

## International Portfolio Choice and Corporation Finance: A Synthesis

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THE STRUCTURE OF THE theory of international finance largely mirrors that of domestic financial theory. Starting from a micro-theory of individual portfolio choice one obtains, via aggregation and market clearing, equilibrium pricing relationships and risk-return tradeoffs. These provide objectives for value maximizing firms from which decision rules can be computed. This analytical sequence is the same whether there is one or more capital markets. To distinguish between the domestic and international settings, one needs an economic concept of nationhood.

Alternative approaches to international economics differ essentially in their conception of what a nation is. Ricardian theory identifies countries by their technologies and consumption preferences. The Heckscher-Ohlin theory of international trade defines nations as zones within which physical factors of production are confined.<sup>1</sup> In monetary economics, individual economic units holding the same currency in their portfolios as a means of payments are recognized as belonging to the same nation, and currencies are distinguished from each other by the fact that they are issued by different central banks.<sup>2</sup>

In portfolio theory, two avenues have so far been explored in an attempt to capture the international dimension. Most of the recent literature stemming from Solnik [179] has been devoted to models where nations are defined as zones of a common purchasing power unit or, more precisely, as subsets of investors

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<sup>1</sup> Generalizations of the Heckscher-Ohlin theory to economies with uncertain production and securities trading may be considered as belonging to the field of international finance but they will not be reviewed here. See Helpman and Razin [85, 86, 87], Baron and Forsythe [20], and Dumas [48, 49].

<sup>2</sup> This approach is germane to portfolio theory and originates from balance-of-payments analysis, especially since the latter took a very strong monetary orientation in the sixties under the influence of Mundell [145], and since the advent of the "portfolio-balance" approach: Branson [28]. The link with portfolio theory is, however, not complete since the latter is only today in the process of introducing money holdings into the portfolios of individual investors (for an early attempt, see Roll [159], Kouri [108], and more recently Fama and Farber [58], Hodrick [90], Poncet [151], and Dumas [49]). The device most commonly employed is the injection of real money balances as a separate argument of the utility functions.

who use the same price index in deflating their anticipated monetary returns. National groups of investors are delineated by deviations from Purchasing Power Parity (PPP) which cause them to evaluate differently the returns from the same security. As will be detailed below, these deviations may arise either from differences in consumption tastes or from differences in the prices of the various commodities to which investors have access. This heterogeneity in individuals' evaluation of returns plays havoc with the standard Separation, Aggregation, and Asset Pricing results of Portfolio Theory. A large part of this paper will be devoted to a review and extension of the several attempts made at restoring these results.<sup>3</sup> Resolving the problem of portfolio choice when investors' real returns differ is a necessary first step towards a truly international theory of finance.

In practice, nations may further be separated by such manifestations of sovereignty as taxes and border controls which constrain private financial transactions between countries. Financial economic theory does not deal easily with such imperfections which tend to segment international capital markets. We therefore relegate the problems associated with segmentation to a single section near the end. This leaves us free largely to adopt a unified world capital market as the paradigm for much of the rest. The survey can then be structured around the derivation of a mean-variance international asset pricing model (IAPM) in which the several nations feature as regions whose residents have different purchasing power indices. This organizing principle mirrors the state of the art in the field, enables us to identify both the contributions and misconceptions in the evolving literature, and leads to the following plan.

The first order of business is to determine whether, in empirical terms, nations can meaningfully be distinguished by PPP deviations. Accordingly, after reviewing briefly the conditions for the existence of price indices, Section I surveys empirical evidence on the stochastic behavior of inflation rates, exchange rates, and PPP deviations. This evidence strongly suggests that PPP deviations are significant as to size, that they last for lengthy but variable periods, and are highly random. It is therefore reasonable to suppose that investors residing in different countries have different yardsticks for measuring real returns and their risks. Consequently, one would expect the compositions of their portfolios also to differ.

Before developing a mean-variance theory, given that utility functions are not universally quadratic, it is further useful to know whether international rates of return are basically normally distributed. Section II addresses first the sketchy evidence concerning this question. Its provisional conclusion is that normality is not an untenable assumption. The section then proceeds to the wider issue of the correlation structure of nominal and real returns.

<sup>3</sup> Note that the PPP-deviations approach to international finance does not in principle presuppose the existence of several currencies. Because the PPP relationship is almost always stated in terms of an exchange rate relating two price levels expressed in two different currencies, the Purchasing Power doctrine has been associated with the equilibrium theory of floating exchange rates. In fact, PPP deviations may very well occur, and typically would occur under fixed exchange rates, or indeed in a world with a unique currency; conversely PPP could conceivably prevail exactly everywhere, causing us to recognize but one "nation" in the world, in the presence of multiple currencies related by randomly fluctuating exchange rates. Empirically, however, we shall find below the most of the PPP deviations are linked to exchange rate movements during floating rate periods.

Section III turns to the problem of optimal portfolio choice in a unified world capital market with no taxes or transactions costs but where investors' consumption preferences are nationally heterogeneous. Every investor, regardless of nationality, has free access to the same menu of assets: all stocks, foreign and domestic, and one default-free, short-term bond per country. All bonds are risky in real terms. Investors are assumed to maximize a time-additive, von Neumann-Morgenstern expected utility of life-time consumption function.<sup>4</sup>

Introducing random exchange rates for translating future returns in one currency into another raises a familiar technical problem. At one level it may be trivial: a change of measurement unit which leaves untouched the underlying economic reality.<sup>5</sup> Nonetheless, currency translation produces products of random variables whose probability distributions are hard to obtain. In particular, the product of two normal variates is not normal. We therefore adopt in Section III the continuous-time methodology of Merton [138, 139, 140] which to a large extent offers a way out of the difficulty. This technique basically justifies the mean-variance paradigm by the approximation reasoning of Samuelson [167] and, by employing Ito processes, effectually transforms products of random variables into sums.<sup>6</sup>

Sections IV and V focus on the international pricing of assets. Capital Asset Pricing Theories involving a multiplicity of measurement units, with the attendant translation problems, have been supplied by Fischer [62], Grauer, Litzenberger, and Stehle [GLS] [79], Friend, Landskroner, and Losq [FLL] [70] and Hodrick [90]. These CAPMs are restatements in terms of nominal returns of the traditional CAPM of Black [23] which dealt implicitly with real (i.e., deflated) returns. GLS assume a complete market and use a state-of-the-world formulation which avoids the problems connected with translation, since this operation can then be performed on a state-by-state basis. Fischer and FLL use the continuous-time methodology; they do not refer explicitly to several currencies but the manner in which they translate real rates of return into nominal ones and vice-versa can equally well be used to translate dollars into francs. Hodrick postulated the existence of one good traded worldwide at the same price and allows for non-stationary returns.

These CAPMs are applicable only when investors use the same price index in

<sup>4</sup> Time additive von Neumann-Morgenstern utility functions have a major drawback. The curvature of the instantaneous utility function ( $U(\cdot)$  in the appendix) simultaneously plays the role of a risk aversion parameter and that of the elasticity of substitution between consumption of different points in time. For an attempt at disentangling these two aspects of preferences see Selden [171].

<sup>5</sup> *No matter what economic setting is chosen*, the decisions of rational economic units (investors, firms, etc.) should be invariant as one changes the unit of measurement or the currency of accounting, in which returns are expressed. This is a minimum criterion of rationality (which may admittedly not be passed by corporate treasurers concerned about the looks of their end-of-year accounting statements). To verify that this criterion is satisfied by a given decision method, one must translate from one currency into another using random exchange rates and check that the decisions obtained are unchanged. If so, the *principle of irrelevance of the measurement currency* is upheld and all is well with the theory at hand.

<sup>6</sup> The method does restrict at each instant the conditional distribution of instantaneous rates of return which must be normal. But rates of return on finite intervals of time are not so restricted since the parameters of the Ito process may themselves be variable.

deflating returns,<sup>7</sup> an assumption which is unrealistic at the international level. Nevertheless, even in this simple setting, one may derive an expression for the forward exchange rate<sup>8</sup> as a function of the distribution of the future spot exchange rate corresponding to the maturity of the forward contract. A strong conclusion emerges: the forward exchange rate generally ceases to be an unbiased predictor of the future spot rate. It differs from the expected value of the spot by two premia. One is the result of the risk aversion of investor-speculators. The other premium would exist under risk neutrality and it arises from the presence of random inflation. We shall obtain the same result, but in the more general setting where PPP is violated.

When PPP does not hold, the heterogeneity in portfolio-choice behaviors limits the aggregation of individual demands into a CAPM. Section IV rescues the standard CAPM to the extent it is possible, by limiting its scope to the pricing of some assets relative to the others. The number of these other assets is equal to the number of countries and, for practical purposes, they are identified with the local Treasury Bills. Section V then deals with the pricing of these remaining assets.

Section VI takes a closer look at the welfare problems that may be connected with the randomness of exchange rates. There has been much debate as to when and whether exchange risk is “nominal” or “real” and whether it matters or not. The issue is complex: it depends on such factors as how money is introduced into each economy and why it is held; on the completeness of capital markets; and on how the government raises revenues and uses the proceeds of money creation.

Progress can be made while ignoring capital market imperfections. Ultimately, however, these must be confronted, possibly more so in the international than in the purely domestic setting. Section VII therefore turns to the literature on segmentation. Some of the papers in this area aim to compute equilibrium conditions. Others discuss the welfare gains from bridging the investment barriers and the possibility for optimal corporate decisions. Empirically, the severity of the market imperfections which tend to produce segmentation and the extent of segmentation itself have yet to be measured. Resolving these matters remains a key challenge for future research.

Section, VIII, finally, turns to questions of corporate policy. It focuses mainly on foreign exchange risk avoidance, i.e., hedging policy. In a complete, perfect, and unified international capital market, corporate hedging would be irrelevant. The section then explores the difficulties of measuring exposure and the hedging decision in the circumstances in which it may matter.

This introduction owes its length to the necessity of setting the bounds of this survey. Apologies are due to authors who have contributed important insights into topics closely linked to those covered here. In particular, we shall discuss very few issues in macroeconomics. Empirical tests of the portfolio-balance

<sup>7</sup> In one part of their article, GLS do allow for PPP deviations but their investors must then have equal risk aversions. The GLS article contributed greatly to the focus on the role of PPP deviations and to the decomposition of exchange risk into a purely “nominal” and purely “real” component. A fuller discussion will be provided in Section VI.

<sup>8</sup> The forward exchange rate is the exchange rate specified in a contract to deliver a foreign currency at a future point in time. See Section V.

approach to capital flows initiated by Branson [28] have thrown some light on the validity of portfolio theory in international finance but we shall not have room to review them. The vast macroeconomic literature on the neutrality of money which is closely linked to the discussion of Section VI, will only be mentioned in passing. The literature on trade is totally omitted; furthermore, even for topics falling directly within the scope of this survey (e.g., the empirical literature on purchasing power parity and on forward rates as predictors of ensuing spot rates), when numerous papers have been published, only a few representative ones will be cited.

### I. Purchasing Power Parity in International Finance

PPP has essentially two uses in capital market theory and international corporate finance. One is as a measure of the similarity of, or difference between, consumption opportunities in different countries. The other is as a possible influence on the cash flows generated by production or trade activities of firms. Our concern in this section is with the former aspect while the latter aspect will be discussed in Section VIII. Deviations from PPP, which our survey reveals to be the rule, serve to differentiate one nation from another. When PPP deviations exist, investors in different regions will measure their real returns differently and desire to hold generally different portfolios.

To fix ideas, let us first distinguish PPP from commodity price parity (CPP), also known as the law of one price. CPP is an instantaneous arbitrage condition which holds between the prices of identical traded goods in two locations in the absence of any barriers to trade. It may also hold between nontraded goods which are close substitutes for (or can be transformed into) traded goods.<sup>9</sup> In contrast, PPP is a relationship between weighted average price levels, not individual commodity prices. In practice, price levels are measured by indices which are calculated relative to some base period; writing the parity of price levels at two different instants and taking the ratio leads to the so-called "Relative PPP." If price levels are arbitrarily set to be equal in some base year, any change in the price of one good relative to the others, will suffice to create a PPP deviation in the ensuing years.

Let us dwell briefly on the conditions required for PPP to hold exactly. First, the consumer price indices computed by the world's national statistical institutes must be a valid representation of the consumption possibilities and preferences of their citizens. The Appendix details the assumptions required. If a nation's consumers are utility maximizers (with time additive utility), they will have a compositionally invariant price index in which budget shares reveal their tastes if their preferences are also homothetic. Given such homothetic preferences, published cost of living indices approximate exact indices and may reasonably be compared across countries.<sup>10</sup>

<sup>9</sup> CPP can be tested directly by comparing absolute prices after translation into a common currency. A second test is whether the relative price of two goods is the same everywhere.

<sup>10</sup> We know of no direct test of the homotheticity of preferences. If the homotheticity assumption is violated, *two variable* price indices (one reflecting "average" and the other "marginal" budget-

For PPP to hold exactly, sufficient conditions include homothetic preferences, as above; CPP with respect to every good included in the index; and, in addition, identical tastes to guarantee that the compositions of different nations' indices will be identical. These conditions are not necessary; it is apparent from the debates of the 1920s that PPP could also emerge despite differences in tastes and the presence of nontraded goods, provided that there existed enough substitutability among goods in consumption and between traded and nontraded goods in production to produce high correlations between the prices of individual commodities. The details need not concern us here.

As noted, the importance of PPP deviations in international finance stems from the way in which investors compute the real returns from a given security. Consider a U.S. investor holding a Japanese security with a given (perhaps random) nominal return. If this nominal (or monetary) return is measured in yen,<sup>11</sup> he will first translate it into dollars and then deflate it using the U.S. Consumer Price Index or, better yet, his own index, expressed in dollars.<sup>12</sup> This generates the U.S. purchasing power of this foreign income. But, consider a Japanese investor holding the same security and expecting the same nominal (yen or dollar) nominal return. His real return is obtained by deflating the yen nominal return by his, or the Japanese, price index measured in yen. If PPP held exactly, i.e., if the two price indices were exactly in line with the exchange rate, the two investors would view the real return identically. Their notions of the real returns from the same security will differ to the extent that their price indices, expressed or translated into a common currency, are different, i.e., to the extent that PPP is violated.

This is not to say, however, that the deviations themselves enter any investor's calculus or, therefore, that they feature in any way as a separate source of risk. Each computes his real returns using his own index, irrespective of any comparison with a foreigner's index. For given distributions of nominal securities returns, the existence of PPP does not remove or alleviate the risk of a foreign investment absolutely or relatively to a home investment.<sup>13</sup>

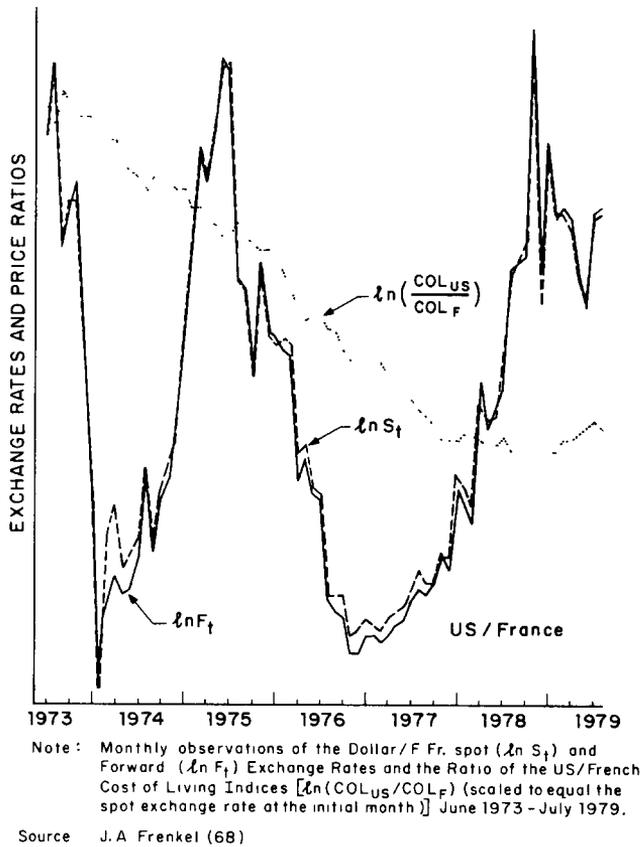
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shares) are necessary to model mean-variance portfolio choices. See the last section of Breeden [30]. See also Stulz [191].

<sup>11</sup> Starting from returns measured in Yen is of no importance. The end result, the real returns, is independent of that fact. If we had specified that the security, or its returns, is *denominated* in Yen, it would have been an indication as to the pay-off structure of the security, namely that the return in Yen is fixed ahead of time. Such a specification would apply to a deposit in a Japanese bank or to a Yen fixed income security. For other securities types, such as stocks, no matter what the currency of payment of the random dividends may be, there is no denomination a priori. But, based on the probabilistic payoff structure of the security, one may establish its implicit denomination. This is the object of exposure computations; see below, in Sections IV and VIII.

<sup>12</sup> Obviously cost-of-living indices given by national statistical institutes have a currency dimension (irrespective of the fact that they are usually quoted as a dimensionless number computed relative to a base year where the index is set at 100). They may accordingly be translated from one currency into the other. One may thus obtain the cost-of-living index of French households expressed in U.S. dollars.

<sup>13</sup> Nonetheless price discrepancies between trading areas will generally affect the trade and production activities of firms and therefore their returns as well as the general welfare (see Sections VI and VIII). But this is a separate issue. We are examining now portfolio choices with given securities returns. In this context, PPP deviations arise only when *comparing* the portfolio choices of different investors, not in examining the risk borne by any one of them.



**Figure 1.** U.S.-French PPP Deviations, 1973-1979

The empirical evidence regarding PPP deviations is important mainly for revealing that investors' real returns do differ and therefore that their portfolio compositions should differ. Figure 1 is typical of the empirical record: similar patterns are displayed by all the other major currencies. The clear implication is that PPP is violated. During the most recent period, PPP deviations have been large and cumulative in the sense that deviations in a given direction have lasted for lengthy but variable periods. Clearly, PPP is a questionable hypothesis for the short-run; is it better in the longer run? In terms of Figure 1, does the fact that the exchange rate seems to cycle irregularly around the price-index ratio mean that PPP deviations are ultimately self-reversing? Let us examine first the evidence regarding the two major causes of PPP deviations:<sup>14</sup> CPP deviations and differences in national consumption baskets.

CPP normally holds to within narrow transaction cost margins for homogeneous goods which are traded on organized auction markets such as commodities exchanges. Gold and other readily arbitrable metals are good examples. Since Cassel proposed the PPP hypothesis, however, other authors including most

<sup>14</sup> We shall not survey exhaustively the voluminous literature on PPP. It has been reviewed already by Balassa [18] and Officer [148] and in the May 1978 issue of the *Journal of International Economics*.

recently Isard [96], Kravis and Lipsey [112], and Richardson [156] have found evidence of persistent and large deviations from CPP (up to 5% during the fixed rate period and up to 20% under floating rates). Exhaustive empirical investigations have been conducted by Katseli-Papaefstratiou [101] and Crouhy-Veyrac et al. [38]. The combined sample includes most classes of traded goods; the deviations are largely unexplained. Their existence has been attributed to imperfections which prevent arbitrage such as taxes, tariffs, and transactions costs; asymmetric information; or monopoly and discriminatory pricing. In part they may be illusory. Goods within even narrow commodity trade classifications are not homogeneous. In addition, there may be measurement errors in prices, especially on long term contracts. All in all, however, CPP violations seem to be the rule rather than the exception.

As far as PPP is concerned, Kravis and Lipsey [112] quoting Kravis et al. [111] have produced comparisons of absolute price levels in 1970. Their Table 1 is reproduced here as our Table I. The comparison is performed in two ways: once using local national weights, and once using uniformly for all countries the same set of weights (*viz.* the U.S. weights) for the various commodity groups. Simple examination shows that deviations are wider when using different weights than when using the same weights. Differences in national consumption tastes therefore do contribute to PPP deviations.

There is a frequently stated view, however, that while PPP may fail at any instant, it is a good long-run hypothesis. This belief appears to be based on the notion that the average deviation, where the average is taken over long periods, seems to tend to zero for most countries. Gailliot [72] calculated the percentage deviations of wholesale price indices from purchasing power parity with the U.S. dollar, from the 1900-4 average to the 1963-7 average; they were, for Canada .04, France -.01, Germany .04, Italy -.11, Japan .26, Switzerland .14, and the United Kingdom .11 or .02, depending on which index he used. Aliber and Stickney [16] calculated annual PPP deviations and found that the average deviation declined

Table I  
Relative Price Levels in 1970 (U.S. = 100)

Country	Own Weights			U.S. Weights		
	Price Levels			Price Levels		
	GDP	Traded Goods	Nontraded Goods	GDP	Traded Goods	Nontraded Goods
Kenya	34.9	58.5	22.3	74.2	96.7	48.9
India	24.0	46.8	10.3	47.6	74.4	17.3
Columbia	31.8	57.6	21.0	62.0	91.6	28.6
Hungary	44.7	62.7	30.7	66.0	95.0	33.0
Italy	66.2	84.0	50.4	82.9	107.0	55.5
Japan	61.1	82.5	46.4	76.4	95.3	55.0
United Kingdom	65.8	83.3	50.2	77.8	97.0	55.9
Germany, F.R.	79.8	99.2	60.9	95.4	114.5	73.5
France	73.8	83.4	63.4	89.1	105.9	70.4
United States	100.0	100.0	100.0	100.0	100.0	100.0

Source: I.B. Kravis, Z. Kenessey, A. Heston, and R. Summers [111].

as the number of annual observations was increased. Such results are to be expected if PPP deviations follow patterns such as those reflected in Figure 1. The question is what they imply for the modeling of commodities price levels and exchange rates in the absolute, and relative to each other. Should we select stochastic processes for these quantities incorporating reversals towards PPP? The answer should, of course, come from a general equilibrium model, but at this stage, we can only discuss the elements of a reasonable formulation.

The available empirical evidence does not exhibit any tendency towards reversal (or non-zero serial correlation). Gailliot [72] claimed on the basis of casual observation that a positive 5-year-average deviation in one decade was followed by a negative deviation in the next, between 1903 and 1967. Such evidence does not seem strong. Roll [161] provided the first test of the existence of serial correlation in PPP deviations. Using monthly IMF data for twenty-three countries between 1957 and 1976, Roll found no evidence of serial correlations in his pooled-sample runs. However, some measure of serial correlation, both positive and negative, emerged for some countries, most notably Iran, Argentina, and Mexico. The individual country results were not analyzed in detail. Other tests by Rogalski and Vinso [158] reached similar conclusions. Adler and Lehmann [10], using distributed-lag regressions, tested the hypothesis that PPP deviations follow a martingale (and therefore exhibit no serial correlation). Various samples of monthly and annual data (for both "fixed" and flexible rate periods) and various durations of lags (up to ten years) all produce similar results: as a rule the martingale model is not rejected.<sup>15</sup>

These empirical results suggest that PPP is violated instantaneously and can be expected to be violated for any forecasting horizon. They therefore suggest the heterogeneity of national consumption tastes as a foundation for international finance.

The economic interpretation of the empirical success of the martingale process is not straightforward, however. A theoretically sound model of PPP deviations would incorporate both the action of costly commodities trading and that of almost costless information-efficient foreign exchange trading. Costly commodities trading would cause the law of one price (CPP) to be violated instantaneously (*ex post*) and would allow the exchange rate to fluctuate within a band (akin to the gold points of the previous century) on either side of its PPP level. On the other hand, costless information-efficient foreign exchange and bond trading causes the current spot exchange rate to be the best predictor of the future exchange rate adjusted for interest rates (see Section V) and further, since interest rates anticipate inflation, it also causes the current real exchange rate to be the best predictor of the future exchange rate adjusted for the anticipated inflation difference between the two countries. This last statement is what may be termed the *ex ante* PPP hypothesis and it leads to the martingale model.<sup>16</sup>

But it is not clear how the commodities and the foreign exchange markets

<sup>15</sup> I.e., the fact that a currency is below its PPP value is no indication that it will subsequently appreciate absolutely or even relative to price levels. Nevertheless, a number of commercial foreign exchange forecasting services sell forecasts based on PPP deviations.

<sup>16</sup> As was stressed by one referee, *ex post* PPP, and not just *ex ante* PPP, would be needed to remove the heterogeneity of perceived real returns across investors.

interact. If the martingale model held strictly, an absolute deviation taking place today would never tend to be corrected; this is surely an undesirable feature of this model.

For the present, all that can be safely said is that the actual behavior of PPP deviations cannot be statistically distinguished from a martingale. More specifically, under floating rates, PPP deviations behave approximately like the exchange rates themselves<sup>17,18</sup> (see Genberg [73] and our Table II) and the latter more or less follow a martingale (Poole [152]).

## II. The Empirical Structure of International Returns

Since much of the research in international finance involves either applications of statistical regression techniques or extensions of mean-variance portfolio theory, we should briefly review the little that is known about the probabilistic structure of international returns. Two separate aspects are covered in this section: one is the question of the probability distribution which best describes these returns; and the second is the matter of estimating empirically the scope for risk reduction through international diversification and, more generally, the validity of market regression models as descriptors of the risk structure of international asset prices. Let us take each in turn.

### A. *The Probability Distribution of International Returns*

The first issue is whether the returns on international investment are normally distributed and, if they are not, what probability distribution best describes them. Unfortunately, no one yet has investigated the distributions of real returns. Farber, Roll, and Solnik [61] examined the distribution of  $R = (S_{t+1}/S_t) [(1 + r_k)/(1 + r_n)] - 1$  where  $S$  is the spot exchange rate and  $r_k$  and  $r_n$  are the foreign and domestic nominal interest rates:  $R$  represents the nominal excess rate of return (over the domestic riskfree rate) on speculation in the money market.<sup>19</sup> For monthly data between 1964 and 1975, they discovered that the distribution of  $R$  departed from normality but less, apparently, than the distribution of the exchange rate change itself: the comparison was not made in great detail.

It is the distribution of exchange rate changes which has received the most attention. Besides Farber, Roll, and Solnik [61], investigations include those of Westerfield [202], Dooley and Shafer [42], Papadia [149] and, most recently, McFarland, Pettit, and Sung [135]. In all cases, severe departures from normality were discovered, generally more so for pre-1973 data than for the floating-rate period. Following the research of Granger and Morgenstern [77] and Fama and

<sup>17</sup> I.e., price levels fluctuate little in comparison to exchange rates. To some extent, this may be due to a diversification effect between commodities prices. See Cornell [35].

<sup>18</sup> This indicates that PPP deviations and the portfolio issues arising from them are properly raised within the context of a multi-currency world. PPP deviations also occur domestically (i.e., across one country) but they are probably less important and less random.

<sup>19</sup> Under interest rate parity (cf. Section V),  $R$  can be rewritten  $R = S_{t+1}/F_t - 1$  where  $F_t$  is the forward rate:  $R$  is then also the nominal return to forward speculation for a contract size equal to one unit of domestic currency. If the domestic interest rate properly anticipates domestic inflation, it is also an estimate of the real rent from a foreign investment position (in excess of the domestic real interest rate).

Table II

Sources of Deviations from PPP: 1971 February–1979 December. Correlations of eight exchange rates (against the U.S. dollar) with their respective PPP levels. 107 observations

The correlations are measured after taking first differences on both sides.

Germany	.996
Belgium	.995
Canada	.923
France	.998
Japan	.938
Netherlands	.994
United Kingdom	.961
Switzerland	.983

Data Source: OECD main economic indicators, various issues.

Roll [59], and the observation that the empirical exchange-rate distributions had fatter tails and greater kurtosis than the normal, further tests suggest that these distributions may be stable-paretian with characteristic exponent between 1.3 and 1.8. The currently prevailing hypothesis, then, is that the distributions of the exchange rates belong to the stable, infinite-variance class.<sup>20</sup>

Alternative views are, however, by no means excluded. One view is that the observed exchange rate variations are drawn from nonstationary normal distributions; i.e., that the exchange rates themselves follow a stochastic process with variable parameters. In order to account for the fat tails in exchange rate variations, this process should have a tendency to diverge or oscillate more widely than does a regular Brownian motion; i.e., the variance would have to grow faster than linearly with time.

One other view is that exchange rates, as a result of government intervention, or for some other reason, undergo discrete jumps. This could be modeled by means of combined Poisson and diffusion process, provided the mean frequency of these jumps is approximately constant. Kouri [107] calculated portfolio choices and some elements of the equilibrium (the forward premium) under the Poisson assumption. Both alternative views lack a good empirical analysis so far.

<sup>20</sup> As Granger [78] points out, it is, of course, difficult actually to prove that any sequence of observed random variables in fact has a stable, non-normal distribution. Strong indication but not proof is provided by tests of the stability of the characteristic exponent as increasing numbers of (log) price or exchange-rate changes are added together, both chronologically and in randomized order. Westerfield tested only the chronological sequences and found the characteristic exponents to be reasonably stable for sums of between one and ten weekly observations: characteristic exponents rose with the length of sequence but reached a maximum of 1.69. McFarland, Pettit, and Sung employ daily data between 1/75 and 7/79. Tests of chronological and randomized sequences both showed a tendency for characteristic exponents to rise slowly but to remain below 1.7 for sequences of up to 15. However, when the chronological sequences were lengthened to 25, at the cost of reduced sample size, the characteristic exponents of six out of eight exchange rates reached 2.00, the critical value for normality. The authors express little confidence in this comforting result.

The major question raised is whether these various distribution assumptions make mean-variance theory totally inapplicable. With regard to the infinite-variance hypothesis, one may note that, even if returns distributions are stable, they may be approximately normal within a finite range; if, further, utility functions are appropriately bounded, the expected utility integral computed over such a normal approximation may be approximately mean-variance. There is a clear need, at this point, for better theories of approximation. The nonstationarity hypothesis would create only minor complications at the theoretical level; the state variables which shift the parameters of the stochastic process must merely be made explicit. Mean-variance theory remains valid locally. The real difficulty is empirical; it would arise when attempting to identify the state variables and estimating the functional link between the parameters to the process and the state variables. Of course, the wider the class of permissible model formulations, the more hazardous the statistical task.<sup>21</sup> Similar comments may be made regarding the possibility of adding on some Poisson jumps.

### *B. Correlations among International Returns and Diversification*

If national financial markets are not perfectly (positively) correlated, investors should be able to reduce their portfolio-variance-risk without sacrificing expected return by international diversification. This simple insight gave rise to a series of papers, including Grubel [80]; Levy and Sarnat [122], who computed also internationally-efficient combinations of stock market indices subject to a short-selling constraint; and Solnik [181] who demonstrated that the additional variance reduction could be obtained with a relatively small number of securities.

Table III illustrates, with data for nine stock markets during the 1971–9 period, the kind of correlation patterns which underlie these earlier studies. Panel A presents correlations between pairs of nominal returns which are not translated into U.S. dollars or adjusted for U.S. inflation: these seem quite similar to those presented in Lessard [117] for the period 1959–73. Translation and deflation do not exchange the coefficients very much: the correlations among real returns in U.S. terms appear in Panel B. The thing to notice about both panels is that the correlations are fairly small. The same pattern holds true for returns measured in other currencies and deflated by other indices.

One other method used to display the potential for risk reduction consists in regressions of individual stock returns or of national market indices on a world market index. This is the route taken by early writers such as Agmon [12], Solnik [180], and Lessard [118]. In Lessard's experiments, the residuals of regressions of the stock's national index or of its local industry index on the world market index proxy were introduced as additional orthogonal factors into the market model test.

In the actual event the fits produced by Agmon's single index, Lessard's multiple factor, and Solnik's "national" and "international" factor models were relatively weak. Generally, at least 40 percent of the variation was left unexplained. This result is not in itself surprising, given the low correlations between

<sup>21</sup> This is especially true when the results of the statistical estimation are to be fed into an optimization program. See the end of Section III.

**Table III**  
**Correlations of Monthly Price Relatives For Nine Major Stock Market Indices:**  
**1/71-12/79**

Panel A: Nominal Monthly Returns in Original Currency											ST.
	GER	BEL	CAN	FRA	JAP	NET	UK	SWI	USA	E.V.	DEV.
GER	1.00									0.005	0.041
BEL	0.48	1.00								0.008	0.038
CAN	0.24	0.45	1.00							0.011	0.051
FRA	0.44	0.60	0.34	1.00						0.009	0.070
JAP	0.35	0.29	0.26	0.13	1.00					0.013	0.050
NET	0.59	0.66	0.52	0.50	0.37	1.00				0.005	0.047
UK	-.03	0.06	-.01	-0.6	0.04	-.04	1.00			0.008	0.820
SWI	0.52	0.64	0.46	0.42	0.37	0.63	-.14	1.00		0.004	0.050
USA	0.28	0.49	0.69	0.34	0.34	0.52	-.005	0.53	1.00	0.005	0.046

Panel B: Real Monthly Returns in \$US, Deflated by U.S. C.P.I.											ST.
	GER	BEL	CAN	FRA	JAP	NET	UK	SWI	USA	E.V.	DEV.
GER	1.00									0.011	0.090
BEL	0.51	1.00								0.012	0.085
CAN	0.25	0.40	1.00							0.004	0.053
FRA	0.41	0.59	0.40	1.00						0.018	0.144
JAP	0.47	0.40	0.28	0.26	1.00					0.011	0.059
NET	0.54	0.65	0.47	0.51	0.39	1.00				0.010	0.104
UK	0.11	0.09	0.66	-.02	0.06	0.12	1.00			0.071	0.764
SWI	0.56	0.70	0.46	0.55	0.48	0.60	-.07	1.00		0.007	0.058
USA	0.34	0.48	0.69	0.38	0.33	0.53	0.02	0.49	1.00	-.002	0.046

Sources: (a) Capital International, *Perspectives*, various issues; (b) OECD Main Economic Indicators.

the indices from various exchanges. It is consistent also with the earlier results of Blume [25] and King [103] for the U.S. market. The size of the firm-specific residual variances in any of these regressions, domestic or international, certainly seem to suggest that considerable scope exists for risk reduction from diversification. However, one should be careful about taking this conclusion much further.

The main limitation of this kind of analysis arises from the choice of the index or indices against which the regression is run. In domestic finance, regressing a security's return on any efficient-portfolio return produces residuals whose variance is easily interpreted as "diversifiable risk," since an investor choosing that portfolio would require no compensation for bearing that risk. Further, in a market which is at equilibrium and in which practically all investors do diversify, the market portfolio is one portfolio known to be efficient (Roll [160]). The normative implication is then fairly simple: diversifiable risk can be measured against the market portfolio. But, when investors' purchasing power units differ by nationality, as we allow below, they will in principle differ in their concept of what an efficient portfolio is (except if they all have logarithmic utilities: see Section III); and there will be no implication that at market equilibrium the market portfolio should be efficient in any sense or, *a fortiori*, that its nominal rate of return measured in any currency should serve as a benchmark for valuation. Any segmentation will compound this effect. As a result, there can be

no simple way to infer the extent to which the gains from variance-risk reduction can be reaped by different nations' citizens merely from analyzing the structure of securities' markets data alone. The data themselves may be consistent with any number of models of portfolio selection and market equilibrium, with or without segmentation. The availability of risk reduction will depend upon which one is true. This point should put an end to all further attempts to base measures of such quantities as "risk reduction" or "diversifiable risk" on sample estimates of the means and variances of and correlations among market or industry indices. The covariance matrix also says nothing about the presence or absence of segmentation.<sup>22</sup>

Suppose, however, that we have an international CAPM where the market portfolio nominal rate of return is the proper benchmark, as is the case, for instance, in Solnik [179]. National or industry factors in this setting are theoretically irrelevant.<sup>23</sup> One may then regress individual stock returns on a single international market index. To an approximation, the residual risk may be diversified away.<sup>24</sup> One may, of course, introduce additional orthogonal factors such as the residuals from regressions of national indices on the world index or from regressions of industry indices on the world index. The expanded specification cannot produce any information regarding the relative benefits of international as opposed to interindustry or purely domestic diversification. This observation underscores the confusion surrounding the question of whether interindustry diversification within countries or international diversification across industries, or some combination of both, can be replicated by international diversification across national indices. This question, which no one has examined systematically, is of considerable practical interest.

In short, the main use of single or multiple-factor market models is probably, much as in Sharpe's [175] diagonal model, the reduction of data requirements for the computation of optimal portfolios. The variability of the world index combined with other factors, or alternatively, the variability of foreign market indices combined with orthogonal factors representing local industry groupings and the world market both leave large fractions of individual assets' variabilities unexplained. The potential for international diversification to reduce risk seems unquestionable. Beyond simplifying data requirements,<sup>25</sup>; however, the available

<sup>22</sup> See Adler and Dumas [3] and Section VII below. Two more criticisms may be leveled against these statistical descriptions. One is potentially devastating: rates of return may not be stationary so that it may not be legitimate to apply correlation or regression analysis to them. A second one may be less important empirically: most of the published work refers to U.S. current dollar returns when in actually deflated returns should have been employed.

<sup>23</sup> This is not a critique of Solnik's [180] empirical procedure. Quite the opposite: he was precisely trying to verify that national factors (orthogonal to the world portfolio) received a zero price.

<sup>24</sup> See, however, Friend and Losq [71] who argued that a simple extension of the CAPM such as Agmon's [12] in which all investors worldwide hold only the world market portfolio, will tend to overstate the gains from international diversification.

<sup>25</sup> These statistics do have descriptive, if not normative, value, however, and they may lead to some more fundamental analysis: one could seek the sources of the correlations between financial markets and the origins and differences between means and variances of return. E.g., is there a connection between these and patterns of trade and specialization?

market-model results offer neither guidelines for the construction of optimally diversified international portfolios nor the basis for evaluating the benefits or value of such diversification. These matters are addressed next.

### III. Portfolio Choice

In view of the fairly low correlation between national financial markets observed in the previous section, it is possible to reap important gains from international portfolio diversification. In the present section we seek the optimal portfolios of worldwide investments which maximize these gains.

Consider a world of  $L + 1$  countries and currencies. Without loss of generality, we measure nominal returns in terms of the  $L + 1$ st currency. Nominal rates of return given in another currency can easily be translated by multiplying one plus the foreign-currency rate of return by the ratio of the end-of-period to the beginning-of-period exchange rate. There are  $N$  nominally risky securities, whose nominal price dynamics in terms of the measurement currency are given by stationary Ito processes (Brownian motions):<sup>26</sup>

$$dY_i/Y_i = \mu_i dt + \sigma_i dz_i \quad i = 1 \dots N \quad (1)$$

where

- $Y_i$  is the market value of security  $i$  in terms of currency  $L + 1$ ;
- $\mu_i$  is the instantaneous expected nominal rate of return on security  $i$ ;
- $\sigma_i$  is the instantaneous standard deviation of the nominal rate of return on security  $i$ ; and
- $z_i$  is a standard Wiener process and  $dz_i$  is the associated white noise.

We also define  $\underline{\Omega}$  as the  $N \times N$  matrix of instantaneous covariances  $\sigma_{i,k}$  of the nominal rates of return on the various securities. Finally, there is one (the  $N + 1$ st) security which is nominally riskless: an interest earning bank deposit or short-term bond denominated in the measurement currency. The instantaneous nominal rate of interest paid on this deposit is denoted  $r$ .

In some applications, it will be useful to distinguish two subsets among the nominally risky securities: the last  $L$  securities may be taken to be the nominal bank deposits denominated in the non-measurement currencies, while the first  $n$  ( $n = N - L$ ) would be stock securities paying a random dividend.<sup>27</sup> If we

<sup>26</sup> See the critique below.

<sup>27</sup> Formally Equation (1) only allows for income in the form of capital gains. A simple change of notation can incorporate dividends. Let  $a$  (constant return-to-scale) dividend  $dx$  be paid in an interval of time  $dt$  with

$$dx/Y = \mu_d dt + \sigma_d dz_d$$

and let the price actually behave as:

$$dY/Y = (\mu - \mu_d) dt + \sigma dz - \sigma_d dz_d \quad (1')$$

then the total rate of return on the stock is given by Equation (1) in the text. Stock price behavior

partition accordingly the covariance matrix  $\underline{\Omega}$ , its southeast block then contains the covariances of exchange rates.

There are  $L + 1$  national investor types, each with homothetic utility functions.<sup>28</sup> The price index  $P^l$  of an investor of type  $l$ , expressed in the measurement currency, follows a stationary process:<sup>29</sup>

$$dP^l/P^l = \pi^l dt + \sigma_\pi^l dz_\pi^l \quad l = 1 \dots L + 1 \quad (2)$$

where  $\pi^l$  and  $\sigma_\pi^l$  are the expected value and standard deviation of the instantaneous rate of inflation as seen by investor  $l$ . We call  $\omega^l$  the  $N \times N$  vector of covariances  $\sigma_{i,\pi}^l$  of the  $N$  risky securities returns with investor  $l$ 's rate of inflation. The superscript  $l$  will be dropped whenever we consider one (generic) investor in isolation.

While they are not strictly inconsistent with the empirical evidence presented in Sections I and II above, one may nevertheless raise strong objections to the *a priori* specification of the functional forms ((1) and (2)) for the dynamics of stock and commodities prices when they should actually be endogenous. Lucas [129] can be interpreted as having shown that the stationary Brownian motion for asset prices in equation (1) is not consistent with the positive risk aversion of (time-additive von Neumann-Morgenstern utility endowed) investors. It would be consistent only with risk neutral investor behavior where  $\mu$  would be the rate of time discount (applicable uniformly to all securities). Under risk aversion, asset prices multiplied by discounted marginal utilities must follow martingales and so discounted asset prices by themselves generally do not, in contradiction with equation (1). For further details, see Lucas [129]. Rosenberg and Ohlson [164] also pointed out that the portfolio choices (9) to be derived from the equations of motion (1) are unreasonable. In a domestic setting at least, these asset demands would imply that the prices of assets relative to each other would all be functions of one and the same random factor (the weighted average of risk tolerances  $\alpha$ ) and so would all be perfectly correlated.

These internal flaws can be corrected by introducing nonstationary Ito processes where the parameters  $\mu$  and  $\sigma$  would be functions of a vector of state variables. The resulting portfolio choices would contain one more (hedge) fund per state variable, in addition to the two funds which appear in (9) below. See Merton [140] or Breeden [30], Stulz [191], and Hodrick [90]. The functions  $\mu(\cdot)$  and  $\sigma(\cdot)$  would in turn have to be endogenized at equilibrium. Procedures for so doing have been provided by Cox, Ingersoll, and Ross [37]. They amount essentially to interpreting the CAPM as a functional equation in these unknown factors, rather than as an algebraic equation giving the expected return on a security. This final step, so far, has only been performed for economies populated with *identical consumer-investors*.

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(1') is still a geometric Brownian motion because

$$(\sigma dz - \sigma_d dz_d)/(\sigma^2 + \sigma_d^2 - 2\rho\sigma\sigma_d)^{1/2}$$

is a Wiener process, for so long as  $\sigma$  and  $\sigma_d$  are constants ( $\rho$  stands here for the correlation between  $dz$  and  $dz_d$ .)

<sup>28</sup> See footnote 10 for references to the more general case.

<sup>29</sup> See the critique below.

For this reason, we are not prepared to propose a complete general-equilibrium model of international capital markets. The heterogeneity in consumption tastes which characterizes international finance would require as yet unknown procedures for computing equilibrium prices. State variables which will ultimately have to be introduced into the model could easily be handled in the portfolio computation below, in the manner of Merton [140] or Breeden [30], but this added complication would not speak to the specific feature of the field which is the heterogeneity in the purchasing power of money. In order to focus on that aspect and to illustrate its implications most vividly, we restrict ourselves to stationary Brownian motions; i.e., to constant  $\mu$ 's and  $\sigma$ 's in Equation (1).

The equation of motion (2) can equally be criticized on the ground that commodities prices should be endogenized. In addition, it might have been preferable to start the analysis with the price dynamics of individual commodities rather than with that of a price index. Were individual prices to follow stationary Brownian motions, price indices generally would not, as expenditure shares fluctuate with relative prices (even if preferences are homothetic<sup>30</sup>). Only when expenditure shares are constant (Cobb-Douglas utility function) can both individual prices and indices simultaneously be Brownian. But we have no reason to favor one assumption over the other. Note that we leave unspecified the reason for the PPP deviations ( $dP^i/P^i - dP^1/P^1$ ); they may equally arise from differences in tastes (as in Stulz [191]) or from CPP deviations. If, as suggested by the empirical evidence of Section I, *ex ante* PPP holds and PPP deviations follow a martingale, the parameters of the stochastic process in (2) can be restricted accordingly.

Assuming homothetic direct utility functions, the material of the Appendix implies that we may express an investor's objective function as:<sup>31</sup>

$$\text{Max } E \int_t^T V(C, P, s) ds \quad (3)$$

where  $C$  is the nominal rate of consumption expenditures,  $P$  is the price level index, and  $V(\cdot)$  is a function homogeneous of degree zero in  $C$  and  $P$  expressing the instantaneous rate of indirect utility. Calling  $w = \{w_i\}$ , the  $(N + 1) \times 1$  vector whose components sum to 1 and which indicates the investor's portfolio choice among the available investment opportunities, his wealth dynamics are:<sup>32</sup>

$$dW = [\sum_{i=1}^N w_i(\mu_i - r) + r]W dt - C dt - \sum_{i=1}^N w_i\sigma_i dz_i \quad (4)$$

where  $W$  is nominal wealth. Denoting by  $J(W, P, t)$  the maximum value of (3) subject to (4), the Bellman principle states that this function must be stationary

<sup>30</sup> Homotheticity only guarantees that indices do not fluctuate with wealth, as relative prices are kept constant. If, in addition, utilities were not homothetic, recall from Breeden [30] and Stulz [191] that two indices (one based on average and the other on marginal expenditure shares) would be needed. A similar observation had been made by Adler and Dumas [6] in the more restrictive context of utilities assuming a quadratic form.

<sup>31</sup> Some bequest function could be added without modifying the results we wish to obtain. Income from sources other than security returns is ruled out.

<sup>32</sup> See Merton [138]. The last portfolio variable  $w_{N+1}$  has been eliminated on the basis of  $\sum_{i=1}^{N+1} w_i = 1$ .

or that its total expected rate of increase must be identically zero:

$$\begin{aligned}
 0 = \text{Max}_{C,w} [ & V(C, P, t) + J_t + J_W \{ [\sum_{i=1}^N w_i(\mu_i - r) + r]W - C \} \\
 & + J_P P \pi + \frac{1}{2} J_{W,W} \sum_{i=1}^N \sum_{k=1}^N w_i w_k \sigma_{i,k} W^2 + \frac{1}{2} J_{P,P} \sigma_\pi^2 P^2 \\
 & + J_{W,P} \sum_{i=1}^N w_i \sigma_{i,\pi} W P ] \tag{5}
 \end{aligned}$$

The homogeneity of degree 0 of the function  $V(C, P, t)$  implies that  $J(W, P, t)$  and  $C(W, P, t)$  which satisfy (5) must be homogeneous of degree zero in  $W$  and  $P$ . By Euler's theorem:

$$J_P \equiv -(W/P)J_W$$

and therefore:<sup>33</sup>

$$\begin{aligned}
 J_{P,W} & \equiv -(1/P)J_W - (W/P)J_{W,W} \\
 J_{P,P} & \equiv -(W/P)J_{W,P} + (W/P^2)J_W \\
 & \equiv 2(W/P^2)J_W + (W/P)^2 J_{W,W}
 \end{aligned}$$

Substituting into (5)

$$\begin{aligned}
 0 = \text{Max}_{C,w} [ & V(C, P, t) + J_t \\
 & + J_W \{ [\sum_{i=1}^N w_i(\mu_i - r) + r - \pi + \sigma_\pi^2 - \sum_{i=1}^N w_i \sigma_{i,\pi}]W - C \} \\
 & + \frac{1}{2} J_{W,W} \{ \sum_{i=1}^N \sum_{k=1}^N w_i w_k \sigma_{i,k} - 2 \sum_{i=1}^N w_i \sigma_{i,\pi} + \sigma_\pi^2 \} W^2 ]
 \end{aligned}$$

The derivatives with respect to the decision variables  $C$  and  $w_i$  are set equal to zero:

$$V_C = J_W \tag{6}$$

$$0 = J_W(\mu_i - r - \sigma_{i,\pi}) + J_{W,W}(\sum_{k=1}^N w_k \sigma_{i,k} - \sigma_{i,\pi})W \tag{7}$$

Defining:  $\alpha = -J_W/J_{W,W}W$  as the investor's risk tolerance,<sup>34</sup> we can rewrite (7) in the form of a required nominal yield on security  $i$ :

$$\mu_i = r + \left(1 - \frac{1}{\alpha}\right)\sigma_{i,\pi} + \frac{1}{\alpha} \sum_{k=1}^N w_k \sigma_{i,k}, \tag{8}$$

an equation we shall have occasion to refer to again. Solving for the optimal portfolio directly in vector notation, we get:

$$w = \alpha \left( \frac{\underline{\Omega}^{-1}(\underline{\mu} - r\underline{1})}{1 - \underline{1}'\underline{\Omega}^{-1}(\underline{\mu} - r\underline{1})} \right) + (1 - \alpha) \left( \frac{\underline{\Omega}^{-1}\underline{\omega}}{1 - \underline{1}'\underline{\Omega}^{-1}\underline{\omega}} \right) \tag{9}$$

where  $\underline{1}$  is an  $N \times 1$  vector of ones and  $\underline{1}'$  its transpose;  $\underline{\mu}$  is the vector of

<sup>33</sup> This procedure has been used by Fischer [62] and Losq [127].

<sup>34</sup> This was not the only possible definition of risk tolerance. An alternative definition (Breedon [30]) is:  $-V_C/V_{C,C}C$ . The two are not equivalent, since the wealth elasticity of consumption is not generally equal to 1.

nominal expected returns,  $\mu$ ;  $\Omega$  is the  $N \times N$  matrix of instantaneous covariances  $\sigma_{i,k}$  of the nominal rates of return on the various securities;  $\varphi$  is the  $N \times 1$  vector of covariances  $\sigma_{i,\pi}$  of the  $N$  risky securities returns with the investor's rate of inflation.

As has been pointed out by Kouri [107] and Losq [127], the optimal portfolio is the combination with weights  $\alpha$  and  $1 - \alpha$  of two component portfolios which we now interpret. As is well known,<sup>35</sup> the logarithmic utility function  $((\rho)^t \ln(C/P))$  implies  $\alpha = 1$ . The first component portfolio (with coefficient  $\alpha$ ) is therefore the portfolio of a logarithmic investor. The formula indicates that its composition is independent of the behavior of commodity prices. This is not a new result:<sup>36</sup>  $\ln(C/P) = \ln C - \ln P$  and therefore commodities prices separate out in the objective function and have no influence on the decisions. As a result, the logarithmic component is the same for all investors, irrespective of nationality; a logarithmic investor would be nationless. Geometrically, this implies that the Markowitz efficient frontiers of all the investors have one point in common (where they are all tangent necessarily). Naturally, the composition is independent of the choice of the measurement currency.<sup>37</sup>

The second component portfolio (with weight  $1 - \alpha$ ) is the portfolio of an investor with zero risk tolerance ( $\alpha = 0$ ). It is therefore, for any given investor, his global minimum variance portfolio in real terms. The formula based on nominal rates of return bears out this interpretation:  $\Omega^{-1}\varphi$  is the vector of regression coefficients of the investor's rate of inflation on the various securities' returns. This portfolio is thus the one whose nominal rate of return is the most highly correlated with the investor's rate of inflation (in measurement currency); or, in other words, it is the best possible hedge against inflation. By Ito's lemma, the random part of real returns is nothing but the random part of nominal returns minus the random part of the rate of inflation. The regression just mentioned minimizes the variance of this difference which is the variance of the real portfolio return. Since this hedge portfolio involves the rate of inflation of commodities' prices, it is investor specific. Its composition is independent of expected nominal returns ( $\mu$ ) since the formula involves only covariances; this is appropriate, since this portfolio minimizes the variance without regard for profitability. The composition is also independent of the choice of measurement currency.

We summarize these results in the following formulae (describing investor  $l$ 's portfolio) and a theorem:

$$w^l = \alpha^l w_{\log} + (1 - \alpha^l) w_h^l \tag{10}$$

$$w_{\log} = \begin{pmatrix} \Omega^{-1}(\mu - r\mathbf{1}) \\ \mathbf{1} - \mathbf{1}'\Omega^{-1}(\mu - r\mathbf{1}) \end{pmatrix} \tag{11}$$

$$w_h^l = \begin{pmatrix} \Omega^{-1}\varphi^l \\ \mathbf{1} - \mathbf{1}'\Omega^{-1}\varphi^l \end{pmatrix} \quad l = 1, \dots, L + 1 \tag{12}$$

<sup>35</sup> Merton [139].

<sup>36</sup> Hakansson [82].

<sup>37</sup> The incredulous reader may check in Sercu [174], Appendix A, where calculations are performed explicitly, using translation of returns, to verify that fact.

**THEOREM.<sup>38</sup>** (*Optimal portfolio strategy for the individual investor*). Every investor in the world holds a combination of:

- the universal logarithmic portfolio with weight  $\alpha$ .
- his personalized hedge portfolio which constitutes the best protection against inflation as he perceives it, with weight  $1 - \alpha$ .

We have computed the logarithmic portfolio<sup>39</sup> as well as the hedge portfolios of U.S. and French investors as they would have been over the years 1971–9, based on *ex post* monthly rates of return. They are displayed in Table IV. The logarithmic portfolio exhibits large positive and negative weights, negative weights implying borrowing or short selling. This is the component portfolio which takes advantage of expected rate of return differentials. There are exact opposite entries in Canadian and U.S. dollar deposits, because, during this period, U.S. interest rates were above Canadian ones without offsetting exchange rate changes.<sup>40</sup> There is a similar pseudo-arbitrage between the Deutsche Mark on the one hand and the Belgian Franc and the Guilder on the other. When comparing stocks and bank deposits, there is a clear tendency for bank deposit entries to be negative in those currencies where the stock entry is positive and vice-versa, although the numbers are by no means exactly opposite. The reason is that exchange rate variations tend not to offset, and to be wider than, stock price variations; hence there is an incentive to hedge stock purchases against currency risk by means of local borrowing.

The hedge portfolios are even more striking. Although we have only shown the U.S. and French hedge portfolios, the pattern is the same for all nationalities.<sup>41</sup> An investor's hedge portfolio is almost entirely made up of a nominal bank deposit (or Treasury Bill or short-term bond) denominated in his home currency. The reason is that exchange rate fluctuations are much wider than price level (CPI) fluctuations, as we observed earlier in Section II. Also, contrary to a

<sup>38</sup> It is known since Black [23] that mean-variance investors who care only about real returns and who are confronted with nationality distinctions, only need (any) two efficient funds. The specific choice of the two funds made here is specially telling in the context of purchasing power differences across investors.

<sup>39</sup> All our statistics and portfolios are conditional. Nominal interest rates presumably contain some *ex ante* information on ensuing inflation rates, exchange rates and stock returns. We have therefore regressed all rates of return on the nine concomitant interest rates of the various currencies and computed all statistics and portfolios from the residuals of the regression. This procedure amounts to treating nominal interest rates as pseudo state variables. As a result, the logarithmic portfolio composition is a (linear) function of the interest rates prevailing and it fluctuates from month to month. We show in Table IV the average portfolio over the entire decade. One referee objected to this statistical treatment on the grounds that it introduces errors in the variables. In practice, the procedure makes little difference: nominal interest rates simply "explain" very little of ensuing variations. It was introduced only in order to ensure that portfolio choices would be exactly invariant with respect to the choice of measurement currency.

<sup>40</sup> The same empirical result is obtained by Braga de Macedo [26] over the period October 1973 to April 1978. It arises, of course, from the strong correlation between the two currencies. But this multicollinearity itself renders unreliable the two figures  $-21.81$  and  $22.01$ . This and other critical statistical problems will be discussed below.

<sup>41</sup> The weights to be placed on their home-currency deposit by the ten national investor types are: for German investors, 1.009; for the Belgians, 1.029; for the Canadians, 0.983; for the French, 0.988; for the Japanese, 1.007; for the Dutch, 0.973; for the British, 1.030; for the Swiss, 1.021; and for U.S. nationals, 0.983.

popular belief, but full in line with the observations of Lintner [125] and Fama and Schwert [60], stocks are not good hedges against inflation, for various reasons. As a consequence, investors who are very risk averse prefer to bear fully their home inflation risk rather than to bear exchange rate uncertainty or stock price uncertainty. It is not clear whether the result would be modified if real estate or commodities such as rare metals were included in the array of possible investments.

The log-portfolio calculations, however, should not be taken at face value. They suggest improbably that investors individually and in the aggregate should short some securities and hold more than 100% of the available supply of others. Beyond the possibility that the calculated weights violate typical short selling constraints such a result, if sustained, would imply that international capital markets are not in equilibrium (a point on which the evidence will be reviewed in sections IV and V).

Moreover the estimates are plagued by major statistical problems which undermine their significance.<sup>42</sup> No statistical theory, to our knowledge, gives the sample distribution of the estimated  $\psi_{\log}$  from equation (11). We are, therefore, unable to build confidence intervals for the optimal log-portfolio composition in Table IV. Simulation experiments using a single risky asset suggest that these confidence intervals are much wider than the hypothesized [0, 1] range. Had we run them with several risky assets, multicollinearity would have compounded the problem. Presumably, it would be possible to develop more efficient estimates of  $\psi_{\log}$  than the one obtained simply by pre-multiplying the estimated average return vector by the inverse of the estimated covariance matrix. Bawa *et al.* [22] have worked on a Bayesian theory aimed in that direction but few applications have been made.

The hedge portfolio composition is, nonetheless, very clearcut and it is doubtful that, as long as one uses CPI's as indicators of inflation, any statistical problem could raise doubts about it. In fact, the small variability of CPI's relative to securities' returns and exchange rates in the countries we have considered provides a rationale for the early work of Solnik [179], generalized recently by Sercu [174],<sup>43</sup> where it is assumed that each investor ignores his home-currency inflation (or assumes it to be null) and therefore considers rates of return expressed in home currency units as being real returns. Quite evidently this is a case of deviation from PPP<sup>44</sup> since different people regard the same securities' returns differently. One consequence of the assumption is that the home currency bank deposit or Treasury Bill is seen as riskless in real terms by the national investors (and only by them). It is not difficult to verify on the basis of equation

<sup>42</sup> This is quite apart from the issue, which we leave aside, of the possible nonstationarity of the rate-of-return distribution.

<sup>43</sup> Solnik assumed independence of exchange rates and nominal local-currency stock returns, and he further made some (internally inconsistent) assumptions regarding exchange rate behavior. Sercu corrected these deficiencies. One referee pointed out that Solnik had produced an appendix to his work where independence was no longer assumed, but he had then reached no simple statement of portfolio strategy.

<sup>44</sup> One controversy arose from Solnik's assumption No. 7 which seemed to imply the absence of trade between countries! This assumption is actually unnecessary, for so long as there are several goods and different consumption tastes, PPP deviations could occur even if trade were unhampered.

**Table IV**  
**Universal Logarithmic Portfolio and Investor Hedge Portfolios for**  
**U.S. and French Investors, Computed from Nominal Rates of**  
**Return February 1971 to December 1979**

	Logarithmic Portfolio (weights sum to 1)	Hedge Portfolios	
		U.S. Investor	French Investor
<b>Stocks</b>			
Germany	-6.18	0.021	0.025
Belgium	6.15	0.000	-0.032
Canada	4.68	0.000	-0.028
France	-1.59	0.000	0.011
Japan	3.01	0.005	0.002
Netherlands	1.34	-0.011	0.014
United Kingdom	0.01	0.000	0.000
Switzerland	0.90	0.001	-0.022
United States	-6.75	-0.020	0.020
<b>Bank Deposits</b>			
Deutsche Mark	11.57	-0.029	-0.047
Belgian Franc	-9.22	-0.003	0.059
Canadian Dollar	-21.81	0.034	0.046
French Franc	3.02	0.004	0.988
Japanese Yen	-0.68	0.034	-0.007
Guilder	-2.79	0.016	-0.009
British Pound	-4.10	-0.024	-0.005
Swiss Franc	1.43	-0.017	0.017
U.S. Dollar	22.01	0.983	-0.032

Data Source: Morgan Guaranty, *World Capital Markets* for one-month deposit rates; Capital International, *Perspectives* for stock price indexes and dividend rates; OECD, *Main Economic Indicators* for U.S. and French CPI's.

Warning: Monthly dividends are taken to be the last 12-month dividend divided by twelve.

(12) that in this case the hedge portfolio reduces to the home deposit. Consider the portfolio problem of an investor of country  $l$ ; since he assumes that his rate of inflation measured in currency  $l$  is zero (or nonrandom), the same rate of inflation translated into the measurement currency  $L + 1$  and to be introduced into the covariance vector  $\omega^l$ , reduces to the rate of change of the  $(L + 1/l)$ th exchange rate; and since the translated rate of return on the currency  $l$  treasury Bill to be introduced into the covariance matrix  $\underline{\Omega}$  is also equal, in its random part, to the same exchange rate change, the regression of inflation on securities which underlies formula (12) will produce a unit coefficient on the currency- $l$  Bill and zeros on all other securities. Hence:<sup>45</sup>

<sup>45</sup> Solnik [179], in one of his separation theorems, had further split up the logarithmic portfolio into two: one whose return was independent of exchange rate changes and one which was fully dependent upon them (the latter being made up of bank deposits only). Sercu [174] did the same but only in order to show that Solnik could be generalized. While this procedure is always open (see also our Section VIII) even in the general case, there is really no point to it since the logarithmic portfolio in its entirety is held by all investors. If we split it into two parts, the two subportfolios will be held in the same proportion by all.

COROLLARY. (*Solnik; Sercu*)

When home inflation (measured in home currency and using home consumption weights) is zero (or nonrandom), every investor in the world holds a combination of:

- the universal logarithmic portfolio with weight  $\alpha$ .
- his home currency Treasury Bill or bank deposit, with  $1 - \alpha$ .

Statements such as these generated considerable interest among macroeconomists (Kouri and de Macedo [110], Dornbusch [43], Krugman [113]) aiming to explain spot exchange rates and, more specifically, to find links between the current account and the exchange rate.<sup>46</sup> Portfolio balance theorists such as Branson [28], Girton and Henderson [75, 76], and Kouri [109] worked with postulated asset demand functions and assumed that investor's exhibited "home habitat" preference: they demand their home-currency denominated asset relatively more than foreigners do. A country's current account surplus which places more wealth in the hands of home investors consequently raises the demand for home currency assets and causes a rise in the value of the home currency.

The corollary above provides the microeconomic foundation for this reasoning: if a shift in wealth in favor of the home country leaves the world logarithmic demand more or less unchanged, it will usually raise the demand for the home Treasury bill. Braga de Macedo [26] and Krugman [113] suggested that the reasoning will be correct only if the home risk tolerance  $\alpha < 1$  (i.e., home investors are holding the home bill rather than borrowing at that rate).<sup>47</sup> Strong further assumptions are required, however, to make this an exact equilibrium argument. These are detailed in the next section. One is tempted to accept Krugman's conjecture that the magnitude of the effect is small.

Actually, in much of this body of literature, the optimal portfolio strategy theorem has been specialized in another way which is often less favorable to the argument. Imagine that a country's *output* prices are fairly stable (or nonrandom) when expressed in the local currency. A given investor consumes in certain proportions goods produced in his home country which have a stable price and goods imported from various foreign countries whose translated prices fluctuate like exchange rates. He composes his purchasing power index accordingly. When, following formula (12), this index is regressed on the various securities' returns, including the translated returns on foreign Treasury Bills, which vary like exchange rates, the weights of the hedge portfolio obviously reconstruct the consumption mix. When the investor spends 10% of his consumption budget on goods imported from France, 10% of his hedge portfolio is devoted to French franc bank deposits as a hedge against the translated price of French imports.<sup>48</sup>

COROLLARY. (*Kouri and de Macedo*)

When the various countries' local-currency output prices are nonrandom, the weights falling on foreign currency Treasury Bills or bank deposits in an invest-

<sup>46</sup> We are grateful to Jeffrey Frankel for bringing this important insight to our attention.

<sup>47</sup> A frequently accepted value is  $\alpha = \frac{1}{2}$  (risk aversion equal to 2).

<sup>48</sup> Kouri and de Macedo actually have a more general model where output prices are allowed to be random. But the theorem to be given, which is our responsibility, captures their contribution most strikingly.

tor's hedge portfolio replicate his consumption mix according to origin of the goods.<sup>49</sup>

The contrast between the two corollaries should serve to highlight the dependence of the hedge portfolio composition on the choice of the commodities price index: the CPI versus one which incorporates explicitly the prices of imports. The reason for this difference is to be found in the odd behavior of national CPI's: they do not seem to reflect immediately the variations in import prices arising from exchange rate changes. Why they do not is an open issue presumably linked to the behavior of importing firms and to the theory of commercial contracting.

#### IV. Partial Pricing: $n$ Assets Priced Relative to $L + 1$ Other Assets

In the tradition of the Capital Asset Pricing Model (CAPM) of Sharpe [176], Lintner [124], and Mossin [143], equilibrium in the capital market is characterized by a relationship between the required yields on the various assets. The demand is assumed to originate from investors who hold optimal portfolios given by Equation (10) and the supply is assumed to be fixed. One asks the question: what return must this security bring relative to another security so that investors are willing to hold both in the proportion in which they are available? In the international context, the heterogeneous perceptions of real returns, due to PPP deviations, will prevent us from answering this question for every security; we shall have to take as given the expected rates of return of as many securities as there are countries ( $L + 1$ ) and price the other securities ( $n = (N + 1) - (L + 1)$ ) relative to these.

The *nominal* yield required on the various securities by an individual investor in order for him to be willing to hold a given portfolio  $w$  is given to us by Equation (8) which we reproduce here for convenience, emphasizing with a superscript the terms which depend on the identity  $l$  of the investor:

$$\mu_i = r + (1 - 1/\alpha^l)\sigma_{i,\pi}^l + (1/\alpha^l) \sum_{k=1}^N w_k^l \sigma_{i,k}; \quad i = 1 \dots N \quad (8 \text{ repeated})$$

As we saw, this equation is equivalent to the formulation of portfolio demand (9) for given yields. It may be rewritten in the following form:<sup>50</sup>

$$\mu_i = r + \sigma_{i,\pi}^l + (1/\alpha^l) \sum_{k=1}^{N+1} w_k^l (\sigma_{i,k} - \alpha_{i,\pi}^l); \quad i = 1 \dots N \quad (13)$$

In this formula, the last term contains the covariance of the nominal return on security  $i$  with the nominal return on security  $k$  minus the covariance between the nominal return on security  $i$  with the rate of inflation; this difference is, in effect, the covariance between the nominal return on security  $i$  with the *real* return on security  $k$ ; hence the summation is the covariance between the nominal return on security  $i$  and the real return on the investor's portfolio.

<sup>49</sup> Here is a comment by Braga de Macedo [26]: "That the effect of an increase in the relative demand for country 1 goods increases the relative demand for country 1 currency is similar to the condition for stability in a flow view of the foreign exchange market, whereby the demand for currency is derived from the supply and demand for exports and imports." Some tests on the external demand for U.S. dollars were run by Dumas and Poncet [50].

<sup>50</sup> Recall that  $\sum_{k=1}^{N+1} w_k = 1$  and that  $\sigma_{i,N+1} = 0$  for all  $i$  since the last security is nominally riskless.

The intuitive meaning of (13) is therefore as follows. A security must bring a nominal return in excess of the nominal rate of interest, which is made up of two premia. The last one is a risk premium proportional to the covariance of the security's nominal rate with the investor's real portfolio return. A covariance with a portfolio is the usual measure in the CAPM literature of the risk contributed by a security to a portfolio (its marginal risk). When investors are concerned with their purchasing powers, they relate the required nominal yield on each asset to the real returns on their benchmark portfolio, much as one would expect.

The first premium in (13) is *not* a risk premium as it would exist even if the investor exhibited zero risk aversion ( $1/\alpha = 0$ ). It reflects the fact that investors predicate their portfolio choices on real returns. The expected real return on a security depends on the expected value of the nominal return, the expected value of inflation, *and* the covariance between the nominal return and inflation. Deflation involving a product, it generates a covariance when computing expected values. We may perhaps call this the inflation premium.

Formulae (8) or (13), although valid for every investor, are unfortunately not usable directly to obtain, in an empirically meaningful fashion, the required yield on the various securities. This is because we cannot observe the individual portfolio holdings,  $w_k^l$ . The only portfolio which is directly observable by reading the prices in the newspaper is the aggregate one, given by the relative market capitalizations of all the securities on the market: the market portfolio<sup>51</sup>  $w^m$  with

$$w_k^m = \sum_l W^l w_k^l / \sum_l W^l$$

where the summation is taken over all the investors and  $W^l$  is investor  $l$ 's nominal wealth. We must therefore transform (8) into an equation valid at the aggregate market level.

The operation of aggregation is typically performed by multiplying (8) by  $\alpha^l$  and taking an average over all investors, where the weights are their relative wealths. But in the present case, with PPP deviations and with investor specific measures of inflation, the result will be disappointing:

$$\mu_i = r + (1 - 1/\alpha^m) \frac{\sum_l (1 - \alpha^l) W^l \sigma_{i,\pi}^l}{\sum_l (1 - \alpha^l) W^l} + (1/\alpha^m) \sum_{k=1}^N w_k^m \sigma_{i,k} \quad i = 1 \dots N \quad (14)$$

where

$$\alpha^m = (\sum_l W^l \alpha^l) / (\sum_l W^l)$$

The disappointment is that the second term of (14) is now unobservable. Indeed, it contains the covariances of security  $i$  with the various investors' rate of inflation, weighted by their wealth *and* by one minus their risk tolerance. It is evidently out of the question to measure each individual's risk tolerance.<sup>52</sup> But

<sup>51</sup> Roll [160] objects that, in a world where the prices of most assets (e.g., real estate or human capital) are observed infrequently, if ever, the market portfolio itself is not observable. Worse yet, no proxy measure of this portfolio will work adequately in a test of the CAPM.

<sup>52</sup> This stumbling block is intrinsic to the international setting of heterogeneous investors' rate of inflation. No model known to us is capable of collapsing the wealth and risk-tolerance weighted

note that the problem may not be as large as it seems. The summation in the second term can be performed in two steps: once over all the individuals of the same nation who use the same deflator and therefore have the same  $\sigma_{i,\pi}$ ; and then once again over the several nations, using the national wealth weighted average risk tolerances in lieu of the individual ones. We therefore have only a sum of  $L + 1$  terms, one for each nation. The result is a CAPM containing  $L + 1$  terms of covariance with inflation in addition to the intercept and the covariance with the market. This "multi-beta" CAPM may be tested directly for so long as the number of data points (i.e., securities) is sufficiently large ( $N + 1 \gg L + 3$ ). The hypotheses to be tested are that the intercept is equal to the nominal measurement-currency interest rate, that the regression coefficients on all the covariance terms sum to one and that the coefficient on the covariance with the market is positive.

There exists, however, another procedure involving a prior analytic transformation of (14), which leads to a useful economic result. When the model is exact, this second approach is strictly equivalent to the previous one. It starts with the observation that the main difficulty arises from the  $L + 1$  national weights (summing to one):

$$(1 - \alpha^l)W^l / \sum_l (1 - \alpha^l)W^l$$

which are not observable. In order to compute them, one may reverse the problem partially; i.e. take the expected yields on  $L$  nominally risky securities (e.g., the last  $L$  ones) as given, assume that they conform exactly to the model and use them to solve for the unknown weights. The weights so obtained can then be substituted back into (14) to compute the required yields on the other  $n$  ( $n = N - L$ ) securities. The effect of this procedure is to make the covariances between rates of return and inflation independent of investors or, equivalently, to set the covariances between rates of return and PPP deviations, which reflect the differences among investors, equal to zero.

To achieve the requisite transformation of (14), we introduce  $\gamma_i$ : this is the vector of regression coefficients from a regression of the returns of security  $i$  ( $i \leq n$ ), on the last  $L$  securities, so specified as to render the residuals independent of PPP deviations. For the given covariance matrix, the definition of  $\gamma_i$  emerges from:<sup>53</sup>

$$\begin{aligned} \sigma_{i,\pi}^l - \sum_{k=n+1}^{n+L} \gamma_{i,k} \sigma_{k,\pi}^l &= \sigma_{i,\pi}^{L+1} - \sum_{k=n+1}^{n+L} \gamma_{i,k} \sigma_{k,\pi}^{L+1}; & i = i, \dots, n, \\ & & l = 1, \dots, L. \end{aligned} \quad (15)$$

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average rate of inflation into one observable number. The same problem arises, for instance, in the consumption CAPM of Breeden [30] and Stulz [191] when PPP is violated.

<sup>53</sup> The technique leading to (15) and (16) is an extension of Sercu [174] to the case with random domestic inflation rates. Pricing stocks relative to bonds requires partitioning and inverting the covariance matrix. The partition of the inverse corresponding to stocks can then be identified as the inverse of a matrix of the *residuals* from regressions of each stock on the set of bonds, so structured as to render these regression residuals themselves independent of PPP deviations. The  $\gamma_i$  emerge naturally as the *coefficients* of these regressions and represent the weights of a portfolio of bonds which immunizes stock  $i$  from the PPP deviations. A stock combined with its associated hedge portfolio is a hedged stock. Equation (15) defines the  $\gamma_i$  vector as the solution to setting the covariance between the hedged stock and PPP deviations equal to zero.

The suggested procedure then leads to the following CAPM:

$$\begin{aligned} \mu_i - \sum_{k=n+1}^{n+L} \gamma_{i,k} \mu_k &= r(1 - \sum_{k=n+1}^{n+L} \gamma_{i,k}) + (1 - 1/\alpha^m)(\sigma_{i,\pi}^l - \sum_{k=n+1}^{n+L} \gamma_{i,k} \sigma_{k,\pi}^l) \\ &+ (1/\alpha^m)[\sum_{j=1}^N w_j^m(\sigma_{i,j} - \sum_{k'=n+1}^{n+L} \gamma_{i,k'} \sigma_{k',j})]; \quad i = 1, \dots, n; \quad \forall l; \end{aligned} \quad (16)$$

where it will be noted that the right-hand side has the same value (by virtue of (15)) no matter which national rate of inflation ( $l = 1, \dots, L + 1$ ) is used.

The economic interpretation of the above is clearest if one visualizes the  $\gamma_i$ 's as side investments in the last  $L$  securities which would accompany negatively each unit investment in security  $i$ , ( $i \leq n$ ). The purpose of these auxiliary investments is revealed by equation (15) which is a condition on the net nominal return from security  $i$  and its associated bundle of securities held short. The left-hand side is the covariance of this net return with investor  $l$ 's purchasing power index and the right-hand side is the covariance with investor  $L + 1$ 's index. Taking the difference between the left and the right-hand sides, equation (15) says that the side investments associated with security  $i$  are chosen in such a way that the net return is linearly independent of the purchasing power deviation between investor  $l$  and investor  $L + 1$  ( $dP^l/P^l - dP^{L+1}/P^{L+1}$ ), and generally independent of all the  $L$  basic PPP deviations which may arise between  $L + 1$  national investor groups. The side investments therefore constitute a hedge of security  $i$  against PPP deviations. It is natural to choose the  $L$ , non-measurement currency bonds as the hedging vehicles. The result is a CAPM which prices stocks relative to the  $L$  bonds, that is, which provides the expected returns on stocks only when all  $L + 1$  nominal interest rates are given.

Consider, then, the following zero-investment bet:

- invest 1 measurement currency unit in security  $i$ ,
- borrow (short sell)  $\gamma_{n,k}$  units in securities  $k = n + 1$  to  $n + L$ ,
- borrow  $1 - \sum_{k=n+1}^{n+L} \gamma_{i,k}$  units riskfree in the measurement currency.

The net expected return (in measurement currency) of this bet is the left-hand side of (16) minus the first term on the right-hand side. According to (16), this net expected return is linearly related to the covariance with inflation and the covariance with the market, exactly as in the nominal CAPM of, e.g., Friend, Landskroner and Losq [70]. Hence:

**THEOREM.** *The net nominal required yield on a security hedged against PPP deviations is given by the traditional nominal Capital Asset Pricing Model.<sup>54</sup>*

As noted, equation (15) specifies the  $\gamma$ 's as coefficients of a regression of the first  $n$  securities on the last  $L$  ones, such that the residuals are independent of PPP deviations. We use the  $L$  nonmeasurement-currency bonds as the reference securities. These securities are nominally risky only because exchange rates are random. In practice, the regression can be estimated by instrumental-variables techniques, [100, p. 278], with the PPP deviations playing the role of

<sup>54</sup> The reader should carefully avoid a misunderstanding regarding the words "hedged against PPP deviations." The theorem means that we, as financial economists, know how to price a security by the standard CAPM once we have associated to it a combination of securities which constitutes a hedge against PPP deviations. There is no implication that PPP deviations are some kind of separate risk against which any one investor would want to hedge. See the remarks at the end of Section I and in Section VI below.

the instrumental variables. This is largely equivalent to a 2 SLS procedure. In the first stage, one regresses the exchange rates on the PPP deviations: in the second stage, one regresses the stock returns on the fitted values from the first stage. The coefficients in the second stage provide estimates of the  $\gamma_i$  vector; and by construction the residuals are independent of the PPP deviations.

In the Solnik-Sercu special case (see Section III) of zero local currency inflation as seen by local investors, PPP deviations are collinear with exchange rates. Hence, there is no distinction between the instrumental variables and the regressors, and the regression reduces to an ordinary least squares regression of the first  $n$  securities (stocks) on exchange rates. In that case, the regression coefficients  $\gamma$  may be interpreted as the sensitivities of the stocks with respect to exchange rates, or as their "exposures" to exchange risks. We shall return to this notion in our Section VIII on corporate policy. The theorem may thus be specialized:

**COROLLARY NO. 1 (Solnik-Sercu).** *When local inflation rates are zero, the net nominal required yield on a security hedged against exchange risks by means of multi-currency borrowing and lending, is given by the traditional nominal<sup>55</sup> Capital Asset Pricing Model.*

There is actually an alternative way to derive and to state this corollary. We saw in Section III that, in this special case, the hedge portfolios are entirely made up to the investors' respective home Treasury bills. Stock securities, that is, receive a zero weight in the hedge portfolios and are therefore held by investors only as part of their logarithmic portfolio. Hence:

**COROLLARY NO. 2.** *When local inflation rates are zero, the world market portfolio of stocks and the stock part of the logarithmic portfolio are proportional to each other. If, in addition, Treasury bills are in zero net supply, we have*

$$\alpha^m w_{i,\log} = w_i^m \quad i = 1, \dots, n$$

Substituting (11) into this relationship and partitioning out the elements corresponding to stocks would directly produce the CAPM alluded to in Corollary No. 1. The  $\gamma$  coefficients would appear as one partitions the inverse covariance matrix. One word of warning is in order: this corollary does not say that the market portfolio as a whole is efficient for anybody.

By way of illustration, we have computed the coefficients  $\gamma$  for the stock market returns included in our sample (described in Section II). This was done by regressing each market index (translated into dollars) on all exchange rates using the instrumental variables technique described above. Because exchange rates are so closely correlated with PPP deviations (cf. Table II), however, we may regard these coefficients as approximate exposures to exchange risk (i.e., OLS regressions on exchange rates). The numbers therefore are of great descriptive interest. They are displayed in Table V. It appears, from the diagonal elements of the array, that most European stock markets and Canada are

<sup>55</sup> In that case, it is immaterial whether one uses a nominal or real CAPM as the second term on the right-hand side of (16) (the covariance with inflation) is now equal to zero.

Table V

The coefficients  $\gamma$  or the deposit combinations needed to hedge one U.S. dollar invested in the national stock securities against PPP deviations. They are also approximately the exposures of the U.S. dollar rates of return on stocks to the national exchange rates

	Deposits (i.e., Exchange Rates)							
	Germany	Belgium	Canada	France	Japan	Netherlands	United Kingdom	Switzerland
<u>Stocks</u>								
Germany	1.540 (0.057)	-0.114 (0.077)	-0.205 (0.253)	0.071 (0.040)	-0.218 (0.126)	0.135 (0.055)	0.331 (0.131)	-0.483 (0.107)
Belgium	-0.033 (0.048)	1.514 (0.064)	-0.311 (0.212)	0.071 (0.034)	-0.136 (0.106)	0.160 (0.047)	0.033 (0.110)	-0.404 (0.090)
Canada	-0.057 (0.095)	0.081 (0.128)	1.931 (0.420)	0.188 (0.067)	-0.264 (0.210)	0.273 (0.092)	-0.048 (0.218)	0.068 (0.177)
France	0.005 (0.098)	-0.115 (0.132)	-1.765 (0.432)	1.771 (0.069)	-0.057 (0.216)	0.099 (0.095)	0.207 (0.224)	-0.344 (0.183)
Japan	0.276 (0.097)	-0.021 (0.131)	0.189 (0.428)	0.047 (0.068)	0.439 (0.214)	0.084 (0.094)	0.167 (0.222)	0.086 (0.181)
Netherlands	0.064 (0.069)	-0.038 (0.092)	0.151 (0.303)	0.118 (0.048)	-0.140 (0.151)	1.576 (0.067)	0.138 (0.157)	-0.559 (0.128)
United Kingdom	2.608 (1.652)	0.758 (2.223)	0.058 (7.289)	-1.673 (1.162)	-1.632 (3.644)	2.481 (1.600)	-9.220 (3.78)	2.621 (3.087)
Switzerland	0.130 (0.079)	0.319 (0.106)	0.112 (0.347)	0.146 (0.055)	-0.250 (0.173)	0.209 (0.076)	0.407 (0.180)	0.269 (0.147)
United States	0.078 (0.084)	0.168 (0.113)	0.803 (0.373)	0.074 (0.059)	-0.109 (0.184)	0.268 (0.082)	0.007 (0.194)	-0.244 (0.158)

Note: "Exposure" to the U.S. dollar (the measurement currency) may be obtained by row complementation to one. E.g., exposure of the U.S. dollar is -0.045. See the text. Actually the array so completed is globally invariant to the choice of the reference currency, i.e., if everything had been measured in terms of francs instead of U.S. dollars, the U.S. dollar column (showing the very same numbers as the row complements to one) would have come out of the regression and the French franc column would have disappeared, but every other number would have been the same. Standard deviations are shown in parentheses.

Sources of data: See Table IV.

overexposed<sup>56</sup> to their respective currency. The United Kingdom, however, provides results which are difficult to rationalize. Japanese and Swiss stock are remarkably well diversified as far as their vulnerability to exchange rate changes are concerned. Finally, United States stocks are mostly exposed to the U.S./Canada exchange rate but very little exposed to the overall posture of the U.S. currency. We cannot offer a theory which explains these results, but it is conceivable that an international extension of Fama [57] would provide one.

Specialized versions of the International Asset Pricing Model (15) have been submitted to empirical tests (Solnik [180] and Agmon [12]). The tests have been inconclusive both from a statistical standpoint and also in view of Roll's [160] general critique of such tests: the world market portfolio is an elusive entity which is probably badly proxied by any available index. In Solnik [180] the IAPM, which puts a price on the systematic risk measured against the world market portfolio, was tested against the alternative hypothesis that national factors (orthogonal to the world market portfolio) also receive a price. The absence of statistical significance was due to the large specific risks of individual securities and to the relatively small share of the variance of returns explained by national factors. For details on empirical tests, see Solnik [182] and the discussion by Dumas [45].

### V. The International Structure of Interest Rates and the Forward Exchange Market

In a world of  $L + 1$  nations, we have so far succeeded in pricing all assets except  $L + 1$  of them, and we take these assets to be the  $L + 1$  local currency bank deposits or Treasury bills. As far as these assets are concerned, we have no choice but to use Asset Pricing Model (14) which, as we have already observed, is not directly testable with the usual data. Letting exchange rates appear explicitly, we have

$$r_i + \theta_i = r_{L+1} + (1 - 1/\alpha^m) (\sum_l (1 - \alpha^l) W^l s_{i,\pi}^l) / \sum_l (1 - \alpha^l) W^l + (1/\alpha^m) \sum_{k=1}^N w_k^m s_{i,k}; \quad i = 1, \dots, L \quad (17)$$

where

- $r_i$  is the nominal interest rate on the currency  $i$  bank deposit;
- $\theta_i$  is the expected value of the instantaneous rate of change of the exchange rate of currency  $i$  against the measurement currency  $L + 1$ ;
- $r_{L+1}$  is the measurement currency interest rate, so far denoted simply  $r$ ;
- $s_{i,\pi}^l$  is the covariance of exchange rate  $i$  with national investor  $l$ 's rate of inflation; and
- $s_{i,k}$  is the covariance of exchange rate  $i$  with the translated return on asset  $k$ , including for  $k = n + 1$  to  $N$ , the covariances with the exchange rates themselves.<sup>57</sup>

<sup>56</sup> One hundred percent exposure would be the case where the array contained ones in the diagonal and zeros everywhere else. Such was implicitly the assumption in the original Solnik [179] model where local-currency rates of return were assumed independent of exchange rates. A one-dollar investment in German stocks would then simply be hedged by borrowing one dollar's worth of Deutsche Marks.

<sup>57</sup>  $s_{i,\pi}^l$  is another notation for  $\sigma_{n+1,\pi}^l$  and  $s_{i,k}$  another notation for  $\sigma_{n+1,k}$ .

Forward exchange contracts are redundant in our model.<sup>58</sup> Were they to exist then, in the absence of impediments to arbitrage, the forward rate would be set by interest rate parity (IRP). Calling  $f_i$  the percentage difference between the forward and spot rates (premium if positive, discount if negative), IRP implies

$$f_i = r_{L+1} - r_i \tag{18}$$

i.e., the forward premium equals the interest rate differential. As a result, at equilibrium we have the so-called “Fisher open” or “uncovered IRP” relationship:<sup>59</sup>

$$f_i = \theta_i - (1 - 1/\alpha^m)[(\sum_l (1 - \alpha^l) W^l s'_{i,\pi}) / \sum_l (1 - \alpha^l) W^l] - (1/\alpha^m) \sum_{k=1}^N w_k^m s_{i,k} \tag{19}$$

or

$$f_i = \theta_i - [(\sum_l (1 - \alpha^l) W^l s'_{i,\pi}) / \sum_l (1 - \alpha^l) W^l] - (1/\alpha^m) \sum_{k=1}^{N+1} w_k^m [s_{i,k} - (\sum_l (1 - \alpha^l) W^l s'_{i,\pi}) / \sum_l (1 - \alpha^l) W^l] \tag{20}$$

Equations (19) and (20) imply that the forward rate is biased predictor of the ensuing spot. Repeating here a comment made previously in various contexts, the spread appearing in (20) between the forward rate and the expected spot is made up of two premia. One would exist even in the absence of risk aversion ( $1/\alpha^m = 0$ ) but would disappear under nonrandom measurement-currency inflation and Purchasing Power Parity (i.e. nonrandom inflation for all when all rates are expressed in the measurement currency). The mathematical and economic reasons for its existence were given following Equation (13). Under risk neutrality, the expected value of the spot rate *deflated* must be equal to the forward rate also deflated.<sup>60</sup> The covariance between the spot rate and the deflator (deflators

<sup>58</sup> Selling francs forward is equivalent to selling borrowed francs spot and investing the proceeds in dollars. Hence the forward rate must be equal to the simultaneous spot rate corrected for the interest rate difference. Otherwise some arbitragers could reap instantaneous riskless profits (aside from default risk). We do not review here the evidence on the Interest Rate Parity relationships. See Frenkel and Levich [69], Herring and Marston [89] and, for a review, Kohlhaugen [106] Section II. See also footnote 68 below.

<sup>59</sup> In the Solnik-Sercu special case of no local inflation as seen by local investors, we have:

$$s'_{i,\pi} = s_{i,n+i} (= \sigma_{n+i,n+i})$$

i.e., translated nonmeasurement currency inflation rates behave like exchange rates.

<sup>60</sup> i.e.:

where

$$FE(1/I) = E(S/I);$$

$F$  = forward exchange rate (in measurement currency units per unit of foreign currency);

$S$  = future spot rate quoted the same way;

$I$  = price index expressed in measurement currency units; and

$E$  = expected value operator

Reasoning in real terms, as we do here, produces the inflation premium and also disposes of the so-called Siegel [177] paradox. Siegel argued that it is impossible simultaneously that  $F = E(S)$  and, after changing measurement currency, that  $1/F = E(1/S)$  since by Jensen’s inequality,  $E(1/S) > 1/E(S)$ . The paradox was quickly dismissed as a trivial mathematical inconvenience without economic or empirical significance. Actually, in our formulation, there is no paradox. Note that in the above equation, the price index  $I$  has a currency dimension. If we switch currencies around, as Siegel did,

in the absence of PPP) is the source of this premium.<sup>61</sup> The second premium in (20) is a plain risk premium and it is linked to the covariance of the exchange rate with “the” real return on the world market portfolio. This real return is computed using an average worldwide rate of inflation, where the weights in the average are, as in the first premium, the national wealths times one minus national risk tolerances. Again, the unobservability of the risk tolerance is the reason for the nontestability of formulae (17), (19), and (20). The world market portfolio composition is given as usual by the relative capitalizations of all the assets. It contains, therefore, all the assets which are not globally in zero net supply; Frankel [64] baptized these “outside assets.” When all assets are “inside” (all in zero net supply) and returns are stationary, there is no risk premium; i.e. the formula reduces to what it would be if  $1/\alpha^m = 0$ .

This equilibrium model of the forward rate is useful in at least two respects. First, it provides a focus for a short review of the empirical literature concerned with the “efficiency” of the exchange markets. Second, it serves to reveal the shortcomings of the so-called “Modern Theory” which features in the conventional account of the forward markets.

There are no direct tests of Equation (20) and it is unlikely that there will be until the problem of estimating risk tolerance under diverse consumption preferences is solved. Roll and Solnik [163] tested a very special version in which the second term of (20) was omitted and equal weights were used in the third. They were unable to establish conclusively that risk premia exist. Frankel [65, 66] must be credited for testing the hypothesis that the risk premium may fluctuate with the supplies of “outside assets” and specifically with the supply of government debts (cumulated government deficits). But he was unable to produce evidence of a significant link with these quantities.

What appears most frequently in the literature is time-series analyses of the nominal difference between the (logarithms of the) forward and subsequent spot rates, uncorrected for inflation. This difference reflects the nominal returns to forward speculation (sell forward, buy at the future spot) or, equivalently, the forecast error if one takes the forward rate as a predictor of the spot. The literature asks two related questions: is the expected forecast error zero and are forecast errors serially uncorrelated? Taken together, the hypothesis is that the

we must also translate the price index ( $I \rightarrow I/S$ ). Then we have:

$$(1/F)E(S/I) = E[(1/S)/(I/S)]$$

which is equivalent to the first equation. So there is no contradiction. To counter one referee’s misunderstanding, note that we are not arguing here that the inflation premium is unimportant; quite the opposite; we say that it takes care of the Siegel paradox.

<sup>61</sup> Solnik [179] overlooked this premium because he was missing the term  $1 \times \sigma_{i,x}^l$  in his version of Equation (11). As a result, his comments regarding the weighting of the various assets in the overall premium are incorrect. This is unfortunate: he had identified the weights with the net investment position of each country. Under his restrictive set of assumptions (recall from footnote 43 that he assumed not only absence of local inflation but also independence of exchange rates and local-currency stock returns; so that:  $s_{i,x}^l = s_{i,n+l}$  and  $s_{i,k} = s_{i,n+k}$ ), his interpretation holds for the weights appearing within the second premium of (23) only.

forecast error follows a random walk.<sup>62</sup> This provides an indirect test of whether the premia in (20) are zero.

Evidence against the random walk hypothesis, which was at first the prevailing one, is mounting.<sup>63</sup> Levich [120] discovered biases which were of opposite sign depending on the direction of change of the spot rate. These he attributed not to risk premia, but to transaction costs which penalize speculation and keep the forward below the expected spot when the latter is rising and vice-versa. Cornell [34], Geweke and Feige [74], Hansen and Hodrick [83], and Cumby and Obstfeld [39] all found in floating rate data, both post-World War I and in the recent past, instances where the unconditional mean bias was significantly different from zero, and whether it was zero or not, other cases where forecast errors were serially correlated. These results do not, of course, imply anything about the efficiency of the forward exchange markets although some of the authors above motivated their tests by appeals to the efficient markets hypothesis.<sup>64</sup> They are, however, consistent with the existence of biases or premia, perhaps of the type described in (20), which fluctuate widely and in a serially correlated fashion. Indeed, Stockman [187] was able to find evidence of variable premia: when he split his sample into two subperiods, the premia which were not significantly different from zero in each subperiod were nevertheless significantly different from each other, for some currencies. There is need for a general equilibrium theory which would identify the exogenous determinants of the premia.

Equation (20) further enables us to confront Tsiang's [196] theory of the forward exchange market, subsequently baptized the Modern Theory (MT) by Stoll [189]. The MT apparently still enjoys currency in the thinking in central banks. In its later manifestations, the MT was used to account for deviations from IRP, to justify the forward rate as a predictor of the future spot, and as an underpinning for official intervention, often righteously termed "counter speculation." In the MT, forward transactions are contracted between two distinct and separate classes of traders. Speculators take open forward positions and link the forward rate to the expected future spot rate. Arbitragers demand forward contracts when the forward rate deviates from IRP and link the forward rate to

<sup>62</sup> Absence of serial correlation does not imply a random walk. But, since the hypothesis is usually couched in the form of a regression model, the assumption of stationarity of the residual term is needed anyway for statistical purposes. The random walk model is really the one being tested.

<sup>63</sup> See Kohlhagen [104] and Dufey and Giddy's [44] "submartingale" model. The literatures on the efficiency of the foreign exchange market prior to and until 1977 has been exhaustively surveyed by Kohlhagen [106]. There is no need to reproduce this work here. Also discussed there are some crucial macroeconomic questions which are related to the matter of efficiency but are too remote from our topic of international portfolio choice to be discussed here; these are: the impact of trade flows on the forward rate due to "hedging pressure" (Levin [121], Dooley [40]); the impact of official intervention (Kohlhagen [105]); and whether speculative activity can be destabilizing ("bandwagon" effects) and, if so, what measurement method would allow us to identify the periods where it is.

<sup>64</sup> By itself, market efficiency does not imply that the forward rate is equal to the expected spot. This used to be a frequent misconception. In the presence of risk aversion, or nonconstant required real returns, there is also no implication that market efficiency leads to serially uncorrelated returns. See Lucas [129]. But in practice, risk aversions are sufficiently low as to produce very low serial correlations. Witness the autocorrelations computed by Fama [56] on New York Stock Exchange data.

interest rates. Hedgers protecting previously-established exposures cannot be modeled in the MT. While their motives are akin to those of arbitrageurs, their calculations are like those of speculators and they are therefore lumped with the latter.

The key flaws in this specification are the assumed specialization of traders and the assumption that interest rates in the MT are purely exogenous, set by the whims of central bankers independently of expectations.<sup>65</sup> When IRP prevails, the equilibrium forward rate in the MT is therefore also exogenous. At IRP, the arbitrage demand is infinitely elastic. Any shift in speculators' expectations of the future spot, due say to government counter-speculation, leaves both the arbitrage schedule and the IRP forward rate unchanged. The arbitrage volume adjusts automatically to meet the speculative demand leaving the false impression that governments can successfully induce capital flows by forward intervention. Due to the exogeneity of interest rates, speculators' and arbitrageurs' demand curves for forward contracts are, implausibly, perfectly independent.

In contrast, there are no separate classes of traders in the theory leading to Equation (20). Investors are identifiable by their price deflators, not by their transactions motives. Their portfolio problem has been solved globally: their demand for forward exchange<sup>66</sup> is an indistinguishable component of their vector asset-demand function. Their motives may include arbitrage, speculation, and hedging when the latter is identified with the need to diversify. Any linear decomposition of forward transactions according to purpose is, however, essentially arbitrary.<sup>67</sup> The probability distribution of the exchange rate is simultaneously a determinant of both speculation and diversification. Because IRP holds, the arbitrage demand is potentially infinite but no arbitrage flows, accommodating or otherwise, will actually occur. More importantly, interest rates are endogenous. Equation (17) makes clear that, given expectations regarding future spot rates, interest rates in various currencies *cannot* be set independently: conversely, interest rate differentials reflect anticipations of future spot rates. The forward rate in (20) is therefore jointly at IRP and equal to the certainty equivalent of the future spot rate.

In the MT, the equilibrium forward rate generally falls between the exogenous IRP level and the expected future spot rate.<sup>68</sup> To forecast the future spot, all one

<sup>65</sup> There are others, quite aside from the difficulty with hedgers. First, speculators will not bet on the difference between the forward and expected spot rates unless they are risk neutral or, if they are risk averse, unless forward contracts are the only available risky asset. Second, in creating the link between the forward market and capital flows, the MT confusingly identified the spot transactions associated with covered interest arbitrage (borrow one currency, sell it spot, invest in a second) with "hot," interest-sensitive, short-term capital movements. In reality, these would not be covered by buying forward the borrowed currency and therefore, unlike arbitrage, will tend to be independent of the forward rate and of deviations from IRP.

<sup>66</sup> This is a figure of speech. As noted in footnote 58, forward contracts are redundant instruments when borrowing and lending is allowed in all currencies. Hence we never introduced (and could not have because of the perfect substitutability) a separate demand for forward exchange. In the present context the expression refers to the equivalent demands for bank deposits or loans. The situation will be different below when arbitrage becomes risky.

<sup>67</sup> Adler and Dumas [5 and 6] showed that the additive separation of the speculation and hedging purposes is possible with a quadratic utility function which generates a linear marginal utility function.

<sup>68</sup> To account for IRP deviations at equilibrium was one of the MT's design objectives. Nowadays,

does is to compare the concurrent actual and IRP forward rates. This demonstrably false proposition is the mechanical result of the MT's identification of forward equilibrium at the intersection of the speculator's inelastic demand curve (with its intercept at the expected spot) and the arbitrageurs' inelastic supply curve for forward contracts (with an intercept at IRP). The majority of the authors cited associated the finite elasticities of the two curves to traders' aversion to a notion of default risk which rises with market volume. To refute the MT's prediction, Adler and Dumas [5, 6] and Kouri [107] modeled the portfolio demand for forward exchange in the presence of an exogenous default risks on both forward contracts and banking transactions, which made interest arbitrage risky. The results generalize Equation (20). Interest rates and the forward rate are endogenized: both are functions of spot rate expectations.<sup>69</sup> IRP may be violated due to the default risks. The demand for forward contracts cannot be decomposed additively among the motives: rather the arbitrage motive acts multiplicatively on the others. And above all, the forward rate need not be bracketed by the IRP rate and the expected spot.

Following the demise of the MT, the theory leading to Equation (20) has taken its place in the tool kit of macroeconomists. It produces as a special case the model, mentioned in Section III, which was used by Krugman [113] to suggest a link between exchange rates and the current account.<sup>70</sup> This link is directly apparent in Equation (17). Assume  $1 - \alpha' > 0$  and  $\alpha' = \alpha^{L+1}$  and take some wealth from country  $L + 1$  and transfer it to country  $i$ . This modified situation may be the result of a history of large current account surpluses or smaller current account deficits for country  $i$  at the expense of country  $L + 1$ .<sup>71</sup> The (comparative-static) transfer induces a drop of the interest differential ( $r_i - r_{L+1}$ ) in favor of currency  $i$  or a drop of its expected rate of appreciation ( $\theta_i$ ).<sup>72</sup> The

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the daily press reveals that IRP holds to within very narrow transaction cost tolerances in the Eurocurrency interbank dealer market even during turbulent periods. Nevertheless, most of the papers in the area presume that IRP is violated. Many reasons have been given: transactions costs (Branson [27], Frenkel and Levich [69]); institutional constraints (Einzig [52], Sohmen [178], Canterbury [32]); interest rates functionally related to the volume of arbitrage (Prachowny [153], Frenkel [67] and Kenen [102]); default risk in arbitrage transactions (Stoll [189], Grubel [81]) or political risk (Aliber [14], Dooley and Isard [41]). Deviations from IRP are frequently observed in comparisons of domestic money market or local government T-bill rates. Internal markets may therefore be segmented while the offshore markets are not. IRP violations depend on where one looks.

<sup>69</sup> In one section of Kouri [107], interest rates are taken to be exogenous but then restrictions correctly follow on the expected behavior of spot rates. The fact that interest rates, as much as forward rates, reflect exchange rate anticipations have been noted before these formal models were constructed. See Branson, Katz, and Willett [29], Pippenger [150], and the literature reviewed in Kohlhaugen [106], Section 11c.

<sup>70</sup> Via a wealth effect rather than an arbitrage effect.

<sup>71</sup> Larger wealth may also be the result of capital gains on country  $i$ 's preferential holding of home-currency assets. These gains would, however, induce a current account deficit.

<sup>72</sup> This conclusion requires a world where local inflation rates are negligible so that  $s_{i,\pi}^{L+1} = 0$  and  $s_{i,\pi}^i = s_{i,\pi+i}$  is the variance of the exchange value of currency  $i$  and is therefore positive. Further the market portfolio of stocks is the stock component of the logarithmic portfolio ( $w_k^s / \alpha^m = w_{k,\log}$ ;  $k = 1 \dots n$ ) which must be held constant. And, the market portfolio of bonds is zero if these are in zero net supply. Simple examination of Equation (17) then produces the stated result. Note, however, that the conclusion may be mitigated or strengthened by changes in the logarithmic portfolio induced by the lower value of  $r_i + \theta_i - r_{L+1}$ .

dynamics of wealth accumulation and exchange rates are yet to be worked out in keeping with this observation.<sup>73</sup>

## VI. Some Welfare Questions Associated with Exchange Risk

The function of capital markets is the allocation of the bearing of risks. Among these, the risk arising from holding nominal assets is of special interest in international finance. To define terms, recall the development of Section I. A resident of a country holding a foreign security with a given probability distribution of foreign-currency rate of return would compute his real return by first translating into home currency and then deflating by means of the home price index. Consequently, the currency risk arising from holding a nominal asset denominated in foreign currency (i.e. paying a fixed amount of foreign money) is linked to the randomness of the exchange rate times the home purchasing power index. Exchange risk, which may be identified with the randomness of the exchange rate, is never borne alone but only in conjunction with home purchasing power uncertainty. The currency risk associated with holding a nominal security denominated in home currency is simply the randomness of the home purchasing power index, if any. The question is under what conditions these currency risks affect welfare and how they are allocated across individuals.

We first consider a world in which money is only a unit of account but is not held in the investors' portfolios and is issued by no one. Exchange rates are then arbitrary (random) numbers, exogenously given, translating one measurement unit into another. It should be clear that the multiplicity of benchmarks for value by itself has no impact on welfare. If there is a welfare issue, it arises from the presence in the financial markets of nominal securities, i.e., securities whose payoff is linked to the fluctuations in the purchasing power of one monetary unit of value. The issue, then, is in what circumstances would such securities be willingly held by investors at equilibrium; for, if they were not held, the randomness of exchange rates and price levels, holding constant the physical payoffs (outputs) on the other securities, could not possibly have an impact on welfare.

When the financial market allows individuals to trade risks and insurance in every conceivable dimension of their choice, the allocation of risk bearing is Pareto optimal. In that case, a strong assertion of financial theory is that all consumption risks are mutualized. Consider a random event which is to cause an individual to lose something to the benefit of another individual (a zero-sum risk). Then it would be optimal for these two risk-averse persons to precontract: the first person would buy insurance from the second one and pay him a fixed premium and, if the risk materializes, its effects would be cancelled by invoking the insurance policy. All personal risks would disappear in this manner and in the end individual consumption (assumed to be the only source of utility) would only be a function of aggregate consumption. Individual investors, that is, would only bear the impact of social (aggregative) risks. This function, relating individ-

<sup>73</sup> This current account argument may seem reminiscent of the "hedging pressure" theory wherein trade flows induced trading firms to hedge in the forward market, thereby affecting the equilibrium rate (Levin [121], Dooley [40]). But really it is not, as we are referring now to the accumulated current account and to the stock of wealth resulting from it rather than to trade flows.

ual and aggregate consumptions, is called an optimal sharing rule.<sup>74</sup> If there are several goods, each person's consumption of each good is a function of the aggregate consumption of all goods. If the investor's utility functions are of the von Neumann-Morgenstern type, the sharing rules are nonrandom functions.

Consider now a pure exchange economy (aggregate consumption of each good equals its aggregate exogenous output) endowed with a Pareto optimal capital market and where the various moneys are only units of account.<sup>75</sup> In this economy,<sup>76</sup> the only securities which would be held are those which achieve the optimal sharing rule, i.e., which serve to allocate the risks of aggregate output of the various goods. But, the various currency risks are not aggregate risk; they arise only in connection with the holding of nominal securities whose payoffs are linked to no underlying physical output.<sup>77</sup> For every borrower there is a lender. The gain of the one is the loss of the other. And, at equilibrium, there will be neither nominal borrower nor nominal lender; nominal securities *will not be held*.<sup>78</sup> This is an important conclusion and a fairly robust one provided one clarifies the phrase "nominal securities will not be held." If a nominal security is issued by a corporation and the proceeds are used to repurchase stock, it will, in the present setting, be bought by the stockholders of that corporation (Modigliani-Miller [141]). This is not to be seen as an exception to the above statement. If a government issues nominal bonds to purchase claims on future output, the bonds will be purchased and the claims on output sold (possibly short) by the taxpayers of that government (Wallace [200]).<sup>79</sup> This is not an exception to the above statement either. In both cases, stockholders or taxpayers will earn gains or losses on their dividends or on their tax bill which exactly offset their losses or gains on the holding of nominal bonds.

If we switch, however, to an exchange economy in which the capital market is not Pareto optimal because of restrictions in the array of tradable securities, then some personal risks will not be hedgable. In that case, capital market participants will look for proxies, i.e., securities which are correlated with the risks they wish to hedge. In general they will make up a portfolio of all the available securities and compose it in such a way as to achieve the best possible correlation. To the extent that nominal securities are available, they will be used for these purposes and will generally be held.<sup>80</sup> Increased currency risk could then reduce or improve welfare.

<sup>74</sup> See Rubinstein [165]. The concept is identical to the one of contract curves in an Edgeworth-box formulation where the edges of the box would represent consumption in the various states of nature.

<sup>75</sup> We must add the following purely technical assumption: there is no nominal security whose payoff in terms of one commodity is perfectly correlated with the aggregate consumption of that commodity. This is only meant to avoid indeterminacy.

<sup>76</sup> This is the kind of economy considered by Grauer, Litzenberger, and Stehle [79]. We do not introduce, however, their assumption of identical consumption tastes for all individuals, which leads to PPP holding exactly (i.e., *ex post*: see Section I).

<sup>77</sup> See *infra* the case of corporate or government bonds.

<sup>78</sup> Unless they are also real in the eyes of some investors, as in the Solnik-Sercu special case.

<sup>79</sup> The government budget deficit (on current account) bites into consumable output and as such constitutes a nonhedgable aggregative risk. The reasoning in the text keeps the deficit constant.

<sup>80</sup> In the model of Sections III, IV, and V the market was generally not Pareto optimal and nominal assets were indeed held at equilibrium. In the Solnik-Sercu special case the market became Pareto optimal but nominal assets also became real and were held for that reason.

Therefore, it appears that currency risks matter or do not matter depending on conditions in the capital market. This was the conclusion reached by Grauer, Litzenberger, and Stehle [79] but in the context of an economy where consumers had identical tastes for the various commodities and PPP prevailed. It should be clear that their conclusion owes nothing to this assumption.

What is crucial, however, to the reasoning is the assumption that, as one varies currency risks, one leaves unchanged the probability distributions of the consumable outputs which are the real aggregate risks to be borne by investors. This assumption may not be tenable in some settings. For instance, in the Solnik-Sercu special case, where local currency inflation in each country is zero but tastes differ, Grauer, Litzenberger, and Stehle make the valid point that, if CPP prevailed for each good, then exchange rates could not fluctuate and currency risks would not exist unless the relative prices between goods and their outputs fluctuated; and this in turn implies that one could not vary currency risks without varying output risks. In that case, currency risks may be said to matter, but only because they vary in step with output risks.<sup>81</sup>

The juxtaposition of the Grauer et al. model where PPP was assumed and currency risks did not matter, with the Solnik model where PPP did not prevail and currency risks mattered, led some astray. It is often stated in the literature (Aliber [15, p. 106], Jaffee [98], Cornell [36]) that, with PPP, exchange rates are not linked with relative commodities prices or that their variations are offset by price levels, so that exchange risk becomes a purely nominal uncertainty which matters to no one. The result is to identify the exchange risk which matters (sometimes called "real exchange risk") with deviations from PPP, and the exchange risk which does not matter ("nominal exchange risk") with fluctuations in the PPP level of exchange rates (i.e., the ratio of price indices). This seems wrong for the above reasons and also because the risk of PPP deviations is not a separate risk which is to be borne by anyone: the foreign deflator does not enter any domestic investor's risk calculus. PPP deviations, as distinct from variations in the purchasing power at home of domestic and foreign securities, will therefore not affect any investor's financial decisions.

At some point, one must stop thinking in terms of price levels and exchange rates reflecting only exogenous random changes in measurement units, and come to grips with the fact that moneys are held by households and issued by Central Banks. But, at that stage, one must also realize that the question of the relevance of exchange risk becomes ill-formulated since exchange rates and price levels are endogenous. The issue then becomes that of the welfare impact or non-neutrality of monetary policies in a multi-currency world. It is a very complex one, for which few statements remain valid outside a particular context or model formulation. The vast macroeconomic literature which addressed this issue recognized at least four channels of influence:<sup>82</sup>

(a) The rate of monetary expansion affects the nominal rate of interest which

<sup>81</sup> Two other instances of links between consumable output and nominal quantities will be encountered below and in Section VIII: monetary policy may have a real effect and the outputs and sales flows of firms may be affected by e.g., a change in the selling price abroad of the finished product, relative to the production cost at home.

<sup>82</sup> In addition, government's fiscal policy directly influences the amount of output available for consumption as the budget deficit (on current account) is a drain on real resources. The deficit may

influences real money balances, consumption, and the demand for securities (savings). If output is kept constant, a rise in the nominal rate of interest increases the cost of the liquidity services of money and lowers welfare (Bailey [17], Barro [21]). If output is allowed to vary, the conclusion depends on the setting chosen (Mundell [146], Fischer [63]);

(b) Monetary policy alters the relative stocks of available assets. As their prices adjust to maintain market equilibrium, the discrepancies which appear between the market prices of physical assets and their replacement costs affect aggregate demand and have therefore real effects (Tobin [194]). This argument is independent of the liquidity services of money. It deals only with the composition of the government's versus the private portfolios. Some irrelevance propositions are being developed (Wallace [200], Chamley and Polemarchakis [33]) which tend to invalidate it;

(c) Random monetary policies induce equivocation in price signals (Lucas [128], Sargent and Wallace [170], Santomero and Seater [169], Weiss [201]), leading to a short-term inflation-employment trade-off;

(d) Exchange rates affect export competitiveness and employment (Laursen and Metzler [115]).

Capital market theory is not, so far, capable of incorporating these effects into a general-equilibrium framework (see Lucas [130, 131] and Helpman and Razin [88]). Attempts at introducing moneys into portfolios have been made in the partial equilibrium context of the capital asset pricing model: Kouri [108], Stockman [187], Fama and Farber [58], Landskroner and Liviatan [114], Stapleton and Subrahmanyam [184], Poncet [151], and Dumas [49]. Some limited welfare issues are discussed in Fama and Farber [58] and Dumas [49]. In both of these, money is a separate argument in investors' utility functions and is held because it yields liquidity services. Fama and Farber [58] make the point that \$1 of money and a nominal (short-term) bond paying \$1 carry the same currency risk. Hence a "separation" exists between the decision to hold money versus nominal bonds and the decision as to the composition of the remainder of the portfolio. People may decide how much they want to hold of nominal assets in their overall portfolio and then divide this amount between money and bonds depending on the amount of liquidity services they wish to use (the cost is as usual the foregone nominal rate of interest). The implication is that the presence of money does not increase or reduce the amount of currency risks people have to bear, as compared to the situation where the government would only issue nominal bonds to finance its purchases. The point is quite general; but, in order to illustrate, consider the simplest case analyzed in Dumas [49] where financial markets are Pareto optimal and money is issued to make transfer payments (or reduce taxes). The random benefits to transfer recipients are exactly equal to the purchasing power losses of money holders. Pre-contracting between the two overlapping groups of households can occur: transfer recipients may borrow in a state-contingent way from money holders and, in exchange, remit the transfers to them later.<sup>83</sup> This possibility makes it unnecessary in this case for anyone to

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be financed by money creation, thereby creating a link between consumable output on the one hand and prices and exchange rates on the other.

<sup>83</sup> If the amount of the transfers are fixed nominally, the market need not be a complete one: nominal bonds of maturities can be used for the purpose.

bear the purchasing-power risk of currencies.

While in full agreement with Fama and Farber on the matter of currency risk, Dumas [49] points out that they overlooked another risk of a monetary origin: the risk of fluctuations in the nominal rate of interest.<sup>84</sup> As has been recognized by macroeconomists (see the monetary channel of influence (a) above), the opportunity cost of holding money balances, measured by the nominal interest rate, is a determinant of aggregate welfare (along with aggregate consumption) and, of course, the next period's nominal interest rate is also a separate argument of the optimal sharing rules. Because nominal interest-rate randomness is an aggregate risk, it cannot be pre-contracted away and each investor must bear a share of it.

While the above discussion has de-emphasized PPP deviations as a measure of exchange risk (especially those arising from differences in investors' consumption tastes), the matter may be different when CPP is violated and individual commodity prices are misaligned across the world. CPP deviations are symptomatic of barriers to free trade. It is then not clear how investors trading contingent claims in a supposedly integrated world capital market will take receipt or make delivery of the physical payoffs resulting from their bets. The notion that the world-wide aggregate amount of each good constitutes a pool of freely allocable resources must be called into question. The concepts of aggregate consumption and of sharing rules may well lose their meaning. To take the issue further requires a model capable of accounting for the CPP deviations. Anticipating the results of such a model, it will undoubtedly remain improper loosely to identify currency risks (or the part of them that matters) with CPP (or PPP) deviations. There is a presumption that the wider the CPP deviations, the larger will be the amount of welfare foregone as a result of insufficient or inefficient trading. But one can only speculate as to how widely-fluctuating, random CPP deviations will be linked empirically to variability in exchange rates or to their product with domestic price levels. Whether increased exchange or currency risks reduce welfare is an open question.

## VII. Segmentation

Segmentations of the international commodities markets which produce CPP deviations may disturb the worldwide allocation of risk: because the goods market is partly segmented, so is the capital market.<sup>85</sup> Independently, however, capital markets may be separated along national lines owing either to investors' inhibitions or official restrictions. Investors may be inhibited by a lack of information, the fear of expropriation or, more generally, discriminatory taxation.<sup>86</sup> Official restrictions may include exchange and border controls which restrict foreigners'

<sup>84</sup> Technically, the reason they overlooked it is that they worked with an indirect utility function of current consumption, current balances, and end-of-period wealth. This is incorrect: when money is held, the indirect utility cannot be a function of future wealth alone; it also depends on the future nominal interest rate.

<sup>85</sup> The CPP deviations need not be random for this to happen. When investors encounter difficulty in collecting payoffs in consumable form, they may be deterred from investing in the first place.

<sup>86</sup> The randomness of exchange rates is not to be conceived as a source of segmentation since by proper hedging, repatriated payoffs may be rendered independent of exchange rates.

access to local capital markets, reduce their freedom to repatriate capital and dividends, and limit the fraction of a local firm's equity that they may own. These manifestations of sovereignty serve in part to define nations as distinct segments of the international capital market.

As a phenomenon, segmentation is not unique to the international arena. It has received attention also at the domestic level, from Rubinstein [166] and, more comprehensively, from Lintner [126]. Capital market segments in these papers generally consist of groups of investors and of groups of securities which each investor class is allowed to trade. The multiplicity of possible groups of investors and securities led the authors to define gradations ranging from complete segmentation to partial segmentation with overlap.<sup>87</sup> The main objective and contribution of these models was to show that a sufficient amount of diversity among investors in different segments would lead to firms having optimal, value maximizing, interior capital structure decisions. The cost is that separation properties generally break down: portfolio separation for individuals and the independence of capital budgeting and financing decisions for firms. Lintner's encyclopedic treatment attempts parenthetically to establish conditions in which separation is restored. Neither paper questions the existence or uniqueness of equilibrium in segmented markets or whether the risk allocation will be Pareto optimal. Both assume that value maximization will be unanimously supported irrespective of the type of segmentation being postulated. Whether this last assumption can be maintained, at least to an approximation, is a question that continues to bedevil this strand of the literature.

Despite an absence of empirical justification, it is possible that segmentation can safely be ignored at the domestic level. Most authors do. Internationally it is harder to avoid if only because, from time to time and place to place, governments try to insulate their capital and goods markets from the rest of the world. To the extent that segmentation exists in practice, international corporations may be able to play an important role by recognizing the causes of the segmentation and by planning transactions to enable their stockholders to reap the welfare gains from integration.<sup>88</sup> When stock markets are segmented, for example, home-country firms can purchase shares in foreign firms and provide an indirect diversification as well as a rate-of-return arbitrage service. Adler [1] and Adler and Dumas [3] calculated the value-maximizing foreign acquisition and the resulting home and foreign market equilibria when both markets separately are described by a CAPM. They calculated that the value maximizing foreign acquisition would not be welfare maximizing from the viewpoint of home investors. This point was pursued by Lee and Sachdeva [116]. They pointed out that what produced the result was the implicit assumption that home firms had monopoly power in the home capital market. When these firms behave at home as pure competitors, as in Ekern and Wilson [53], home welfare is maximized

<sup>87</sup> Rubinstein further remarked that segmentation might not be an essential concept. Incomplete diversification of domestic portfolios can result from other causes such as nontraded assets, default risks in borrowing, taxes, and transaction costs. Internationally, however, such factors may operate to separate financial markets from each other. We do not therefore attempt any further distinction between segmentation and the imperfections that may cause it.

<sup>88</sup> The welfare gains from integration have been evaluated by Subrahmanyam [193].

while the welfare of investors in the host country is generally minimized. Their work contributed an essential insight. When conditions in the domestic market are those leading to the Modigliani and Miller theorem, there is no optimal foreign acquisition decision for any single, individual home-firm. What exists is an optimum for the total amounts of foreign shares to be acquired by all the firms in the home-market segment. The allocation of this total acquisition among home firms is irrelevant for investors holding a diversified home market portfolio.

Stapleton and Subrahmanyam [133] calculated numerically optimal decisions and capital market equilibrium in a variety of stock market settings. Adler and Dumas [4] established the principle that for each kind of segmentation there is a corresponding domestic value-maximizing decision for unrestricted home firms: when the bond market is segmented, there is an optimal foreign versus home borrowing decision; and there is an optimal forward contracting decision when stockholders' access to the forward exchange market is restricted. As before, these decisions will not be firm-specific if the domestic market is free of imperfections and bankruptcy costs.

The acquisitions of foreign shares that firms make for the purpose of providing their owners with international diversification should be distinguished clearly from private foreign direct investment (PFDI), although they might accidentally be so classified in the balance of payments statistics. For one thing, PFDI involves the purchases of control, a consideration excluded from the acquisition decisions discussed above. Segmentation among the financial markets does not seem particularly important in this connection.<sup>89</sup> Most explanations of MNC's behavior and of synergy, however, appeal to imperfections in, and segmentations of, not the financial markets but of the markets for products, factors, and technology. To paraphrase Kindleberger, direct investment falls more within the province of industrial organization and monopoly theories than financial theory.<sup>90</sup>

Nevertheless, the fact that MNC's foreign operations can be viewed as (more

<sup>89</sup> Ragazzi [154] offers an intriguing theory based on segmentation within countries which are financially underdeveloped. In such countries, he hypothesizes two capital markets, one for the trading of very large, controlling blocks of shares and one for the trading of minority holdings. Expected returns for a given level of risk would be higher in the market for control. Ragazzi suggests that MNC's might finance themselves in the minority market, repackage the funds, and enter the oligopolistic market for controlling interests, thus reaping the difference in rates of return.

<sup>90</sup> A full review of the determinants of PFDI is beyond the scope of this survey. Segmentation of product markets underlies Adler and Stevens's [10] analysis of MNC's exporting and investment decisions. Early accounts of the spread of international investment, summarized in Hufbauer and Adler [95] and elsewhere, emphasized the migration of labor-intensive industries to low wage countries: such investment bridges a segmentation in the factor market. By the same token, however, capital intensive industries should concentrate in, and preferably export capital intensive goods from, countries like the U.S. where the cost of capital may be relatively low. This proposition has proved empirically questionable. The failure of the factor proportions account led to the analysis of PFDI as a channel for the transfer of technology: Hufbauer [92, 93, 94] and Vernon [199]. The product life cycle account follows from the view that the market for technology is segmented and monopolistic. This notion also underlies the influential proposal by Johnson [99] and Magee [132] that MNC's be modeled as monopolistic producers, not of goods, but of information. Their efforts to appropriate the externalities tend to segment the information market between private information held by the innovating firms and public information. Other explanations of PFDI resort to economies of scale in the production of goods and information or in the ability of large firms to negotiate concessions from host governments. Few feature financial market segmentations as key contributing factors.

or less diversified) portfolios of controlling shares raised the hypothesis that they may offer a (partial) diversification service. Can one replicate true international diversification by purchasing a portfolio of MNC stocks? Agmon and Lessard [13] regressed the returns of 217 U.S. MNC's on the U.S. index and an international factor. In second pass regressions, they found the coefficient of the world factor to be correlated with a sales measure of MNC's international involvement and, therefore, suggested that perhaps one can. However, using a sample of 40 European and 23 U.S. firms between 1966 and 1974, Jacquillat and Solnik [97] concluded that one cannot. Basically, a multiple regression of MNC returns on various national indices showed that these firms correlate highly with their respective national indices and very little with foreign stock markets. These negative results were confirmed by Brewer [31] and Senschak and Beedles [173]. The issue is not easy to settle empirically, as noted in Adler [2]. If investors can diversify costlessly into the shares of foreign firms with equal access to the same projects as U.S. MNC's, geographical diversification of projects ceases to be a service that MNC's can valuably perform on behalf of their stockholders. It may be impossible to detect diversification benefits if MNC's operate in markets where individuals also can trade. If any "foreign investment" effect is to be observed then, based on the theory of PFDI, it is likely to be the result of MNC's monopoly advantages abroad. Errunza and Senbet [54] independently followed this reasoning. They found that a measure of MNC's monopoly returns, i.e., market value minus replacement cost, was significantly correlated with a sales measure of MNC's foreign involvement. The clear implication is that it is hard empirically to unravel the effects of monopoly from potential diversification benefits.

While segmentation within and among capital markets may not be central to explanations of PFDI, the possibility that it exists is important and perhaps crucial in connection with the analysis of corporate financial decisions. Unfortunately, there is as yet no definitive empirical method for determining whether and to what extent the international capital market is segmented. There appear to be four conceivable avenues of investigation, not all of which have been followed in the literature.

The first is misguided. Section II reviewed several studies of the correlations between national stock markets. A theme sometimes encountered in these papers is that low correlations indicate segmentation on the grounds that integrated national markets would tend to fluctuate together. This inference, however, is incorrect. There are national random factors (politics, etc.) which affect selectively the production activities of any one country. They are reflected in stock returns but this is no evidence of segmentation. Further, output mixes vary considerably among countries partly as a consequence of the specialization induced by international trade. Random shocks may affect selectively specific industrial sectors. They may, therefore, have a relatively heavy impact on those stock markets where these sectors are large but not in others. Small correlations among national stock market indices are generally consistent with perfect capital market integration.

Prospectively, a better approach for detecting segmentation is to analyze the correlations among national consumption rates. As was pointed out in Section

VI, consumption risks (as opposed to production risks) are mutualized in an integrated and Pareto optimal capital market. Consequently, small random (unanticipated) fluctuations in national consumption rates should in such a market be perfectly correlated with the aggregate random consumption rate. There are some difficulties in defining aggregate consumption risk when differences in tastes lead to PPP deviations. These are not impossible to overcome.<sup>91</sup> The problem of CPP violations may, however, be more severe for reasons already given at the end of the previous section.

The third possible method is, like the first, based on an analysis of securities' prices. Rather than trying to infer segmentation directly from the correlation structure, however, the idea is to derive competing capital asset pricing models, with and without segmentation, and to confront them with data to see which one fits best.<sup>92</sup> Stehle [195] attempted to test the hypothesis that the U.S. market is completely isolated against the null that it is completely integrated with the rest of the world. To avoid the problems associated with PPP deviations, he assumed a world of logarithmic investors ( $\alpha' = 1$  and, therefore,  $\alpha^m = 1$  in Equation (17)). With complete segmentation, the proper market portfolio for U.S. investors would be represented by the U.S. index whereas under complete integration a world index would be appropriate. Unfortunately, his empirical evidence did not significantly discriminate between the two competing models.<sup>93</sup>

The last approach, which also relies on CAPM concepts, was initiated by Black [24]. A variation and extension was recently proposed by Stulz [190]. Rather than postulate the extremes of either complete segmentation or none at all, these papers employ a continuous parameter of segmentation in the form of a proportional tax. In Black, the tax is on an investor's net holdings (longs minus shorts) of risky foreign assets while in Stulz the tax on both long and short positions is positive. Borrowing at home and abroad is riskless and untaxed in both models. This apparently minor difference in specification, nevertheless, produces two quite different CAPMs with a common feature: the world portfolio will not be efficient for any investor in either one. The segmentation test in each case would essentially consist of fitting the derived CAPM to stock price data and either estimating the value of the implied tax rate or detecting its effects via security market line analysis. So far, this approach remains in the realm of theory.

<sup>91</sup> Breeden [30] is credited for pointing out that the mutualization of consumption risks implies the perfect correlation of consumption rates. Admittedly, (Stulz [191]) consumption mixes vary across the world, leading to PPP deviation, and destroying the perfect correlation. But examining correlations of consumption rates of *individual* goods would eliminate the problem. Such a route was not open to the authors who studied the stock market returns: examining industry rates of return would have resolved the specialized issue but would not have dealt with the issue of country-specific production risks (which are not supposed to be mutualized).

<sup>92</sup> An alternative method, in the same spirit, would use the Arbitrage Pricing Theory (Roll and Ross [162]) and test whether random factors which are common to stocks of different countries receive the same price in the different national stock markets (an APT in the absence of PPP would be needed for this purpose). This suggestion was made by one referee. We are thankful to him.

<sup>93</sup> Solnik [182] following the Roll [160] methodology argues in favor of comparing actual and optimal portfolios (the latter computed on the basis of actual returns) rather than comparing the actual return statistics to their theoretical values (19). He claims that optimal aggregate portfolios under segmentation and under integration would not be sufficiently different (because of the low correlation of returns across countries) to permit a clear-cut conclusion.

### VIII. International Corporate Financial Decisions: Hedging Policy

The discussion of the previous section clearly implies that the analysis of international corporate financial decisions rapidly becomes problematic once capital market imperfections and segmentations enter the picture. As these are often hard to ignore in practice, the literature in this sub-field is correspondingly slim. We shall therefore focus on the subject which has received the most attention, hedging policy, after the briefest of reviews of other decision problems.

A few papers address long-term decisions. Mehra [137] confirms that all the Modigliani and Miller (M & M) propositions continue to hold in a perfectly integrated, two country capital market with random exchange rates, identical investors but no (or identical) taxes and no inflation. The capital budgeting criterion in such a world is independent of the choice of measurement currency, the nationality of the investing firm, and of the financing decision. The same conclusion regarding investment and financial decisions will also emerge, however, from any of the (quasi-) complete-market, tax-free international asset pricing models such as Solnik's [179], GLS's [79] or, for that matter, our own in Section IV above. Adler and Dumas [4] and Senbet [172] introduce different tax rates at home and abroad with the result that there emerges a value-maximizing foreign borrowing decision, the investment and financing decisions become interdependent and planning becomes a programming problem. Adding other sources of segmentation does not change this general picture but complicates it considerably. Exact solutions depend very much on the specific provisions of the assumed tax regime and on the specific constraints imposed by the assumed imperfections. Lessard [119] offers a pragmatic compromise which treats the decisions separately. One would like to know how good an approximation his procedure is.

Exchange rate variations can affect firms along several dimensions: through their impact on short and long-term monetary assets and liabilities and on physical assets; and via their effect on the volume of sales and the associated production plans. Exchange risk is but one of many environmental risks with which firms contend. Isolating its effects is a matter of decomposing the variability of some measure of the firm's results among the various risk sources. Once the extent of the firm's vulnerability to foreign currency risk is determined, it may readily be modified using financial hedging instruments such as forward contracts, swaps (borrowing one currency and lending the proceeds in another), and a variety of insurance schemes. Two questions then arise. Should a firm hedge at all? If there are circumstances in which it should, then by how much? While these issues are by no means completely settled, we may review the progress to date and the problems that remain.

In a perfectly integrated world capital market like that of Section IV, where the M & M propositions all hold, Baron [19] and Dumas [46] show that corporate hedging, like the choice of debt versus equity, is irrelevant *regardless* of the firm's exposure. In such a world, firms need not do what investors can do equally well for themselves. This irrelevance proposition is independent of the existence of risk and inflation premia in the relationship between the forward and expected future spot rates. The equilibrium forward rate in Equation (23) is set precisely

at the level where no value is gained or lost by hedging. When a firm hedges, there is a change in its risk posture which is precisely offset by a value-preserving change in its expected return. What the proposition depends on, in other words, is the dual assumption of symmetric information and the (quasi-) completeness of the world's financial markets. Integration of the foreign exchange and stock markets guarantees that the same (linear) valuation functional is used in both. This allows the value additivity principle to be applied: the value of a firm with a forward contract is equal to the value of the same firm without a contract plus the value of a contract. In the absence of transactions costs, forward contracting involves no exchange of money between the parties: the forward rate adjusts until the value of the contract at the time it is entered is exactly zero. The values of the hedged and unhedged firms are therefore equal on the hedging-decision date.

We should emphasize that the market conditions which guarantee that this variant of the M & M theorem will hold are the only reasons for hedging to be irrelevant. Hedging is also claimed to be unnecessary by followers of Aliber [15] on the generally faulty ground that the net gain from it (equal to the forward rate minus the ensuing spot) is sometimes positive and sometimes negative and tends to zero over long periods. Aliber's own argument was empirical and more restrictive: he observed that the average deviation of the forward from the future spot rate tended towards zero as the number of observations included in the average increased.<sup>94</sup> He proceeded to deduce that long-term nominal foreign-currency assets are not exposed in the long run, and therefore that these assets, at least, need not be hedged. The argument is flawed for several reasons. The most important is that it ignores risks whose expected values are zero. It is wrong to base on reasoning dealing with averages the theoretical argument that risk avoidance is irrelevant.<sup>95</sup> There is no reason to suppose that stockholders or corporate managers are risk-neutral and care only about long-run expected values. If hedging is useless, it can only be for the reasons given in the previous paragraph.

As is the case with most financial decisions that do not matter in M & M's theory, practitioners do devote time and resources to hedging exchange risk. Even those who ignore balance sheet translation exposures substitute some other target. What accounts for this activity? One reason may be that the market is not as integrated as the theory requires. Segmentation and other imperfections, including bankruptcy costs in addition to the ones already mentioned, may be part of the explanation; but their impact is hard to measure. Information may not be symmetrically distributed. Firms do not publish their exposures by currency so treasurers must be better informed in this regard than investors. Perhaps there is room for managers to make hedging decisions for stockholders.<sup>96</sup>

<sup>94</sup> Note that the long term-average observation would only show that the expected value of the forward rate is equal to the expected value of the spot, *not* that the forward rate is equal to the conditional expected value of the spot.

<sup>95</sup> Also, when risk is considered, the exposure of long-term nominal foreign-currency assets is found to have little to do with deviations of the forward from the future spot rate: we discuss exposure below.

<sup>96</sup> Asymmetric information raises thorny problems. If managers attempt to enter forward transactions on behalf of shareholders, what should they assume regarding the forward contracting

Organization and accounting theories may provide additional clues. Managers may not willingly further the interests of stockholders. Many of them in fact object to value maximization partly on the generally questionable grounds that it is unfair to evaluate their performances relative to market prices over which they have no control. Instead, they prefer or may be required to serve some accounting objective such as minimizing the exchange losses reported in financial statements. This last is reflected in Rodriguez's [157] survey and would account for the popularity of Aliber's averaging arguments. There is for the present no comprehensive theoretical framework for dealing convincingly with this issue. We can offer some preliminary notions as a step in that direction.

For a point of departure, let us postulate that "market-value stabilization" is the objective.<sup>97</sup> A firm's exposure to a given foreign exchange rate can then be defined as the sensitivity of the domestic real market value of the firm's equity, as of a *given future date*, to the concomitant random variations in the future domestic purchasing power of the foreign currency on that date. The measure of sensitivity is an amount of foreign currency: it is the amount deposited in the bank which would render the firm as vulnerable to foreign currency risk at the target date, as does its commercial activity. A perfect hedge consists of the forward transaction in foreign currency for the said maturity required to render the random variations in the *future* real domestic market price of the stock independent of the randomness of the future purchasing power of the foreign currency. As one performs this sensitivity measure vis à vis all currencies simultaneously,<sup>98</sup> the result is an equivalent portfolio of foreign (and also domestic) currencies which the firm is implicitly holding on account of its commercial activity. An optimal hedge is equal to the simple difference between the implicit pre-existing portfolio and some desired portfolio which meets the postulated objective.

To state a theoretical definition of exposure is to reveal also its practical limitations. Economic theory has not yet progressed to the point where it is possible in the case of a firm, security, or commodity accurately to associate a specific future market price and its probability with each possible level of the future exchange rate. Much of the managerial literature (Lietaer [123], Makin [133], Adler and Dumas [7]) seeks to avoid the problems involved with market value objectives by relying on accounting numbers.<sup>99</sup> These suffer from at least

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behavior of the stockholders themselves? The answer is especially difficult to provide when stockholders' nationalities differ. Asymmetric information is not the only rationale for corporate hedging: one referee pointed out that hedging entails fixed (information and transactions) costs which would become prohibitive if stockholders hedged on their own account on a day-to-day basis.

<sup>97</sup> Perhaps, this objective would be derived from the desire to minimize the probability of default (default being the circumstance where the value of equity is zero).

<sup>98</sup> If the postulated objective function can be put in the mean-variance form (as, for instance, when the market value of the equity is normally distributed), the exposures to the various currencies are coefficients of a multiple regression (across states of nature) of the market value of the equity on the purchasing powers of all foreign currencies (Dumas [46]). The same procedure applies, of course, to any asset or security. Approximate stock market exposures were displayed in Table V.

<sup>99</sup> Accountants themselves somewhat arbitrarily classify balance sheet items into exposed and non-exposed. Exposure is then simply the net amount of exposed assets minus liabilities in today's balance

one deficiency:<sup>100</sup> they do not incorporate the delayed effects of an exchange rate change on the firm's ensuing cash flows.<sup>101</sup> To remedy this limitation and to accommodate managerial concerns, Adler and Dumas [7] proposed that treasurers be concerned with the mean and variance of the consolidated net worth or the consolidated cash balance<sup>102</sup> (in the company's functional currency), measured at some cut-off date. This date would be chosen posterior to the target date at which the exchange rate changes being analyzed are to take place.<sup>103</sup> Either nominal objective may be justified as a signal of default risk. A firm is equally bankrupt when the real or nominal value of its equity reaches zero; and cash balances can be compared to their deficiency levels, irrespective of the measurement unit.

Equipped with such a mean-variance objective, the firm could proceed as before. Levels of the cut-off-date net worth or cash balance would be associated, perhaps by simulation, with levels of the target-date exchange rates after taking account of the responses the firm might plan to make in each state. Exposures to each currency will be represented by the coefficients in a regression (across states) of the target variable on the set of exchange rates.

The exposures measured in this way or on the basis of the earlier market-value reasoning will be global in the sense that they encompass the sensitivity of the firm as a whole to exchange rates. They can, however, be decomposed among components which represent the different kinds of influence that exchange rates can have. There are at least five categories of these: (1) the impact on short-term nominal net assets with maturity equal to the target date; (2) the impact on longer term nominal net assets with maturities falling beyond the target date; (3) the impact on the salvage value of existing physical assets and on the replacement cost or purchase price of physical assets to be replaced or acquired; (4) the impact on sales prices and unit costs; and (5) the indirect impact via sales prices on the volume of sales and consequently on the planned volume of production and other physical activities. Accounting measures of exposure have imperfectly dealt with only the first two of these effects. The fifth has so far been completely ignored. There is some confusion regarding the rest.

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sheet: the time-dated forward-looking nature of exposure is lost. Under FASB 8 exposure was equal to foreign accounts receivable plus cash minus long-term debt, accounts payable, and short-term debt; it was generally negative. Under FASB 52, all items are considered exposed; exposure is then equal to net foreign assets and it is necessarily positive.

<sup>100</sup> One other deficiency is that accounting numbers are nominal in nature and no price index seems inherently satisfactory to deflate them as has been made clear by the endless debate on inflation accounting. For a questionable attempt at solving the firm's numeraire problem, see Eaker [51]

<sup>101</sup> These would have been included in a market-value based measure of exposure.

<sup>102</sup> This approximation, too, is ad hoc. It suffers from the same potential problems of non-optimality and inconsistency as any firm-specific quantity. It has, however, the major virtue of being able to bring all the potential effects of the exchange rate into the picture. There is little to choose between book net worth and the cash balance on this count. The former accommodates long-term debt while the latter focuses on treasurers' professed main concern, cash, and is independent of any accounting rule.

<sup>103</sup> The cut-off date only determines the precision of the measurement whereas the target date is an essential parameter of exposure measures.

Adler and Dumas [8] considered channel (2); i.e., nominal long-term, foreign currency bonds. The market values of such bonds may be exposed more or less than 100% in the sense that their exposures, and therefore the amount of shorter-term forward contracts required for a perfect hedge, may be larger or smaller than their face or redemption values. Whether they are over- or under-exposed depends on the degree of serial dependence in anticipated exchange rates (and not on the difference between the forward and ensuing spot exchange rates, as in Aliber [15]).

Aliber and Stickney [16] linked channels (3) and (4) to the existence of deviations from commodity price parity (CPP) between countries. Their specific contention was that physical assets are not exposed because the average deviation from purchasing power parity (relative to a base period) empirically tends to zero over the long haul. In Section (2), this evidence was seen to be consistent with the hypothesis that PPP deviations follow a martingale, hardly a situation of no risk. Their argument is also highly questionable for additional reasons. As far as channel (3) is concerned, it is clear that exchange rates may influence the reference currency *market* value of a physical asset located in any country once exchange rates and goods prices are correlated. Physical assets regardless of their location are indeed generally exposed to exchange risk. It is equally clear, however, that the issue of comparing the exposures of (identical) physical assets located in different countries does not arise when exposure is properly defined. Hence comparative prices (as in CPP) have no role to play. Traditional accounting trailed this red herring: the original cost book value of physical assets located at home does not fluctuate and is by definition not exposed. The CPP misconception presumably arose from an implicit comparison with home assets.

As for channel (4), it is clear that exchange rate variations may erode or improve the firm's competitiveness abroad. But note that a firm generally purchases goods and services in one country, transforms them, and may sell them in some other country. Except in the case of a pure shipping firm, the goods bought in one place and sold in another are never the same. Even if CPP held exactly, there remains the possibility that the prices of the output good and the input goods will have different exposures. Profitability and net cash flows would then be affected by exchange rates. It is small solace to know that on the average over the long run goods prices may be internationally linked. The firm should not be concerned with averages when it is planning its risk-bearing strategy.

## IX. Conclusion

The best conclusion of a survey paper is undoubtedly one filled with directions for research prompted by the shortcomings of existing theory. The shortcomings have probably been apparent to the reader, but it may be useful to recapitulate.

Deviations from parity of individual commodities prices are a phenomenon about which existing microeconomics has very little to say. A model of international goods markets must be constructed before we can say anything serious regarding the financial side. We have mentioned that segmentation of the goods market can induce segmentation of capital markets. The manner in which the

foreign exchange market reacts to CPP or PPP deviations (see Section I) can and will be properly analyzed only when the physical events (shipments, production, etc.) which are being anticipated by this market are themselves made explicit. Macroeconomists would also benefit from such models, since fluctuations in employment may be linked to the international competitiveness of a country's products.

Disregarding possible inconsistencies, we dealt with portfolio choices and asset pricing in a unified worldwide capital market with PPP deviations. As in much of finance theory so far, the functional form of the dynamics of securities prices was postulated *ab initio*, leaving only the cross-sectional relationship between risk and return parameters to be determined in market equilibrium. Following Lucas [129] and Cox, Ingersoll, and Ross [37], the stochastic process of asset prices should instead be fully endogenized by dynamic methods borrowed from functional analysis and by imposing an assumption of rational expectations.<sup>104</sup> Such a project would come at an auspicious time since balance-of-payments theorists, who have so far postulated *ad hoc* asset demands, have lately become interested in utility maximization (see Obstfeld [167]). The introduction of money balances in portfolios which we discussed briefly in Section VI would tie together the concerns of financial micro- and macroeconomists, which are becoming remarkably convergent. We are on the threshold of a true stochastic theory of the balance of payments. One difficulty, however, looms large. In a complete financial market with available instruments for all maturities and investors holding rational expectations, there is no need for portfolio revisions; prices adjust but not portfolios. One way to account for international capital flows under rational expectations is if there remain unhedgable or unexpected risks for at least some maturities. Other avenues for introducing portfolio revisions may prove fruitful. Delicate analytical choices will have to be made.

On the empirical side, tests and measures of the degree and specific sources of segmentation of international capital markets are becoming essential. It is almost impossible to progress without having some knowledge of the true meaning of national borders in finance. Initial tests should be based not on an analysis of securities prices, but on the principle that in integrated capital markets all consumption risks are mutualized. It is to be feared, however, that data on consumption behavior are not sufficiently reliable to be submitted to stochastic analysis. In particular, it may prove difficult to distinguish in a relatively stable fashion the expected and unexpected variations of consumption rates or to trace the effects of specific imperfections.

As far as policy implications are concerned, corporate financial behavior still awaits a proper paradigm which will have to be provided by general financial theory. In the meanwhile, some mean-variance portfolio choice will serve as a framework for exchange risk hedging decisions. The thrust should be towards practicality and towards a good understanding of the dynamics of the problem: should one hedge stocks or flows and how to long-term and short-term hedging instruments interact?

<sup>104</sup> In order for the theory to have content, the state variables upon which prices and decisions are contingent should be fully specified. Stulz [191] stopped short of this goal. In addition, the difficult problems associated with aggregating across investors' diverse preferences will have to be solved.

## APPENDIX

## Conditions for the Existence of Price Indices

As in standard portfolio theory, we assume that investors maximize a time-additive von Neumann-Morgenstern expected-utility of lifetime consumption function. But we introduce here several commodities so that there is some difficulty in defining what is meant by the consumption rate. The basic objective function of the investor must now be formulated as a function of the several consumption rates achieved for the various commodities:

$$\text{Max } E \int_t^T U(\underline{c}(s); s) ds \quad (1)$$

where

$E(\cdot)$  = the expected-value operator conditional on the information available at time  $t$ ; and

$\underline{c}(s) = \{c_g(s); g = 1, \dots, G\}$  is the vector of consumption rates for the  $G$  goods at time  $s$

Optimization of the consumption mix at each point in time leads to an equivalent objective in terms of the indirect utility function  $V(\cdot)$ :

$$\text{Max } E \int_t^T V(C(s); \underline{P}(s); s) ds \quad (2)$$

where

$C(s)$  is the rate of nominal consumption expressed in some arbitrary monetary unit per unit of time; and

$\underline{P}(s) = \{P_g(s); g = 1, \dots, G\}$  is the vector of prices for the  $G$  goods at time  $s$ , expressed in the same monetary unit, and

$$V(C; \underline{P}; s) \equiv \text{Max } U(\underline{c}; s) \quad \text{s.t.} \quad \underline{c} \cdot \underline{P} = C, \quad \underline{c} \geq 0 \quad (3)$$

Assuming that the function  $U(\cdot)$  is sufficiently well behaved for the function  $V(\cdot)$  to exist, be unique and – for the purposes of later derivations – to be continuous and twice differentiable, the function  $V(\cdot)$  satisfies a relationship known as Roy's identity (see Varian [198], page 93):

$$\underline{V}_p = -\underline{c} V_c \quad (4)$$

where

$\underline{V}_p$  is the  $1 \times G$  vector of partial derivatives of  $V$  with respect to the price of goods;

$\underline{c}$  is here the  $1 \times G$  vector of *optimal* consumption rates; and  $V_c$  is the partial derivative of  $V$  with respect to the consumption budget  $C$ .

The formulation (2) proves somewhat cumbersome when it comes to portfolio choices because all prices of consumption goods appear separately in the objective function. A more compact, albeit special, formulation is possible when the function  $U(\cdot)$  is such that there exists an invariant price index, i.e., a compression

of the price vector  $\underline{P}$  into a single scalar leading to the same decisions and valid at all levels of the consumption budget  $C$ . Precisely:

**DEFINITION.** *There exists an invariant price index when the indirect utility function  $V(\cdot)$  either satisfies the property*

$$V(C; \underline{P}; s) \equiv C(V(1; \underline{P}; s)) \quad (5)$$

or can be transformed by some monotonic transformation into a function which satisfies (5).

The condition to be satisfied by the direct utility function  $U(\cdot)$  in order for there to exist an invariant price index was pointed out by Samuelson and Swamy [168]:

**LEMMA.** *A necessary and sufficient condition for the indirect utility function  $V(\cdot)$  to satisfy property (5) is that the direct utility function  $U(\cdot)$  be homogeneous of degree one with respect to the consumption rates  $\underline{c}$ .*

*Proof:* The proof is given in Varian ([198], pages 14 and 42) apropos cost and production functions exhibiting constant returns to scale.

A homothetic function is defined as the composition of a monotonic transform with a function homogeneous of degree one.

**THEOREM (Samuelson and Swamy).** *A necessary and sufficient condition for there to exist an invariant price index is that the direct utility function  $U(\cdot)$  be homothetic with respect to the vector of consumption rates  $\underline{c}$ .*

*Proof:* It is obvious from the definition (3) of  $V(\cdot)$  that applying a monotonic transformation to  $U(\cdot)$  applies the same transformation to  $V(\cdot)$  and vice versa.

When property (5) is satisfied,  $V(1; \underline{P}; s)$  is evidently one over the invariant price index (or, it is the purchasing power index). It can be computed in practice only if one knows explicitly the function  $V(\cdot)$ . However, one may expect that for small percentage changes in prices, the local log-linear approximation of the function  $V(\cdot)$  may be revealed by the consumer's budget allocation. Indeed, when (5) holds, one may write Roy's identity (4) as

$$C\underline{V}_p(1; \underline{P}; s) \equiv -\underline{c}V(1; \underline{P}; s) \quad (6)$$

so that, for each commodity  $g$ ,

$$[P_g/V(1; \underline{P}; s)]V_{P_g}(1; \underline{P}; s) \equiv c_g P_g/C$$

**PROPOSITION.** *The elasticities of the index function with respect to prices are revealed by budget shares.*

This result is not affected if a monotonic transformation is applied to the function  $V(\cdot)$ , as the derivative of the monotonic transformation would only appear as a factor on both sides of Equation (6). In practice it means that percentage variations of the price index may be computed as an average of the percentage variations of individual commodity prices weighted by budget shares. This is, approximately, the way in which national statistical institutes across the world compute cost-of-living indices, a procedure which is valid under homothetic preferences only.

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