Is Everything Neutral?

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In his well-known analysis of the national debt, Robert Barro introduced the notion of a "dynastic family." This notion has since become a standard research tool, particularly in the areas of public finance and macroeconomics. In this paper, we critique the assumptions on which the dynastic model is predicated and argue that this framework is not a suitable abstraction in contexts in which the objective is to analyze the effects of public policies. We reach this conclusion by formally considering a world in which each generation consists of a large number of distinct individuals as opposed to one representative individual. We point out that family linkages form complex networks, in which each individual may belong to many dynastic groupings. The resulting proliferation of linkages between families gives rise to a host of neutrality results, including the irrelevance of all public redistributions, distortionary taxes, and prices. Since these results are not at all descriptive of the real world, we conclude that, in some fundamental sense, the world is not even approximately dynastic. These observations call into question all policy-related results based on the dynastic framework, including the Ricardian equivalence hypothesis.

I. Introduction

Over the last decade, there has been a growing awareness that many important public policy issues turn critically on the assumed nature of

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economic relationships within the family. This awareness is largely attributable to seminal papers by Barro (1974) and Becker (1974). Barro's paper ostensibly concerns the national debt, but its implications are much more far-reaching. Specifically, Barro supplemented the traditional overlapping generations model with intergenerational altruism and argued, in essence, that voluntary transfers between parents and children cause the representative "dynastic" family to behave as though it is a single, infinite-lived individual. Policies that fail to affect the family's real opportunities are neutralized through private actions; thus Ricardian equivalence and related propositions (concerning the irrelevance of government debt and social security) follow directly.[1] The dynastic family model has since become a standard research tool, particularly in the areas of public finance and macroeconomics (see, e.g., Chamley 1981; Abel 1984; Judd 1985).[2]

In this paper, we critique the dynastic family as a modeling tool and argue that it is not a suitable abstraction in contexts in which the objective is to analyze the effects of various public policies. Our criticism differs fundamentally from those offered by previous commentators (see, e.g., Feldstein 1976; Tobin and Buiter 1980) in that we assail neither the logic nor the assumptions employed by studies that invoke the dynastic framework. Rather, we take these at face value and show that they lead to untenable conclusions.

We reach these conclusions by formally considering a world in which each generation consists of a large number of distinct individuals as opposed to one "representative" individual. While the notion of a representative consumer is always somewhat objectionable, here it is especially pernicious in that it obscures considerations arising from the biological structure of families. For the human species, propagation requires the participation of two traditionally unrelated individuals. Thus family linkages form complex networks in which

[1] While Barro's paper has been a centerpiece of the debate concerning the neutrality of national deficits, the earliest modern reference to the Ricardian proposition seems to be Bailey (1962).

[2] No doubt its popularity in part reflects considerations of analytic convenience. Overlapping-generations models not only are generally less tractable but also often give rise to equilibria with undesirable properties. Specifically, equilibria in overlapping-generations models may fail to be either efficient or locally unique (see Balasko and Shell 1980; Kehoe and Levine 1985). Failure of local uniqueness is particularly troubling in any exercise involving comparative statics or dynamics. Thus Judd (1985) unabashedly attributes his adoption of the dynastic framework to analytic convenience. Unfortunately, this advantage may be illusory. In a recent paper, Gale (1985) has pointed out that, while Barro's dynastic solution is an equilibrium for the model he considers, this model also generally gives rise to a continuum of subgame-perfect intergenerational equilibria (see Selten 1965, 1975), many of which are inefficient. By adopting dynastic assumptions, one therefore does not necessarily succeed in avoiding the problems that arise in the standard overlapping-generations framework.

each individual may belong to many dynastic groupings. In this paper, we argue that the resulting proliferation of linkages *between* families gives rise to incomparably stronger neutrality properties under *weaker* conditions than those imposed by Barro. In particular, no government transfer (including those between unrelated members of the same generation) has any real effect, and all tax instruments (including so-called distortionary taxes) are equivalent to lump-sum taxes. In essence, the government can affect the allocation of real resources only by altering real expenditures. The efficiency role of government is thus severely limited, and the distributional role is entirely eliminated. More generally, we argue that if all linkages between parents and children are truly operative, then market prices play no role in the resource allocation process: the distribution of all goods is determined by the nature of intergenerational altruism.

If taken literally, these results would have profound implications for the study of economics. We hardly intend to suggest that such extreme conclusions are warranted. Rather, when results stretch the bounds of credibility past the breaking point, it is natural to question the validity of underlying assumptions. We must therefore emphasize that we have obtained these results under relatively weak conditions and that these same conditions are the fundamental building blocks of the dynastic model. Thus refusal to accept the practical implications of our results is tantamount to a rejection of the dynastic framework and calls into serious question the results (such as Ricardian equivalence) that follow from it. Further analyses of economic policies, such as the effects of government debt, require us to specify quite explicitly which of the underlying assumptions fails and how it fails.

The paper is organized as follows. In Section II, we consider some simple examples that illustrate the principles driving our neutrality results. Section III presents the general model. In Section IV, we discuss the notion of operative private transfers and argue that parent-child linkages are alone sufficient to interconnect virtually the entire population. Section V contains the central neutrality theorem. We describe various extensions and qualifications in Section VI. Finally, in Section VII, we clarify the nature of our results, reexamine the central assumptions, and consider various interpretations.

II. Examples

The linear structure of "dynastic" families in Barro (1974) allows him to model a family as, essentially, a single, infinite-lived consumer with dynamically consistent preferences. In particular, he specifies the well-being of generation t (u_t) as a function of t's own consumption and the utility of t's immediate successor: $u_t = v_t(c_t, u_{t+1})$. Popular intuitive explanations of Ricardian equivalence are closely tied to this formulation: since the dynastic family chooses an optimal program, it will simply offset any exogenous, intertemporal redistributions of resources that might displace it from the optimum.[3]

In practice, families are not independent linear units but rather complex, interlocking networks, in which unrelated individuals share common descendants. As a result, it is in general impossible to represent any particular family (or set of families) as a single, utility-maximizing agent, even when the well-being of each individual is assumed to depend only on his own consumption and the well-being of his children, as above.[4] It is perhaps somewhat surprising that this observation does not invalidate Ricardian equivalence. As we shall see, neutralization of public transfers depends only on the existence of operative, altruistically motivated private transfers, and not on any particular pattern of linkages or specification of preferences. Yet therein lies the difficulty. For if, as we argue in Section IV, virtually all the population is interconnected through chains involving parent-child linkages, then Ricardian equivalence is merely one manifestation of a much more powerful and implausible neutrality theorem.

We begin our analysis with two simple examples that serve to illustrate basic concepts.

Example 1

Suppose that there are three individuals, 1, 2, and 3. Individuals 1 and 2 have quasi-concave preferences of the form $u_i(c_i, c_3)$, $i = 1, 2$, while 3's preferences are simply $u_3(c_3)$. We may think of 3 as a common descendant of 1 and 2, who are unrelated. Each consumer i is endowed with wealth w_i. Individuals 1 and 2 divide this wealth between own consumption c_i and a nonnegative transfer to 3, b_i, $i = 1, 2$. Individual 3 consumes $w_3 + b_1 + b_2$.

There are, of course, a variety of ways in which 1 and 2 might determine the magnitude of their transfers to 3. For the purpose of

[3] Barro (1974, p. 1097) explains that "current generations act effectively as though they were infinite-lived when they are connected to future generations by a chain of operative intergenerational transfers." Subsequent papers reinforced the notion that Ricardian equivalence is somehow tied to the dynamically consistent formulation of family preferences; see, e.g., Buiter and Carmichael's (1984) dispute with Burbidge (1983, 1984).

[4] Equilibria are quite generally inefficient in models with interlocking families. One important reason has been emphasized by Nerlove, Razin, and Sadka (1984): when unrelated individuals share a common descendant, the consumption of that descendant is a public good.

this illustration, we will assume that the exogenous environment is such that 1 and 2 must make simultaneous, noncooperative choices. Accordingly, it is perhaps most natural to consider Nash equilibria in transfers to 3. Suppose that there exists a Nash equilibrium in which 1 and 2 both make positive transfers. The reader may easily verify through a direct argument that private transfers will then neutralize the effects of all sufficiently small lump-sum redistributional policies,[5] despite the fact that this extended family does *not* act as a single, utility-maximizing individual.[6] Throughout this paper, we use a more powerful but less direct line of argument, which works as follows.

We have described an environment in which two agents, 1 and 2, play a simple game. Each agent chooses an action (transfer) b_i subject to the constraint $b_i \geqq 0$ and receives a payoff of $u_i(w_i - b_i, w_3 + b_i + b_j)$. By transferring z from 1 to 2, the government alters this game as follows. Each agent still chooses an action b_i subject to the constraint $b_i \geqq 0$ but receives different payoffs: $u_1(w_1 - z - b_1, w_3 + b_1 + b_2)$ and $u_2(w_2 + z - b_2, w_3 + b_1 + b_2)$.

Now we introduce the following change of variables: $\beta_1 = b_1 + z$ and $\beta_2 = b_2 - z$. That is, we think of agent 1 (2) as choosing β_1 (β_2) subject to the constraint $\beta_1 \geqq z$ ($\beta_2 \geqq -z$) and receiving payoffs $u_i(w_i - \beta_i, w_3 + \beta_i + \beta_j)$. Note that this differs from the original game in only two respects. First, the same abstract action has a different practical interpretation in each case. For example, we associate the choice "$b_1 = 5$" with the interpretative label "1 transfers \$5 to 3," while we associate "$\beta_1 = 5$" with the label "1 transfers \$5 $- z$ to 3." Since all standard solution concepts have the property that "strategically equivalent"[7] games give rise to equivalent sets of equilibria, changing the interpretations of abstract actions is inconsequential.[8]

[5] In fact, one can think of agent 3 as a public project financed by voluntary contributions, in which case the analysis of Bergstrom, Blume, and Varian (1984) establishes the neutrality result. We are aware that Laurence Kotlikoff also derived this result independently. These authors did not, however, note the strategic equivalence of the pre- and posttransfer games (which makes the result substantially more general), nor did they discover the neutrality of so-called distortionary taxes (and the implications, discussed in Sec. VI, for the role of prices in resource allocation). In addition, the framework employed by Bergstrom et al. is substantially more restrictive than the general model considered in Sec. III (their neutrality result was established for networks of interpersonal linkages with a very specific structure). We also note that Carmichael (1982) had previously recognized that the key to Barro's theorem concerns the preservation of opportunity sets rather than the specification of altruism.

[6] In this example, equilibria are generally inefficient because of the public good problem noted in n. 4.

[7] Two games are strategically equivalent if they have the same extensive forms.

[8] Our central result depends critically on the assumption that strategically equivalent games yield equivalent equilibria. Essentially, this implies that we can think entirely in terms of abstract actions, ignoring the primitive actions to which these actually correspond. Yet in many situations, primitive actions may play a role in determining behav-

Second, agents' opportunity sets differ between the two games. Since this difference is potentially substantive, we conclude that private actions neutralize the effects of government transfer policies as long as the original equilibrium is insensitive to perturbations in the agents' constraints. This simple condition is, in principle, easily verifiable. Suppose, for example, that we have an initial Nash equilibrium with $\hat{b}_i > 0$, $i = 1, 2$. Under the assumption that utility is quasi-concave, equilibrium behavior is insensitive to small perturbations of the constraints, so neutrality follows as an immediate corollary.[9]

This alternative line of reasoning also allows us to conclude that neutrality will hold for a wide range of solution concepts.[10] Indeed, it is natural to expect that interior equilibria will typically satisfy the basic condition unless the corner constraints play a special role in defining the relevant solution concept. Suppose, for example, that 1 and 2 can form binding contracts so that transfers are determined through bargaining. If the relevant threat point is $b_1 = b_2 = 0$, then the Nash bargaining solution will *not* give rise to neutralizing behavior. However, if a breakdown in negotiations is followed by noncooperative behavior (so that a noncooperative Nash equilibrium prevails) and if this threat point entails positive transfers, then the basic condition will generally hold, and private actions will neutralize small transfer policies.

While it would be interesting to identify more primitive exogenous conditions under which linkages are operative (in the sense that transfers are positive and equilibria are robust with respect to perturbations of corner constraints), this is not our current objective. Instead, we critique the dynastic model by taking its central premises at face value, thereby implicitly restricting attention to the set of environments that give rise to operative linkages.

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ior. The most obvious role arises when the game yields multiple equilibria: players may gravitate toward equilibria in which their choices are close to certain focal alternatives (e.g., zero transfers or transfers prior to the policy perturbation). It is, however, difficult to see how this possibility could invalidate our result without simultaneously rendering the dynastic framework inapplicable.

[9] Quasi concavity does not play a significant role in establishing this result. As long as u_i is continuous and i strictly prefers $\hat{b}_i$ to zero, the basic robustness condition is satisfied. Since indifference between $\hat{b}_i$ and zero is an extremely unlikely outcome (formally, one can show that it is a measure zero event in the space of potential preferences), the existence of an equilibrium with positive transfers is generally sufficient to guarantee that private actions will neutralize sufficiently small government transfer policies.

[10] Consider, e.g., refinements of the Nash equilibrium concept. As long as an interior Nash equilibrium strictly satisfies (violates) certain refined criteria, small perturbations of the corner constraints will not generally cause it to violate (satisfy) these criteria. Indeed, the purpose of many refinements is to rule out equilibria that are sensitive to perturbations either in the environment or in the rules governing behavior (see Fudenberg, Kreps, and Levine 1986).

Example 2

Suppose that everything is as in example 1, except that individual 1 chooses labor supply (l_1) prior to the choices of consumption and transfers. Further assume that his utility is given by $u_1(c_1, c_3, l_1)$ and that his wealth is $w_1 = w_1^0 + wl_1 - z(l_1)$, where w_1^0 is nonlabor income, w is the wage rate, and z is a tax schedule, used for redistributing wealth from 1 to 2. Thus 2's wealth is $w_2 = w_2^0 + z(l_1)$.

The extensive form of this game is represented schematically in figure 1*a*. First, 1 chooses his labor supply; then 1 and 2 play a "simultaneous move-transfer game," as in example 1. Thus if 1 chooses labor supply l_1^1, 1 and 2 play a transfer game in which their endowments are $w_1^0 + wl_1^1 - z(l_1^1)$ and $w_2^0 + z(l_1^1)$, respectively (this game is denoted G_1). Similarly, if 1 chooses labor supply l_1^2, 1 and 2 play the corresponding transfer game G_2.

Suppose that the government contemplates an arbitrary change in the tax-transfer schedule from z to z'. At first, this may appear to alter

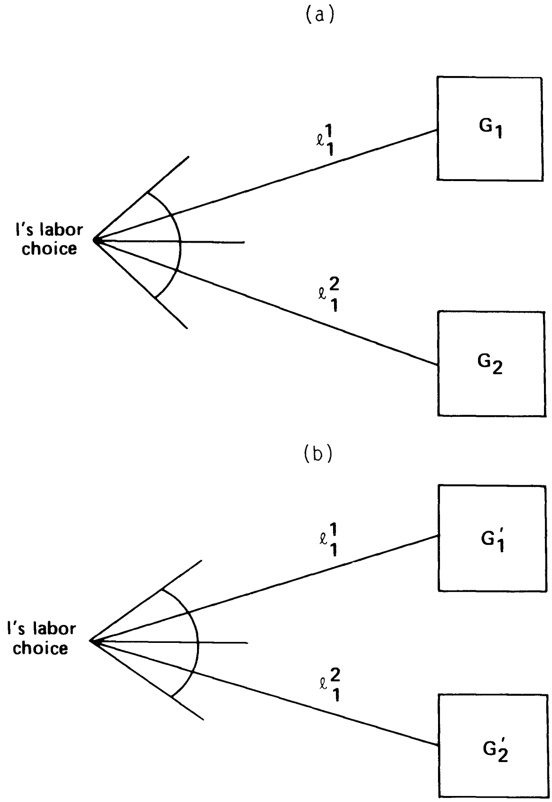

FIG. 1.—Schematic representation of example 2. *a*, Policy z. *b*, Policy z'

the game in a fundamental way. For instance, when 1 chooses l_1^1, this induces a simultaneous move-transfer game between 1 and 2, where endowments are now $w_1^0 + wl_1^1 - z'(l_1^1)$ and $w_2^0 + z'(l_1^1)$, respectively (this game is denoted G_1' in fig. 1*b*). Yet the total resources of 1 and 2 are identical in G_1 and G_1'. Thus by the argument given in example 1, if we appropriately perturb the corner constraints in G_1, we obtain a game that is strategically equivalent to G_1'. Clearly, this same reasoning applies regardless of 1's labor supply decision (for instance, an appropriate perturbation in the corner constraints of G_2 yields a game that is strategically equivalent to G_2'). Consequently, the only substantive difference between these two environments consists of perturbations in corner constraints. If the original equilibrium (or set of equilibria) is robust with respect to such perturbations, then private actions will offset sufficiently small policy changes.

Again, we would like to identify circumstances under which this robustness condition is likely to hold. For purposes of illustration, we consider "subgame-perfect" Nash equilibria (see Selten 1965, 1975).[11] This solution concept demands that agents act in their own best interests at all times and serves to rule out threats that are not credible, in the sense that agents would not be willing to carry them out. Formally, a Nash equilibrium is subgame perfect if strategies form Nash equilibria in every proper subgame. In the current context, this implies that every choice of l_1 must be followed by Nash behavior in the ensuing simultaneous move-transfer game. Our discussion in example 1 establishes that, as long as the initial equilibrium entails positive transfers in some particular subgame, perturbations of the corner constraints will not generally alter behavior in that subgame. If this condition holds for every subgame, then the original set of perfect equilibria will indeed be robust with respect to arbitrary perturbations of the corner constraints.[12]

[11] It is interesting to contrast these results with those that follow from use of the unrefined Nash concept. There is typically a continuum of Nash equilibria for the game described here, which we construct as follows. If 1 selects some l_1^0, 1 and 2 play Nash choices in the ensuing subgame. For any other choice of l_1, 2 subsequently plays $b_2 = 0$ (1's choices in these subgames are irrelevant). Effectively, 2 induces 1 to choose l_1^0 by threatening to hurt 3 unless 1 complies (of course, if 2 must hurt himself to carry out this threat, 1 might choose to call 2's "bluff"; accordingly, these equilibria are not subgame perfect). Clearly, 2's ability to punish 1 through 3 determines the set of labor supply choices that are sustainable in some equilibrium. Any transfer from 1 to 2 strengthens this ability and therefore expands the range of potential outcomes. There is then no guarantee that private actions neutralize redistributional policies; indeed, if 2 can select a threat, redistributions ordinarily have real effects. It is, however, important to reiterate that in such circumstances the central properties of dynastic behavior are also lost (see Sec. IV).

[12] The assumption that transfers are positive for every choice of l_1 is, of course, quite strong. However, the basic result typically holds as long as transfers are positive following any choice in some neighborhood of the original equilibrium selection, $\hat{l}_1$. By the

The irrelevance of an apparently distortionary tax may, at first, seem counterintuitive. Indeed, readers who are unfamiliar with abstract game-theoretic arguments may wish to verify this result directly through standard comparative statics.[13] The imposition of a tax schedule certainly appears to change the relative price of 1's leisure; should this not affect his decision? The fundamental insight here is that the price of 1's leisure is not simply w since he must also consider the effect of his labor-leisure choice on 2's transfer to 3. Thus he faces some "shadow" wage, and it is this shadow wage that is invariant with respect to tax policy.

This invariance is easiest to understand in a single-consumer world. As long as the government must balance its budget, no tax can distort behavior since the individual knows that all revenues must be returned to him at some point. By way of contrast, in a representative consumer world each consumer is thought of as small relative to the economy so that the fraction of marginal revenues distributed to any one consumer is negligible and (in the absence of altruistic linkages) can be ignored. The point of our analysis is that, as long as consumers are linked through operative transfers, all marginal revenues associated with the taxation of a particular individual are, regardless of population size, eventually returned to that same individual, just as in a single-consumer world.

III. The Model

We consider a discrete-time ($t = 1, 2, \ldots$), infinite-horizon overlapping-generations model. For simplicity, we assume that there is one composite good that can be either consumed or invested. Current output is determined through a constant-returns-to-scale production technology as a function of current labor inputs and investment from the previous period. Markets are perfectly competitive; firms earn zero profits and pay factor inputs their marginal products. Labor and capital are each homogeneous so that in period t all labor receives a wage rate of w_t and capital yields a gross return of α_t^*, paid in period $t + 1$ (the net rate of return is equal to $\alpha_t^* - 1$).

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preceding argument, policy changes will not alter the consequences of 1's choices in this neighborhood; hence, $\hat{l}_1$ remains a local optimum. Furthermore, if 1 strictly prefers $\hat{l}_1$ to alternatives outside of this neighborhood and if equilibrium outcomes following such alternatives vary continuously with the values of corner constraints, then $\hat{l}_1$ remains a global optimum for sufficiently small perturbations. This last condition (continuity) is relatively weak and, e.g., follows from quasi concavity of u_i.

[13] Details of such calculations are available from the authors on request. The reader may also wish to consult Bernheim (1986).

Let I_t be the set of individuals born in period t. We suppose that every individual lives for $M + 1$ periods. Thus I^t, the set of individuals living at time t, is given by

$$I^t = \bigcup_{k=0}^{M} I_{t-k}.$$

We will use N_t and N^t to denote the number of individuals in I_t and I^t, respectively.

At time t, each individual $i \in I^t$ chooses consumption (c_i^t), labor supply (l_i^t), transfers to other living individuals $(b_{ij}^t, j \in I^t - \{i\})$,[14] purchases of physical capital (s_i^t), and purchases of short-term (single-period) bonds (d_i^t). Let α_t denote the gross rate of return on period t bonds ($\alpha_t - 1$ is the interest rate). It is convenient to define

$$\alpha^t = \prod_{\tau=1}^{t-1} \alpha_t.$$

Throughout our analysis, we employ the following notation. Let $\mathbf{b}_i^t$ denote the vector of i's transfers in period t:

$$\mathbf{b}_i^t \equiv (b_{ij}^t)_{j \in I^t - \{i\}}.$$

We will use $\mathbf{B}^t$ to indicate the sequence of all transfer choices up to period $t - 1$:

$$\mathbf{B}^t \equiv \left((\mathbf{b}_i^1)_{i \in I^1}, \ldots, (\mathbf{b}_i^{t-1})_{i \in I^{t-1}}\right)$$

(similarly for $\mathbf{C}^t$, $\mathbf{L}^t$, $\mathbf{S}^t$, and $\mathbf{D}^t$). We define a "t-history" of the economy as a complete record of all choices made through the end of period $t - 1$: $\mathbf{H}^t = (\mathbf{C}^t, \mathbf{L}^t, \mathbf{B}^t, \mathbf{S}^t, \mathbf{D}^t)$.

The government participates in this economy by financing a stream of real expenditures through tax levies and bond sales. Throughout, we assume that the stream of real expenditures (g_t, $t \geqq 1$) is fixed and focus on the effects of alternative financial policies. We allow the government to specify period t taxes on individual i as an arbitrary function, $z_i^t(\mathbf{H}^t)$, of the observed t-history.[15] Note that this very general specification subsumes taxes on labor income, capital income, and transfers. It also allows for idiosyncratic provisions, such as income

[14] One could also allow consumers to lock in transfers for a number of years, including transfers to unborn generations. This would change nothing of substance.

[15] Note that we do not allow z_i^t to depend on *current* (period t) choices. Effectively, the government collects tax revenues at the "end" of each time period (in the last period of life, revenues must be collected from the individual's estate or, equivalently, from his heirs). This aspect of our model is an artifact of discrete time. We model the government policy in this way so that private and public transfers are on an equal footing (within a single period, both may be conditioned on the same behavior).

averaging. The government may also condition one individual's taxes on another's actions. In short, virtually any action may be taxed, and the corresponding rate schedule may be chosen without restriction.

Given a tax policy and real expenditure stream, the government balances its budget by issuing debt. Specifically, the supply of government bonds evolves as follows:

$$d^t(\mathbf{H}^t) + \sum_{i \in I^t} z_i^t(\mathbf{H}^t) = \alpha_{t-1} d^{t-1}(\mathbf{H}^{t-1}) + g_t.$$

For arbitrary taxes and expenditures, the implied deficit profile may, of course, be infeasible (i.e., debt might eventually exceed economic resources). We implicitly exclude such policies from consideration.

Prevailing prices and government financial policy determine the opportunity constraint of each consumer. Specifically, for each $i \in I^t$,

$$c_i^t + s_i^t + d_i^t + \sum_{j \in I^t - \{i\}} b_{ij}^t + z_i^t(\mathbf{H}^t)$$
$$= w_t l_i^t + \alpha_{t-1}^* s_i^{t-1} + \alpha_{t-1} d_i^{t-1} + \sum_{j \in I^t - \{i\}} b_{ji}^t.$$

In addition, i confronts a number of feasibility constraints: $0 \leqq l_i^t$, $0 \leqq s_i^t$, and $\underline{b}_{ij}^t(\mathbf{H}^t) \leqq b_{ij}^t$.

Note that we do not impose any constraints on i's purchases of bonds. In particular, it is possible to have $d_i^t < 0$, which signifies that i borrows at the competitive rate, α_t.[16] Thus we have implicitly assumed that capital markets are perfect.[17]

Note also that we allow the lower bounds on i's period t transfers, $\underline{b}_{ij}^t(\mathbf{H}^t)$, to depend on the evolution of prior decisions, $\mathbf{H}^t$. Ordinarily, we would expect these lower bounds to be invariant with respect to $\mathbf{H}^t$ (generally, they equal zero). The more general formulation adopted here allows us to contemplate a wider class of perturbations to the corner constraints and, correspondingly, a wider class of alternative financial policies.

We complete the model by assuming that it is possible to represent the preferences of each consumer i by a utility function defined over choices of consumption and labor: $u_i(\mathbf{C}^\infty, \mathbf{L}^\infty)$. Since we allow for dependence on the entire history of choices, this specification is ex-

[16] We will, however, impose the restriction that $d_i^t \geqq 0$ in the final period of life so that i's ability to bequeath debts is determined solely by the lower bounds on b_{ij}^t.

[17] It is straightfoward to relax this assumption by artificially limiting borrowing; as long as liquidity constraints are generally nonbinding, our central results continue to hold. Since the absence of binding liquidity constraints is fundamental to the dynastic formulation, it is appropriate for us to focus our attention on this case.

tremely general.[18] By imposing additional restrictions, one can obtain various formulations employed by other authors, such as Barro's (1974) dynastic specification. We do, however, explicitly rule out direct dependence of preferences on the levels of transfers. This restriction is, of course, essential.

As in the preceding section, our central result will depend on a hypothesis about the sensitivity of equilibria to perturbations in the corner constraint. Since we generally expect interior equilibria to satisfy this hypothesis under a wide range of solution concepts and since the hypothesis is in principle verifiable in any particular context, we wish to avoid tying our analysis directly to a particular notion of equilibrium. Thus we will describe behavior within this economy in as general terms as possible. We assume that consumers take the wage rates and interest rates as fixed. A given profile of factor prices induces a game in which in each period t consumers choose consumptions, labor supplies, transfers, purchases of capital, and purchases of bonds. Each distinct t-history $\mathbf{H}^t$ identifies a distinct subgame originating in period t. Players may condition their period t choices on the actual t-history that has resulted from previous play. Thus strategies consist of functions mapping t-histories to current choices.

Individuals are, of course, constrained to select strategies that satisfy their opportunity constraints in each period t for all feasible t-histories and concurrent choices made by their contemporaries. This requirement is more demanding than it might at first appear. In particular, individual i cannot simply specify c_i^t, l_i^t, $(b_{ij}^t)_{j\in I^t-\{i\}}$, s_i^t, and d_i^t as functions of $\mathbf{H}^t$ subject to budget balance since concurrent deviations by contemporaries (e.g., a change of b_{ji}^t for some j) might render these choices infeasible. Rather, he must allow one of these variables to be determined as a residual. As long as we confine our attention to pure strategy equilibria, i has no basis for preferring one residual variable to another (he always assumes that other players will select their equilibrium choices). For the purposes of our analysis, it is convenient to assume that consumption is always the residual variable.[19]

For any particular solution concept, there is a (potentially empty) set of equilibria among consumers for the game induced by each profile of possible factor prices and interest rates. We will say that a

[18] Note in particular that we do not require the utility of each individual to vary with the choices of all other individuals; indeed, u_i would ordinarily be insensitive to changes in most of its potential arguments.

[19] For this reason, we cannot (as a formal matter) constrain consumption to be nonnegative in all subgames. Under weak conditions it will, however, be positive on the equilibrium path. While selecting some other residual variable might well alter the specific set of equilibrium outcomes (since this changes the consequences of deviating from equilibrium actions for the deviating player), it would not affect our central arguments.

particular price profile is a "full equilibrium" if there exists an equilibrium among consumers relative to these prices such that, along the equilibrium path, (i) consumers demand in aggregate the amount of bonds supplied by the government and (ii) the aggregate demand for capital (labor) equals the aggregate supply of capital (labor). Condition ii holds as long as full employment of factor supplies generates marginal products equal to the assumed factor prices.

IV. Operative Links and the Linkage Hypothesis

Barro's (1974) formulation of the dynastic family employs both a restrictive specification of preferences and a restrictive notion of equilibrium (see n. 2). In addition, he assumes that successive generations are operatively linked, in the sense that parents make positive, discretionary transfers to their children. Within his framework, this condition is sufficient to guarantee that equilibrium behavior is insensitive to perturbations in the lower bounds that constrain specific transfer decisions.

In less restrictive models, perturbations of corner constraints could conceivably alter equilibrium behavior, even when the corresponding transfers are strictly positive.[20] This raises an important question: Do Barro's results require the irrelevance of such perturbations, or do they depend only on the weaker requirement that transfers are positive? In Bernheim and Bagwell (1986), we posed this question in a representative consumer model much like Barro's, except that we allowed for a larger class of preferences. Under the supposition that an arbitrarily small perturbation to the corner constraint corresponding to some parent-child linkage would have real effects, we demonstrated that one could design an arbitrarily small deficit policy that would also have real effects.[21] We conclude that the validity of the

[20] One might initially hope to rule out this possibility by imposing appropriate convexity conditions, as in Sec. II. Unfortunately, when the preferences of successive generations conflict, convexity of an individual's decision problem depends not only on the characteristics of utility functions and budget constraints but also on the properties of equilibrium strategies. In general, it is extremely difficult to guarantee that these strategies are well behaved (see, e.g., Kohlberg 1976; Bernheim and Ray 1983, 1987; Leininger 1986). Furthermore, in various kinds of noncooperative equilibria, one individual may condition his transfer on another's behavior to exert influence. If the first individual can credibly threaten to sever financial ties, then the location of his corner constraint will affect behavior even if one observes positive discretionary transfers in equilibrium (see Bernheim, Shleifer, and Summers 1985). It is very difficult to rule out this possibility by imposing restrictions either on the exogenous environment or on the notion of equilibrium.

[21] This result actually follows as a corollary of the arguments in Sec. V of this paper since there we establish a one-to-one relationship between perturbations to corner constraints and fiscal policies.

dynastic formulation depends on the irrelevance of perturbations to corner constraints; in general, the observation that transfers are positive does not by itself guarantee dynastic properties.[22]

It is therefore appropriate to formalize the notion of an operative link as follows. A "potential link" is a triplet, (i, j, t), such that $i \neq j$ and $i, j \in I^t$ (since i and j are both alive in period t, it is conceivable that i could transfer resources to j at that time). Let Λ be the set of all potential links. Consider some $\lambda \subset \Lambda$, and choose some vector of real numbers $\boldsymbol{\epsilon} = (\epsilon_{ij}^t)_{(i,j,t)\in\lambda}$. The vector of functions $(\underline{\beta}_{ij}^t)_{(i,j,t)\in\lambda}$ is an $\boldsymbol{\epsilon}$-perturbation of the original constraints $(\underline{b}_{ij}^t)_{(i,j,t)\in\lambda}$ if, for all $(i, j, t) \in \lambda$ and $\mathbf{H}^t$, $|\underline{\beta}_{ij}^t(\mathbf{H}^t)| < \epsilon_{ij}^t$. Subsequent to some perturbation, the constraints become

$$b_{ij}^t \geqq \underline{b}_{ij}^t(\mathbf{H}^t) + \underline{\beta}_{ij}^t(\mathbf{H}^t)$$

for all $\mathbf{H}^t$ and $(i, j, t) \in \lambda$. We will say that $\lambda \subset \Lambda$ is a set of "jointly operative links" if there exists some $\boldsymbol{\epsilon} > 0$ such that no $\boldsymbol{\epsilon}$-perturbation of $(\underline{b}_{ij}^t)_{(i,j,t)\in\lambda}$ alters the set of equilibria in the game induced by the prevailing profile of factor prices.

The dynastic model is founded on the assumption that all parent-child linkages are jointly operative. Rather than search for a set of exogenous conditions that guarantees this result, we simply take the assumption at face value and investigate its implications. As a first step, we argue that parent-child linkages alone should ordinarily suffice to interconnect the entire population.[23]

For illustrative purposes, suppose that all individuals marry, that each marriage produces two children, and that parents are operatively linked to their children. Then when redundancies are ignored,[24] every individual is indirectly linked through common descendants in the next G generations to

$$\phi(G) = \sum_{i=1}^{G-1} 2^{2i-1}$$

members of its own generation. If we assume (as seems natural) that spouses are operatively linked to each other, then one fewer genera-

[22] This conclusion has important empirical implications in that it becomes very difficult to establish whether links are, in fact, operative. Bernheim et al. (1985) have, e.g., presented evidence indicating that corner constraints often matter despite the fact that transfers are positive.

[23] In practice, people are connected through operative transfers not just to children but also to siblings, nieces, nephews, cousins, charities, political organizations, and so forth. Thus we strongly suspect that, even accounting for the childless, very few individuals or groups of individuals are truly isolated in the sense discussed here.

[24] For example, the possibility that siblings have different in-laws. For large populations and small values of G, this is presumably an excellent approximation.

tion is required to establish the same links. Accordingly, to appreciate the importance of links spanning two, three, and four generations, we note that for $G = 2$, 3, and 4, $\phi(G)$ is 2, 10, and 42, respectively.

This, however, is only the "tip of the iceberg." Once we have established that one household is connected to another of the same generation, we may extend the chain further by moving up and down the family tree as many times as desired. Thus operative linkages form complex networks, perhaps interconnecting large segments of the population. Indeed, if each couple is connected to 10 others (through grandchildren), then the probability of finding a "cycle" (a set of interconnected individuals who are isolated from the rest of the population) seems quite small.

Although we were unable to obtain any formal results along these lines, we did conduct a large number of Monte Carlo simulations. In each simulation, we fixed the number of households (N) in the initial generation and, under the assumption that each household produced two children, arranged marriages between these children. We then repeated this procedure for grandchildren.[25] We took all marriages to be equally likely[26] and, in particular, did not rule out marriages between siblings. Out of 100 simulations with $N = 20$, the population was completely interconnected in 96 cases. For $N = 50$, the figure was 100 out of 100, and for $N = 100$, it was 98 out of 100. We also conducted 20 simulations for $N = 1{,}000$ and found that the population was completely interconnected in every case. Furthermore, every instance of incomplete interconnection resulted from the existence of a single, completely incestuous family (i.e., siblings married siblings in two consecutive generations).[27]

[25] We also conducted a separate set of simulations in which we limited consideration to linkages between the first two generations (i.e., we omitted grandchildren). Although complete interconnection was relatively rare, a large number of people were nevertheless linked together. In 100 simulations for $N = 20$, the largest connected component averaged 61 percent of the population; for $N = 50$ and 100, the figure was 63 percent.

[26] In reality, not all links are equally probable. Individuals are most likely to marry others who live in the same geographic areas, and some communities, such as the Amish, are almost entirely self-contained. Note, however, that even with a near-perfect caste system, it takes only one "intermarriage" to link the entire population. In practice, marital links between identifiable population subgroups are probably quite common, particularly since many groups overlap. As a result, we suspect that our assumption is probably innocuous.

[27] In Bernheim and Bagwell (1986), we have also discussed some related results from the theory of random graphs. Unfortunately, this framework is not ideally suited to our current purposes in that it does not impose much of the structure implied by family relationships. Nevertheless, calculations based on known asymptotic distributions (see Erdos and Renyi 1959) corroborate our simulation results. In one respect, the random-graphs framework is superior to the stylized model of family relationships considered in this paper. Specifically, the number of edges terminating at each node is determined randomly. For purposes of interpretation, one can think of the resulting heterogeneity

Note that the complexity of family networks renders perhaps the majority of interpersonal linkages redundant: typically, two individuals will be connected through several distinct channels. Consequently, complete interconnection of the population ordinarily follows under weaker conditions than those imposed by Barro in that all parent-child linkages need not be operative.

On the basis of the preceding observations, we henceforth confine attention to equilibria satisfying a certain strong condition, which we designate the "linkage hypothesis."

LINKAGE HYPOTHESIS. There exists a set of jointly operative links λ and an integer T such that for each $t \geqq 1$ the following property holds. For all $i, j \in I^t$, there exists a finite integer p and sequences $(i_1, \ldots, i_p)$ and $(\tau_1, \ldots, \tau_{p-1})$ with $i = i_1$ and $j = i_p$ such that, for $k = 1, \ldots, p - 1$, $t \leqq \tau_k \leqq t + T$, and either $(i_k, i_{k+1}, \tau_k) \in \lambda$ or $(i_{k+1}, i_k, \tau_k) \in \lambda$.

Loosely, this hypothesis implies that, in each period t, one can find a chain of operative linkages connecting any two living individuals, with each link consisting of a transfer made sometime between periods t and $t + T$.

V. The Central Result

We now demonstrate that, when the linkage hypothesis is satisfied, sufficiently small but otherwise arbitrary perturbations of government fiscal policy are irrelevant. We define a policy perturbation as follows. Consider some vector of real numbers $\boldsymbol{\eta} = (\eta^1, \eta_0^1, \eta^2, \eta_0^2, \ldots)$; $\langle \delta^t, (\xi_i^t)_{i \in I^t} \rangle_{t=1}^{\infty}$ is an $\boldsymbol{\eta}$-perturbation of some initial policy $\langle d^t, (z_i^t)_{i \in I^t} \rangle_{t=1}^{\infty}$ if

$$|\delta^t(\mathbf{H}^t)| < \eta^t,$$

$$|\xi_i^t(\mathbf{H}^t)| < \eta_0^t,$$

and

$$\delta^t(\mathbf{H}^t) + \sum_{i \in I^t} \xi_i^t(\mathbf{H}^t) = \alpha_{t-1} \delta^{t-1}(\mathbf{H}^{t-1})$$

for all t, $i \in I^t$, and t-histories $\mathbf{H}^t$.[28] Subsequent to the perturbation, the

as reflecting differences in the number of children. Isolated nodes are then naturally interpreted as childless households. As demonstrated by Erdos and Renyi, asymptotically one obtains (with probability one) one completely interconnected set plus isolated nodes.

[28] Note that we have allowed the government to alter only its taxes and level of borrowing; it cannot change the gross rate of interest that it offers on government bonds (α_t). This involves no loss of generality since the government can effectively change this rate by taxing or subsidizing interest income.

government's deficit and tax policies are given by $d^t(\mathbf{H}^t) + \delta^t(\mathbf{H}^t)$ and $z_i^t(\mathbf{H}^t) + \xi_i^t(\mathbf{H}^t)$, respectively.

For the following result, we will assume that the government taxes neither interest nor transfers. That is, for all i, t, $\mathbf{H}^t$, and $\hat{\mathbf{H}}^t$ with $(\mathbf{C}^t, \mathbf{L}^t, \mathbf{S}^t) = (\hat{\mathbf{C}}^t, \hat{\mathbf{L}}^t, \hat{\mathbf{S}}^t)$, $z_i^t(\mathbf{H}^t) = z_i^t(\hat{\mathbf{H}}^t)$, $\xi_i^t(\mathbf{H}^t) = \xi_i^t(\hat{\mathbf{H}}^t)$, $d^t(\mathbf{H}^t) = d^t(\hat{\mathbf{H}}^t)$, and $\delta^t(\mathbf{H}^t) = \delta^t(\hat{\mathbf{H}}^t)$. The introduction of taxes on interest income and transfers substantially complicates the analysis. We defer consideration of such policies to Section VI.

PROPOSITION. Suppose that some full equilibrium satisfies the linkage hypothesis. For some $\eta > 0$ and every η-perturbation of the government's fiscal policy, there exists a full equilibrium in which factor prices, labor supplies, consumption decisions, and purchases of physical capital are all unaffected. In such an equilibrium, the policy perturbation simply induces offsetting private transfers and bond purchases.[29]

We establish this proposition by showing that, subsequent to the policy perturbation, there must exist an equilibrium for the game induced by the original profile of factor prices in which consumers select the same levels of consumption and factor supplies as in the original full equilibrium and in which the aggregate demand for government bonds changes to match supply in every period. The desired conclusion follows immediately.

We proceed by first considering three simple classes of policy perturbations, labeled A, B, and C. These provide the building blocks for analyzing more complex fiscal policies.

Class A

This class of perturbations consists of transfers between pairs of individuals who are directly linked. That is, we choose some $(i, j, t) \in \lambda$, select ξ_i^t arbitrarily, and set $\xi_j^t(\mathbf{H}^t) = -\xi_i^t(\mathbf{H}^t)$ for all $\mathbf{H}^t$. All other aspects of fiscal policy remain unchanged.

To demonstrate the irrelevance of this policy, we consider the following change of variables for i's transfer choice. For each history $\mathbf{H}^t$, let

[29] Technically, our result says nothing about the size of η; conceivably, only very small policy perturbations might be irrelevant. However, when one employs the dynastic framework, one assumes that its basic premise—including the assumption that particular links are operative—is robust with respect to an interesting range of environments (otherwise, the model would be inapplicable if the environment changed slightly). Fiscal policy is certainly one aspect of the environment. If the dynastic assumptions hold for a wide range of fiscal policies, then policy changes within this range are irrelevant.

$$\beta_{ij}^t(\mathbf{H}^t) = b_{ij}^t + \xi_i^t(\mathbf{H}^t)$$

(i.e., we adopt a different change of variables in each subgame). Clearly, the same numerical choices yield the same payoffs prior to the perturbation and in the transformed game after the perturbation. However, subsequent to the perturbation, the transfer constraint becomes

$$\beta_{ij}^t(\mathbf{H}^t) \geqq \underline{b}_{ij}^t(\mathbf{H}^t) + \xi_i^t(\mathbf{H}^t).$$

This game is therefore strategically equivalent to one in which the perturbation $\underline{\beta}_{ij}^t(\mathbf{H}^t) \equiv \xi_i^t(\mathbf{H}^t)$ is applied to $\underline{b}_{ij}^t$. Since i and j are operatively linked, this perturbation of the corner constraint will not affect behavior as long as $\underline{\boldsymbol{\beta}}_i^t$ is sufficiently small. But then the policy perturbation is also irrelevant in the sense that it alters only i's transfer to j.

Class B

In this class of policy perturbations, the government issues debt, distributes the proceeds to some individual i, and retires the debt in the subsequent period by taxing the same individual. That is, we choose $i \in I^t \cap I^{t+1}$, select ξ_i^t arbitrarily, and set $\delta^t(\mathbf{H}^t) = -\xi_i^t(\mathbf{H}^t)$ and $\xi_i^{t+1}(\mathbf{H}^{t+1}) = -\alpha_t\xi_i^t(\mathbf{H}^t)$ for all $\mathbf{H}^t$. All other aspects of fiscal policy remain unchanged.

To demonstrate the irrelevance of this policy, we consider the following change of variables for i's bond purchases. For each history $\mathbf{H}^t$, let

$$\sigma_i(\mathbf{H}^t) = d_i^t - \delta^t(\mathbf{H}^t).$$

Clearly, the same numerical choices yield the same payoffs prior to the perturbation and in the transformed game after the perturbation. Indeed, these two games are strategically equivalent since there are no constraints on borrowing or lending. Equilibrium therefore entails the same numerical choices. This implies that i simply increases his bond purchases by $\delta^t(\mathbf{H}^t)$.

Class C

In this class of perturbations, we consider transfers between pairs of individuals who are alive at the same point in time. That is, we choose $i, j \in I^t$, select ξ_i^t arbitrarily, and set $\xi_j^t(\mathbf{H}^t) = -\xi_i^t(\mathbf{H}^t)$ for all $\mathbf{H}^t$. All other aspects of fiscal policy remain unchanged.

Let $i_1, \ldots, i_p$ be the sequence of individuals described in the linkage hypothesis. For each $k = 1, \ldots, p - 1$, define the following policy perturbations:

$$\underline{\xi}^{\tau_k}_{i_k}(\mathbf{H}^{\tau_k}) = \xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau_k}}{\alpha^t},$$
$$\bar{\xi}^{\tau_k}_{i_{k+1}}(\mathbf{H}^{\tau_k}) = -\xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau_k}}{\alpha^t}. \tag{1}$$

For $k = 0, \ldots, p$, let

$$\tilde{\xi}^{\tau_k}_{i_{k+1}}(\mathbf{H}^{\tau_k}) = \xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau_k}}{\alpha^t},$$
$$\tilde{\xi}^{\tau_{k+1}}(\mathbf{H}^{\tau_{k+1}}) = -\xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau_{k+1}}}{\alpha^t}, \tag{2}$$
$$\delta^{\tau}_k(\mathbf{H}^{\tau}) = \operatorname{sgn}(\tau_k - \tau_{k+1})\xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau}}{\alpha^t},$$
$$\min(\tau_k, \tau_{k+1}) \leqq \tau < \max(\tau_k, \tau_{k+1}),$$

where $\tau_0 \equiv \tau_{p+1} \equiv t$.

First, we note that the cumulative effect of all these component policies is equivalent to the effect of the original policy. In particular, the effect on i_k in period τ_{k-1} is

$$\bar{\xi}^{\tau_{k-1}}_{i_k}(\mathbf{H}^{\tau_{k-1}}) + \tilde{\xi}^{\tau_{k-1}}_{i_k}(\mathbf{H}^{\tau_{k-1}}) = \begin{cases} 0, & k = 2, \ldots, p, \\ \xi^t_i(\mathbf{H}^t), & k = 1. \end{cases}$$

Similarly, the effect on i_k in period τ_k is

$$\underline{\xi}^{\tau_k}_{i_k}(\mathbf{H}^{\tau_k}) + \tilde{\xi}^{\tau_k}_{i_k}(\mathbf{H}^{\tau_k}) = \begin{cases} 0, & k = 1, \ldots, p-1, \\ -\xi^t_i(\mathbf{H}^t), & k = p. \end{cases}$$

Finally, the change in period τ bond issues is

$$\sum_{k=0}^{p} \delta^{\tau}_k(\mathbf{H}^{\tau}) = \xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau}}{\alpha^t}\left[\sum_{\substack{k \text{ s.t.} \\ \tau_{k+1} \leqq \tau < \tau_k}} \operatorname{sgn}(\tau_k - \tau_{k+1})\right]$$
$$= 0$$

since $\tau_0 = \tau_{p+1}$.

Now note that, for each k, equation (1) is a class A policy and equation (2) is a collection of class B policies. We know that class B policies are always irrelevant. We can therefore focus on the class A policies. Reasoning as before, we see that adoption of all the class A policies described in (1) yields a game that is equivalent to one that is induced by perturbing the original transfer constraints as follows: if $(i_k, i_{k+1}, \tau_k) \in \lambda$,

$$\underline{\beta}^{\tau_k}_{i_k, i_{k+1}}(\mathbf{H}^{\tau_k}) \equiv \xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau_k}}{\alpha^t};$$

if $(i_{k+1}, i_k, \tau_k) \in \lambda$,

$$\underline{\beta}^{\tau_k}_{i_{k+1}, i_k}(\mathbf{H}^{\tau_k}) \equiv -\xi^t_i(\mathbf{H}^t)\,\frac{\alpha^{\tau_k}}{\alpha^t}.$$

Since no link need appear twice in this chain, we can obviously make the composite perturbation to transfer constraints arbitrarily small by taking ξ^t_i small. Since a small perturbation of transfer constraints has no effect on behavior, the corresponding policy perturbation must be irrelevant.

Having analyzed cases A, B, and C, we are now prepared to consider an arbitrary policy perturbation, $\langle \delta^t, (\xi^t_i)_{i\in I^t}\rangle_{t=1}^{\infty}$. We begin by decomposing this into component parts. For each t, we define $N^t - N_{t-M-1}$ policy perturbations as follows. For all $i \in I^t - I_{t-M}$, let

$$\begin{aligned}
\underline{\xi}^t_i(\mathbf{H}^t) &= \frac{-\delta^t(\mathbf{H}^t)}{N^t - N_{t-M}},\\
\overline{\xi}^{t+1}_i(\mathbf{H}^{t+1}) &= \frac{\alpha_t\delta^t(\mathbf{H}^t)}{N^t - N_{t-M}},\\
\tilde{\delta}^t_i(\mathbf{H}^t) &= \frac{\delta^t(\mathbf{H}^t)}{N^t - N_{t-M}}.
\end{aligned} \tag{3}$$

Next, define $N^t - 1$ policy perturbations as follows. Choose some $i^* \in I_{t-M}$ and, for all $i \in I^t - \{i^*\}$, let

$$\begin{aligned}
\hat{\xi}^t_i(\mathbf{H}^t) &= \xi^t_i(\mathbf{H}^t) - \underline{\xi}^t_i(\mathbf{H}^t) - \overline{\xi}^t_i(\mathbf{H}^t),\\
\hat{\xi}^{ti}_{i^*}(\mathbf{H}^t) &= \underline{\xi}^t_i(\mathbf{H}^t) + \overline{\xi}^t_i(\mathbf{H}^t) - \xi^t_i(\mathbf{H}^t).
\end{aligned} \tag{4}$$

We now establish that the cumulative effect of these component policies is equivalent to the effect of the original policy. The effect on debt at time t is clearly

$$\frac{(N^t - N_{t-M})\delta^t(\mathbf{H}^t)}{N^t - N_{t-M}} = \delta^t(\mathbf{H}^t).$$

The effect on $i \in I^t - \{i^*\}$ is

$$\hat{\xi}^t_i(\mathbf{H}^t) + \underline{\xi}^t_i(\mathbf{H}^t) + \overline{\xi}^t_i(\mathbf{H}^t) = \xi^t_i(\mathbf{H}^t).$$

Finally, the effect on i^* is

$$\sum_{i \in I^t - \{i^*\}} \hat{\xi}_{i^*}^{ti}(\mathbf{H}^t) = \sum_{i \in I^t - \{i^*\}} [\underline{\xi}_i^t(\mathbf{H}^t) + \bar{\xi}_i^t(\mathbf{H}^t) - \xi_i^t(\mathbf{H}^t)]$$

$$= -\delta^t(\mathbf{H}^t) + \alpha_{t-1}\delta^{t-1}(\mathbf{H}^{t-1}) - \sum_{i \in I^t - \{i^*\}} \xi_i^t(\mathbf{H}^t)$$

$$= \xi_{i^*}^t(\mathbf{H}^t),$$

where the last equality follows from the government's budget constraint.

Note that, for each $i \in I^t - I_{t-M}$, (3) is a class B policy. Recall once again that class B policies are completely irrelevant. We can therefore confine attention to the class C policies described in (4). We know that each class C policy induces a game that, after a change of variables, is equivalent to one in which we have perturbed corner constraints. Similarly, the entire policy induces a game that, after all component changes in variables are made, is equivalent to one in which we have made all component perturbations in the appropriate corner constraints. We need only verify that the transformed game (i.e., the one in which variables have been changed) is well defined and that the corresponding aggregate perturbation to corner constraints can be made arbitrarily small by taking the policy to be small.

Fix some period t and consider a link, $(i, j, t) \in \lambda$. This link might appear in any chain connecting any two individuals living in periods $t - T$ through t. However, it does not appear in any other chain. The total number of potential appearances is therefore finite; specifically, it is bounded by $\Sigma_{\tau=t-T}^{t} (I^\tau - 1)$.[30] Consequently, the composite change of variables is well defined. The corresponding total perturbation to the link (i, j, t) is just equal to the sum of the component perturbations. We can clearly make this sum arbitrarily small by taking $(\eta^{t-T}, \eta_0^{t-T}, \ldots, \eta^t, \eta_0^t)$ sufficiently small. Now choose ϵ such that no ϵ-perturbation to the corner constraints affects equilibrium behavior. Since there are a finite number of individuals living in period t, we can find some $(\hat{\eta}_t^{t-T}, \hat{\eta}_{0,t}^{t-T}, \ldots, \hat{\eta}_t^t, \hat{\eta}_{0,t}^t) > 0$ such that, for $\boldsymbol{\eta}$ satisfying

$$(\eta^{t-T}, \eta_0^{t-T}, \ldots, \eta^t, \eta_0^t) \leqq (\hat{\eta}_t^{t-T}, \hat{\eta}_{0,t}^{t-T}, \ldots, \hat{\eta}_t^t, \hat{\eta}_{0,t}^t),$$

[30] Given the redundancy of interpersonal linkages noted in Sec. IV, it is very likely that the number of potential appearances is vastly greater than the number of actual appearances. This observation is of some importance because $\boldsymbol{\eta}$—the bound on policy perturbations—depends on the number of actual appearances.

the aggregate perturbation to each $\underline{b}_{ij}^t$, β_{ij}^t, implied by any $\boldsymbol{\eta}$-perturbation of fiscal policy satisfies $|\beta_{ij}^t(\mathbf{H}^t)| \lessgtr \epsilon_{ij}^t(\mathbf{H}^t)$ for all $\mathbf{H}^t$ and i, j with $(i, j, t) \in \lambda$. Thus, by choosing $\boldsymbol{\eta}$ such that

$$(\eta^t, \eta_0^t) = (\min\{\hat{\eta}_{t-T}^t, \ldots, \hat{\eta}_t^t\}, \min\{\hat{\eta}_{0,t-T}^t, \ldots, \hat{\eta}_{0,t}^t\}) > 0$$

for all t, we guarantee that the aggregate perturbation to each $\underline{b}_{ij}^t$, $\underline{\beta}_{ij}^t$, implied by any $\boldsymbol{\eta}$-perturbation of fiscal policy satisfies $|\beta_{ij}^t(\mathbf{H}^t)| < \epsilon_{ij}^t(\overline{\mathbf{H}}^t)$ for all $\mathbf{H}^t$ and $(i, j, t) \in \lambda$. The proposition is therefore established.

VI. Extensions and Qualifications

So far we have confined our analysis to policies consisting of taxes levied on earnings, consumption, and income from physical capital. The introduction of taxes on interest income and transfers complicates our arguments significantly. Rather than provide a completely general extension of our result, we choose instead to illustrate basic principles through a simple example.

Consider an economy consisting of three individuals, 1, 2, and 3, whose decision variables and preferences are given as in example 1 (Sec. II). However, in this case suppose that 2 chooses his transfer to 3 (b_2) after observing 1's transfer (b_1) (i.e., 1 makes his choice in period 1, 2 makes his choice in period 2, and each agent i consumes in period i). Suppose further for simplicity that the net interest rate is zero. We allow for arbitrary taxes as in Section III. Thus the government collects z_1 from 1 in period 1, so that 1 consumes $w_1 - b_1 - z_1$. Since H^1 is degenerate, z_1 cannot be conditioned on any actions. In period 2, the government collects $z_2(b_1)$ from 2 (b_1 completely describes H^2), so that 2 consumes $w_2 - b_2 - z_2(b_1)$. Finally, in period 3, the government collects $z_3(b_1, b_2)$ from 3 (b_1 and b_2 completely describe H^3). Budget balance requires that $z_3(b_1, b_2) = -z_1 - z_2(b_1)$, so that 3 consumes $w_2 + [b_1 + z_2(b_1)] + b_2 + z_1$. This suggests a natural interpretation: z_1 is a lump-sum tax on 1, the proceeds of which are distributed to 3, while $-z_2(b_1)$ is a tax on 1's transfer to 3, the proceeds of which are distributed to 2.

Now consider a policy perturbation, $(\xi_1, \xi_2(\cdot))$. For the new game, employ the following change of variables:

$$\beta_1 = b_1 + \xi_1,$$

$$\beta_2 = b_2 + [z_2(\beta_1 + \xi_1) - z_2(\beta_1)] + \xi_2(\beta_1 + \xi_1).$$

It is easy to check that the same numerical choices produce the same levels of consumption in the original game and in the transformed, perturbed game. The only difference is that, in the latter, the constraints are

$$\beta_1 \geqq \xi_1$$

and

$$\beta_2 \geqq [z_2(\beta_1 + \xi_1) - z_2(\beta_1)] + \xi_2(\beta_1 + \xi_1).$$

Clearly, if the policy perturbation is small and if z_2 is continuous, then the corresponding perturbation to the corner constraints is also small. Under the linkage hypothesis, the policy is therefore irrelevant. Thus lump-sum redistributions from 1 to 3 have no effects despite the fact that 1's transfer to 3 is taxed, and changes in the transfer schedule are also inconsequential. Unfortunately, a general formulation of this result is extremely complex.[31]

While we have couched this discussion entirely in terms of fiscal policy, our analysis also implies that prices are locally indeterminate and that sufficiently small changes in prices have no effect on the allocation of real resources. This conclusion follows directly from the observation that changing a price is formally analogous to imposing a tax on one party to a transaction and distributing the proceeds to the other party.[32]

If prices are irrelevant, then our competitive pricing assumption is plainly inessential. Even the assumption of price-taking behavior becomes vacuous since one can change the price for any given transac-

[31] This is so for two reasons. First, as is evident from our example, the correspondence between perturbations to policies and corner constraints is significantly more complex than in Sec. V (see the formula for the lower bound on β_2). Second, elaborate private transfers are required to offset even the simplest government redistributions (e.g., lump-sum transfers between operatively linked individuals). Indeed, one must rule out cases in which it is logistically impossible for the private sector to neutralize public redistributions (e.g., if the government imposes a 100 percent tax rate on all transfers to and from some individual, then redistributions involving this individual will typically have real effects).

[32] Technically, this argument raises some subtle issues. First, the formal analogy between price changes and tax-transfer schemes does not hold if agents envision themselves as trading with a fictitious entity called "the market" rather than with other market participants. For prices to be irrelevant, each agent must realize that changes in his sales or purchases necessarily alter the sales or purchases of other agents. Clearly, a rational agent cannot believe otherwise. Traditionally, one ignores these effects in competitive models on the grounds that each agent is small relative to the market just as one ignores budget-balancing distributions of marginal tax revenues. Analogous to our discussion of taxes, this leads one astray under the current set of assumptions, regardless of population size. Second, the general irrelevance of prices follows only if one assumes that firms act in the interests of their owners rather than as profit-maximizing automata (profit maximization is simply not a sensible objective in the world described here). See Bernheim and Bagwell (1986) for a more complete discussion of these issues.

tion without effect. It is therefore not surprising that one can also introduce market power without altering our central result.

Suppose, for example, that in addition to a nonmonopolized consumption good, *c*, there is also a monopolized consumption good, *x*. The monopolist's price will, of course, be indeterminate, at least within some range. He will exercise his market power by deciding who will consume *x* and in what amounts. We can think of the monopolist as making operative transfers of *x* to others (the recipients of *x* may in turn pass some of it on). At the same time, there will be a network of operative transfers in *c*. Redistributions of *c* (*x*) between people who are operatively linked in *c* (*x*) will be irrelevant. If the linkage hypothesis is satisfied for *c*, then our central result applies with respect to redistributions of *c*. One may even condition such redistributions on transfers of *x* (i.e., tax the exercise of market power) without effect. Thus if government fiscal policy entails redistribution of units of account (dollars), the relevant question is whether the linkage hypothesis is satisfied for units of account. However, even if it is satisfied, market power will still matter: changing the identity of the monopolist will necessarily alter the pattern of operative linkages in *x* (certainly, the original monopolist will be driven to corners) and will therefore have real allocative effects.

By now, it should be clear that other restrictive features of the model are not central to our analysis. It is, for example, relatively easy to disaggregate consumption, capital, and labor. One can then establish that excise taxes and various partial factor taxes are irrelevant. One may also dispense with the assumption of constant returns to scale; we maintained this assumption simply to avoid the necessity of accounting for distributed profits. Perhaps the most important restrictions concern uncertainty and information. Our model describes a deterministic world in which all individuals are perfectly informed. For the most part, we believe that these restrictions are also inessential.

First, suppose we introduce uncertainty concerning length of life, outputs, wages, or gross returns. One could simply view nature as a "player," who selects current values of these variables according to some random scheme. One would then include nature's choices in the description of a *t*-history and proceed as before.[33] Similar remarks apply to uncertainty concerning future government policy. Even if the government randomizes its actions, individuals may condition

[33] Insurance effects, such as those described in Barsky, Mankiw, and Zeldes (1986), would not materialize since interpersonal transfers would neutralize government redistributive policies for each realization.

transfers on policy realizations. Thus each realization induces a game that is strategically equivalent to the "no-policy" game with perturbed corner constraints. Clearly, randomization between equivalent games changes nothing of substance.

Next, suppose that individuals have incomplete information about each other's preferences. As long as an agent makes a transfer with probability one, perturbations to the corresponding corner constraint will not ordinarily affect his choice. A slight modification of the argument in Sections II and V then establishes the desired result.

A somewhat more subtle issue concerns information about operative linkages. The chains that connect different individuals may be complex; indeed, two individuals may not know how they are connected. Yet it is not clear that this knowledge is at all essential. As long as individuals correctly perceive the effects of their own actions on payoffs, it does not matter if they understand the process that generates these payoffs. Thus if we prescribe equilibrium actions that offset the effects of some transfer policy, individuals will be willing to abide by these prescriptions. However, this sidesteps a deep and difficult question: How do individuals arrive at the new prescriptions? This issue is completely analogous to the observation that if no single agent knows the "big picture" and coordinates actions, there is no guarantee that an economy will reach a standard competitive equilibrium. To resolve this issue, we would require a theory of how agents achieve equilibria; unfortunately, this important problem is poorly understood. One could envision an iterative process, wherein each individual would reactively adjust his transfers, with the property that stationary points correspond to equilibria. To the extent that individuals acknowledge the irrelevance of fiscal policy, the process of adjustment following a policy change might actually be very simple: all agents hold real activities (consumption or production) fixed and allow transfers to absorb all residual resources. Finally, if one is unpersuaded by these arguments and is unwilling to dogmatically accept the implications of equilibrium theory, then one must also regard the dynastic model with considerable skepticism.

These waters become still murkier if one allows for uncertainty concerning future linkages (e.g., those arising from marriages formed after some individual's death). However, we have argued in Section IV that the linkage hypothesis is likely to be satisfied for small T: one need only use links spanning a few generations so that most of these links might well be known at the relevant point in time. Even when this is not the case, one can show that the central result continues to hold as long as, for each pair of individuals, one can devise an algorithm that describes transfers as a function of realized linkages (e.g., marriage) and that connects this pair with probability one. Given the huge number of linkages known to exist at each point in time, this condition does not seem very demanding. Of course, the process by which agents achieve such an equilibrium is again problematic.

Finally, suppose that individuals cannot observe some set of actions taken by others. Asymmetric information of this sort may interfere with the neutralization of distortionary taxes. In the proof of our central result, we transformed variables differently for subgames differentiated according to prior choices of taxed activities. For this to be valid, players with transformed actions must be able to distinguish between these subgames; that is, they must be able to observe taxed activities. Note, however, that these issues do not bear on the neutrality of arbitrary lump-sum redistributions. Indeed, this weaker form of neutrality does not even require individuals to observe each other's transfers.[34]

VII. Interpretations and Conclusions

What should one make of the rather perplexing conclusions reached in Sections II–VI? Several points of interpretation require further discussion. First, what general principles drive our neutrality results? Second, what does this analysis teach us about "real" economics? Third, where do we go from here?

With respect to the first question, one might initially suspect that our result follows from the fact that each consumer acts as though he is part of a "big happy family," which behaves as if it maximizes a single utility function. Certainly, the discussions of Barro (1974) and Becker (1974) have this flavor. Yet this suspicion is simply *false* since our results concern neutrality, not optimality. In equilibrium, chosen actions may well be inefficient, yet redistributions contingent on these choices will not affect behavior.[35]

Aside from delimiting the scope of neutrality, this observation also implies that our results have no normative implications. Although all taxes are equivalent to lump-sum taxes, lump-sum taxation may not be desirable. Since the equilibrium ordinarily entails preexisting distortions (due to intrafamily conflict), the government should wish to

[34] In example 1, we have assumed that 1 and 2 select transfers simultaneously. This is equivalent to letting 1 choose first and assuming that 2 then selects a transfer without having observed 1's choice.

[35] Recall example 2 of Sec. II. There we argued that a labor income tax is neutral since it leaves 1's effective wage unaltered. We did not, however, assert that 1 chooses his labor supply optimally in the initial equilibrium. Specifically, we did not say that 1's effective wage is w. Rather, he faces some shadow wage, which reflects the effect of his labor supply choices on subsequent transfers. Since the shadow wage need not equal w, his choice is generally inefficient. We argued only that this shadow wage is insensitive to tax and transfer policies.

engage in second-best taxation. However, paradoxically, second-best tax instruments are unavailable. The government can introduce countervailing distortions only by conditioning real expenditures on consumer behavior.

With respect to the second question, our analysis casts serious doubt on the usefulness of the dynastic framework as an analytic tool for studying public policy issues. Accordingly, one must regard any conclusions derived within this framework with considerable skepticism. Barro's Ricardian equivalence results, which concern the neutrality of public deficits and social security, are probably the best-known implications that follow from dynastic assumptions. Yet our criticism also applies to numerous other studies that adopt Barro's model.[36]

A natural response to our criticism is that one ought to view the dynastic formulation as only an approximation to reality; one should therefore expect properties such as Ricardian equivalence to hold only as approximations. Taking the premises of this model literally is simply unfair and is bound to generate some untenable results.

We find this position completely unsatisfactory in that both the degree and nature of the approximation clearly matter a great deal. If we agree that taxes, transfers, and prices are not even close to being irrelevant, then we must also agree that in some important, policy-relevant sense the world is not even close to being dynastic. One cannot simply assert that the model holds as a good approximation in one context but not in another. It is essential to describe the approximation explicitly so that analysts can identify a new set of assumptions and elucidate their implications. In practice, it is extremely difficult to modify the model in a plausible way that preserves Ricardian equivalence (at least as an approximation) while eliminating the untenable neutrality results without introducing new and equally troubling difficulties (see Abel and Bernheim [1986] and Bernheim and Bagwell [1986] for discussions).

We devote the remainder of this section to the third question: Where do we go from here? Clearly, constructive analysis of public policy must be based on a model that departs from ours in some fundamental way. It is therefore natural to begin by summarizing our central premises. First, we assume that operative linkages are quite common. Second, we assume that individuals care only about the consequences of giving, and not directly about the amount given. To

[36] For example, Abel (1984) demonstrates that social security may have real effects in a dynastic world by inducing redistributions between families. In the light of our analysis, it is clear that, once one adopts dynastic assumptions, distributional questions are ill posed. Chamley (1981) and Judd (1985) study the welfare effects of capital income taxation in dynastic models. Yet the premises of these models may imply neutralization of the very distortion they purport to study.

establish the irrelevance of all fiscal policies and prices, we must also assume that actions are publicly observable. If one relaxes this last assumption and supposes instead that capital markets are perfect, it is then possible to demonstrate that all lump-sum redistributions are irrelevant.[37]

The last two assumptions (public observability and/or perfect capital markets) are certainly objectionable on empirical grounds. However, this does not fully account for our skepticism concerning the model's implications. Suppose, for example, that the government adopted and effectively enforced (through enormous penalties) a new law requiring public disclosure of all private financial decisions. We would not expect fiscal policy and prices to become irrelevant as a consequence. We conclude that an important source of our disbelief must lie elsewhere.

The second assumption might fail for several reasons. Generosity may be inherently fulfilling. Alternatively, individuals might be myopic with respect to the actions of their heirs and simply take the size of transfers as a proxy for well-being. Both views are somewhat appealing, but neither leads to a satisfactory theory of transfers. For example, it is difficult to know why an individual would care about the magnitude of his transfer if it truly did not affect any real outcome. In both cases, the specification of the transfer motive is necessarily ad hoc.

Violations of the first assumption fall into two categories: either a large number of people fail to make positive transfers or corners matter despite the fact that transfers are positive. Many commentators have indeed claimed that corner constraints bind for most individuals. Barro (1984) offers a theoretical reason for expecting this outcome but does not elucidate implications for policy. Our analysis suggests a somewhat different reason, which raises some intriguing possibilities.

To illustrate, consider a world in which there are three successive generations ($t = 1, 2, 3$), each consisting of N households. For purposes of interpretation, one should think of each household as a married couple. Each member of generation $t = 1, 2$ has two children, but, of course, children are shared (a child household is formed by the marriage of two individuals who come from two different parent households). Suppose that the children of the ith household in generation 1 belong to the $m(i)$th and $f(i)$th households in generation 2 (where these indices are assigned so that everyone in generation 2 has two forebears in generation 1). Further suppose that the children

[37] It may also be possible to dispense with the assumption that capital markets are perfect (see Hayashi 1985; Yotsuzuka 1986).

of the ith household in generation 2 belong to the ith and $i + 1$st households in generation 3 (with the exception that N's children belong to the Nth and first households). We select this stylized pattern of linkages in order to guarantee that there is a chain that interconnects the entire population.

We will assume that all households are identical within generations. The utility of household i in generation t is given by

$$u_i^t = \begin{cases} u(c_i^1) + k[u_{f(i)}^2 + u_{m(i)}^2], & t = 1, \\ u(c_i^2) + k(u_i^3 + u_{i+1}^3), & t = 2, \\ u(c_i^3), & t = 3. \end{cases}$$

This individual is endowed with initial wealth, w^t.

Behavior unfolds as follows. First, each member of generation 1 chooses its own consumption and transfers to its children. Next, each member of generation 2 does the same. Finally, members of generation 3 consume their endowments plus all transfers received.

Suppose that along some symmetric subgame-perfect Nash equilibrium path, all members of generation 2 make operative transfers to their children.[38] The analysis of Section V then establishes that the consumption of any household in either generation 2 or 3 depends only on the total resources available to all members of those generations. If N is large, then the marginal propensity to consume from wealth for any given individual must be close to zero (this point is analogous to Sugden's [1982] observation concerning the provision of charity). Thus each member of generation 1 knows that his gifts will have a negligible impact on the consumption of his descendants. In contrast, gifts involve a nonnegligible sacrifice of his own consumption. Thus, under relatively weak conditions, no member of generation 1 will make an operative transfer.

The main point raised by this discussion is that in large populations in which preferences are dynastic and decisions are sequential, large numbers of individuals *must* end up at corners. In addition, the particular model considered here produces endogenous cycles: one generation acts altruistically, making transfers to its children, while the next generation, despite being identical to the first, acts selfishly (this remains true as one adds generations). While we do not seriously propose this particular pattern as descriptive of the real world, our analysis does suggest that endogenous behavior may well give rise to patterns of operative linkages that do not generate standard dynastic results, such as Ricardian equivalence.

While there are both empirical and theoretical reasons for doubting

[38] One can derive relatively weak conditions under which this occurs.

that most individuals make positive transfers, we are unable to fully attribute our disbelief to this assumption. We suspect that, the thrill of victory aside, most individuals would prefer winning $1,000 in a lottery to learning that one of their siblings has won $1,000, despite the expectation of future transfers from the parent. Yet dynasticism implies that one should be indifferent.

We are therefore led to reexamine the other aspect of our first assumption: corners matter, even though they do not bind, in the traditional sense. As far as we know, the only fully elaborated theories that are compatible with this view envision transfers as a means of facilitating exchange within families (see, e.g., Kotlikoff and Spivak 1981; Bernheim et al. 1985). Accordingly, we believe that subsequent policy analyses should consider more carefully the implications of nonstandard alternatives to the dynastic transfer motive.

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