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Introductory Price as a Signal of Cost in a Model of Repeat Business

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A two-period game between firms and consumers is considered. Firms are privately informed about their individual costs, and consumers must pay a search cost in order to learn a firm's current price. Consumers thus have incentive to use introductory price as a signal of cost and, hence, second period price. Recent refinements of the sequential equilibrium concept are employed, and the resulting equilibria involve low introductory prices (introductory sales).

I. INTRODUCTION

In recent years, much research has focused on the strategic pricing decisions of firms that face rational consumers. In particular, a number of economists have investigated the conditions under which an introductory or cyclical sale might be part of an optimal pricing strategy.

At least three such conditions have been identified. First, if consumers have incomplete information about product quality, then an introductory sale may be used to signal a high level of quality (Bagwell (1985*a*), Milgrom and Roberts (1986)). A separate literature links sales to consumer heterogeneity. Here, a sale emerges as a means of discriminating among heterogeneous consumers. Consumers may differ in their information (Varian (1980)), search characteristics (Salop (1977), Shilony (1977)), or reservation prices (Conlisk, Gerstner, and Sobel (1984), Sobel (1984)). Finally, some research has suggested that durable (Conlisk *et al.* (1984), Irvine (1981), Sobel (1984)) or storable (Salop and Stiglitz (1984)) products are especially likely to be offered at sale prices.

In this article, a new motivation for introductory sales is presented. Indeed, it is argued that introductory sales constitute rational, profit maximizing activity, even when quality is fixed and costlessly known, consumers are identical, and the product is non-durable and non-storable.

Working with a two-period model, I explore the simple hypothesis that a firm may have an introductory sale in order to signal to consumers that its price will be low in the second period. In this way, an introductory sale can lead to repeat business and, possibly, gains in overall profit.

The two crucial assumptions of the paper concern the information held by consumers. First, it is assumed that price information is costly. Specifically, a consumer is assumed able to learn a firm's current price only after incurring a (transportation or time-loss) cost associated with visiting the firm's store. Clearly, a consumer will not wish to incur this expense unless he expects to find a sufficiently low price.

The second assumption is that cost information is incomplete. Thus, a firm knows whether its production costs are high or low, but consumers do not observe costs. While not completely informed, consumers are rational and they therefore correctly anticipate that a low (high)-cost firm will charge a low (high) price in the second period.

Tying these strands together, imagine now a firm, privately informed about its costs (high or low), facing a group of rational consumers in a market with costly price information. Suppose that consumers visit the firm's store in period one. What introductory price should the firm charge?

If consumers take the introductory price to signal low costs, then consumers expect the second period price to be low, and they therefore return to the firm's store. If, however, the introductory price is believed to signal high costs, then a high second period price is expected, and so consumers do not return to the firm's store. Repeat business is (not) forthcoming if the introductory price signals low (high) costs. Thus, whether it actually has high or low costs, a firm will hope to use its introductory price in a way which persuades consumers that costs are low.

An equilibrium will take one of two forms. In a *separating* equilibrium, the introductory price selected by a firm depends on the firm's cost type. Thus, introductory price perfectly signals cost in a separating equilibrium. A *pooling* equilibrium is said to exist if the firm's introductory price is independent of its cost type. In a pooling equilibrium, introductory price provides no information.

After eliminating dominated strategies (Milgrom and Roberts (1986)), I show that only one separating equilibrium can exist. In this equilibrium, the firm offers an introductory sale if its costs are low and charges the high-cost monopoly price if its costs are high. Intuitively, a low-cost firm makes more profit in period two than would a high-cost firm, and so the low-cost firm can separate from its high-cost ghost by publicly sacrificing some of these second period profits in the introductory period. An introductory sale involves a financial sacrifice, and the low-cost firm is therefore able to separate with such a sale.

Thus, in the separating equilibrium, an introductory sale is profitable because it encourages repeat business, and the sale encourages repeat business because it signals low costs and, therefore, the incentive to charge a low future price.

Depending on the model's parameters, pooling equilibria may also exist. If low-cost firms are thought to be rare, then consumers are naturally "cautious" about returning to a given store. In this case, pooling equilibria cannot exist and the separating equilibrium discussed above is the unique "refined" equilibrium. In the converse case, where high-cost firms are thought to be rare, consumers are naturally "inclined" to return to a given store. A continuum of pooling equilibria exists in this case. However, if one applies Kreps' "intuitive criterion" (Kreps (1984)), many—but not all—of these equilibria can be argued to involve unreasonable beliefs. If in addition one discards equilibria which are Pareto dominated (over firm types) by other equilibria, then the unique equilibrium is characterised by pooling in the introductory period at the low-cost monopoly price. Here, the high-cost firm is thought of as having a sale in order to prevent consumers from inferring its costs.

I develop the formal results in a model with a monopoly and a single consumer. This simple framework can be extended in a number of directions. In particular, assuming that consumers do not communicate price information, I show that all of these results extend exactly to a multi-firm, many consumer world. Moreover, a free-entry condition can be imposed. From this perspective, the model generalises Diamond's (1971) seminal search model to include incomplete information about production costs and firm-specific price reputations.

The basic idea—today's price signals, through cost, tomorrow's price—also has application to the case in which the consumer of the monopolist's product is in fact a downstream firm. Consider an upstream monopolist that offers to supply an input to a downstream firm. When the downstream firm can use the monopolist's product only after an idiosyncratic investment is sunk (say, in machines needed to utilise the input, or in learning on the part of employees), appropriable quasi-rents are generated and a mutually beneficial relationship is threatened. As is well known (Williamson (1979), Klein, Crawford, and Alchian (1978)), contracts and, in the extreme case, vertical integration can often enable the efficient transaction. I show that signalling issues can also play an important role in determining whether or not the beneficial relationship develops. Specifically, an introductory sale signals a desire to extract relatively few future rents and therefore encourages the idiosyncratic investment.

The paper is organised in seven sections. The formal model is described in Section 2, and, in Section 3, a "refined" equilibrium is defined. In Section 4, the consumer is assumed "inclined" to visit, and the above described pooling equilibrium is featured. Consumers are assumed "cautious" in Section 5, and the aforementioned separating equilibrium is shown to be the unique "refined" equilibrium. In Section 6, various extensions of the model are discussed. Section 7 then concludes the paper.

II. THE MODEL

Time is divided into two periods. There is one consumer and one firm, each living two periods. The product we consider is non-durable and non-storable.

In each period, the consumer is endowed with an income I. A composite commodity, x^c, represents alternative uses for the consumer's income. The composite commodity price, P^c, is assumed fixed and costlessly known.

Assume that the monopolist is privately informed at the beginning of the two-stage game as to whether it is a high-cost or low-cost firm. It is common knowledge that Prob $(C = C^H) = \delta$ and Prob $(C = C^L) = 1 - \delta$, where $\delta \in (0, 1)$. The consumer, however, does not know which cost type is realised.

The game proceeds as follows. In stage one of the game, the consumer informed only of δ decides whether or not to visit the monopolist's store. He does not know the price that the monopolist will charge until he arrives at the store. Visiting the monopolist's store involves a cost, d, representing time-inconvenience, transportation expenses, etc. If the consumer does visit the store, then the monopolist quotes a price.[1] The consumer then chooses the quantity of the monopolist's good which he desires to buy at that price (this quantity may be zero). In the second stage of the game, the consumer again decides whether or not to visit the store. If he visited in the first period, he uses the monopolist's introductory price as a signal of cost and, hence, second period price. If he expects the second period price to be "high," then he doesn't return to the store. If in fact second period visitation does occur, the monopolist again quotes a price and the consumer responds with a demand for the corresponding quantity.

A rational consumer has foresight and maximises the expected value of a personally discounted sum of utility. The resulting behaviour can, of course, differ from that which would occur were the consumer to maximise instantaneous expected utility in each period. For, in the instantaneous (myopic) case, the consumer has no incentive to visit *in order to* obtain information which enables a better second period choice. Nevertheless, we will assume that the consumer maximises instantaneous expected utility in each period, given all available information. Information is used but not sought. This assumption will increase the clarity of the exposition (since reservation prices can now be related to single period indirect utility functions) and, as we will see, comes at absolutely no cost.[2]

Assume P^c, $I-d>0$. Let P be the price of the monopolist's product. Denote this product as x. Consider only $P>0$. Let $U(x^c, x)$ be the consumer's single period utility function. Assume that, for all $(x^c, x) \geqq 0$, U is differentiable, strictly increasing in each argument, and strictly quasi-concave. These assumptions are sufficient to give continuous, downward sloping, and strictly convex indifference curves. Assume also that indifference curves cross the x^c axis and, thus, that reservation prices for x can be defined (see below).

It is convenient to define a visit-contingent indirect utility function:

$$V(P) \equiv \max U(x^c, x) \quad \text{s.t. } P^c \cdot x^c + P \cdot x \leqq I - d.$$

$V(P)$ gives the maximum single period utility available to a consumer, given that the consumer has visited the monopolist's store. Let $X(P)$ be the corresponding visit-contingent demand function for the consumer. $X(P)$ is derived from the above maximisation problem. Under our assumptions, $X(P)$ is continuous (see, e.g. Varian (1978)). I will argue below that there exists $\hat{P}$ such that $X(P)>0$ for $P<\hat{P}$ and $X(P)=0$ for $P \geqq \hat{P}$. Assume that $X(P)$ is strictly decreasing for $P<\hat{P}$.

The points which I wish to now make can best be understood via a simple graph. A more formal analysis is contained in Bagwell (1985*b*).

Consider Figure 1. I first analyse the visit-contingent demand for the consumer who learns P only after incurring a cost d. Suppose this consumer walks into the store. What is the maximum price he will pay? Define $\hat{P}$ by

$$\frac{\hat{P}}{P^c} = U_2\left(\frac{I-d}{P^c}, 0\right) \Big/ U_1\left(\frac{I-d}{P^c}, 0\right).$$

Thus, at $P=\hat{P}$, the indifference curve becomes tangent to the budget line at the corner point $(x^c, x) = ((I-d)/P^c, 0))$. For $P<\hat{P}$, the visiting consumer buys a positive quantity, $X(P)$. For $P \geqq \hat{P}$, $X(P)=0$. Notice that $V(\hat{P}) \equiv U((I-d)/P^c, 0)$.

Going into the second period, a consumer who visited the monopolist in period one may know which type of monopolist is operative (if introductory price signalled cost). The consumer is then "perfectly informed" in that he knows, prior to visiting, the second period price which will be charged (in equilibrium). Consequently, we need to analyse the (pre-visit) reservation price associated with the demand of the perfectly informed consumer.

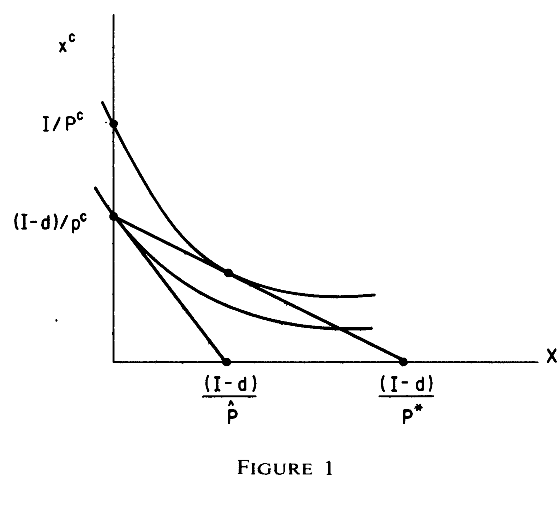


FIGURE 1

Consider again Figure 1. If the consumer knew P before visiting the monopolist's store, then his budget line would be $I/P^c \rightarrow (I-d)/P^c \rightarrow (I-d)/P$. Define P^* by

$$V(P^*) \equiv U(I/P^c, 0).$$

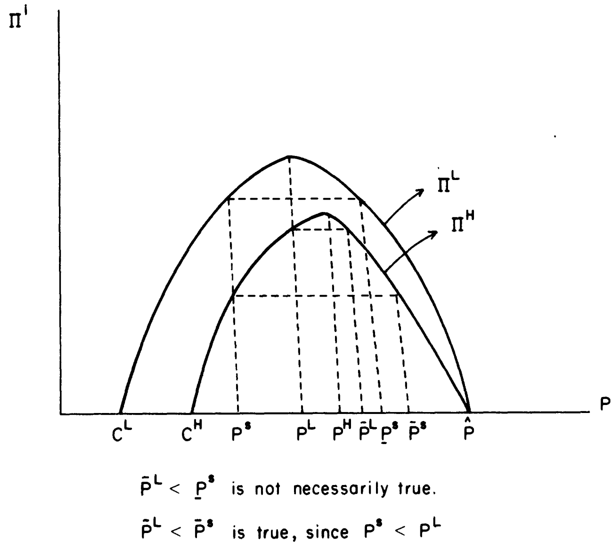
At $P = P^*$, the consumer with perfect information is exactly indifferent between applying an income $I-d$ to both the composite commodity and the monopolist's product (visiting the store) and applying an income I solely to the composite commodity (not visiting the store). If $P > P^*$, the informed consumer does not visit; if $P < P^*$, he visits and demands according to the visit-contingent demand function, $X(P)$. Notice that $\hat{P} > P^*$. The assumption that the consumer must pay d in order to be told the monopolist's price therefore results in a consumer who is willing to pay more once he's arrived at the store than he would have been willing to pay had he known the price before leaving home.

The monopolist either has high or low costs: $C^H = c^H \cdot x$ or $C^L = c^L \cdot x$, where $c^H > c^L$.[3] Define Π^i, $i = H, L$, as the single-period profit function of a monopolist of type i, given a visiting consumer. Thus,

$$\Pi^i(P) \equiv (P - c^i) \cdot X(P), \qquad i = H, L$$

Π^i is clearly continuous. Note also that $\Pi^i(c^i) = 0$, $\Pi^i(\hat{P}) = 0$, and $\Pi^H(P) \leqq \Pi^L(P)$, for all $P \leqq \hat{P}$ (with equality holding only at $\hat{P}$). Assume Π^i is strictly increasing for $P < P^i$, maximised at P^i, strictly decreasing for $P > P^i$, and $\Pi^i(P^i) > 0$. Thus, letting $i = H$, P^H is the single-period monopoly price. P^L is interpreted analogously. Assume $P^H > P^L$.[4] Notice that $\Pi^i(P^i) > 0$ implies $\hat{P} > P^H$, P^L. Thus, $\hat{P} > P^H > P^L$. Figure 2 illustrates the profit functions.

We now complete the model with three important assumptions on consumer preferences. Normalise so that $V(P^*) \equiv U(I/P^c, 0) = 0$. The utility of not visiting is zero.



$\bar{P}^L < \underline{P}^*$ is not necessarily true.

$\bar{P}^L < \bar{P}^*$ is true, since $P^* < P^L$

$\underline{P}^* < \bar{P}^*$ is true (see Lemma 2, Appendix 4)

FIGURE 2

Assumption A. $V(P^L) > 0$.

Assumption B. $V(P^H) < 0$.

Since the monopolist will charge its monopoly price (P^i) in the second and final period, Assumptions A and B imply that the consumer would not visit in the second period if he knew the monopolist's costs were high, while he would visit if he knew the costs were low. These assumptions can be stated equivalently as $P^L < P^* < P^H$.

The consumer is free to leave the monopolist's store, without purchasing anything, if he so desires. We assume, however, that once the consumer has paid the cost of visiting the monopolist's store, he's better off to buy some of the monopolist's product at the price P^H than to go home empty-handed with an income $I - d$. Formally,

Assumption C. $V(P^H) > U((I-d)/P^c, 0) \equiv V(\hat{P})$.

This assumption is equivalent to $P^H < \hat{P}$ or $\Pi^H(P^H) > 0$, which we've already assumed. Assumption C is repeated here in order to highlight its relation to Assumptions A and B. Thus, in sum, we have $P^L < P^* < P^H < \hat{P}$.

Since $P^H > P^*$, the consumer will not visit in the second period if he infers from the introductory price that the operative monopolist is "likely" to have high costs. Consequently, the high-cost monopolist has an incentive to offer an introductory price which prevents the consumer from inferring that its costs are high. Likewise, since $P^L < P^*$, the low-cost monopolist has an incentive to use its introductory price to convince the consumer that its costs are, in fact, low. Repeat business occurs only if the consumer is "sufficiently" persuaded that the operative firm has low costs. The two monopoly types therefore have conflicting incentives. In the next two sections, we will see that this conflict leads to an introductory sale.

III. SEQUENTIAL EQUILIBRIUM AND REFINEMENTS

We employ the sequential equilibrium concept introduced by Kreps and Wilson (1982). A sequential equilibrium is defined (informally) as a system of beliefs and a vector of strategies such that:

(1) strategies are sequentially rational in that each player's strategy maximizes his expected payoff (starting from any of the player's information sets), given his beliefs and the equilibrium strategies of the other players, and
(2) beliefs are consistent in that on the equilibrium path, they are computed using Bayes' rule.[5]

The players in the game we consider are the consumer and the two monopoly types. We will consider only pure strategies.

Let $\delta \in (0, 1)$, $C \equiv \{C^L, C^H\}$, and $\underline{P} \equiv (0, \hat{P}]$.[6] Let v_1 symbolize the visitation strategy of the consumer in period one. v_1 is thus a function of δ, and v_1 maps into the set $\{0, 1\}$, where 0 indicates "don't visit" and 1 indicates "visit".

Second period visitation strategy can be represented as follows:

$$\text{If } v_1 = 1, \quad v_2 : (0, 1) \times \underline{P} \to \{0, 1\}$$

$$\text{If } v_1 = 0, \quad v_2 : (0, 1) \to \{0, 1\}.$$

Only by visiting in the first period can the consumer condition his second period rule on first period prices. If the consumer does not visit the store in period one, then he conditions his second period rule only on δ.

Each monopoly type chooses a first period price (${}^1P$) and a second period price (${}^2P$). ${}^1P(C^i)$ is then interpreted as the first period price that a monopolist of cost-type i would charge, in the event that the consumer visits the store in period one. ${}^2P(C^i)$ can be interpreted analogously.

Some simplifying observations can now be made. In any sequential equilibrium, ${}^2P(C^i)$ must be selected so as to maximize second period profit, given that the consumer has visited the store. Thus, in any sequential equilibrium,

$$\begin{aligned} {}^2P(C^i) &= \operatorname{argmax} \Pi^i({}^2P) \\ &= P^i. \end{aligned} \tag{1}$$

A monopolist of cost-type i chooses the i-cost monopoly price in period two.

Consider now the consumer. Let $q(C=C^H \mid {}^1P)$ be the consumer's posterior belief function. After visiting the monopolist's store in period one and observing ${}^1P$, the consumer believes that the monopolist has high costs with probability $q(C=C^H \mid {}^1P)$. If the consumer does not visit the monopolist's store in period one, then he observes no signal and continues to believe that the monopolist has high costs with probability δ.

The consumer maximizes expected utility. Since ${}^2P(C^i)=P^i$ in any sequential equilibrium, it follows that the equilibrium strategy for the consumer satisfies:

If $v_1=1$, then $v_2=1$ if and only if

$$q(C=C^H \mid {}^1P)\cdot V(P^H)+(1-q(C=C^H \mid {}^1P))\cdot V(P^L)>0.^7 \tag{2}$$

If $v_1=0$, then $v_2=1$ if and only if

$$\delta\cdot V(P^H)+(1-\delta)\cdot V(P^L)>0. \tag{3}$$

The second period of the game is now "solved." Consider the first period. Equilibrium behaviour requires:

$$v_1=1 \quad \text{if and only if } \delta\cdot V({}^1P(C^H))+(1-\delta)\cdot V({}^1P(C^L))>0. \tag{4}$$

(The myopic rationality assumption is implicit here.)

Remaining to consider is the first period monopoly pricing strategy. Define a repeat business function $r({}^1P)$ as follows: $r({}^1P)=0$ if and only if $v_2=0$ when $v_1=1$, and $r({}^1P)=1$ if and only if $v_2=1$ when $v_1=1$. This function is well-defined in any sequential equilibrium (via condition (2)). It then follows that equilibrium first period pricing strategies satisfy:

$${}^1P(C^i)\in \operatorname{argmax} \Pi^i({}^1P)+\alpha\cdot r({}^1P)\cdot \Pi^i(P^i), \tag{5}$$

where $\alpha\in(0,1)$ is the discount factor.

Conditions (1)-(5) are sequential rationality conditions. In a sequential equilibrium, beliefs must also be consistent:

$$\text{If } {}^1P(C^L)={}^1P(C^H), \quad \text{then } q(C=C^H \mid {}^1P(C^H))=\delta. \tag{6}$$

$$\text{If } {}^1P(C^L)\neq{}^1P(C^H), \quad \text{then } q(C=C^H \mid {}^1P(C^H))=1 \tag{7}$$

$$\text{and } q(C=C^H \mid {}^1P(C^L))=0.$$

A *sequential equilibrium* can now be defined as an assessment $\{v_1, v_2, {}^1P(C^H), {}^1P(C^L), {}^2P(C^H), {}^2P(C^L), q(C=C^H \mid {}^1P)\}$ satisfying (1)-(7).

Unfortunately, a number of equilibria exist. Some equilibria, however, are less plausible than others, and we'll attempt to discard these "implausible" equilibria. To this end, we employ three refinements of the sequential equilibrium concept.

Our first refinement involves the elimination of dominated strategies (Milgrom and Roberts (1986)).

Definition 1. ${}^1P$ is *dominated* for a firm of cost-type i if $\Pi^i(P^i) > \Pi^i({}^1P) + \alpha \cdot \Pi^i(P^i)$.

Definition 1 is interpreted as follows. Suppose the consumer visits in period one. Then ${}^1P$ is dominated if P^i generates more profit under the worst of possible future conditions (zero repeat business) then ${}^1P$ could generate under the best of possible future conditions (repeat business).

Refinement 1. A sequential equilibrium is said to be *undominated* if, upon seeing a price ${}^1P$ which is dominated for a firm of cost-type i, the visiting consumer believes that the firm of cost-type i is operative with probability zero.

One additional comment is required. There may be prices which are dominated for both firm types. Clearly, such prices will not be played in any sequential equilibrium. We can specify beliefs for "doubly-dominated" prices as we like—they will never be played, regardless of beliefs.

A second refinement is called the *intuitive criterion* (Kreps (1984)). Consider any sequential equilibrium $(v_1, v_2, {}^1P(C^H), {}^1P(C^L), {}^2P(C^H), {}^2P(C^L), q(C = C^H \mid {}^1P))$. We will be interested in the subgame defined when the consumer visits in period one and equilibrium strategies are played immediately thereafter.[8] Do the equilibrium strategies dictate plausible behaviour in the subgame?

Let $\tilde{\Pi}^i$ be the game profit to a firm of cost-type i, given that the game is initialized after the consumer visits and that equilibrium strategies are played immediately thereafter. Thus, $\tilde{\Pi}^i = \Pi^i({}^1P(C^i)) + \alpha \cdot r({}^1P(C^i)) \cdot \Pi^i(P^i)$.

Refinement 2. Consider any sequential equilibrium $\{v_1, v_2, {}^1P(C^H), {}^1P(C^L), {}^2P(C^H), {}^2P(C^L), q(C = C^H \mid {}^1P)\}$. Define $\tilde{\Pi}^L$ and $\tilde{\Pi}^H$ as above. This equilibrium is said to *fail* the intuitive criterion if there exists $P' \neq {}^1P(C^L), {}^1P(C^H)$ such that:

Condition 1. The low-cost firm could make more by charging P' than by charging ${}^1P(C^L)$, if by charging P' it could convince the consumer of its true type; i.e. $\Pi^L(P') + \alpha \cdot \Pi^L(P^L) > \tilde{\Pi}^L$.

Condition 2. The high-cost firm would do worse with P' than with ${}^1P(C^H)$, regardless of what the consumer believes about firm-type upon observing P'; i.e. $\Pi^H(P') + \alpha \cdot \Pi^H(P^H) < \tilde{\Pi}^H$.

If there does not exist such a P', then we say that the sequential equilibrium is *intuitive*.

Notice that the consumer is required to make a best response, given his beliefs. Thus, in Condition 1, the low-cost firm gets repeat business when its true type is revealed with P'.

Refinement 2 has the following interpretation. If Conditions 1 and 2 were met, then, in the event that the consumer initially visits the store, the low-cost firm could charge P' and thereby convince the consumer of its type. Implicit is the assumption that the consumer could figure out that the high-cost firm would never charge P'. Condition 1 indicates that the low-cost firm would, in fact, charge P', causing the initial equilibrium to break up. It is this lack of stability which the intuitive criterion rejects.[9]

The third refinement we consider involves Pareto dominance and the set of equilibria. Let E and E_0 be two different sequential equilibria. Let $\tilde{\Pi}^L$, $\tilde{\Pi}^H$ be defined as above for E, and let $\tilde{\Pi}_0^L$, $\tilde{\Pi}_0^H$ be defined as above for E_0.

Refinement 3. Consider any sequential equilibrium E. This equilibrium is said to be *Pareto superior* if there does not exist another sequential equilibrium E_0 such that $\tilde{\Pi}_0^L \geqq \tilde{\Pi}^L$ and $\tilde{\Pi}_0^H \geqq \tilde{\Pi}^H$ (with one inequality strict).

This is a strong refinement, and we will use it only after exhausting all of the implications of Refinements 1 and 2. Notice that Refinement 3 defines Pareto optimality in reference to the *firm* only. Consumer welfare is not considered. The underlying idea is that a firm won't be wastefully competitive with itself (i.e. over its types).

IV. THE POOLING EQUILIBRIUM

We consider first the pooling equilibrium. Here, the high-cost firm charges the same price as the low-cost firm in the first period. This, it turns out, will involve a sale on the part of the high-cost firm. In short, the high-cost firm offers its product at a reduced price, thereby temporarily halting the growth of its reputation as a high-cost monopoly. The resulting revenue from repeat business can be used to offset relative losses incurred in period one due to the sale.

Our approach will be to list the needed assumptions, prove that pooling behaviour is indeed a sequential equilibrium, and, in a sense made clear below, show that this equilibrium is unique. We will then consider what happens when the assumptions are not met.

Assumption 1. $\Pi^H(P^H) < \Pi^H(P^L) + \alpha \cdot \Pi^H(P^H)$.

If the consumer visits in period one, it is more profitable for the high-cost firm to charge P^L in the first period and have the consumer return in the second period than to charge P^H and have the consumer not return in period two. Notice that Assumption 1 implies $P^L > c^H$ (so that $\Pi^H(P^L) > 0$).

Our second assumption is that visiting is worthwhile if no information has been received to the contrary. That is, if ${}^1P(C^H) = {}^1P(C^L)$ so that $q(C = C^H \mid {}^1P(C^H)) = \delta$, the consumer who visited in the first period will visit again in the second period. Second period visitation follows first period pooling.

Assumption 2. $\delta \cdot V(P^H) + (1-\delta) \cdot V(P^L) > 0$.

We can interpret Assumption 2 as saying that the consumer is "inclined" to visit or willing to gamble (over monopoly prices).

Before formulating our theorems, we require some definitions. Figure 2 may be helpful. Define $\bar{P}^L$ as the price such that $\Pi^H(P^L)=\Pi^H(\bar{P}^L)$. Define P^s as the price below P^H solving

$$\Pi^H(P^s)+\alpha\cdot\Pi^H(P^H)\equiv\Pi^H(P^H). \tag{8}$$

From our assumptions on the shape of Π^H, $\bar{P}^L>P^H$. From Assumption 1, $P^s<P^L$, and, from the definition of $P^s, \Pi^H(P^s)>0$; i.e. $P^s>c^H$. P^s (the separating or sale price) is thus the price at which the high-cost firm would be indifferent between pooling in period one with the low-cost firm, thereby getting repeat business, and profit maximising in the first period and losing repeat business. If the low-cost firm is to separate with a sale, it must charge P^s or less in the first period (assuming there isn't a high price $\bar{P}^s$ that separates "better"—see below). Otherwise, the high-cost firm would find it profitable to mimic. Define $\bar{P}^s(\underline{P}^s)$ as the "flip-side" of P^s for the high (low)-cost firm. That is, $\Pi^H(P^s)=\Pi^H(\bar{P}^s)$ and $\Pi^L(P^s)=\Pi^L(\underline{P}^s)$, where $c^H<P^s<P^L$, $\bar{P}^s>P^H$, and $\underline{P}^s>P^L$. Notice that prices below P^s or above $\bar{P}^s$ are dominated for the high-cost firm.

We are now ready to state our first theorem. The equilibrium in Theorem 1 is pooling, has the consumer buying from the monopolist in each period, and can be interpreted as characterising an introductory sale for the high-cost firm (since ${}^1P(C^H)=P^L<P^H$).

Theorem 1. *Under Assumptions* 1 *and* 2, *a pooling sequential equilibrium is*:

Beliefs:

$$q(C=C^H\,|\,{}^1P=P^L)=\delta,$$
$$q(C=C^H\,|\,{}^1P\leqq P^s \text{ or } {}^1P\geqq\bar{P}^s)=0,$$
$$q(C=C^H\,|\,{}^1P\in(P^s,\bar{P}^s), {}^1P\neq P^L)=1.$$

Consumer strategy:

$v_1=1$

If $v_1=1$, *then* $v_2=1$ *if and only if* ${}^1P\leqq P^s$, ${}^1P\geqq\bar{P}^s$, *or* ${}^1P=P^L$.

If $v_1=0$, *then* $v_2=1$.

Monopoly strategy:

$${}^1P(C^L)=P^L, \qquad {}^2P(C^L)=P^L$$
$${}^1P(C^H)=P^L, \qquad {}^2P(C^H)=P^H.$$

The proof is contained in Appendix 1. Beliefs have been selected so as to put zero weight on dominated strategies. Notice that neither firm type will cut price below P^L, regardless of how we might specify beliefs for ${}^1P<P^L$ (a price cut would decrease period one profit with no possibility of increasing period two profit). The price pooling equilibrium is therefore robust to any specification of beliefs for ${}^1P<P^L$.

The next theorem indicates that the above pooling equilibrium is "refined". Since the intuitive criterion is a relatively recent concept, I will include in the text the proofs that utilise this criterion.

Theorem 2. *Under Assumptions* 1 *and* 2, *the pooling sequential equilibrium of Theorem* 1 *is undominated and intuitive.*

Proof. We show here that the equilibrium is intuitive. Proof that it is also undominated is found in Appendix 2.

Recall Condition 1 of an equilibrium that fails to be intuitive: There must exist a price P' such that $P' \neq P^L$ (recall ${}^1P(C^L) = {}^1P(C^H) = P^L$) and the low-cost firm could do better with P' than with P^L, if by charging P' it were taken to be itself. This condition is not met because the low-cost firm cannot possibly hope to make more than $\Pi^L(P^L) + \alpha \cdot \Pi^L(P^L)$, the maximum possible profit for a low-cost firm. ||

The equilibrium in Theorem 1 is but one of many: separation can occur at low (${}^1P(C^L) \leqq P^s$) or possibly high (${}^1P(C^L) \geqq \bar{P}^s$) prices, and pooling can be supported over a wide range of prices. The next three theorems will indicate restrictions which can be placed on equilibrium behaviour when Refinements 1, 2, and 3 are invoked. It will be shown that the equilibrium strategies in Theorem 1 are the only possible equilibrium strategies in an intuitive, undominated, and Pareto superior sequential equilibrium (when Assumptions 1 and 2 hold).

Theorem 3. *The following properties must hold:*

(1) *Under Assumption* 1, *neither* ${}^1P(C^L) < P^s$ *nor* ${}^1P(C^L) > \bar{P}^s$ *can happen in any undominated sequential equilibrium.*

(2) *Under Assumption* 2, *in any intuitive sequential equilibrium, it cannot be that* ${}^1P(C^L) = {}^1P(C^H) \in (P^L, \bar{P}^L)$.

Proof. The proof of part 1 is contained in Appendix 3.

Consider then a sequential equilibrium in which ${}^1P(C^L) = {}^1P(C^H) \in (P^L, \bar{P}^L)$. Then $\tilde{\Pi}^i = \Pi^i({}^1P(C^L)) + \alpha \cdot \Pi^i(P^i)$. (Assumption 2 has been used here.) Since $\Pi^L(P^L) + \alpha \cdot \Pi^L(P^L) > \tilde{\Pi}^L$, the low-cost firm could make more with P^L than with ${}^1P(C^L)$, if it were taken to be low-cost in charging P^L. Since $\Pi^H(P^L) + \alpha \cdot \Pi^H(P^H) < \tilde{\Pi}^H$, the high-cost firm prefers pooling at ${}^1P(C^H)$ as compared to charging P^L, even if in charging P^L it were believed to be low-cost. Conditions 1 and 2 are both met for $P' = P^L$, and the equilibrium therefore fails to be intuitive. ||

Separating equilibria also exist. Theorem 4 indicates that there is only one pricing strategy which can be part of an undominated, separating sequential equilibrium. This strategy involves an introductory sale on the part of the low-cost firm. The sale price (P^s) is just low enough that the high-cost firm finds it unprofitable to mimic. The consumer is therefore correct in conjecturing that only a low-cost firm could survive such a sale. The same strategies will arise (uniquely) in Section V, where it is assumed that Assumption 2 fails.

Theorem 4. *Under Assumption* 1, *if in an undominated sequential equilibrium* ${}^1P(C^L) \neq {}^1P(C^H)$, *then* ${}^1P(C^L) = P^s$ *and* ${}^1P(C^H) = P^H$.

The proof is contained in Appendix 4.

Refinements 1 and 2 have taken us a long way. Separation can only occur at P^s and P^H, and pooling cannot occur in $(P^L, \bar{P}^L)$. Nevertheless, there may still be undominated, intuitive sequential equilibria in which pooling occurs between P^s and P^L or between $\bar{P}^L$ and $\bar{P}^s$. Notice that pooling equilibria in which ${}^1P(C^L) = {}^1P(C^H) \in [\bar{P}^L, \bar{P}^s]$ are such that the consumer would not visit (since $\bar{P}^L > P^H$). In such equilibria, the consumer correctly anticipates that optimal pricing behaviour, given his visitation, involves a "high" price, and he therefore never visits. Thus, in some sense, these "high" price pooling equilibria would never be observed. The remaining equilibria—separation with P^s and P^H or pooling in $[P^s, P^L]$—involve a sale for one or both firm types. Therefore, in any undominated, intuitive sequential equilibrium in which first period visitation occurs, neither firm type charges a price above its monopoly price and at least one type has an introductory sale (prices below its monopoly price).

We have yet to use Refinement 3. Theorem 5 indicates that, together, the three refinements select a unique sequential equilibrium.[10]

Theorem 5. *Under Assumptions* 1 *and* 2, *the only undominated, intuitive, and Pareto superior sequential equilibrium pricing strategy is the one presented in Theorem* 1.[11]

Proof. The proof is short. From Theorem 4, separation can occur in equilibrium only if ${}^1P(C^L) = P^s$ and ${}^1P(C^H) = P^H$. Since $\Pi^L(P^s) + \alpha \cdot \Pi^L(P^L) < \Pi^L(P^L) + \alpha \cdot \Pi^L(P^L)$ and $\Pi^H(P^H) < \Pi^H(P^L) + \alpha \cdot \Pi^H(P^H)$ (by Assumption 1), this separating equilibrium is Pareto inferior to the equilibrium of Theorem 1.

Suppose then that ${}^1P(C^L) = {}^1P(C^H) \equiv \tilde{P}$. By Theorem 3, we need only consider prices in $[P^s, P^L]$ or $[\bar{P}^L, \bar{P}^s]$. If $\tilde{P} < P^L$, both firm types clearly make more with the equilibrium of Theorem 1. If $\tilde{P} \geqq \bar{P}^L$, the low-cost firm makes strictly less with $\tilde{P}$ than in the equilibrium of Theorem 1, while the high-cost firm can at best do no better with $\tilde{P}$ than in the equilibrium of Theorem 1 (it does strictly worse if $\tilde{P} > \bar{P}^L$). ||

Even a monopolist, therefore, may have incentive to offer a sale. The key idea is that while a monopolist is not competing with other firms, it is, in a sense, competing with itself (more correctly, with its other cost-type). In the equilibrium of Theorem 1, the low-cost firm can afford to ignore this potential competition and profit maximise in each period. Repeat business will occur even if the high-cost firm can turn this potential competition to its advantage by disguising itself as a low-cost firm.

Assumption 1 was certainly needed to provide the high-cost firm with sufficient incentive to mimic the low-cost firm.

When Assumption 2 is violated, the low-cost firm has incentive to separate. It is to this case that we now turn.

V. THE SEPARATING EQUILIBRIUM

In Section IV, we got what, in retrospect, seems the natural equilibrium. If the consumer is "inclined" to visit, then the low-cost firm is in the "strongest" position. It needs only to make sure that the consumer does not mistake it for something which it is not (viz, a high-cost firm). The high-cost firm, by contrast, must sufficiently "deceive" or fool the consumer in order to get repeat business. In an equilibrium characterised by consumer rationality, it is not surprising that it is the high-cost firm which must offer a sale.

In this section, we switch some of the signalling responsibility to the low-cost firm. Specifically, suppose that instead of Assumption 2, we have

Assumption 2′. $\delta \cdot V(P^H) + (1-\delta) \cdot V(P^L) < 0.$

The low-cost firm can no longer tolerate first period pooling, for the consumer will not return to the monopolist's store unless he receives some (perhaps slight) indication that costs are low. The "burden of proof" now falls strongly on the low-cost firm: it must convince the consumer that its future price will be low; otherwise, repeat business will not be forthcoming.

The resulting equilibrium is again a natural outcome to expect. The low-cost firm offers an introductory sale. It cuts price to a level just low enough that the high-cost firm would not find it profitable to mimic. The consumer who conjectures that an introductory sale is a sign of a low future price (P^L) will find his conjecture correct.

We require an additional assumption. Define $\bar{P}$ as the solution to

$$\delta \cdot V(P^H) + (1-\delta) \cdot V(\bar{P}) \equiv 0. \tag{9}$$

If ${}^1P(C^H) = P^H$, then ${}^1P(C^L)$ must be less than or equal to $\bar{P}$; otherwise, first period visitation will not occur in equilibrium. By Assumption 2′, $\bar{P} < P^L$. The next assumption can be interpreted as a strengthening of Assumption 1.

Assumption 3. $P^s < \bar{P}$.

This assumption ensures that $\delta \cdot V(P^H) + (1-\delta) \cdot V(P^s) > 0$. That is, Assumption 3 ensures that the consumer will visit in the first period when ${}^1P(C^L) = P^s$ and ${}^1P(C^H) = P^H$. Observe that $P^s < \bar{P} < P^L$. Since $P^s < P^L$, Assumption 1 holds *necessarily.*

Theorem 6. *Under Assumptions* 2′ *and* 3, *a separating sequential equilibrium is*:

Beliefs:

$$q(C = C^H \,|\, {}^1P \leqq P^s \text{ or } {}^1P \geqq \bar{P}^s) = 0$$
$$q(C = C^H \,|\, {}^1P \in (P^s, \bar{P}^s)) = 1.$$

Consumer strategy:

$v_1 = 1$

If $v_1 = 1$, *then* $v_2 = 1$ *if and only if* ${}^1P \leqq P^s$ *or* ${}^1P \geqq \bar{P}^s$.

If $v_1 = 0$, *then* $v_2 = 0$.

Monopoly strategy:

$${}^1P(C^L) = P^s, \qquad {}^2P(C^L) = P^L$$
$${}^1P(C^H) = P^H, \qquad {}^2P(C^H) = P^H.$$

Moreover, this is an undominated, intuitive sequential equilibrium, and the pricing strategy is the only pricing strategy which can arise in an undominated sequential equilibrium.

The proof of Theorem 6 is in Appendix 5. Notice that this equilibrium also arose under Assumptions 1 and 2 (see Theorem 4). As explained above, the low-cost monopolist separates itself from its high-cost counterpart with an introductory sale (P^s). An introductory sale is used to signal a low future price (P^L).

What would happen if Assumption 1 held but Assumption 3 failed? Curiously, the industry would shut down. If the consumer visited in period one, then, we have seen, ${}^1P(C^L) = P^s$ is the optimal response. The low-cost firm has no way of committing to ${}^1P(C^L) \leqq \bar{P}$. Consequently, the consumer would not visit in period one. But, by Assumption 2′, he would also not visit in period two. The unique, undominated sequential equilibrium is the one in which no transactions occur (with "off-equilibrium"—or visiting—behaviour governed by the beliefs and strategies in Theorem 6). In essence, the consumer sinks resources in visiting the store and, consequently, appropriable quasi-rents are generated. If $P^s > \bar{P}$, the low-cost firm can't be trusted not to appropriate those rents in such a way as to make the initial gamble less valuable (in terms of expected utility) than simply remaining at home and buying only the composite commodity.

At this point, it is instructive to review the role of the assumption that the consumer behaves myopically and maximises instantaneous expected utility. The above theorems give uniqueness results for the subgame that arises when $v_1 = 1$. Notice that this subgame is unaffected by the myopia assumption.[12] Thus, the results derived for ${}^1P(C^i)$, ${}^2P(C^i)$, v_2, and q are all independent of the myopia assumption. The only influence that consumer foresight has on the above discussion is that it may increase the consumer's desire to visit in period one.[13] Since, in the unique equilibria described above, v_1 is already unity, consumer myopia can be assumed at no loss of generality.

VI. EXTENSIONS

The model presented above can be extended in a number of directions. The present section contains discussion of some of these extensions.

Many consumers

Suppose now that there are M identical consumers, each living two periods. One would hope that the single-consumer equilibria described above continue to hold in a multi-consumer model (recall that returns to scale are constant and consumers are identical). This is, in fact, the case.

There are two ways of modeling a multi-consumer market. One approach is to allow the firm to "price off of its demand curve". In this formulation, the monopolist is allowed to condition its price on the number of current visitors (level of demand). Surprisingly, there are a continuum of asymmetric sequential equilibria in which the monopolist's price depends on the number of visitors. However, in Bagwell (1985*b*), I show that the three refinements applied to each possible subgame remove these asymmetries and give as the unique equilibria those described in Theorems 1 and 6 (with price necessarily independent of the number of visitors).[14]

An alternative approach, perhaps more suitable when the monopolist sells to many consumers, is to assume that the monopolist "posts" the price (say, on his door) and sells at that price to all visitors. One can then think of price and visitation strategies as being selected simultaneously. All of the above theorems can easily be shown to hold in such a world.[15]

Multi-firm markets

Suppose N firms open simultaneously, each having high costs with probability δ. Suppose also that a consumer can visit only one of these stores per period and that consumers do not communicate with one another.

Again, all of the above theorems apply. Each consumer initially selects a firm at random. Consider a particular firm, f. Since consumers are assumed unable to communicate, f can, in choosing its introductory price, completely ignore the consumers that did not visit it. These non-visiting consumers will never observe f's introductory price, and their future shopping strategies are therefore independent of this price. Consequently, f acts as if its initial group of consumers forms the entire consumer population. In this way, the market decomposes into N independent monopolists, with each firm pricing as specified in the above equilibria. This "local monopoly" result complements the work of Diamond (1971). I add to Diamond's model a truly dynamic structure (with "localised" price reputations) and incomplete information about costs.

Suppose, for example, that Assumptions 2′ and 3 hold, so that the unique "refined" equilibrium is separating. Consumers initially select firms at random. If ${}^1P = P^s$ at the selected firm, the consumer realises that he has picked a winner, and he returns to the store in period two. If ${}^1P = P^H$, the consumer drops out of the market (this comes from Assumption 2′). In equilibrium, ${}^1P(C^L) = P^s$, ${}^1P(C^H) = P^H$, ${}^2P(C^H) = P^H$, and ${}^2P(C^L) = P^L$. Notice that there is price dispersion in each period.[16] Similarly, when Assumptions 1 and 2 hold, the unique "refined" equilibrium is pooling, with all firms charging P^L as the introductory price.

A free-entry condition can be imposed to determine N endogenously. To bound entry, a start up cost F is needed. It is necessary to assume that F is incurred prior to the firm's knowledge of its cost-type. Indeed, F can be thought of as the cost of obtaining that information. It is only after all employees have been hired and all equipment bought that a firm knows how efficiently its employees and equipment work together as a unit. F, then, is the cost of gathering inputs. It applies only to the first period. (I am assuming that entry can only occur in the first period.)

Suppose that there are M consumers. Each firm expects to get $1/N$-th of the market (or M/N consumers). Thus, expected game profit for a firm of cost-type i is $(M/N)\cdot\tilde{\Pi}^i - F$. Choose N so that the expected profit of entry is zero: $\delta\cdot(M/N)\cdot\tilde{\Pi}^H + (1-\delta)\cdot(M/N)\cdot\tilde{\Pi}^L - F = 0$. Thus, in a free-entry equilibrium, $(M/N)\cdot\tilde{\Pi}^L - F > 0 > (M/N)\cdot\tilde{\Pi}^H - F$. High-cost firms therefore make negative game profit. They remain open, however, because $\tilde{\Pi}^H > 0$.[17,18] (F, recall, is sunk prior to the learning of cost information.)

I have shown that, if consumers do not communicate and if the entry decision comes prior to the learning of cost information, then the above theorems extend to a multi-firm, multi-consumer model with free-entry. This result, of course, rests on the assumption that both current and past price information is costly. More realistically, consumer communication may enable a consumer to learn the introductory price of firms which he did not visit. This possibility considerably changes the strategic flavour of the game. A firm can no longer ignore the influence of its introductory price on non-visiting consumers, and visiting consumers generate a positive information externality for non-visiting consumers.

Many periods, recurrent sales

Perhaps the greatest limitation of the model is its two period framework. Intuition suggests that the same forces should be at work in a multi-period, overlapping generations model. In particular, recurrent sales might arise as a way to signal to new consumers that the firm's cost structure is low.

Alternatively, if one assumes that a single set of consumers live throughout the T-period game (with $T \geqq 2$), then the resulting equilibria involve a "sale" in every period but the last. For example, in the pooling equilibrium, ${}^tP(C^L) = {}^tP(C^H) = P^L$, for $t = 1, \ldots, T-1$, and ${}^TP(C^L) = P^L$ and ${}^TP(C^H) = P^H$ (the t-superscript is a time index).

From this perspective, a sale never really occurs at all; rather, incomplete information about costs exerts continual downward pressure on the monopolist's price.

New equilibria emerge if past pricing information is costless. In particular, punishment strategies can support the separating equilibrium (Theorem 6) in a multi-period game. For example, low-cost firms may begin with a sale (P^s) and then price at P^L for the remainder of the game. High-cost firms will not mimic this behaviour if low-cost firms credibly threaten to respond to such mimicry with sale prices in subsequent periods.[19]

Upstream and downstream firms

An interesting extension of the model is to the relationship between an upstream monopolist and a downstream "consumer" or firm. Consider the following scenario. An upstream monopolist emerges on the market with a product that can be used by the downstream firm as an input. But, to use this input in any period t, the downstream firm must at the beginning of this period make an investment in machinery or labour that is idiosyncratic to the input. We can think of this sunk cost as the exact analogue of the search cost d in the consumer model. After the sunk cost is incurred for the period (or simultaneous to the incurrence of this cost), the monopolist announces the current price.

Formally, this model is equivalent to the consumer model described above.[20] The upstream firm has incentive to offer an introductory sale in order to convince the downstream firm that the second period price will be low (P^L). If the downstream firm is "sufficiently" convinced that costs are low, then it again makes the sunk investment in period two. An introductory sale therefore encourages repeat business (i.e. downstream entry).

By signalling its costs, the upstream firm is signalling the extent to which it will appropriate the second period quasi-rents of the downstream firm.[21] The downstream firm would like to continue transacting with a low-cost monopoly (since P^L extracts relatively few of these rents); however, it will not incur the second period sunk cost if it thinks upstream costs are high (since P^H extracts too much of these rents). Thus, the presence of appropriable quasi-rents, coupled with incomplete information about costs, threatens the mutually beneficial relationship between the low-cost monopolist and the downstream firm. However, by signalling its costs, the low-cost monopolist can enable the efficient transaction. Like contracts and vertical integration (see Williamson (1979), Klein *et al.* (1978)), signals have a role in enabling mutually beneficial relationships when appropriable quasi-rents exist.[22]

Price information

I have assumed throughout that price information is costly. Why, in the consumer model, don't consumers simply pick up the phone and ask the merchant what his current price is? From a purely game theoretic view, phone calling capabilities could be added to the model without destroying any of the results. Barring any legal ramifications for lying over the phone (and assuming no incomplete information about the "honesty" of the firm), any price that the firm might announce over the phone is meaningless. Since a "phone-signal" costs the firm nothing, there is no sense in which the message can be credibly conveyed.

Of course, in reality, credible price advertisements do occur. Billboards, newspaper ads, and flyers often advertise prices. These advertisements have legal credibility. They are also costly to the firm, and their very existence may therefore be a signal (Bagwell (1985c), Milgrom and Roberts (1986)). The present model is not equipped to deal with price advertising of this type, though it seems a promising area for future research.[23]

VII. CONCLUSION

We have analysed a new motivation for introductory sales. Specifically, by offering a sale, a firm can signal to consumers that its costs are low and, hence, that its future price will be low. In this way, repeat business is generated and the sale emerges as a rational, profit maximising activity.

The model differs from traditional search models in that consumers rationally use available information as a signal of price. In addition, out of equilibrium behaviour and beliefs are dealt with explicitly. The search model developed here, however, is quite limited. Much interesting work remains in understanding the role of signalling and price reputations in search models.

APPENDIX 1

Consider first consumer behaviour. Since $V(P^L)>0$, visiting in period one is optimal. By Assumption 2, the consumer also visits in period two if ${}^1P=P^L$ is observed or if, for some reason, he didn't visit in period one. If ${}^1P\leqq P^s$ or ${}^1P\geqq \bar{P}^s$, $v^2=1$ is clearly optimal, given beliefs.

Consider now the monopolist. Given the consumer's strategy, ${}^1P(C^L)=P^L$ is clearly optimal for the low-cost firm. Second period prices are of course optimal for both firm types. ${}^1P\leqq P^s$ and ${}^1P\geqq \bar{P}^s$ are (weakly) dominated by P^H for the high-cost firm. Also, given consumer beliefs, P^H is better than any other ${}^1P\in(P^s, \bar{P}^s)$ such that ${}^1P\neq P^L$ for the high-cost firm. But, by Assumption 1, ${}^1P=P^L$ is better for the high-cost firm than ${}^1P=P^H$. ||

APPENDIX 2

We prove here that the equilibrium in Theorem 1 is undominated. By the definition of P^s and $\bar{P}^s$, the consumer in Theorem 1 believes that the high-cost monopolist plays dominated strategies with probability zero.

Consider now the low-cost monopolist. Define P^0 as the price below P^L such that $\Pi^L(P^0)+\alpha\cdot\Pi^L(P^L)\equiv\Pi^L(P^L)$. Define $\underline{P}^0$ as the "flip-side" of P^0; i.e. $\Pi^L(\underline{P}^0)\equiv\Pi^L(P^0)$. Prices below P^0 or above $\underline{P}^0$ are dominated for the low-cost firm.

Thus, if $P^0<P^s$, we require $q(C=C^H\,|\,{}^1P\in[P^0, P^s))=0$. (${}^1P<P^0$ are "doubly-dominated" and beliefs can arbitrarily be specified for such ${}^1P$.) This specification is consistent with Theorem 1.

Lemma 1. *If $P^s<P^L$, then $\Pi^L(P^s)+\alpha\cdot\Pi^L(P^L)>\Pi^L(P^L)$ and, thus, $P^0<P^s$.*

The proof of Lemma 1 is not difficult (see Bagwell (1985b)). Notice that Assumption 1 guarantees that $P^s<P^L$.

$\underline{P}^0$ may or may not exceed $\bar{P}^s$. ${}^1P>\max(\underline{P}^0, \bar{P}^s)$ are "doubly-dominated", and, for such prices, beliefs can be specified arbitrarily. If $\underline{P}^0\geqq\bar{P}^s$, then we require $q(C=C^H\,|\,{}^1P\in(\bar{P}^s, \underline{P}^0])=0$, as in Theorem 1. If instead $\bar{P}^s>\underline{P}^0$, then $q(C=C^H\,|\,{}^1P\in(\underline{P}^0, \bar{P}^s])=1$ is required. This specification differs from the one given in Theorem 1 (at the price $\bar{P}^s$). However, changing beliefs at $\bar{P}^s$ from 0 to 1 does not affect the incentive to deviate; i.e. ${}^1P(C^L)={}^1P(C^H)$ is still an equilibrium. So, with the understanding that the beliefs should be changed at ${}^1P=\bar{P}^s$ when $\bar{P}^s>\underline{P}^0$, the beliefs specified in Theorem 1 satisfy the dominance refinement. ||

APPENDIX 3

We prove here part 1 of Theorem 3.

Recall from Appendix 2 that $q(C=C^H\,|\,{}^1P\in[P^0, P^s))=0$ is necessary in an undominated sequential equilibrium under Assumption 1. ${}^1P(C^L)<P^0$ is dominated for the low-cost firm and won't be played in

equilibrium. ${}^1P(C^L)\in[P^0, P^s)$ will also not be played, given the above necessary beliefs, since ${}^1P(C^L)+\varepsilon < P^s$ would give more current profit without sacrificing repeat business.

If $\underline{P}^0 \leqq \bar{P}^s$, then ${}^1P(C^L) > \bar{P}^s$ is dominated for the low-cost firm and won't be played in equilibrium. If $\underline{P}^0 > \bar{P}^s$, then $q(C = C^H \mid {}^1P \in (\bar{P}^s, \underline{P}^0]) = 0$ in an undominated sequential equilibrium (as argued in Appendix 2). Thus, ${}^1P(C^L) \in (\bar{P}^s, \underline{P}^0]$ will not be played, given such beliefs, since ${}^1P(C^L) - \varepsilon > \bar{P}^s$ would give more current profit without sacrificing repeat business. (Lemma 2 of Appendix 4 can be used to prove $\bar{P}^s > P^L$). ||

APPENDIX 4

We prove here Theorem 4.

Suppose ${}^1P(C^L) \neq {}^1P(C^H)$. From Theorem 3, ${}^1P(C^L) \in [P^s, \bar{P}^s]$. Also, ${}^1P(C^L) \notin (P^s, \bar{P}^s)$; otherwise, the high-cost firm would mimic. Thus, ${}^1P(C^L) = P^s$ or $\bar{P}^s$. Finally, ${}^1P(C^H) = P^H$ is necessary in a separating equilibrium: If separation occurs, repeat business is foregone for the high-cost firm, and the only equilibrium possibility is ${}^1P(C^H) = P^H$.

We have the following:

Lemma 2. *If $P^s < P^L$, then $\bar{P}^s > \underline{P}^s$.*

$P^s < P^L$ follows from Assumption 1. The proof of Lemma 2 is in Bagwell (1985*b*).

The low-cost firm can separate with either P^s or $\bar{P}^s$, as argued above. Since $\bar{P}^s > \underline{P}^s$, $\Pi^L(\bar{P}^s) < \Pi^L(\underline{P}^s) = \Pi^L(P^s)$. P^s thus seems the "better" price with which to separate.

Could an undominated sequential equilibrium exist in which ${}^1P(C^L) = \bar{P}^s$? The answer is "no". To see this, assume instead that ${}^1P(C^L) = \bar{P}^s$. In an undominated equilibrium, $q(C = C^H \mid {}^1P = P^s - \varepsilon) = 0$, since $P^s - \varepsilon$ is dominated for the high-cost firm. Since Π^L is continuous, $\Pi^L(P^s - \varepsilon) > \Pi^L(\bar{P}^s)$. Thus, the low-cost firm would deviate from $\bar{P}^s$ to $P^s - \varepsilon$, thereby increasing period one profit without losing repeat business. ||

APPENDIX 5

We prove here Theorem 6.

Beliefs are clearly consistent. $v_1 = 1$ is optimal since $\delta \cdot V(P^H) + (1-\delta) \cdot V(P^s) > 0$. Given beliefs, v_2 is an optimal strategy. If, for some reason, $v_1 = 0$, then $v_2 = 0$ by Assumption 2′.

By the definitions of P^s and $\bar{P}^s$, the high-cost firm has no incentive to deviate. ${}^2P(C^L) = P^L$ and ${}^2P(C^H) = P^H$ are clearly optimal.

Consider now the low-cost firm. It has two choices: promote repeat business (P^s and $\bar{P}^s$ are clearly the best two candidates) or maximize short-run profit (P^L is the best choice here). By Lemma 1, P^s is better than P^L. By Lemma 2, P^s is also better than $\bar{P}^s$. Thus, ${}^1P(C^L) = P^s$ is optimal, completing the proof that the given strategies and beliefs form a sequential equilibrium.

We show next that the equilibrium is intuitive. Let us find the P''s satisfying Condition 1. For any $P' < P^s$ or $P' \geqq \underline{P}^s$, Condition 1 is clearly not met. For $P' \in (P^s, \underline{P}^s)$, Condition 1 does hold. But since $\underline{P}^s < \bar{P}^s$, $P' \in (P^s, \underline{P}^s)$ implies $P' \in (P^s, \bar{P}^s)$. Thus, for $P' \in (P^s, \underline{P}^s)$, $\Pi^H(P') + \alpha \cdot \Pi^H(P^H) > \Pi^H(P^s) + \alpha \cdot \Pi^H(P^H) = \Pi^H(P^H)$. That is, if the high-cost firm could be taken as a low-cost firm by charging $P' \in (P^s, \bar{P}^s)$, then it would do so. For any P' such that Condition 1 holds, Condition 2 fails. The equilibrium is therefore intuitive.

We have left to show that the equilibrium is undominated and that the pricing strategy of Theorem 6 is the only pricing strategy that can arise in an undominated sequential equilibrium.

The first step is to show that pooling cannot occur when Assumption 2′ holds. Suppose to the contrary that $P(C^L) = P(C^H) \equiv \tilde{P}$. Then $q(C = C^H \mid \tilde{P}) = \delta$. By Assumption 2′, repeat business is lost. Thus, each firm would deviate to its respective monopoly price in period one.

Next, recall that Assumption 1 necessarily holds. Thus, we may use Theorem 4 to show that the only possible undominated sequential equilibrium pricing strategy is ${}^1P(C^L) = P^s$ and ${}^1P(C^H) = P^H$.

We therefore need only to show that the equilibrium presented in Theorem 6 is in fact undominated.

Since $P^0 < P^s$, $q(C = C^H \mid {}^1P \leqq P^s) = 0$ is in accord with the dominance refinement. If $\underline{P}^0 \geqq \bar{P}^s$, then $q(C = C^H \mid {}^1P \geqq \bar{P}^s) = 0$ satisfies dominance requirements. If instead $\underline{P}^0 < \bar{P}^s$, then $q(C = C^H \mid {}^1P \in (\underline{P}^0, \bar{P}^s]) = 1$ is required. This specification is different from the beliefs specified in Theorem 6 at the price $\bar{P}^s$. However, this change does not alter the incentive to deviate. Thinking of this change as being applicable when $\underline{P}^0 < \bar{P}^s$, then, completes the proof that the equilibrium is undominated. ||

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NOTES

1. One can also think of the monopolist's price and the consumer's visitation strategy being selected simultaneously. All of the below equilibria survive this alteration. However, a new class of sequential (but not perfect) equilibria arise, in which the monopolist chooses a very high price and the consumer does not visit. By letting the consumer move first, we force the monopolist to choose price optimally, *given* visitation. This eliminates the imperfect equilibria.

2. It is also true that, given the consumer has visited the store and observed the current price, he will demand the bundle which maximises current utility. Since the monopolist has complete information about consumer preferences, false revelation of demand serves no potential purpose (whether the consumer is myopic or farsighted).

3. A start-up cost can be added at no loss of generality. For the single consumer model, the below results hold for a general class of cost functions. However, constant returns to scale are needed for uniqueness results in the multi-consumer model.

4. Differentiation of the first order conditions indicates that $P^H > P^L$ if $X(P)$ is differentiable and concave. In general, there are assumptions on Π' and on $X(P)$, and one may wonder if they are consistent. Fortunately, it is quite easy to construct examples satisfying all of the assumptions of the model.

5. Off-equilibrium path beliefs in a sequential equilibrium must be structurally consistent, that is, consistent with the structure of the game. This requirement is always met in the current game. See, however, Kreps and Ramey (1985) for problems with structural consistency in more complicated games.

6. Since no consumer will buy at $P > \hat{P}$, we can restrict attention to $P \in (0, \hat{P}]$.

7. We will ignore the case where expected utility is zero.

8. The uninformed party moves first in this game (unlike in Kreps (1984)). I apply the intuitive criterion to the subgame that emerges given that the consumer has visited.

9. There is no need in phrasing Condition 1(2) in terms of the high (low)-cost firm. The high-cost firm will never do better than in equilibrium by revealing itself. See Bagwell (1985*c*), Milgrom and Roberts (1986), and Kreps (1984) for other applications of the intuitive criterion.

10. Pareto superiority (over firm-types) is the least appealing of the three refinements. However, as I argue below, it does generate the "natural" equilibrium. For those reluctant to use this refinement, notice (as I argue above) that Refinements 1 and 2 give rise to equilibria in which the only observed introductory prices are sales.

11. Strategies are unique, beliefs need not be.

12. This is because the second period is the last period. To see one period ahead, is to see all that there is.

13. A farsighted consumer may have extra incentive to visit in period one if the observation of ${}^1P$ tells him something about costs and thereby enables a more informed v_2 decision. Of course, in a pooling equilibrium, this incentive is removed and myopia is rational.

14. For this result, one must assume consumers do not communicate (constant communicative returns to scale), the product is non-resaleable, and that consumers too can "look around the store" and count the number of buyers (so that costs are the only source of information asymmetry). Asymmetric equilibria arise because consumers can believe that the monopolist's price strategy depends on the number of visitors, and these beliefs can, in equilibrium, "force" the monopolist to price in precisely that manner.

15. There is, however, again the problem of sequential (but not perfect) "no-visit" equilibria. See footnote 1. An additional iteration of eliminating dominated strategies can remove these equilibria.

16. The high-cost firms can be thought of as charging P^H in period two; however, they receive no second period visitors in the separating equilibrium.

17. In the pooling equilibrium, $\tilde{\Pi}^L = \Pi^L(P^L)\cdot(1+\alpha)$ and $\tilde{\Pi}^H = \Pi^H(P^L) + \alpha\cdot\Pi^H(P^H)$. In the separating equilibrium, $\tilde{\Pi}^L = \Pi^L(P^s) + \alpha\cdot\Pi^L(P^L)$ and $\tilde{\Pi}^H = \Pi^H(P^H)$. In either case, $\tilde{\Pi}^L > \tilde{\Pi}^H > 0$.

18. If entry were informed (i.e. decided upon after the learning of cost information), no free-entry equilibrium would exist. Only low-cost firms would enter, consumers would always return to stores, and then, in a cyclical disequilibrium process, high-cost firms would enter, etc.

19. It is necessary to assume that the number of low (N_L) and high (N_H) cost firms is known. Consumers refuse to believe that a firm has low costs unless it has a sale simultaneously with N_L or fewer firms. Thus, if a high-cost firm mimics the sale (so that N_L+1 firms have a sale), the low-cost firms must again have a sale in order to separate. This is a sketch of the way in which retaliatory sales become credible. See also Green and Porter (1984) and Sobel (1984).

20. For the equivalence to be precise, it is necessary to imagine the downstream firm operating subject to a budget constraint. Also, a "composite" input should exist.

21. If non-linear prices are possible, then all of the quasi-rents will be appropriated in the second period, regardless of cost-type. The incentives for signalling are then eliminated.

22. For the story I have told, one can imagine a period by period rental of a durable machine (or a recurrent purchase of a non-durable machine). Interesting future work might focus on the following sequence of moves: (1) the monopolist quotes an introductory price, (2) the downstream firm decides whether or not to buy a durable machine, and (3) if the machine is bought, the monopolist sells today at the announced price and tomorrow at the appropriate monopoly price. An example of these types of stories is Alcoa, who tried to get downstream firms to forget about steel and to use instead aluminum. Interestingly, Alcoa eventually lowered its introductory price. See also Stuckey (1983).

23. The results in Bagwell (1985*c*) suggest that a "non-informative" (but costly) advertising signal could be included in the model, but that it would not be used in a "refined" equilibrium.

On a related note, this research suggests the possibility of loss-leaders emerging in a rational world. Might a firm advertise a low price on one product in order to signal to consumers that its costs are low and, hence, that it will have low prices for its other (unadvertised) products?

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