Investor Protection, Diversification, Investment, and Tobin's q^*

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Abstract

We develop a dynamic incomplete-markets model where an entrenched insider, facing imperfect investor protection and non-diversifiable illiquid business risk, makes interdependent consumption, portfolio choice, expropriation, corporate investment, ownership, and business exit decisions. Unlike in the first-best, the insider's tradeoff between private benefits and under-diversification costs leads to the following results: (1) the firm either over- or under-invests, depending on firm size; (2) the insider's private valuation fundamentally differs from diversified investors valuation;(3) conditional CAPM holds for outside equity; (4) the insider demands an additional idiosyncratic risk premium; (5) the exit option and ownership dynamics are important for the insider to manage business risk.

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In contrast to the common belief that corporations are widely held (Berle and Means (1932)), many corporations around the world, including large publicly traded companies, have controlling shareholders such as founders, founding family members, and States. La Porta, López-de-Silanes, and Shleifer (1999) document controlling shareholders' concentrated ownership in large firms around the world.[1] With weak investor protection, controlling shareholders, whom we also interchangeably refer to as insiders, become entrenched and pursue private benefits at the expense of outside investors. By "investor protection," we broadly refer to features of institutional, legal, political, regulatory, and market environments as well as corporate governance mechanisms at the firm level, which facilitate financial contracting and contractual enforcement, and protect investors against expropriation by corporate insiders.

Agency problems take a variety of forms including outright stealing from the firm, selling the firm's output to a related party at below market prices, hiring unqualified friends, and self-serving value-destroying investment, just to name a few.[2] It is difficult to verify and contract on decisions such as corporate investment, since they often involve managerial discretion and judgment. Penalizing self-serving insiders based on value-destroying investment is difficult, especially under weak investor protection. We take private benefits and corporate investment as non-contractible in our analysis. By holding a concentrated ownership, insiders alleviate agency conflicts with outside investors but incur an under-diversification cost due to imperfect risk sharing, illiquidity, and incomplete markets frictions.

We incorporate the key frictions, imperfect investor protection and the insider's lack of diversification, in an integrated dynamic framework, where the entrenched insider makes *interdependent* business decisions (private benefits, corporate investment, business exit) and household decisions (consumption-saving and portfolio choice). Using this framework, we address the following questions: What determines corporate investment in firms run by controlling shareholders? How do private benefits of control influence corporate investment and

[1] Claessens, Djankov, and Lang (2000) and Faccio and Lang (2002) document concentrated ownership for large public firms in East Asian countries and Western European countries, respectively.

[2] For example, see La Porta et al. (2000a) for such a statement in an influential survey on investor protection and corporate governance.

valuation? What determines the insider's private valuation and outside investors' public valuation (Tobin's average q)? What drives the wedge between private marginal q for insiders and public marginal q for outsiders? What determines the idiosyncratic risk premium for the insider? What are the effects of frictions on the cost of outside equity capital? How do frictions influence firm size, inside ownership, growth, and welfare? How does the insider manage the dynamics of ownership?

Investor protection and under-diversification frictions have opposing effects on firm investment. On the one hand, the insider under-invests in illiquid (but productive) business in order to lower the idiosyncratic business risk exposure.[3] On the other hand, the insider has incentives to over-invest in business because private benefits in each period are proportional to contemporaneous firm size. A forward-looking insider thus has a preference to build a bigger firm under weaker investor protection, *ceteris paribus*. Both over-investment and under-investment may thus occur. Intuitively, for a firm with a sufficiently large size, the insider's concern about under-diversification outweighs incentives to pursue private benefits, leading to under-investment. In contrast, for a sufficiently small firm, the opposite holds and hence the insider over-invests.

Since a key focus of our study is corporate investment, we naturally start with the neoclassical (Tobin's) q theory of investment, and incorporate the key frictions discussed above into the q theoretic framework.[4] Specifically, under the Modigliani-Miller (MM) assumption, our first-best benchmark extends the seminal Hayashi (1982), a widely-used neoclassical q-theoretic model, to a stochastic setting with risk premia by incorporating independently and identically distributed (iid) productivity and capital shocks.[5] In this first-best benchmark, the optimal

[3]Panousi and Papanikolaou (2012) find that the firm's investment falls as its idiosyncratic risk rises, and more so when the manager owns a larger fraction of the firm and hence is more exposed to the firm's non-diversifiable idiosyncratic risk.

[4]Brainard and Tobin (1968) and Tobin (1969) define the ratio between the firms market value to the replacement cost of its capital stock, as q and propose to use this ratio to measure the firms incentive to invest in capital. This ratio has become known as Tobin's average q. Hayashi (1982) provides conditions under which average q is equal to marginal q. Abel and Eberly (1994) develop a unified q theory of investment in neoclassic settings. Lucas and Prescott (1971) and Abel (1983) are important early contributors. See Caballero (1999) for a survey on investment.

[5]The productivity shocks are used in q-theoretic models. The permanent shocks to the level of capital have been widely used. See Cox, Ingersoll, Jr., and Ross (1985), Obstfeld (1994), and Albuquerque and Wang (2008),

investment-capital ratio is constant, Tobin's average q equals marginal q, and the capital asset pricing model (CAPM) holds. Because the properties of our first-best benchmark are so strikingly simple, any interesting new dynamics and properties that our model generates are thus attributed to the *interaction* of the frictions, investor protection and illiquidity/incomplete markets.

The insider also has a timing option to exit from the business. While being worthless under complete markets, this exit option is valuable for the insider in our incomplete-markets setting as it is a valuable risk management tool for the insider. By relinquishing control, the insider forgoes future private benefits and also gives up an efficient production technology but importantly, diversifies all idiosyncratic business risks. The insider's exit timing decision relates to the literature on the timing of initial public offerings (IPOs). Pástor, Taylor, and Veronesi (2009) model the decision of entrepreneurs to go public as the tradeoff between diversification benefits and the costs of losing private control in a setting where the firm learns about its future profitability in a complete-markets framework.[6]

In the normal region where firm size is not very large, the exit option is effectively out of the money, the investment-capital ratio decreases in firm size because the insider's diversification benefits increases with firm size in a convex way, and the private benefits increase linearly in firm size. However, importantly, as the exit option becomes deeper in the money (near the endogenous exit boundary), the insider's diversification benefit of under-investment lowers, and the investment-capital ratio thus increases in firm size. Therefore, investment-capital ratio is non-monotonic in firm size despite a constant-returns-to-scale production technology.

With imperfect investor protection, public firm value is unambiguously lower than the first-best value. However, the effect of investor protection on the cost of capital for outside equity is not at all obvious. We derive a simple formula to calculate conditional beta and the cost of capital using public average q and marginal q. We show that the cost of capital under

and Barro (2009) for example.

[6]Benninga, Helmantel, and Sarig (2005) develop a simple binomial model to highlight the insight on the timing of IPO with the tradeoff between diversification benefits and private benefits. They do not explicitly model the entrepreneur's preference, agency frictions, or other decisions.

imperfect investor protection can be either higher or lower than the first-best framework value. We also calculate the controlling shareholder's idiosyncratic risk premium. For our calibration, at the moment of exit, the annual idiosyncratic risk premium is about 1.5%. Additionally, we endogenize ownership and the initial firm size. We show that firm size is larger and ownership is less concentrated under weaker investor protection.

A large empirical literature documents that under weaker investor protection, (1) private benefits of control are higher (Zingales (1994), Dyck and Zingales (2004), and Nenova (2003)); (2) dividend payout is smaller (La Porta et al. (2000a)); (3) firm value is lower (La Porta et al. (2002) and Claessens et al. (2002)); (4) corporate ownership is more concentrated (La Porta et al. (1999) and Claessens et al. (2000)); (5) financial markets are smaller and less developed (La Porta et al. (1997) and Demirgüc-Kunt and Maksimovic (1998));[7] and (6) firm size is smaller (La Porta et al. (1999)). Our model's predictions are consistent with these empirical findings. Additionally, our model also generates predictions on time-varying investment dynamics that are purely due to the frictions, rather than changing investment opportunities. We show that frictions matter for corporate investment and valuation, both conceptually and quantitatively.

Related literature. Our paper links to several strands of literature in finance, macroeconomics, and entrepreneurship. Shleifer and Wolfenzon (2002) develop a static equilibrium model of an entrepreneur's going public decision under imperfect investor protection. La Porta et al. (2002) provide a static model to explain their empirical findings of lower firm values in countries with weaker investor protection.[8] Both papers assume risk neutral controlling shareholders in static settings and thus have implications on neither dynamics nor risk/return tradeoffs. Himmelberg, Hubbard, and Love (2002) develop a two-period model where the risk-averse entrepreneur chooses ownership concentration by trading off the benefit of diversification with the cost of raising capital under imperfect investor protection. Unlike Himmelberg et al.

[7]See La Porta et al. (2000b) for a survey. Gompers et al. (2003) and Black et al. (2006) study the impact of firm-level corporate governance on firm value.

[8]Stulz (2005) constructs a twin agency model where rulers of sovereign states and corporate insiders pursue their own interests to explain the limit of financial globalization.

(2002), we study how investor protection affects corporate investment, public firm valuation for diversified investors, private firm valuation for the under-diversified insider, cost of capital for inside and outside equity, as well as the idiosyncratic risk premium for the insider's equity in a unified dynamic incomplete-markets q-theoretic framework. Additionally, we also analyze the insider's consumption-saving, portfolio choice, business exit and ownership dynamics.

Albuquerque and Wang (2008) develop an equilibrium model of investment and asset pricing under imperfect investor protection.[9] They show that the firm over-invests, the cost of capital is higher, and Tobin's q is lower when investor protection is weaker. The investment-capital ratio, risk premium, and Tobin's q for both insiders and outside investors are all constant. Unlike their model, we focus on a single firm's investment, cost of capital, and firm valuation for both insiders and outsiders. While their work focuses on general equilibrium results, we study dynamic implications of agency on corporate investment, private and public firm value, risk/return tradeoffs for both the under-diversified insider and diversified outsiders for a firm run by an under-diversified entrenched controlling shareholder in an incomplete-markets q-theoretic framework.[10]

Our paper relates to entrepreneurship dynamics and dynamic entrepreneurial finance. Building on Leland (1994), Chen, Miao, and Wang (2010) develop a model of entrepreneurial finance where a risk-averse entrepreneur chooses capital structure as well as consumption and portfolio choices under incomplete markets. Miao and Wang (2007) develop a real-options model of investment for an entrepreneur who is under-diversified. These papers do not study the implications of lacking investor protection and do not model investment and capital accumulation dynamics.

Dynamic corporate finance literature including both investment-based and capital-structure-focused models is fast growing.[11] Almost all models in this literature assume that either the

[9]Dow, Gorton, and Krishnamurthy (2005) study the effects of agency conflicts on asset prices and investment by integrating managerial empire building into a neoclassical asset pricing model.

[10]Li (2010) studies the effects of corporate governance on cross-sectional stock returns in a managerial agency model where the manager derives non-pecuniary private benefits from empire building.

[11]See Whited (1992), Gomes (2001), Hennessy and Whited (2007), Riddick and Whited (2009), and Bolton, Chen, and Wang (2011), among others, for models with investment and financial frictions. Fischer, Heinkel, and Zechner (1989), Leland (1994), and Goldstein, Ju, and Leland (2001) are examples of contingent-claim-style

firm is risk neutral or investors price the firm using a stochastic discount factor. Zwiebel (1996), Morellec (2004), and Lambrecht and Myers (2008) develop dynamic capital structure models with managerial entrenchment, building on Jensen (1986) and Stulz (1990).[12] Unlike existing work, we model the *interactive* effects of managerial agency and risk aversion in a dynamic incomplete-markets-based q theory of investment. Our model distinctively allows us to study the impact of frictions on both marginal q and average q for both the insider and outsiders, corporate investment, as well as the cost of capital for inside and outside equity.[13] Our model also relates to the optimal dynamic contracting literature.[14]

Finally, our paper also contributes to the literature on ownership dynamics. Admati, Pfleiderer, and Zechner (1994) develop a model where a risk-averse large shareholder trades off the enhanced incentives to monitor the firm's performance against the increased exposure to the firm's idiosyncratic risk. DeMarzo and Urosevic (2006) develop a dynamic model of ownership for the large shareholder in light of the trade-off between monitoring incentives and diversification. We model the insider's tradeoff between private benefits of control and diversification. As in the literature, the insider also faces time inconsistency in our model.[15] Unlike the existing work in the literature, we explicitly incorporate a cost function for the insider's ownership adjustments, as often done for equity/debt issuance in dynamic capital structure literature.

1 Model

An entrepreneur, the insider, has a proprietary profitable production technology/business, and is critical for the operation of the business. However, the insider has incentives to pursue private benefits and, moreover, is not well-diversified. We next incorporate imperfect

capital structure models.

[12]Morellec, Nikolov, and Schürhoff (2011) and Nikolov and Whited (2011) estimate dynamic capital structure models with managerial agency.

[13]Lambrecht and Myers (2011) assume risk-averse managers and generate a Lintner-type payout, but do not study investment dynamics, firm valuation, and the cost of capital for outside equity.

[14]See DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), and DeMarzo, Fishman, He, and Wang (2011), for example.

[15]Stoughton and Zechner (1998) study time consistency in a two-period model and consider applications to initial public offering (IPO) underpricing.

investor protection and non-diverisifiable idiosyncratic illiquidity risk into our newly proposed, incomplete-markets-based q-theoretic model of investment.

Physical production and investment technology. The entrepreneur's profitable business uses capital to produce output. Let K and I denote the firm's capital stock and investment, respectively. Capital stock accumulates and stochastically depreciates.[16] We write the dynamics of capital stock as follows,

$$dK_t = (I_t - \delta_K K_t)dt + \sigma_K K_t dZ_t^K \,, \tag{1}$$

where Z^K is a standard Brownian motion, $\delta_K \geq 0$ is the expected rate of depreciation, and σ_K is volatility for capital shocks/depreciation. The firm's operating revenue over $(t,\, t+dt)$ is proportional to its time-t capital stock K_t, and is given by $K_t dA_t$, where dA_t is the firm's productivity shock over the same time period. The productivity shock dA_t is assumed to be independently and identically distributed (iid), and is given by

$$dA_t = \mu_A dt + \sigma_A dZ_t^A, \tag{2}$$

where Z^A is a standard Brownian motion. Here, $\mu_A > 0$ and $\sigma_A > 0$ are the mean and volatility parameters of the productivity shock, respectively.[17] Our simple model ignores the persistence of productivity shocks so as to focus on the effects of investor protection and illiquidity/incomplete markets on investment, firm value, and risk/return. The economic mechanism emphasized here continues to play an important role in a richer and more realistic economic environment.

Changing capital stock often incurs adjustment costs. For example, installing new equipment or upgrading capital may disrupt production lines, and require additional time and resources. Costly capital adjustments are empirically plausible and widely assumed in the in-

[16] For stochastic capital depreciation in macro and finance, see Cox, Ingersoll, Jr., and Ross (1985), Obstfeld (1994), Albuquerque and Wang (2008), and Barro (2009) for examples.

[17] This continuous-time specification is a stochastic formulation of the "AK" technology in Hayashi (1982). For applications of this technology specification in dynamic corporate finance, see DeMarzo, Fishman, He, and Wang (2011), for example.

vestment literature.[18] Let $\Phi(I, K)$ denote the capital adjustment cost function. As is standard in the q-theory literature, we assume that $\Phi(I, K)$ is convex in investment I. Moreover, following Hayashi (1982) and Lucas and Prescott (1971), we assume that $\Phi(I, K)$ is homogeneous of degree one in investment I and capital K. That is, we may write $\Phi(I, K) = \phi(i)K$, where i denotes the investment-capital ratio and $\phi(i)$ is increasing and convex. While our model applies to a well-behaved function $\phi(i)$, for simplicity, we specify $\phi(i)$ as follows,

$$\phi(i) = \frac{\theta_i}{2} i^2 , \tag{3}$$

where the constant $\theta_i > 0$ is the adjustment cost parameter. A higher value θ_i implies a more costly adjustment. With homogeneity, average q is equal to marginal q without frictions. The non-diversifiable risk drives a wedge between average q and marginal q for the controlling shareholder. For simplicity, we assume that the firm's capital shock Z^K and its productivity shock Z^A are uncorrelated.

Investor protection. Because investor protection is imperfect and the insider has control rights, firm profits are not shared on a *pro rata* basis between the insider and outsiders. The insider can pursue private benefits at a personal cost, which is socially inefficient. It may take a variety of forms such as excessive salary, transfer pricing, employing unqualified relatives and friends, just to name a few.[19] We assume that the insiders' discretion does not depend on their cash flow rights, provided that their equity ownership in the firm exceeds $\underline{\alpha}$, a lower bound.[20]

By diverting the amount sK from the firm, the insider incurs a cost $\Psi(s, K)$, which is assumed to be increasing and convex in s as in La Porta et al. (2002), Johnson et al. (2000), and Stulz (2005). Additionally, we assume that $\Psi(s, K)$ is homogeneous in diversion amount sK and K, $\Psi(s, K) = \psi(s)K$, analogous to the one for the capital adjustment cost function

[18] See Hayashi (1982), Abel (1983), and Abel and Eberly (1994) on the role of adjustment costs on investment and the value of capital. See Caballero (1999) for a survey.

[19] See Barclay and Holderness (1989), Dyck and Zingales (2004), and Albuquerque and Schroth (2010) on the empirical evidence in support of private benefits of control.

[20] We can extend our model to assume that the the controlling shareholder's power (and hence ability to pursue private benefits) also depends on ownership. The controlling shareholders can achieve full control of the firm with far less than majority cash flow rights via dual class shares, pyramidal structure, a controlled board, and/or other strategies.

$\Phi(I, K)$. For simplicity, we specify

$$\psi(s) = \frac{\theta_s}{2} s^2 , \tag{4}$$

where θ_s is the investor protection parameter as in La Porta et al. (2002). Given our focus on the economic and financial consequences of lacking investor protection, for simplicity, we take investor protection as exogenously given. However, in reality, investor protection is at least partly chosen by the insider.[21]

The insider's exit option. While enjoying private benefits of control, the insider also faces significant non-diversifiable risk from the exposure to the firm. It may thus sometimes be optimal for the insider to exit from the firm by giving up the control and collecting the sales proceeds of the *pro rata* share in the firm. Under the new management, firm value (net of all transaction costs due to the exit) is a fraction of the firm size, lK, where l is a constant. This exit may take the form of public offering or a private sale arrangement. The exit decision resembles an American-style call option on the underlying non-tradable firm. Since markets are incomplete for the insider, we cannot use the standard financial option pricing model but, instead, need to value this exit option by solving the insider's interdependent dynamic optimization problem.

Payouts to outside shareholders and the insider. Let τ denote the stochastic and endogenously chosen time that the controlling shareholder exits from the firm. Before exiting ($t < \tau$), the payout to outside shareholders is given by

$$dY_t = K_t dA_t - I_t dt - \Phi(I_t, K_t)dt - s_t K_t dt , \quad t < \tau , \tag{5}$$

where the last term is the firm's output diverted by the controlling shareholder. The cash flow accruing to the controlling shareholder, dM_t over the period $(t, t + dt)$, is given by the sum of *pro rata* cash flow and diverted output less the cost of diversion,

$$dM_t = \alpha dY_t + s_t K_t dt - \Psi(s_t,\, K_t)dt , \quad t < \tau , \tag{6}$$

[21] See Bergman and Nicolaievsky (2007) for one such model.

where dY_t given by (5) is the payout to outside shareholders.

Financial investment opportunities. As outside shareholders, the controlling shareholder has standard liquid financial investment opportunities, a risk-free asset which pays a constant rate of interest r and a risky market portfolio. As in Merton (1971), the incremental return dR_t of the market portfolio over the period $(t, t+dt)$ is iid and given by

$$dR_t = \mu_R dt + \sigma_R dB_t \,, \tag{7}$$

where μ_R and $\sigma_R > 0$ are mean and volatility parameters of the market portfolio return process, and B is a standard Brownian motion. Let η denote the Sharpe ratio of the market portfolio, which is given by

$$\eta = \frac{\mu_R - r}{\sigma_R} \,. \tag{8}$$

Let ρ_A be the correlation coefficient between the firm's productivity shock dZ_t^A and the return of the market portfolio dR_t. Similarly, let ρ_K denote the correlation between the firm's capital shock dZ_t^K and dR_t. The firm faces both productivity and capital shocks. With either $|\rho_A| < 1$ or $|\rho_K| < 1$, the controlling shareholder cannot fully diversify the idiosyncratic business risk. Non-diversifiable idiosyncratic risk and investor protection jointly play critical roles in the controlling shareholder's decision making and implied private as well as public valuation.

Let X and Π denote the controlling shareholder's financial wealth and the investment amount in the market portfolio, respectively. Thus, $(X - \Pi)$ is the amount invested in the risk-free asset. Before exiting, the controlling shareholder's wealth X evolves as,

$$dX_t = r\,(X_t - \Pi_t)\,dt + (\mu_R dt + \sigma_R dB_t)\,\Pi_t - C_t dt + dM_t \,, \qquad t < \tau \,, \tag{9}$$

where the first and second terms give the returns from investments in the risk-free asset and in the risky market portfolio respectively, the third term gives the consumption outflow, and the last term dM_t given by (6) is the income (the sum of *pro rata* share of equity payout and private benefits of control net of diversion costs) for the controlling shareholder. At the exit time τ, the insider's wealth changes from $X_{\tau-}$ to

$$X_\tau = X_{\tau-} + \alpha l K_\tau \,. \tag{10}$$

After exiting, the controlling shareholder's wealth evolves as follows,

$$dX_t \quad = \quad r\,(X_t - \Pi_t)\,dt + \mu_R \Pi_t dt + \sigma_R \Pi_t dB_t - C_t dt\,, \qquad t > \tau\,. \tag{11}$$

The insider's optimization problem. The controlling shareholder chooses consumption C_t, allocation to the market portfolio Π_t, corporate investment I_t, diversion s_t, and stochastic exit time τ to maximize utility given by

$$\max_{C,\,\Pi,\,I,\,s,\,\tau} \quad \mathbb{E}\left[\int_0^\infty e^{-\zeta t}\,U(C_t)\,dt\right]\,, \tag{12}$$

where $\zeta > 0$ is the subjective discount rate and $U(C)$ is an increasing and concave function. Our setup applies to any well-behaved utility function $U(C)$. For tractability, we adopt the constant absolute risk averse (CARA) utility for the remainder of the paper, $U(C) = -e^{-\gamma C}/\gamma$, where $\gamma > 0$ is the CARA coefficient. Our model by construction misses the wealth effect implications due to the CARA utility assumption. However, the main insight of our paper is robust, since the key mechanism of our model operates through the *interactive* and opposing effects between the controlling shareholder's precautionary savings demand and incentives to pursue private benefits.

The outside shareholders' perspective. Outside shareholders hold a diversified investment portfolio and thus demand risk premia for systematic risks, not for *idiosyncratic* risks. There are two sources of systematic risks, the productivity shock induced cash flow risk and the capital shock induced capital gains risk. We will show that the conditional capital asset pricing model (CAPM) holds in our model.

2 Solution

We first solve the controlling shareholder's optimization problem. Then, we report results for the special case where markets are complete.

2.1 The controlling shareholder's optimality

After exiting, the insider solves the standard consumption and portfolio choice problem. Following Merton (1971), we may write the post-exit value function $F_0(X)$ as

$$F_0(X) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2}\right)\right]. \tag{13}$$

Before exiting, the insider's value function $F(X,K)$ is given by

$$F(X,K) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2} + \alpha P(K)\right)\right], \quad t < \tau, \tag{14}$$

where $P(K)$ can be interpreted as the insider's private (certainty equivalent) valuation of the firm. The following theorem characterizes $P(K)$ and the insider's decision rules.

Theorem 1 *The controlling shareholder's private valuation of capital per unit of ownership, $P(K)$, solves the following ordinary differential equation (ODE),*

$$\begin{aligned} rP(K) &= (\nu_A + b(\alpha))\,K - \delta K P'(K) + \frac{(P'(K)-1)^2}{2\theta_i}K + \frac{\sigma_K^2 K^2 P''(K)}{2} \\ &\quad -\frac{\alpha\gamma r K^2}{2}\left[(1-\rho_A^2)\sigma_A^2 - 2\rho_A\rho_K\sigma_A\sigma_K P'(K) + (1-\rho_K^2)\sigma_K^2 P'(K)^2\right], \end{aligned} \tag{15}$$

where the net private benefit of control per unit of ownership, $b(\alpha)$, is given by

$$b(\alpha) = \frac{(1-\alpha)^2}{2\alpha\theta_s}, \tag{16}$$

the risk-adjusted expected productivity ν_A is given by

$$\nu_A = \mu_A - \rho_A \eta \sigma_A, \tag{17}$$

and the risk-adjusted capital depreciation rate, δ, is given by

$$\delta = \delta_K + \rho_K \eta \sigma_K. \tag{18}$$

We solve the ODE (15) subject to the following boundary conditions,

$$P(0) = 0, \tag{19}$$

$$P(\overline{K}) = l\overline{K}, \tag{20}$$

$$P'(\overline{K}) = l. \tag{21}$$

The optimal investment-capital ratio $i = I/K$ is given by

$$i(K) = \frac{P'(K) - 1}{\theta_i} . \tag{22}$$

The optimal diversion s is given by

$$s(\alpha) = \frac{1 - \alpha}{\theta_s} , \tag{23}$$

and the optimal consumption rule is given by

$$C(X, K) = r \left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2} + \alpha P(K) \right) . \tag{24}$$

The optimal investment amount in the risky market portfolio is given by

$$\Pi(K) = \frac{\eta}{\gamma r \sigma_R} - \frac{\rho_A \sigma_A}{\sigma_R} \alpha K - \frac{\rho_K \sigma_K}{\sigma_R} \alpha K P'(K) . \tag{25}$$

The controlling shareholder collects private benefits at the rate of $(1-\alpha)sK$ and incurs cost at the rate of $\psi(s)K$. Thus, per unit of capital and ownership, the net private benefit per period is $[(1 - \alpha)s - \psi(s)]/\alpha = b(\alpha)$. The consumption rule (24) is similar to other CARA-utility-based permanent-income/precautionary-saving models. The optimal investment-capital ratio is now determined by the controlling shareholder's (private) marginal q, $P'(K)$. The optimal portfolio rule (25) has the standard mean-variance demand as well as the dynamic hedging demands against both productivity and capital shocks. The ODE (15) characterizes $P(K)$, the controlling shareholder's certainty equivalent valuation of the firm in the interior region of K. The left boundary $K = 0$ is absorbing and hence $P(0) = 0$. Conditions (20)-(21) are the value-matching and smooth-pasting conditions for $P(K)$ at the optimal exit boundary $\overline{K}$.

It proves useful to define the private average q and marginal q for the under-diversified entrenched insider. Since $P(K)$ is the insider's certainty equivalent valuation for the firm per unit of ownership, we may thus naturally refer to $P'(K)$ as the insider's (private) marginal q. The insider's (private) average q is given by

$$p(K) = \frac{P(K)}{K} . \tag{26}$$

2.2 A special case: Imperfect investor protection and CM

For the special case with complete markets (CM), we have closed-form solutions. With imperfect investor protection, insiders and outsiders have different valuation for the firm. Importantly, as long as markets are complete, perfect risk sharing is feasible, hence neither the insider nor outsiders demand idiosyncratic risk premium. The following proposition summarizes the main results for the CM case.

Proposition 1 *The insider values the firm $P^{CM}(K) = p^{CM}(\alpha)K$, where $p^{CM}(\alpha)$ is the insider's average q as well as marginal q, given by*

$$p^{CM}(\alpha) = 1 + \theta_i \, i^{CM}(\alpha) \,, \tag{27}$$

and the firm's investment-capital ratio $i^{CM}(\alpha)$ is given by

$$i^{CM}(\alpha) = (r+\delta) - \sqrt{(r+\delta)^2 - \frac{2}{\theta_i}\left(\nu_A + b(\alpha) - (r+\delta)\right)} \,. \tag{28}$$

Here, $b(\alpha)$ is given in (16), ν_A is given by (17), and δ is given by (18). Outside investors value the firm at $V^{CM}(K) = q^{CM}(\alpha)K$, where Tobin's q is given by

$$q^{CM}(\alpha) = \frac{\nu_A - s(\alpha) - i^{CM}(\alpha) - \phi(i^{CM}(\alpha))}{r + \delta - i^{CM}(\alpha)} \,. \tag{29}$$

The homogeneity property implies that marginal q equals average q for the insider, which we denote by $p^{CM}(\alpha)$. Outside investors have rational expectations, and thus price the firm accordingly at $q^{CM}(\alpha)$, which is the public investors' marginal q as well as their average q, which we denote by $q^{CM}(\alpha)$. Because the insider gains at the expense of outside investors, we have $p^{CM}(\alpha) > q^{CM}(\alpha)$.

The risk-adjusted expected productivity ν_A is lower than the expected productivity μ_A by the risk premium $\rho_A \eta \sigma_A$. Similarly, capital is subject to shocks and thus the risk-adjusted capital depreciation rate δ is larger than the expected depreciation rate δ_K by the size of the risk premium $\rho_K \eta \sigma_K$. Investment is positive if and only if

$$\nu_A + b(\alpha) > r + \delta \,. \tag{30}$$

Intuitively, when the sum of the firm's risk-adjusted expected productivity ν_A and net private benefits $b(\alpha)$, $\nu_A + b(\alpha)$, is greater than the cost of investing $r + \delta$ given by the sum of the interest rate r and the risk-adjusted capital depreciation rate δ, the insider invests $i^{CM}(\alpha) > 0$ and earns rents in equilibrium, $p^{CM}(\alpha) > 1$. With CM, neither insiders nor outsiders demand idiosyncratic business risk premia for holding the firm. Therefore, for both the insider and outsiders, CAPM holds but with different betas. The following Corollary reports the results.

Corollary 1 *With CM, the expected returns, μ_{in}^{CM} and μ_{out}^{CM} for the insider and outsiders respectively, are given by CAPM with respective betas, in that*

$$\mu_n^{CM} = r + \beta_n^{CM} \left(\mu_R - r\right) \,, \quad \textit{where} \quad n = in,\ out\,. \tag{31}$$

Here, $n = in$ and $n = out$ refer to the insider and outsiders, respectively, and

$$\beta_n^{CM} = \beta_K^{CM} + \beta_{A,n}^{CM} \,, \tag{32}$$

where

$$\beta_K^{CM} = \frac{\rho_K \sigma_K}{\sigma_R} \,, \tag{33}$$

$$\beta_{A,in}^{CM} = \frac{\rho_A \sigma_A}{\sigma_R} \frac{1}{p^{CM}(\alpha)} \,, \qquad \beta_{A,out}^{CM} = \frac{\rho_A \sigma_A}{\sigma_R} \frac{1}{q^{CM}(\alpha)} \,. \tag{34}$$

The productivity shock beta is lower for the insider than for outsiders, $\beta_{A,\,in}^{CM} < \beta_{A,\,out}^{CM}$, because the firm is more valuable for the insider than for outsiders, $p^{CM}(\alpha) > q^{CM}(\alpha)$, and thus a realized negative cash flow yield matters less for the insider, *ceteris paribus*. The insider and outsiders value the capital shock risk in the same way with beta, β_K^{CM}, given by (33). Adding the two sources of risks, the firm is thus less risky for the insider than for outsiders, $\beta_{in}^{CM} < \beta_{out}^{CM}$, and the expected return for the insider is lower than that for outsiders, $\mu_{in}^{CM} < \mu_{out}^{CM}$. In sum, even with CM, the costs of capital, $\mu_{in} < \mu_{out}$, differ for inside and outside equities. This is due to the fact that contracts are incomplete and control matters for firm value which in turn influences the costs of capital.

The first-best (FB) benchmark. With perfect investor protection ($\theta_s = \infty$) and CM, the insider pursues no private benefits, $s = 0$, and outsiders have the same valuation as the insider does, $p^{CM}(\alpha) = q^{CM}(\alpha) = q^{FB}$, where

$$q^{FB} = 1 + i^{FB}\theta_i\,, \tag{35}$$

and the first-best investment-capital ratio i^{FB} is given by

$$i^{FB} = (r+\delta) - \sqrt{(r+\delta)^2 - \frac{2}{\theta_i}\left(\nu_A - (r+\delta)\right)}\,. \tag{36}$$

The standard q theory of investment holds and firm value is given by $V^{FB}(K) = q^{FB}K$. Asset pricing implications follow from Corollary 1 for the CM case by using (35).

3 Firm investment and the insider's valuation

We now explore the implications of imperfect investor protection and non-diversifiable idiosyncratic risk on investment and the controlling shareholder's private valuation.

Parameter choice. When applicable, parameter values are annualized. The risk-free rate is $r = 5\%$. For the market portfolio return, the risk premium and the volatility are $\mu_R - r = 6\%$ and $\sigma_R = 20\%$, with an implied Sharpe ratio of $\eta = 30\%$. Using the sample of large firms in Compustat from 1981 to 2003, Eberly, Rebelo, and Vincent (2009) provide empirical evidence in support of Hayashi (1982). Using their work as a guideline, we set the expected productivity $\mu_A = 22.8\%$, the volatility of productivity shocks $\sigma_A = 25\%$, and the expected capital depreciation rate $\delta_K = 8\%$. We choose the adjustment cost parameter $\theta_i = 3$, which is in the range of estimates used in the literature.[22] We set the volatility of the capital shock $\sigma_K = 20\%$, and the correlation coefficients $\rho_A = \rho_K = 0.5$. The implied risk-adjusted productivity $\nu_A = 19\%$ and the risk-adjusted depreciation rate $\delta = 11\%$. In the first-best benchmark, $q^{FB} = 1.256$, $i^{FB} = 0.085$, $\beta_A^{FB} = 0.5 = \beta_K^{FB} = 0.5$, which implies the firm's beta, $\beta^{FB} = 1$.

The insider also has an exit option, which can be valuable for diversification and risk management purposes. Exit may take the form of a public offering, a private sale arrangement,

[22] See Whited (1992), Hall (2004), Riddick and Whited (2009), and Eberly et al. (2009).

or a buyout. We set $l = 1.15$ so that outside investors collect 1.15 per unit of capital (net of transaction costs) upon exiting.[23] The insider's ownership is set at $\alpha = 0.25$ (Dahlquist et al. (2003)). Using the calibration in Albuquerque and Wang (2008) as a reference, we set the investor protection parameter $\theta_s = 350$, which implies that the diversion amount in each period is $s(0.25) = (1 - 0.25)/350 = 0.21\%$ of the firm's capital stock, or equivalently 0.94% of the firm's average output.

For risk aversion, researchers have views about sensible values of *relative* risk aversion, but not *absolute* risk aversion. While researchers may disagree on the exact value of relative risk aversion, they agree that a sensible range of relative risk aversion is likely between one to five. For CARA utility, we may approximate the coefficient of relative risk aversion by γX_0, where X_0 is the entrepreneur's initial wealth. We choose the CARA coefficient $\gamma = 2$, which implies that the coefficient of *relative* risk aversion is around 3.2 in our calibration. See Section 7 for details for the calibration of γ. Table 3 summarizes all variables and baseline parameter values used in the paper.

The insider's private marginal q, $P'(K)$, and private average q, $p(K)$. Panel A of Figure 1 plots the private marginal q, $P'(K)$ and the private average q, $p(K) = P(K)/K$. First, for a firm whose size is infinitesimal ($K \to 0$), the idiosyncratic risk is negligible for the insider. Hence, $p(0) = P'(0) = p^{CM} = 1.304$, where p^{CM} is the CM solution of Section 2.2. Second, at the endogenously chosen exit boundary $\overline{K} = 30.8$, both the insider's private marginal q and the private average q equal to the exit value l, i.e. $P'(\overline{K}) = p(\overline{K}) = l = 1.15$, implied by the insider's optimality, more specifically, the value-matching and smooth pasting conditions, (20) and (21) respectively.

Third, in the interior region $0 < K < \overline{K} = 30.8$, as firm size K increases, the insider's idiosyncratic business risk exposure increases, the insider's under-diversification cost thus also increases, and the private average q, $p(K)$, decreases. Fourth, because the insider's average q,

[23] While the value for each unit of capital $l = 1.15$ is larger than unity, the firm's setup and other adjustment costs rule out arbitrage.

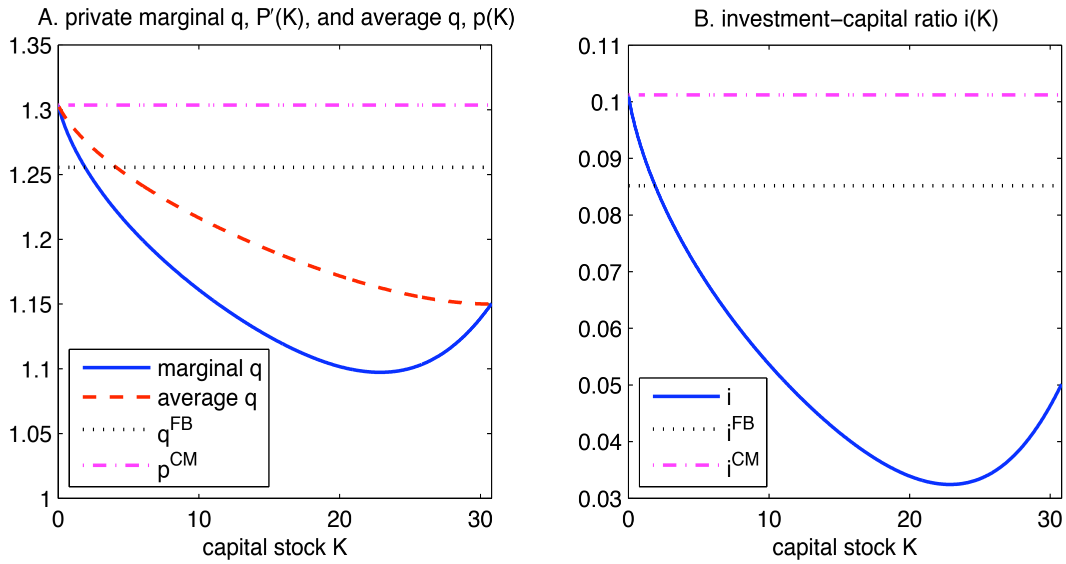

Figure 1: The controlling shareholder's private marginal q, $P'(K)$, private average q, $p(K)$, and the investment-capital ratio $i(K)$.

$p(K)$, decreases with K, the private marginal q must be lower than private average q for all K, i.e. $P'(K) < p(K)$. This result follows from

$$P'(K) - p(K) = Kp'(K) < 0\,. \tag{37}$$

Intuitively, in order for the average q to decrease with K, the incremental value of K for the insider , which is the private marginal q, must be lower than the private average q to pull down the average. This relation is analogous to the one between the marginal cost and average cost in micro theory. Panel A shows that other than at the boundaries, $K = 0$ and $K = \overline{K}$, where $P'(0) = p(0)$ and $P'(\overline{K}) = p(\overline{K})$, the private marginal q is strictly lower than the private average q, i.e. $P'(K) < p(K)$ for $0 < K < \overline{K}$.

Fifth, $P'(K)$ is non-monotonic in firm size K as the *positive* wedge between average q and marginal q, $p(K) - P'(K)$, must vanish at the two boundaries $K = 0$ and $K = \overline{K}$. Panel A of Figure 1 shows that the wedge, $p(K) - P'(K)$, first widens as K increases and then narrows as K approaches $\overline{K}$. For $K \leq 22.9$, the under-diversification cost increases with K, making $P'(K)$ decrease in K. For $22.9 < K \leq 30.8$, the under-diversification cost continues to increase but the flexible exit option becomes increasingly deep in the money, causing $P'(K)$ to increase in K.

The firm's investment-capital ratio $i(K)$. Panel B of Figure 1 plots the investment-capital ratio $i(K)$. Since the insider optimally equates the private marginal q, $P'(K)$, with the private marginal cost of investing, $1 + \theta_i \, i(K)$, $i(K)$ is an affine function of $P'(K)$ and effectively traces the shape of $P'(K)$. Compared with the first-best level, both over- and under-investment occur. The private benefits of control lead to over-investment while under-diversification discourages investment. When the private benefits motive is stronger than the diversification one ($K \leq 1.90$), the firm over-invests. Otherwise, it under-invests. Additionally, $i(K)$ is non-monotonic in firm size K in the same way as $P'(K)$ is. In the region $K \leq 22.9$, as K increases, the insider becomes increasingly concerned with under-diversification and decreases $i(K)$. In the region $22.9 < K \leq 30.8$, the insider's option value of exiting is sufficiently in the money. The positive volatility effect on the exit option value gives the insider incentives to increase $i(K)$ from $i(22.9) = 0.032$ at $K = 22.9$ to $i(30.8) = 0.050$ at $\overline{K} = 30.8$.

Importantly, under-diverisification, the flexible exit option, and investor protection jointly generate rich investment dynamics, even when production features constant-returns-to-scale technology as in Hayashi (1982).

4 Public firm value, average q, and marginal q

Outside investors take the insider's decisions as given, and rationally price the firm. Unlike the insider, diversified outsiders only demand systematic risk premia.

Proposition 2 *Firm value $V(K)$ for outside investors solves the ODE,*

$$rV(K) = (\nu_A - s(\alpha) - i(K) - \phi(i(K)))\, K + (i(K) - \delta)\, KV'(K) + \frac{\sigma_K^2 K^2 V''(K)}{2}\,, \tag{38}$$

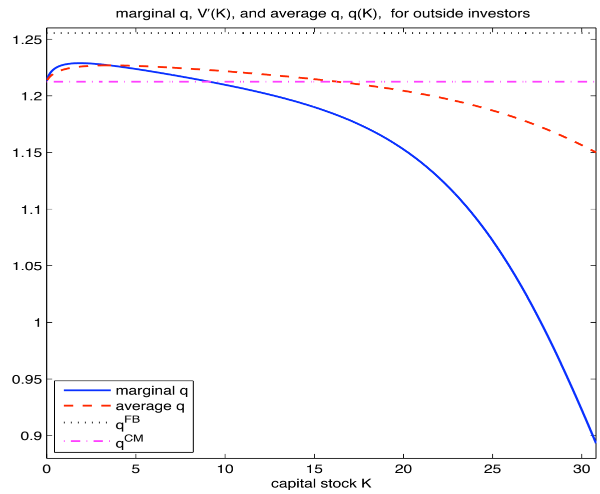

Figure 2: Public marginal q, $q_m(K) = V'(K)$ and public average q, $q_a(K) = V(K)/K$

subject to the following boundary conditions,

$$V(0) = 0\,, \tag{39}$$

$$V(\overline{K}) = l\overline{K}\,. \tag{40}$$

For outside investors, the firm's public average q and public marginal q are

$$q_a(K) \equiv \frac{V(K)}{K}\,, \quad \text{and} \quad q_m(K) \equiv V'(K)\,. \tag{41}$$

Figure 2 plots public average q, $q_a(K)$, and public marginal q, $q_m(K)$. Public average $q_a(K)$ is lower than the first-best benchmark value $q^{FB} = 1.256$ due to agency costs.

In the limit as $K \to 0$, the insider faces no idiosyncratic business risk. Therefore, both average q and marginal q approach the CM-benchmark value q^{CM}, $q_a(0) = q_m(0) = q^{CM} = 1.213$. Outsiders rationally anticipate the expropriation by the insider and hence price the firm accordingly, i.e. the insider's q is larger than the first-best q^{FB}, which in turn is higher than outsiders' public q, in that $p^{CM} = 1.304 > q^{FB} = 1.256 > q^{CM} = 1.213$. As a fraction of the first-best value q^{FB}, the total social discount is

$$q^{FB} - \left[\alpha p^{CM}(\alpha) + (1-\alpha) q^{CM}(\alpha)\right] = 0.020\,, \tag{42}$$

which is 1.6% of the first-best q^{FB}.

Average q, $q_a(K)$, first increases in firm size K and then decreases in K. This is in sharp contrast to the private average q, $p(K) = P(K)/K$, which monotonically decreases with K. For $K < 3.6$, as K increases, the insider's idiosyncratic business risk increases and thus the investment-capital ratio $i(K)$ decreases. This decrease of $i(K)$ is value enhancing for outsiders because the insider's over-investment motive is mitigated by under-diversification costs. For $K > 3.6$, public average q, $q_a(K)$, decreases with K as the under-investment motive is sufficiently strong and dominates the insider's over-investment motive.

Since the average $q_a(K)$ is increasing in the region $K < 3.6$, public marginal $q_m(K)$ must be higher than public average $q_a(K)$ in order to pull up average $q_a(K)$ as K increases. This relation between average q and marginal q again is analogous to the average cost and marginal cost in micro theory.[24] Similarly, for $K > 3.6$, public marginal $q_m(K)$ has to fall below average $q_a(K)$, $q_m(K) < q_a(K)$ to cause public average $q_a(K)$ to decrease with K. Average $q_a(K)$ is maximized at $K = 3.6$. At the exit threshold $\overline{K} = 30.8$, public marginal q equals $q_m(30.8) = 0.89$, 22.6% lower than public average q, $q_a(30.8) = 1.15$. This wedge between the two q's also reflects agency conflicts. We next explore the asset pricing implications for outsiders.

5 Beta and the cost of outside equity capital

Using Ito's formula, we show that the incremental return dR_t^V for outside equity is given by the sum of dividend yield dY_t/V_t and capital gains dV_t/V_t,

$$dR_t^V \equiv \frac{dY_t + dV_t}{V_t} = \mu_V(K_t)dt + \frac{\sigma_A}{q_a(K_t)}dZ_t^A + \frac{q_m(K_t)}{q_a(K_t)}\sigma_K dZ_t^K \,, \tag{43}$$

where the expected return, $\mu_V(K)$, is also referred to as the cost of capital for outside equity. The following proposition summarizes the asset pricing implications for investors.

Proposition 3 *The conditional CAPM holds for outside equity and $\mu_V(K)$ satisfies*

$$\mu_V(K) = r + \beta(K)(\mu_R - r) \,, \tag{44}$$

[24] Using $q_a'(K) = (q_m(K) - q_a(K))/K$ and $q_m(0) = q_a(0)$, we have $q_a'(K) > 0$ if and only if $q_m(K) > q_a(K)$.

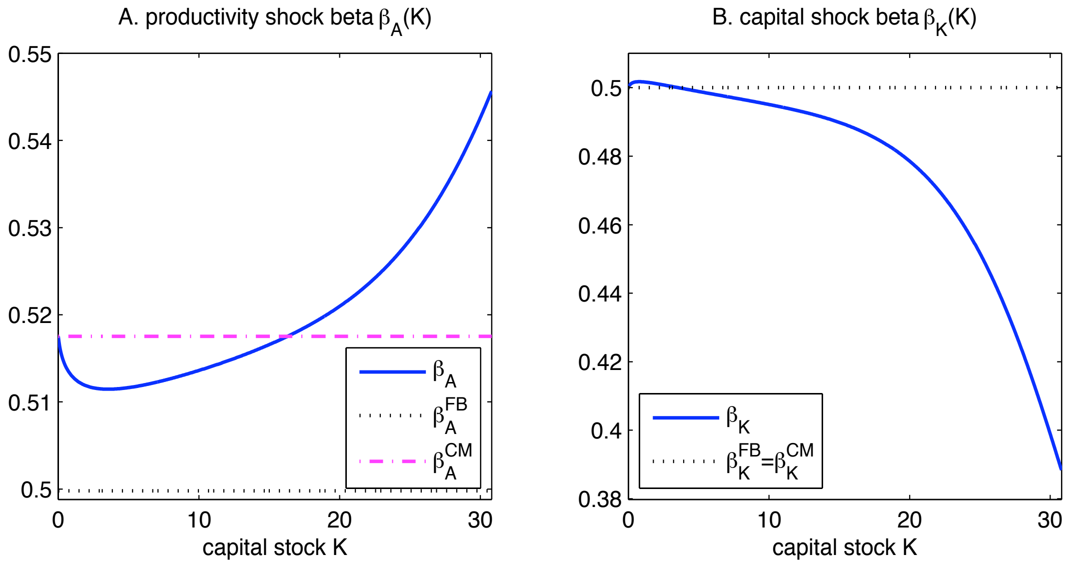

Figure 3: Productivity shock beta $\beta_A(K)$ and capital shock beta $\beta_K(K)$

where the conditional beta for outside equity, $\beta(K)$, is given by

$$\beta(K) \quad = \quad \beta_A(K) + \beta_K(K)\,, \tag{45}$$

and

$$\beta_A(K) \quad = \quad \frac{\rho_A \sigma_A}{\sigma_R} \frac{1}{q_a(K)}\,, \tag{46}$$

$$\beta_K(K) \quad = \quad \frac{\rho_K \sigma_K}{\sigma_R} \frac{q_m(K)}{q_a(K)}\,. \tag{47}$$

While our model has both productivity and capital shocks, our conditional CAPM only has one factor, capital stock. The productivity shock carries a firm-size dependent risk premium, because average q, $q_a(K)$, depends on firm size K.

Panels A and B of Figure 3 plot $\beta_A(K)$ and $\beta_K(K)$, respectively. The productivity shock beta, $\beta_A(K)$, is stochastic and depends inversely on Tobin's $q_a(K)$. Under imperfect investor protection, average $q_a(K)$ is lower than q^{FB}. Thus, the formula (46) for $\beta_A(K)$ implies $\beta_A(K) > \beta_A^{FB}$. The lower Tobin's $q_a(K)$, the higher $\beta_A(K)$. Intuitively, the weaker investor protection, the lower firm value, which in turn makes productivity shock riskier, and hence a

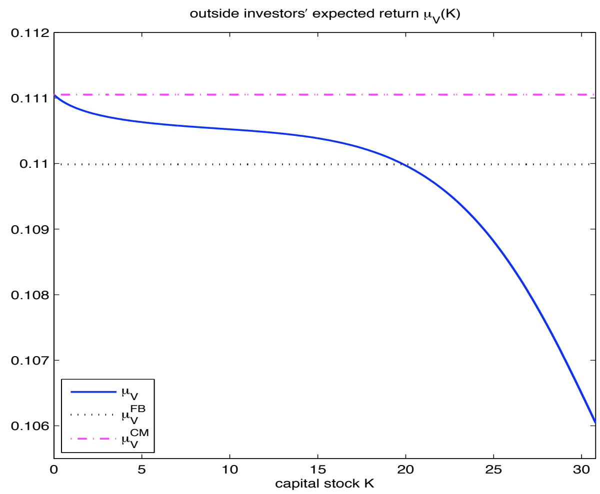

Figure 4: The cost of capital $\mu_V(K)$ for outside equity

higher $\beta_A(K)$. As we have discussed, $q_a(K)$ is non-monotonic in firm size K, thus productivity shock $\beta_A(K)$ is also non-monotonic in K. In the limit as $K \to 0$, the controlling shareholder faces no idiosyncratic business risk exposure, hence the productivity shock beta approaches $\beta_A^{CM} = 0.500 + 0.517 = 1.017$.

The capital shock beta, $\beta_K(K)$ given in (47), depends on the ratio between the firm's marginal q, $q_m(K)$, and average q, $q_a(K)$, which can be equivalently expressed as the elasticity of firm value $V(K)$ with respect to capital K, since $d\ln V(K)/d\ln K = q_m(K)/q_a(K)$. Intuitively, capital growth is stochastic and co-varies with the aggregate risk, which induces a risk premium described by capital shock beta, $\beta_K(K)$. When $K < 3.6$, $q_m(K) > q_a(K)$, and hence $\beta_K(K) > \beta_K^{FB}$. In contrast, when $K > 3.6$, $q_m(K) < q_a(K)$ and $\beta_K(K) < \beta_K^{FB}$.

Figure 4 plots $\mu_V(K)$, the expected rate of return for outside investors, which is given by the conditional CAPM (44). Recall that under the first-best benchmark, the firm's beta is constant and the expected return thus also remains constant. In our example, $\beta^{FB} = 1$ and $\mu_V^{FB} = 11\%$ (see the dotted line). Frictions cause the expected return $\mu_V(K)$ to be either higher or lower than the expected return $\mu_V^{FB} = 11\%$ under the first-best benchmark. The insider has both over-investment and under-investment motives due to private benefits and under-diversification discount. In sum, while frictions lower firm value, the effect of frictions on the cost of capital is far from obvious. For our example, the cost of capital for outside equity is higher than the first-best benchmark value $\mu_V^{FB} = 11\%$ for $K < 19.8$, and $\mu_V(K) < \mu_V^{FB} = 11\%$ when $K > 19.8$.

6 Idiosyncratic risk premium for the insider

We next develop an analytically tractable and operational method to calculate the insider's idiosyncratic risk premium. The insider holds the illiquid business until endogenous exit. We measure the insider's cost of capital via the internal rate of return (IRR) over the stochastic holding period. Let ξ denote the insider's IRR, which solves

$$P(K_0) = \frac{1}{\alpha}\mathbb{E}\left[\int_0^\tau e^{-\xi t} dM_t + \alpha e^{-\xi\tau} l K_\tau\right] , \tag{48}$$

where τ is the stochastic liquidation time. The right side of (48) is the insider's present discounted value (PDV) using the IRR ξ as the rate, per unit of ownership. The left side is the insider's "private" firm value, $P(K_0)$. We write the IRR as $\xi(K_0)$, a function of initial firm size K_0. We solve the IRR $\xi(K_0)$ via an ODE in the Appendix.

The IRR contains both the systematic and the idiosyncratic risk premia. If markets are complete, the cost of capital for inside equity, μ_{in}^{CM}, is given by the unconditional CAPM (31) for the insider. The idiosyncratic risk premium $\omega(K_0)$, defined as the difference between $\xi(K_0)$ and μ_{in}^{CM}, is given by

$$\omega(K_0) \quad = \quad \xi(K_0) - \mu_{in}^{CM} . \tag{49}$$

There is much debate in the empirical literature about the significance of this *private* equity risk premium. For example, Moskowitz and Vissing-Jørgensen (2002) document the risk-adjusted returns to investing in U.S. nonpublicly traded equity are not higher than the returns to private equity, while Mueller (2011) finds the opposite. Our model provides an analytical formula to calculate this private equity idiosyncratic risk premium for controlling shareholders. Figure 5 plots the idiosyncratic risk premium for the controlling shareholder. As firm size K increases, the idiosyncratic risk premium also increases because firm-specific risk

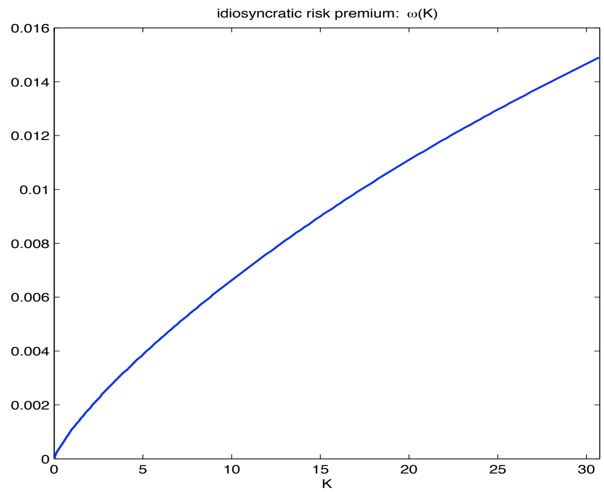

Figure 5: Idiosyncratic risk premium for the insider

increases. At the moment of exit, $\overline{K} = 30.8$, the idiosyncratic risk premium approaches 1.5%, which is quite significant.

7 Endogenous ownership and firm size

We now endogenize the initial firm size K_0 and the insider's ownership.

Model setup. We assume that the external financing is equity as in Shleifer and Wolfenzon (2002). The entrepreneur needs a sufficiently high equity stake in the firm to retain full control. Let $\underline{\alpha}$ denote the minimal level of ownership for the entrepreneur to have full control rights. We focus on the economically interesting case where it is optimal for the entrepreneur to hold a controlling stake in the firm, $\alpha \geq \underline{\alpha}$.[25]

Let K_e and K_m denote the controlling shareholder's and outside shareholders' contribution to the initial firm size K_0, respectively. The initial firm size K_0 is

$$K_0 = K_e + K_m\,. \tag{50}$$

[25]It is conceivable that the entrepreneur's control rights within the firm depend on ownership α. In that case, the entrepreneur has an additional tradeoff margin between diversification and the degree of control. We leave this extension for future research.

Under perfectly competitive capital markets, outside shareholders break even in risk-adjusted present value, which implies

$$K_m = (1-\alpha)\,V(K_0;\alpha)\,. \tag{51}$$

Here, we explicitly note the dependence of firm value $V(K)$ on ownership α.

Setting up the firm is costly for the entrepreneur. We take a broad interpretation for the setup cost. It may represent dilution costs as in Myers and Majluf (1984), the real setup cost including legal, accounting, and compliance costs, indirect costs such as the forgone wages earned elsewhere, as well as the difficulty of raising funds or other barriers to be an entrepreneur. Let $\Lambda(K_0)$ denote this setup cost as a function of initial firm size K_0. Intuitively the larger the firm size K_0, the higher the setup cost $\Lambda(K_0)$ and also the higher the marginal setup cost $\Lambda'(K_0)$. While our model applies to any increasing and convex cost function $\Lambda(K)$, for simplicity, we assume

$$\Lambda(K) = \lambda_1 K + \frac{\lambda_2 K^2}{2}\,, \tag{52}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$.

Let X_0 denote the entrepreneur's total liquid wealth just prior to setting up the firm. At time 0, the entrepreneur invests amount K_e in the firm, raises external equity K_m for the firm, chooses ownership $\alpha \geq \underline{\alpha}$, pays the setup cost $\Lambda(K_0)$, and allocates the remaining amount, $X_0 - K_e - \Lambda(K_0)$, between the risk-free asset and the risky market portfolio to maximize lifetime utility (12) subject to outside shareholders' break-even condition (51) and the accounting identity (50) for initial firm size K_0.

Optimization problem. Because outsiders break even *ex ante*, the insider internalizes the net benefits of setting up the firm, and the optimality can be written as

$$\max_{K_0,\,\alpha\geq\underline{\alpha}} W(K_0;\alpha) - (K_0 + \Lambda(K_0))\;. \tag{53}$$

Here, $W(K)$ is given by

$$W(K;\alpha) = \alpha P(K;\alpha) + (1-\alpha)V(K;\alpha)\,. \tag{54}$$

Note that $W(K)$ is the sum of the insider's private valuation, $\alpha P(K)$, and outsiders' public valuation, $(1-\alpha)V(K)$. We refer to $W(K_0;\alpha) - (K_0 + \Lambda(K_0))$ as the net social surplus. The next proposition summarizes the insider's optimality at time 0.

Proposition 4 *For a given $\alpha \geq \underline{\alpha}$, firm size K_0^* as a function of α, $K_0^*(\alpha)$, solves*

$$W_K(K_0^*;\alpha) \;\; = \;\; 1 + \Lambda'(K_0^*)\,. \tag{55}$$

The controlling shareholder's optimal ownership α^* satisfies

$$W_\alpha(K_0^*(\alpha^*);\alpha^*) \leq 0. \tag{56}$$

Additionally, if $\alpha^* > \underline{\alpha}$, (56) holds with equality.

The FOC (55) states that the insider's marginal benefit of capital, $W_K(K_0^*;\alpha)$, equals the marginal setup cost, $1+\Lambda'(K_0^*)$. Inequality (56) states that $W_\alpha(K;\alpha)$ cannot be positive at optimally chosen K_0^*. Otherwise, increasing ownership α raises $W(K;\alpha)$, contradicting the insider's optimality. Moreover, if the optimal α is interior, $\alpha^* > \underline{\alpha}$, (56) holds with equality. Using the optimal ownership α^* and firm size K_0^*, we then obtain the amount of external capital K_m^* via the outsiders' break-even condition (51).

Parameter choice and calibration. We set the minimal level of ownership for the entrepreneur to retain full control of the firm at $\underline{\alpha} = 20\%$. Various mechanisms such as dual class shares, cross holdings, and pyramidal structure allow the insider with cash flow rights substantially less than a majority position to achieve full control.[26] For the setup cost, we choose $\lambda_1 = 8\%$ and $\lambda_2 = 6\%$. To choose the coefficient of absolute risk aversion γ, we use the invariance result in Proposition 5 in the Appendix.

We choose the CARA coefficient $\gamma = 2$ based on the following argument. For a given value of γ, say $\gamma = 2$, we use two empirically motivated identifying assumptions, (1) the

[26] La Porta et al. (1999) use ownership data on large corporations in 27 wealthy countries to show that the controlling shareholders often have power that significantly exceeds their cash flow rights, primarily through the use of pyramids and active managerial participation. Claessens et al. (2000) provide evidence for pyramidal class shares and cross holdings in nine Eastern Asian countries/regions: Hong Kong, Indonesia, Japan, South Korea, Malaysia, the Philippines, Singapore, Taiwan, and Thailand.

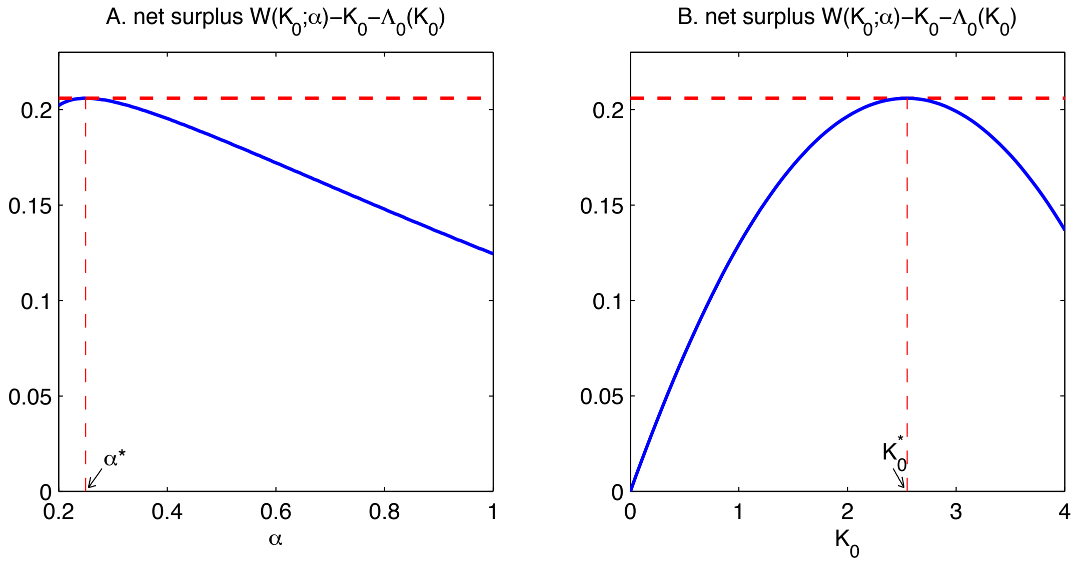

Figure 6: **The controlling shareholder's net surplus,** $W(K_0;\alpha)-K_0-\Lambda(K_0)$**, ownership** α**, and firm size** K_0**.** Panel A plots net surplus as a function of α by fixing $K_0 = K_0^* = 2.55$. Panel B plots net surplus as a function of K by fixing $\alpha_0 = \alpha_0^* = 25\%$.

entrepreneur's certainty equivalent wealth for the business is about 40% of total assets[27] and (2) the controlling shareholder's inside ownership is $\alpha = 25\%$.[28] The implied value for our proxy of relative risk aversion, $\overline{\gamma} = \gamma X_0 = 2 \times 1.8 = 3.6$, which is within the range of values for the coefficient of relative risk aversion commonly used in quantitative and calibration exercises.[29] All other parameter values are given in Table 3.

Ownership and firm size. The insider contributes $K_e^* = 0.20$, and raises a substantially larger amount, $K_m^* = 2.35$, from outside investors. The initial firm size is thus $K_0^* = K_e^* + K_m^* = 2.55$. The break-even condition for outsiders implies that 75% of firm equity belongs to outsiders, and the remaining 25% is inside equity. The public average q is $q_a(2.55) = 1.227$. The optimal exit boundary is $\overline{K} = 30.8$. The insider internalizes the setup cost, $\Lambda(2.55) = 0.40$.

[27] Gentry and Hubbard (2004) report that active businesses account for about 41.5% of entrepreneurs' total assets using the survey of consumer finance (SCF). We use 40% for our calibration.

[28] This estimate is within the range reported by La Porta et al. (1999), Claessens et al. (2000), and Dahlquist et al. (2003).

[29] The first condition gives $40\% = \alpha P(K_0^*)/(\alpha P(K_0^*) + X_0 - K_e^* - \Lambda(K_0^*))$. Later in this section, we show that the optimal firm size $K_0^* = 2.55$, inside equity contribution $K_e^* = 0.20$, and the controlling shareholder's private average q is $p(K_0^*) = 1.269$. Based on these numbers, the calibrated value for initial wealth is $X_0 = 1.8$.

The marginal setup cost is $\lambda_1 + \lambda_2 K_0^* = 0.233$. Evaluated at optimal ownership $\alpha^* = 25\%$ and firm size $K_0^* = 2.55$, the insider's *net surplus* $W(K_0^*; \alpha^*) - K_0^* - \Lambda(K_0^*)$ equals 0.21.

Figure 6 illustrates the insider's tradeoff when choosing ownership α and firm size K_0. Panel A plots the insider's net surplus as a function of α by holding firm size fixed at $K_0^* = 2.55$. The net surplus increases with α for $20\% < \alpha < 25\%$, and then decreases with α for $\alpha > 25\%$. Intuitively, for a given K_0, the higher the inside ownership, the more capital contribution by the insider, the better incentive alignment but the more costly idiosyncratic risk exposure. Panel B plots the insider's net surplus as a function of firm size K_0, by holding ownership fixed at $\alpha^* = 25\%$. The net surplus increases with K_0 for $K_0 < 2.55$ and then decreases for $K_0 > 2.55$. For a given α, the larger firm size K_0, the more capital that the insider contributes, the larger the firm and the better incentive alignment but also the more idiosyncratic risk exposure.

The effects of investor protection on ownership and firm size. La Porta et al. (1999) and Claessens et al. (2000) document that ownership is more concentrated under weaker investor protection. La Porta et al. (2000a) and Demirgüc-Kunt and Maksimovic (1998) find that financial markets are smaller and less developed in countries with weaker investor protection. Table 1 shows that the insider's ownership α^* decreases with investor protection θ_s. For example, $\alpha^* = 25\%$ when $\theta_s = 350$ and $\alpha^* = \underline{\alpha} = 20\%$ when $\theta_s = 700$, i.e. the ownership constraint $\alpha \geq \underline{\alpha}$ for full control binds with $\theta_s = 700$. Firm size K_0^* also increases with investor protection θ_s. For example, K_0^* increases by 6% from 2.55 to 2.70 as we increase θ_s from 350 to 700. While firm size increases with investor protection, the composition between inside and outside capital changes. Inside capital K_e^* falls by 90% from 0.20 to 0.02 and outside capital, K_m^*, increases from 2.35 to 2.68, as we increase θ_s from 350 to 700. As a result, the net surplus increases approximately by 5% from 0.21 to 0.22. We demonstrate that the value of improving investor protection can be quantitatively significant.

Table 1: **Investor protection, ownership, and firm size.** The parameter values are: $r = 5\%$, $\mu_R = 11\%$, $\sigma_R = 20\%$, $\mu_A = 22.8\%$, $\sigma_A = 25\%$, $\delta_K = 8\%$, $\theta_i = 3$, $\sigma_K = 20\%$, $\rho_A = \rho_K = 0.5$, $\lambda_1 = 8\%$, $\lambda_2 = 6\%$, $\gamma = 2$, and $l = 1.15$.

investor protection θ_s	300	350	400	500	700
ownership α^*	28%	25%	23%	21%	20%
inside capital K_e^*	0.28	0.20	0.14	0.05	0.02
external capital K_m^*	2.22	2.35	2.46	2.60	2.68
firm size K_0^*	2.50	2.55	2.60	2.65	2.70
net surplus	0.20	0.21	0.21	0.22	0.22

8 Ownership dynamics

In reality, an important channel through which the insider manages the idiosyncratic business risk exposure is ownership adjustments. The insider sometimes retains business control with fewer votes less than majority but induces agency costs. Adjusting ownership can be costly for the insider due to adverse selection, managerial agency, transaction costs, and other frictions. For simplicity, we follow the dynamic capital structure literature and assume a cost function for ownership adjustment, rather than explicitly modeling the underlying economic frictions. Let N denote the total share value that the insider trades. Let $D(N)$ denote the direct cost of trading the amount N in equity markets. We assume that $D(N)$ is given by

$$D(N) = d_0 + d_1 N + \frac{d_2}{2} N^2 \,, \tag{57}$$

where d_0, d_1, and d_2 are the fixed, linear, and quadratic cost parameters, respectively. The fixed cost component d_0 deters the insider from continuous trading. DeMarzo and Urosevic (2006) develop a dynamic model of ownership structure where a large shareholder trades off monitoring benefits against diversification costs. The time consistency issue for the large shareholder arises and acts as additional source of frictions.[30]

[30] See also Admati, Pfleiderer, and Zechner (1994) and Stoughton and Zechner (1998). Leland and Pyle (1977) is a classic (static) signaling model in Corporate Finance where the risk-averse entrepreneur with a higher quality of the project holds a more concentrated ownership. Gomes (2000) develop a dynamic model of ownership dynamics with asymmetric information and agency.

Table 2: **The effects of ownership adjustments.** The parameter values are: $r = 5\%$, $\mu_R = 11\%$, $\sigma_R = 20\%$, $\mu_A = 22.8\%$, $\sigma_A = 25\%$, $\delta_K = 8\%$, $\theta_i = 3$, $\theta_s = 350$, $\sigma_K = 20\%$, $\rho_A = \rho_K = 0.5$, $\alpha_1 = 0.4$, $d_0 = 1\%$, $d_1 = 4\%$, $d_2 = 2.5\%$ and $l = 1.15$.

	risk aversion	model	ownership	threshold	investment	private q	public q
	γ		α_2	$\widehat{K}$	$i(K_0)$	$p(K_0)$	$q_a(K_0)$
FB	-	both	0.4	∞	0.085	1.256	1.256
CM	-	both	0.4	∞	0.091	1.273	1.230
Incomplete Markets	0.5	static	-	-	0.083	1.233	1.261
	0.5	dynamic	0.35	20.8	0.083	1.233	1.261
	2	static	-	-	0.068	1.232	1.234
	2	dynamic	0.25	5.8	0.069	1.232	1.235
	4	static	-	-	0.054	1.226	1.208
	4	dynamic	0.2	3.3	0.060	1.228	1.212

Parameter choice and calibration. We set the initial ownership for the entrepreneur at $\alpha_1 = 0.4$. We choose $d_0 = 0.01$, $d_1 = 0.04$, and $d_2 = 0.025$. All other parameters remain the same as those in our baseline calibration. For simplicity, we assume that the insider adjusts ownership just once. The total trading amount $N = 1.07$, and the total cost is $D = 0.067$, which is about 6.3% of N. The fixed cost d_0 is about 16% of the total cost D. These results are close to the empirical estimation for equity issuance costs as reported in Altinkilic and Hansen (2000).

The effects of ownership adjustments. Table 2 analyzes the effects of ownership adjustments on investment, insider's private valuation and outsiders' public valuation. First, we review the results for two important special cases, the first-best (FB) benchmark and the complete-markets (CM) case. In the FB benchmark, the investment-capital ratio and Tobin's (marginal and average) q for both the insider and outsiders are all constant, $i^{FB} = 0.085$ and $q^{FB} = 1.256$.

For the CM case with agency costs, the investment-capital ratio $i(K)$, the insider's private q,

$p(K)$, and the outside investors' public q, $q_a(K)$ are also constant at all times. With $\alpha = 40\%$, $i^{CM} = 0.091$, which is higher than $i^{FB} = 0.085$ because the insider over-invests for private benefits and is able to fully diversify the business risk. Intuitively, the insider's private q is $p^{CM} = 1.273$, larger than $q^{FB} = 1.256$, which in turn is larger than outside investors' public q under CM, $q_a^{CM} = 1.23$, i.e. $p^{CM} > q^{FB} > q^{CM}$ due to the insider's (socially inefficient) expropriation of outside investors.

Importantly, the insider has no incentives to change ownership even with no cost, $D(N) = 0$. With CM, the insider has no diversification benefits by selling equity. Additionally, as outsiders price equity at new "anticipated" ownership level and hence capture the surplus generated via change of ownership, the insider thus cannot gain by changing ownership. This argument is analogous to the free-rider's argument in Grossman and Hart (1980), but adapted to our dynamic context.

Importantly, with incomplete markets, there are potential gains from adjusting ownership. With $\gamma = 2$, the insider optimally trades from the initial 40% equity to 25% by selling the 15% equity to diversified investors when firm size K reaches $\widehat{K} = 5.8$. Intuitively, the insider's diversification benefit becomes sufficiently large. More risk averse insiders sell equity sooner (i.e. a lower value of $\widehat{K}$) and sell a larger equity stake to outsiders, because diversification benefits are higher. The insider reduces under-investment due to the anticipation of future equity sale to outside investors.

The insider's private valuation at inception, $p(K_0)$, decreases in risk aversion γ. The more risk averse the insider is, the lower the insider's certainty equivalent valuation, *ceteris paribus*. This can be viewed as the direct effect of risk aversion γ on private q, $p(K)$. Additionally, risk aversion also influences the insider's decisions including ownership adjustments, investment, and exit. This effect of risk aversion on decisions further influences the insider's private valuation as an indirect effect.

Interestingly, outside investors' public average q, $q_a(K_0)$, is non-monotonic in risk aversion γ. For low γ, as γ increases, over-investment incentives decrease, and hence $q_a(K_0)$

increases. This is the region where the two frictions, imperfect investor protection and under-diversification, offset each other and yield a higher valuation for investors as γ increases. However, for high γ, as γ increases, the dominant under-diversification costs continue to increase and hence public average $q_a(K_0)$ decreases as γ increases. Therefore, it is possible that outside investors may prefer having an insider with moderate level of risk aversion γ.

We also note that outside investors may value the firm more than the insider does. For example, with $\gamma = 0.5$, $q_a(K_0) = 1.261$, which is larger than $p(K_0) = 1.233$. Outsiders' public q is high because investment is close to being socially optimal and the insider's private benefits are moderate. The insider's private q, $p(K_0)$, is lower than public q because the former needs to bear non-diversifiable idiosyncratic business risk which is not sufficient to offset the private benefits of control.

9 Conclusion

Many firms including large publicly traded ones around the world are run by entrenched controlling shareholders, who extract private benefits and choose non-value maximizing investment decisions. Concentrated business ownership mitigates agency conflicts between insiders and outside investors, but exposes insiders to substantial idiosyncratic illiquid business risks. We incorporate imperfect investor protection and under-diversification, two key frictions, into a tractable dynamic q-theory of investment framework where the insider makes *interdependent* consumption-saving, portfolio choice between a risky asset and a risk-free asset, private benefits, corporate investment, ownership, and flexible business cash-out/exit timing decisions. Our model extends the modern q-theory of investment, e.g. Hayashi (1982), along two important dimensions, incomplete markets and imperfect investor protection.

Two opposing forces work in our model. On the one hand, the weaker investor protection, the more private benefits to collect and the stronger the incentives to over-invest as private benefits increase with firm size. On the other hand, incomplete markets discourage insiders from investing in their firms as under-diversification costs increase with firm size. The insider optimally over-invests when the firm is small and under-invests when the firm becomes sufficiently large. Investor protection, flexible exit option and incomplete markets jointly generate a non-monotonic investment-capital ratio even with a constant-returns-to-scale production technology as in Hayashi (1982).

Our model allows us to (1) solve for the insider's private valuation and diversified outsiders' public valuation, both marginal and average; (2) calculate the cost of capital, i.e. risk/return implications for both the insider and outsiders; (3) compute the idiosyncratic risk premium for the under-diversified insider; and (4) characterize corporate investment decisions for a firm run by an under-diversified entrenched insider and link to the insider's private marginal valuation; (5) quantify the value of exit options and ownership adjustment for the insider.

We further derive valuation and asset pricing implications for outside equity and inside equity. While clearly reducing firm value for outside investors, agency costs have ambiguous effects on the cost of capital. The cost of outside equity varies with firm size and can be either higher or lower than the first-best benchmark value. Our model thus provides empirically testable predictions on corporate governance and cross-sectional returns.[31] We also infer the additional idiosyncratic risk premium for the illiquid inside equity. Moreover, our model generates rich predictions on time-varying investment dynamics purely driven by imperfect investor protection and incomplete markets, rather than changing investment opportunities.

Our model generates predictions that are broadly consistent with existing empirical findings, which include higher private benefits, smaller dividend payout, lower firm value, more concentrated corporate ownership, smaller and less developed financial markets, and smaller firm size under weaker investor protection, *ceteris paribus.*

For simplicity, we have taken investor protection as exogenously given. However, insiders may choose governance and investor protection so as to maximize their values. A critical issue in firms run by controlling shareholders is the succession of power.[32] We plan to incorporate these important issues in our future work.

[31] See Gompers, Ishii, and Metrick (2003) for an influential empirical study using US data.

[32] Burkart, Panunzi, and Shleifer (2003) develop a model of family firms.

Table 3: Summary of Key Variables and Parameters

This table summarizes the symbols for the key variables used in the model and the parameter values.

Variable	Symbol	Parameter	Symbol	Value
Productivity shock	dA	Depreciation rate	δ_K	8%
Capital stock	K	Expected output rate	μ_A	22.76%
Business investment	I	Volatility of output	σ_A	25%
Capital adjustment cost	Φ	Volatility of capital stock	σ_K	20%
Diversion cash-capital ratio	s	Capital adjustment cost	θ_i	3
The controlling shareholder's diversion cost	Ψ	Investor protection parameter	θ_s	350
The controlling shareholder's private value	P	Risk-free rate	r	5%
The controlling shareholder's value function	F	Market portfolio expected return	μ_R	11%
Cash flow to controlling shareholder	dM	Market portfolio volatility	σ_R	20%
Incremental return of market portfolio	dR	Market portfolio Sharpe ratio	η	30%
The controlling shareholder's liquid wealth	X	Correlation(productivity/market)	ρ_A	50%
Firm value	V	Correlation(capital/market)	ρ_K	50%
Payout to outside shareholders	dY	Risk-adjusted capital depreciation rate	δ	11%
Utility function	U	Risk-adjusted expected output rate	ν_A	19%
Social welfare	W	Ownership of equity	α	25%
Consumption	C	Cash-out parameter	l	1.15
Portfolio allocation	Π	Risk aversion	γ	2
Public Tobin's average q	q_a	Minimal ownership requirement for full control	$\underline{\alpha}$	20%
Public Tobin's marginal q	q_m	Firm's setup cost parameter	λ_1	8%
Controlling shareholder's average q	p	Firm's setup cost parameter	λ_2	6%
Firm's setup cost	Λ	Ownership adjustment parameter	d_0	1%
Ownership adjustment cost	D	Ownership adjustment parameter	d_1	4%
Trading amount when adjusting ownership	N	Ownership adjustment parameter	d_2	2.5%

Appendices

A Technical details

Proof of Theorem 1. The standard dynamic programming argument implies that $F(X,K)$ satisfies the following HJB equation

$$\begin{aligned}\zeta F = \max_{C,\Pi,I,s}\ & U(C) + (I-\delta_K K)F_K + \frac{(\sigma_K K)^2}{2}F_{KK} + \rho_K\sigma_K\sigma_R K\Pi F_{KX} \\ & + [rX + \Pi(\mu_R - r) - C + \alpha(\mu_A K - I - \Phi(I,K)) + (1-\alpha)sK - \Psi(s,K)]F_X \\ & + \frac{(\alpha\sigma_A K)^2 + 2\rho_A\sigma_A\sigma_R\alpha K\Pi + \sigma_R^2\Pi^2}{2}F_{XX}\,. \end{aligned} \tag{A.1}$$

Using the FOC for diversion s, corporate investment I, the insider's consumption C, and market portfolio allocation Π respectively, we obtain

$$\Psi_s(s,K) = (1-\alpha)K\,, \tag{A.2}$$

$$1+\Phi_I(I,K) = \frac{F_K(X,K)}{\alpha F_X(X,K)}\,, \tag{A.3}$$

$$U'(C) = F_X(X,K)\,, \tag{A.4}$$

$$\Pi = -\frac{\eta}{\sigma_R}\frac{F_X(X,K)}{F_{XX}(X,K)} - \frac{\rho_A\sigma_A}{\sigma_R}\alpha K - \frac{\rho_K\sigma_K}{\sigma_R}\frac{KF_{KX}(X,K)}{F_{XX}(X,K)}\,. \tag{A.5}$$

For a quadratic diversion cost as given in (4), we have a simple diversion rule as given by (23). We then conjecture that the controlling shareholder's value function is given by (14). Substituting the value function (14) into the FOCs (A.3), (A.4), and (A.5) for investment, consumption, and portfolio, respectively, we obtain the firm's investment-capital ratio $i(K)$ given by (22), consumption rule $C(X,K)$ given by (24), and portfolio allocation rule $\Pi(K)$ given by (25). Substituting the consumption rule (24), portfolio allocation rule (25), and value function (14) in the HJB equation (A.1) and simplifying, we have ODE(15).

We now turn to analyzing the boundary conditions. At the instant of exit, the controlling shareholder's value function is continuous,

$$F(X,K) = F_0(X+\alpha lK)\,, \tag{A.6}$$

where $F_0(X)$ is given by (13). Because the exit boundary $\overline{K}(X)$ is optimally chosen by the insider, the following smooth-pasting conditions along both X and K margins hold:

$$F_X(X,\overline{K}(X)) = F_0'(X+\alpha l\overline{K}(X))\,, \tag{A.7}$$

$$F_K(X,\overline{K}(X)) = \alpha lF_0'(X+\alpha l\overline{K}(X))\,. \tag{A.8}$$

Substituting the pre-exit and the post-exit value functions (14) and (13) into the insider's value-matching condition (A.6) and the smooth-pasting conditions (A.7) and (A.8), we obtain (20) and (21) in terms of the certainty equivalent $P(K)$.

Proof of Proposition 1. Proposition 1 is a special case of Theorem 1 where the controlling shareholder's risk aversion approaches $\gamma = 0$.

Proof of Proposition 2. Using $i(K)$ given in (22), the exit boundary $\overline{K}$, and the diversion policy $s(\alpha)$ given in (23), we have

$$\begin{aligned} dY_t &= (\mu_A - s(\alpha) - i(K) - \phi(i(K)))\, K_t dt + \sigma_A K_t dZ_t^A \\ &= (\nu_A - s(\alpha) - i(K_t) - \phi(i(K_t)))\, K_t dt + \sigma_A K_t d\widetilde{Z}_t^A \,, \end{aligned} \tag{A.9}$$

where $\nu_A = \mu_A - \rho_A \eta \sigma_A$, and

$$d\widetilde{Z}_t^A = dZ_t^A + \rho_A \eta dt \,. \tag{A.10}$$

The firm's capital stock accumulates as follows,

$$dK_t = (i(K_t) - \delta)\, K_t dt + \sigma_K K_t d\widetilde{Z}_t^K \,, \tag{A.11}$$

where $\delta = \delta_K + \rho_K \eta \sigma_K$, and

$$d\widetilde{Z}_t^K = dZ_t^K + \rho_K \eta dt \,. \tag{A.12}$$

For outside investors, firm value $V(K)$ is then given by the present discounted value of all future cash flows under the risk neutral measure,

$$V(K) = \widetilde{\mathbb{E}} \left(\int_0^\infty e^{-rt}\, dY_t \right) \,. \tag{A.13}$$

Firm value then solves the ODE (38). At the boundary $\overline{K}$, firm value equals $l\overline{K}$, implying (40). Equation (39) states that firm is worthless at $K = 0$.

Proposition 5 *Define $\overline{\gamma} = \gamma X_0$ and $k = K/X_0$, where X_0 is the entrepreneur's initial wealth. We have $P(K) = g(k) X_0$, where $g(k)$ solves the following ODE,*

$$\begin{aligned} rg(k) &= (\nu_A + b(\alpha))\, k - \delta k g'(k) + \frac{(g'(k) - 1)^2}{2\theta_i} k + \frac{\sigma_K^2 k^2 g''(k)}{2} \\ &\quad - \frac{\alpha \overline{\gamma} r k^2}{2} \left[(1 - \rho_A^2) \sigma_A^2 - 2\rho_A \rho_K \sigma_A \sigma_K g'(k) + (1 - \rho_K^2) \sigma_K^2 g'(k)^2 \right] \,, \end{aligned} \tag{A.14}$$

subject to the following boundary conditions,

$$g(0) = 0\,, \tag{A.15}$$

$$g(\overline{k}) = l\overline{k}\,, \tag{A.16}$$

$$g'(\overline{k}) = l\,. \tag{A.17}$$

Here, $\overline{k}$ is the ratio between the optimal exit threshold $\overline{K}$ and initial wealth X_0.

Proof of Proposition 5. Let $k = K/X_0$ and $g(k) = P(K)/X_0$, where X_0 is the entrepreneur's initial wealth. Substituting $k = K/X_0$ and $g(k) = P(K)/X_0$ into (15), we obtain ODE (A.14) for $g(k)$. Substituting $g(k) = P(K)/X_0$ into the boundary conditions (19)-(21) gives (A.15)-(A.17).

The entrepreneur's certainty equivalent value of business, as a fraction of the initial wealth X_0, is $\alpha P(K_0)/X_0 = \alpha g(k_0)$, where $k_0 = K_0/X_0$. Note that $g(k)$ depends only on $\overline{\gamma} = \gamma X_0$, the product of CARA coefficient γ and the entrepreneur's initial wealth X_0. This property is useful for our calibration and quantitative analysis because researchers often have views on values of *relative* risk aversion rather than *absolute* risk aversion.

Proposition 5 also implies that the certainty equivalent valuation, $P(K) = g(k)X_0$, is proportional to initial wealth X_0, given $k = K/X_0$ and $\overline{\gamma} = \gamma X_0$. To illustrate, for a value of $\overline{\gamma}$, if an entrepreneur with $X_0 = \$200$M values a firm with capital $K_0 = \$100$M at $P(K_0) = g(0.5)X_0 = \$120$M, our model implies that this entrepreneur with $X_0 = \$100$M values the same firm with $K_0 = \$50$M at $P(K_0) = g(0.5)X_0 = \$60$M. In general, this invariance to the unit of account does not hold under incomplete markets but it is a desirable property of our model from a valuation perspective.

A valuation equation for the idiosyncratic risk premium. For a given constant value of ξ, we denote the following present value,

$$\widetilde{P}(K_t;\xi) = \frac{1}{\alpha}\mathbb{E}\left[\int_t^\tau e^{-\xi(v-t)}dM_v + \alpha e^{-\xi(\tau-t)}lK_\tau\right]. \tag{A.18}$$

Using the standard martingale representation, we have the following ODE for $\widetilde{P}(K;\xi)$,

$$\xi\widetilde{P}(K) = \left(\mu_A + b(\alpha) - i(K) - \phi(i(K))\right)K + \left(i(K) - \delta_K\right)K\widetilde{P}'(K) + \frac{\sigma_K^2 K^2 \widetilde{P}''(K)}{2}\,, \tag{A.19}$$

subject to the following boundary conditions,

$$\widetilde{P}(0) = 0\,, \tag{A.20}$$

$$\widetilde{P}(\overline{K}) = l\overline{K}\,. \tag{A.21}$$

Dynamics of ownership. Our framework allows us to analyze the dynamics of ownership adjustment. For expositional simplicity, we illustrate the dynamics for the first round of trading ownership from α_1 to α_2. At the moment of trading, the controlling shareholder's liquid wealth changes from X to $X+N-D(N)$, where N is the gross trading amount, $N=(\alpha_1-\alpha_2)V_2(K)$, and $D(N)$ is the trading cost incurred by the controlling shareholder. At the moment of trading, the controlling shareholder's value function satisfies

$$F_1(X,K) = F_2(X+N-D(N),K)\,. \tag{A.22}$$

Because both the trading boundary for capital $\widehat{K}$ and the trading share $\alpha_1-\alpha_2$ are optimally chosen by the controlling shareholder, we thus have the following smooth-pasting conditions along both X and K margins:

$$\left.\frac{\partial F_1(X,K)}{\partial K}\right|_{K=\widehat{K}(X)} = \left.\frac{\partial F_2(X+N-D(N),K)}{\partial K}\right|_{K=\widehat{K}(X)}, \tag{A.23}$$

$$0 = \left.\frac{\partial F_2(X+N-D(N)),K)}{\partial \alpha_2}\right|_{K=\widehat{K}(X)}. \tag{A.24}$$

We conjecture that

$$F_n(X,K) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(X+\alpha_n P_n(K)+\frac{\eta^2}{2\gamma r^2}+\frac{\zeta-r}{\gamma r^2}\right)\right],\ n=1,2. \tag{A.25}$$

where $P_n(K)$ is the controlling shareholder's certainty equivalent wealth per unit of ownership when inside ownership is α_n and capital stock is K. The value-matching and smooth-pasting conditions can be expressed in terms of the certainty equivalent as follows,

$$\alpha_1 P_1(\widehat{K}) = \alpha_2 P_2(\widehat{K}) + (\alpha_1-\alpha_2)V_2(\widehat{K}) - D((\alpha_1-\alpha_2)V_2(\widehat{K}))\,, \tag{A.26}$$

$$\alpha_1 P_1'(\widehat{K}) = \alpha_2 P_2'(\widehat{K}) + (\alpha_1-\alpha_2)V_2'(\widehat{K})(1-D'((\alpha_1-\alpha_2)V_2(\widehat{K})))\,, \tag{A.27}$$

$$\begin{aligned} 0 = {} & P_2(\widehat{K}) + \alpha_2\frac{\partial P_2(\widehat{K})}{\partial \alpha_2} \\ & + \left((\alpha_1-\alpha_2)\frac{\partial V_2(\widehat{K})}{\partial \alpha_2} - V_2(\widehat{K})\right)(1-D'((\alpha_1-\alpha_2)V_2(\widehat{K})))\,. \end{aligned} \tag{A.28}$$

Taking the controlling shareholder's decisions as given, we are now equipped to value the firm for outside investors. Outside investors rationally anticipate managerial agency and price the firm at its fair value. The controlling shareholder's private valuation of capital per unit of ownership, $P_n(K)$, and firm value for the outside investors, $V_n(K)$, jointly solve the following ordinary differential equations (ODEs):

$$\begin{aligned} rP_n(K) &= (\nu_A + b(\alpha_n))\,K - \delta K P_n'(K) + \frac{(P_n'(K)-1)^2}{2\theta_i}K + \frac{\sigma_K^2 K^2 P_n''(K)}{2} \\ &- \frac{\alpha_n \gamma r K^2}{2}\left[(1-\rho_A^2)\sigma_A^2 - 2\rho_A\rho_K\sigma_A\sigma_K P_n'(K) + (1-\rho_K^2)\sigma_K^2 P_n'(K)^2\right], \qquad \text{(A.29)} \\ rV_n(K) &= (\nu_A - s(\alpha_n) - i_n(K) - \phi(i_n(K)))\,K \\ &+ (i_n(K)-\delta)\,KV_n'(K) + \frac{\sigma_K^2 K^2 V_n''(K)}{2}, \qquad \text{(A.30)} \end{aligned}$$

where the net private benefit of control per unit of ownership, $b(\alpha_n)$, is given by

$$b(\alpha_n) = \frac{(1-\alpha_n)^2}{2\alpha_n\theta_s}. \qquad \text{(A.31)}$$

We solve the ODEs (A.29)-(A.30) subject to the following boundary conditions:

$$\begin{aligned} P_1(0) &= P_2(0) = V_1(0) = V_2(0) = 0, \qquad &\text{(A.32)} \\ V_1(\widehat{K}) &= V_2(\widehat{K}), \qquad &\text{(A.33)} \\ P_2(\overline{K}) &= V_2(\overline{K}) = l\overline{K}, \qquad &\text{(A.34)} \\ P_2'(\overline{K}) &= l. \qquad &\text{(A.35)} \end{aligned}$$

The optimal investment-capital ratio $i_n = I_n/K$ is given by

$$i_n(K) = \frac{P_n'(K)-1}{\theta_i}. \qquad \text{(A.36)}$$

If the conditions (A.26)-(A.28) do not admit an interior solution satisfying $\alpha_{n+1}^a > \underline{\alpha}$, the optimal ownership is given by the minimal stake for the entrepreneur, $\alpha_{n+1}^a = \underline{\alpha}$ and condition (A.28) is not necessary.

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