing ability. If selling covered calls is possible, then α_2 may be significant, but yet the manager may have no valuable information. We suggest calculating the value as $(\alpha_0/R_f) + \alpha_2 V(X_I^2)$ where $V(X_I^2)$ is the value of a contingent claim which pays X_I^2.

Another example is provided by the analysis of market timing in Henriksson and Merton (1981). Suppose, as in Henriksson and Merton, the portfolio manager is a market timer, and that $X_p = \alpha X_m$, where α is either one or zero. Further, suppose that:

$$E[\alpha|F_T] = p_2 \quad \text{if } X_m > 0$$

$$1 - p_1 \quad \text{if } X_m \leqq 0 \ .$$

Then,

$$E[ZX_p] = E[ZX_m\alpha] = E[ZX_m E[\alpha|F_T]]$$

$$= p_2 E[ZX_m] + E[Z(p_1 + p_2 - 1)(-\min(X_m, 0))]$$

$$= (p_1 + p_2 - 1)PV$$

where PV is the value of a one period put on the stock index with current value of 1 and exercise price equal to one plus the risk-free rate.

It is unlikely that the specification of $E[\alpha|F_T]$ is exactly correct, even when the portfolio manager is a pure market timer. Rather, it may be viewed as a convenient approximation to the true relation between α and X_m. This is brought home

by consideration of Henriksson and Merton's parametric test $\beta_0 + \beta_1 X_m + \beta_2(-\min(X_m, 0))$ the value of which is given by $(\beta_0/R_f) + \beta_2 PV$. It is important to stress, however, that the manager need not be a market timer as assumed by Henriksson and Merton. One can view this as a "one-knot spline" approximation to what may be a more complicated relation between X_p and X_m. Similarly, Jensen's alpha calculation can be seen as a linear approximation to the true relation, and the approach suggested by Admati et al. (1986) can be viewed as a quadratic approximation to the true relation. Even when we choose more than one index and project the portfolio excess returns on the index returns, as the above analysis suggests, the true projection need not be linear. The Connor and Korajczyk (1986) approach to performance evaluation can be thought of as a linear approximation to the true functional relation between managed portfolio excess returns and the set of index excess returns.

2.3.2. Semi parametric estimation

On purely statistical grounds, a technique based on Gallant, Rossi and Tauchen (1992) may well be superior. The primary advantage it offers is the ability to fit a wide range of stochastic relations between random variables, even with a fairly small set of parameters. As the following shows, however, the contingent claim that this technique delivers is difficult to interpret and to value.

Define Y by $Y = (X_P, R_I)'$. Following Gallant, Rossi, and Tauchen (1992) the density of $Y, f(.)$, is proportional to:

$$P_n(R^{-1}(Y-m))^2 \phi(R^{-1}(Y-m))/\det(R),$$

where P_n is a polynomial of order n, m is the mean vector, $m = (m_P, m_I)'$, R is such that RR' is the variance covariance matrix of Y and ϕ is the standard bivariate normal density. Denote by $f_{P|I}$ and f_I, respectively, the conditional density of X_P given R_I and the marginal density of R_I derived from $f(.)$. We seek the expectation of X_P given R_I derived from $f(.)$:

$$e(R_I) = \int x f_{P|I}(x|R_I)\mathrm{d}x = \frac{\int x f(x, R_I)\mathrm{d}x}{\int f(x, R_I)\mathrm{d}x}.$$

Define $r(R_I)$ to be the linear regression of X_P on R_I and a constant, and let SE be the standard deviation of the residual from this regression. Consider the normalization of Y:

$$R^{-1}(Y-m) = ((X_P - r(R_I))/SE), \quad ((R_I - m_I)/\sigma_I)' = (Z_P, Z_I)'$$

where σ_I is the standard deviation of R_I. Notice that Z_P and Z_I are orthogonal and have zero mean and unit variance. Then, $e(R_I)$ is given by:

$$e(R_I) = r(R_I) + SE \frac{\int \dfrac{(x - r(R_I))}{SE} f(x, R_I) dx}{\int f(x, R_I) dx}.$$

Fix R_I at some value t. Then given the normalization above:

$$e(t) = r(t) + SE \frac{E\left[Z P_n\left(Z, \dfrac{(t - m_I)}{\sigma_I} \right)^2 \right]}{E\left[P_n\left(Z, \dfrac{(t - m_I)}{\sigma_I} \right)^2 \right]},$$

where Z is a standard normal random variable. The numerator of the second term will be a polynomial of degree $2n - 1$, while the denominator will be a polynomial of degree $2n$. For example, consider $n = 1$. In this case, $e(R_I)$ is given by:

$$e(R_I) = b_0 + b_1 R_I + 2SE(g_0 + g_1 R_I)/(1 + (g_0 + g_1 R_I)^2)$$

for some constants to be estimated $\{b_i, g_i, SE; i = 0, 1\}$.

As with the polynomial case, the second term on the right is, in theory, a reasonable contingent claim. It is not a contingent claim that we observe and hence valuing with confidence is difficult. Even if one is willing to make the Black–Scholes assumptions, valuation requires numerical integration.

Given current pricing and statistical methodologies, we prefer approximating the function $e(.)$ by splines. We believe that this approach will be empirically preferable to nonparametric methods when the analyst is constrained to work with relatively short time series of data. Also, we prefer splines to polynomials and semi parametric methods since splines can be interpreted as options and hence are easier for practitioners to understand and value.

3. Ability to approximate the true function using a limited number of options

3.1. The portfolio manager has no abilities

Like all other approximations that have been suggested in the literature on asset pricing, the approximations we have suggested are not uniform. If the manager knows that the analyst is using a specified number of options to evaluate performance, the manager (if he is allowed to do so) can show spurious

superior performance by engaging in certain trading strategies. This is not a limitation only of our method. This is true of all performance evaluation methods. This weakness arises from the fact that every known empirically implementable asset pricing model is itself an approximation and misprices a certain subset of assets. Hence an important question that needs to be addressed before choosing the number and type of options used to approximate the managed portfolio payoff is whether the approximation is adequate.

In this section we show that payoffs from certain option trading strategies (which exhibit substantial curvature) can be approximated sufficiently well by using only three options. We suggest using similar methods to evaluate the adequacy of the number and type of options used in the specific application the practitioner is interested in, taking into consideration the tradeoff between Type I and Type II errors.

Our motivation for examining the ability of the payoff on a portfolio of options to approximate the payoff on certain arbitrarily chosen call options arises from the following observation. One way that a portfolio manager can appear to be providing valuable management services is through the use of dynamic trading strategies which replicate payoffs from options. According to the theory developed above, such use of trading strategies based on common knowledge should not show value. Strictly speaking, this requires knowing the functional form of the projection. To investigate how well various approximations work, we assume that the managed portfolio returns are generated in the following way: Each month, the manager purchases a three-month call option on the market index at the Black–Scholes price, holds it for one month and sells it at the Black–Scholes price. We assume that monthly continuously compounded index returns are normally distributed, with a mean of 0.0109 and a standard deviation of 0.0608, corresponding to the sample moments of the stock index portfolio during 1968 to 1982. The risk-free rate is assumed constant each period at 0.54 percent, corresponding to the average continuously compounded treasury bill rate during the above period.

We examine 5 portfolio strategies, ranging from buying a call with an exercise price equal to 0.95 of the value of the index to buying a call with an exercise price equal to 1.2 of the value of the index. For each portfolio strategy we look at five specifications of the projection:

1. $X_p = \beta_0 + \beta_1 X_I + \varepsilon$

2. $X_p = \beta_0 + \beta_1 X_I + \beta_2 \max(X_I, 0) + \varepsilon$ (Henriksson–Merton)

3. $X_p = \beta_0 + \beta_1 X_I + \beta_2 \max(X_I - t, 0) + \varepsilon$

4. $X_p = \beta_0 + \beta_1 X_I + \beta_2 \max(X_I - t_1, 0) + \beta_3 \max(X_I - t_2, 0) + \varepsilon$

5. $X_p = \beta_0 + \beta_1 X_I + \beta_2 \max(X_I - t_1, 0) + \beta_3 \max(X_I - t_2, 0)$
$$+ \beta_4 \max(X_I - t_3, 0) + \varepsilon$$

We compute the true parameters (including the location of the knots) by minimizing the sum of the squared errors of the projections for each of the five models above, using 20,000 simulated observations. Numerical methods were used to calculate the minimizing parameters. We calculate the values using Black–Scholes procedures. The value of 1 is $1/R_f$. The value of X_I is 0 and the remaining terms in expressions 2 through 5 are valued using the Black–Scholes price of the indicated option.

Table 1 presents the result of the simulations. The linear approximation is inadequate even for calls close to being at the money. The one-knot spline does substantially better than the Henriksson–Merton approach (that is, a one-knot spline with the knot located at the risk-less return). Higher order splines with optimally located knots do better than a one-knot spline. A two-knot spline appears to provide a reasonable approximation even for a way out of the money call (with a strike price to spot index ratio of 1.10 to 1.20). These results suggest that even in those situations where a one-knot spline may be adequate, the location of the knot will generally not be at the risk-free return as in the Henriksson–Merton method. However, the three-knot value estimates were not critically dependent on the location of the knots. For example, the values obtained when the expected number of observations between knots was equalized were still close to zero. For options with exercise prices of 0.9, 1, 1.1, and 1.2 the values were, respectively (annualized and in percent) -0.29, -0.9, -2, and 2.25.

Table 1

Various approximations to $e(R) = E[X|R]$. R is lognormally distributed with $E[\log(R)] = 0.010891$, $SD(\log(R)) = 0.060799$. The excess return, X, over the risk-free rate $(\log(R_F) = 0.005419)$ on the managed portfolio is generated by buying a 3 month call on the index and holding it for one month. The estimated values of X_p are annualized in percent and based on 20,000 simulated observations.

| | | Estimated value | | | |
| | | Number of knots in the spline | | | |
Exercise Price	H–M	Zero	One	Two	Three
0.95	− 0.35	− 3.34	− 0 21	− 0 05	0 13
1 00	− 0.40	− 7 55	− 0.17	− 0.12	− 0 11
1 05	− 1 54	− 8.73	− 0 40	− 0.39	− 0.44
1.10	− 6 06	− 21.19	− 3.05	− 0.46	− 0.38
1 15	− 10.75	− 28.01	− 2.01	− 1.69	− 0.36
1 20	− 17 68	− 66.16	− 13.92	− 1.17	− 1.26

Note that the true value of X, the excess return is zero, since we assume frictionless trading.

3.2. The manager has ability

The next question that arises is that of power, that is, whether our approach is capable of detecting ability when it does exist. In what follows we examine a situation where the portfolio manager has superior information of a particular type and uses it in a particular way. Since power can only be examined against specific alternatives, the practitioner is advised to examine the power function of the particular approximation he chooses for the specific alternatives of interest to him or her.

We suppose that the market index return is lognormally distributed with its historical mean and standard deviation. Now suppose that there is a manager who sees a signal, S, given by $S = \log(R_m) + \varepsilon$, where ε is independent of R_m and normally distributed with mean zero and variance equal to $\sigma^2(1 - RSQ)/RSQ$, where σ^2 is the variance of $\log(R_m)$ and RSQ is the R-Square of the regression of $\log(R_m)$ on S in the population. Assume, further, that the manager invests in the market if $E[R_m|S] > R_f$ and invests in treasury bills otherwise. We also assume that the manager is extremely small relative to the "liquidity" of the market and hence does not affect market prices due to his trading activity. After some tedious but straightforward calculations it can be shown that the value of the information when used in this way is $\Phi(c_2) - \Phi(c_1)$ where Φ is the standard normal cumulative distribution function and c_2 and c_1 are given by:

$$c_2 = \{(\mu - r)[(1 - RSQ)/RSQ] + \sigma^2/(2RSQ)\}/(\sigma(1/RSQ)^{0.5}),$$

$$c_1 = c_2 - \sigma/(1/RSQ)^{0.5},$$

$$\mu = E[\log(R_m)], \quad \sigma^2 = \mathrm{Var}(\log(R_m)), \quad r = \log(R_f).$$

We simulated 100 samples of 40 observations each, for values of RSQ ranging from 0.05 to 1. The results for one- and three-knot splines are reported in Table 2. We have insufficient data to optimally place the knots and hence we placed the knots so that, in expectation, there would be an equal number of observations between the knots. Thus, for three knots, the knots were placed at the 25, 50, and 75 percent points of the distribution of R_I. As can be seen, the power function is fairly steep even for 40 observations, and the one-knot spline (knot at the median of the R_I distribution) appears to do a satisfactory job.

3.3. Choosing the index

Unfortunately, the methodology we have been discussing does not get around the problem of the correct choice of an index. Fortunately, however, we can show that if the index choice is not too bad, and as long as we can correctly value

Table 2

Various approximations to $e(R) = E[X|R]$, where R is lognormally distributed; $E[\log(R)]$ $= 0.010891$, $SD(\log(R)) = 0.060799$ The excess return, X, is given by $X = I(R - Rf)$ where Rf is the risk-free rate (0.54159%), I is 1 if $E[R|S] > Rf$ and zero otherwise, where $S = \log(R) + \varepsilon$ and ε is normally distributed, independent of R with mean zero and variance equal to $(1 - RSQ)/RSQ$ times the variance of $\log(R)$. That is, RSQ is the asymptotic R-Square from the regression of $\log(R)$ on S. Each model was estimated 100 times with 40 simulated observations each True values and average estimated values are annualized, in percent. The column titled "number of rejections" shows the number of times, out of 100, that the estimated value was significantly larger than zero at the indicated levels. The knots were placed so as to equalize the expected number of observations between the knots.

		One-knot spline			Three-knot spline		
RSQ	True value	Average estimated value	Number of rejections 0 05	0.01	Average estimated value	Number of rejections 0.05	0 01
1.00	29.10	29 10	100	100	29 10	100	100
0.90	27 61	27.53	100	100	27 61	100	100
0.80	26 02	26.14	100	100	26.21	100	100
0 70	24.33	24.25	100	100	24.04	100	100
0.60	22.50	22 80	100	100	22.49	100	100
0.50	20.51	19.95	98	97	21.00	100	100
0.40	18.29	16.97	96	87	18.73	100	100
0.30	15.75	15.20	92	77	15.25	90	78
0.20	12.72	12.80	82	59	12.11	76	56
0.10	8 68	8.37	50	35	9 42	56	40
0 05	5 71	5.79	34	16	4 92	31	15

the contingent claim, then the resulting value estimate will be close to the true value. Formally, suppose that R_I is the correct index return and the average value of the contingent claim, $e(R_I) = E[X_p|R_I]$, is the correct average value of the managed portfolio. Suppose we pick an incorrect index return, R, and estimate the contingent claim $e'(R'_I) = E[X_p|R'_I]$. The following proposition shows that if R'_I explains most of the variation of Z or if given R'_I, R_I explains little of the remaining variation in X_p, then the true average value $E[Ze(R_I)]$ will be close to average value arrived at using the incorrect index R'_I, $E[Ze'(R'_I)]$.

Proposition 3. If the variance of Z unexplained by R'_I,

$$E[(Z - E[Z|R'_I])^2] = (1 - RSQ(Z; R'_I))\text{var}(Z)$$

is small, or the difference between the variance of X_p explained by R_I and R'_I and the variance of X_p explained by R'_I,

$$E[(E[X_p| R_I, R'_I] - e'(R'_I))^2] = (RSQ(X_p; R_I, R'_I) - RSQ(X_p; R'_I))\text{var}(X_p)$$

is small, then the squared valuation error,

$$(E[Ze(R_I)] - E[Ze'(R_I')])^2$$

is small.

Proof.

$$(E[Ze(R_I)] - E[Ze'(R_I')])^2 = (E[ZX_p] - E[Ze'(R_I')])^2$$

$$= (E[Z(E[X_p|R_I, R_I'])] - e'(R_I')])^2$$

$$= (E[(Z - E[Z|R_I']))(E[X_p|R_I, R_I'] - e'(R_I')])^2$$

$$\leqslant \{E[(Z - E[Z|R_I'])^2]\}\{E[(E[X_p|R_I, R_I']$$

$$- e'(R_I'))^2]\}$$

$$= \mathrm{var}(Z)(1 - RSQ(Z; R_I'))$$

$$\times \mathrm{var}(X_p)(RSQ(X_p; R_I, R_I')$$

$$- RSQ(X_p; R_I')). \qquad \square$$

As with other continuity results of this sort [Green (1986), for example] the bound is not uniform over all possible managed portfolios. For example, levering up a managed portfolio will increase the divergence from the true value. Thus, as Green (1986) points out in the context of evaluating the performance of managers using Jensen's alpha, performance ranks evaluated using one proxy can easily be reversed using another proxy that is close to the original proxy. Thus, the same problems noted by Roll (1978) with ranking portfolios will appear with the methodology examined here. The positive aspect of the proposition is that no matter what the true index is, if the chosen index and the functional form of the projection explain most of the variation in X_p, then the valuation error will be small.

This proposition suggests that different indices may be used to evaluate different portfolios. For example, if a manager switches funds between bills and some index with return R', then in the absence of knowledge of the true index, the index being timed may be the most appropriate index to use. The next proposition provides a condition under which the correct value can be inferred from the use of an incorrect index.

Proposition 4. Suppose that the portfolio excess return is given by $X_p = w(R' - R_f) \equiv w \cdot X'$, where R' is the return on some portfolio of traded security and R_f the risk-less return. The scalar random variable, w, takes on the value of one or zero, and is a function of a (possibly multi-dimensioned) signal, S, which in turn is a function of X' and observational noise v which is independent of X' and the return, R, on the true index. Then the true average value of the managed portfolio, $E[ZX_p]$, equals the average value of the contingent claim $E(X_p|X')$, $E[ZE[X_p|X']]$.

Proof.

$$E[ZX_p] = E[ZwX'] = E[ZX'E[w|R, X']] = E[ZX'E[w|X']]$$

$$= E[ZE[X_p|X']]. \quad \square$$

These results suggest that knowledge of what the manager is doing will most likely help in choosing the right functional form and the right index to achieve the right mix of Type I and Type II errors.

In the empirical finance literature, it is common practice to use the return on the equally weighted index portfolio of stocks on the NYSE as the market index portfolio (or the single pre-specified factor in linear beta pricing models). However, Proposition 4 suggests that if the portfolio manager behaves as though he allocates the funds between cash and a particular portfolio of securities, then it may be advisable to use such a portfolio of securities as the index. There are a priori reasons to believe that while the portfolio of stocks held by some portfolio managers resembles the value weighted index of stocks traded on the exchange, others resemble the equally weighted index. Hence in our empirical study of mutual funds to illustrate the application of our methods we will examine how sensitive our conclusions are to the choice of the index.

4. Evaluating the performance of mutual funds

Lehmann and Modest (1987) examine the performance of 130 mutual funds over the period 1968 to 1982. Their primary concern was the sensitivity of performance evaluation to the choice of the index. They find that a substantial number of fund returns exhibit option-like features (that is, nonlinearly related to the return on the index, see Tables X and XI, pages 254–55) which provides a priori justification for using our valuation method. They conclude that (a) the choice of the index matters in the sense that the number of rejections of the hypothesis of zero value changes, in some cases substantially, and (b) there is evidence of negative value in the mutual fund returns. Lehmann and Modest's analysis was fairly wide ranging, looking at not only the equally and value

weighted indices, but various APT indices as well. Our analysis, using the Lehmann–Modest data set will be more modest. Our intent is to illustrate our methodology on actual portfolio returns and examine how the choice of the index and the number of knots matters. Unlike Lehmann and Modest, who examine the statistical significance of the Treynor–Black (1972) appraisal ratios and the coefficient on the quadratic term, we examine the statistical significance of the total value of the fund manager's abilities.

For the analysis of the mutual funds, we used a slight modification of the spline estimation discussed above. Specifically, we regressed the excess portfolio return divided by one plus the interest rate on a spline function of the gross index return divided by the gross interest rate. That is, defining X_{pt}^* to be $X_{pt}/(1 + r_{ft})$ and R_{It}^* to be $R_{It}/(1 + r_{ft})$ where R_{It} is the gross (one plus) rate of return on the index, the one-knot spline estimation is:

$$X_{pt}^* = a_0 + a_1 R_{It}^* + a_2 MAX(R_{It}^* - k, 0) + e_t .$$

This was done for two reasons. First, R_{It}^* is more likely than R_{It} to be stationary. Second, the valuation of the resulting projection of X_{pt}, conditional on the interest rate, is independent of the interest rate. This can be seen by multiplying the estimated projection by $(1 + r_{ft})$ to get:

$$e_t = a_0(1 + r_{ft}) + a_1 R_{It} + a_2 MAX(R_{It} - k(1 + r_{ft}), 0) .$$

The value of the first two terms is $a_0 + a_1$. Using the Black–Scholes formula, the value of the third term is:

$$a_2(N(d_1) - N(d_2)k(1 + r_{ft})/(1 + r_{ft})) = a_2(N(d_1) - kN(d_2)) ,$$

where

$$d_1 = [\log(1) - \log(k(1 + r_{ft})) + 0.5v^2 + \log(1 + r_{ft})]/v$$

$$= -\log(k)/v + v/2$$

and $d_2 = d_1 - v$. In these expressions v is the standard deviation of the continuously compounded index return, while $N(.)$ is the standard normal distribution function. When the one-knot spline was estimated, the knot was put at $k = 1$. When three-knot splines were estimated, the knots were placed at 1 and at $\exp(m_I^* \pm 0.67\sigma_I^*)$ where m_I^* and σ_I^* are respectively the mean and standard deviation of $\log(R_{It}) - \log(1 + r_{ft})$ and 0.67 is approximately the 75 percent point of the standard normal distribution.

Our results agree with both the conclusions of Lehmann and Modest in the sense that there are substantial differences in the value estimates obtained using

Table 3

Mutual fund performance

Cross-sectional distribution of the value estimates for 130 mutual funds using various specifications Value is calculated with respect to the NYSE value weighted index. Values are annualized and in percent

	One-knot spline	Three-knot spline
Mean	− 0.091	− 0.171
Standard deviation	2.692	2.703
Deciles min	− 8.645	− 8.704
1	− 3.036	− 3.291
2	− 2.144	− 2.182
3	− 1.195	− 1.297
4	− 0.905	− 0.984
5	− 0 334	− 0 400
6	0 340	0 360
7	1.098	1.088
8	2 029	1 987
9	3 233	3 112
Max	10 097	10.094

the two indices. Furthermore there is a preponderance of negative values. However, we find that the two value estimates are highly correlated and the rank correlation is also very high.

Our methods classify some funds as providing "valuable service" independent of whether we use the equally weighted or the value weighted index. This finding is consistent with that of Lee and Rahman (1990). In classifying fund managers as having superior ability we cannot use the standard 5 percent significance level as a cutoff. This is because if none of the funds managers have any abilities and the estimation errors are uncorrelated then 5 percent of the funds should indeed show superior performance if the test statistic is right. To take this into account, we use the Bonforoni p-values. Also, the evidence supports the view that for the mutual fund portfolios we consider, there is not much gain to be obtained from using more than one option (in addition to the risk-free asset) along with the index return to approximate the excess return on mutual funds.

Tables 3 and 4 present the cross-sectional distribution of value estimates with respect to the value weighted and equally weighted index respectively for the one-knot and three-knot spline cases. There are two conclusions to be drawn from these tables. First, the one-knot and three-knot spline specifications produce very similar results, suggesting that use of the one-knot spline approximation may be sufficient for this particular application. In fact, the correlation between the values obtained using the two indices is never less than 0.995. Second, use of the equally weighted index produces lower values in general. It also appears that the use of the equally weighted index just shifts the distribution of values to the left by a constant amount, where the constant approximately

Table 4

Mutual fund performance

Cross-sectional distribution of value estimates for 130 mutual funds using various specifications. Value is calculated with respect to the AMEX and NYSE equally weighted index. Values are annualized and in percent.

	One-knot spline	Three-knot spline
Mean	− 3.055	− 2 678
Standard deviation	2.752	2 723
Deciles min	− 12.065	− 11.421
1	− 6 100	− 5.756
2	− 5 336	− 4.973
3	− 4.535	− 4 204
4	− 3.763	− 3 321
5	− 2.849	− 2.666
6	− 2.140	− 1.922
7	− 1.712	− 1.361
8	− 0 825	− 0.480
9	0.166	0.567
Max	7.585	7.775

equals the value assigned to the value weighted portfolio of stocks in the NYSE when the equally weighted portfolio of stocks is used as the index. We suspect that this may be due to the fact that mutual fund portfolios resemble the value weighted index portfolio and the value weighted index is under-valued by the equally weighted index.

Tables 5 and 6 report the cross-sectional distribution of the t-statistics (computed allowing for the presence of conditional heteroscedasticity) for the hypothesis of zero value for the mutual fund manager's abilities, for the value weighted and equally weighted indices respectively. An examination of the minimum and maximum t-statistics with an application of the Bonforoni bound suggests that when using the value weighted index there is evidence of both positive and negative value while there is evidence only of negative value when value is estimated using the equally weighted index.

Despite the significant differences in value estimates obtained using the two indices and different number of knots, the value estimates themselves are highly correlated. Table 7 shows that the correlation between any two value estimates is never less than 0.95.

The top performer by any measure (both one- and three-knot specifications with either index) was the Templeton Growth fund. One interesting observation is that the shape of the fit tends to be concave with a positive intercept. See Figure 1. The shape is similar to a fund that uses written covered calls in its portfolios, but obtains a better than fair value for its written calls.

In our analysis we used the Black–Scholes model to evaluate the value of the estimated options. Hence to some extent our conclusions depend on the validity

Table 5

Mutual fund performance

Cross-sectional distribution of 130 *t*-statistics. Each *t*-statistic is for the hypothesis that the associated fund is providing zero net value returns. Value is calculated with respect to the NYSE value-weighted index The "Bonforoni *p*" is the *p*-value of the associated *t*-statistic multiplied by 130.

	One-knot spline	Three-knot spline
Min *t*	− 4.01	− 4.05
Bonforoni *p*	0.008	0 007
Average *t*	− 0.09	− 0 13
Max *t*	4.21	4 18
Bonforoni *p*	0.003	0.004
Number with *t*-statistics		
< −2.326	8	8
− 2.326 < *t* < − 1 96	4	5
− 1.96 < *t* < − 1.645	7	7
− 1.645 < *t* < 0	52	51
0 < *t* < 1.645	41	41
1.645 < *t* < 1.96	9	9
1.96 < *t* < 2.326	4	4
2.326 < *t*	5	5

Table 6

Mutual fund performance

Cross-sectional distribution of 130 *t*-statistics. Each *t*-statistic is for the hypothesis that the associated fund is providing zero net value returns Value is calculated with respect to the AMEX and NYSE equally-weighted index. The "Bonforoni *p*" is the *p*-value of the associated *t*-statistic multiplied by 130

	One-knot spline	Three-knot spline
Min *t*	− 4.16	− 4.32
Bonforoni *p*	0 004	0.002
Average *t*	− 1.19	− 1.11
Max *t*	2.94	2.98
Bonforoni *p*	0.427	0.375
Number with *t*-statistics		
< − 2 326	15	15
− 2.326 < *t* < − 1.96	12	12
− 1 96 < *t* < − 1 645	15	14
− 1 645 < *t* < 0	72	70
0 < *t* < 1.645	14	17
1.645 < *t* < 1 96	1	0
1.96 < *t* < 2.326	0	1
2 326 < *t*	1	1

of the Black–Scholes model. However, it should not be difficult to modify our procedure to make use of any other option pricing model that the analyst considers appropriate. If prices on traded options are available, the arguments in this paper can be extended to justify the use of the Linear Factor Model

Table 7

Mutual fund performance

Correlations and rank correlations of 130 value estimates obtained from the NYSE value weighted index and the AMEX and NYSE equally weighted index for various specifications of the relations between the portfolio excess return and the index excess return

	Correlation	Rank correlation
One-knot spline	0 964	0.958
Three-knot spline	0.978	0.971

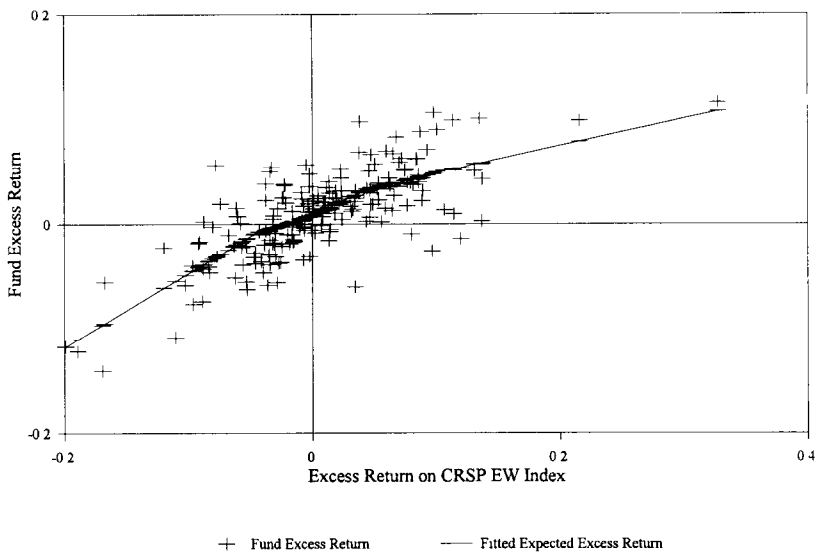+ Fund Excess Return   —— Fitted Expected Excess Return

Fig 1 Temptleton growth fund.

analogue of Jensen's alpha suggested by Connor and Korajczyk (1986), when excess return on a few bench mark options are used as "factors" in addition to the excess return on the factors suggested by Connor and Korajczyk.

Our analysis is almost certainly affected by a survivorship bias—we restrict attention to those funds in operation between 1968 and 1982. Consideration of this bias merely strengthens our conclusion that there is a preponderance of negative values. It is of course possible that the positive performers were merely the survivors of an originally large family of competing mutual funds. It is not clear why the survivor problem should bias our analysis of the index choice, however. Still, for the evaluation of mutual funds this is an important area of further research [see Brown, Goetzmann, Ibbotson, and Ross (1992)].

5. Conclusion

The purpose of this paper is to develop a fairly general methodology for valuing the performance of managed portfolios. The general approach is to decompose the payoff on the managed portfolio into two components: the first is a function of the return on some index portfolio (or a set of index portfolios), and the second is the residual that is left out. We assume that for some choice of the index portfolio the residual has zero value. Hence the value of the payoff from the managed portfolio is the value of the contingent claim on the index portfolio defined by the first part of the decomposition.

In practice, the form of the projection is not known. We suggest approximating the form of the contingent claim using low order linear splines or polynomials. Our results suggest that a three-knot linear spline may be adequate to capture some types of nonlinearities—namely, that may arise due to portfolio insurance and market timing strategies of portfolio managers, so long as sufficiently long time-series of data is available. While approximating the function using a one-knot spline is operationally similar to the Henriksson Merton (1981) method, there are two differences. First, the theoretical support for our model arises from entirely different arguments. Second, the knot is not necessarily located at the risk-free rate. Even though it may not be possible to separate abilities into "timing" and "selectivity," the total value estimated by the Henriksson–Merton method may be reasonably close to the true value. Hence our results can be viewed as supporting the use of the Henriksson–Merton method for detecting superior performance, but not for identifying the source of this superior performance. Similarly the value of the portfolio manager's abilities estimated by the Treynor–Mazuy quadratic regression method can be viewed as a polynomial approximation. Jensen's alpha can be viewed as a linear approximation.

The choice of an index is problematic, since theory does not provide adequate guidance in this respect. We present results suggesting that if the index chosen is reasonably close to the true index, or if the managed portfolio return that is not explained as a (nonlinear) function of the return on some index portfolio is relatively small, then the estimated value will be close to the true value.

Our procedure gives the value at the margin and hence is sensitive to scaling, in the sense that scaling up the size of the operations by leverage will increase value. Hence the value computed according to this procedure cannot in general be used to compare different fund managers, or to predict the demand for the portfolio. This is a common shortcoming of performance evaluation procedures. If a stand can be taken on investor preferences, and the index the investor will hold in the absence of access to the managed portfolio, Bayesian extensions to the valuation method described here can be used to arrive at an estimate of the value of the manager's abilities.

Our approach also helps in appreciating the major difficulties involved in evaluating performance. First, the assumption that the ability of the portfolio manager varies over time in a sufficiently systematic stochastic fashion severely restricts the applicability of our method as well as the other methods that have been suggested in literature. Second, the form of the contingent claim has to be estimated. As the figures in Table 1 suggest, no one particular choice for the functional form of the contingent claim may be able to capture all types of nonlinearities involved adequately. One possible way out of this difficulty is to choose the functional form after extensive discussions with the portfolio manager to understand how the manager operates. To minimize the moral hazard problems involved, it will be necessary to monitor the manager ex post to ensure that the operations were consistent with what was agreed upon earlier.

We examined the performance of 130 mutual funds during 1968–82 to illustrate the use of our method. We found that while the value estimates depend on the choice of the index, the relative rankings were not all that sensitive to the choice of the index. Our analysis using mutual funds suggest that use of one-knot spline may be adequate. We found that while superior performance is indeed rare, there still were either a few superior performers or lucky survivors.

Our results can be viewed as supporting the use of the multi-factor analogue of Jensen's alpha suggested by Connor and Korajczyk (1986), by modifying their approach to include the excess returns on certain selected options on stock index portfolios as additional "factor excess returns."

Appendix

We used monthly returns on 130 mutual funds during the period January 1968 through December 1982. This data set was originally compiled by Roy Henriksson and updated by Bruce Lehmann and David Modest. We are grateful to Bruce Lehmann and David Modest for letting us use their data set. A detailed description of this data set appears in Lehmann and Modest (1987).

We also used monthly returns on the value weighted index and equally weighted index of stocks in the American and New York Stock Exchanges from the CRSP monthly tapes available from the Center for Research in Security Prices, University of Chicago.

All computations were done using GAUSS version 1.49b and RATS version 2.0.

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