RAND Journal of Economics
Vol. 19, No. 1, Spring 1988

Advertising and limit pricing

Kyle Bagwell*

and

Garey Ramey**

We enrich Milgrom and Roberts' (1982) limit-pricing model to allow an incumbent to signal his costs with both price and advertisements. Our fundamental result is that a cost-reducing distortion occurs, in that the incumbent behaves as if there were complete information but his costs were lower than they are. Preentry price is therefore distorted downward, and demand-enhancing advertising is distorted upward, as a consequence of signalling. If advertising is a purely dissipative signal, it is not used, nor therefore distorted. Recent refinements of the sequential equilibrium concept are featured.

1. Introduction

■ When an incumbent firm has private information about the profitability of entry, a potential entrant has reason to use the incumbent's actions as signals of this information. In particular, as argued by Milgrom and Roberts (1982), an incumbent's price may be used as a signal of his costs. Their fundamental finding is that the entrant's effort to infer cost information creates a downward distortion in preentry pricing.[1] Bain's (1956) notion of limit pricing was thus shown to have an equilibrium foundation.

In this article we enrich Milgrom and Roberts' model by allowing the incumbent to signal his costs with price *and* advertisements. Specifically, we consider a two-period model in which the incumbent is privately informed as to whether his costs are high or low. In the first period the incumbent chooses a price and a level of advertising, and these choices determine his first-period profit. A single entrant observes the price-advertising selection and attempts to infer the incumbent's costs. We assume that entry can occur in the second period only and that entry is profitable if and only if the incumbent has high costs.

In such a setting a low-cost incumbent has an incentive to separate from his high-cost counterpart. That is, a low-cost incumbent will tend to choose price-advertising pairs that

---

* Northwestern University.

** University of California, San Diego.

We thank Michael Riordan, John Roberts, Alvin Klevorick, an Associate Editor, and an anonymous referee for helpful comments. We also gratefully acknowledge comments from seminar participants at the University of California at Berkeley, Boston University, Carnegie-Mellon University, Queens University, Stanford University, Toronto University, and York University. This article extends an earlier paper with the same title by the first author to allow for positive advertising effects.

[1] See Ramey (1985) for an extension of Milgrom and Roberts' model that allows for capacity choice by the entrant. Cho (1987) refines the equilibrium set of the limit-pricing game, when price is the only possible signal.

would be especially unattractive were the incumbent to have high costs. It then follows that the low-cost incumbent's choice will typically differ from the choice he would make in an environment with complete information. The intriguing question concerns the direction of the distortion.

Using the sequential equilibrium concept (Kreps and Wilson, 1982), we find that a continuum of separating equilibria typically exists. This multiplicity problem arises because the equilibrium concept does not place sufficient structure on disequilibrium beliefs. Most notably, the entrant is allowed to believe that the high-cost incumbent might play a strictly dominated strategy. As initially proposed by Milgrom and Roberts (1986) and Moulin (1981), we therefore refine the equilibrium set by eliminating dominated strategies. A surprising but intuitive result then emerges. Once dominated strategies are eliminated, there can exist at most one separating equilibrium, and in this equilibrium the low-cost incumbent acts as if there were complete information but his costs were lower. Put differently, in the unique undominated separating equilibrium, the low-cost incumbent chooses a price-advertising pair that would maximize first-period profit if his costs were even lower and there were no threat of entry. The fundamental distortion occurring in the undominated separating equilibrium is therefore a cost-reducing distortion.

We next consider the possibility of pooling equilibria, in which the incumbent's price-advertising selection is independent of his cost type. Even after dominated strategies are eliminated, a great number of pooling equilibria remain. When, however, we require beliefs to be intuitive in the sense of Cho and Kreps (1987), the set of pooling equilibria is easily characterized. Specifically, in any intuitive pooling equilibrium, the incumbent chooses the complete-information optimal selection for some cost level no higher than that of the low-cost incumbent. The fundamental distortion is again a cost-reducing distortion.

Thus, in both separating and pooling equilibria, the incumbent acts as if his costs were lower and then chooses the corresponding monopoly price and advertising levels. Consequently, if advertising enhances demand,[2] then an upward distortion in advertising occurs; but if advertising is a dissipative signal that does not increase demand, then the advertising signal is not used and, therefore, is not distorted. In either case preentry price is distorted downward. The possibility of signalling with advertisements does not destroy the incumbent's incentive to limit price.

Our ideas relate to a large volume of previous research. Comanor and Wilson (1967) have argued for a correlation between advertising volume and industry profit that is consistent with our analysis. Bain (1956) regards product differentiation as the most important entry barrier and suggests that this barrier can be erected with an appropriate advertising campaign. This view implicitly derives from a model of brand loyalty. Salop (1979) makes a similar point. He assumes that the incumbent's preentry advertising choice must be matched by the entrant in the postentry period and shows that a high level of advertising can deter entry. As Schmalensee (1983) argues, however, an incumbent may also choose to "underinform" consumers of his existence to threaten credibly an aggressive response to entry.

Unlike in previous research on advertising as an entry deterrent, which has emphasized the direct influence of preentry advertising on the postentry game, we assume that the postentry game is independent of preentry behavior and focus on advertising as a signal of the profitability of entry. Furthermore, our model does not establish advertising as an entry barrier. We find that profitable entry is deterred only in pooling equilibria, where the entrant's uncertainty about the incumbent's cost is not resolved. A positive level of advertising is not fundamental to this possibility.

---

[2] Advertising might increase demand if it provides information about the existence of the incumbent's product (Butters, 1977; Schmalensee, 1983; Grossman and Shapiro, 1984), if it persuades consumers to buy the product (Dixit and Norman, 1978), if it indirectly informs consumers about the product's quality (Nelson, 1970, 1974; Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986), or if it indirectly informs consumers about prices when search costs exist (Bagwell, 1987). For a survey on advertising, see Schmalensee (1986).

Our work is also related to the product-quality literature. Nelson (1970, 1974), Kihlstrom and Riordan (1984), and Milgrom and Roberts (1986) have argued that dissipative advertising can signal product quality in markets without entry. The recent Milgrom and Roberts article is particularly noteworthy, as it also features sequential equilibrium refinements in a model with price and advertising signals. In contrast to our results, however, Milgrom and Roberts find that dissipative advertising may occur in a refined equilibrium. The key difference between the two models is that the level of sales in the first period can directly influence second-period profit in the Milgrom and Roberts model but not in ours.

The remainder of the article is organized in five sections. We describe the basic model in Section 2. In Section 3 we examine separating equilibria. Pooling equilibria are investigated in Section 4, and concluding thoughts appear in Section 5.

2. The model

■ Consider the following situation. Two firms, an incumbent and a potential entrant, interact for two periods in a market for a homogeneous good. In the initial period the incumbent monopolizes the market, while at the start of the second period the entrant may choose to enter the market. If entry occurs, the firms earn duopoly profits for the second period, while if entry does not occur, the incumbent remains a monopolist. The entrant makes his choice without having complete knowledge about the incumbent's production costs, though he might be able to infer cost information by observing the incumbent's first-period price and advertising decisions. This process of inference might, in turn, distort the incumbent's incentives for choosing these variables.

Consumer behavior is summarized by a market demand function $X(P, A)$, where $P \geqslant 0$ denotes price and $A \geqslant 0$ denotes advertising. The function $X(P, A)$ is assumed to be continuous. Production costs are linear in quantity, and fixed costs are zero. Although the entrant does not directly observe the incumbent's unit cost, he does know that it is one of two possible levels, C^L and C^H, with $0 < C^L < C^H$.[3] Let $\rho \in (0, 1)$ be the entrant's prior probability assessment of the event that the incumbent's unit cost is C^H.

In the first period the incumbent observes his unit cost, either C^L or C^H, and chooses the first-period price and advertising levels. First-period profits are

$$\Pi^i(P, A) = (P - C^i)X(P, A) - A, \qquad i = L, H.$$

Let (P^i, A^i) be the unique maximizer of $\Pi^i(P, A)$.

We use Π^i_D, $i = L, H$, to denote the incumbent's duopoly profits in the second period if entry occurs, and we assume that $\Pi^i(P^i, A^i) > \Pi^i_D$ and that $\Pi^L_D > \Pi^H_D > 0$. Let $F > 0$ represent the start-up cost for the entrant, and denote by Π^i_E the entrant's duopoly profits when the incumbent's unit costs are C^i. Assume that the entrant would desire to enter if the incumbent's unit costs were high, but not if they were low: $\Pi^H_E > F > \Pi^L_E > 0$.

While the entrant cannot observe unit cost before making his entry choice, he can observe the incumbent's first-period price and advertising decisions. Let $\hat{\rho}(P, A) \in [0, 1]$ be the entrant's posterior belief that unit cost is high when he observes P and A.

As a formal matter, we model this situation as an extensive-form game having four stages. First, "Nature" chooses the incumbent's unit costs, with ρ being the probability that C^H is chosen. Next, the incumbent observes C^i and chooses P and A; these must be non-negative real numbers. The entrant then observes P and A, but not C^i, and chooses either to enter or not to enter. Finally, the firms earn duopoly profits in the second period, if entry has occurred, and otherwise the incumbent receives monopoly profits.[4] We shall consider

[3] Our general result of a cost-reducing distortion is *not* limited to the case of linear costs. Indeed, one can demonstrate that our results hold if the cost of producing x units for an incumbent of type t is $c(t, x)$ with $c_{tx} > 0$.

[4] As in Milgrom and Roberts (1982), postentry profits are assumed to be independent of the entrant's belief at the time of entry. One interpretation is that the incumbent's costs become commonly known upon entry. Cho

only pure-strategy sequential equilibria of this game.[5] Let the entrant's strategy be denoted by $\hat{R}(P, A) \in \{0, 1\}$, where $\hat{R} = 1$ indicates entry. The collection $\{(\hat{P}^i, \hat{A}^i)_{i=L,H}, \hat{R}(P, A), \hat{\rho}(P, A)\}$ is an *equilibrium* if the following three conditions are satisfied.

Condition 1: optimality for incumbent. For $i = L, H$,

$$(\hat{P}^i, \hat{A}^i) \in \underset{(P,A)}{\operatorname{argmax}} \{\Pi^i(P, A) + \delta[\hat{R}(P, A)\Pi^i_D + (1 - \hat{R}(P, A))\Pi^i(P^i, A^i)]\},$$

where $\delta \in (0, 1)$ is the incumbent's discount factor.

Condition 2: optimality for entrant. For all (P, A), $\hat{R}(P, A) = 1$ if and only if

$$\hat{\rho}(P, A)\Pi^H_E + (1 - \hat{\rho}(P, A))\Pi^L_E > F.$$

Condition 3: Bayes' consistency of beliefs. If $(\hat{P}^L, \hat{A}^L) \neq (\hat{P}^H, \hat{A}^H)$, then $\hat{\rho}(\hat{P}^L, \hat{A}^L) = 0$ and $\hat{\rho}(\hat{P}^H, \hat{A}^H) = 1$. If $(\hat{P}^L, \hat{A}^L) = (\hat{P}^H, \hat{A}^H)$, then $\hat{\rho}(\hat{P}^L, \hat{A}^L) = \rho$.

In other words, Condition 3 requires the entrant's posterior beliefs about C^i to be obtained from his prior beliefs by using Bayes' rule together with the incumbent's equilibrium strategies. For a (P, A) pair that the incumbent does not choose under either cost level, Bayes' rule cannot be used, and thus $\hat{\rho}(P, A)$ may take any value; it is here that arbitrary off-equilibrium-path beliefs are allowed.

3. Separating equilibria

■ If $(\hat{P}^L, \hat{A}^L) \neq (\hat{P}^H, \hat{A}^H)$, then observing price and advertising allows the entrant to become fully informed of the incumbent's unit cost before making his entry decision; this is called a *separating equilibrium.* In this section we begin with a characterization of the set of separating equilibria. A large class of possible price and advertising levels can arise in separating equilibria; the incumbent's incentives might be distorted in any direction. We show that a unique separating equilibrium emerges when dominated strategies are eliminated.

In any separating equilibrium the incumbent will make his optimal one-period monopoly choice (P^H, A^H) if $C^i = C^H$, since any $(\hat{P}^H, \hat{A}^H) \neq (P^H, A^H)$ would yield

$$\Pi^H(\hat{P}^H, \hat{A}^H) + \delta\Pi^H_D < \Pi^H(P^H, A^H) + \delta[\hat{R}(P^H, A^H)\Pi^H_D + (1 - \hat{R}(P^H, A^H))\Pi^H(P^H, A^H)],$$

which violates Condition 1. Let us exploit the arbitrariness of off-equilibrium-path beliefs by setting $\hat{\rho}(P, A) = 1$ for all $(P, A) \neq (\hat{P}^L, \hat{A}^L)$, which means that the entrant always enters unless he observes the equilibrium choices of the low-cost incumbent. In this case we can be sure that Condition 1 is satisfied for $i = H$ as long as

$$\Pi^H(\hat{P}^L, \hat{A}^L) \leqslant (1 - \delta)\Pi^H(P^H, A^H) + \delta\Pi^H_D \equiv \bar{\Pi}^H. \tag{1}$$

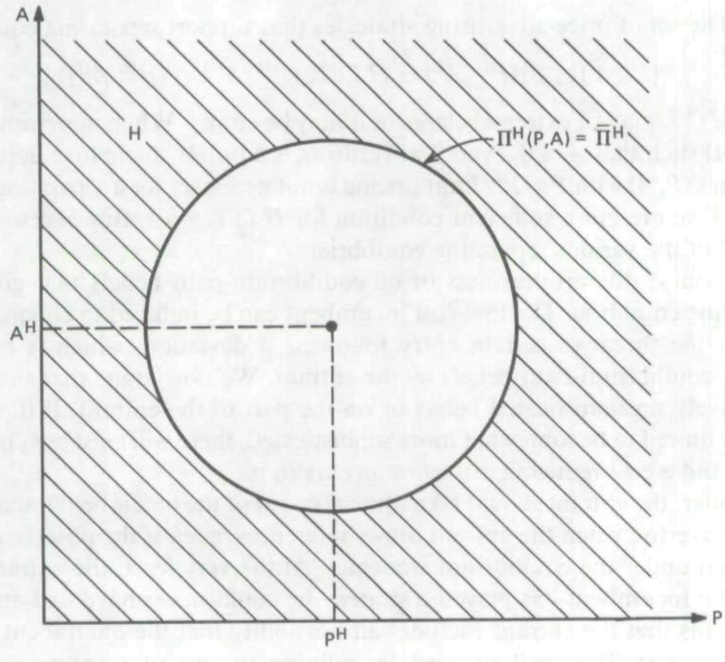
Thus, $(\hat{P}^L, \hat{A}^L)$ must give the incumbent sufficiently low first-period profits under $C^i = C^H$ to discourage him from choosing it and deterring entry. Figure 1 depicts a possible shape for the isoprofit curve $\Pi^H(P, A) = \bar{\Pi}^H$, so that the shaded region H then gives the set of possible $(\hat{P}^L, \hat{A}^L)$ that satisfy (1).

Moreover, if the incumbent does not prefer $(\hat{P}^L, \hat{A}^L)$ under $C^i = C^L$, then his optimal choice, given the beliefs we have specified, will clearly be (P^L, A^L). This means that Condition 1 will be satisfied for $i = L$ if

(1987) and Bagwell and Ramey (1987) have relaxed the complete-information assumption to study the preentry distortion associated with the incentive to manipulate the postentry game. The latter paper builds on the present work and finds that a postentry price (quantity) game causes a cost-increasing (decreasing) distortion in preentry variables.

[5] Although the sequential equilibrium concept is formally defined for games with finitely many actions, the definition of equilibrium we offer is the obvious extension of this concept to our game. See Kreps and Wilson (1982) and Kreps and Ramey (1987) for further discussion.

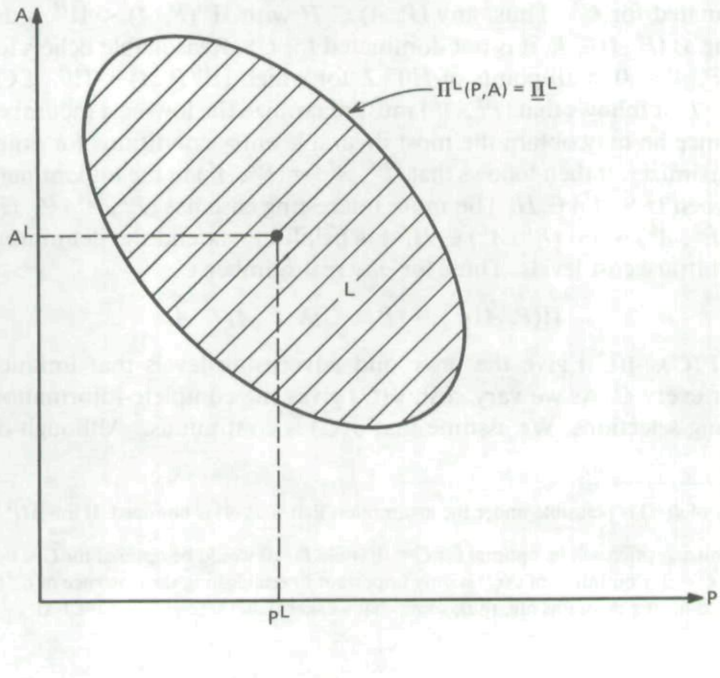
FIGURE 1
PRICE–ADVERTISING PAIRS THAT C^H WOULD NEVER CHOOSE



$$\Pi^L(\hat{P}^L, \hat{A}^L) \geqslant (1 - \delta)\Pi^L(P^L, A^L) + \delta\Pi^L_D \equiv \underline{\Pi}^L. \tag{2}$$

The set of $(\hat{P}^L, \hat{A}^L)$ satisfying (2) is depicted as the shaded region L in Figure 2. As long as $(\hat{P}^L, \hat{A}^L) \in H \cap L$, it is certain that Condition 1 is satisfied under our specification of beliefs. Further, if either $(\hat{P}^L, \hat{A}^L) \notin H$ or $(\hat{P}^L, \hat{A}^L) \notin L$, then Condition 1 will be violated for either

FIGURE 2
PRICE–ADVERTISING PAIRS THAT C^L MIGHT CHOOSE



$i = H$ or $i = L$, respectively, for *any* specification of beliefs, and $(\hat{P}^L, \hat{A}^L)$ cannot then yield a separating equilibrium. This completes the proof of the following theorem.

Theorem 1. The set of price-advertising strategies that support separating equilibria is

$$\{(\hat{P}^i, \hat{A}^i)_{i=L,H} \mid (\hat{P}^L, \hat{A}^L) \in H \cap L, (\hat{P}^H, \hat{A}^H) = (P^H, A^H)\}.$$

The set $H \cap L$ may be extremely large or it may be empty. When nonempty, it generally includes (P, A) such that $A > 0$, even if advertising is a purely dissipative activity. Further, the set often has (P, A) with $P > P^L$: limit pricing is not necessary for a separating equilibrium. In Theorem 3 we present a sufficient condition for $H \cap L \neq \varnothing$. But first we examine the "plausibility" of the various separating equilibria.

It is, of course, the arbitrariness of off-equilibrium-path beliefs that gives rise to so many separating equilibria. The low-cost incumbent can be induced to choose any $(\hat{P}^L, \hat{A}^L)$ in $H \cap L$ by the threat of certain entry following a deviation, which is based on very optimistic off-equilibrium-path beliefs by the entrant. We now argue that such beliefs represent excessively unsophisticated behavior on the part of the entrant. If the entrant's inferences are required to be somewhat more sophisticated, there will exist only one separating equilibrium, and a cost-reducing distortion occurs in it.

In particular, the entrant should take some account of the incumbent's actual incentives to price and advertise when the entrant draws inferences, even if the observed (P, A) could not have arisen under the equilibrium strategies. At the very least, the entrant should not believe that the incumbent has played a strategy he could never have had an incentive to play. This means that the entrant excludes all possibility that the incumbent plays strictly dominated strategies. We shall proceed by refining the set of separating equilibria by eliminating dominated strategies, as proposed by Moulin (1981) and Milgrom and Roberts (1986).

To this end, the pair (P, A) will be called *dominated for* C^i if

$$\Pi^i(P, A) + \delta\Pi^i(P^i, A^i) < \Pi^i(P^i, A^i) + \delta\Pi^i_D.$$

Thus, (P, A) is dominated for C^i if it yields less profit under the best entry conditions than does the one-period monopoly optimum under the worst conditions. An equilibrium will be called *undominated* if $\hat{\rho}(P, A) = 0$ whenever (P, A) is dominated for C^H but not for C^L.

Observe that $H \cap L$ is the set of equilibrium (P, A) that are weakly dominated for C^H and not dominated for C^L. Thus, any $(P, A) \in H$ with $\Pi^H(P, A) < \bar{\Pi}^H$ is dominated for C^H, but as long as $(P, A) \in L$, it is not dominated for C^L. Reasonable beliefs for the entrant then entail $\hat{\rho}(P, A) = 0$ at all points of $H \cap L$ for which $\Pi^H(P, A) < \bar{\Pi}^H$. If Condition 1 is satisfied for $i = L$, it follows that $(\hat{P}^L, \hat{A}^L)$ must maximize the low-cost incumbent's preentry profit on H, since he may obtain the most favorable entry conditions for points arbitrarily close to the maximizer. It then follows that $(\hat{P}^L, \hat{A}^L) = (P^L, A^L)$ in the undominated separating equilibrium when $(P^L, A^L) \in H$. The more interesting case has $(P^L, A^L) \notin H$.

To find $(\hat{P}^L, \hat{A}^L)$ when $(P^L, A^L) \notin H$, it is helpful to extend the definition of the profit function to arbitrary cost levels. Thus, for *any* real number C,

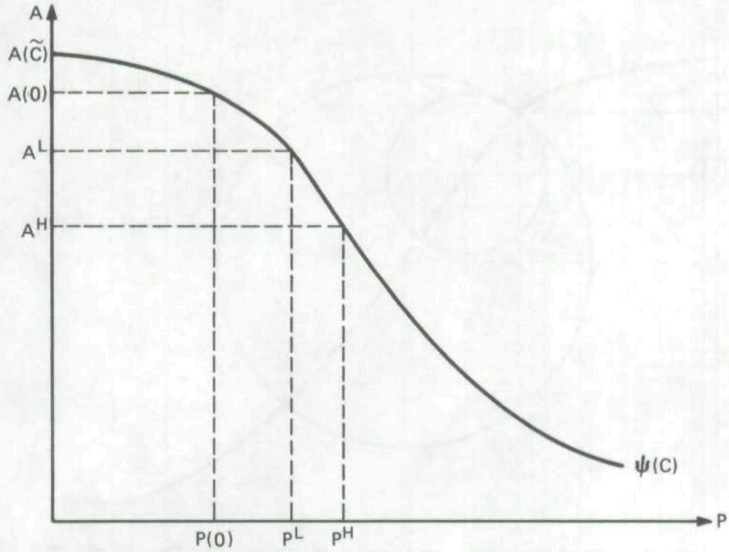
$$\Pi(P, A \mid C) = (P - C)X(P, A) - A.$$

Let $\psi(C) = (P(C), A(C))$ give the price and advertising levels that uniquely maximize $\Pi(P, A \mid C)$ for every C. As we vary cost, $\psi(C)$ gives the complete-information, monopoly price-advertising selections. We assume that $\psi(C)$ is continuous.[6] Although our proofs do

[6] Continuity of $\psi(C)$ is plausible under the assumption that $X(P, A)$ is bounded. If $\lim_{P\to 0} X(P, A) = +\infty$, then the revenue-maximizing price will be optimal for $C = 0$, while $P = 0$ would be optimal for $C < 0$; this may create a discontinuity at $C = 0$. Continuity of $\psi(C)$ is only important in establishing the existence of C^o (see below). The assumptions needed for the theorems are, thus, somewhat weaker than those given in the text.

FIGURE 3

COMPLETE–INFORMATION PRICE–ADVERTISING SELECTIONS



not require $\psi(C)$ to be monotonic, the natural assumption is that $P(C)$ is strictly increasing and $A(C)$ is strictly decreasing when advertising is positively related to demand.[7] Of course, if advertising is a purely dissipative signal having no effect on demand, then $A(C) \equiv 0$. Note that $\psi(C)$ intersects the $P = 0$ axis for some cost level $\tilde{C} < 0$, as shown in Figure 3.

With this structure established, we prove in the Appendix that the element of H that maximizes $\Pi^L(P, A)$ is the point of intersection of $\psi(C)$ and $\bar{\Pi}^H$, shown as $(P(C^o), A(C^o))$ in Figure 4. In other words, in the unique undominated separating equilibrium the low-cost incumbent acts as if he were maximizing single-period profit and his costs were $C^o < C^L$.

Theorem 2. If $(P^L, A^L) \notin H$, then there exists at most one undominated separating equilibrium, and in it $(\hat{P}^L, \hat{A}^L) = (P(C^o), A(C^o))$ for some $C^o < C^L$.

The distortion associated with efficient signalling is therefore fundamentally a cost-reducing distortion. This leads, in turn, to a downward distortion in preentry pricing and an upward distortion in preentry, demand-enhancing advertising. If advertising is a purely dissipative activity, in which case $\psi(C)$ is horizontal at $A(C) = 0$, then the low-cost incumbent does not employ, nor therefore distort, the advertising signal. In general, the net effect of signalling is that demand is increased relative to complete information.

We now provide a sufficient condition for the existence of an undominated separating equilibrium.

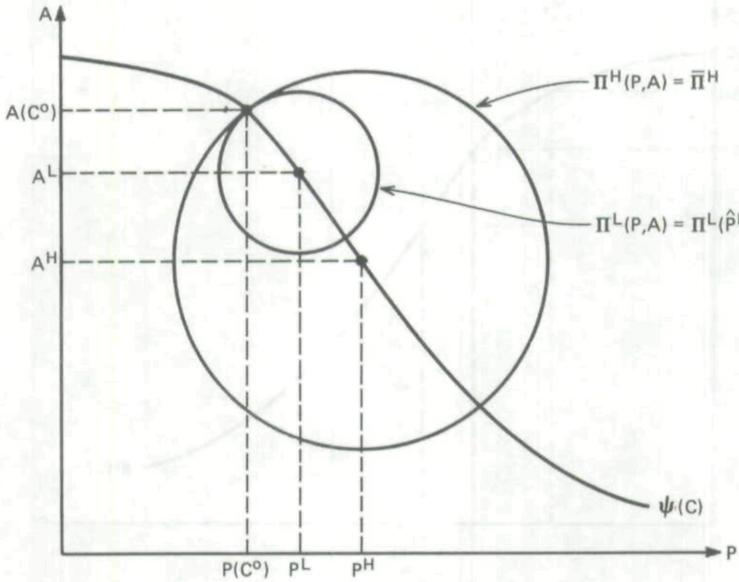
Theorem 3. If $\Pi^L(P^L, A^L) - \Pi^L_D \geq \Pi^H(P^H, A^H) - \Pi^H_D$, then an undominated separating equilibrium exists.

The proof of this theorem is also in the Appendix. Intuitively, the condition of the theorem is that the low-cost incumbent has more to gain from entry deterrence than does the high-

[7] It is easy to see that a reduction in C has a direct effect that tends to reduce $P(C)$ and to raise $A(C)$, since increases in demand are more profitable when the markup is higher. But the cross effects of advertising on the profitability of price increases and *vice versa* could counteract the direct effect, thereby possibly leading cost reductions to have net effects on $P(C)$ and $A(C)$ that are in the same direction. By assuming that the cross effects are of sufficiently small magnitude, we can be sure that $P(C)$ strictly increases and that demand-enhancing $A(C)$ strictly decreases in C when $P(C), A(C) > 0$, at least for $C \leq C^H$.

FIGURE 4

THE UNIQUE UNDOMINATED SEPARATING EQUILIBRIUM



cost incumbent, which is to say that the low-cost incumbent is willing to incur a larger sacrifice to deter entry than is his high-cost counterpart. Since signalling entails a financial sacrifice, the condition is sufficient for the existence of the equilibrium.

4. Pooling equilibria

■ Thus far we have considered equilibria in which the entrant becomes fully informed, but there can also exist equilibria in which the entrant learns nothing at all from observing price and advertising. These *pooling equilibria* are characterized by $(\hat{P}^L, \hat{A}^L) = (\hat{P}^H, \hat{A}^H)$. It is immediate, though, that pooling equilibria cannot exist when entry would be favorable under the entrant's prior beliefs. For suppose we have

$$\rho\Pi_E^H + (1-\rho)\Pi_E^L > F. \tag{3}$$

Then $\hat{R}(\hat{P}^L, \hat{A}^L)$ is 1 in a pooling equilibrium, and for any specification of off-equilibrium-path beliefs, the incumbent could always do better under at least one cost level by deviating to his one-period optimum. Under (3), then, the entrant will always gain information in equilibrium.

When (3) does not hold, however, many price-advertising pairs can arise in pooling equilibria, even after the elimination of dominated strategies. Yet, the beliefs supporting these equilibria need not be "reasonable." To illustrate these points, we consider momentarily the special case in which advertising is purely dissipative and $A^i = 0$, for $i = H, L$. Define $\underline{P}^H$ to be the price below P^H solving $\Pi^H(P, 0) = \bar{\Pi}^H$. To make the example interesting, assume that $(P^L, 0) \notin H$ and $(\underline{P}^H, 0) \in L$.[8] Choose A^o to satisfy

$$0 < A^o \leqslant \Pi^L(P^L, 0) - \Pi^L(\underline{P}^H, 0). \tag{4}$$

It then follows that (P^L, A^o) is a common incumbent strategy in an undominated pooling equilibrium. First, Condition 2 will be satisfied if $\hat{R}(P^L, A^o) = 0$. For all (P, A) such that

[8] $\Pi^L(P^L, 0) - \Pi_D^L \geqslant \Pi^H(P^H, 0) - \Pi_D^H$ is sufficient for $(\underline{P}^H, 0) \in L$.

$\Pi^H(P, A) < \bar{\Pi}^H$, we may set $\hat{\rho}(P, A) = 0$ in an undominated equilibrium, and $\hat{\rho}(P, A) = 1$ may be specified for all the remaining $(P, A) \neq (P^L, A^o)$.

Second, the incumbent with low costs prefers $(\underline{P}^H, 0)$ with no entry to $(P^L, 0)$ followed by entry, since we have assumed that $(\underline{P}^H, 0) \in L$. Thus, by (4) he will certainly prefer (P^L, A^o) to $(P^L, 0)$. In other words, (P^L, A^o) is in region L, and Condition 1 will be satisfied for $i = L$. Assuming that demand is negatively related to price, it is easy to establish that the isoprofit curve $\Pi^L(P, A) = \Pi^L(\underline{P}^H, 0)$ lies strictly below $\bar{\Pi}^H$ for all $P > \underline{P}^H$, as shown in Figure 5. This guarantees that (P^L, A^o) is not in H, and under high costs the incumbent will prefer (P^L, A^o) with no entry to his next best choice, which is $(P^H, 0)$ followed by entry. Thus, Condition 1 is satisfied for $i = H$. We have an undominated pooling equilibrium with positive dissipative advertising.

The equilibrium derived above does not require the entrant to believe that the incumbent takes action that could never possibly be in his interest. Still, the entrant is not very sophisticated in taking account of the incumbent's incentives. Suppose that the entrant anticipates an equilibrium in which the incumbent will choose (P^L, A^o) under either cost level, but then unexpectedly observes the values $(\tilde{P}, \tilde{A})$ shown in Figure 5. What should the entrant then believe? With high costs the incumbent would have preferred the equilibrium choice (P^L, A^o) followed by no entry to $(\tilde{P}, \tilde{A})$, no matter what entry decision the latter elicited. This fact could easily be deduced by the entrant, so that it does not seem reasonable for him to entertain the possibility that the high-cost incumbent had deviated to $(\tilde{P}, \tilde{A})$. Thus, $\hat{\rho}(\tilde{P}, \tilde{A}) = 0$ gives the appropriate inference. But in this case the incumbent would prefer $(\tilde{P}, \tilde{A})$ under low costs, and the equilibrium would collapse.

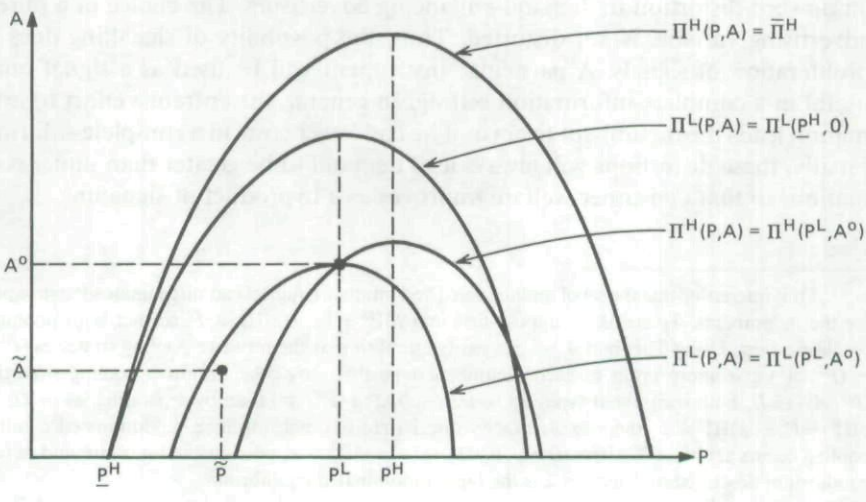
To eliminate equilibria based on unreasonable beliefs of this sort, we must further refine the equilibrium concept. Returning now to the general model of advertising, we follow Cho and Kreps (1987) and call a set of equilibrium beliefs *unintuitive* if there exists $(\tilde{P}, \tilde{A}) \neq (\hat{P}^L, \hat{A}^L), (\hat{P}^H, \hat{A}^H)$ such that

$$\Pi^H(\tilde{P}, \tilde{A}) + \delta\Pi^H(P^H, A^H) < \Pi^H(\hat{P}^H, \hat{A}^H) + \delta[\hat{R}(\hat{P}^H, \hat{A}^H)\Pi^H_D + (1 - \hat{R}(\hat{P}^H, \hat{A}^H))\Pi^H(P^H, A^H)] \quad (5)$$

$$\Pi^L(\tilde{P}, \tilde{A}) + \delta\Pi^L(P^L, A^L) > \Pi^L(\hat{P}^L, \hat{A}^L) + \delta[\hat{R}(\hat{P}^L, \hat{A}^L)\Pi^L_D + (1 - \hat{R}(\hat{P}^L, \hat{A}^L))\Pi^L(P^L, A^L)]. \quad (6)$$

FIGURE 5

AN UNINTUITIVE POOLING EQUILIBRIUM WHEN ADVERTISING IS DISSIPATIVE



The right-hand sides of (5) and (6) give the equilibrium profits of the incumbent under high and low costs, respectively. By (5) the high-cost incumbent prefers his equilibrium choice to $(\tilde{P}, \tilde{A})$, even if the latter leads to the most favorable entry situation. Inequality (6) indicates that the low-cost incumbent would prefer $(\tilde{P}, \tilde{A})$ to the equilibrium, as long as choosing $(\tilde{P}, \tilde{A})$ convinced the entrant that the incumbent's true cost level was low. Thus, the equilibrium could be supported only by an "unintuitive" inference of $\hat{\rho}(\tilde{P}, \tilde{A}) > 0$. We call an equilibrium *intuitive* if it can be supported by beliefs that are not unintuitive.

In the case of a pooling equilibrium, we know that $\hat{R}(\hat{P}^L, \hat{A}^L) = 0$, so that equilibrium beliefs fail to be intuitive if there exists $(\tilde{P}, \tilde{A}) \neq (\hat{P}^L, \hat{A}^L) = (\hat{P}^H, \hat{A}^H)$ such that $\Pi^H(\tilde{P}, \tilde{A}) < \Pi^H(\hat{P}^H, \hat{A}^H)$ and $\Pi^L(\tilde{P}, \tilde{A}) > \Pi^L(\hat{P}^L, \hat{A}^L)$. We now have the following theorem.

Theorem 4. In any intuitive pooling equilibria with strategies $(\hat{P}, \hat{A})$, $(\hat{P}, \hat{A}) = \psi(\hat{C})$ for some $\hat{C} \in [C^o, C^L]$.

This theorem, which we prove in the Appendix, establishes that a cost-reducing distortion also occurs in intuitive pooling equilibria. Thus, whether or not the equilibrium is informative, efficient signalling entails a downward distortion in preentry prices, an upward distortion in preentry demand-enhancing advertising, and no distortion of a dissipative advertising signal.

We next provide a sufficient condition for the existence of intuitive pooling equilibria.

Theorem 5. Suppose that $\Pi^L(P^L, A^L) - \Pi^L_D \geqslant \Pi^H(P^H, A^H) - \Pi^H_D$. Then for every $\hat{C} \in [C^o, C^L]$ there exists an intuitive pooling equilibrium with strategies $(\hat{P}, \hat{A}) = \psi(\hat{C})$.

Intuitive pooling equilibria are not unique.[9] To understand the theorem, note that C^H will choose the pooling strategy $\psi(\hat{C})$ only when $\hat{C} \geqslant C^o$. The supposition of the theorem is simply to ensure that C^L is also willing to pool at such price-advertising pairs. The remainder of the proof, which appears in the Appendix, is to show that a point on $\psi(\hat{C})$ with $\hat{C} \in [C^o, C^L]$ is itself supportable as an intuitive pooling strategy.

7. Conclusion

■ We have extended Milgrom and Roberts' (1982) model to the case in which the incumbent firm uses both price and advertising to signal his costs. Our analysis indicates that there is a downward distortion in price, whether advertising is dissipative or demand-enhancing. Limit pricing can therefore be expected to occur, even when the incumbent has the option of signalling costs with advertising. We have also argued that the signalling process causes an upward distortion in demand-enhancing advertising. The choice of a purely dissipative advertising variable is not distorted. Thus, the possibility of signalling does not lead to a proliferation of signals. A particular instrument will be used as a signal only if it is also useful in a complete-information setting. In general, the entrant's effort to infer cost information leads the incumbent to act as if he had lower costs in a complete-information setting. Finally, these distortions will always lead demand to be greater than under complete information, so that consumer welfare improves as a byproduct of signalling.

---

[9] It is interesting that the set of undominated and intuitive equilibria admits a unique Pareto-optimal equilibrium for the incumbents. To see this, suppose first that $\rho\Pi^H_E + (1 - \rho)\Pi^L_E \leqslant F$, so that both pooling and separating equilibria exist. Using Theorem 4, we can easily establish that the intuitive pooling strategies $(\hat{P}^L, \hat{A}^L) = (\hat{P}^H, \hat{A}^H) = (P^L, A^L)$ give more profit to each incumbent type than any other intuitive pooling strategies. Moreover, if $(P^L, A^L) \in H$, both incumbent types do better pooling at (P^L, A^L) than by separating, as in Theorem 2. Thus, if $\rho\Pi^H_E + (1 - \rho)\Pi^L_E \leqslant F$, there exists exactly one Pareto optimal, intuitive, undominated equilibrium, in which pooling occurs at (P^L, A^L). Alternatively, if $\rho\Pi^H_E + (1 - \rho)\Pi^L_E > F$, pooling cannot occur, and in fact the separating equilibrium described in Theorem 2 is the only undominated equilibrium.

Appendix

■ The proofs of Theorems 2–5 follow.

Proof of Theorem 2. We first establish the existence of an intersection point $(P(C^o), A(C^o))$ of $\psi(C)$ and $\Pi^H(P, A) = \bar{\Pi}^H$ with $P(C^o), A(C^o) > 0$. Since $\Pi^H(P^H, A^H) > \bar{\Pi}^H > 0 > \Pi^H(0, A)$, $\psi(C)$ and $\Pi^H(P, A)$ are continuous, and $\psi(C)$ intersects the $P = 0$ axis, it is sufficient to show that $\Pi(\psi(C)|C^H)$ is strictly increasing in C for $C < C^H$.

To this end, choose $\underline{C} < \bar{C} < C^H$. Let $(\bar{P}, \bar{A}) \equiv (P(\bar{C}), A(\bar{C}))$ and $(\underline{P}, \underline{A}) \equiv (P(\underline{C}), A(\underline{C})$. Then

$$(\bar{P} - \bar{C})X(\bar{P}, \bar{A}) - \bar{A} - (\underline{P} - \bar{C})X(\underline{P}, \underline{A}) + \underline{A} > 0 \tag{A1}$$

$$(\underline{P} - \underline{C})X(\underline{P}, \underline{A}) - \underline{A} - (\bar{P} - \underline{C})X(\underline{P}, \underline{A}) + \bar{A} > 0.$$

Adding these inequalities gives

$$(\bar{C} - \underline{C})[X(\underline{P}, \underline{A}) - X(\bar{P}, \bar{A})] > 0,$$

whence $X(\underline{P}, \underline{A}) > X(\bar{P}, \bar{A})$. Using (A1), we may write

$$(\bar{P} - C^H)X(\bar{P}, \bar{A}) - \bar{A} - (\underline{P} - C^H)X(\underline{P}, \underline{A}) + \underline{A} > 0,$$

which proves monotonicity and establishes the existence of the intersection point $(P(C^o), A(C^o))$. Notice that $C^o < C^L$, since $\Pi^H(P^L, A^L) > \bar{\Pi}^H$.

Second, we argue that $\Pi^H(\hat{P}^L, \hat{A}^L) = \bar{\Pi}^H$. For suppose that $\Pi^H(\hat{P}^L, \hat{A}^L) < \bar{\Pi}^H$. Let $(P(C^o), A(C^o)) \equiv (P^o, A^o)$. Then

$$(P^o - C^H)X(P^o, A^o) - A^o - (\hat{P}^L - C^H)X(\hat{P}^L, \hat{A}^L) + \hat{A}^L > 0$$

$$(\hat{P}^L - C^L)X(\hat{P}^L, \hat{A}^L) - \hat{A}^L - (P^o - C^L)X(P^o, A^o) + A^o \geqslant 0, \tag{A2}$$

where (A2) follows from the fact that $(\hat{P}^L, \hat{A}^L)$ must maximize $\Pi^L(P, A)$ on H. Adding gives

$$(C^H - C^L)[X(\hat{P}^L, \hat{A}^L) - X(P^o, A^o)] > 0,$$

whence $X(\hat{P}^L, \hat{A}^L) > X(P^o, A^o)$. But using (A2) and $C^o < C^L$, we obtain

$$(\hat{P}^L - C^o)X(\hat{P}^L, \hat{A}^L) - \hat{A}^L > (P^o - C^o)X(P^o, A^o) - A^o,$$

which is a contradiction.

We now complete the proof by showing that (P^o, A^o) uniquely maximizes $\Pi^L(P, A)$ over (P, A) such that $\Pi^H(P, A) = \bar{\Pi}^H$. Pick any $(P, A) \neq (P^o, A^o)$ such that $\Pi^H(P, A) = \bar{\Pi}^H$. Then

$$(P^o - C^o)X(P^o, A^o) - A^o - (P - C^o)X(P, A) + A > 0$$

$$(P - C^H)X(P, A) - A - (P^o - C^H)X(P^o, A^o) + A^o = 0. \tag{A3}$$

Adding these yields

$$(C^H - C^o)[X(P^o, A^o) - X(P, A)] > 0,$$

so that $X(P^o, A^o) > X(P, A)$. Using (A3), we may write

$$(P^o - C^L)X(P^o, A^o) - A^o - (P - C^L)X(P, A) + A = (C^H - C^L)[X(P^o, A^o) - X(P, A)] > 0.$$

This proves that (P^o, A^o) uniquely maximizes $\Pi^L(P, A)$ on H. *Q.E.D.*

Proof of Theorem 3. The theorem is immediate if $(P^L, A^L) \in H$. Suppose then that $(P^L, A^L) \notin H$. From the proof of Theorem 2 there exists $C^o < C^L$ such that $P(C^o), A(C^o) > 0$ and $\Pi^H(P(C^o), A(C^o)) = \bar{\Pi}^H$. Thus,

$$(P(C^o), A(C^o)) \in H.$$

Letting $(P^o, A^o) \equiv (P(C^o), A(C^o))$, notice that

$$\begin{aligned}
&\Pi^L(P^o, A^o) + \delta\Pi^L(P^L, A^L) - \Pi^L(P^L, A^L) - \delta\Pi^L_D \\
&= \Pi^L(P^o, A^o) + \delta\Pi^L(P^L, A^L) - \Pi^L(P^L, A^L) - \delta\Pi^L_D - \Pi^H(P^o, A^o) - \delta\Pi^H(P^H, A^H) + \Pi^H(P^H, A^H) + \delta\Pi^H_D \\
&= [\Pi^L(P^o, A^o) - \Pi^H(P^o, A^o)] - [\Pi^L(P^L, A^L) - \Pi^H(P^H, A^H)] + \delta[\Pi^L(P^L, A^L) - \Pi^L_D - \Pi^H(P^H, A^H) + \Pi^H_D] \\
&\geqslant [\Pi^L(P^o, A^o) - \Pi^H(P^o, A^o)] - [\Pi^L(P^L, A^L) - \Pi^H(P^H, A^H)] \\
&> [\Pi^L(P^o, A^o) - \Pi^H(P^o, A^o)] - [\Pi^L(P^L, A^L) - \Pi^H(P^L, A^L)] \\
&= (C^H - C^L)[X(P^o, A^o) - X(P^L, A^L)].
\end{aligned}$$

But we know that

$$(P^o - C^o)X(P^o, A^o) - A^o - (P^L - C^o)X(P^L, A^L) + A^L > 0$$

$$(P^L - C^L)X(P^L, A^L) - A^L - (P^o - C^L)X(P^o, A^o) + A^o > 0.$$

Adding, we obtain

$$(C^L - C^o)[X(P^o, A^o) - X(P^L, A^L)] > 0,$$

whence $X(P^o, A^o) > X(P^L, A^L)$, which establishes that $(P^o, A^o) \in L$. By Theorem 1 $(\hat{P}^L, \hat{A}^L) = (P^o, A^o)$ and $(\hat{P}^H, \hat{A}^H) = (P^H, A^H)$ can thus be supported as separating equilibrium strategies. Since from the proof of Theorem 2 $\Pi^L(P, A)$ is uniquely maximized over H at (P^o, A^o), the above strategies also support an undominated separating equilibrium. *Q.E.D.*

Proof of Theorem 4. Suppose that $(\hat{P}, \hat{A})$ with $(\hat{P}, \hat{A}) \not\in \Psi(I\!R)$ gives pooling equilibrium strategies. Since

$$\Pi^H(P^H, A^H) > \Pi^H(\hat{P}, \hat{A}) \geqslant \bar{\Pi}^H,$$

we can argue as above to establish the existence of $\hat{C} < C^H$ such that

$$(\hat{P} - C^H)X(\hat{P}, \hat{A}) - \hat{A} = (P(\hat{C}) - C^H)X(P(\hat{C}), A(\hat{C})) - A(\hat{C}) \tag{A4}$$

$$(P(\hat{C}) - \hat{C})X(P(\hat{C}), A(\hat{C})) - A(\hat{C}) > (\hat{P} - \hat{C})X(\hat{P}, \hat{A}) - \hat{A}.$$

Adding these inequalities gives

$$(C^H - \hat{C})(X(P(\hat{C}), A(\hat{C})) - X(\hat{P}, \hat{A})) > 0,$$

so that $X(P(\hat{C}), A(\hat{C})) > X(\hat{P}, \hat{A})$. Using (A4), we obtain

$$(\hat{P} - C^L)X(\hat{P}, \hat{A}) - \hat{A} < (P(\hat{C}) - C^L)X(P(\hat{C}), A(\hat{C})) - A(\hat{C}).$$

Thus, since $\Pi^i(P, A)$ is continuous and $\Pi^H(P, A)$ has strict monotonicity properties on $\Psi(C)$, there exists $(\tilde{P}, \tilde{A})$ near $(P(\hat{C}), A(\hat{C}))$ such that $\Pi^H(\hat{P}, \hat{A}) > \Pi^H(\tilde{P}, \tilde{A})$ and $\Pi^L(\hat{P}, \hat{A}) < \Pi^L(\tilde{P}, \tilde{A})$. Intuitive beliefs then imply that $\hat{\rho}(\tilde{P}, \tilde{A}) = 0$, and Condition 1 is violated for $i = L$.

Now suppose $(\hat{P}, \hat{A}) = (P(\hat{C}), A(\hat{C}))$ for $\hat{C} > C^L$. Under our assumptions it is easy to establish the existence of $C' < C^L$ such that

$$(\hat{P} - C^L)X(\hat{P}, \hat{A}) - \hat{A} = (P(C') - C^L)X(P(C'), A(C')) - A(C') \tag{A5}$$

$$(P(C') - C')X(P(C'), A(C')) - A(C') > (\hat{P} - C')X(\hat{P}, \hat{A}) - \hat{A}.$$

Adding these inequalities gives

$$(C^L - C')(X(P(C'), A(C')) - X(\hat{P}, \hat{A})) > 0,$$

or $X(P(C'), A(C')) > X(\hat{P}, \hat{A})$. From (A5) we have

$$(\hat{P} - C^H)X(\hat{P}, \hat{A}) - \hat{A} > (P(C') - C^H)X(P(C'), A(C')) - A(C').$$

Thus, for some $(\tilde{P}, \tilde{A})$ near $(P(C'), A(C'))$, $\Pi^H(\hat{P}, \hat{A}) > \Pi^H(\tilde{P}, \tilde{A})$ and $\Pi^L(\hat{P}, \hat{A}) < \Pi^L(\tilde{P}, \tilde{A})$. $(\hat{P}, \hat{A})$ cannot then give intuitive pooling equilibrium strategies. Finally, $(P(C), A(C))$ is dominated for C^H when $C < C^o$. *Q.E.D.*

Proof of Theorem 5. Pick any $\hat{C} \in [C^o, C^L]$. Since $\Pi(\psi(C)|C^H)$ is strictly increasing in C for $C < C^H$ and since $\Pi^H(P(C^o), A(C^o)) = \bar{\Pi}^H$, we have that $\Pi^H(P(\hat{C}), A(\hat{C})) + \delta\Pi^H(P^H, A^H) \geqslant \Pi^H(P^H, A^H) + \delta\Pi^H_D$. Furthermore, from the proof of Theorem 3 we have that $(P(C^o), A(C^o)) \in L$. Thus, since $\Pi(\psi(C)|C^L)$ is strictly increasing in C for $C < C^L$, it follows that $\Pi^L(P(\hat{C}), A(\hat{C})) + \delta\Pi^L(P^L, A^L) \geqslant \Pi^L(P^L, A^L) + \delta\Pi^L_D$. A pooling equilibrium therefore exists in which $(\hat{P}^H, \hat{A}^H) = (\hat{P}^L, \hat{A}^L) = (P(\hat{C}), A(\hat{C}))$, $\hat{\rho}(\hat{P}^H, \hat{A}^H) = \rho$, and $\hat{\rho}(P, A) = 1$ for all $(P, A) \neq (\hat{P}^H, \hat{A}^H)$.

To establish that the equilibrium is intuitive, assume to the contrary that there exists $(\tilde{P}, \tilde{A}) \neq (P(\hat{C}), A(\hat{C}))$ such that

$$(\tilde{P} - C^L)X(\tilde{P}, \tilde{A}) - \tilde{A} - (P(\hat{C}) - C^L)X(P(\hat{C}), A(\hat{C})) + A(\hat{C}) > 0 \tag{A6}$$

$$(P(\hat{C}) - C^H)X(P(\hat{C}), A(\hat{C})) - A(\hat{C}) - (\tilde{P} - C^H)X(\tilde{P}, \tilde{A}) + \tilde{A} > 0.$$

Adding inequalities gives

$$(C^H - C^L)[X(\tilde{P}, \tilde{A}) - X(P(\hat{C}), A(\hat{C}))] > 0,$$

whence $X(\tilde{P}, \tilde{A}) > X(P(\hat{C}), A(\hat{C}))$. But $\hat{C} \leqslant C^L$ and (A6) give

$$(\tilde{P} - \hat{C})X(\tilde{P}, \tilde{A}) - \tilde{A} - (P(\hat{C}) - \hat{C})X(P(\hat{C}), A(\hat{C})) + A(\hat{C}) > 0,$$

which is a contradiction. *Q.E.D.*

References

BAGWELL, K. "Introductory Price as a Signal of Cost in a Model of Repeat Business." *Review of Economic Studies*, Vol. 54 (1987), pp. 365–384.

——— AND RAMEY, G. "Pricing and Advertising When Entry Is Certain." Unpublished Manuscript, Northwestern University, 1987.

BAIN, J. *Barriers to New Competition.* Cambridge: Harvard University Press, 1956.

BUTTERS, G. "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies,* Vol. 44 (1977), pp. 465–491.

CHO, I. "Equilibrium Analysis of Entry Deterrence: A Reexamination." Unpublished Manuscript, University of Chicago, 1987.

——— AND KREPS, D. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics,* Vol. 102 (1987), pp. 179–221.

COMANOR, W. AND WILSON, T. "Advertising Market Structure and Performance." *Review of Economics and Statistics,* Vol. 49 (1967), pp. 423–440.

DIXIT, A. AND NORMAN, V. "Advertising and Welfare." *Bell Journal of Economics,* Vol. 9 (1978), pp. 1–18.

GROSSMAN, G. AND SHAPIRO, C. "Informative Advertising with Differentiated Products." *Review of Economic Studies,* Vol. 51 (1984), pp. 33–52.

KIHLSTROM, R.E. AND RIORDAN, M.H. "Advertising as a Signal." *Journal of Political Economy,* Vol. 92 (1984), pp. 427–450.

KREPS, D. AND RAMEY, G. "Structural Consistency, Consistency, and Sequential Rationality." *Econometrica,* Vol. 55 (1987), pp. 1331–1348.

——— AND WILSON, R. "Sequential Equilibria." *Econometrica,* Vol. 50 (1982), pp. 863–894.

MILGROM, P. AND ROBERTS, J. "Limit Pricing and Entry under Incomplete Information: A General Equilibrium Analysis." *Econometrica,* Vol. 50 (1982), pp. 443–459.

——— AND ———. "Price and Advertising Signals of Product Quality." *Journal of Political Economy,* Vol. 94 (1986), pp. 796–821.

MOULIN, H. *Game Theory for Social Sciences.* New York: New York University Press, 1981.

NELSON, P. "Information and Consumer Behavior." *Journal of Political Economy,* Vol. 78 (1970), pp. 311–329.

———. "Advertising as Information." *Journal of Political Economy,* Vol. 81 (1974), pp. 729–754.

RAMEY, G. "Limit Pricing Equilibria." Unpublished Manuscript, Stanford University, 1985.

SALOP, S. "Strategic Entry Deterrence." *American Economic Review,* Vol. 69 (1979), pp. 335–338.

SCHMALENSEE, R. "Advertising and Entry Deterrence: An Exploratory Model." *Journal of Political Economy,* Vol. 91 (1983), pp. 636–653.

———. "Advertising and Market Structure" in J. Stiglitz and G. Mathewson, eds., *New Developments in the Analysis of Market Structure,* Cambridge: MIT Press, 1986, pp. 373–396.