

Earnings Inequality and the Minimum Wage: Evidence from Brazil*

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Abstract

We show that a rise in the minimum wage accounts for a large decline in earnings inequality in Brazil since 1994. To this end, we combine rich administrative and survey data with an equilibrium model of the Brazilian labor market. Our results imply that the minimum wage has far-reaching spillover effects on wages higher up in the distribution, accounting for one-third of the 25.9 log point fall in the variance of log earnings in Brazil since 1994. At the same time, the minimum wage's effects on employment and output are muted by reallocation of workers toward more productive firms.

Keywords: Inequality, Wage Distribution, Minimum Wage, Worker and Firm Heterogeneity, Equilibrium Search Model, Monopsony, Spillover Effects, Reallocation, Employment, Brazil, Linked Employer-Employee Data

JEL classification: E24, E25, E61, E64, J31, J38

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1 Introduction

In light of historically high levels of income inequality in many places, understanding the effects of labor market policies on the distribution of income and employment is seen as increasingly important. Several countries have recently implemented higher minimum wages in an attempt to aid low-income workers. Yet the benefits and costs of minimum wage policies remain controversial. In the U.S., for example, there is an active debate over the connection between the decline in the real minimum wage and the rise in income inequality over the last decades. A lesser known case is that of Brazil, which among other Latin American countries—has seen a remarkable decline in income inequality since the 1990s. Over the same period, Brazil's real minimum wage more than doubled. This raises a question: Is the minimum wage an effective tool to reduce income inequality?

The main contribution of our paper is to quantify the effects on inequality and unemployment of a large increase in the minimum wage in Brazil from 1996 to 2012, using a combination of theory and measurement. By exploiting variation in the effective bindingness of the federal minimum wage across states (Lee, 1999; Autor et al., 2016), we show that a higher minimum wage is associated with lower inequality up to at least the 70th percentile of the wage distribution. At the same time, we find little evidence of negative effects on employment. To understand the impact of the minimum wage, we develop an equilibrium model of a frictional labor market subject to a minimum wage, with a particular focus on the role played by heterogeneous firms in mediating such a policy. The minimum wage compresses firm pay differences and affects wages higher up in the distribution. At the same time, it leads to worker reallocation from less to more productive employers, countering the effect of a (modest) employment decline on aggregate output. We conclude, based on our reduced-form and structural analysis, that the minimum wage was a key factor behind Brazil's remarkable decline in wage inequality over this period.

Our analysis proceeds in three steps. In the first step, we empirically dissect Brazil's inequality decline and link it to the minimum wage. To this end, we decompose the variance of log wages in separate time windows, using a variant of the two-way fixed effects model created by Abowd, Kramarz and Margolis (1999, henceforth AKM). While firms account for 26 percent of the variance of log wages around 1996, 43 percent of the reduction in the variance over time was due to declining firm pay differences for identical workers. To assess what fraction of the aggregate decline in inequality is accounted for by the minimum wage, we exploit cross-sectional variation in the effective bindingness of the federal minimum wage across states. Motivated by the fact that especially lower-tail inequality declined by more in initially lower-income regions, we estimate the effects of the minimum wage throughout the wage distribution, building on Lee's (1999) seminal econometric framework and the recent contribution by

Autor et al. (2016). We find robust evidence of spillover effects of the minimum wage up to at least the 70th wage percentile and a large negative effect on the variance of wages. At the same time, we find no significant effects of the minimum wage on employment, formality, and other labor market outcomes.

To understand these patterns, in the second step, we develop and estimate an equilibrium model of Brazil's labor market subject to a minimum wage. Our model extends the popular Burdett and Mortensen (1998) framework to include, in a tractable manner, unobserved worker heterogeneity, firm productivity dispersion, minimum wage jobs, and endogenous job creation. We show that a relatively simple extension of this framework can be operationalized to speak to worker and firm pay differences in the data and to quantify the equilibrium effects of the minimum wage. In our model, workers permanently differ in their ability and value of leisure, as well as their time-varying on-the-job search efficiency and separation rate. They engage in random search in frictional labor markets segmented by worker type. Differentially productive firms operate a linear technology in labor and choose recruiting intensity and what wage to offer in each market. The model allows for a flexible account of worker and firm pay differences, including a mass point in the wage distribution at the minimum wage. We estimate the model via the simulated method of moments (SMM) based on our linked employer-employee data and find that, despite its simplicity, it provides a parsimonious account of salient empirical patterns in Brazil.

In the third and final step, we use the model to quantify the effects of the observed increase in the minimum wage on the distribution of wages, employment, and aggregate output. To this end, we feed the empirical increase in Brazil's minimum wage between 1996 and 2012 into the estimated model. We find that the increased minimum wage reduces the variance of wages by 8.6 log points, or one-third of the empirical decline over this period. A critical factor behind these large effects on inequality is that the rise in the minimum wage induces firms above the new minimum wage to raise pay to maintain their rank in the wage distribution. Indeed, such spillover effects reach up to the 90th wage percentile, though wage gains decline monotonically and fall below 2.5 percent above the median. We demonstrate that the magnitudes of our estimated effects of the minimum wage on inequality are driven by how binding the minimum wage is, together with the extent of firm productivity dispersion in Brazil. At the same time, we find muted negative effects of the minimum wage on employment and aggregate output, owing to the heterogeneous effects of the minimum wage across the firm productivity distribution. Lower-productivity firms cut vacancy creation as the minimum wage squeezes their profit margins. The easier recruiting environment in turn induces higher-productivity firms to increase hiring. As a result, the minimum wage reallocates employment primarily from lower- to higher-productivity firms rather than to unemployment. Finally, we confront the model-implied effects of the minimum wage with cross-

sectional evidence that lends additional support to our model predictions.

Related literature. This paper contributes to three strands of the literature. First, much research has been devoted to the reduced-form measurement of minimum wage effects on labor market outcomes.¹ A large number of these studies are concerned with the employment effects of the minimum wage (e.g., [Card and Krueger, 1994](#)). A complementary set of papers assesses the distributional consequences of the minimum wage in the U.S. and other high-income countries ([Grossman, 1983](#); [DiNardo et al., 1996](#); [Machin et al., 2003](#); [Teulings, 2003](#); [Butcher et al., 2012](#); [Fortin and Lemieux, 2015](#); [Brochu et al., 2018](#); [Firpo et al., 2018](#); [Rinz and Voorheis, 2018](#); [Cengiz et al., 2019](#); [Fortin et al., 2021](#)). In a seminal contribution to this literature, [Lee \(1999\)](#) finds significant effects of the minimum wage in the lower half of the U.S. wage distribution. By extending this methodology and data series, [Autor et al. \(2016\)](#) argue that spillover effects of the minimum wage are indistinguishable from measurement error using household survey data from the U.S. Current Population Survey (CPS). Relative to these papers, ours exploits administrative data to quantify the effects of a large increase in the minimum wage in a developing country, Brazil. We find robust evidence of spillovers throughout large parts of the wage distribution, which we link to the relatively greater bindingness of the minimum wage and dispersion in firm pay policies in Brazil.

Second, a separate literature has developed and estimated structural models to assess the impacts of a minimum wage. [Van den Berg and Ridder \(1998\)](#), [Bontemps et al. \(1999, 2000\)](#), and [Manning \(2003\)](#) highlight the contribution of firms in imperfectly competitive labor markets toward wage dispersion for identical workers, based on the seminal framework by [Burdett and Mortensen \(1998\)](#). A theoretical prediction of this framework is that the minimum wage has spillover effects on higher wages through the equilibrium response of firm pay policies. Perhaps surprisingly, the magnitude of these spillover effects has, before our work, not been quantified using linked worker-firm data. Related research abstracts from firms and instead models match-level heterogeneity in the context of minimum wage policies to study endogenous contact rates ([Flinn, 2006](#)) and the nature of wage setting ([Flinn and Mullins, 2018](#)). Relative to these works, ours shows that a model of multi-worker firms has distinct predictions for the reallocation of workers across heterogeneous employers and for changes in firm pay policies in response to a minimum wage. In this sense, our findings connect to recent work on the reallocative effects of minimum wages ([Aaronson et al., 2018](#); [Berger et al., 2019](#); [Harasztosi and Lindner, 2019](#); [Dustmann et al., 2020](#); [Clemens et al., 2021](#)).² Our contribution is to show that a relatively simple extension of

¹See [Card and Krueger \(1995\)](#) and [Neumark and Wascher \(2008\)](#) for comprehensive overviews of this literature.

²Other mechanisms that could give rise to spillover effects include skill assignments with comparative advantage ([Teulings, 1995](#)), hierarchical matching ([Lopes de Melo, 2012](#)), fairness considerations ([Card et al., 2012](#)), educational investment ([Bárány,](#)

the seminal framework by [Burdett and Mortensen \(1998\)](#) provides a strikingly good description of the Brazilian labor market and is well suited to incorporating minimum wages into the recent literature exploring firms' role in labor market outcomes ([Clemens, 2021](#)).³ The model also helps reconcile our finding of large distributional consequences of the minimum wage with its small disemployment effects ([Teulings, 2000](#)) and sheds light on the determinants of the magnitude of these effects ([Neumark, 2017](#)).

Third, our paper relates to a literature that aims to understand the evolution of wage inequality in Brazil over the past decades, as summarized by [Firpo and Portella \(2019\)](#). [Alvarez et al. \(2018\)](#) document the role of falling firm pay differences in a large inequality decline in Brazil between 1996 and 2012, for which our current paper provides a structural explanation: the rise of the minimum wage. Previous reduced-form work by [Fajnzylber \(2001\)](#), [Neumark et al. \(2006\)](#), and [Lemos \(2009\)](#) studies the distributional effects of Brazil's minimum wage over an earlier period before the minimum wage rapidly increased. Subsequent work by [Haanwinckel \(2020\)](#) also quantifies the contribution of the minimum wage toward the decline in wage inequality in Brazil. Although his task-based model differs from ours in several dimensions, his main conclusion is consistent with our results regarding the inequality-reducing effect of the minimum wage through spillovers higher up in the wage distribution. As is the case in other developing countries, the informal sector plays an important role in the Brazilian labor market ([Ulyssea, 2018, 2020](#); [Dix-Carneiro et al., 2021](#)). While our estimated model accounts for informality in a simple manner, the richer model by [Meghir et al. \(2015\)](#) allows for interactions between formal and informal firms, suggesting that policies like the minimum wage may affect pay and employment in both sectors ([Jales, 2018](#)).

Outline. The paper proceeds as follows. Section 2 introduces the data and dissects Brazil's inequality decline. Section 3 presents reduced-form evidence for the effects of the minimum wage on wages and employment. Section 4 develops a structural equilibrium model of Brazil's labor market subject to a minimum wage. Section 5 estimates the model. Section 6 uses the estimated model to quantify the effects of the minimum wage on the distribution of wages and employment. Section 7 validates our model predictions using cross-sectional evidence. Finally, Section 8 concludes.

2016), hedonic compensation ([Phelan, 2018](#)), and the union threat ([Taschereau-Dumouchel, 2020](#)).

³See also [Davis and Haltiwanger \(1991\)](#), [Abowd et al. \(1999\)](#), [Card et al. \(2013\)](#), [Barth et al. \(2016\)](#), and [Song et al. \(2019\)](#) for empirical studies of firms in the labor market, and see [Postel-Vinay and Robin \(2002\)](#), [Dey and Flinn \(2005\)](#), [Cahuc et al. \(2006\)](#), [Lise and Robin \(2017\)](#), [Bilal et al. \(2019\)](#), [Elsby and Gottfries \(2019\)](#), [Gouin-Bonenfant \(2020\)](#), [Bilal and Lhuillier \(2021\)](#), and [Jarosch \(2021\)](#) for structural advances in this area.

2 The decline in wage inequality in Brazil

2.1 Data

Our main data source is an administrative linked employer-employee data set that covers nearly the universe of formal sector workers between 1985 and 2014, called *Relação Anual de Informações Sociais* (RAIS). It consists of annual employment records that employers are required to report to the Ministry of the Economy (formerly the Ministry of Labor), and it allows tracking workers across employers over time.⁴ Our analysis also exploits two household surveys: the *Pesquisa Nacional por Amostra de Domicílios* (PNAD) and the *Pesquisa Mensal de Emprego* (PME). PNAD is a nationally representative household survey that covers all individuals, regardless of labor market status, in repeated cross sections between 1996 and 2012. PME is a longitudinal household survey that tracks individuals in a rotating monthly panel structure similar to the CPS in the U.S. It covers Brazil's six largest metropolitan regions between 2002 and 2012. Appendix A.1 discusses the three data sets in more detail.

Variables and sample selection. RAIS contains the start and end dates of all formal job spells during a given calendar year. We use as our income concept in RAIS the mean monthly earnings in multiples of the current minimum wage—henceforth referred to as "wages". These are consistently reported over the period from 1985 to 2014. RAIS also contains unique individual and employer identifiers, gender, age, educational attainment, contractual weekly work hours, and six-digit occupation codes.

One feature that distinguishes the two household surveys, PNAD and PME, from RAIS is that they do not contain employer identifiers. Instead, they ask respondents questions about the job they held during a reference week preceding the interview, including their work status. Following Meghir et al. (2015), we classify as informal all self-employed individuals and those in remunerated employment without an official work permit.

For our empirical analysis, we restrict attention to male workers between the ages of 18 and 54. We exclude women and individuals outside of this age range to focus on a subpopulation that is relatively attached to the (formal) labor market.⁵ Among this subpopulation in RAIS, we restrict attention to the largest leave-one-out connected set of workers and firms, as in Kline, Saggio and Sølvsten (2020, henceforth KSS). A connected set is defined as a set of all workers and firms that are linked through worker mobility across firms during a given time period. A leave-one-out connected set is a connected set that remains connected when eliminating worker-firm matches one at a time. No such restriction is

⁴All of our analysis is at the level of the establishment, which we interchangeably refer to as the firm or the employer.

⁵For a separate study of men and women in Brazil's labor market, see Morchio and Moser (2020).

necessary or possible in PNAD or PME.⁶

Summary statistics. Table 1 summarizes our sample from the three data sets. The RAIS data show that between 1996 and 2012, Brazil experienced an 18 log points increase in mean formal sector wages at the same time that there was a striking fall in inequality, with the standard deviation declining by 19 log points. While the age distribution remained somewhat stable, there was a significant increase in educational attainment over this period. Using the PNAD survey data, we find congruent trends in the formal sector wage distribution. Relative to those in the formal sector, informal wages are initially characterized by lower levels but similar relative dispersion. Throughout 2012, the informal sector wage distribution saw an increase in its mean, accompanied by mild compression. At the same time, the employment rate remained stable, while the formal employment share rose by 8 percentage points. Consistent with the increase in formality, the longitudinal PME data show a rise in the flow rate from formal into formal employment and a decline in the flow rate from formal into informal jobs.⁷

Panels A and B of Figure 1 show histograms of log wages in 1996 and 2012, respectively.⁸ Evidently, Brazil's inequality decline was associated with relatively greater compression in the left tail of the wage distribution over this period. Indeed, panel C shows that when measured by the P90/P50 log wage percentile ratio, lower-tail wage inequality fell by significantly more compared with upper-tail inequality. While both tails of the wage distribution experienced some compression, lower-tail inequality fell by almost 40 percent between 1996 and 2012, whereas upper-tail inequality fell by around 20 percent over the same period. The main conclusion we draw from examining the raw data is that there was a significant decrease in wage dispersion, with little sign of displacement among formal sector workers in Brazil between 1996 and 2012. Additional summary statistics are presented in Appendix A.2.

2.2 Dissecting Brazil's decline in earnings inequality: The role of firms

To understand the decline in wage inequality in Brazil, we follow Alvarez et al. (2018) in implementing a statistical decomposition of wages among formal sector workers in Brazil. Motivated by the fact that a large share of empirical wage dispersion is within detailed worker groups based on observable characteristics—as demonstrated in Appendix A.4—we estimate two-way fixed effect specifications

⁶The PME is the same data source as used in Meghir et al. (2015), though we apply slightly different selection criteria (e.g., ages 18 to 54 instead of ages 23 to 65), use a longer period (from 2002 to 2012 instead of from 2002 to 2007), and measure employment transitions slightly differently (counting any month-to-month transition over the 16-months rotating panel instead of counting months until the first transition or until four months without transition have passed).

⁷In Appendix A.3, we show that official labor force statistics are compatible with the sample sizes in the RAIS.

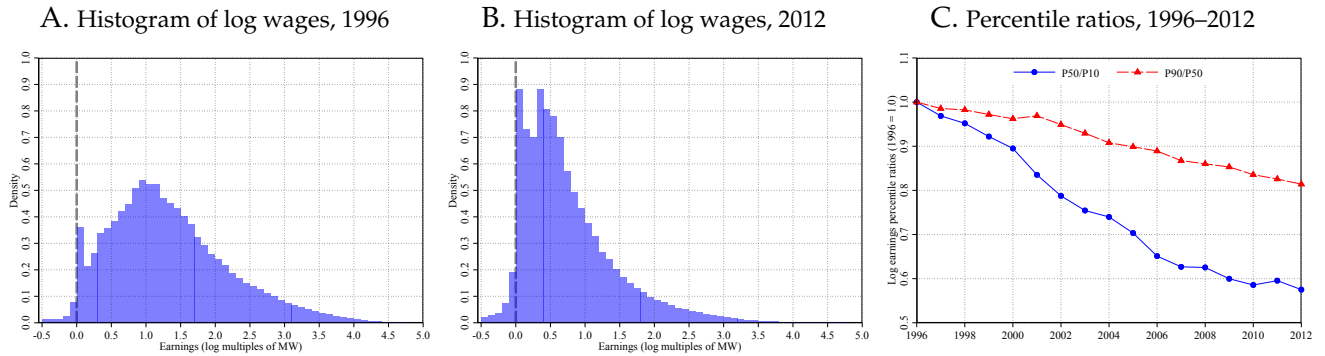
⁸Appendix B.1 shows histograms of wages in multiples of the current minimum wage (Figure B.1) and of log wages (Figure B.2) for each year between 1996 and 2012.

Table 1. Summary statistics for three data sets, 1996 and 2012

	1996		2012	
	Mean	Std. dev.	Mean	Std. dev.
<i>Panel A. Administrative linked employer-employee data (RAIS)</i>				
Age	32.74	9.30	33.48	9.68
Years of education	8.87	3.91	10.53	3.23
Real wage (log BRL 2012, formal sector)	5.96	0.86	7.15	0.69
Observations (millions)	17.2		30.8	
<i>Panel B. Cross-sectional household survey data (PNAD)</i>				
Real wage (log BRL 2012, formal sector)	7.01	0.81	7.13	0.62
Real wage (log BRL 2012, informal sector)	6.26	0.81	6.56	0.78
Employment rate	0.95		0.95	
Formal employment share	0.68		0.76	
Observations (thousands)	74.5		86.0	
<i>Panel C. Longitudinal household survey data (PME)</i>				
Transition rate nonemployed-employed	0.08		0.10	
Transition rate employed-nonemployed	0.05		0.04	
Observations (thousands)	94.3		121.2	

Notes: "Real wage" refers to mean (in RAIS) or usual (in PNAD) monthly earnings. "Employment" comprises domestic workers, employees, and the self-employed. Formal employment is employment with a legal work permit. Monthly transition rates are between employment (i.e., formal employment) and nonemployment (i.e., informal employment + unemployment). Source: RAIS, PNAD, PME, 1996 and 2012.

Figure 1. Lower- and upper-tail inequality, 1996–2012



Notes: Panels A and B show histograms of log wages in multiples of the current minimum wage based on 60 equi-spaced bins for the population of male workers aged 18–54 for 1996 and 2012, respectively. Panel C plots lower- and upper-tail wage inequality, as measured by the P50/P10 and the P90/P50 log wage percentile ratios between 1996 and 2012, normalized to 1.0 in 1996. Source: RAIS, 1996–2012.

based on the econometric framework by AKM. Specifically, we decompose log wages w_{ijt} of individual i working at firm j in year t within five-year periods as

$$w_{ijt} = \alpha_i + \psi_j + X_{it}\beta + \varepsilon_{ijt}, \quad (1)$$

where α_i denotes a worker fixed effect, ψ_j denotes a firm fixed effect, X_{it} is a vector of time-varying worker characteristics—including education-specific age dummies restricted to be flat between ages 45 and 49, education-specific year dummies, contractual work hours dummies, and six-digit occupation dummies—and ε_{ijt} is a residual satisfying a strict exogeneity condition. Equation (1) is identified off workers switching employers within the largest set of firms connected through worker mobility. While ordinary least squares (OLS) estimates of individual coefficients in equation (1) are unbiased, the variance and covariance terms based on these coefficients generally are biased in finite samples. To correct for this bias, we adopt the leave-one-out estimator developed by KSS, which yields unbiased estimates of the variance components of log wages based on equation (1).⁹

Table 2 presents a decomposition of the variance of log wages based on the AKM wage equation (1). It does so separately for a five-year period centered on 1996 (i.e., from 1994 to 1998) and a five-year period centered on 2012 (i.e., from 2010 to 2014). For each period, we report results from four estimations: one without KSS correction and without controls in columns (1) and (5), one with KSS correction and without controls in columns (2) and (6), one without KSS correction and with controls in columns (3) and (7), and one with KSS correction and with controls in columns (4) and (8). The last four columns report the change between periods for each of the four sets of estimates.

During 1994–1998 (columns 1–4), out of the total variance of wages of 70.9 log points, between 46 and 25 percent are attributable to the variance of person fixed effects. The inclusion of worker controls reduces this share by around one-third, and the KSS correction further reduces it. Between 30 percent (column 1) and 26 percent (column 4) of the total variance of log wages is attributed to the firm pay component, with little variation in this share with and without KSS correction or controls. There is significant positive worker-firm sorting, as measured by the correlation between worker and firm fixed effects of 0.330, including the KSS correction and controls. The associated value of two times their covariance term equals 0.120, which accounts for an additional 17 percent of the total variance (column 4).

During 2010–2014 (columns 5–8), the total variance of wages is 45.3 log points, which is 25.6 log points lower than 1994–1998. While worker heterogeneity is the most important factor behind the cross-sectional wage variance, a drop in the variance of firm fixed effects constitutes between 48 percent (comparing columns 5 and 1) and 43 percent (comparing columns 8 and 4) of the total decline. A lower variance of person fixed effects accounts for between 27 percent (comparing columns 5 and 1) and 17

⁹There has been a fruitful debate around the benefits and drawbacks of estimating AKM wage equations, including the contributions of Andrews et al. (2008), Eeckhout and Kircher (2011), Lopes de Melo (2018), Card et al. (2018), Bonhomme et al. (2019), Bonhomme et al. (2020), and Borovičková and Shimer (2020). In related work, Alvarez et al. (2018) and Gerard et al. (2020) present a battery of robustness checks, which suggest that the AKM equation is well suited for describing the Brazilian data during this period.

Table 2. Decomposition of the variance of log wages over time

	Variance (%), 1994–1998				Variance (%), 2010–2014				Change (%), 1994–1998 to 2010–2014			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(5)–(1)	(6)–(2)	(7)–(3)	(8)–(4)
$Var(w_{ijt})$	0.709 (100%)	0.709 (100%)	0.709 (100%)	0.709 (100%)	0.453 (100%)	0.453 (100%)	0.453 (100%)	0.453 (100%)	–0.256 (100%)	–0.256 (100%)	–0.256 (100%)	–0.256 (100%)
$Var(\hat{\alpha}_i)$	0.323 (46%)	0.279 (39%)	0.217 (31%)	0.176 (25%)	0.253 (56%)	0.228 (50%)	0.154 (34%)	0.133 (29%)	–0.070 (27%)	–0.051 (20%)	–0.063 (25%)	–0.043 (17%)
$Var(\hat{\psi}_j)$	0.212 (30%)	0.198 (28%)	0.201 (28%)	0.187 (26%)	0.088 (19%)	0.082 (18%)	0.082 (18%)	0.077 (17%)	–0.124 (48%)	–0.116 (45%)	–0.119 (46%)	–0.110 (43%)
$2 \times Cov(\hat{\alpha}_i, \hat{\psi}_j)$	0.140 (20%)	0.163 (23%)	0.098 (14%)	0.120 (17%)	0.090 (20%)	0.100 (22%)	0.061 (13%)	0.069 (15%)	–0.050 (20%)	–0.063 (25%)	–0.037 (14%)	–0.051 (20%)
$Var(\hat{\varepsilon}_{ijt})$	0.034 (5%)	0.070 (10%)	0.033 (5%)		0.021 (5%)	0.043 (9%)	0.020 (4%)		–0.013 (5%)	–0.027 (11%)	–0.013 (5%)	
$Corr(\hat{\alpha}_i, \hat{\psi}_j)$	0.267	0.347	0.234	0.330	0.302	0.364	0.272	0.339				
R^2	0.951	0.902	0.953		0.953	0.905	0.957					
Obs. (mm)	67.8	67.8	67.8	67.8	125.7	125.7	125.7	125.7				
KSS correction	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓
Controls	✗	✗	✓	✓	✗	✗	✓	✓	✗	✗	✓	✓

Notes: This table shows plug-in and bias-corrected variance components of log wages based on estimating AKM equation (1) for the population of male workers ages 18–54 in 1994–1998 and 2010–2014. $Var(w_{ijt})$ denotes the variance of log wages, $Var(\hat{\alpha}_i)$ denotes the variance of estimated person fixed effects, $Var(\hat{\psi}_j)$ denotes the variance of estimated firm fixed effects, $2 \times Cov(\hat{\alpha}_i, \hat{\psi}_j)$ denotes two times the sum of the covariance between estimated person fixed effects $\hat{\alpha}_i$ and estimated firm fixed effects $\hat{\psi}_j$, and $Var(\hat{\varepsilon}_{ijt})$ denotes the variance of estimated residuals. For columns (3)–(4) and (7)–(8), omitted terms include the variance of the estimated component of log wages due to observable worker characteristics, $Var(X_{it}\hat{\beta})$, and two times the sum of covariance terms involving observable worker characteristics. $Corr(\hat{\alpha}_i, \hat{\psi}_j)$ denotes the correlation between estimated person fixed effects $\hat{\alpha}_i$ and estimated firm fixed effects $\hat{\psi}_j$. Variance shares are in parentheses in columns (1)–(8). Share of total change in the variance of log wages is in parentheses in the last four columns. Observations are in millions of worker-years. "KSS correction" refers to leave-one-out estimators of variance components by Kline et al. (2020). The variance of estimated residuals, $Var(\hat{\varepsilon}_{ijt})$, and the coefficient of determination, R^2 , are not reported in columns (4) and (8) because of their omission in the KSS leave-one-out estimation with controls. Controls include education-specific age dummies restricted to be flat between ages 45 and 49, education-specific year dummies, contractual work hours dummies, and occupation dummies. Source: RAIS, 1994–1998 and 2010–2014.

percent (comparing columns 8 and 4) of the total decline. Lower covariance terms and residual variance account for the remaining decline. Between the two periods, the correlation between worker and firm fixed effects, as well as the coefficient of determination, slightly increased after accounting for the KSS correction and controls.

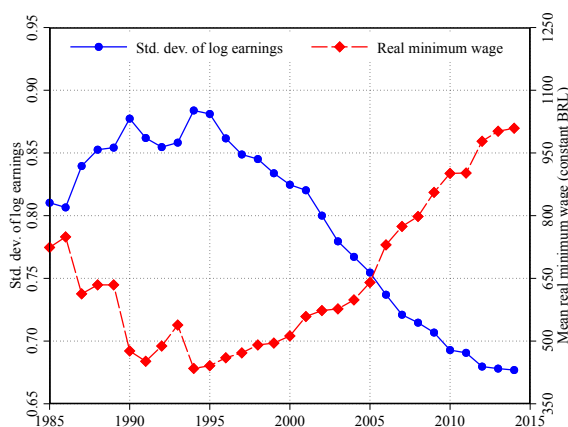
In summary, Brazil saw a remarkable decline in wage inequality between 1994 and 2014, which was driven partly by a reduction in pay differences for identical workers across firms. We interpret this as evidence for the hypothesis that the decline in inequality over this period was the result of changes in firms' pay policies and was not due solely to changes in worker composition.

3 Evidence on the effects of the minimum wage in Brazil

3.1 Evolution of Brazil's minimum wage over time

Motivated by the decline in wage inequality in Brazil, we now turn to a salient change in the labor market over this period: the rise in the minimum wage.¹⁰ Brazil's statutory minimum wage is set at the federal level and stated in terms of a floor on monthly nominal earnings. There are no provisions for legal subminimum wages or differentiated minimum wages across demographics or economic subdivisions (Lemos, 2004). The minimum wage applies to workers with full-time contracts of 44 hours per week and is proportionately adjusted for part-time workers.¹¹ Brazil's real minimum wage deteriorated under high inflation in the years leading up to 1996, when a switch in government ignited a gradual ascent of the wage floor by 119 percent in real terms. By 2012, it reached 622 BRL per month, or 410 purchasing power parity-adjusted USD. If we account for aggregate productivity growth, this corresponds to a 44 log points rise in the minimum wage. To put these numbers into context, the minimum wage as a fraction of the median wage increased from around 34 percent in 1996 to 60 percent in 2012. Figure 2 shows a strong negative comovement between the minimum wage and the variance of log wages between 1985 and 2014, with a correlation of -0.92 .

Figure 2. Evolution of wage inequality and the real minimum wage, 1985–2014



Notes: Statistics are for males ages 18–54. Real minimum wage is the annual mean of the monthly time series. The correlation between the two time series is -0.92 . Source: RAIS and IPEA, 1985–2014.

¹⁰While Brazil enacted other social policies during the mid-2000s, such as a transfer program for needy families (*Bolsa Família*) launched in 2003, the minimum wage predates many of these policies.

¹¹Using information on hours in the RAIS data, we find a small share of part-time workers. Special labor contracts allow for parts of the minimum wage to be paid in-kind in the form of accommodation and food, although in the PNAD data, only 0.8 percent of workers report that they received nonmonetary remuneration in 1996, and 0.3 percent of workers report having done so in 2012.

3.2 Cross-sectional heterogeneity and the minimum wage

While the correlation between the minimum wage and aggregate wage inequality is striking, we caution against interpreting this pattern as causal. For example, the changes in wage inequality over this period might have been driven by simultaneous changes in macroeconomic conditions or secular trends in the wage distribution that were unrelated to Brazil's federal minimum wage. We address this simultaneity problem by exploiting spatial variation in the bindingness of the federal minimum wage across states in Brazil, building on the seminal econometric framework by [Lee \(1999\)](#) and the recent contribution by [Autor et al. \(2016\)](#). This approach allows us to filter out changes in national macroeconomic conditions and secular trends. In this sense, the fact that inequality decreased in Brazil over this period is neither necessary nor sufficient for our conclusions regarding the effects of the minimum wage on wage inequality.

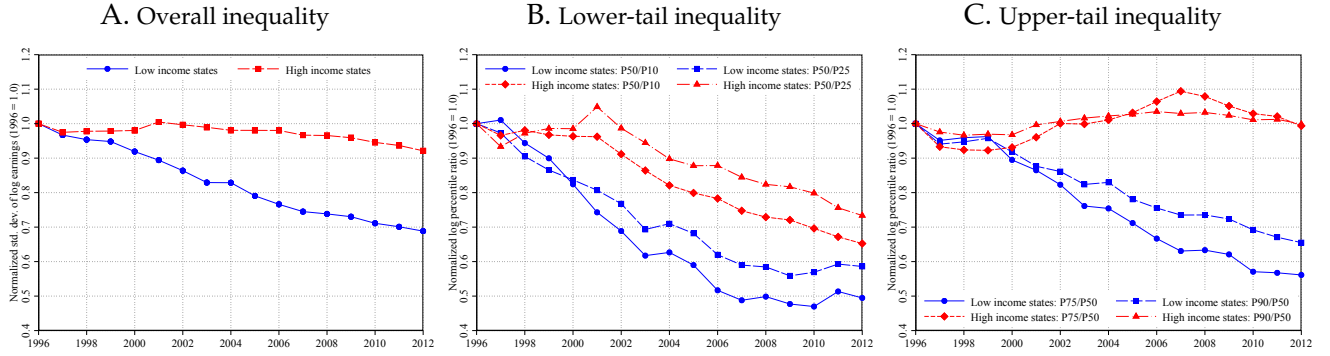
Motivating evidence on state-level heterogeneity. To motivate our econometric analysis, we start by noting that although wage inequality declined overall during this period, it fell disproportionately in initially low-income regions for which the federal minimum wage was relatively more binding. [Figure 3](#) plots normalized wage inequality measures between 1996 and 2012 for the three lowest ("low income") and three highest ("high income") among Brazil's 27 states ranked by average wages in 1996. [Panel A](#) shows that the variance of log wages drops by more than half in initially low-income states, but by less than one-fifth in initially high-income states. [Panel B](#) shows that lower-tail inequality drops especially in initially low-income states, with the P50–P10 and P50–P25 for this group declining by 50 and 40 percent, respectively, but it falls by markedly less for initially high-income states. In contrast, upper-tail inequality, measured by the P75–P50 or the P90–P50, falls only in initially low-income states, as shown in [panel C](#).¹²

These empirical patterns yield three take-aways. First, Brazil's inequality decline was due to factors that matter more at lower income levels. Second, the inequality decline was associated with compression, particularly in the bottom of the wage distribution. Third, the compression in the wage distribution reaches from the bottom to above the median of the wage distribution. This motivates our study of the minimum wage.

Econometric framework. To correlate the minimum wage with wage inequality, we follow [Lee \(1999\)](#) and [Autor et al. \(2016\)](#) in exploiting heterogeneous exposure across states that differ in their bindingness

¹²Appendix [B.5](#) shows that the inverse relationship between the effective bindingness of the minimum wage and wage inequality generalizes to the full set of states.

Figure 3. Evolution of wage inequality across rich and poor states, 1996–2012



Notes: For this figure, we assign the three lowest-income and three highest-income states, ranked by their mean log wage in 1996, into a “low income” group and a “high income” group, respectively. The three panels then plot various wage inequality measures by state group between 1996 and 2012, normalized to 1.0 in 1996. Panel A shows the variance of log wages, panel B shows lower-tail percentile ratios (P50/P10 and P50/P25) of log wages, and panel C shows upper-tail percentile ratios (P75/P50 and P90/P50) of log wages. Source: RAIS, 1996–2012.

with respect to Brazil’s federal minimum wage. To this end, we define the generalized p -Kaitz index as $gkatz_{st}(p) \equiv \log w_t^{min} - \log w_{st}(p)$ — that is, the log difference between the minimum wage prevailing in year t , w_t^{min} , and the p th percentile of the log wage distribution of state s in year t , $w_{st}(p)$.¹³ We are interested in how various inequality measures at the state-year level covary with the generalized p -Kaitz index for p high enough such that the p th percentile of the wage distribution is not (directly or indirectly) affected by the minimum wage. To assess this, we regress outcome variable $y_{st}(p'; p)$ specific to wage percentile p' with respect to some base percentile p in state s and year t on the generalized p -Kaitz index, using the same base percentile p and state-year controls:

$$y_{st}(p'; p) = \sum_{n=1}^N \beta_n(p') gkatz_{st}(p)^n + \gamma_{st}(p') + \varepsilon_{st}(p'), \quad (2)$$

where N is the polynomial order of the generalized p -Kaitz index, $\beta_n(p')$ is the percentile p' -specific coefficient on the n th power of the generalized p -Kaitz index, $\gamma_{st}(p')$ denotes a set of percentile p' -specific state-year controls—year dummies, state dummies, or state dummies in addition to state-specific quadratic time trends—and $\varepsilon_{st}(p')$ is a percentile p' -specific error term, which we assume satisfies the strict exogeneity condition $\mathbb{E}[\varepsilon_{st}(p') | gkatz_{st}(p), \dots, gkatz_{st}(p)^n, \gamma_{st}(p')] = 0$.

After estimating equation (2) separately for each wage percentile p' using a baseline percentile p , we

¹³Figure B.12 in Appendix B.4 shows that variation across Brazilian states in the generalized p -Kaitz index, for $p = 50$, is large initially and decreases as the minimum wage increases, while approximately preserving the ranking of states over time.

estimate the marginal effect of the minimum wage throughout the wage distribution,

$$\rho(p', p) \equiv \sum_{n=1}^N n\beta_n(p') g_{kaltz_{st}}(p)^{n-1}, \quad (3)$$

evaluated at the worker-weighted median value of the generalized p -Kaitz index across states and years. Allowing for polynomials of order at least $N = 2$ is important to capture the nonlinear effects of the minimum wage as it becomes more binding. In practice, after experimenting, we set $N = 2$.¹⁴

We first consider as outcome variables in equation (2) a set of measures of global or local wage inequality. As a global inequality measure, we consider the variance of log wages, $y_{st}(p'; p) = \text{Var}(w_{st})$. As local inequality measures, we use, for various values of p' , the log ratio between wage percentile p' and a base percentile p , so that $y_{st}(p'; p) = \log[w_{st}(p')/w_{st}(p)]$. Here, p is the same percentile as in the generalized p -Kaitz index, with p chosen to be high enough so as to be (directly and indirectly) unaffected by the minimum wage. Prior studies of the minimum wage in the U.S. context have used $p = 50$ —that is, the median—while appealing to the fact that, ex-post, their findings suggest insignificant spillover effects at or above that point in the wage distribution. For Brazil, where the minimum wage is more binding than in the U.S., we report results for the same value of $p = 50$ and consistently find a statistically significant correlation with outcomes above the median of the wage distribution.¹⁵ Therefore, we also report results for an alternative, preferred normalization using $p = 90$.

When analyzing the correlation between the minimum wage and log wage percentile ratios, the inclusion of the p th wage percentile in both the dependent and the independent variable may induce a spurious correlation that results in biased estimates of the coefficient $\beta_n(p')$, and thus the marginal effect $\rho(p', p)$, in the presence of measurement error or other transitory shocks (Autor et al., 2016). While measurement error is plausibly a lesser concern in large administrative data such as ours, we address this issue by implementing a variant of the solution proposed by Autor et al. (2016). Specifically, we instrument the generalized p -Kaitz index and its square using an instrument set that consists of the log real statutory minimum wage, its square, and the log real statutory minimum wage interacted with the mean of the log real median wage for each state over the full sample period. The motivation for this instrument set is that the current level of the statutory minimum wage in relation to the long-term average income level within a state affects the concurrent bindingness of the minimum wage (i.e., instrument relevance). In addition, it has an effect on concurrent wage inequality only through its effect on the concurrent bindingness of the minimum wage (i.e., the exclusion restriction) by being essentially decoupled

¹⁴Using polynomials of order $N > 2$ yields results that are substantially the same as those presented below.

¹⁵See Appendix B.11 for a comparison of the relative bindingness of the minimum wage, as proxied by left-tail wage inequality, between Brazil and the U.S.

from transitory wage fluctuations. Since Brazil has a federal minimum wage but, in contrast to the U.S., no state-level minimum wages, we include as controls in our instrumental variables (IV) specification state-specific quadratic time trends instead of a set of year dummies as in [Autor et al. \(2016\)](#).

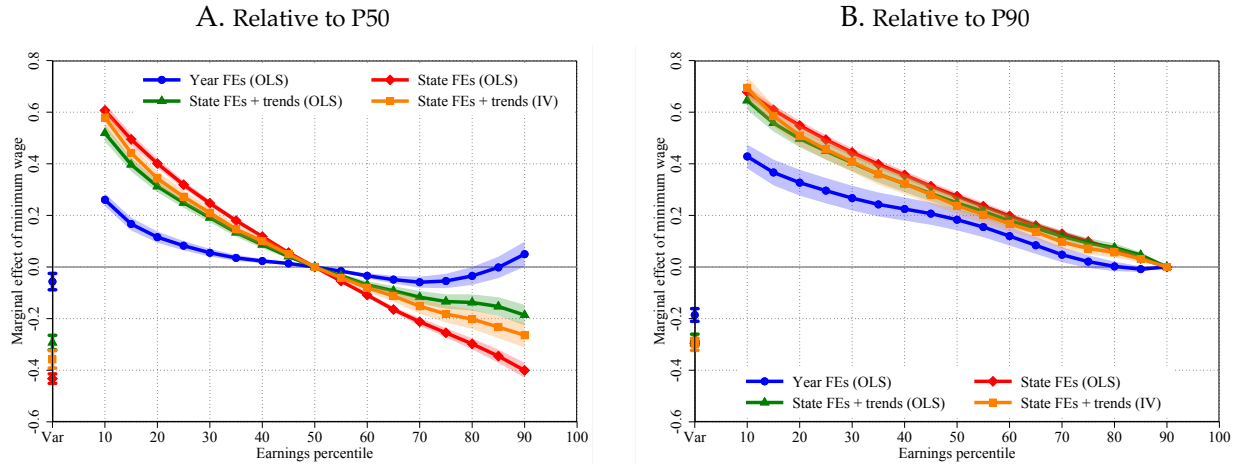
Result 1: Effects of the minimum wage on wage inequality. Figure 4 shows the results obtained from estimating equation (2) in RAIS over the full sample period from 1985 to 2014. We report results for four different specifications (colored lines and shaded areas), which are estimated with the base percentile being either $p = 50$ (panel A) or $p = 90$ (panel B) estimated across Brazil's 27 states. The four specifications include as controls, year fixed effects (blue line), state fixed effects (red line), state fixed effects in addition to state-specific quadratic time trends (green line)—all three of which are estimated using OLS—and the last specification estimated using the IV strategy described above (orange line). The corresponding shaded areas represent 99 percent confidence intervals based on regular standard errors at the state level.¹⁶ For each of the four specifications in a given panel, we report the estimated marginal effect on the variance of log wages ("Var" on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution ("10" to "90" on the x-axis) relative to the base wage p .

The results show a strong correlation between the minimum wage and inequality throughout most of the wage distribution. If we use the median as a base percentile (panel A), the estimated marginal effects of the minimum wage are monotonically decreasing between the 10th and at least the 70th percentile, and they are statistically significant at the 1 percent level up to at least the 75th percentile. The marginal effects are tightly estimated with the exception of the specification with only year fixed effects, consistent with there being large regional heterogeneity. The inclusion of state fixed effects leads to significantly greater estimates of minimum wage spillovers, reaching essentially all the way up to the 90th percentile of wages. The inclusion of state-specific quadratic time trends attenuates our spillover estimates somewhat. Finally, compared with the OLS estimates, the IV strategy yields slightly more pronounced spillover effects.

The statistically significant correlation between the minimum wage and inequality outcomes above the median motivates our alternative, preferred normalization using the 90th wage percentile (panel B). We consistently find significant marginal effects of the minimum wage up to at least the 70th percentile. For the specifications including state fixed effects, the marginal effects are tightly estimated and statistically significant all the way up to the 90th percentile of wages, regardless of the specification.

¹⁶Appendix B.6 shows that spillover effects of the minimum wage are similarly strong when estimated at a more granular level across Brazil's 137 mesoregions. Appendix B.7 reports equally strong or stronger results from two specifications in differences. Appendix B.8 shows the robustness of our results with respect to the choice of the polynomial order of region-specific time trends. Although the number of clusters at the state level falls below common thresholds for clustering ([Cameron and Miller, 2015](#)), Appendix B.9 also presents results with standard errors clustered at the state level.

Figure 4. Estimated minimum wage effects on the distribution of wages



Notes: Figure plots estimated marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from four different specifications (colored lines and error bars or shaded areas) with different state-year controls: year fixed effects (blue line), state fixed effects (red line), state fixed effects in addition to state-specific quadratic time trends (green line)—all three of which are estimated using OLS—and the last specification estimated using an IV strategy (orange line). The IV strategy instruments the generalized p -Kaitz index and its square using an instrument set that consists of the log real statutory minimum wage, its square, and the log real statutory minimum wage interacted with the mean of the log real median wage for the state over the full sample period. Each panel shows the estimated marginal effect of the minimum wage on the variance of log wages (“Var” on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution (“10” to “90” on the x-axis) relative to some base wage p . Panel A uses the 50th percentile as the base wage (i.e., $p = 50$), while panel B uses the 90th percentile as the base wage (i.e., $p = 90$). Both panels are estimated across Brazil’s 27 states. The four error bars and four shaded areas represent 99 percent confidence intervals based on regular standard errors at the state level. Source: RAIS, 1985–2014.

In terms of the correlation between the minimum wage and the overall variance of log wages, the eight sets of estimates—consisting of the combinations of four specifications and two wage percentile normalizations—yield fairly consistent results. Our preferred estimates, which use the 90th wage percentile as a base, suggest a semi-elasticity in the range of 0.17 to 0.32. This means that a 1 percent increase in the nominal minimum wage, holding fixed the median 90th percentile of wages, is associated with a decrease in the variance of wages of between 0.17 and 0.32 percent points. Although caution is warranted when extrapolating from cross-sectional regressions to aggregate trends, the estimates would suggest a decline in the variance of wages in the range between 7.5 and 14.2 log points, compared with the actual decline in the variance of wages of 25.6 log points in the raw data, in response to the 44.4 log point labor productivity-adjusted increase in the minimum wage seen in Brazil between 1996 and 2012.

Our finding of a correlation between the minimum wage and inequality outcomes up to the 90th percentile of the wage distribution may seem surprising. For comparison, Autor et al. (2016) show spillovers up to the 20th percentile of the wage distribution in the U.S. In light of this, we make five observations. First, our large-scale administrative data plausibly admit less measurement error than the CPS, alleviating concerns about bias in the estimates of $\beta_n(p)$ in equation (2) and allowing us to measure spillover effects with greater accuracy than previously possible. Second, the minimum wage in Brazil

during this period was more binding compared with that in the U.S. over the last decades (Autor et al., 2016). Because of the nonlinear nature of spillover effects, this is expected to lead to greater effects throughout the wage distribution.¹⁷ Third, while a relatively small fraction of Brazilian workers earn the minimum wage in any given year during our sample period, we find that a significant fraction of workers throughout the wage distribution ever (currently, in the past, or in the future) earn the minimum wage during our sample period. This may suggest that the minimum wage in Brazil acts as an important stepping stone, even for workers who eventually find themselves high up in the wage distribution.¹⁸ Fourth, the minimum wage in Brazil is particularly salient given Brazil's volatile economic history. While indexation of wages to the minimum wage is not allowed by Brazilian labor laws and not supported by the government, the minimum wage still serves as an important reference point in wage setting mechanisms (Neri and Moura, 2006).¹⁹ Fifth and finally, compared with the U.S., Brazil's workforce is heavily skewed toward low-skill workers, as measured by educational attainment. Around the 70th–90th percentile of the wage distribution, there is a sharp increase in the share of workers with either a high school or a college degree, and wage levels increase sharply across wage quantiles.²⁰ Therefore, it seems natural that the minimum wage would have a greater impact among lower-skill workers, who make up a larger population share in Brazil than their American counterparts do in the U.S. population.

Result 2: Effects of the minimum wage on employment. So far, we have focused on the correlation between the minimum wage and inequality. We now extend our regression framework to investigate the link between the minimum wage and employment outcomes—in both the formal and informal sectors—over our period of study. To this end, we supplement the administrative data from RAIS with household survey data from PNAD and PME to estimate variants of the specification in equation (2) with the dependent variable $y_{st}(p'; p) = y_{st}$ varying only at the region-year level.²¹

Consistent with previous evidence by Lemos (2009), results from the PNAD survey data in panel A in Table 3 show that the minimum wage has precisely estimated zero effects on the population size, labor force participation rate, employment rate, and formal employment share, all of which are insignificant at

¹⁷Appendix B.11 shows that between 1985 and 2012, the minimum wage in Brazil relative to that in the U.S. has gone from less binding to significantly more binding.

¹⁸Appendix B.2 shows that a relatively small fraction of Brazilian workers have wages exactly equal to, less than, or around the minimum wage at any given point in time between 1996 and 2012. Appendix B.3 studies minimum wage earners.

¹⁹Our model in Section 4 rationalizes the view of the minimum wage as a reference point as an equilibrium outcome due to frictional inter-firm competition for workers. In the data, as in our model, the link between the minimum wage and the wage distribution is imperfect—not all wages move one-for-one with the minimum wage. Thus, the wage distribution compresses as the minimum wage is increased. Appendix B.12 compares the distribution of (changes in) wages in nominal values and in multiples of the current minimum wage.

²⁰See Appendix A.2 for details.

²¹A region corresponds to one of Brazil's 27 states in RAIS and PNAD and to one of the six largest metropolitan areas in PME.

conventional levels. Specifically, there is little evidence of cross-state differences in population or labor force dynamics linked to the minimum wage. If anything, the rise in the minimum wage is associated with a rise in log population size that is statistically significant only at the 10 percent level. Results from the PME data in panel B show small estimated marginal effects of the minimum wage on transition rates from nonformal to formal employment as well as from formal to nonformal employment. While both point estimates are negative, they are also statistically insignificant at conventional levels.²² Finally, panel C shows the estimated effects of the minimum wage on other labor market outcomes in RAIS. Mean hours worked do not vary significantly with the Kaitz index, suggesting that the hours margin of adjustment to the minimum wage (Doppelt, 2019) is not of prime importance for understanding the Brazilian context. Mean firm size correlates positively with the minimum wage, consistent with the idea that the minimum wage induces small firms to shrink or exit in favor of larger, more productive competitors. The estimated effect on the probability of a worker's remaining employed until next year is positive, suggesting that the minimum wage might have retained more workers in the formal sector.²³

Table 3. Effects of the minimum wage on employment worker transitions

	Marginal effect (s.e.)
<i>Panel A. Cross-sectional household survey data (PNAD)</i>	
Log population size	0.057 (0.030)
Labor force participation rate	0.009 (0.016)
Employment rate	0.014 (0.015)
Formal employment share	0.024 (0.020)
<i>Panel B. Longitudinal household survey data (PME)</i>	
Transition rate nonformal-formal	−0.003 (0.017)
Transition rate formal-nonformal	−0.005 (0.009)
<i>Panel C. Administrative linked employer-employee data (RAIS)</i>	
Mean log hours worked	−0.006 (0.004)
Mean log firm size	0.203 (0.040)
Probability of remaining employed	0.042 (0.007)

Notes: Table shows predicted marginal effects with standard errors in parentheses evaluated at the worker-weighted mean across Brazil's 27 states. Each cell corresponds to the estimated coefficient and standard error from one regression with the relevant dependent variable (row) and various controls (columns). The underlying regressions are variants of equation (2) including state fixed effects and state-specific quadratic time trends. Source: PNAD, 1996–2012, PME, 2002–2012, and RAIS, 1985–2014.

²²An increase in the minimum wage may affect both formal and informal employment (Jales, 2018). While we view this as an interesting avenue for future research, a structural assessment of the minimum wage on inter-firm competition for workers between Brazil's formal and informal sectors, as in Meghir et al. (2015), is beyond the scope of the current paper.

²³See also Appendix B.13 for a more detailed analysis of the correlation between the minimum wage and hours worked. Appendix B.14 shows that these results are robust across specifications with region-specific time trends of various polynomial orders. Appendix B.15 also shows that the effects of the minimum wage on population dynamics, labor force participation, the employment rate, and formality are small for all education groups.

3.3 A call for an equilibrium model

The above findings suggest that Brazil's minimum wage has had far-reaching effects on the wage distribution but limited, negative effects on employment. That the inequality-decreasing effects of the minimum wage are so large may seem surprising in light of past findings of smaller effects in the U.S. by [Lee \(1999\)](#) and [Autor et al. \(2016\)](#). Yet there exists little theoretical guidance on how strong we should expect spillover effects of the minimum wage to be and their costs. Furthermore, reduced-form estimates based on cross-sectional variation recover only the relative, not the absolute, effects of the minimum wage—a problem that is compounded if spillovers are present throughout most of the wage distribution.²⁴ Finally, there may remain concerns about confounding factors not controlled for in our econometric analysis, such as the concurrent rollout of social security programs and the expansion of education in Brazil.

To address these issues, we develop and estimate an equilibrium model of the Brazilian labor market subject to a minimum wage. Such a model, while based on certain assumptions, can lend additional credibility to our reduced-form estimates, which rely on a very different set of assumptions. Another benefit of a structural model is that it can aggregate the effects of the minimum wage estimated based on cross-sectional variation in the data, while using counterfactual simulations to shed light on the mechanisms by which the minimum wage affects the labor market.

4 Equilibrium model of a labor market subject to a minimum wage

We now develop an equilibrium model of the Brazilian labor market subject to a minimum wage. Our framework is essentially a series of heterogeneous [Burdett and Mortensen \(1998\)](#) economies separated by worker types. Our contribution is to provide empirical content to this framework by integrating unobserved worker heterogeneity, minimum wage jobs, and endogenous job creation in a tractable manner. The extended framework is geared toward estimation on linked employer-employee data and an analysis of the equilibrium effects of the minimum wage on the distribution of wages and employment.

4.1 Environment

Consider a continuous-time economy in steady state populated by a unit mass of workers and a mass M of firms, both infinitely lived and with risk-neutral preferences over consumption discounted at rate ρ .

²⁴This is a variant of the “missing intercept” problem highlighted in recent micro-to-macro literature ([Nakamura and Steinsson, 2018](#)).

Worker types. At any point in time, a worker can be either employed or nonemployed. We think of nonemployment as a simple way of capturing either unemployment or informal employment with associated utility flow value $ab(a)$ that depends on permanent worker ability $a \sim \Psi(\cdot)$, with $a \in [\underline{a}, \bar{a}]$. That the informal market offers a constant flow utility substantially simplifies the analysis. We think of the dependence of this flow utility on ability a as reflecting individual traits that are valued not just in formal employment but also in informal employment or home production. This ability parameter corresponds to both observable and unobservable worker characteristics, which Appendix A.4 shows matter for explaining empirical wage dispersion.

Workers also differ in their relative on-the-job search efficiency, $s \in [\underline{s}, \bar{s}]$, which captures the fact that different workers have unequal access to professional networks or have different propensities to switch employers; for example, a worker may have family circumstances that prevent a geographic move. While we do not endogenize the on-the-job search efficiency, we allow it to change both between workers and within workers over time.²⁵ In particular, an employed worker of type (a, s) becomes nonemployed at Poisson rate $\delta(a, s)$, at which point the search efficiency is updated according to a first-order Markov process with transition probability $\pi(s'|a, s)$. The assumption that search efficiency updates only when a worker transitions into nonemployment avoids added complexity from the entry of worker type transition hazards into firms' problem.

Technology. Firms are heterogeneous in their permanent productivity $z \sim \Gamma(z)$, with $z \in [\underline{z}, \bar{z}]$. A firm that employs $l(a, s)$ workers of each type (a, s) produces output according to the linear technology

$$y\left(z, \{l(a, s)\}_{a, s}\right) = z \int_{\underline{a}, \underline{s}}^{\bar{a}, \bar{s}} al(a, s) da ds.$$

To hire workers, firms post vacancies v in each market (a, s) at convex increasing cost $c(v|a, s)$, which reflects the cost of advertising the job, screening applicants, and training workers for the job.

Search and matching. Both nonemployed and employed workers search for jobs at random in labor markets that are segmented by worker type (a, s) . Let $p(a, s)$ denote the Poisson arrival rate of job offers per unit of search efficiency in market (a, s) . A job offer is an opportunity to work for a fixed piece rate w for the duration of a job. Therefore, a worker of ability a employed at piece rate w receives flow

²⁵In a framework with endogenous search intensity as in Lentz (2010), we hypothesize that a rise in the minimum wage would have two opposing effects on incentives to search. On the one hand, it would render employment more attractive since it pays better on average, which incentivizes search. On the other hand, it flattens the wage ladder and reduces job vacancies, which disincentivizes search. Given these offsetting forces and existing evidence that worker search effort is rather inelastic (Engbom, 2020), we focus here on a model with exogenous search effort.

value wa . Let $F(w|a, s)$ denote the distribution of piece rates offered in market (a, s) . While workers take offer arrival rates and the offer distribution as given, both are determined in equilibrium by firms' vacancy and wage posting decisions. In particular, if firms post total vacancies $V(a, s)$ in a given market (a, s) and workers' aggregate search intensity is $S(a, s) = u(a, s) + se(a, s)$, where $u(a, s)$ is the number of nonemployed workers and $e(a, s)$ is the number of employed workers of type (a, s) , then the total number of worker-firm contacts in market (a, s) is given by $\chi V(a, s)^\alpha S(a, s)^{1-\alpha}$. Here, $\chi > 0$ is the match efficiency and $\alpha \in (0, 1)$ is the match elasticity with respect to aggregate vacancies.

4.2 Worker's problem and the distribution of workers over the job ladder

Let $U(a, s)$ denote the value to a nonemployed worker with ability a and search efficiency s . Let $W(w, a, s)$ be the value to a worker with ability a and search efficiency s from being employed at piece rate w . The value $U(a, s)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho U(a, s) = ab(a) + p(a, s) \int_{\underline{w}(a, s)}^{\bar{w}(a, s)} \max \{W(w, a, s) - U(a, s), 0\} dF(w|a, s). \quad (4)$$

For nonemployed workers, there exists a reservation threshold $r(a, s)$ such that $W(r(a, s), a, s) = U(a, s)$. A nonemployed worker of type (a, s) accepts any piece rate offer $w \geq r(a, s)$ and rejects any offer $w < r(a, s)$. In equilibrium, firms make offers only with $w \geq r(a, s)$.

The value $W(w, a, s)$ of a worker of type (a, s) employed at piece rate w is given by the HJB equation:

$$\begin{aligned} \rho W(w, a, s) = & wa + sp(a, s) \int_w^{\bar{w}(a, s)} (W(w', a, s) - W(w, a, s)) dF(w'|a, s) \\ & + \delta(a, s) \left(\int_{\underline{s}}^{\bar{s}} U(a, s') \pi(s'|a, s) ds' - W(w, a, s) \right) \end{aligned} \quad (5)$$

A worker of type (a, s) employed at piece rate w receives outside offers at rate $sp(a, s)$, which the worker accepts if the associated piece rate offer w' satisfies $w' > w$. If an employed worker rejects an outside offer, the worker remains employed in his or her current job. Employed workers become nonemployed at exogenous rate $\delta(a, s)$, in which case the worker's search efficiency updates according to the Markov process $\pi(s'|a, s)$.

Let $G(w|a, s)$ denote the steady-state distribution of employed workers of type (a, s) over piece rates w . Appendix C shows that this distribution satisfies

$$G(w|a, s) = \frac{p(a, s)F(w|a, s)}{\delta(a, s) + sp(a, s)(1 - F(w|a, s))} \frac{u(a, s)}{e(a, s)}. \quad (6)$$

4.3 Firms' problem

Under the assumption that the discount rate tends to zero, $\rho \rightarrow 0$, firms' dynamic problem reduces to maximizing flow profits. Firms choose, market by market, how many job openings to advertise, $v \geq 0$, and what piece rate to pay, w , subject to a minimum wage constraint, $wa \geq w^{\min}$:

$$\max_{w \geq w^{\min}/a, v} \{a(z-w)l(w, v|a, s) - c(v|a, s)\}, \quad (7)$$

where $l(w, v|a, s)$ is the number of workers of type (a, s) that a firm posting piece rate w and vacancies v attains in equilibrium. In particular, Appendix C.2 shows that

$$l(w, v|a, s) = \frac{vu(a, s)p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))^2} \quad (8)$$

Let $v(z|a, s)$ denote the optimal vacancy policy of a firm with productivity z in market (a, s) and $w(z|a, s)$ its optimal wage policy. Given these policies, the equilibrium offer distribution is given by

$$F(w(z|a, s)|a, s) = \frac{M}{V(a, s)} \int_{\underline{z}}^z v(\tilde{z}|a, s) d\Gamma(\tilde{z}), \quad \text{where } V(a, s) = M \int_{\underline{z}}^{\bar{z}} v(\tilde{z}|a, s) d\Gamma(\tilde{z})$$

Henceforth, we assume that the vacancy cost takes an isoelastic form, $c(v, a, s) = ac(a, s)v^{1+\eta}/(1+\eta)$. Define $h(z|a, s) = F(w(z|a, s)|a, s)$ so that $f(w(z|a, s)|a, s) = h'(z|a, s)/w'(z|a, s)$.

4.4 Equilibrium

Appendix C defines the equilibrium, which is characterized by a system of differential equations,

$$\begin{aligned} w'(z|a, s) &= (z - w(z|a, s)) \frac{2sp(a, s)h'(z|a, s)}{\delta(a, s) + sp(a, s)(1 - h(z|a, s))}, \\ h'(z|a, s) &= \gamma(z) \frac{M}{V(a, s)} \left(\frac{1}{c(a, s)} (z - w(z|a, s)) \frac{u(a, s)}{V(a, s)} p(a, s) \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - h(z|a, s)))^2} \right)^{\frac{1}{\eta}}, \end{aligned} \quad (9)$$

subject to boundary conditions $\lim_{z \rightarrow \underline{z}(a, s)} w(z|a, s) = \max\{r(a, s), w^{\min}/a\}$ and $\lim_{z \rightarrow \underline{z}(a, s)} h(z|a, s) = 0$, where $\underline{z}(a, s)$ is the lowest productivity active in market (a, s) , so $\underline{z}(a, s) = \max\{\underline{z}, \max\{r(a, s), w^{\min}/a\}\}$. Equilibrium requires that the total number of vacancies, $V(a, s)$, is such that $\lim_{z \rightarrow \bar{z}} h(z|a, s) = 1$.

5 Estimation

We estimate the model by targeting empirical moments from the preperiod 1994–1998. The goal is to use the estimated model to quantify the equilibrium effects of the observed increase in the minimum wage.

5.1 Estimation strategy

To accommodate unobserved heterogeneity among workers and firms, our model features a continuum of parameters. To reduce the dimensionality of the estimation problem, we make some simplifying assumptions. We first discretize both worker ability a and firm productivity p . We then parameterize how worker heterogeneity varies across ability levels a and how firm productivity p is distributed. Subsequently, we proceed in three steps. First, we preset three parameters based on standard values in the literature. Second, we directly infer three parameters, which the model maps one-to-one to three empirical moments. Third, we estimate 12 remaining parameters using the SMM via indirect inference.

Preset parameters. We adopt a monthly frequency and set the discount rate to the equivalent of an annual real interest rate of 5 percent. We normalize matching efficiency to $\chi = 1$, since without data on vacancies, it is not separately identified from the intercept in the vacancy cost function. Based on standard values in the literature (Petrongolo and Pissarides, 2001), we set the elasticity of matches with respect to vacancies to $\alpha = 0.5$, which is at the upper end of the range considered by Meghir et al. (2015). For robustness, we consider alternative values for α in Section 6.5.

Directly inferred parameters. The mass of firms, M , can be directly chosen to target a mean firm size in RAIS of 11.8 workers. Under the assumption that the separation rate of workers with zero on-the-job search efficiency, $\delta(a, 0)$, is constant across ability levels, we can equate this parameter to the empirical separation rate of workers earning the minimum wage, which equals 6.5 percent per month. We assume that the job finding rate $p(a, s) = \lambda$ is independent of worker ability a and relative on-the-job search efficiency s . We set the auxiliary parameter λ to target a monthly nonemployment-to-employment (NE) rate of 4.4 percent.²⁶ Of course, λ is an equilibrium outcome, but we can treat it as an auxiliary parameter since the cost of creating jobs, $c(a, s)$, can be chosen to rationalize any positive value of λ in equilibrium.

²⁶Brazil's NE rate is low in an international comparison (Engbom, 2021), likely because we include informal workers in our definition of nonemployment. This is not a prime concern for us, however, because the key factor affecting firm wages is how fast workers move up and fall off the job ladder, which relates to job-to-job (EE) and employment-to-nonemployment (EN) rates. In contrast, the NE rate impacts the economy primarily through the stock of nonemployed workers, as we confirm in robustness exercises in Section 6.6.

Hence, we pin down the structural parameters $c(a, s)$ flexibly in each market such that the equilibrium job finding rate is λ .

Internally estimated parameters. We estimate the remaining model parameters by the SMM via indirect inference. Specifically, we choose the parameter vector $\mathbf{p}^* \in \mathcal{P}$ that minimizes the sum of weighted squared percentage deviations between a set of moments in the model and in the data:

$$\mathbf{p}^* = \arg \min_{\mathbf{p}} \sum_{p \in \mathcal{P}} \sum_{m \in \mathcal{M}(p)} w_m \left(\frac{m^{\text{model}} - m^{\text{data}}}{m^{\text{data}}} \right)^2.$$

While all parameters are jointly determined, we assign to each parameter p a set of moments $\mathcal{M}(p)$ that are particularly informative for p as we compare the model-based moments m^{model} against their data equivalent m^{data} with weight w_m . We discuss our choice of moments and weights in greater detail below.

To further simplify the problem, we impose some flexible parametric restrictions based on inspection of the data vis-à-vis the model output. We assume that log worker ability is distributed according to a double exponential distribution with mean μ and shape parameter σ . Firm productivity is Pareto distributed with shape parameter ζ and a scale parameter normalized to one.

We restrict search efficiency to fluctuate between $s = 0$ and a positive value $s(a) > 0$, which depends on ability a . In equilibrium, firms offer the reservation wage $r(a, s)a$ to workers with $s = 0$ who do not search on the job (Diamond, 1971). If the minimum wage binds with $w^{\min} = r(a, s)a$ for a positive measure of workers, then our model produces a spike at the minimum wage in the wage distribution. We assume that an employed worker with search efficiency $s(a) > 0$ who becomes unemployed transitions to $s(a) = 0$ with probability π and retains $s(a)$ with complementary probability $1 - \pi$.²⁷ A worker with $s(a) = 0$ who becomes unemployed transitions to $s(a) > 0$ with probability 1. That is,

$$\pi(x|a, s(a) > 0) = \begin{cases} 1 - \pi & \text{if } x = s(a) \\ \pi & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases} \quad \pi(x|a, 0) = \begin{cases} 1 & \text{if } x = s(a) \\ 0 & \text{otherwise.} \end{cases}$$

For a worker with search efficiency $s(a) > 0$, the exogenous separation rate is assumed to be an affine transformation of a worker's ability rank, $\delta(a, s) = \delta_0(1 + \delta_1\Psi(a))$. The relative on-the-job search efficiency among workers in the regular state is $s(a) = \phi_0(1 + \phi_1(\exp(\Psi(a)) - 1))$. These parametric forms are guided by what appears to fit the data well.

²⁷While the probability of transiting to $s = 0$ upon separating to unemployment is independent of a , our model features a lower incidence of minimum wage jobs among higher-ability workers, since they are less likely to separate to unemployment.

Next, we posit a reduced-form relationship for the reservation wage among workers with positive search efficiency given by $ar(a, s > 0) = r_0 + r_1(a - \underline{a})$. The reservation wage is an endogenous outcome, but the flow value of leisure $b(a)$ is a free parameter, allowing us to treat $r(a, s > 0)$ —or, in this case, r_0 and r_1 —as auxiliary parameters to be estimated. We then choose $b(a)$ so as to reproduce the estimated reservation piece rate $r(a, s)$ as an equilibrium outcome.²⁸ This approach allows us to solve the model using the system of differential equations in (9) without reference to workers' value functions (4)–(5), leading to a great reduction in computational time.

Model solution, simulation, and estimation. We solve the model in continuous time over 50 grid points for ability and 500 grid points for productivity. We then simulate the model at monthly frequency over a period of five years for a large number of workers, starting from the ergodic distribution. To match the empirical residual wage dispersion conditional on worker and firm heterogeneity, we assume that the logarithm of measured wages, $\log \tilde{w}$, equals the sum of the logarithm of the true wage, $\log w$, and measurement error, κ , so $\log \tilde{w} = \log w + \kappa$. We let $\kappa \sim \mathcal{N}(0, \varepsilon)$ with variance ε and values drawn independently and identically distributed across worker-firm matches. Motivated by the empirical existence of a (relatively small) spike in the wage distribution at the minimum wage, we assume that measurement error is identically zero for minimum wage jobs. One interpretation of this is that employers offering exactly the minimum wage are well aware of its statutory level and the penalties for violations, which induce them to make accurate reports. We construct monthly and annual data sets based on model simulations, using the same sample selection criteria and variable construction as in the data.

These assumptions leave us with 12 parameters to estimate by the SMM via indirect inference:²⁹

$$\mathbf{p} = \{\mu, \sigma, \zeta, \eta, \varepsilon, \delta_0, \delta_1, \phi_0, \phi_1, \pi, r_0, r_1\}.$$

While all parameters are jointly determined, it is useful to provide a heuristic discussion of which data moments are particularly informative for each parameter. We verify this intuition in Appendix D.3. The scale of the ability distribution, μ , is informed by the log ratio of the median to minimum wage. Greater μ means that wage distribution is further removed from the wage floor. This moment plays a key role in our analysis and we assign it a weight of $w_m = 5$. For the ability shape parameter, σ , we target log wage

²⁸We verify that all worker types ($a, s = 0$) prefer being employed at the minimum wage over unemployment under our estimated parameter values. Note that in markets in which the minimum wage is binding, the minimum wage provides an upper bound on the latent reservation wage. Since the impact of a simulated minimum wage increase—unlike that of a decrease—is invariant to the level of the flow value of leisure, $b(a)$, in markets where the minimum wage is initially binding, we assume that $b(a)$ equated to the value of unemployment in those markets.

²⁹Recall that our directly inferred estimate of λ is associated with an implied vacancy cost scalar $c(a, s)$ for each market (a, s), and each value of $r(a, s > 0)$ corresponding to our estimates of (r_0, r_1) is associated with an implied flow value of leisure $b(a)$.

percentile ratios relative to the median in increments of five (i.e., P5-50, P10-50, ..., P95-P50). We assign each of the 18 percentile ratios a weight of $w_m = 1$.

For the remaining parameters, we connect our equilibrium model to reduced-form estimates from the AKM wage equation in Section 2.2. The AKM wage equation does not have a structural interpretation in our general framework. Nevertheless, Appendix D.3 shows that this indirect inference approach disciplines the distributions of unobserved worker and firm heterogeneity in our model vis-à-vis the data.³⁰

The shape of the Pareto distribution for firm productivity, ζ , is informed by the standard deviation of AKM firm fixed effects. Lower values of ζ are associated with greater dispersion in productivity and firm pay. We match the curvature of the vacancy cost, η , to the share of employment at firms with 50 or more workers. For lower values of η , it is cheaper for firms to scale up vacancies, which results in more productive firms growing relatively larger. Both moments are assigned a weight of $w_m = 1$.

The variance of measurement error $w_m = 1$ intuitively maps into the variance of residuals in the AKM wage equation. We assign this moment a weight of $w_m = 1$.

For the separation rate's intercept, δ_0 , and slope, δ_1 , we target the EN rate by AKM worker fixed effect deciles. The intercept δ_0 steers the average EN rate, while the slope in ability, δ_1 , steers heterogeneity in EN rates across AKM worker fixed effects. Moments for each of the AKM worker fixed effect deciles receive a weight of $w_m = 1/10$, which results in a unit cumulative weight.

The intercept, ϕ_0 , and slope, ϕ_1 , of the relative on-the-job search intensity, $s(a)$, maps into the EE rate by AKM worker fixed effect decile. Again, each of these 10 moments receives a weight of $w_m = 1/10$.

The probability $\pi(0|a, s)$ that a displaced worker transitions from $s > 0$ to $s = 0$ maps into the spike at the minimum wage in the wage distribution. This moment also receives a weight of $w_m = 1$.

For the auxiliary parameters governing reservation wages, r_0 and r_1 , we target the P5 of log wages by AKM worker fixed effect decile. Intuitively, r_0 guides the minimum wage bindingness for all markets, while r_1 guides the relative bindingness across AKM worker fixed effect deciles. Again, each of these 10 moments receives a weight of $w_m = 1/10$, which results in a unit cumulative weight.

5.2 Parameter estimates and model fit

Table 4 presents the three preset, three directly inferred, and 12 internally estimated parameter values along with their targeted moments. A few comments are in order. Let us first address the set of parameters related to the wage distribution. The model closely replicates the empirical median-to-minimum

³⁰For this indirect inference estimation step, both in our model and in the data, we drop minimum wage workers, do not apply a KSS bias correction, and do not include additional controls. Importantly, we treat the model and the data identically.

log wage ratio (related to $\mu = 0.960$) and the general shape of the log wage distribution (related to $\sigma = 0.258$) shown in Figure 6 and to be discussed shortly. A tail index of the firm productivity distribution of $\zeta = 3.503$ allows the model to match well the variance of AKM firm fixed effects. To match the share of workers employed at firms with at least 50 employees, the model requires a relatively low curvature of the vacancy cost, $1 + \eta = 1.467$.³¹ Finally, most of the AKM residual variance is accounted for by measurement error, $\varepsilon = 0.215$, as opposed to violations of log additivity of the wage equation.

We now turn to a set of parameters related to employment transitions. In the RAIS data, around 4 percent of workers leave formal employment in the subsequent month, a rate that is close to the EN rate in the U.S. Note that this number includes workers who leave for informal employment not recorded in RAIS. Furthermore, the data show a steep negative gradient between NE rates and AKM person fixed effect deciles. Together, these empirical moments lead us to estimate $\delta_0 = 0.074$ and $\delta_1 = -0.815$ —see Figure D.3A in Appendix D.2 for details. Next, an average of 1.8 percent of workers make an EE transition each month, a number that is again close to the corresponding number in the U.S.³² Because the EE rate is high relative to the NE rate in Brazil, we infer a high average relative search efficiency, $s(a)$. This does not mean that Brazilian labor markets are highly efficient but merely that EE transitions are not as rare as NE transition rates may suggest. The resulting parameter estimates $\phi_0 = 0.436$ and $\phi_1 = 1.055$ match the empirical EE transition rates shown in Figure D.3B of Appendix D.2. The estimated value of the transition rate to minimum wage jobs, $\pi = 0.019$, leads our model to generate a realistic spike in the wage distribution at the minimum wage.

The two remaining parameters relate to workers' outside option value. The parameters $r_0 = -0.078$ and $r_1 = 1.127$ capture the empirical feature of the P5 of log wages rising steeply across AKM person fixed effect deciles—see Figure D.3D in Appendix D.2.

We now discuss the mapping between the estimated auxiliary parameters (λ , r_0 , r_1) and the corresponding structural parameters of the model. Panel A of Figure 5 plots the implied vacancy cost scalars $c(a, s)$ across markets (a, s) . The implied per-ability-unit vacancy cost is nonmonotonic, initially decreasing in ability and subsequently increasing. Because the overall recruiting cost for workers of type (a, s) equals $ac(a, s)$, the overall recruiting cost turns out to be relatively flat among low ability levels and

³¹Because η also governs the elasticity of vacancy creation with respect to firm profitability, a low value of η implies that firms' employment responds relatively flexibly to the minimum wage—see Figure D.4D in Appendix D.3 for details.

³²Using survey data from the PME, Meghir et al. (2015) report a quarterly EE rate of 1.58 percent and 2.49 percent, in the Brazilian metropolitan regions of São Paulo and Salvador, respectively. There are several differences between the way we estimate EE transitions for our purposes and how Meghir et al. (2015) estimate them. Our estimates are based on a different data set, RAIS, which has wider geographic coverage. RAIS, unlike PME, also records the exact employment start and end dates, mitigating concerns about time aggregation bias (Shimer, 2012). Reporting issues are likely also a lesser concern in administrative data like RAIS than they are in survey data like PME. Finally, regarding right censoring, the RAIS data allow us to estimate transition rates over a longer panel of 60 months, compared with the four-month panel in PME. See Engbom et al. (2021) for a detailed comparison between the PME and RAIS data sets.

Table 4. Parameter estimates

Parameter	Description	Estimate	Targeted moment	Data	Model
<i>Panel A. Preset parameters</i>					
ρ	Discount rate	0.004	Annual real interest rate of 4 percent		
χ	Matching efficiency	1.000	Normalization		
α	Elasticity of matches w.r.t. vacancies	0.500	Petrongolo and Pissarides (2001)		
<i>Panel B. Directly inferred structural and auxiliary parameters</i>					
M	Mass of firms	0.069	Average firm size	11.8	13.1
$\delta(a, 0)$	Separation rate of those with $s = 0$	0.065	EN rate from minimum wage jobs	0.065	0.065
λ	Job finding rate	0.044	NE rate	0.044	0.044
<i>Panel C. Internally estimated structural and auxiliary parameters</i>					
μ	Mean of worker ability	0.960	Median to minimum wage	1.224	1.192
σ	Shape of worker ability	0.258	Percentiles of wage distribution	See Figure 6	
ζ	Shape of productivity distribution	3.503	Variance of AKM firm FEs	0.217	0.195
η	Curvature of vacancy cost	0.467	Employment share of firms with 50+ empl.	0.589	0.583
ε	Variance of noise	0.215	Variance of AKM residual	0.032	0.035
δ_0	Separation rate, intercept	0.074	EN rate	See Figure D.3	
δ_1	Separation rate, slope	-0.815	EN rate	See Figure D.3	
ϕ_0	Relative search intensity, intercept	0.436	EE rate	See Figure D.3	
ϕ_1	Relative search intensity, slope	1.055	EE rate	See Figure D.3	
π	Transition rate to $s = 0$	0.019	Share of employment at minimum wage	0.012	0.011
r_0	Reservation wage, intercept	-0.078	P5 of log wages	See Figure D.3	
r_1	Reservation wage, slope	1.127	P5 of log wages	See Figure D.3	

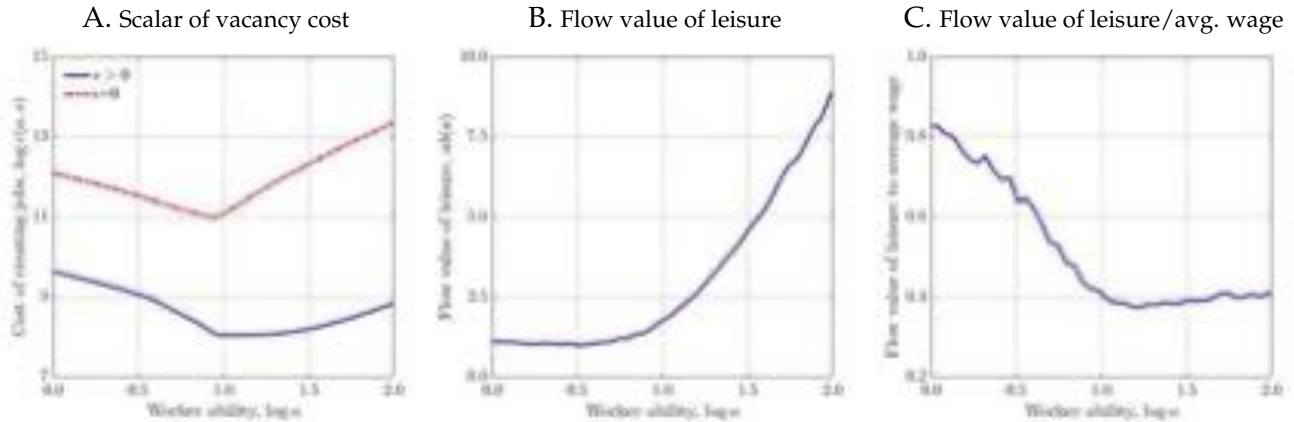
Notes: Parameter estimates are expressed at a monthly frequency, when applicable. Source: Model and RAIS, 1994–1998.

then sharply increasing toward higher ability levels. Conditional on worker ability a , recruiting costs are uniformly higher in the markets with $s = 0$.

Panel B of Figure 5 shows the flow value of leisure $b(a)a$ across ability types a , which is first flat and then upward sloping, consistent with the idea that higher-ability workers are also better at home production or at work in the informal sector. Panel C shows for each ability type the flow value as a fraction of mean wages, $\bar{w}(a)$, which varies from around 80 percent among low ability levels to around 40 percent at medium and high ability levels. Appendix D.1 shows that these estimates give rise to a model-implied mean-to-min wage ratio ([Hornstein et al., 2011](#)) of between 1.3 at low ability levels and 3.0 at the top. Thus, the estimated model suffers less from the critique raised by [Hornstein et al. \(2011\)](#) that many search models require unrealistically low (or indeed negative) flow values of leisure to generate realistic levels of frictional wage dispersion. This result obtains for two reasons. First, the high relative on-the-job search efficiency means that job acceptance out of unemployment forgoes a lesser option value. Second, a large share of the variance of wages in our linked employer-employee data is due to unobserved worker heterogeneity corresponding to ability differences in our model. Conventional survey data would attribute this variation partly to residual, or frictional, wage dispersion—see Appendix

A.4 for details.

Figure 5. Estimated vacancy costs and flow values of leisure



Notes: Parameter estimates are expressed at a monthly frequency, when applicable. Panel A shows the scalar $c(a, s)$ of the vacancy cost function $ac(a, s)v^{1+\eta}/(1+\eta)$. Panel B shows the flow value of leisure $b(a)a$ that workers of ability a receive when not formally employed. Panel C shows the flow value of leisure, $b(a)$, relative to the ability specific average wage, $\bar{w}(a) = \int_s \int_z w(z|a, s)dG(z|a, s)d\Phi(s|a)$, where $\Phi(s|a)$ is the conditional distribution of workers of ability a over search efficiency, s . Source: Model.

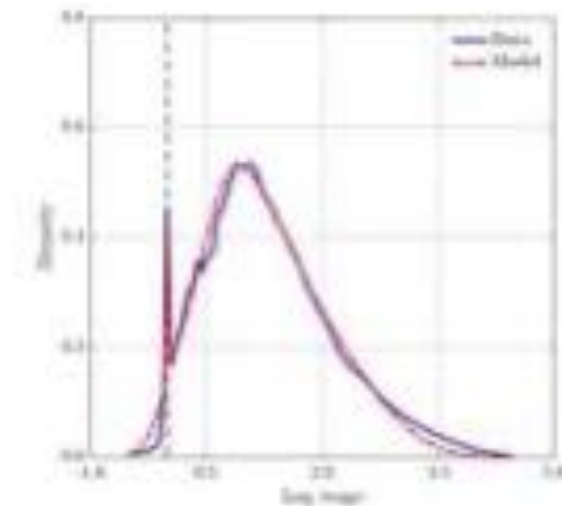
We now turn to an important dimension of our model's performance — namely, its fit vis-à-vis the empirical wage distribution. Figure 6 compares the distribution of log wages in the data and in the model. Overall, the model-generated wage distribution matches several salient features of the empirical wage distribution. These include its mode, dispersion, skewness, a spike at the minimum wage, and a mass below the minimum wage. At the same time, the model fit is less than perfect. For example, the model underpredicts the mass in the far right tail of the wage distribution. It also overpredicts the number of workers below the minimum wage. While the model matches well the spike exactly at the minimum wage—see Table 4 above—it slightly understates the number of workers earning just above the minimum wage.³³ We postulate that more flexible parametric forms or richer wage setting mechanisms such as those in Flinn and Mullins (2018) would help match these features.³⁴ We note, however, that such extensions would come at a significant increase in computational time, which is already substantial.³⁵

³³Appendix E.4 shows that our results are robust to varying parameters to better fit these features of the data in isolation.

³⁴Flinn and Mullins (2018) show that the presence of wage bargaining, in addition to wage posting, can change the predicted spillover effects of the minimum wage. We think that our wage posting model provides a good approximation for our problem at hand for two reasons. First, low-skill workers in the U.S. have been shown to be less likely to bargain over wages. (Hall and Krueger, 2012). Given that the average skill level is significantly lower in Brazil, it is reasonable to expect wage bargaining to be relatively rare for most of Brazil's labor force. Second, we show in Section 7.1 that our model predicts minimum wage spillovers in line with our reduced-form estimates, suggesting that an added degree of freedom from integrating a parameter that guides the trade-off between posting and bargaining would marginally improve the model's predictive power for our context.

³⁵Appendix D.2 presents further details of the model's fit to the data. Appendix D.3 contains additional estimation diagnostics. We find that most of the parameters are well identified. The only exception is the intercept in the reservation wage, r_0 ,

Figure 6. Distribution of wages in estimation period, model vs. data



Notes: Log monthly earnings, expressed as a multiple of the current minimum wage, and constructed as the sum of earnings from a given employer over the five year sample period divided by the sum of months worked for that employer over the five year sample period. Sample selection and variable construction criteria for the model are chosen to match those of the data. Source: Model and RAIS, 1994–1998.

5.3 Worker-firm sorting and firm pay

It will be instructive to lay out the mechanics of the estimated model with regard to worker-firm sorting and firm pay. Regarding sorting, panel A of Figure 7 shows that higher-ability workers work at more productive firms, which rationalizes the positive correlation between AKM worker and firm fixed effects we documented in Section 2.2. This is the case even without log-complementarities in the production technology, since we estimate that higher-skill workers are more efficient at climbing the job ladder. However, a binding minimum wage causes the assortative matching to be negative near the bottom of the ability distribution because it renders matches between low-skill workers and low-productivity firms unviable. Figure D.8 in Appendix D.4 provides reduced-form evidence consistent with this prediction.

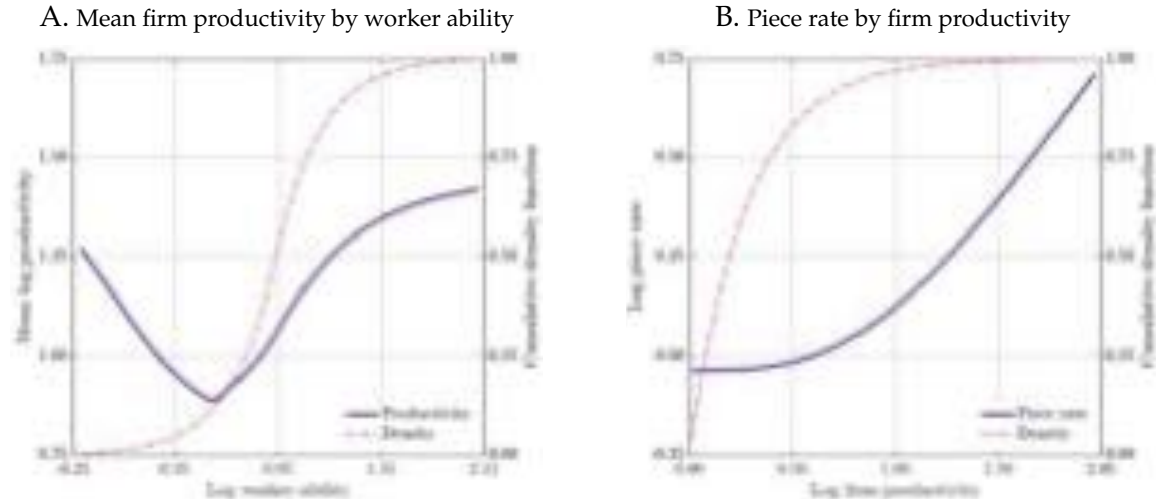
Panel B of Figure 7 shows piece rates across firm productivity levels for a group of workers most affected by the minimum wage—specifically, the first percentile of worker ability. More productive firms pay identical workers more to grow larger. At the same time, pay increases less than one-for-one with productivity. Consequently, higher productivity firms have a lower labor share (Gouin-Bonenfant, 2020).

6 The equilibrium effects of the minimum wage

Having estimated the model to a preperiod from 1994 to 1998, we simulate the equilibrium effects of the observed increase in the effective minimum wage in Brazil between 1996 and 2012. Specifically, we

which Appendix E.4 shows has a negligible impact on the predicted effects of the minimum wage.

Figure 7. Model mechanics



Notes: Panel A shows the average log firm productivity by worker ability, $\int_{\mathbb{Z}} \log z dG(z|a, s)$, in $s(a) > 0$ market by worker ability. Panel B shows log piece rates, $\log w(z|a, s)$, offered by firms to the first percentile of the worker ability distribution in market for $s(a) > 0$ workers. Source: Model.

consider a productivity-adjusted increase in the minimum wage by 44.4 log points, which corresponds to the rise in the productivity growth-adjusted real minimum wage between 1996 and 2012.

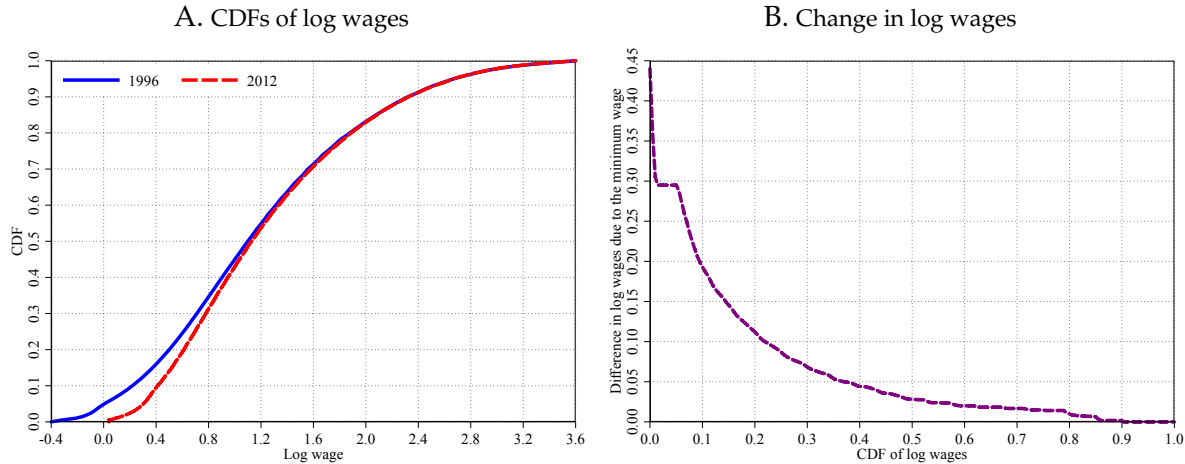
6.1 The impact of the minimum wage on wage inequality

Our main interest lies in the impact of the minimum wage on wage inequality. Figure 8 illustrates the impact of the minimum wage increase on wages throughout the distribution. Panel A shows that the cumulative distribution function (CDF) in 2012 first-order stochastically dominates that in 1996. The right shift of the CDF is particularly evident for the lower half of the wage distribution, reflecting the bottom-driven impact of the minimum wage. Panel B plots the difference in log wages between 1996 and 2012 conditional on the CDF in each year. Naturally, the minimum wage pushes up wages one-for-one at the bottom of the distribution. More surprisingly, it also affects wages strictly above the bottom, up to around the 90th percentile. The wage increase is around 20, 8, 3; less than 2; and close to 0 log points at the 10th, 25th, 50th, 75th, and 90th percentiles, respectively. Thus, while spillover effects of the minimum wage are far reaching, their absolute magnitude is moderate above the median.

Table 5 compares the model-implied effects of the minimum wage on wage inequality with the raw data for two time windows, one centered on 1996 and another on 2012. The rise in the minimum wage accounts for about one-third of the empirical decline in the variance of log wages over this period.³⁶

³⁶Appendix E.1 shows the contribution of the minimum wage toward changes over time in an AKM wage decomposition. Through the lens of the reduced-form AKM wage equation, the minimum wage acts through a combination of compression in

Figure 8. Impact of the minimum wage throughout the wage distribution in the model



Notes: Impact of a 44.4 log point increase in the minimum wage in the estimated model. Panel A shows the CDFs of log wages in 1996 and 2012, conditional on wages at or above the minimum wage. Panel B shows the change in log wages due to the minimum wage conditional on the CDF in each year. Source: Model.

Consistent with the observed data pattern, the minimum wage causes a greater absolute reduction in lower-tail inequality relative to upper-tail inequality. It also accounts for a larger share of the decline in lower-tail inequality measures, varying from 60.1 to 34.4 percent. The minimum wage still has effects on upper-tail inequality, explaining between 11.3 and 9.7 percent of the empirical compression. The reason for this is that spillover effects reach above the median of the wage distribution.

Table 5. Impact of minimum wage on wage inequality, model versus data

	1994–1998		2010–2014		Change		
	Data	Model	Data	Model	Data	Model	Due to MW
Variance	0.704	0.600	0.444	0.514	−0.259	−0.086	33.1%
P50-P5	1.086	1.092	0.611	0.807	−0.475	−0.286	60.1%
P50-P10	0.894	0.874	0.537	.704	−0.358	−0.170	47.4%
P50-P25	0.488	0.484	0.316	0.424	−0.172	−0.059	34.4%
P75-P50	0.614	0.600	0.453	0.582	−0.161	−0.018	11.3%
P90-P50	1.301	1.195	1.045	1.173	−0.256	−0.022	8.4%
P95-P50	1.737	1.532	1.501	1.510	−0.236	−0.023	9.7%

Notes: Table shows estimated impact of a 44.4 log point increase in the minimum wage in the model as well as the raw data. Percentile ratios of log wages are constructed as the sum of wages from a given employer over the five year sample period divided by the sum of months worked for that employer over each five year period. Model and data sample selection and variable construction are identical. See text for details. Source: Model and RAIS, 1994–1998 and 2010–2014.

person fixed effects, compression in firm fixed effects, and a declining covariance between the two.

6.2 Understanding the distributional effects of the minimum wage

To understand the impact of the minimum wage on wage inequality, we write the variance of log wages as the sum of between- and within-worker components:

$$\begin{aligned} \text{Var}(w) &= \int_{a,s} \int_z \left(w(z|a,s) - \bar{w} \right)^2 dG(z|a,s) \frac{e(a,s)}{E} d\Omega(a,s) \\ &= \underbrace{\int_{a,s} \left(\bar{w}(a,s) - \bar{w} \right)^2 \frac{e(a,s)}{E} d\Omega(a,s)}_{\text{between-worker component}} + \underbrace{\int_{a,s} \int_z \left(w(z|a,s) - \bar{w}(a,s) \right)^2 dG(z|a,s) \frac{e(a,s)}{E} d\Omega(a,s)}_{\text{within-worker component}}, \end{aligned} \quad (10)$$

where $\Omega(a,s)$ is the joint distribution over worker ability a and search efficiency s , $E = \int_{a,s} e(a,s) d\Omega(a,s)$ is aggregate employment, $\bar{w} = \int_{a,s} \int_z w(z|a,s) dG(z|a,s) (e(a,s)/E) d\Omega(a,s)$ is the population mean log wage, and $\bar{w}(a,s) = \int_z w(z|a,s) dG(z|a,s)$ is the mean log wage of type- (a,s) workers. The between-worker component captures average differences across worker types, while the within-worker component reflects wage differences among workers of the same type due to employer heterogeneity.

Building on the decomposition in equation (10), we consider two counterfactual experiments. First, fixing the initial allocation of workers, $e(a,s)$ and $g(z|a,s)$, we let firms' wage policies given by $w(z|a,s)$ adjust in response to the minimum wage. We label this the "rent channel" because it captures redistribution of rents from firms to workers. Second, fixing firms' wage policies, $w(z|a,s)$, we let the allocation of workers given by $e(a,s)$ and $g(z|a,s)$ adjust to the higher minimum wage. We call this the "reallocation channel" because it reflects changes in the wage distribution due to worker reallocation across firms.

Table 6 presents the results from these counterfactual exercises.³⁷ We find that 61 percent of the overall variance of log wages is between worker types, while 39 percent is within worker types across firms. The higher minimum wages cause both the between- and the within-worker components to decline, making up 87 and 13 percent, respectively, of the overall decline. The rent channel—firms raising pay for identical workers—is the most important factor behind compression in both the within- and the between-worker components. The reallocation channel also matters for the compression in the between-worker component but less so for the compression in the within-worker component.

To shed further light on the rent and reallocation channels of the minimum wage, we zoom in on a group of workers most affected by the minimum wage—specifically, the first percentile of worker ability. Figure 9 plots changes in firms' piece rate offers and vacancies against log firm productivity. Panel A shows that the minimum wage causes all firms to raise pay. Because low-ability workers' pay rises

³⁷The numbers presented here are based on the analytical solution for wages in the model, while Table 5 uses simulated data.

Table 6. Decomposition of effect of minimum wage on wages, model

	1994–1998	2010–2014	Change
Total variance	0.608	0.517	−0.090
Rent channel		0.530	−0.077
Reallocation channel		0.578	−0.030
Between variance	0.369	0.291	−0.078
Rent channel		0.309	−0.060
Reallocation channel		0.337	−0.031
Within variance	0.239	0.227	−0.012
Rent channel		0.224	−0.015
Reallocation channel		0.240	0.001

Notes: Table shows estimated impact of a 44.4 log point increase in the minimum wage. Decomposition of log wages is based on (10) using exact (nonsimulated) model wages (i.e., without measurement error κ and not aggregated to the annual level following our empirical approach). The rent channel is the counterfactual impact of letting the wage policies $w(z|a, s)$ adjust while holding fixed the allocation of workers $\{g(z|a, s), e(a, s)\}$. The reallocation channel is the counterfactual impact of letting the allocation of workers $\{g(z|a, s), e(a, s)\}$ adjust while holding fixed wage policies $w(z|a, s)$. Source: Model.

across the board, between-worker inequality falls. Moreover, low-productivity firms raise pay by more than high-productivity firms, consistent with the empirical decline in pass-through from firm productivity to pay over this period (Alvarez et al., 2018). Therefore, the minimum wage also reduces the within-component of wage inequality. Panel B shows that low-productivity firms cut vacancy creation, as their profit margins are squeezed. In contrast, the most productive firms actually increase their recruiting intensity, for reasons to be discussed soon. Consequently, employment reallocates toward more productive, higher-paying firms. Since this leads to less positive assortative matching between workers and firms in lower-skill markets, between-worker inequality also falls.³⁸

6.3 Aggregate effects of the minimum wage

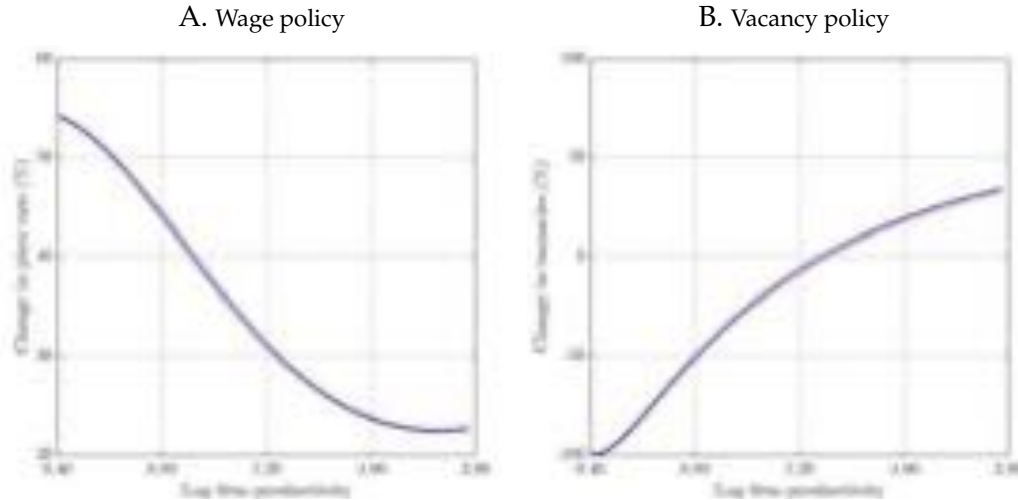
Having understood the changes in rent sharing and worker reallocation at the micro level, we now turn to the aggregate consequences of the minimum wage. Table 7 shows that the aggregate employment rate falls, consistent with the intuition that a higher wage floor discourages job creation. However, the fall is a modest 0.4 percent.³⁹ At the same time, aggregate output and labor productivity increase by 0.7 and 1.7 percent, respectively. Net output increases by a larger amount because of lower aggregate recruiting costs. The total wage bill increases by 1.0 percent.⁴⁰ Profits decrease by less than 0.1 percent. As a result,

³⁸We provide empirical support for this model prediction in Appendix E.

³⁹This number masks significant heterogeneity. Among the lowest-skill workers, employment falls by over 13 percent, while employment is essentially unaffected for workers above the bottom third of the ability distribution. See Appendix E.4 for details.

⁴⁰As shown in Figure 8B, wages increase by much more at the bottom. However, the aggregate wage bill is dominated by the top of the distribution, where wages change by little.

Figure 9. Changes in firms' wage and vacancy policy in low-ability market, model



Notes: Impact of a 44.4 log point (i.e. 55.9 percent) increase in the minimum wage in the estimated model among workers with positive search efficiency, $s(a) > 0$, in the first percentile of the worker ability distribution. Panel A shows the percentage change in firms' wage policy, $w(z|a, s)$, by unweighted (i.e., not employment weighted) productivity, z . Panel B shows the percentage change in firms' vacancy policy, $w(z|a, s)$, by unweighted (i.e., not employment weighted) productivity, z . Source: Model.

the labor share increases by a modest 0.2 percent.

To summarize, labor reallocation across firms mediates the effects of the minimum wage in three ways. First, it buffers the disemployment effects. Second, it increases employment-weighted productivity and output. Third, it shifts workers to firms with higher profits and lower labor shares. As a consequence, the aggregate effects of the minimum wage are relatively muted. These rich predictions regarding worker reallocation depend critically on our model incorporating firms and would be missed by a one-worker-per-firm matching model of the labor market.

Table 7. Impact of minimum wage on aggregate outcomes, model

	1994–1998	2010–2014	Due to MW
Employment rate	0.549	0.545	−0.004
Aggregate output	1.747	1.753	0.007
Labor productivity	2.379	2.396	0.017
Aggregate cost of recruiting	0.258	0.263	0.005
Aggregate output minus recruiting costs	1.491	1.498	0.007
Log wage bill	1.082	1.093	0.010
Total profits	0.399	0.399	−0.000
Labor share	0.515	0.517	0.002

Notes: Table shows estimated impact of a 44.4 log point increase in the minimum wage on aggregate outcomes in the simulated economy. Employment rate is $E = \int e(a, s) d\Omega(a, s)$. Aggregate output is $\log Y = \log (\int a z dG(z|a, s) e(a, s) d\Omega(a, s))$. Log labor productivity is $\log(Y/E)$. Log aggregate recruiting cost is $\log C = \log \left(M \int a c(a, s) \frac{v(z|a, s)^{1+\eta}}{1+\eta} d\Gamma(z) da ds \right)$. Log aggregate output minus recruiting costs is $\log(Y - C)$. Log wage bill is $\log W = \log (\int a w(z|a, s) dG(z|a, s) e(a, s) d\Omega(a, s))$. Log profits, $\log(Y - W - C)$. Labor share is W/Y Source: Model.

6.4 Understanding the employment effects of the minimum wage

To understand the muted employment effects of the minimum wage, we now dissect both firms' recruiting response and the aggregate employment response in our model.

Firms' recruiting response. Log-differentiating the first-order condition for optimal vacancy creation in market (a, s) by a firm of productivity z with respect to the minimum wage, we get

$$\underbrace{\frac{d \log v(z|a, s)}{d \log w^{\min}}}_{\text{firms' recruiting response}} = \underbrace{\frac{1}{\eta} \frac{d \log (z - w(z|a, s))}{d \log w^{\min}}}_{\text{profit channel}} + \underbrace{\frac{1}{\eta} \frac{d \log \left(q(a, s) \left(\frac{u(a, s)}{S(a, s)} + \frac{se(a, s)}{S(a, s)} G(z|a, s) \right) \right)}{d \log w^{\min}}}_{\text{fill channel}} \quad (11)$$

$$+ \underbrace{-\frac{1}{\eta} \frac{d \log (\delta(a, s) + sp(a, s)(1 - F(z|a, s)))}{d \log w^{\min}}}_{\text{retention channel}}.$$

Because our estimated curvature of the vacancy cost function is rather low with $\eta \approx 0.5$ and optimal vacancies scale with $1/\eta$, firms' recruiting response to the minimum wage is relatively elastic. In spite of this, we find a quantitatively small response of firm-level employment to the minimum wage due to three offsetting channels in equation (11). The first is the profit channel, which captures changes in pay at firms with constant productivity, which affect profits. The second is the fill channel, which captures changes in the fill rate of jobs due to inter-firm competition. The fill rate depends on the rate $q(a, s) = (V(a, s)/S(a, s))^{\alpha-1}$ at which a vacancy contacts a worker, the unemployed share $u(a, s)/S(a, s)$, and the employed share's earnings distribution $G(z|a, s)$. The third is the retention channel, which captures changes in match duration due to changes in the rate of poaching by other firms, $sp(a, s)(1 - F(z|a, s))$.

Panel A of Figure 10 shows the results across productivity levels from decomposing firms' recruiting response to the minimum wage based on equation (11). To illustrate the forces at work, we focus again on a group of workers most affected by the minimum wage—specifically, the first percentile of worker ability. The profit channel reduces vacancy creation for low-productivity firms with smaller profit margins to begin with. The fill rate channel is positive throughout, U-shaped, and varies less across productivity levels. Finally, the retention channel varies in sign, follows an inverse-U shape, and is not far from zero throughout. Summing over all three channels, we find that firms' recruiting response to the minimum wage is increasing and concave, negative at the bottom, and positive at higher productivity levels.

Aggregate employment response. The aggregate employment response in market (a, s) is identically

$$\underbrace{\frac{d \log e(a, s)}{d \log w^{\min}}}_{\text{aggregate employment response}} = \underbrace{\frac{d \log e(a, s)}{d \log p(a, s)}}_{\text{job finding channel}} \times \underbrace{\frac{d \log p(a, s)}{d \log V(a, s)}}_{\text{congestion channel}} \times \underbrace{\frac{d \log V(a, s)}{d \log w^{\min}}}_{\text{vacancy channel}}. \quad (12)$$

This allows us to decompose the aggregate employment response to the minimum wage into three terms. First, the job finding channel captures the impact of the job finding rate, $p(a, s)$, on employment, $e(a, s)$. Under the simplifying assumption that the probability of a type transition upon job loss is small,

$$\frac{d \log e(a, s)}{d \log p(a, s)} = \frac{\delta(a, s)}{\delta(a, s) + p(a, s)} \approx 0.6,$$

assuming approximations of $\pi \approx 0.00$, $\delta(a, s) \approx 0.07$, and $p(a, s) \approx 0.04$. Second, the congestion channel captures the impact of aggregate vacancies, $V(a, s)$, on the job finding rate, $p(a, s)$. To simplify,

$$\frac{d \log p(a, s)}{d \log V(a, s)} = \alpha \frac{1}{\left(1 - \alpha \frac{(1-s(a))\delta(a, s)p(a, s)}{(\delta(a, s) + s(a)p(a, s))(\delta(a, s) + p(a, s))}\right)} \approx \alpha = 0.5,$$

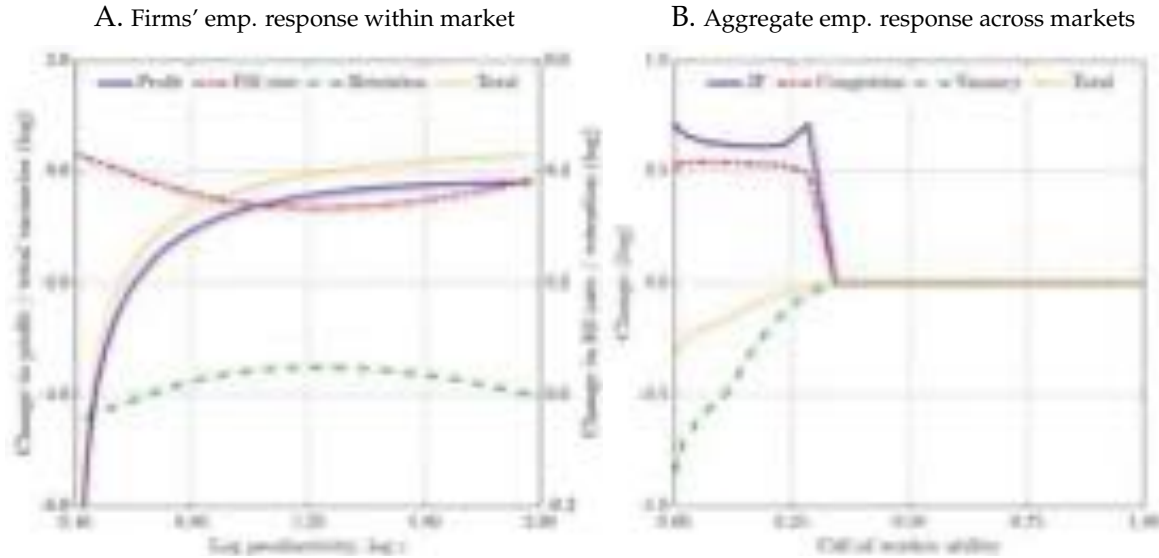
assuming an approximation of $s(a) \approx 1.0$ and using $\alpha = 0.5$. Third, the vacancy channel captures the impact of the minimum wage w^{\min} on aggregate vacancies, $V(a, s)$. This channel simply equals the integral over firms' recruiting responses to the minimum wage corresponding to equation (11) above.

Panel B of Figure 10 shows the results across ability ranks from decomposing the aggregate employment response to the minimum wage based on equation (12). Only workers in the bottom third of the ability distribution are affected. The job finding and congestion channels are roughly constant and positive. The vacancy channel is negative and increases from around -0.8 to 0.0 . Combining the channels yields an aggregate employment response that ranges from around -0.3 to 0.0 log points.

6.5 Robustness of the small employment response

In light of our finding of modest aggregate disemployment effects of the minimum wage, we now investigate the robustness of our conclusions. In order for us to find a larger aggregate employment response, one or more of the channels in our decomposition based on equation (12) would need to be larger in magnitude. The channels' magnitude in turn relates to a set of estimated parameters and equilibrium objects that includes the job finding rate, $p(a, s)$; the separation rate $\delta(a, s)$; the on-the-job search efficiency, $s(a)$; the elasticity of the vacancy cost function, η ; and the elasticity of the matching function, α .

Figure 10. Decomposing the effect on employment



Notes: Panel A shows a decomposition of firms' recruiting response to a 44.4 log point increase in the minimum wage based on equation (11) for the market with $(a = a, s > 0)$. Panel B shows a decomposition of the aggregate employment response across ability markets based on equation (12) for $s > 0$. Both panels show log changes in each component (e.g., 0.2 \approx 20 percent). JF stands for job finding. Source: Model.

We estimate a job finding rate, $p(a, s)$, and a separation rate $\delta(a, s)$ for Brazil that are relatively low compared with common values for the U.S.. All else equal, this would lead to a smaller job finding channel in the U.S. than what we find in Brazil. We estimate an on-the-job search efficiency, $s(a)$, that is comparatively higher than the ones in the U.S.. To see that this high value has limited effects on our results, consider the opposite extreme, with $s(a) \approx 0$. Then, the congestion channel approximately equals $\alpha / (1 - 0.4\alpha) = 0.6$. Hence, even in this extreme case, the congestion channel is a significant moderation force.

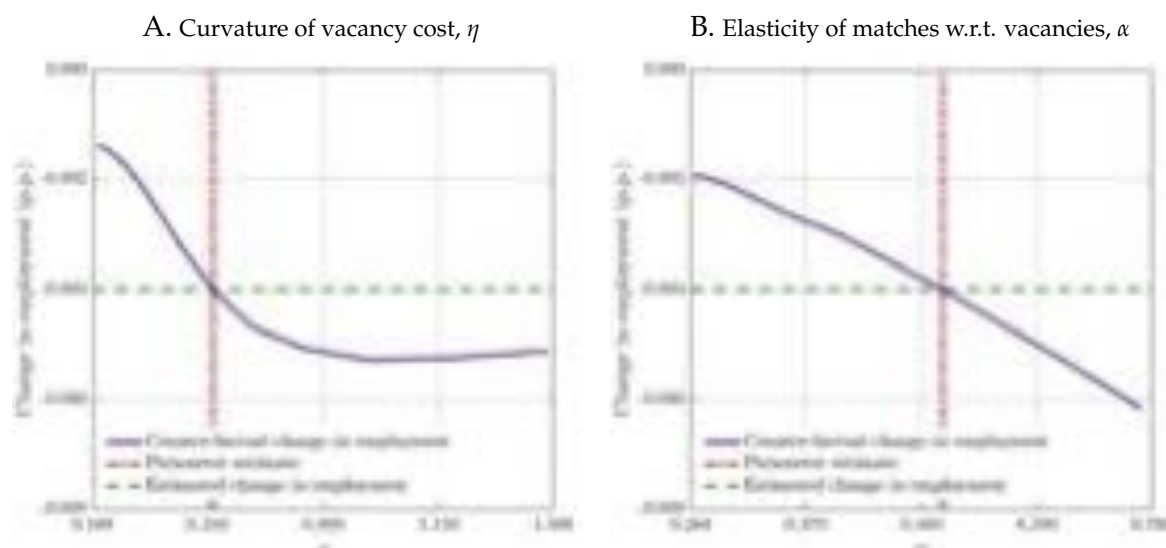
The sensitivity of the aggregate employment response with respect to the elasticity of the vacancy cost function, η , and the elasticity of the matching function, α , are shown in Figure 11.⁴¹ Panel A shows that, perhaps surprisingly, a higher value of η actually amplifies the disemployment effects of the minimum wage. This is due to the equilibrium reallocation effects. Under a higher value of η , the minimum wage leads to smaller employment cuts among unproductive firms but also reduces the scaling up of more productive firms. Quantitatively, the latter force outweighs the former.

Panel B shows that higher values of λ are associated with greater disemployment effects. But our preset value of $\alpha = 0.50$ is high compared to other values in the literature. For example, Meghir et al. (2015) estimate $\alpha = 0.34$ using the Brazil's PME data, Shimer (2005) estimates $\alpha = 0.28$ for the U.S., and Mortensen and Nagypal (2007) argue that a reasonable value is $\alpha = 0.40$. Therefore, our results likely

⁴¹Appendix E.4 presents the same sensitivity analysis with respect to the other model parameters.

represent an upper bound on the aggregate disemployment effects of the minimum wage.

Figure 11. Sensitivity of the employment effect of the minimum wage to parameter values



Notes: Estimated impact of a 44.4 log point increase in the minimum wage on aggregate employment as the parameter in question varies between 25percent and 300 percent of its estimated value, holding all other parameters fixed at their estimated values. Source: Model.

6.6 When are the effects of the minimum wage on wage inequality large?

Maybe our most striking finding is the large inequality reduction due to the minimum wage. This result is so striking because previous work on the distributional effects of the minimum wage has found smaller effects in the U.S. (Lee, 1999; Autor et al., 2016; Fortin et al., 2021), Canada (Fortin and Lemieux, 2015; Brochu et al., 2018), and the U.K (Butcher et al., 2012). To reconcile these differences, we explore the sensitivity of our results with respect to four model parameters—the mean of worker ability, μ ; the tail index of the productivity distribution, ζ ; the separation rate intercept, δ_0 ; and the job finding rate, λ .⁴²

Panel A of Figure 12 shows that a higher mean worker ability, μ , significantly reduces the distributional effects of the minimum wage. Higher values of μ imply that the minimum wage is less binding initially, so the marginal effect of an increase in the minimum wage is smaller. In Appendix B.11, we show that the bindingness of the minimum wage, measured by the P10-P50 log wage percentile ratio, is up to 26 log points higher in Brazil than in the U.S. Counterfactually reducing μ by 26 log points to mimic the U.S. moment indicates that the effects on the variance of log wages are around 50 percent higher in Brazil than in the U.S. due to the relatively greater initial bindingness of the minimum wage in

⁴²Appendix E.4 shows the same comparative statics results with respect to other model parameters. Naturally, our analysis comes with the caveat that, for these experiments, we are considering a movement in only one parameter, while holding all other parameters fixed at their estimated values.

Brazil.⁴³

Panel B shows that the a weaker inequality-reducing effect of the minimum wage for higher values of the productivity tail parameter ζ . While our estimate of $\zeta = 3.5$ corresponds to a variance of AKM firm fixed effects of 19.5 log points, Song et al. (2019) report that variance to be 6.7 log points in the U.S. from 1994 to 2000. For our model to replicate the U.S. moment would require ζ to be equal to 5.8 (see panel B of Appendix Figure D.6), which would imply that the effects on the variance of log wages are around 18 percent higher in Brazil than in the U.S. due to the relatively greater productivity dispersion in Brazil.

Panel C shows that a higher job finding rate λ amplifies the effects of the minimum wage on inequality, although the gradient is flatter than in the previous two cases. By comparison, the inequality reduction due to the minimum wage is relatively invariant to the separation rate intercept δ_0 (panel D).

Besides our parameter estimates discussed above, other reasons for why we find relatively large effects of the minimum wage on inequality in Brazil may include the nature of wage setting. Our model assumes that all wages are posted, which is consistent with existing evidence that lower-skill jobs are more likely to post—rather than bargain over—wages (Hall and Krueger, 2012). In related work, Flinn and Mullins (2018) show that spillover effects of the minimum wage can be smaller in an economy in which wages are sometimes bargained over, which is likely more so the case in the U.S. than in Brazil.

7 Confronting the model with cross-sectional evidence

We now confront the predictions of our structural model with cross-sectional evidence on the reach of spillover effects and the inequality-reducing effects of the minimum wage.

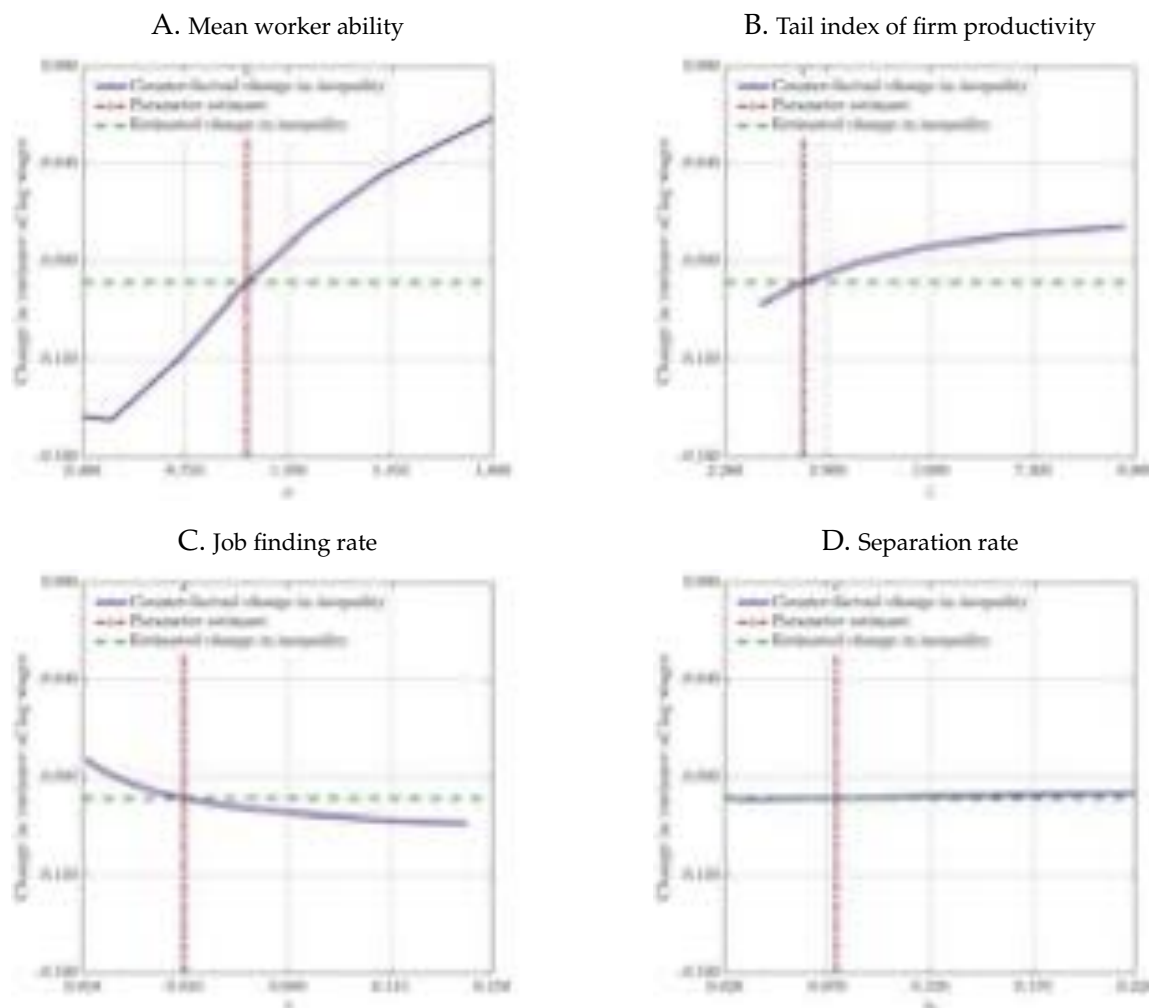
7.1 Spillover effects of the minimum wage in the model vs. in the data

To compare the model-predicted effects of the minimum wage on inequality with estimates from our reduced-form approach following Lee (1999) and Autor et al. (2016), we simulate “state-year level” data from our estimated model by varying only the level of the minimum wage relative to mean worker ability in order to replicate the empirical distribution of the Kaitz index across Brazil’s 27 states over time.⁴⁴ We then run the same regression (2) on our model simulations that we ran on the data in Section 3.2.

⁴³Note that because of spillover effects of the minimum wage, μ would need to change by even more than 26 log points in order to change the P10-P50 log wage percentile ratio by 26 log points.

⁴⁴We treat each model state as its own isolated economy, with no worker or firm mobility between them. An interesting avenue for future work would be to incorporate into our model a richer spatial structure, like the one in Zhang (2018).

Figure 12. Minimum wage effects on wage inequality across selected model parameters



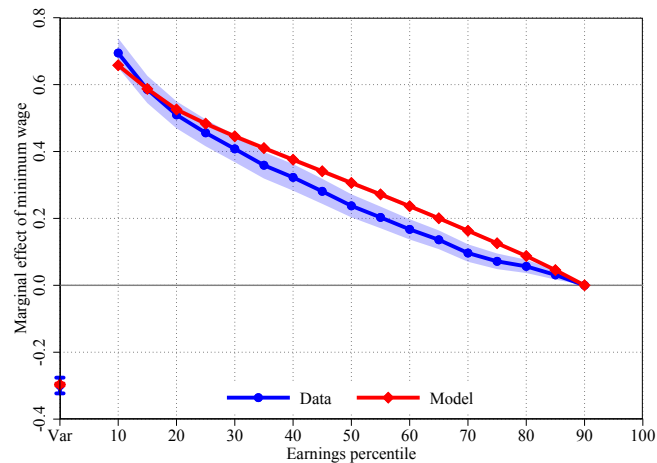
Notes: Estimated impact of a 44.4 log point increase in the minimum wage across different parameter values, varying one parameter at a time and holding fixed all other parameters at their estimated values. Source: Model.

Figure 13 shows that the estimated effects of the minimum wage in the model and in the data broadly match. The specification shown is our preferred one using an IV strategy, state fixed effects, state-specific time trends, and $p = 90$ as the base wage.⁴⁵ The point estimates in the model fall within the 99 percent confidence interval of the empirical estimates for roughly the lower third of the wage distribution. Between the 35th and the 80th percentile, the model predicts slightly larger effects than the data, although both follow a similar trend. Above the 80th percentile, the model estimates again coincide with the data. As a result, the model-predicted effect on the variance of log wages falls in the middle of the already narrow data confidence interval. It is worth noting that these two sets of estimates were derived under

⁴⁵ Appendix F.1 compares the model and the data under the four specifications from Section 3.2 and two additional difference specifications.

very different assumptions and informed by different moments of the data.

Figure 13. Estimated minimum wage effects throughout the wage distribution, model versus data



Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2) estimated across Brazil's 27 states. Results from two separate estimates are shown; those based on the RAIS data (blue line and error bar or shaded area) and those based on model-simulated data (red line and error bar or shaded area). Both sets of estimates use a specification that includes state fixed effects in addition to state-specific quadratic time trends, which is estimated using an IV strategy. The IV strategy instruments the generalized p -Kaitz index and its square using an instrument set that consists of the log real statutory minimum wage, its square, and the log real statutory minimum wage interacted with the mean of the log real median wage for the state over the full sample period. The estimated marginal effect of the minimum wage on the variance of log earnings ("Var" on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution ("10" to "90" on the x-axis) relative to a base wage $p = 90$ are shown. The two error bars and two shaded areas represent 99 percent confidence intervals based on regular standard errors. Source: RAIS, 1985–2014, and model.

7.2 Effects of the minimum wage on inequality in the model vs. in the data

To compare the model predictions regarding the determinants of the distributional effects of the minimum wage with the data, we study heterogeneity across simulations in our model and Brazilian states in RAIS. We show comparative statics with respect to the four structural parameters considered in Section 6.6: mean worker ability, μ ; the productivity shape parameter, ζ ; the job finding rate, λ ; and the separation rate intercept, δ_0 . We compute four reduced-form moments across states in 1994–1998 corresponding to these four structural parameters: the median of log wages, the variance of AKM firm fixed effects, the probability of starting a formal job, and the probability of leaving a formal job. For each state, we also compute the long difference in the variance of log wages between 1996 and 2012.

Figure 14 compares the results for these four metrics for worker-year weighted states in the data and those for equally weighted "states" in the model.⁴⁶ First, the strongest model prediction is that a higher mean of worker ability, μ , leads to a smaller reduction in wage inequality. Panel A confirms this intu-

⁴⁶Appendix F.2 shows the same relationships with equal weights both in the data and in the model. Appendix F.3 shows analogous results for the comparative statics of the employment effects of the minimum wage in the data and in the model.

ition by showing that the empirical decline in wage inequality was smaller in states with initially higher median log wages. Second, according to the model, lower values of the productivity tail parameter, ζ , are associated with a greater reduction in inequality. Panel B shows that, qualitatively, inequality declines by more in regions with higher dispersion in AKM firm fixed effects in the data as in the model. Quantitatively, however, the relationship is stronger in the data. Third, the model predicted that a high job finding rate, λ , means that the minimum wage has a greater negative impact on inequality. In the data, we indeed find a negative relationship between initial EN transition rates and subsequent inequality changes, as shown in panel C; this is also the case in the model. Again, the relationship is stronger in the data than in the model. Fourth, the model predicted a comparably flat relationship between the job separation rate, δ , and the decline in wage inequality. Panel D paints a mixed picture of the relationship between EN transition rates and changes in inequality, with a negative estimated slope but lots of idiosyncratic dispersion in the data.⁴⁷

While key model predictions are consistent with the reduced-form evidence, our model misses other important features of the data. One reason for this might be that starting in the early 2000s, Brazil has seen a period of rapid formalization, which our model is silent on (Haanwinckel and Soares, 2020; Engbom et al., 2021). Another reason is that over this period, educational attainment in Brazil rose rapidly for reasons entirely outside of our model (Binelli et al., 2008; Mak and Siow, 2017). The country also underwent rapid trade liberalization for unmodeled reasons (Dix-Carneiro, 2014; Dix-Carneiro and Kovak, 2017, 2019; Dix-Carneiro et al., 2018). Finally, the country has experienced rapid technological change and structural transformation that has also affected the organization of labor (Bustos et al., 2016). For these reasons, our model estimates should be interpreted as corresponding to a counterfactual in which none of these other changes happened concurrently.⁴⁸ Nevertheless, we believe the exercises presented above are useful because they demonstrate empirical support for key mechanisms of our model while also pointing us in the direction of areas for further investigation.

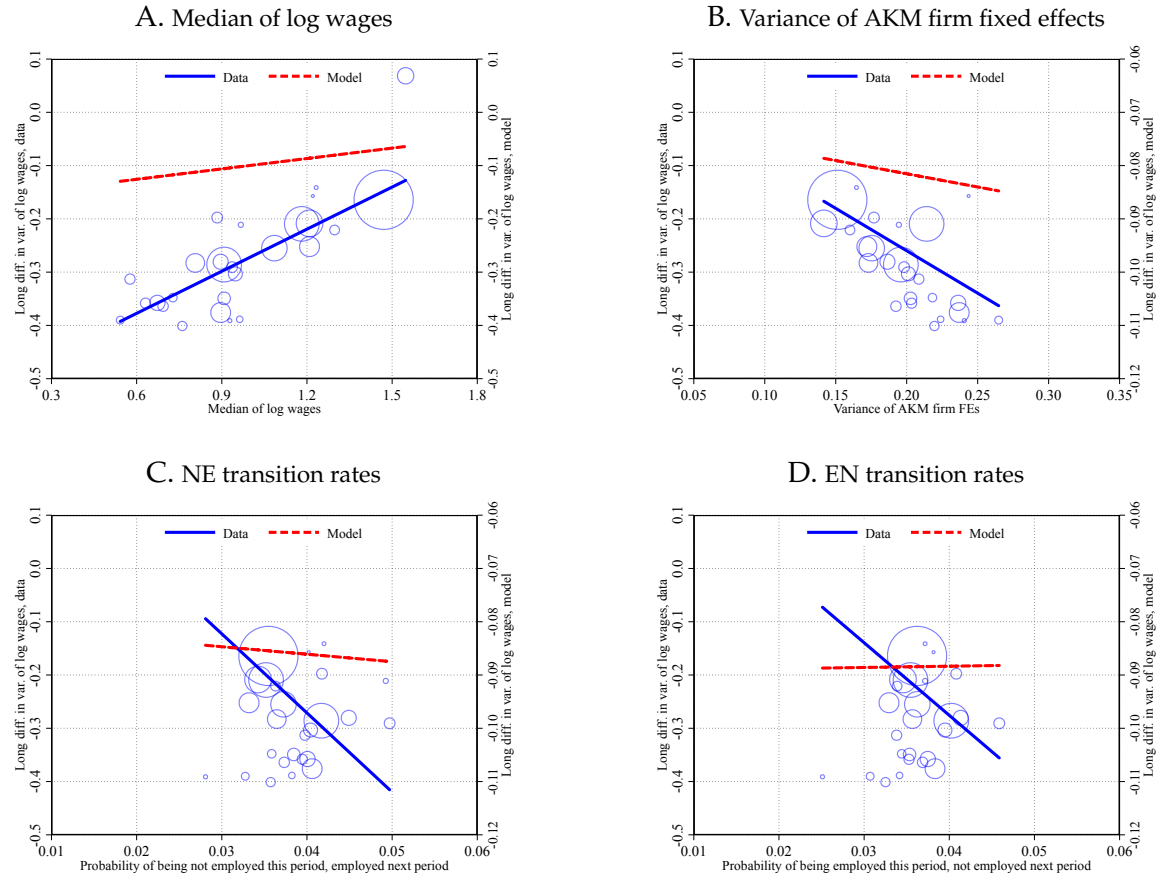
8 Conclusion

There remains a great debate over the potential for labor market institutions to affect wage inequality. To shed new light on this debate, in this paper, we study a large increase in the minimum wage in Brazil, using rich administrative and household survey data together with an equilibrium model. Both our

⁴⁷One reason why the empirical relationship between EN (or NE) rates and the change in inequality might be less informative is that we may misclassify true EE transitions in the data if they end (or start) in the informal sector.

⁴⁸Haanwinckel (2020) studies the effects of the Brazil minimum wage in a model that incorporates some of these additional features and reaches conclusions consistent with ours.

Figure 14. Changes in inequality across observable labor market characteristics, model vs. data



Notes: Figure plots long differences between 1996 and 2012 in wage inequality—measured by the variance of log wages—across states in the data (blue solid line) and across simulations in the model (red dashed line). Variables on the x-axis are the median of log wages (panel A), the variance of AKM firm fixed effects (panel B), the probability of a transition from nonemployment into employment (panel C), and the probability of a transition from employment into nonemployment (panel D). The median of log wages is computed in 1996, while transition probabilities are computed in a balanced monthly panel from 1994 to 1998. States are represented as hollow circles, with their area proportional to their sample population in 1996. Linear best fit line is shown for the data (blue solid line) and the model (red dashed line), using OLS regression of inequality measures on the respective variable on the x-axis. Data is weighted by number of worker-years in each state, while the model is equally weighted across states. Source: RAIS, 1994–1998 and 2012.

reduced-form analysis, based on variation in the bindingness of the minimum wage across Brazilian states, and our estimated structural model indicate significant scope for the minimum wage to compress the distribution of wages, while having only modest disemployment effects. Through the lens of our equilibrium model, and consistent with our reduced-form findings, these results are due to far-reaching spillover effects of the minimum wage and worker reallocation across heterogeneous firms.

Our study points to several fruitful avenues for future research. First, while our structural model incorporates a rather simple view of informality, it would be interesting to quantify spillovers of the minimum wage in Brazil's formal sector to jobs in the informal sector, which is not directly constrained by the policy—Neri and Moura (2006) refer to these as the "lighthouse effect". Second, given our findings on the

prominent role played by firms in the labor market, it is worth revisiting the effects of other labor market policies and institutions—including unions, unemployment benefits, and noncompete agreements—on the distribution of pay and employment in other settings. While such labor market institutions and policies may directly affect only a small share of workers, they may lead to sizable equilibrium effects of the kind we find in Brazil. Finally, our work stops short of an analysis of optimal minimum wage policies in a frictional environment, though our results will be an important ingredient for any such venture.

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Online Appendix—Not for Publication

The overview of the appendices is as follows. Appendix [A](#) gives further details on the datasets introduced in Section [2](#). Appendix [B](#) presents additional empirical results building on Section [3](#). Appendix [C](#) adds to the description of our equilibrium model from Section [4](#). Appendix [D](#) provides more information on the model estimation routine and results extending what is presented in Section [5](#). Appendix [E](#) shows further results related to the simulated impact of the minimum wage that we consider in Section [6](#). Finally, Appendix [F](#) contains the results of additional model validation exercises complementing the materials in Section [7](#).

A Data Appendix

This appendix provides further details on the data sets introduced in Section 2, including subsections on further data description (Appendix A.1), additional summary statistics (Appendix A.2), a comparison of official labor force statistics and sample sizes in the RAIS administrative data (Appendix A.3), an exploration of explained wage dispersion when (not) controlling for observable and unobservable heterogeneity (Appendix A.4), and additional AKM wage decomposition results (Appendix A.5).

A.1 Dataset description

Administrative linked employer-employee data (RAIS). Our main data source is the *Relação Anual de Informações Sociais* (RAIS), a linked employer-employee register by the Brazilian Ministry of the Economy (*Ministério da Economia*). We use the RAIS microdata with person and firm identifiers covering the period 1985–2014. These data are available to us under a confidentiality agreement with the Brazilian ministry.

Firms' survey responses are mandatory, and misreporting is deterred through audits and threat of fines. The earliest available data go back to 1985, with coverage becoming near universal from 1994 onward. The data contain detailed information on job characteristics, with 73 million formal sector employment spells recorded in 2012. Although reports are annual, we observe for every job spell the date of accession and separation in addition to average monthly earnings. We keep for each worker the highest paid among each year's longest employment spells. Because Brazil's minimum wage is set in terms of monthly earnings, we henceforth refer to this income concept interchangeably as "earnings" or "wages."

The main text presents results of both plug-in and leave-one-out bias corrected variance components of log wages, following the methodology and code by KSS. For our main analysis, we restrict attention to the largest leave-one-out connected set but do not impose any additional restrictions on earnings, firm size, or the minimum number of switchers across firms. For a set of additional estimation results of the AKM wage equation (1), we keep workers with earnings not equal to or weakly above the minimum wage. For some additional results, following the argument in Andrews et al. (2008) and subsequent work, we also restrict attention to firms with a minimum of 10 employees in a year on average or a minimum number of workers switching into and out of each firm in order to mitigate concerns about limited-mobility bias in our estimates.

We devise our own cleaning procedure for these data, starting with the raw text files. We have benefitted from guidance by the data team at IPEA. Our cleaning procedure consists of three stages. The first stage reads in and standardizes the format of the raw data files that were transmitted to us at the region-year level, saving a set of compatible region-year files. The second stage reads in all region files within a year and applies a set of cleaning and recoding procedures to the data to make them consistent within each year, saving a set of yearly files. The third stage reads in all yearly files and applies a set of cleaning procedures to the data to make them consistent across years. Whenever possible, we use the official crosswalks provided by IBGE to convert industry (IBGE, CNAE 1.0, and CNAE 2.0 classifications), occupation (CBO 1994 and CBO 2002), and municipality codes (IBGE classification).

Cross-sectional household survey data (PNAD). A substantial fraction of Brazil's working-age population is not formally employed and hence not covered by the RAIS. To address this gap, we complement our analysis using data from the *Pesquisa Nacional por Amostra de Domicílios* (PNAD), a nationally representative annual household survey. Respondents are asked to produce a formal work permit (*Carteira de Trabalho e Previdência Social assinada*). Following Meghir et al. (2015), we classify as "informal" all self-employed individuals and those in remunerated employment without a work permit.

The PNAD data collection consists of a double-stratified sampling scheme by region and municipality. The survey interviews representatives from households in Brazil. The it asks the household head to respond on behalf of all family members and answer a rich set of demographic and employment-related questions. More specifically, the survey asks a question about whether the respondent holds a legal work permit. We use the answer to this survey question to identify whether individuals work in the formal or in the informal sector. Survey questions regarding income and demographics of the respondent household members are comparable to the U.S. CPS. We keep only observations that satisfy our selection criteria and have non-missing observations for labor income, whose variable definition we harmonize across years.

The raw microdata starting from 1996 are publicly available for download at ftp://ftp.ibge.gov.br/Trabalho_e_Rendimento/. For basic cleaning, starting with the raw data in text format, we use the standardized cleaning procedures adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at <http://www.econ.puc-rio.br/datazoom/english/index.html>. From there, we apply a set of procedures to clean and recode key variables used in our analysis.

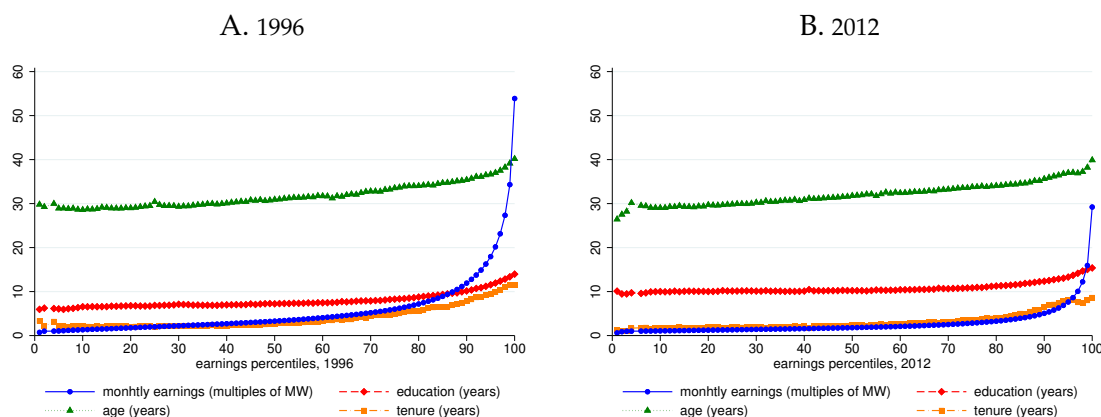
Longitudinal household survey data (PME). We also use a second household survey, the *Pesquisa Mensal de Emprego (PME)*, conducted in Brazil's six largest metropolitan regions: Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, Salvador, and São Paulo. The advantage of this data set is that for every respondent, it features two continuous four-month interview spells separated by an eight-month pause. Starting in 2002, this short panel component allows us to compute workers' monthly transition rates between different employment states, including formal and informal employment. For presentation purposes, we label formal sector workers as "employed" and pool informal sector workers and the unemployed under the label "nonemployed." We distinguish between the disaggregated categories in our empirical analysis of minimum wage effects later.

The raw microdata starting from March 2002 are publicly available for download at ftp://ftp.ibge.gov.br/Trabalho_e_Rendimento/. For basic cleaning, starting with the raw data in text format, we use the standardized cleaning procedures adopted from the Data Zoom suite developed at PUC-Rio and available online for replication at <http://www.econ.puc-rio.br/datazoom/english/index.html>. From there, we apply a set of procedures to clean and recode key variables used in our analysis, similar to the procedures that we applied to the PNAD data.

A.2 Summary statistics

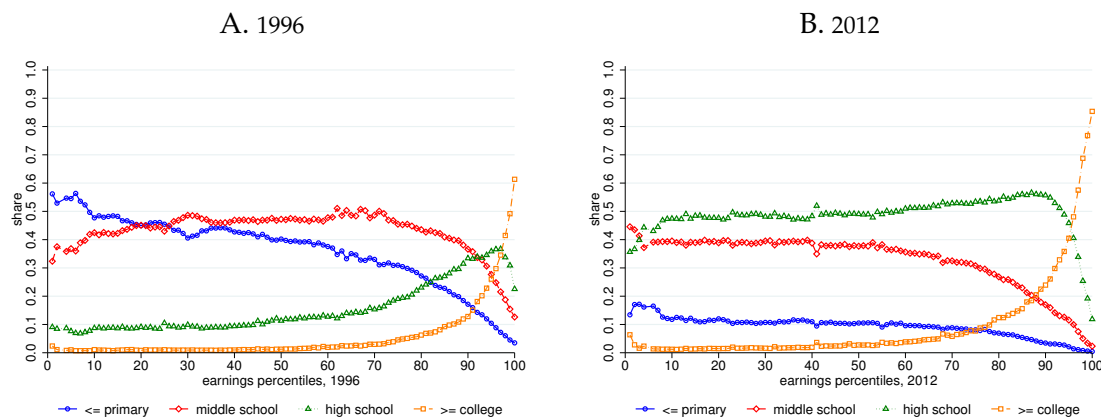
Summary statistics for administrative linked employer-employee data (RAIS). Figure A.1 shows mean values of basic descriptive variables—monthly earnings in multiples of the minimum wage, years of education, age in years, and job tenure in years—throughout the earnings distribution for 1996 in panel A and for 2012 in panel B. Zooming in on educational attainment, which increased significantly over this period, Figure A.2 shows the distribution of education degrees, grouped by individuals who complete the following levels of education: primary school or lower levels, middle school, high school, and college or higher. It shows the distribution for 1996 in panel A and for 2012 in panel B.

Figure A.1. RAIS cross-sectional summary statistics, 1996 and 2012



Notes: Figure shows mean monthly earnings (“wages”), years of education, age, and tenure across wage percentiles for 1996 in panel A and for 2012 in panel B. All statistics are for adult male workers ages 18–54. Source: RAIS, 1996 and 2012.

Figure A.2. RAIS education degree shares, 1996 and 2012



Notes: Figure shows shares of education degrees across wage percentiles for 1996 in panel A and for 2012 in panel B. All statistics are for adult male workers ages 18–54. Source: RAIS, 1996 and 2012.

Summary statistics for cross-sectional household survey data (PNAD). Table A.1 presents summary statistics on the PNAD data.

Table A.1. Summary statistics for cross-sectional household survey data (PNAD)

	# Workers	Real wage (formal)		Real wage (informal)		Employment rate	Formal share
		Mean	Std. dev.	Mean	Std. dev.		
1996	74,487	7.01	0.81	6.26	0.81	0.95	0.68
1997	78,731	7.02	0.79	6.26	0.82	0.94	0.68
1998	79,060	7.03	0.78	6.26	0.81	0.93	0.67
1999	81,230	6.97	0.77	6.21	0.79	0.93	0.66
2000							
2001	89,102	6.93	0.74	6.20	0.81	0.93	0.66
2002	90,855	6.90	0.73	6.19	0.81	0.93	0.66
2003	91,490	6.84	0.71	6.12	0.77	0.92	0.67
2004	94,526	6.85	0.69	6.15	0.77	0.94	0.68
2005	97,348	6.89	0.67	6.19	0.77	0.93	0.68
2006	97,757	6.94	0.66	6.25	0.76	0.94	0.69
2007	95,598	6.97	0.65	6.30	0.78	0.94	0.71
2008	93,677	7.00	0.65	6.35	0.76	0.95	0.72
2009	95,170	7.02	0.63	6.36	0.76	0.94	0.73
2010							
2011	84,910	7.07	0.62	6.51	0.75	0.95	0.76
2012	86,031	7.13	0.62	6.56	0.78	0.95	0.76

Notes: Table shows summary statistics on wages, employment rates, and formal employment shares between 1996 and 2012. All statistics are for adult male workers ages 18–54. Real wages are measured in 2012 BRL and in logs. Surveys are not available for census years 2000 and 2010.
Source: PNAD, 1996–2012.

Summary statistics for longitudinal household survey data (PME). We present summary statistics on the PME data in Table A.2.

Table A.2. Summary statistics for longitudinal household survey (PME)

		Transition rate	Transition rate
	# Workers	employed-nonemployed	employed-nonemployed
2002	94,280	0.08	0.05
2003	140,734	0.09	0.06
2004	146,847	0.08	0.05
2005	154,159	0.08	0.05
2006	153,646	0.08	0.04
2007	154,338	0.09	0.05
2008	150,104	0.10	0.05
2009	149,762	0.10	0.04
2010	150,443	0.10	0.04
2011	145,012	0.11	0.04
2012	121,211	0.10	0.04

Notes: Table shows number of workers and monthly transition rates between employment (i.e., formal employment) and nonemployment (i.e., informal employment + unemployment). All statistics are for adult male workers ages 18–54. *Source:* PME, 2002–2012.

A.3 Details of sample selection and sample size

Table A.3 shows official labor force statistics (panel A) and different sample sizes computed in the RAIS administrative data (panel B). Comparing the numbers of formal sector workers, we find that the two data sources are compatible. In 1996, official statistics stated that—depending on the definition of "formality"—there were between 26,055,801 and 29,015,225 formal sector workers in Brazil and 29,600,720 unique workers recorded in RAIS. In 2012, official statistics stated that there were between 48,261,135 and 50,264,428 formal sector workers in Brazil and 58,738,850 unique workers recorded in RAIS. That RAIS captures a somewhat larger number of workers is not surprising, given that official statistics are based on survey data with respect to a reference week or month, while RAIS covers any employment spells during the entire calendar year. Cumulatively applying our selection criteria based on gender (only males), age (18–54), and nonmissing key information (earnings, worker and employer identifiers, and employment dates), we find that our RAIS sample comprises 17,201,101 workers in 1996 and 30,762,127 workers in 2012.⁴⁹

Table A.3. Official labor force statistics and sample sizes in RAIS administrative data

	1996	2012
<i>Panel A. Official labor force statistics</i>		
Labor force	69,626,959	97,597,798
Unemployment rate	0.076	0.067
Informality rate (definition 1)	0.578	0.470
Informality rate (definition 2)	0.595	0.463
Informality rate (definition 3)	0.549	0.448
Total formal employment (definition 1)	27,149,501	48,261,135
Total formal employment (definition 2)	26,055,801	48,898,546
Total formal employment (definition 3)	29,015,225	50,264,428
<i>Panel B. Sample sizes in RAIS administrative data</i>		
Total number of jobs	34,260,198	73,326,485
Number of unique workers	29,600,720	58,738,850
Number of unique + male workers	18,940,516	34,203,034
Number of unique + male + prime-age workers	17,377,682	31,114,528
Number of workers satisfying additional selection criteria	17,201,101	30,762,127

Notes: Table shows official labor force statistics (panel A) and sample sizes calculated based on the RAIS administrative data for 1996 and 2012 (panel B). Official labor force statistics are made available by the World Bank (indicator ID: SL.TLF.TOTL.IN) and are derived using data from the International Labour Organization's, ILOSTAT database. Unemployment rate is from IPEA. Informality is classified according to three definitions, which—relative to a baseline definition (definition 1)—differ in whether they take into account unpaid workers (definition 2) and employers (definition 3); all data are from IPEA. Additional selection criteria in the last line of the table include a requirement of nonmissing values for earnings, worker identifiers, employer identifiers, and employment dates. Source: World Bank, IPEA, and RAIS, 1996 and 2012.

The main difference between the number of workers reported in the official labor force statistics and our final sample in RAIS is due to our selection based on gender (only males) and age (18–54). The minimum wage is significantly more binding for women and, separately, for men outside of this age range. Thus, our analysis focuses on a population subgroup that is relatively less affected by the minimum wage. In this sense, our results may understate the effects of the minimum wage on Brazil's

⁴⁹In a previous version of the paper, we also excluded workers with earnings below the minimum wage and those at firms below a minimum employment threshold, leading to a slightly smaller sample size than that reported here.

full population.

A.4 The importance of unobserved worker heterogeneity in wages

Let i index workers and t index years, and let $edu(i) \in \{e_1, e_2, \dots, e_N\}$ be the educational attainment group of individual i , which in our data is a permanent worker characteristic. The log wage of individual i in year t is denoted by y_{it} . Then, we can decompose the total variance of log wages as

$$Var(y_{it}) = \underbrace{Var(\mathbb{E}[y_{it}|edu(i)])}_{\text{Between-education-group variance}} + \underbrace{Var(y_{it} - \mathbb{E}_i[y_{it}|edu(i)])}_{\text{Within-education-group variance}}, \quad (\text{A.1})$$

The first term on the right-hand side of equation (A.1) is the variance of education-mean log wages, which we call the "between-education-group" variance. The second term is the variance of worker-year level deviations from the education-mean log wages, which we call the "within-education-group" variance. For education to be a meaningful skill proxy, we require the between-education-group variance to make up a significant share of the total variance of log wages. Implementing the decomposition in equation (A.1) on the RAIS data from 1994 to 1998, we find that out of a total variance of wages of 75.0 log points, around 14.1 log points (19 percent) are due to the between-education-group variance component, while 60.9 log points (81 percent) are due to the within-education-group variance component. From this, we conclude that while education is a significant predictor of wages in the data, the vast majority of wage dispersion is within education groups.

We can extend our analysis to other worker and job attributes as observable skill proxies. To this end, we estimate a sequence of Mincerian wage equations with cumulatively added controls for observable worker characteristics; unobservable, time-invariant worker characteristics; and unobservable, time-invariant firm characteristics. Observable worker characteristics include education-specific age dummies, education-specific year dummies, hours dummies, and occupation dummies. Table A.4 shows the results from four different specifications estimated on the RAIS data. With rich controls for observable worker characteristics (column 1), the coefficient of determination (R^2) is 48.6 percent, while the root mean squared error (RMSE) is 0.623. This suggests that the largest share of wage heterogeneity and significant dispersion in absolute terms is not explained by observables. Adding firm dummies to the first specification (column 2) leads to an R^2 of 76.4 percent, suggesting that firms are an important in wage determination. However, as AKM have argued, some of this explained variance may itself be attributable to unobserved worker heterogeneity. When we add worker dummies instead of firm dummies to the first specification (column 3), we find an R^2 of 88.3 percent and an RMSE of 0.347, meaning that the explanatory power is almost twice as high when controlling worker observable and unobservable characteristics relative to when controlling only for observables. Together, the estimation results from this sequence of wage equations suggest that unobservable worker characteristics constitute an important share of overall wage dispersion, more important than education and a number of other observable characteristics.⁵⁰

⁵⁰ All wage equations here are estimated on the largest available set of workers, whereas Section 2.2 of the main text reports estimates based on the leave-one-out connected set of workers and firms. Note also that all numbers reported here are with respect to the plug-in estimators of the R^2 and the RMSE. However, Section 2.2 presented leave-one-out corrected estimates of variance components, which leads to small changes relative to the plug-in estimates.

Table A.4. Explained wage dispersion when (not) controlling for observable and unobservable heterogeneity

	(1)	(2)	(3)
Coefficient of determination (R^2)	0.486	0.764	0.883
Root mean square error (RMSE)	0.623	0.427	0.347
Observations (mm)	83.2	82.9	78.2
Education-specific age dummies	✓	✓	✓
Education-specific year dummies	✓	✓	✓
Hours dummies	✓	✓	✓
Occupation dummies	✓	✓	✓
Firm dummies	✗	✓	✗
Worker dummies	✗	✗	✓

Notes: This table shows the coefficient of determination (R^2) and root mean square error (RMSE) for different specifications that project log wages on various controls: observable worker characteristics (column 1), observable worker characteristics and firm dummies (column 2), and observable worker characteristics and worker dummies (column 3). Observable worker characteristics include education-specific age dummies, education-specific year dummies, hours dummies, and occupation dummies. Note that the number of observations varies across specifications because we drop singletons based on combinations of the controls included in each specification. *Source:* RAIS, 1994–1998.

A.5 Additional results on the decomposition of the variance of log wages

The following table presents additional decompositions of the variance of log wages based on the AKM wage equation (1). It presents them separately for a five-year period centered on 1996 (i.e., from 1994 to 1998) and a five-year period centered on 2012 (i.e., from 2010 to 2014), similar to Table 2 in Section 2.2 of the main text. Specifically, Table 1 conducts the same variance decomposition for various selection criteria based on whether workers have earnings not equal to or weakly above the minimum wage, and also based on minimum thresholds for the number of employees or the number of switchers at each firm, with and without additional controls. The main takeaway is that the decomposition results are stable across these different specifications and population subgroups.

Table A.5. Additional decompositions of the variance of log wages over time

	1994–1998					2010–2014					Change (%), 1994–1998 to 2010–2014				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(6)–(1)	(7)–(2)	(8)–(3)	(9)–(4)	(10)–(5)
$Var(w_{ijt})$	0.713 (100%)	0.704 (100%)	0.691 (100%)	0.685 (100%)	0.685 (100%)	0.446 (100%)	0.444 (100%)	0.460 (100%)	0.451 (100%)	0.451 (100%)	–0.267 (100%)	–0.260 (100%)	–0.231 (100%)	–0.234 (100%)	–0.234 (100%)
$Var(\hat{\alpha}_i)$	0.331 (46%)	0.333 (47%)	0.346 (50%)	0.340 (50%)	0.195 (28%)	0.251 (56%)	0.252 (57%)	0.276 (60%)	0.267 (59%)	0.146 (32%)	–0.080 (30%)	–0.081 (30%)	–0.070 (31%)	–0.073 (31%)	–0.049 (21%)
$Var(\hat{\alpha}_j)$	0.224 (31%)	0.217 (31%)	0.180 (26%)	0.177 (26%)	0.166 (24%)	0.091 (20%)	0.090 (20%)	0.075 (16%)	0.075 (17%)	0.070 (16%)	–0.133 (50%)	–0.127 (49%)	–0.105 (45%)	–0.102 (44%)	–0.096 (41%)
$Var(\hat{\gamma}_t)$			0.000 (0%)	0.000 (0%)	0.001 (0%)			0.000 (0%)	0.000 (0%)	0.000 (0%)		0.000 (0%)	0.000 (0%)	0.000 (0%)	–0.001 (0%)
$Var(\text{age}_{it} \times \hat{\beta}_a)$					0.021 (3%)				0.021 (5%)	0.021 (5%)					0.000 (0%)
$Var(\text{hours}_{it} \times \hat{\beta}_h)$					0.000 (0%)				0.000 (0%)	0.000 (0%)					0.000 (0%)
$Var(\text{occ}_{it} \times \hat{\beta}_o)$					0.007 (1%)				0.007 (2%)	0.007 (2%)					0.000 (0%)
$Var(\text{exp}_{it} \times \hat{\beta}_e)$					0.014 (2%)				0.014 (1%)	0.005 (1%)					–0.009 (4%)
$2 \times \sum Cov(\cdot, \cdot)$	0.126 (18%)	0.122 (17%)	0.136 (20%)	0.138 (20%)	0.252 (37%)	0.084 (19%)	0.082 (18%)	0.091 (20%)	0.091 (20%)	0.185 (41%)	–0.042 (16%)	–0.040 (15%)	–0.045 (19%)	–0.047 (20%)	–0.067 (29%)
$Var(\hat{\varepsilon}_{ijt})$	0.032 (4%)	0.032 (5%)	0.029 (4%)	0.030 (4%)	0.029 (4%)	0.020 (4%)	0.020 (5%)	0.018 (4%)	0.018 (4%)	0.017 (4%)	–0.012 (4%)	–0.012 (5%)	–0.011 (5%)	–0.012 (5%)	–0.012 (5%)
$Corr(\hat{\alpha}_i, \hat{\alpha}_j)$	0.233	0.229	0.272	0.282	0.207	0.276	0.273	0.317	0.324	0.278					
R^2	0.955	0.955	0.958	0.956	0.958	0.955	0.955	0.961	0.960	0.962					
Obs. (mm)	74.4	73.7	60.7	62.1	62.1	135.8	134.3	106.6	112.4	112.4					
MW selection	None	≠MW	≥MW	≥MW	≥MW	None	≠MW	≥MW	≥MW	≥MW	None	≠MW	≥MW	≥MW	≥MW
Min. employees	None	None	10	None	None	None	None	10	None	None	None	None	10	None	None
Min. switchers	None	None	None	10	10	None	None	None	10	10	None	None	None	10	10

Notes: This table shows plug-in variance components of log wages based on estimating AKM equation (1) for the population of male workers ages 18–54 in 1994–1998 and 2010–2014. $Var(w_{ijt})$ denotes the variance of log wages, $Var(\hat{\alpha}_i)$ denotes the variance of estimated person fixed effects, $Var(\hat{\psi}_j)$ denotes the variance of estimated firm fixed effects, $Var(\hat{\gamma}_t)$ denotes the variance of estimated education-specific year fixed effects, $Var(\text{age}_{it} \times \hat{\beta}_a)$ denotes the variance of estimated education-specific age fixed effects, $Var(\text{hours}_{it} \times \hat{\beta}_h)$ denotes the variance of estimated hours fixed effects, $Var(\text{occ}_{it} \times \hat{\beta}_o)$ denotes the variance of estimated occupation fixed effects, $Var(\text{exp}_{it} \times \hat{\beta}_e)$ denotes the variance of estimated actual-experience fixed effects, $2 \times \sum Cov(\cdot, \cdot)$ denotes two times the sum of all covariance terms, and $Var(\hat{\varepsilon}_{ijt})$ denotes the variance of estimated residuals. $Corr(\hat{\alpha}_i, \hat{\alpha}_j)$ denotes the correlation between estimated person fixed effects α_i and estimated firm fixed effects ψ_j . Variance shares are in parentheses in columns (1)–(10). Share of total change in the variance of log wages is in parentheses in the last five columns. Observations are in millions of worker-years. “MW selection” indicates selection based on wage in current job, selecting either all job spells (“None”), jobs with earnings not exactly equal to the minimum wage (“≠MW”), or jobs with earnings weakly above the minimum wage (“≥MW”). “Min. employees” refers to selecting a subsample of firms based on a minimum number imposed on their average employment across all years that they appear in the data during the sample period. “Min. switchers” refers to selecting a subsample of firms based on a minimum number imposed on their total number of workers switching either into or out from the employer during the sample period. Age dummies are restricted to being flat between ages 45 and 49. Source: RAIS, 1994–2014.

B Empirical Appendix

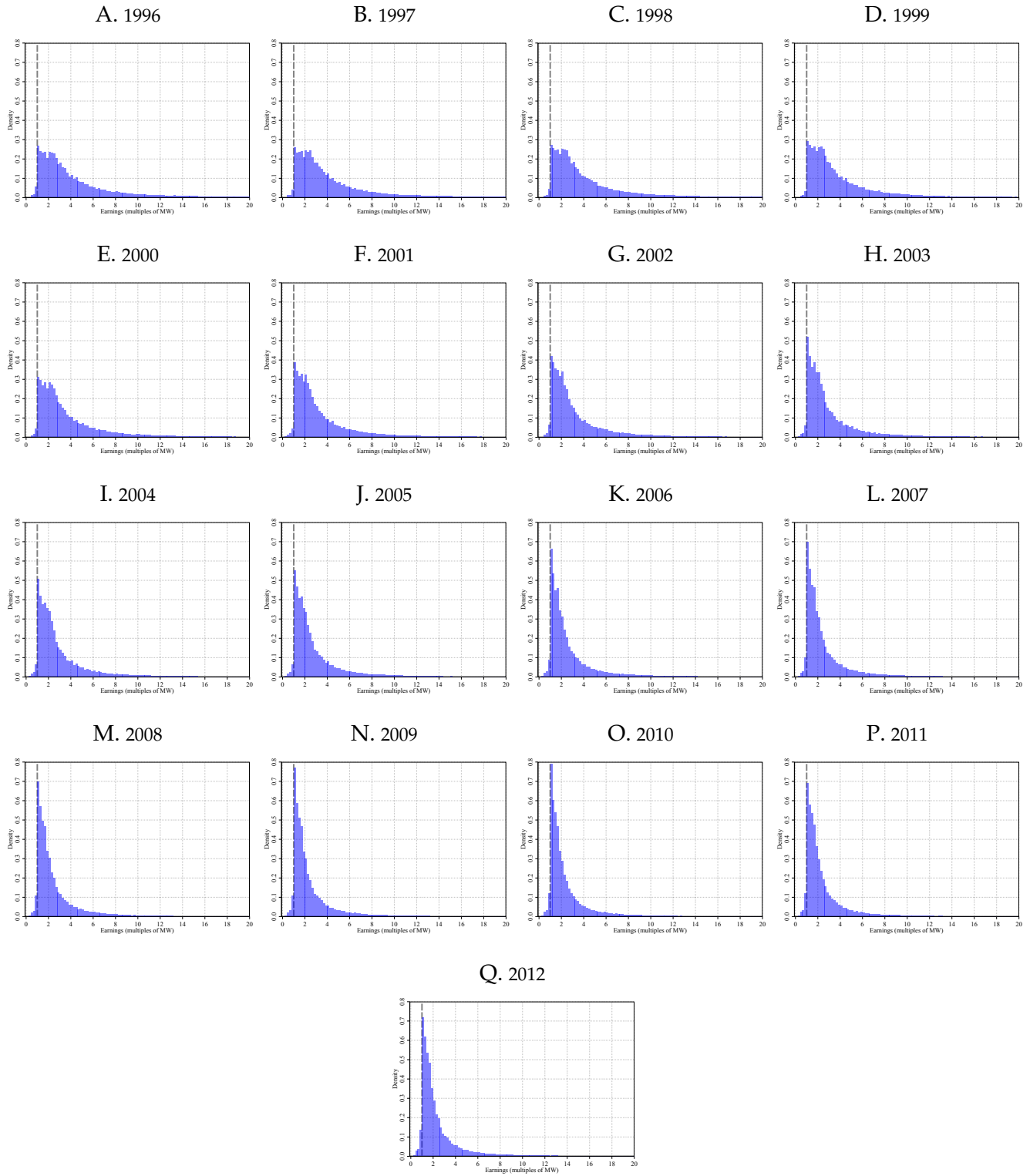
This appendix provides further details on the empirical exercises conducted in Section 3, including subsections on the wage distribution over time (Appendix B.1), the minimum wage spike (Appendix B.2), the incidence of minimum wage jobs (Appendix B.3), the evolution of the effective bindingness of the minimum wage (Appendix B.4), additional details on the effects of the minimum wage on wage inequality (Appendix B.5), additional regression results across mesoregions (Appendix B.6), additional Lee (1999)-style regression specifications in differences (Appendix B.7), additional regression specifications using various polynomial orders for region-specific time trends (Appendix B.8), additional regression specifications that cluster standard errors at the state level (Appendix B.9), a comparison between our findings and subsequent work by Haanwinckel (2020) (Appendix B.10), a comparison of the relative bindingness of the minimum wage in Brazil and in the U.S. (Appendix B.11), a comparison between the distributions of (changes in) log wages in current BRL and in multiples of the current minimum wage (Appendix B.12), hours worked in relation to the bindingness of the minimum wage (Appendix B.13), robustness of our results of the effects of the minimum wage on employment and firm dynamics (Appendix B.14), and an analysis of the employment effects of the minimum wage across education groups (Appendix B.15).

B.1 The wage distribution over time

Distribution of wages in levels. Figure B.1 shows histograms of wages in multiples of the minimum wage separately for each year between 1996 and 2012. Over this period, there is notable compression in the left tail of the wage distribution.

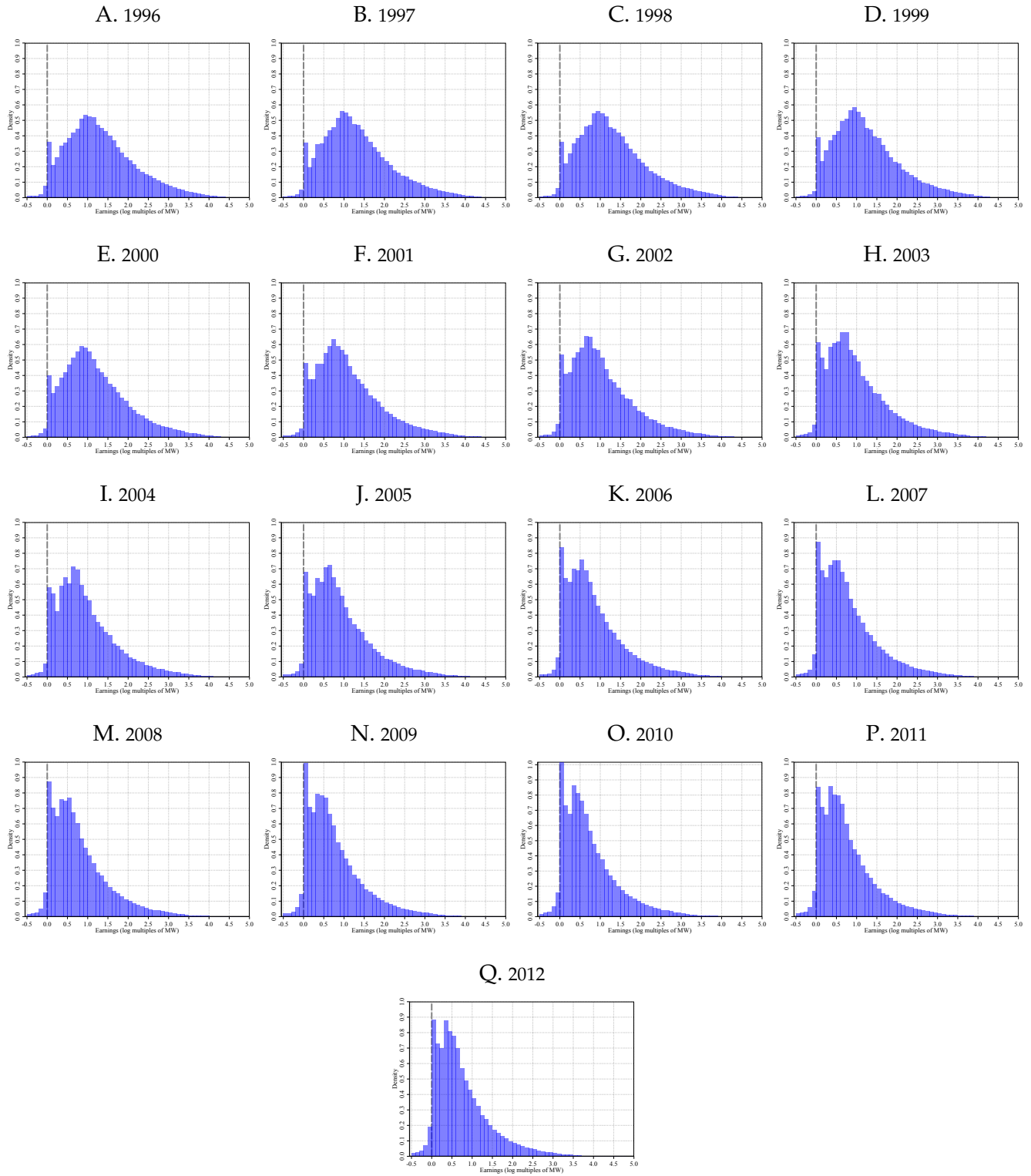
Distribution of wages in logarithms. Figure B.2 shows histograms of log wages separately for each year between 1996 and 2012. As in the histograms of wage levels, there is visible compression in the left tail of the wage distribution over time.

Figure B.1. Distribution of wages by year, 1996–2012



Notes: Each panel shows a histogram of wages in multiples of the current minimum wage based on 60 equi-spaced bins for population of male workers ages 18–54 for one year between 1996 and 2012. Source: RAIS.

Figure B.2. Distribution of log wages by year, 1996–2012



Notes: Each panel shows a histogram of log wages in multiples of the current minimum wage based on 60 equi-spaced bins for population of male workers ages 18–54 for one year between 1996 and 2012. Source: RAIS.

B.2 The (relatively small) spike at the minimum wage

Much of the previous empirical literature has interpreted the mass of workers employed at the minimum wage as a measure of the bindingness of the wage floor (Flinn, 2006, 2010). However, theoretical labor market models in the spirit of Burdett and Mortensen (1998) predict that frictional wage dispersion for identical workers can be sustained absent any mass points in the wage distribution, including at the minimum wage. This suggests that any spike of workers at the minimum wage may be thought of independently from, or maybe in addition to, the effect of the minimum wage on the rest of the wage distribution. Although there is substantial heterogeneity in the empirical bindingness of the minimum wage across population subgroups in our administrative data from Brazil, we robustly find a relatively small spike at the wage floor for male workers between the age of 18–54. The remainder of this subsection is dedicated to studying the (relatively small) spike at the minimum wage in Brazil, both in the cross section and over time.

Share of workers earning exactly, below, or around the minimum wage. Panel A of Figure B.3 shows that across states, an average of approximately 2 percent of workers earn exactly the minimum wage in 1996 and 2012 against the relative bindingness of the minimum wage measured by the Kaitz index, defined as the log minimum-to-median wage.⁵¹ We find a weak positive correlation between the Kaitz index and the share of workers earning the minimum wage, but the fraction remains at relatively small levels, even in states where the minimum wage is most binding.

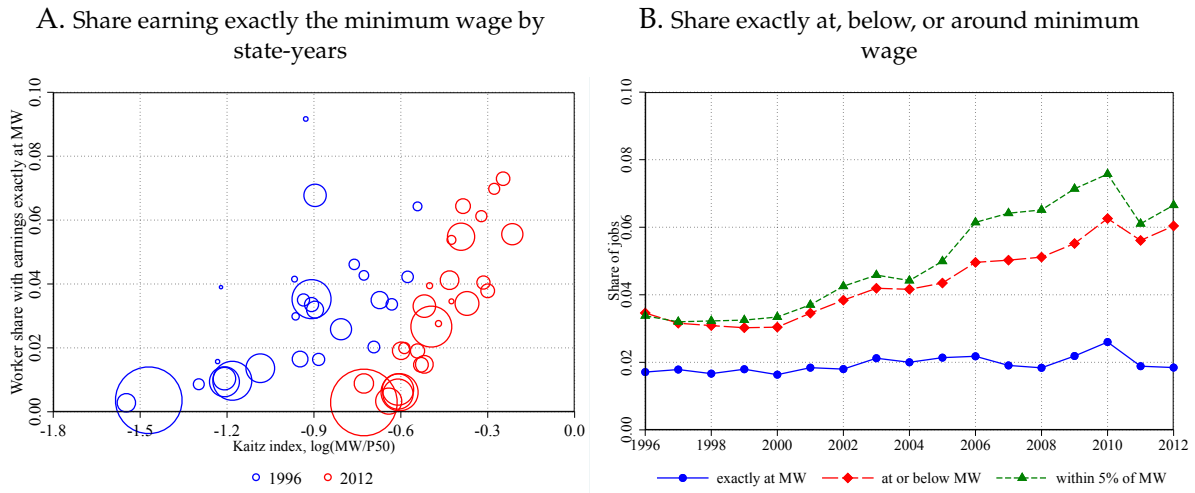
We can broaden our definition of “mass point” to three measures whose evolution from 1996 to 2012 is depicted in panel B of Figure B.3. The share of workers earning exactly the minimum wage, shown by the blue line, remains flat at 2 percent. A little more than 1.5 percent of workers in 1996 and around 3 percent of workers in 2012 report earning less than the minimum wage, shown by the red line. These observations are likely due to a mix of legal exceptions, misreporting, and illegal employment. Our most generous definition includes workers within a 5 percent band around the minimum wage, shown in green. This most generous measure rises from 3.5 to 7 percent over this period, far from the roughly 30 percent of workers between the old and the new minimum wage.

Figure B.4 shows that there is substantial heterogeneity in the share of workers earning exactly the minimum wage across states (panel A) and mesoregions (panel B). At the same time, only a small fraction of regions has a share of workers earning exactly the minimum wage above 7 percent.

Histograms of wages around the minimum wage. Figure B.5 shows a histogram of wages in multiples of the minimum wage in 1996 and 2012. Figure B.6 shows a similar histogram for wages in logarithms.

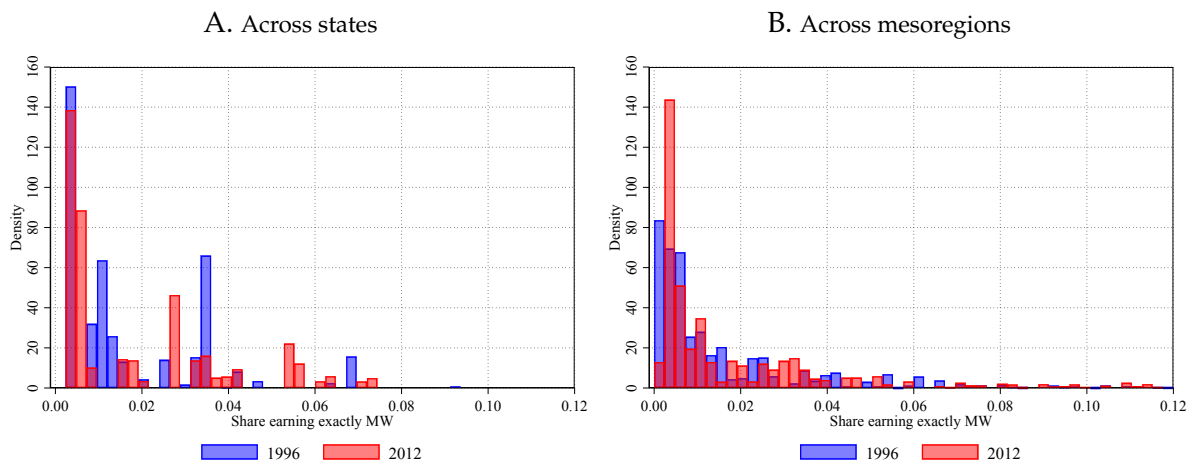
⁵¹For comparison, in 2015 3.3 percent of hourly paid workers in the U.S. earned the prevailing federal minimum wage or less (U.S. Bureau of Labor Statistics, 2017).

Figure B.3. Share of workers at and around the minimum wage, 1996–2012



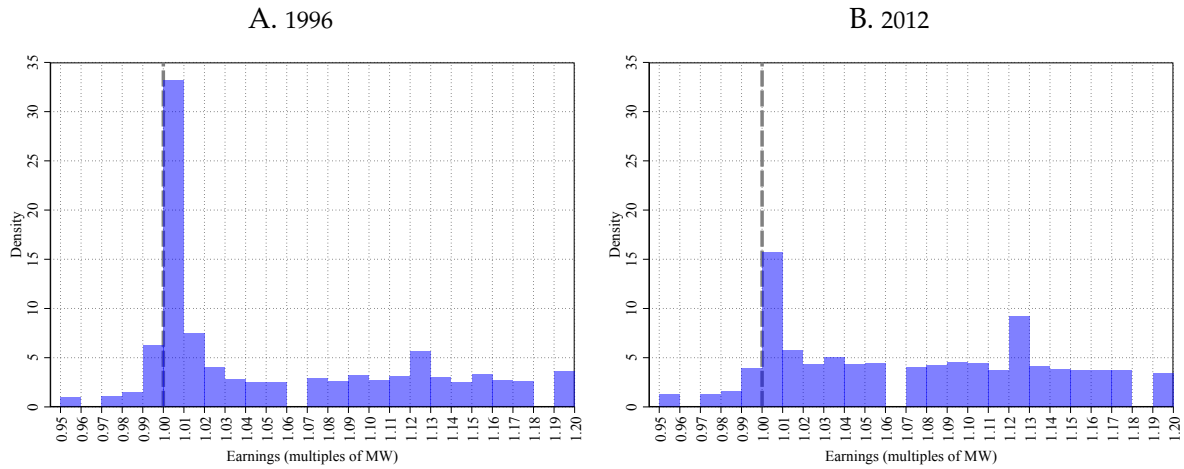
Notes: This figure shows the share of workers with earnings exactly at, below, or around the minimum wage. Panel A shows share of male workers ages 18–54 earning exactly the minimum wage against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, across states in 1996 and 2012. Area of circles is proportional to population size. In panel B, the blue line shows share of workers earning exactly the minimum wage, the red line shows share at or below the minimum wage, and the green line plots share within 5 percent of the minimum wage. Source: RAIS, 1996–2012.

Figure B.4. Histogram of share of workers earning exactly the minimum wage, 1996 and 2012



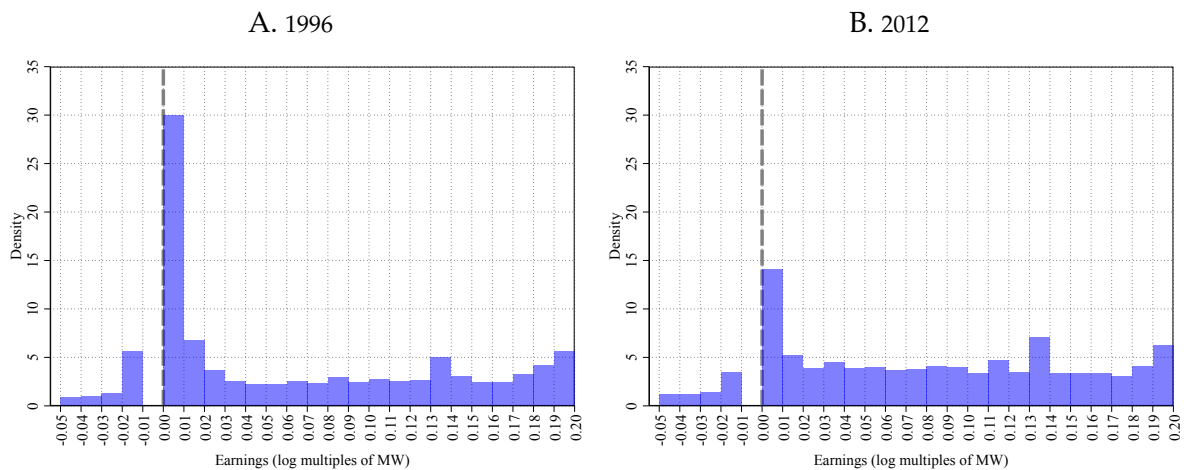
Notes: This figure shows histograms of the share of workers earning exactly the minimum wage across different population subgroups by state (Panel A) and by mesoregion (Panel B). Blue bars show the distribution in 1996, while red bars show the distribution in 2012. Source: RAIS, 1996 and 2012.

Figure B.5. Histogram of wages around the minimum wage, 1996 and 2012



Notes: This figure shows histograms of the wage distribution, measured in multiples of the minimum wage, zoomed in around the minimum wage (dashed vertical line) for 1996 (Panel A) and 2012 (Panel B). Bar width is set to 0.01, i.e., one centavo (subdivision of Brazilian Reais). *Source:* RAIS, 1996 and 2012.

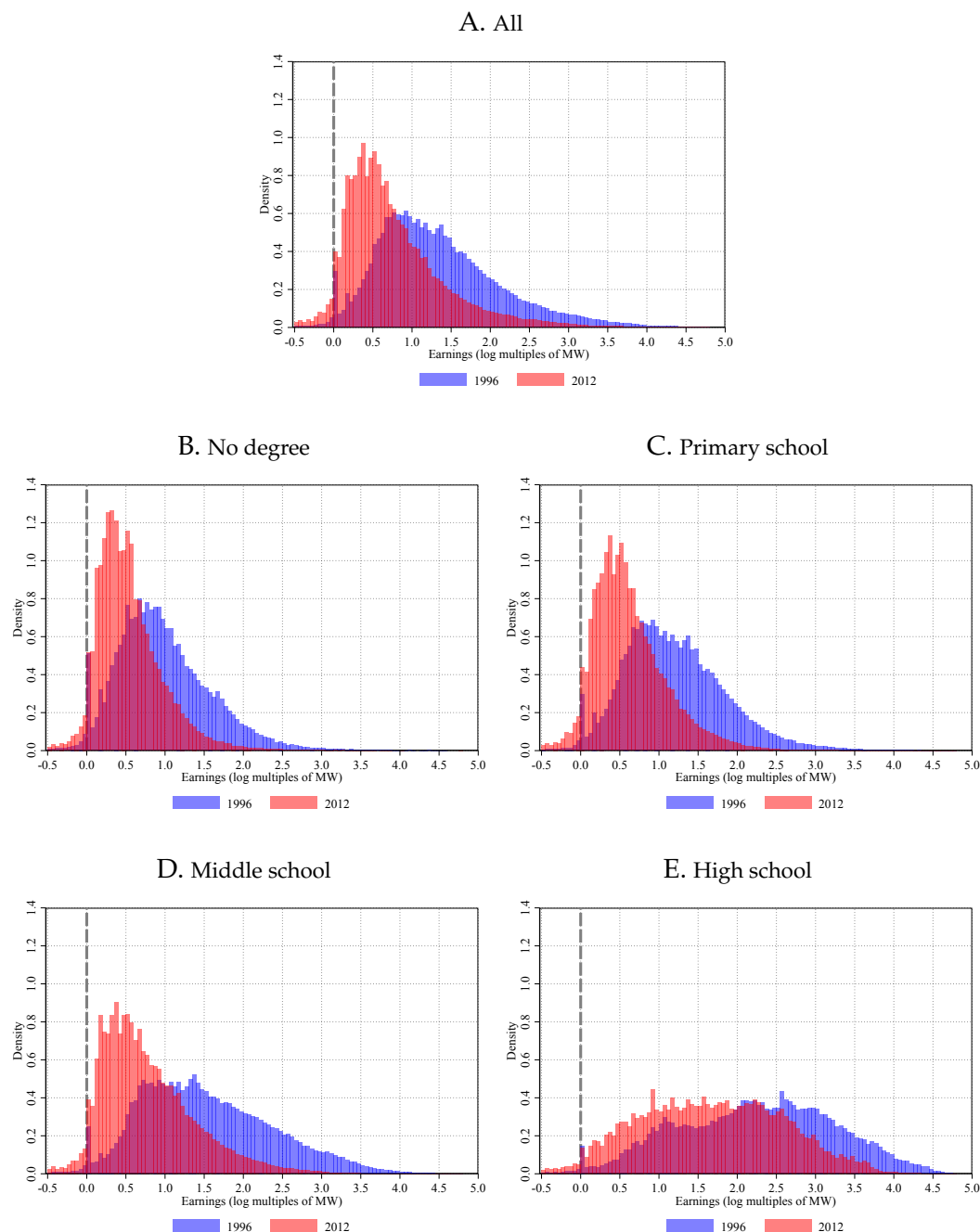
Figure B.6. Histogram of log wages around the minimum wage, 1996 and 2012



Notes: This figure shows histograms of the log wage distribution, measured in log multiples of the minimum wage, zoomed in around the minimum wage (dashed vertical line) for 1996 (Panel A) and 2012 (Panel B). Bar width is set to 0.01, i.e., approximately 1 percent. *Source:* RAIS, 1996 and 2012.

Comparison with Haanwinckel (2020). Figure B.7 shows histograms of log wages for the Brazilian state of Rio Grande do Sul, which is the focus of subsequent work by Haanwinckel (2020), for our subsample of prime-age men. The different panels show the distribution of log wages for the state's overall population (panel A) and separately for four education groups (panels B–E). We note that the histograms are characterized by a spike at the minimum wage and a general shape of the distribution that is not too dissimilar from that of the overall population of Brazil shown in Appendix B.1.

Figure B.7. Wage distributions for the Brazilian state of Rio Grande do Sul, 1996 and 2012

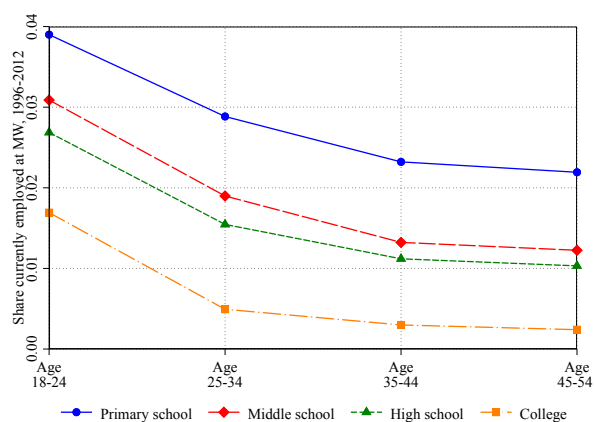


Notes: This figure shows histograms of log wages by education group in the Brazilian state of Rio Grande do Sul for 1996 (blue bars) and 2012 (red bars). The different panels show the histogram of log wages for the whole population (Panel A), for workers without a primary school degree (Panel B), workers with a primary school degree (Panel C), workers with a middle school degree (Panel D), and workers with at least a high school degree (Panel E). This figure corresponds to panel (a) of Figure D4 in Appendix D.5 of [Haanwinckel \(2020\)](#). *Source:* RAIS, 1996 and 2012.

B.3 Who earns the minimum wage in Brazil?

Figure B.8 shows the share of workers currently earning the minimum wage by education group. The share of minimum wage earners is higher at younger ages and lower educational attainments. Yet the maximum share of minimum wage earners—namely, that of workers ages 18–24 with at most a primary school degree—is around 4 percent.

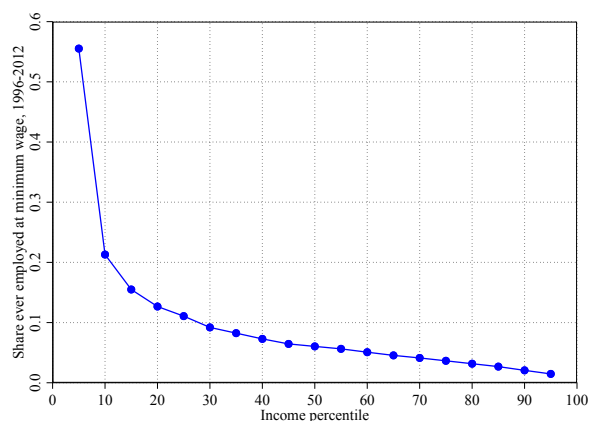
Figure B.8. Share of workers currently earning the minimum wage, by education group



Notes: Figure shows share of workers earning exactly the minimum wage by education groups (colored lines) and age groups (x-axis values) during the period from 1996 to 2012. Source: RAIS, 1996–2012.

A striking feature of the Brazilian labor market is that in spite of a relatively small share of workers earning the minimum wage at any point in time, a surprisingly large share of workers ever—currently, in the past, or in the future—earn the minimum wage. Figure B.9 plots the share of workers who have ever earned the minimum wage during our sample period of 1996–2012.

Figure B.9. Share of workers who ever earned the minimum wage from 1996–2012

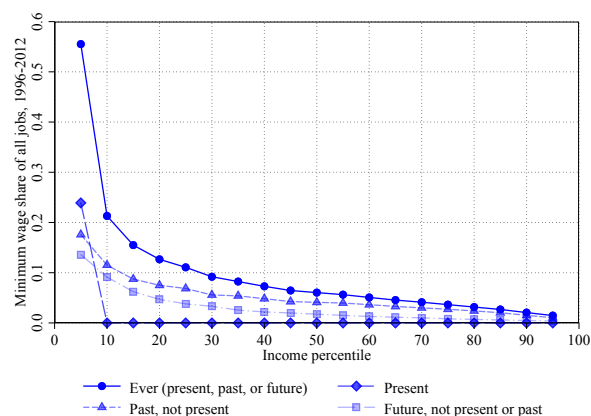


Notes: Figure shows the share of workers who have ever (currently, in the past, or in the future) earned exactly the minimum wage by current income percentile. Current income percentiles are created by ranking workers within a given year according to their current wage for each year between 1996 and 2012. Source: RAIS, 1996–2012.

Figure B.10 decomposes the share of workers between 1996 and 2012 who ever earned the minimum wage into those who earn the minimum wage currently, those who have earned it in the past, or those

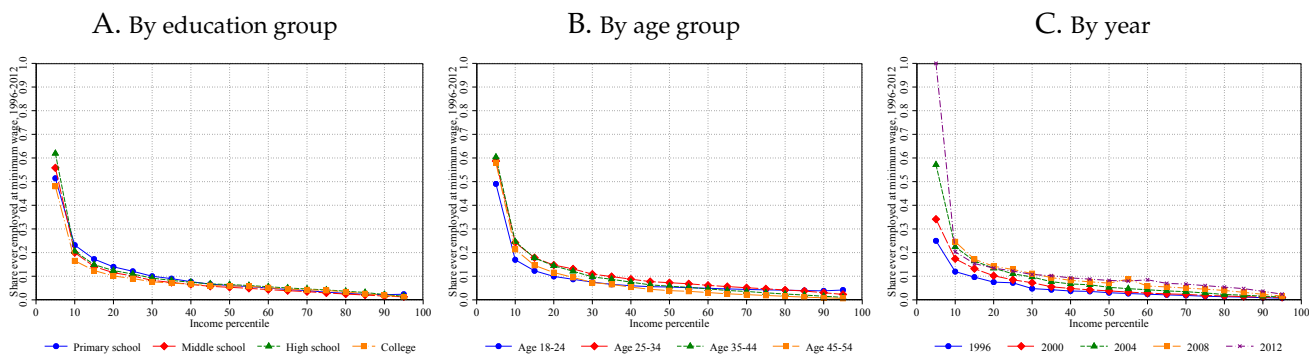
who will earn it in the future.

Figure B.10. Decomposition of share of workers who ever earned the MW, 1996–2012



Notes: Figure shows the share of workers who have ever (Currently, in the past, or in the future) earned exactly the minimum wage. The different colored lines show the share of workers who ever—currently, in the past, and in the future—earn the minimum wage across current income percentiles. Current income percentiles are created by ranking workers within a given year according to their current wage for each year between 1996 and 2012. Source: RAIS, 1996–2012.

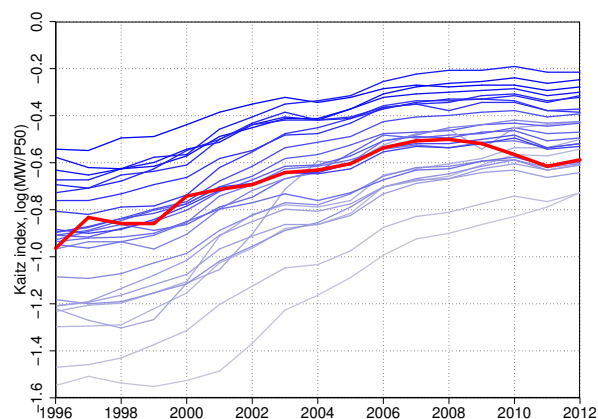
Figure B.11. Share ever employed at minimum wage, by subgroups



Notes: Figure shows the share of workers who have ever (currently, in the past, or in the future) earned exactly the minimum wage by current income percentile, separately by education group (Panel A), by age group (Panel B), and by year (Panel C). Current income percentiles are created by ranking workers within a given year according to their current wage for each year between 1996 and 2012. Source: RAIS, 1996–2012.

B.4 Evolution of the Kaitz index by state

Figure B.12. Data: Evolution of the Kaitz index by state, 1996–2012



Notes: Kaitz index is defined as $kaitz = \log(\text{minimum wage}) - \log(\text{median wage})$. Each blue line represents one of Brazil's 27 states. Red line marks weighted mean across states. Source: RAIS.

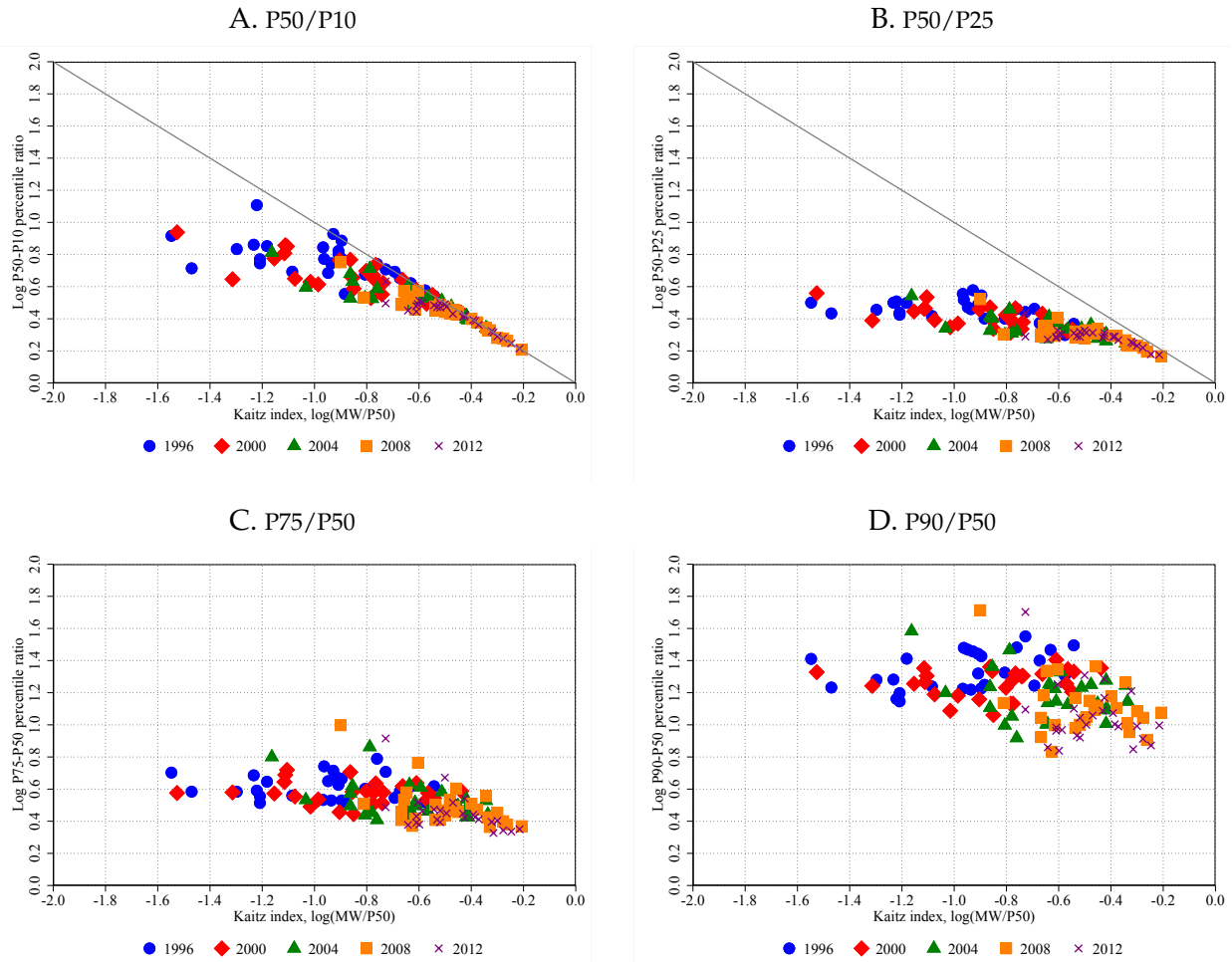
B.5 Additional results on the effects of the minimum wage on wage inequality

We now show that the inverse relationship between state-year level bindingness of the minimum wage and wage inequality generalizes to the full set of states over time. To see this, we define the *Kaitz index* as $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}(50)$ —that is, the log difference between the minimum wage prevailing at time t , w_t^{min} , and the median wage of subgroup s at time t , $w_{st}(50)$.⁵² Figure B.13 plots the relation between different log wage percentile ratios and the Kaitz index. Panel A plots empirical lower-tail inequality, measured by the P50/P10, against the Kaitz index across Brazilian states over time. The negative 45-degree line marks states where the minimum wage is binding at the tenth percentile of the wage distribution. Panel B repeats the same exercise for the P50/P25 ratio. Both plots show a negative relationship between lower-tail inequality and the Kaitz index; it grows more pronounced for more binding states in the cross section and over time. For comparison, the remaining two panels show a weaker relationship between top inequality, measured by the P75/P50 in panel C and by the P90/P50 in panel D, and the Kaitz index.

Figures B.14–B.17 show that our conclusions are robust to considering a broader set of wage percentile ratios and to running the analysis at a more granular level for Brazil’s 559 mesoregions.

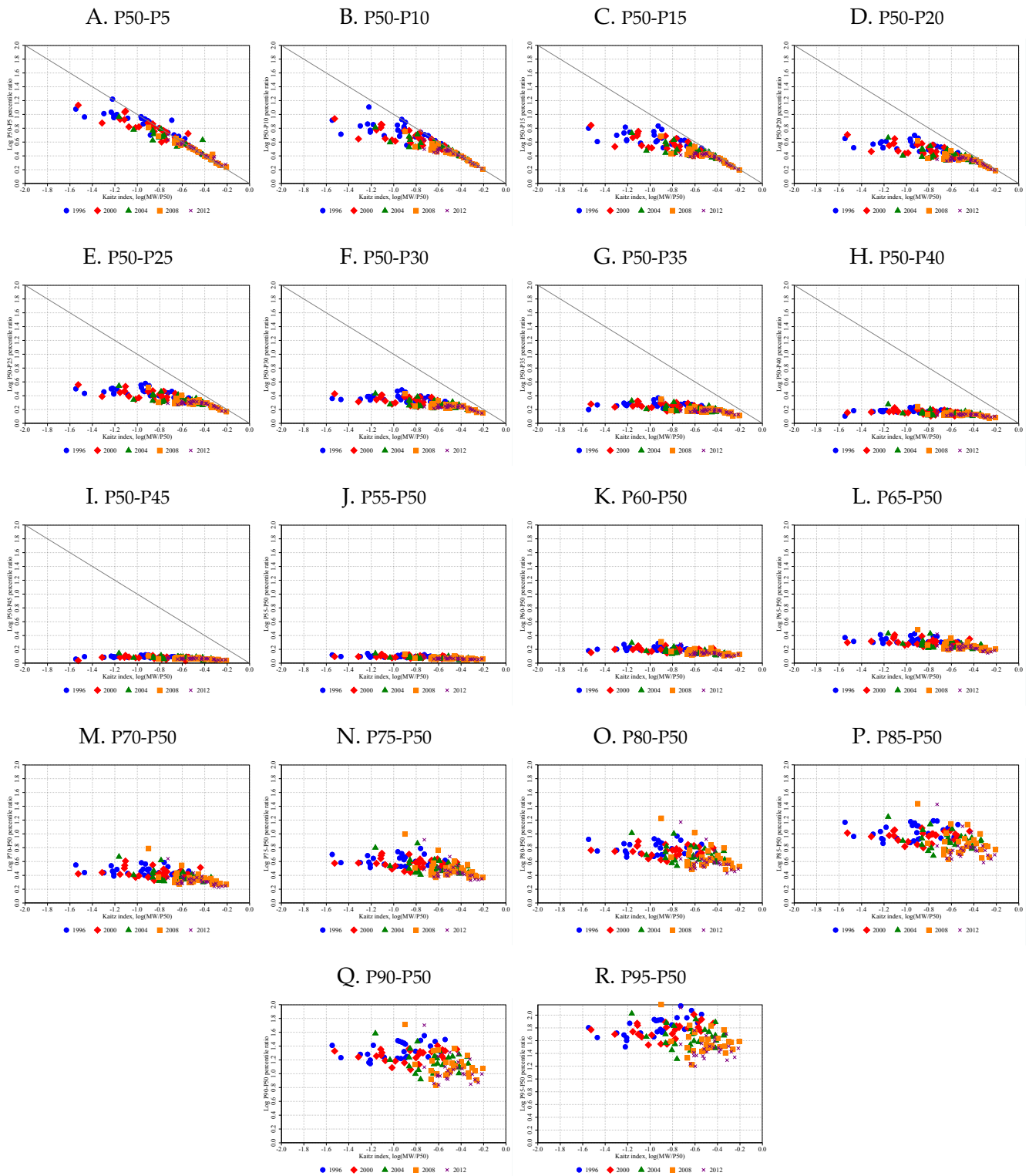
⁵²Recall that Figure B.12 in Appendix B.4 shows that variation in the Kaitz index across Brazilian states is large initially and decreases as the minimum wage increases, while approximately preserving the ranking of states over time.

Figure B.13. Log wage percentile ratios across Brazilian states over time, 1996–2012



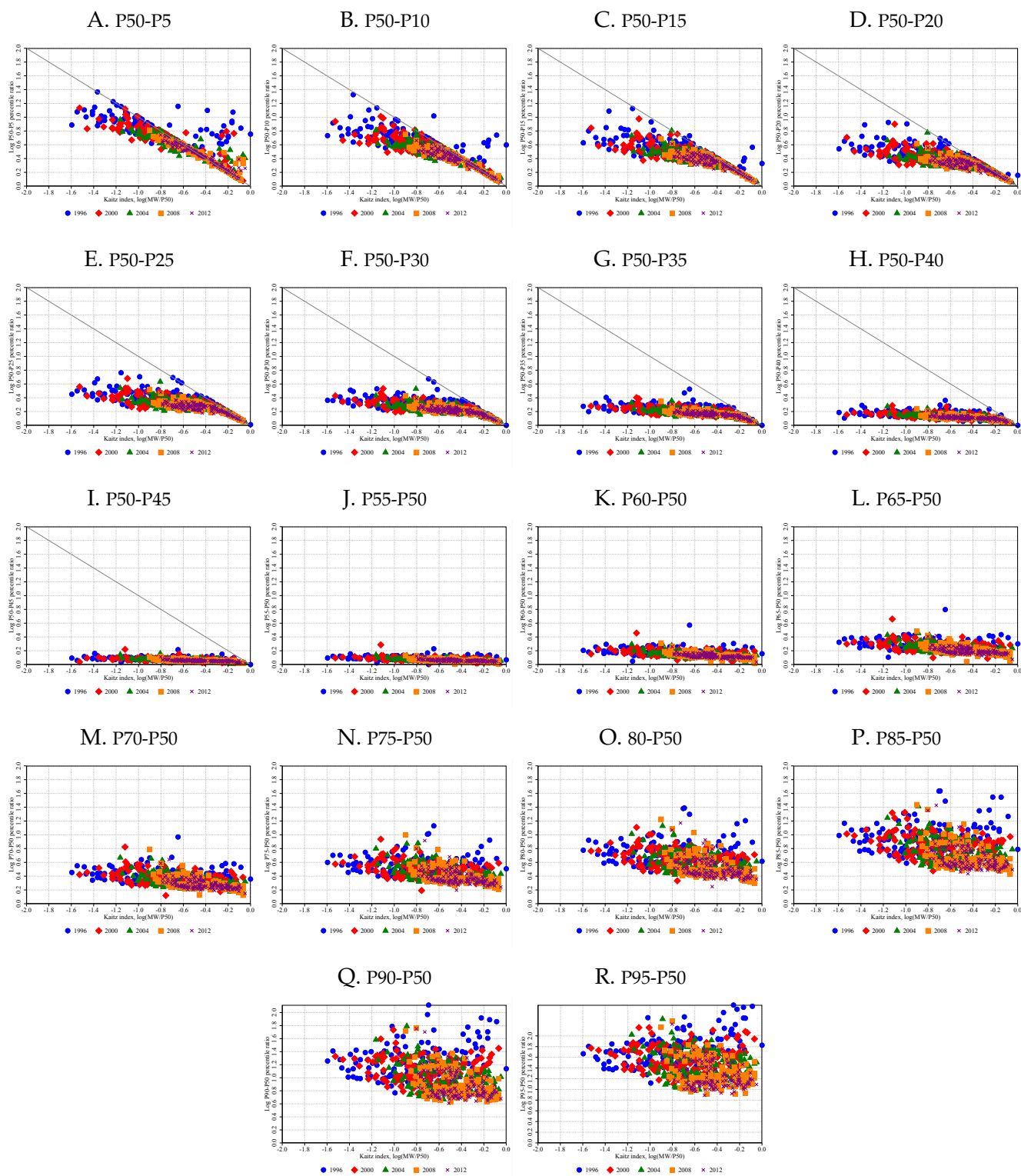
Notes: Figure plots various log wage percentile ratios against the Kaitz index, defined as $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}(50)$, where w_t^{min} is the minimum wage prevailing at time t and $w_{st}(50)$ is the median wage in subgroup s at time t . Each marker represents a combination of a state s and year t for each of Brazil's 27 states between 1996 and 2012. Source: RAIS, 1996–2012.

Figure B.14. Data: Wage percentile ratios across Brazilian states over time, 1996–2012



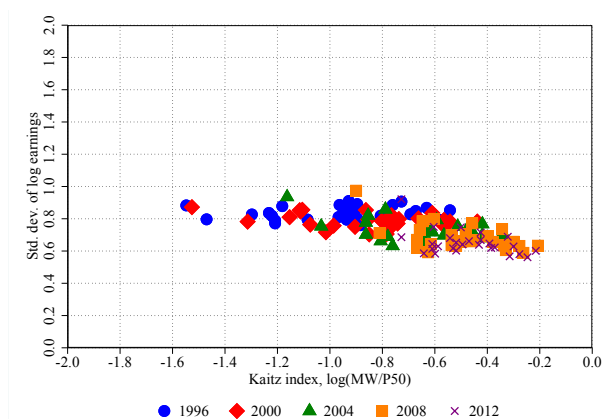
Notes: Figure plots different wage percentile ratios against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one state-year combination for each of Brazil's 27 states from 1996 to 2012. Source: RAIS.

Figure B.15. Data: Wage percentile ratios across Brazilian mesoregions over time, 1996–2012



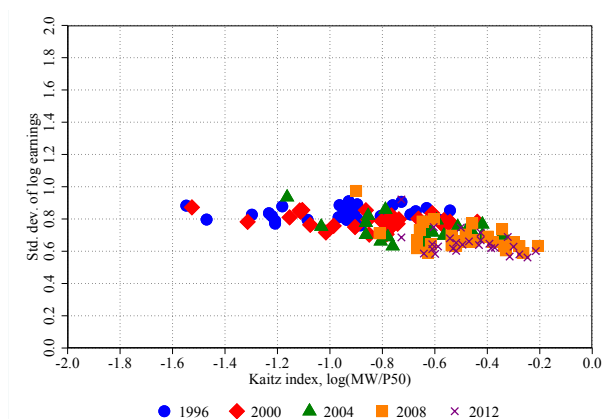
Notes: Figure plots different wage percentile ratios against the Kaitz index, $kaitz_{st} \equiv \log w_{st}^{min} - \log w_{st}^{median}$, with each marker representing one mesoregion-year combination for each of Brazil's 137 mesoregions from 1996 to 2012. Source: RAIS.

Figure B.16. Data: Standard deviation of log wages across states over time, 1996–2012



Notes: Figure plots the standard deviation of log wages against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one state-year combination for each of Brazil's 27 states from 1996 to 2012. Source: RAIS.

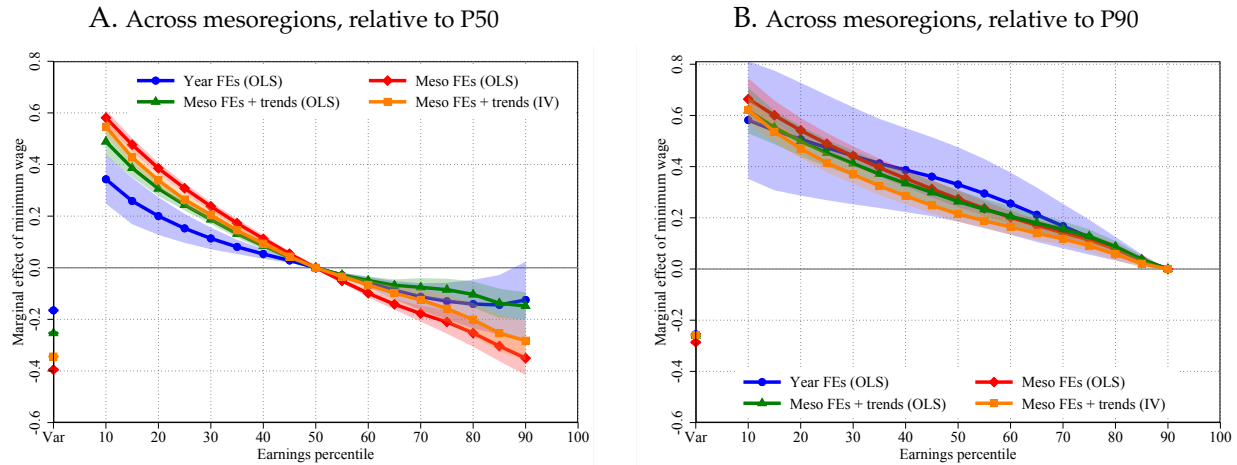
Figure B.17. Data: Standard deviation of log wages across mesoregions over time, 1996–2012



Notes: Figure plots the standard deviation of log wages against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one mesoregion-year combination for each of Brazil's 137 states from 1996 to 2012. Source: RAIS.

B.6 Additional regression results: Across mesoregions

Figure B.18. Estimated minimum wage effects on the wage distribution

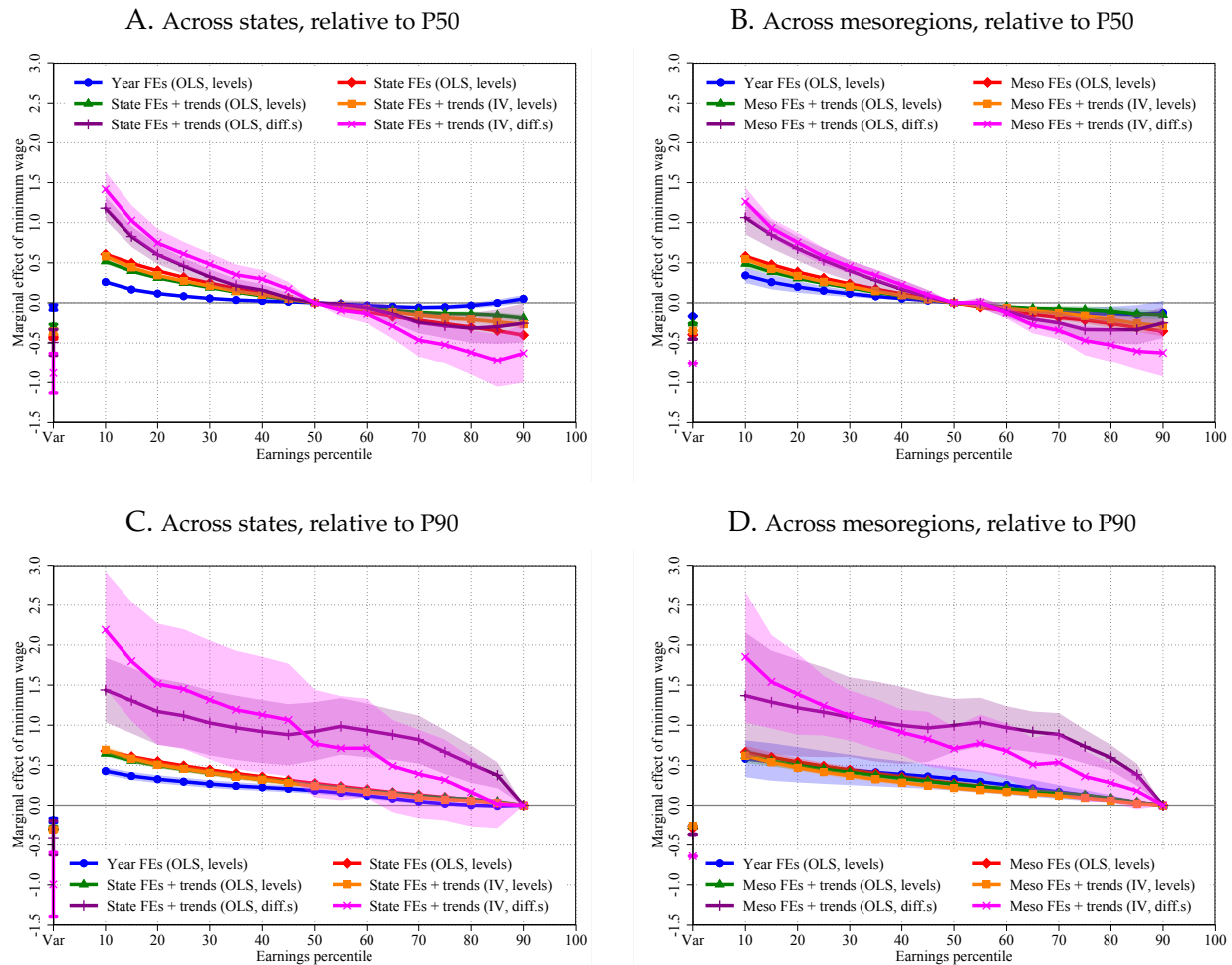


Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from four different specifications (colored lines and error bars or shaded areas) with different region-year controls: year fixed effects (blue line), region fixed effects (red line), region fixed effects in addition to region-specific quadratic time trends (green line)—all estimated using OLS—and the latter specification estimated using an IV strategy (orange line). The IV strategy instruments the generalized p -Kaitz index and its square using an instrument set that consists of the log real statutory minimum wage, its square, and the log real statutory minimum wage interacted with the mean of the log real median wage for the region over the full sample period. Within each panel, the estimated marginal effect of the minimum wage on the variance of log wages (“Var” on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution (“10” to “90” on the x-axis) relative to some base wage p are shown. Panel A uses the 50th percentile as the base wage (i.e., $p = 50$), while panel B uses the 90th percentile as the base wage (i.e., $p = 90$). Both panels are estimated across Brazil’s 137 mesoregions. The four error bars and four shaded areas represent 99 percent confidence intervals based on clustered standard errors at the mesoregion level. *Source:* RAIS, 1985–2014.

B.7 Additional regression results: Difference specifications

Figure B.19 shows additional specifications based on the econometric methodology in Section 3.2. Specifically, we compare the specifications in levels presented in the main text with additional specification in differences. We note that standard error bands tend to be significantly wider in difference specifications compared with level specifications. At the same time, our main conclusions remain unchanged.

Figure B.19. Estimated minimum wage effects on the wage distribution, difference specifications



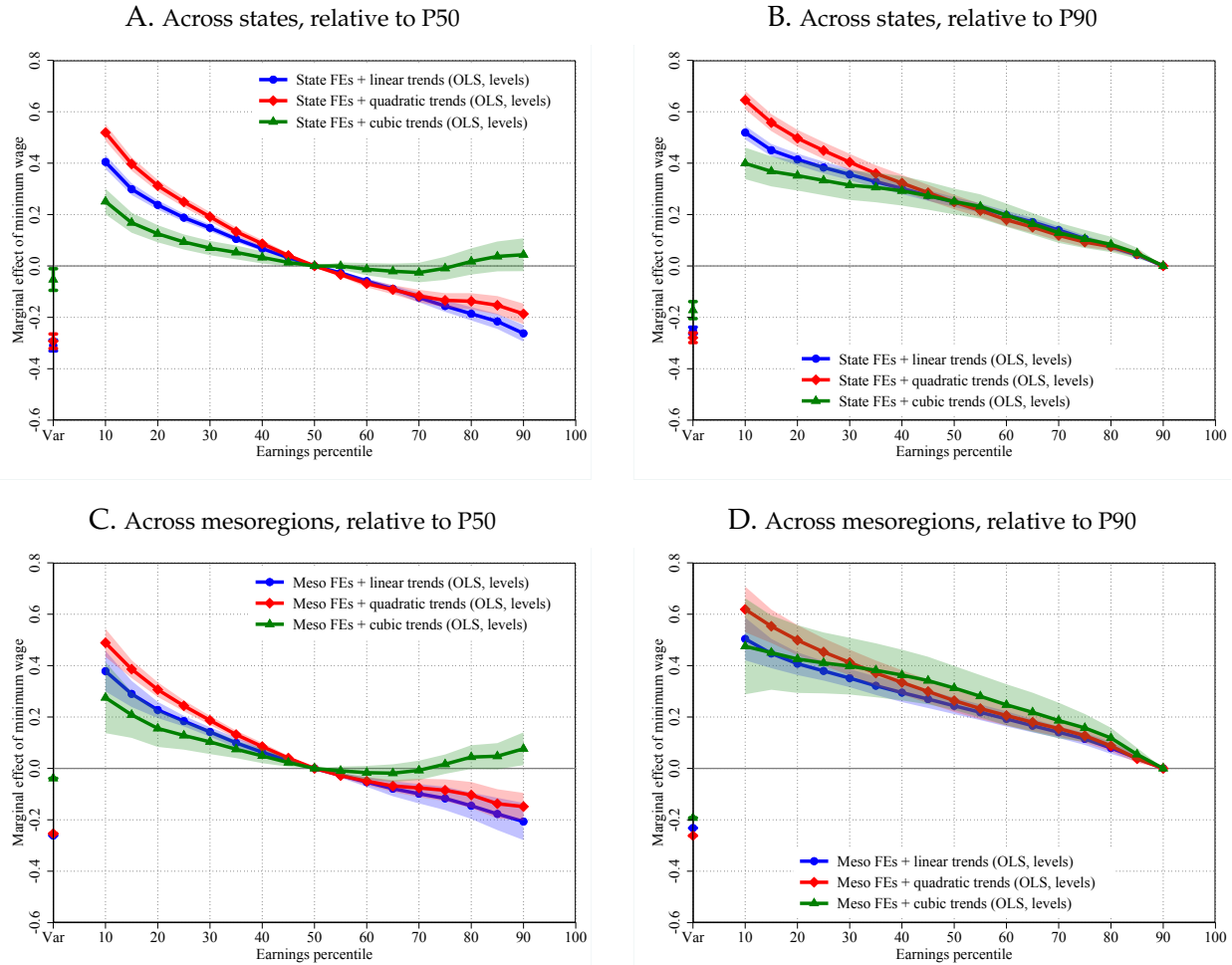
Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from four different specifications (colored lines and error bars or shaded areas) with different region-year controls: year fixed effects (blue line), region fixed effects (red line), region fixed effects in addition to region-specific quadratic time trends (green line)—all estimated using OLS—and the latter specification estimated using an IV strategy (orange line). While the previous four specifications are also shown in the main text, this figure presents results from two additional specifications in each panel: region fixed effects in addition to region-specific quadratic time trends estimated in differences (purple line)—all estimated using OLS—and the latter specification estimated using an IV strategy in differences (pink line). The IV strategy instruments the generalized p -Kaitz index and its square using an instrument set that consists of the log real statutory minimum wage, its square, and the log real statutory minimum wage interacted with the mean of the log real median wage for the region over the full sample period. Within each panel, the estimated marginal effect of the minimum wage on the variance of log earnings (“Var” on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution (“10” to “90” on the x-axis) relative to some base wage p are shown. Panels A and B use the 50th percentile as the base wage (i.e., $p = 50$), while panels C and D use the 90th percentile as the base wage (i.e., $p = 90$). Panels A and C are estimated across Brazil’s 27 states, while panels B and D are estimated across Brazil’s 137 mesoregions. The four error bars and four shaded areas represent 99 percent confidence intervals based on regular standard errors at the state level and clustered standard errors at the mesoregion level. *Source:* RAIS, 1985–2014.

B.8 Additional regression results: Polynomial orders for region-specific trends

There has been a fruitful debate concerning the potential benefits and harms from including region-specific time trends in econometric studies of the minimum wage—see, for example, [Neumark et al. \(2014\)](#), [Allegretto et al. \(2017\)](#), and [Neumark and Wascher \(2017\)](#). To demonstrate the robustness of our findings to the concerns raised by this debate, Figures B.20 and B.21 show additional specifications based on the econometric methodology in Section 3.2. Specifically, we vary the polynomial degree order for region-specific time trends between one and three (with the degree-zero specification being that with only region fixed effects presented in the main text). Figure B.20 estimates these specifications using OLS, while Figure B.21 estimates these specifications using the same IV strategy we introduced in Section 3.2 of the main text.

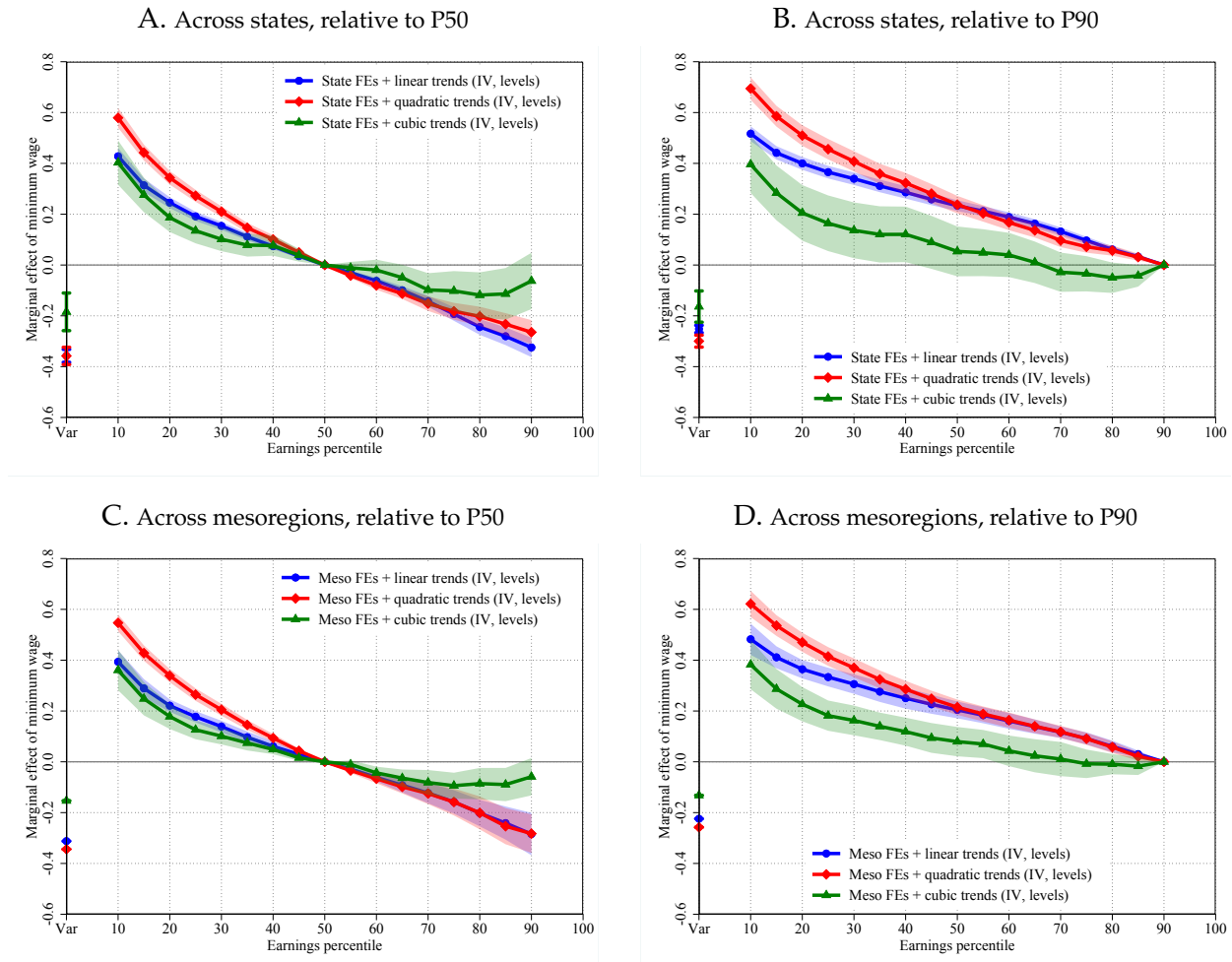
Starting with a description of Figure B.20, we note that specifications with either a linear or quadratic trend show similar patterns in terms of estimated marginal effects, particularly in the upper tail of the wage distribution. The estimated patterns are also similar to the specification with only region fixed effects presented in the main text. The specification with cubic trends delivers less precisely estimated and closer-to-zero point estimates of the marginal effect of the minimum wage, but it remains significant up to between the 40th and 85th percentile of the distribution, depending on whether we normalize to P50 or P90 of the wage distribution. Figure B.21 shows the same set of specifications estimated using an IV strategy. Although standard error bands are larger than before, we find significant spillover effects throughout most of the distribution for linear and quadratic time trends, and up to between the 40th and 85th wage percentile for cubic time trends. While our finding of far-reaching spillover effects of the minimum wage is robust to the choice of region-specific time trends and specifications, we note that including region-specific cubic time trends in this specification may possibly be asking for too much from our comparably short region-year panel of 30 years.

Figure B.20. Estimated minimum wage effects on the wage distribution, various polynomial orders for region-specific time trends using OLS



Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from three different specifications (colored lines and error bars or shaded areas) with different polynomial degrees of the region-specific time trends: linear (blue line), quadratic (red line), and cubic (green line) polynomials in the calendar year—all estimated using OLS in levels. Within each panel, the estimated marginal effect of the minimum wage on the variance of log earnings (“Var” on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution (“10” to “90” on the x-axis) relative to some base wage p are shown. Panels A and C use the 50th percentile as the base wage (i.e., $p = 50$), while panels B and D use the 90th percentile as the base wage (i.e., $p = 90$). Panels A and B are estimated across Brazil’s 27 states, while panels C and D are estimated across Brazil’s 137 mesoregions. The four error bars and four shaded areas represent 99 percent confidence intervals based on regular standard errors at the state level and clustered standard errors at the mesoregion level. Source: RAIS, 1985–2014.

Figure B.21. Estimated minimum wage effects on the wage distribution, various polynomial orders for region-specific time trends using IV

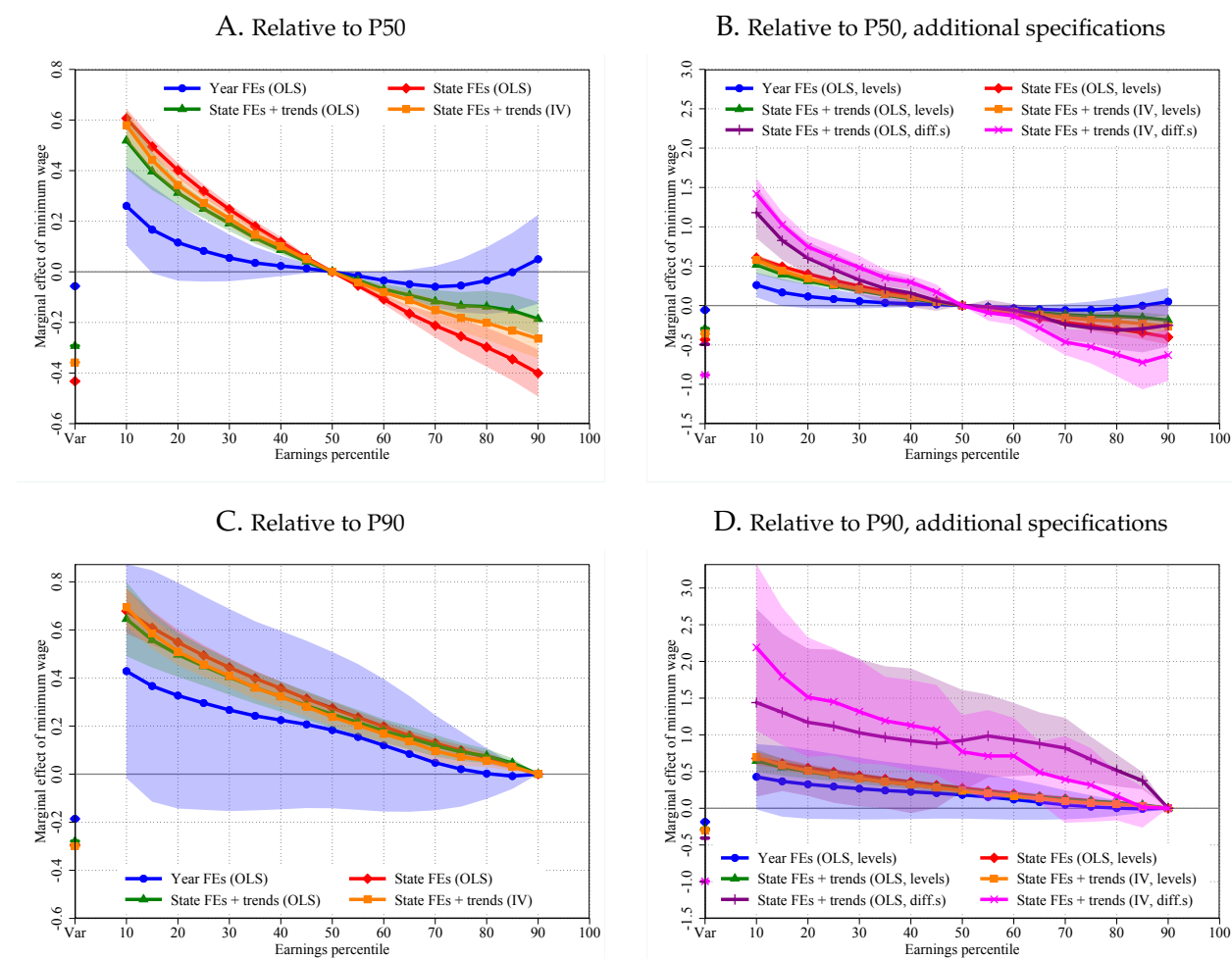


Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from three different specifications (colored lines and error bars or shaded areas) with different polynomial degrees of the region-specific time trends: linear (blue line), quadratic (red line), and cubic (green line) polynomials in the calendar year—all estimated using IV in levels. Within each panel, the estimated marginal effect of the minimum wage on the variance of log earnings ("Var" on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution ("10" to "90" on the x-axis) relative to some base wage p are shown. Panels A and C use the 50th percentile as the base wage (i.e., $p = 50$), while panels B and D use the 90th percentile as the base wage (i.e., $p = 90$). Panels A and B are estimated across Brazil's 27 states, while panels C and D are estimated across Brazil's 137 mesoregions. The four error bars and four shaded areas represent 99 percent confidence intervals based on regular standard errors at the state level and clustered standard errors at the mesoregion level. Source: RAIS, 1985–2014.

B.9 Additional regression results: Clustering standard errors at the state level

Figure B.22 shows additional specifications based on the econometric methodology in Section 3.2 of the main text. Specifically, although the number of clusters at the state level falls below common thresholds for clustering (Cameron and Miller, 2015), in this section we cluster standard errors at the state level. In contrast, in Section 3.2, we presented results based on unadjusted standard errors. The main take-away from Figure B.22 is that standard errors become somewhat larger under state-level clustering, especially for the specification with only year fixed effects. At the same time, our conclusions regarding the reach of spillovers remain unchanged under our preferred specification with state fixed effects and state-specific quadratic time trends, estimated using an IV strategy in levels.

Figure B.22. Estimated minimum wage effects on the wage distribution, clustered standard errors at the state level



Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from four or six different specifications (colored lines and error bars or shaded areas) with different sets of controls, all clustered at the state level: year fixed effects (blue line), region fixed effects (red line), region fixed effects in addition to region-specific quadratic time trends (green line)—all estimated using OLS—the last of these specifications estimated using an IV strategy (orange line), region fixed effects in addition to region-specific quadratic time trends estimated in differences (purple line)—all estimated using OLS—and the last of these specifications estimated using an IV strategy in differences (pink line). Within each panel, the estimated marginal effect of the minimum wage on the variance of log earnings (“Var” on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution (“10” to “90” on the x-axis) relative to some base wage p are shown. Panels A and B use the 50th percentile as the base wage (i.e., $p = 50$), while panels C and D use the 90th percentile as the base wage (i.e., $p = 90$). Panels A and C show only the first four specifications, while panels B and D show two additional specifications. The error bars and shaded areas represent 99 percent confidence intervals based on standard errors that are clustered at the state level. Source: RAIS, 1985–2014.

B.10 Comparison with Haanwinckel (2020)

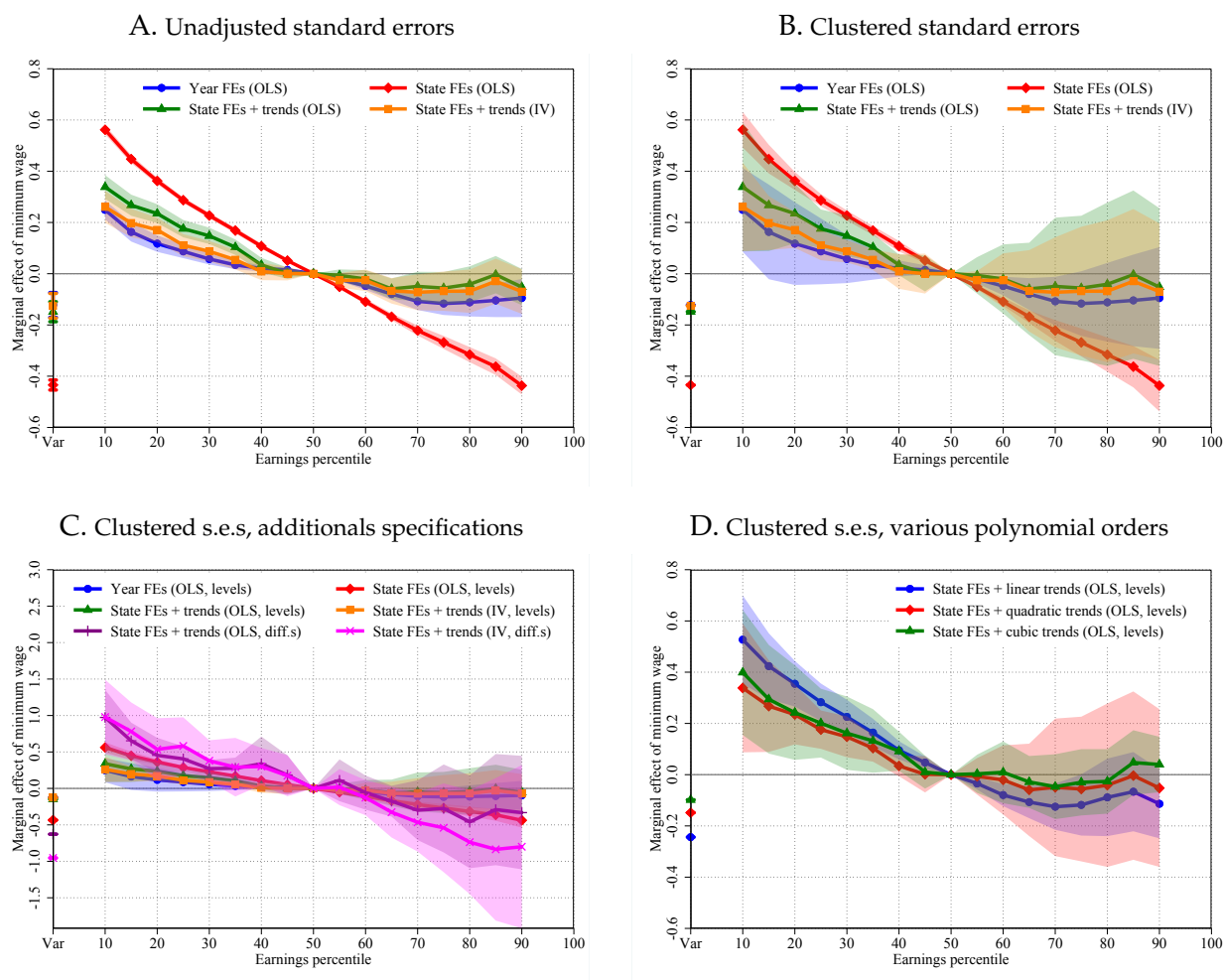
In related work, Haanwinckel (2020) follows our approach of estimating spillover effects of the minimum wage in Brazil using a methodology based on the seminal framework by Lee (1999) and the more recent contribution by Autor et al. (2016). The paper by Haanwinckel (2020) is the first to estimate a variant of our econometric framework that incorporates an IV strategy similar to that proposed by Autor et al. (2016). In Section 3.2, we show that moving from OLS to IV specifications changes little regarding our conclusions about the reach of spillover effects estimated in our sample. Using an IV strategy in differences on a pooled subset of 14 years of the RAIS data from 1996 to 2001, 2005 to 2009, and 2011 to 2013, Haanwinckel (2020) reports significant spillover effects of the minimum wage up to the 40th wage percentile with a 95 percent confidence interval, and up to the 30th wage percentile with a 99 percent confidence interval. The standard error bands estimated by Haanwinckel (2020) are relatively larger above the median, rendering the point estimates indistinguishable from zero. In contrast, our results based on the full sample of 30 years of the RAIS data from 1985 to 2014 robustly indicate significant spillovers up to at least the 70th wage percentile with comparably narrow standard error bands. Therefore, it is natural to suspect that using a subset of years, that accounts for less than half of the number of total years that we use in our analysis may explain the differences in findings between Haanwinckel (2020) and our paper.

To reconcile these differences, Figure B.23 shows additional specifications based on the econometric methodology presented in Section 3.2 of the main text but restricting our sample to the years 1996–2001, 2005–2009, and 2011–2013; this selection replicates the one in Haanwinckel (2020). To be explicit, relative to our baseline results presented in Section 3.2 based on the complete RAIS data from 1985 to 2014, this specification excludes 16 out of the 30 years—namely, 1985–1995, 2002, 2003, 2004, 2010, and 2014.

The results in Figure B.23, which are based on this subset of years from our data, show substantial differences relative to our baseline results based on the complete sample from 1985–2014. With unadjusted standard errors (panel A), estimated spillover effects are significant up to at least the 35th percentile. Under our preferred specification with state fixed effects and state-specific quadratic time trends estimated using an IV strategy in differences, point estimates remain close to zero and insignificant—with the exception of the marginal effect for the 65th percentile—from the 40th wage percentile onwards. Clustering standard errors (panel B) leads to significant effects up to, but not beyond, the 35th wage percentile under the same specification. Estimating the same specification in differences (panel C) yields similar insights; marginal effects are significant up to but not beyond the 30th wage percentile. Finally, estimating a similar specification in OLS across different polynomial orders for the state-specific time trends (panel D) reveals that the baseline estimates with quadratic trends (red line) are not robust to inclusion of either linear (blue line) or cubic time trends (green line).

From this, we conclude that estimating our specification on a subset of years that accounts for less than half the number of total years used in our baseline analysis leads to significantly more noisy estimates that replicate the findings of minimum wage spillovers up to between the 30th and the 40th wage percentile, as reported in Haanwinckel (2020). In contrast, considering the full set of years in the data allows us to exploit significantly more variation in the effective bindingness of the federal minimum wage—see Appendix B.11—which leads us to find minimum wage spillovers that robustly reach up to at least the 70th wage percentile. This finding is reassuring, given that our structural model, as well as the alternative structural model developed by Haanwinckel (2020), predicts spillover effects throughout most of the wage distribution.

Figure B.23. Estimated minimum wage effects on the wage distribution, alternative selection of years replicating Haanwinckel (2020)



Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2). Each panel shows the results from four or six different specifications (colored lines and error bars or shaded areas) with different sets of controls, all based on the subset of years of RAIS data used in Haanwinckel (2020) and clustered at the state level: year fixed effects (blue line), region fixed effects (red line), region fixed effects in addition to region-specific quadratic time trends (green line)—all estimated using OLS—the last of these specifications estimated using an IV strategy (orange line), region fixed effects in addition to region-specific quadratic time trends estimated in differences (purple line)—all estimated using OLS—and the last of these specifications estimated using an IV strategy in differences (pink line). Within each panel, the estimated marginal effect of the minimum wage on the variance of log earnings (“Var” on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution (“10” to “90” on the x-axis) relative to some base wage p are shown. Panels A and C use the 50th percentile as the base wage (i.e., $p = 50$), while panels B and D use the 90th percentile as the base wage (i.e., $p = 90$). Panels A and B show only the first four specifications, while panels C and D show two additional specifications. The error bars and shaded areas represent 99 percent confidence intervals based on standard errors that are clustered at the state level. Source: RAIS, 1996–2001, 2005–2009, and 2011–2013.

B.11 Comparison of relative bindingness of the minimum wage, Brazil versus U.S.

Table B.1 compares the relative bindingness of the minimum wage, as proxied by lower-tail wage inequality, between Brazil and the U.S.

Table B.1. Lower-tail wage inequality in Brazil and in the U.S.

Year	Mean log ($P10/P50$)	
	Brazil	U.S.
1985	−0.64	−0.74
1986	−0.68	−0.74
1987	−0.75	−0.73
1988	−0.70	−0.72
1989	−0.71	−0.72
1990	−0.83	−0.72
1991	−0.80	−0.71
1992	−0.78	−0.72
1993	−0.74	−0.73
1994	−0.83	−0.71
1995	−0.81	−0.71
1996	−0.76	−0.71
1997	−0.73	−0.69
1998	−0.71	−0.69
1999	−0.69	−0.69
2000	−0.66	−0.68
2001	−0.62	−0.68
2002	−0.58	−0.69
2003	−0.54	−0.69
2004	−0.53	−0.70
2005	−0.50	−0.71
2006	−0.46	−0.70
2007	−0.44	−0.70
2008	−0.44	−0.71
2009	−0.43	−0.74
2010	−0.42	−0.73
2011	−0.43	−0.72
2012	−0.42	−0.74
2013	−0.43	
2014	−0.44	
Min.	−0.83	−0.74
Max.	−0.42	−0.68
Range	0.41	0.06
Std. dev.	0.15	0.02

Notes: This table compares the relative bindingness of the minimum wage, as proxied by lower-tail wage inequality, between Brazil and the U.S. between 1985 and 2014. Lower-tail inequality is measured using the mean log wage percentile ratio $P10/P50$ computed across states in a given year. “Min.” denotes the minimum of the mean log wage percentile ratio taken across all years in a given country. “Max.” denotes the maximum of the mean log wage percentile ratio taken across all years in a given country. “Range” denotes the range (i.e., the maximum minus the minimum) of the mean log wage percentile ratio taken across all years in a given country. “Std. dev.” denotes the standard deviation of the mean log wage percentile ratio taken across all years in a given country. The data for Brazil are the same as those used in the analysis presented in the main text, which covers 1985 to 2014. The data for the U.S. are from Autor et al. (2016), who report these statistics between 1979 and 2012. *Source:* RAIS, 1985–2014, and Autor et al. (2016).

B.12 Comparison of wages in nominal terms and in multiples of the minimum wage

In this subsection, we compare the distributions of (changes in) wages across two numeraire: current BRL and the current minimum wage. We demonstrate that while the minimum wage does share certain attributes of a numeraire (Neri and Moura, 2006), it does so imperfectly. Consequently, it is not purely mechanical that the minimum wage affects wages throughout the wage distribution, as we document in Section 3.2.

To see why the distinction between the two numeraires matters, consider the following example. Suppose worker A gets paid 2.0 times the minimum wage throughout his or her employment spell, which lasts from January to June of a given calendar year. Suppose worker B also gets paid 2.0 times the minimum wage, but this worker's employment spell lasts from January to December. Now suppose that the statutory minimum wage stands at BRL 400 from January to June, then increases to BRL 500 from July to December. Then the two workers' wages are the same in multiples of the current minimum wage (i.e., both receive constant pay equal to 2.0 times the current minimum wage) but differ in terms of nominal BRL (i.e., worker A has a mean wage of BRL 400 that consists of a constant stream, while worker B has a mean wage of BRL 450 that changes levels over time). Therefore, the numeraire matters for our measure of wage dispersion within and across individuals.

We begin our analysis with the variable in RAIS that contains each job's mean wage in multiples of the current minimum wage in each year, which is the one underlying our entire analysis in the main text. To obtain a wage measure in nominal BRL, we multiply the mean wage in multiples of the current minimum wage by the months-of-employment-weighted mean of the minimum wage prevailing during that year, using each job's start and end months. Our measurement likely contains some noise because both wages and the minimum wage itself can change over time within a job spell.

Comparing log wages in current BRL and multiples of the current minimum wage, we note that only 2.4 percent of all workers have a constant wage in terms of multiples of the current minimum wage between two consecutive years. For comparison, that number is 0.2 percent in nominal terms. That the latter number is smaller than the former suggests that the minimum wage does serve some numeraire function. At the same time, both shares are small, highlighting the importance of idiosyncratic wage changes.

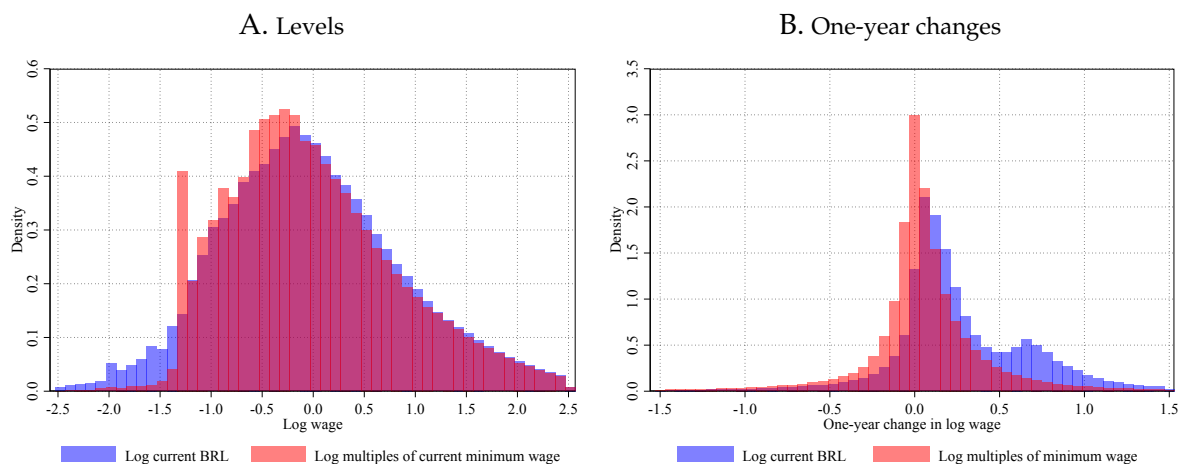
Figure B.24 shows histograms of (changes in) wages in each of the two numeraires. Panel A shows that the distribution of log wages has a relatively more pronounced spike in multiples of the current minimum wage, which is our baseline wage measure. The distribution of one-year changes in log wages in panel B shows less dispersion in multiples of the minimum wage, particularly in the right tail. However, both distributions are significantly dispersed, indicating that both wage measures are far from fixed at a constant multiple of the numeraire.

Another way to assess the minimum wage's numeraire function is a simple variance decomposition in which we decompose some outcome variable of interest, y_{it} , across individuals i and years t as

$$Var(y_{it}) = \underbrace{Var(\mathbb{E}[y_{it}|i])}_{\text{Variance of individual-level means}} + \underbrace{Var(y_{it} - \mathbb{E}[y_{it}|i])}_{\text{Variance of dispersion around individual-level means}}. \quad (\text{B.1})$$

In this case, the outcome variable y_{it} can be either log wages or the one-year changes in log wages. Table B.2 shows the results from the variance decomposition in equation (B.1) for different population subgroups (all workers, only stayers in the same occupation, only stayers at the same employer), for log wages versus one-year changes in log wages, and for the two numeraires: current BRL and the current minimum wage. Starting with wage levels in panel A, the total variance of log wages is slightly higher in current BRL than in multiples of the current minimum wage, 88.8 log points compared with 75.0

Figure B.24. Histograms of (changes in) log wages, nominal versus multiples of minimum wage



Notes: Figure shows levels of (panel A) and one-year changes in (panel B) log wages in terms of two numeraire: current BRL and the current minimum wage. That is, each panel shows the distribution of earnings in log current BRL (blue bars), calculated as the logarithm of the mean multiples of the current minimum wage recorded during each calendar year multiplied by the mean minimum wage prevailing during the months of employment during that year, and the log mean multiples of the current minimum wage (red bars). For panel A, log wages in each numeraire are normalized to be mean zero. Source: RAIS, 1994–1998.

log points. Around 25 percent of the total variance is attributable to the variance of dispersion around individual-level means when measured in current BRL, compared with 12 percent when measured in multiples of the current minimum wage. This suggests that the minimum wage does serve as a numeraire for wages in the economy. Turning now to one-year changes in log wages, we see that, at the same time, there is substantial dispersion in wages from one year to the next, regardless of the numeraire, with variances of 25.1 log points in current BRL and 17.9 log points in multiples of the current minimum wage. A similar share of the total variance of one-year changes in log wages is due to the variance of dispersion around individual-level means, around 73 percent in current BRL, compared with 70 percent in multiples of the current minimum wage. This tells us that earnings are not fixed, not even in terms of multiples of the current minimum wage. Finally, Panels B and C show the same variance decompositions for workers who remain in the same occupation (panel B) or at the same employer (panel C) between years, leading us to draw broadly similar conclusions.

We conclude that, while the minimum wage does share certain attributes of a numeraire (Neri and Moura, 2006), it does so imperfectly, and thus our empirical results in Section 3.2 are unlikely to be driven by this phenomenon.

Table B.2. Comparison of wages in nominal terms and in multiples of the minimum wage

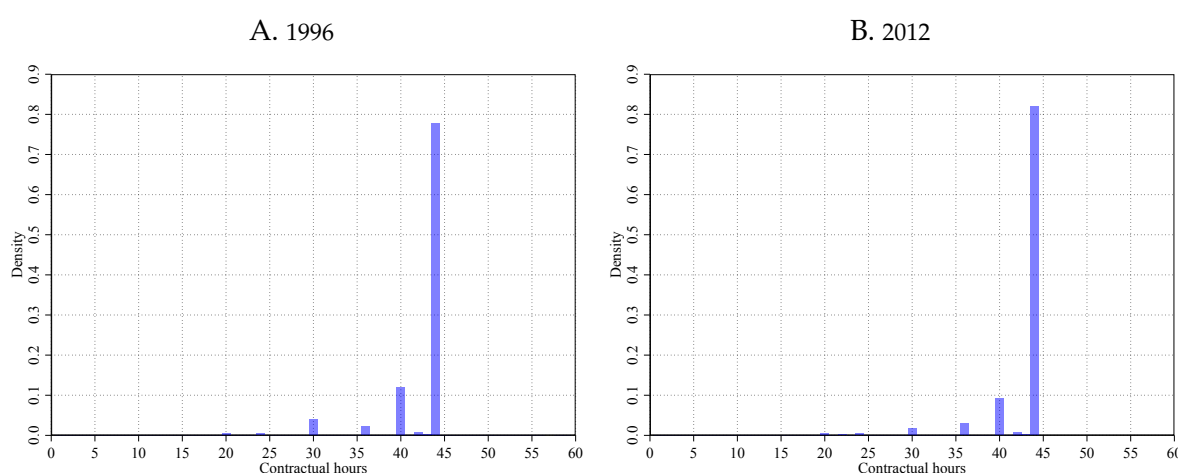
	Log wages		One-year changes in log wages	
	Current BRL	Current MW	Current BRL	Current MW
<i>Panel A. All workers</i>				
Total variance	0.888	0.750	0.251	0.179
Variance of individual-level means	0.667 (75%)	0.661 (88%)	0.069 (27%)	0.054 (30%)
Variance of dispersion around individual-level means	0.221 (25%)	0.088 (12%)	0.182 (73%)	0.125 (70%)
<i>Panel B. Only stayers in same occupation</i>				
Total variance	0.791	0.768	0.174	0.115
Variance of individual-level means	0.727 (92%)	0.728 (95%)	0.071 (41%)	0.055 (48%)
Variance of dispersion around individual-level means	0.064 (8%)	0.040 (5%)	0.103 (59%)	0.060 (52%)
<i>Panel C. Only stayers at same employer</i>				
Total variance	0.791	0.778	0.157	0.101
Variance of individual-level means	0.739 (93%)	0.741 (95%)	0.065 (41%)	0.052 (51%)
Variance of dispersion around individual-level means	0.052 (7%)	0.036 (5%)	0.091 (58%)	0.049 (49%)

Notes: This table shows the total variance of log wages and of one-year changes in log wages based on equation (B.1). Panel A shows the results for the whole population, while panel B shows those for workers who stay in the same occupation between consecutive years. Panel C shows those for workers who stay at the same employer between years. Source: RAIS, 1994–1998.

B.13 Hours distribution and its relation to the bindingness of the minimum wage

Most adult males in Brazil's formal sector work under a full-time contract, defined as either 40 work hours (spread across 5 days) or 44 work hours (spread across 6 days) per week. Figure B.25 shows the raw distribution of hours for this population for 1996–2000 in panel A, and for 2008–2012 in panel B. In the initial period, around 74 percent of all workers work 44 hours per week and another 15 percent work 40 hours per week, constituting around 89 percent of workers in full-time employment. Only around 5 percent of all employees are in employment arrangements with less than 35 contractual work hours per week.⁵³ Furthermore, the comparison between panels A and B show that there is no evidence over time—as the minimum wage increases—of a shift towards shorter work weeks in the aggregate. In contrast, there was a small reduction in the share of workers with 30 hour contracts that in the aggregate shifted toward 44 hour contracts.

Figure B.25. Histogram of contractual hours

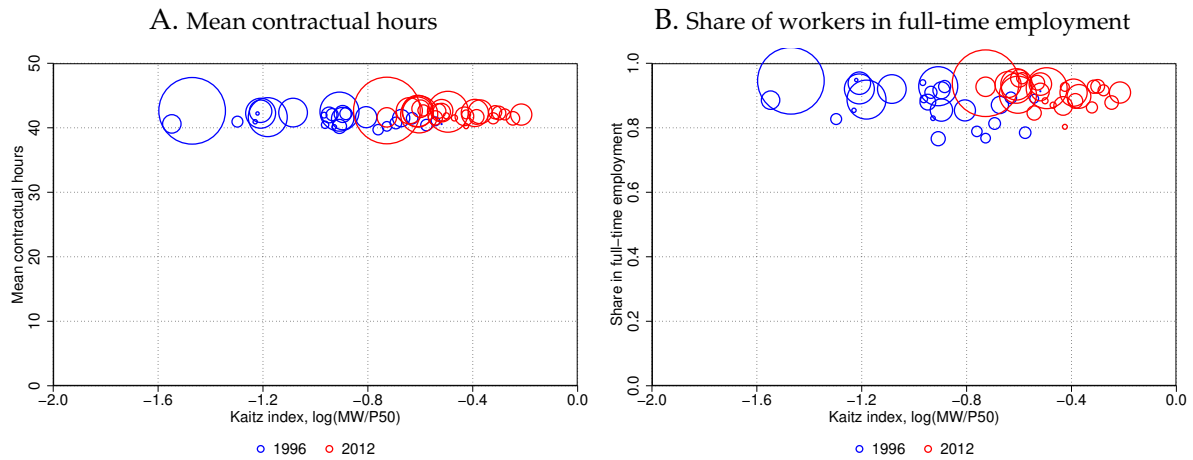


Notes: Figure shows density of contractual work hours for period 1996–2000 in panel A and for 2012 in panel B. A small number of observations reporting more than 60 hours are omitted from the graphs. Source: RAIS.

Going beyond the aggregate statistics, there is also little systematic covariation between the relative bindingness of the minimum wage and work hours across Brazilian states. Figure B.26 shows that both the share of full-time employment in panel A and the mean number of hours in panel B stay constant as the minimum wage increases between 1996 and 2012. While in 1996 there is a weak systematic negative relationship between the full-time worker share and the bindingness of the minimum wage, measured by the Kaitz index, this appears driven almost entirely by transitory fluctuations and permanent state-specific heterogeneity, rather than a correlation with the rising minimum wage over time within states.

⁵³According to the Bureau of Labor Statistics, a higher share, around 12 percent of employees, worked part-time (less than 35 hours) in the U.S. in 2017.

Figure B.26. Relation between contractual hours and bindingness of the minimum wage



Notes: Figure shows for male workers ages 18–49 the share in full-time employment, defined as working 40 hours or more (panel A), and the mean number of contractual work hours (panel B), against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, across states in 1996 and 2012. Area of circles is proportional to population size. Source: RAIS.

B.14 Robustness of effects of the minimum wage on employment and firm dynamics

Table B.3. Effects of the minimum wage on employment and firm dynamics

	(1)	(2)	(3)	(4)	(5)
<i>Panel A. Cross-sectional household survey data (PNAD)</i>					
Log population size	−2.225 (0.255)	0.468 (0.019)	0.244 (0.023)	0.057 (0.030)	0.071 (0.028)
Labor force participation rate	−0.058 (0.005)	0.038 (0.007)	−0.093 (0.012)	0.009 (0.016)	−0.020 (0.016)
Employment rate	0.002 (0.004)	0.015 (0.004)	−0.036 (0.008)	0.014 (0.015)	−0.010 (0.014)
Formal employment share	−0.450 (0.013)	0.134 (0.010)	−0.094 (0.016)	0.024 (0.020)	−0.007 (0.019)
<i>Panel B. Longitudinal household survey data (PME)</i>					
Transition rate nonformal-formal	0.043 (0.008)	0.085 (0.007)	−0.021 (0.015)	−0.003 (0.017)	−0.011 (0.016)
Transition rate formal-nonformal	0.026 (0.004)	−0.021 (0.004)	−0.005 (0.007)	−0.005 (0.009)	−0.002 (0.008)
<i>Panel C. Administrative linked employer-employee data (RAIS)</i>					
Mean log hours worked	−0.033 (0.003)	0.019 (0.001)	0.013 (0.003)	−0.006 (0.004)	−0.012 (0.009)
Mean log firm size	−0.474 (0.066)	−0.311 (0.041)	0.762 (0.038)	0.203 (0.040)	0.920 (0.062)
Probability of remaining employed	0.028 (0.005)	−0.048 (0.004)	−0.051 (0.006)	0.042 (0.007)	0.056 (0.013)
Year fixed effects	✓	✗	✗	✗	✗
Region fixed effects	✗	✓	✓	✓	✓
Region-time trend polynomial	✗	✗	Linear	Quadratic	Cubic

Notes: Table shows predicted marginal effects with standard errors in parentheses evaluated at the worker-weighted mean across Brazil's 27 states. Each cell corresponds to the estimated coefficient and standard error from one regression with the relevant dependent variable (row) and various controls (columns). Underlying regressions are variants of equation (2) including year fixed effects (column 1), state fixed effects (column 2), and state fixed effects in addition to state-specific time trends of various polynomial orders (columns 3–5). *Source:* PNAD, 1996–2012, PME, 2002–2012, and RAIS, 1985–2014.

B.15 Subgroup analysis of effects of the minimum wage on employment

Table B.4. Effects of the minimum wage on employment and firm dynamics

	(1)	(2)	(3)	(4)	(5)
<i>Panel A. Cross-sectional household survey data (PNAD)</i>					
Log population size	0.057 (0.030)	−0.396 (0.304)	0.073 (0.059)	0.126 (0.070)	0.047 (0.112)
Labor force participation rate	0.009 (0.016)	−0.000 (0.013)	−0.001 (0.019)	0.027 (0.016)	−0.013 (0.008)
Employment rate	0.014 (0.015)	−0.011 (0.020)	0.030 (0.018)	0.023 (0.019)	−0.030 (0.018)
Formal employment share	0.024 (0.020)	0.076 (0.050)	0.033 (0.030)	0.002 (0.028)	−0.049 (0.043)
<i>Panel B. Longitudinal household survey data (PME)</i>					
Transition rate nonformal-formal	−0.003 (0.017)	0.013 (0.018)	0.004 (0.028)	−0.021 (0.030)	0.034 (0.053)
Transition rate formal-nonformal	−0.005 (0.009)	−0.006 (0.017)	−0.002 (0.015)	−0.005 (0.010)	−0.002 (0.017)
Sample	Overall	< primary school	Primary school	High school	College
Year fixed effects	✗	✗	✗	✗	✗
Region fixed effects	✓	✓	✓	✓	✓
Region-time trend polynomial	Quadratic	Quadratic	Quadratic	Quadratic	Quadratic

Notes: Table shows predicted marginal effects with standard errors in parentheses evaluated at the worker-weighted mean across Brazil's 27 states. Each cell corresponds to the estimated coefficient and standard error from one regression with the relevant dependent variable (row) for various education groups (columns). Underlying regressions are variants of equation (2) including state fixed effects and state-specific quadratic time trends. *Source:* PNAD, 1996–2012; PME, 2002–2012; and RAIS, 1985–2014.

C Model Appendix

This appendix provides further details on the equilibrium model presented in Section 4, including subsections on the model-implied employment distribution (Appendix C.1), the steady-state firm size mapping (Appendix C.2), the equilibrium definition (Appendix C.3), and the numerical solution algorithm (Appendix C.4).

C.1 The employment distribution

The employment distribution, $G(z|a, s)$, is given by the Kolmogorov Forward Equation (KFE),

$$0 = -\left(\delta(a, s) + sp(a, s)(1 - F(z|a, s))\right)G(z|a, s)e(a, s) + p(a, s)u(a, s)F(z|a, s)$$

The first term reflects the number $G(z|a, s)e(a, s)$ of workers employed at firms with productivity of at most z . These workers flow into unemployment at rate $\delta(a, s)$. They receive outside offers at rate $sp(a, s)$, which in equilibrium they accept if they come from more productive firms, $1 - F(z|a, s)$. The last term reflects inflows into firms with productivity at most z . Since no employed worker accepts a job offer from a less productive firm, the only inflows are from unemployment. In particular, the unemployed receive job offers at rate $p(a, s)$. Rearranging, we get the expression in the paper,

$$G(z|a, s) = \frac{p(a, s)F(z|a, s)}{\delta(a, s) + sp(a, s)(1 - F(z|a, s))} \frac{u(a, s)}{e(a, s)}$$

C.2 The steady-state firm size mapping

The size $l(w, v|a, s)$ of a firm posting vacancies v with wage w in market (a, s) is given by the KFE:

$$0 = -\left(\delta(a, s) + sp(a, s)(1 - F(z|a, s))\right)l(w, v|a, s) + vq(a, s) \left(\frac{u(a, s)}{S(a, s)} + \frac{se(a, s)}{S(a, s)}G(z|a, s)\right)$$

The firm loses workers to unemployment and up the job ladder. Each vacancy contacts a potential hire at rate $q(a, s)$; the potential hire is unemployed with probability $u(a, s)/S(a, s)$ and employed with complementary probability. Employed workers are hired iff they are at a less productive firm. Noting that $q(a, s) = p(a, s)S(a, s)/V(a, s)$, and substituting and rearranging,

$$l(w, v|a, s) = \frac{v}{V(a, s)}p(a, s) \frac{u(a, s) + se(a, s)G(z|a, s)}{\delta(a, s) + sp(a, s)(1 - F(z|a, s))}$$

Using equation (6) to substitute for the employment distribution $G(\cdot)$,

$$\begin{aligned} l(w, v|a, s) &= \frac{v}{V(a, s)}p(a, s) \frac{u(a, s) \frac{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))}{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))} + s \frac{p(a, s)F(w|a, s)}{\delta(a, s) + sp(a, s)(1 - F(w|a, s))} u(a, s)}{\delta(a, s) + sp(a, s)(1 - F(w|a, s))} \\ &= \frac{vu(a, s)p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)(1 - F(w|a, s)) + sp(a, s)F(w|a, s)}{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))^2} \\ &= \frac{vu(a, s)p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))^2} \end{aligned}$$

C.3 Equilibrium definition

In this section, we define an equilibrium of the model economy presented in Section 4.

Definition 1. *An equilibrium of our economy consists of*

- *a set of wage and vacancy posting policies $\{w(z|a, s), v(z|a, s)\}$ that solve firms' problem;*
- *a reservation wage $r(a, s)$ that solves workers' problem; and*
- *aggregate states $\{G(z|a, s), e(a, s), u(a, s), V(a, s), p(a, s), q(a, s)\}$ that are consistent with their laws of motion in steady-state as well as the matching technology.*

To characterize the equilibrium, we start by substituting our assumed iso-elastic cost function into firms' problem (7) and taking first-order conditions,

$$c(a, s)v(z|a, s)^\eta = (z - w) \frac{\partial l(w, v|a, s)}{\partial v} \quad (\text{C.1})$$

$$l(w, v|a, s) = (z - w) \frac{\partial l(w, v|a, s)}{\partial w}; \quad (\text{C.2})$$

Differentiating the equilibrium size (8) with respect to vacancies,

$$\frac{\partial l(w, v|a, s)}{\partial v} = \frac{u(a, s)p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))^2};$$

Substituting this into the first-order condition for vacancies (C.1),

$$c(a, s)v(z|a, s)^\eta = (z - w(z|a, s)) \frac{u(a, s)p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - F(w(z|a, s)|a, s)))^2}; \quad (\text{C.3})$$

Differentiating the equilibrium size (8) with respect to wages,

$$\frac{\partial l(w, v|a, s)}{\partial w} = 2sp(a, s)f(w|a, s) \frac{vu(a, s)p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - F(w|a, s)))^3};$$

and substituting this into the first-order condition for wages (C.2) and cancelling terms,

$$\delta(a, s) + sp(a, s)(1 - F(w(z|a, s)|a, s)) = (z - w(z|a, s)) 2sp(a, s)f(w(z|a, s)|a, s). \quad (\text{C.4})$$

As in [Burdett and Mortensen \(1998\)](#), more productive firms post higher wages. Consequently,

$$F(w(z|a, s)|a, s) = \frac{M}{V} \int_{\underline{z}}^z v(\tilde{z}|a, s) d\Gamma(\tilde{z}), \quad f(w(z|a, s)|a, s)w'(z|a, s) = \frac{M}{V} v(z|a, s)\gamma(z).$$

Define $h(z|a, s) = F(w(z|a, s)|a, s)$ so that $f(w(z|a, s)|a, s) = h'(z|a, s)/w'(z|a, s)$. Substituting in (C.4),

$$w'(z|a, s) = (z - w(z|a, s)) \frac{2sp(a, s)h'(z|a, s)}{\delta(a, s) + sp(a, s)(1 - h(z|a, s))}. \quad (\text{C.5})$$

We also have that $h'(z|a, s) = \frac{M}{V(a, s)} v(z|a, s) \gamma(z)$ so that $v(z|a, s) = \frac{V(a, s)}{M} \frac{h'(z|a, s)}{\gamma(z)}$. Substituting in (C.3),

$$h'(z|a, s) = \frac{M}{V(a, s)} \gamma(z) \left(\frac{1}{c(a, s)} (z - w(z|a, s)) \frac{u(a, s) p(a, s)}{V(a, s)} \frac{\delta(a, s) + sp(a, s)}{(\delta(a, s) + sp(a, s)(1 - h(z|a, s)))^2} \right)^{\frac{1}{\eta}} \quad (\text{C.6})$$

Equations (C.5)–(C.6) constitute a system of differential equations in the two functions $w(z|a, s)$ and $h(z|a, s)$. The first boundary condition is that wages of the least productive firm must equal the lowest possible pay in the market, $\lim_{z \rightarrow \underline{z}(a, s)} w(z|a, s) = \max \left\{ r(a, s), \frac{w^{\min}}{a} \right\}$. The second boundary condition is that the CDF of the offer distribution is zero for the least productive firm, $\lim_{z \rightarrow \underline{z}(a, s)} h(z|a, s) = 0$, where $\underline{z}(a, s)$ is the least productive firm active in market (a, s) : $\underline{z}(a, s) = \max \left\{ \underline{z}, \max \left\{ r(a, s), \frac{w^{\min}}{a} \right\} \right\}$. Finally, the key equilibrium consistency condition is that the total number of vacancies, $V(a, s)$, is such that the CDF of offered wages integrates to one, $\lim_{z \rightarrow \bar{z}} h(z|a, s) = 1$.

C.4 Numerical solution algorithm

Recall that the parameter vector \mathbf{p} includes the reduced-form job finding rate λ . Hence, given a parameter vector, we know all worker flows, since \mathbf{p} also includes $\{\delta_0, \delta_1, \phi_0, \phi_1, \pi\}$. We use the flows to get the implied stock of workers, $\{u(a, s), e(a, s)\}$, and based on that, we recover the required aggregate number of vacancies $V(a, s)$ consistent with the job finding rate $p(a, s) = \lambda$. Recall that the parameter vector \mathbf{p} also includes the reduced-form parameters fully characterizing the reservation wage, $r(a, s)$. Hence, we also know the boundary conditions.

To solve for the equilibrium, we start by solving the system of differential equations (9) for a low cost $c_0(a, s)$ and a high cost, $c_1(a, s)$. Recall that the key consistency condition is $\lim_{z \rightarrow \bar{z}} h(z|a, s) = 1$. We require that under the low cost, firms create too many jobs, $\lim_{z \rightarrow \bar{z}} h_0(z|a, s) > 1$, while under the high cost, firms create too few jobs, $\lim_{z \rightarrow \bar{z}} h_1(z|a, s) < 1$. If this is not true under $c_0(a, s)$ ($c_1(a, s)$), we adjust $c_0(a, s)$ ($c_1(a, s)$) down (up) until it is true. After we have found $c_0(a, s)$ and $c_1(a, s)$ such that both of these conditions hold, we apply a bisection to find the cost $c(a, s)$ such that $\lim_{z \rightarrow \bar{z}} h(z|a, s) = 1$.

We subsequently simulate a monthly approximation to the model, starting workers off from the steady-state distribution. We follow exactly our empirical procedure to construct both a monthly and an annual data set based on the simulated data, including how to select a main employment spell and compute all outcome variables of interest. In particular, we estimate an AKM regression based on the simulated annual data set restricted to the largest connected set, which as in the data covers the vast majority of employment spells.

The steps above are sufficient to estimate the model. Having obtained an estimated parameter vector \mathbf{p}^* , we compute the implied flow value of leisure, $b(a)$, such that the reservation wage is consistent with our reduced-form parameter or alternatively makes workers indifferent between working at the minimum wage and unemployment.

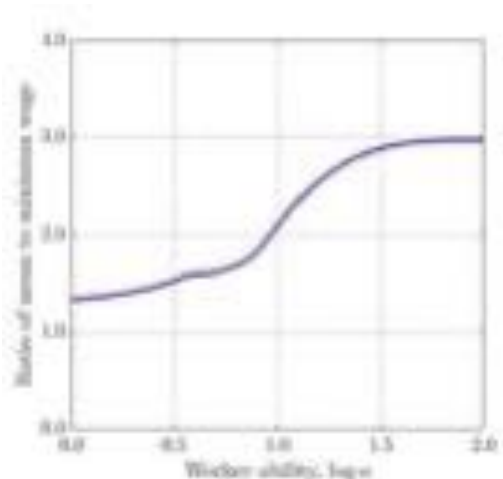
To subsequently solve the model for alternative levels of the minimum wage, we instead hold the cost $c(a, s)$ fixed at its estimated value, as well as the flow value $b(a)$. Applying a similar bisection as above, we instead guess a low job finding rate $p_0(a, s)$ and a high job finding rate $p_1(a, s)$, solve for the equilibrium number of vacancies $V(a, s)$ consistent with these job finding rates, and solve the system of differential equations (9). Since the job finding rate $p(a, s)$ is inversely related to the worker finding rate $q(a, s)$, we require that firms want to create too many jobs under the low job finding rate $p_0(a, s)$ (i.e., high worker finding rate $q_0(a, s)$), and vice versa. If not, we adjust the initial guesses $\{p_0(a, s), p_1(a, s)\}$ until this holds. After that, we apply a bisection for the job finding rate $p(a, s)$ until the key equilibrium consistency condition $\lim_{z \rightarrow \bar{z}} h(z|a, s) = 1$ holds.

D Estimation appendix

This section provides additional details on the model estimation that build on the material presented in Section 5, including subsections on the mean-to-min wage ratio in the estimated model (Appendix D.1), the implied parameter estimates and additional model fit (Appendix D.2), details of the identification (Appendix D.3), and additional model validation (Appendix D.4), as well as a discussion of the structure of the unbalanced panel (Appendix D.5).

D.1 Mean-to-min wage ratio in the estimated model

Figure D.1. Mean-to-min wage ratio in the estimated model



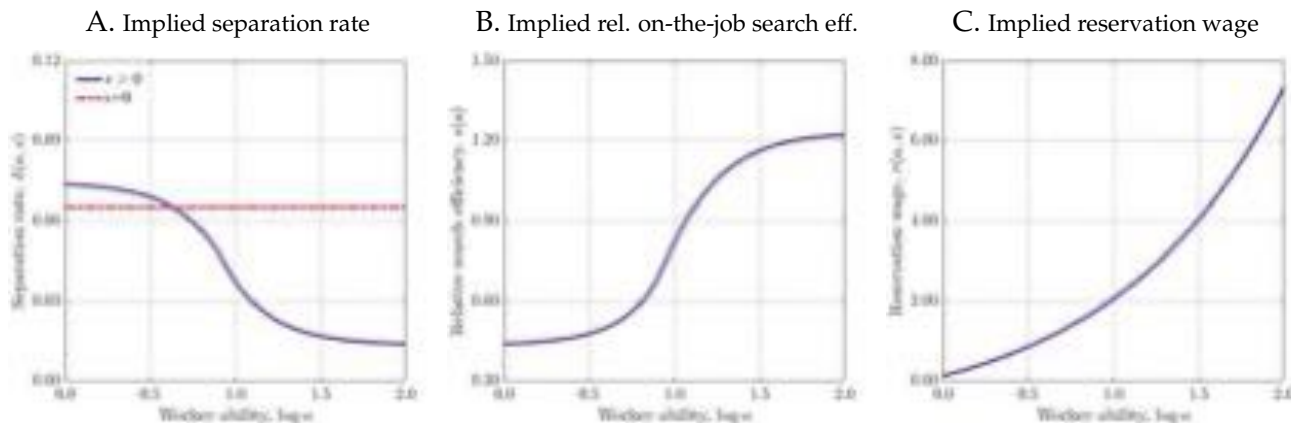
Notes: Figure shows the mean-to-min wage ratio $\bar{w}(a)/w^*(a)$, as defined in [Hornstein et al. \(2011\)](#), from our estimated model. All statistics are computed from model solution before applying the estimated measurement error. The mean wage $\bar{w}(a)$ is defined as the arithmetic mean over the realized distribution of wages in levels for a given ability type a . The min wage $w^*(a)$ is defined as the lowest accepted wage in levels by workers of ability a , which might be either the statutory minimum wage w^{min} or the workers' reservation wage if the latter exceeds the statutory minimum wage. Source: Model and RAIS, 1994–1998.

D.2 Implied parameter estimates and additional model fit

This section discusses additional details regarding the implied parameter estimates and model fit. This follows up on the results presented in Section 5.2 of the main text, specifically those in Table 4 and Figure 4.

Figure D.2 shows the implied parameter values for the separation rates, $\delta(a, s > 0)$ and $\delta(a, s = 0)$; relative on-the-job search efficiency, $s(a)$; and reservation piece rate, $r(a)$, across the distribution of worker ability, a , based on the estimates presented in Table 4 of the main text.

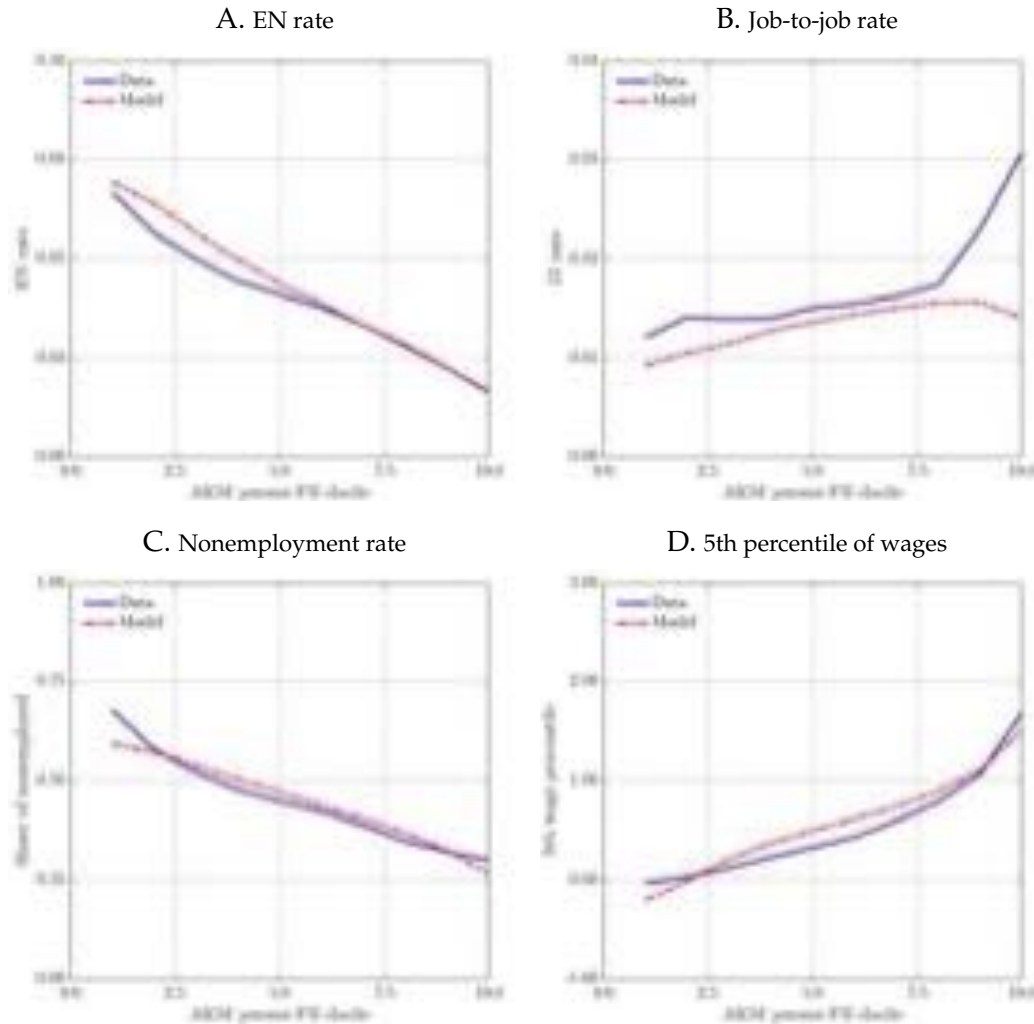
Figure D.2. Implied parameter estimates



Notes: This figure shows the implied parameter values of the separation rates, $\delta(a, s > 0)$ and $\delta(a, s = 0)$, relative on-the-job search efficiency, $s(a)$, and reservation piece rate, $r(a)$, across the distribution of worker ability, a , based on the estimates in Table 4. Source: Model.

Figure D.3 illustrates the model's ability to replicate labor market stocks and flows by AKM worker fixed effects. Panel A shows that the model matches well the lower EN rate among higher paid workers, as a result of the decline in the separation rate, $\delta(a, s)$, with worker ability. Panel B highlights that we understate somewhat the level of job-to-job mobility as well as its gradient with AKM worker fixed effect. Although underlying search efficiency $s(a)$ rises monotonically with worker ability and higher-ability workers are less likely to be minimum wage workers, the resulting job-to-job rate is not monotone in AKM worker fixed effects. The reason is that the separation rate $\delta(a, s)$ declines in ability, such that higher-ability workers are higher up the job ladder. Since workers higher up the job ladder are less likely to accept an outside offer, the realized job-to-job rate is non-monotone in ability. Panel C shows that the model matches well the nonemployment rate by AKM worker fixed effect decile. Although we do not target this in estimation—in fact, we impose the same job finding rate λ across worker types—the model matches well the empirical pattern, because the separation rate, $\delta(a, s)$, declines with worker ability. Panel D shows that the model matches well the fifth percentile of wages by AKM worker fixed effect decile.

Figure D.3. Worker outcomes by AKM person FE, model vs. data



Notes: Panel A shows the share of employed workers who are nonemployed in the subsequent month. Employment refers to formal sector employment in the data; nonemployment is everything else (unemployment, not in the labor force and informal sector employment). Panel B shows the share of employed workers who are employed at a different main employer in the subsequent month. Panel C shows the share of population that is not working in the formal sector. Panel D shows the 5th percentile of log wages. All panels are by decile of AKM person fixed effects based on a regression of log monthly earnings on person fixed effects and firm fixed effects. Model and data sample selection and variable construction are identical—see text for details. Source: Model and RAIS, 1994–1998.

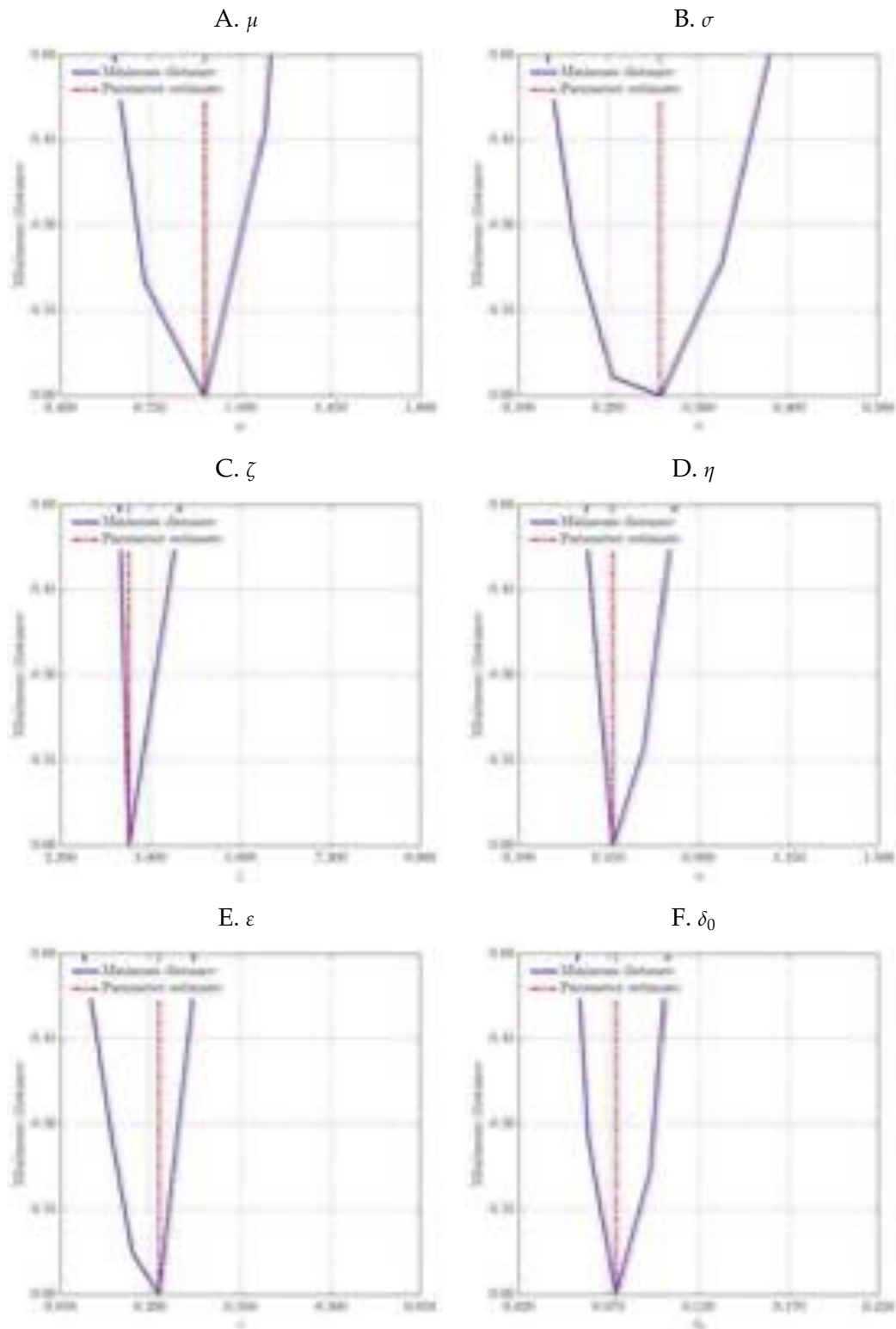
D.3 Details of the identification

This subsection provides a further discussion of identification of the 12 internally estimated parameters of our model. We consider two exercises. The first plots how the minimum distance objective function changes as each parameter varies around its estimated value, holding all other parameters fixed at their estimated values. This exercise is local in nature, in the sense that an envelope condition ensures that the other parameters remain optimal as one parameter varies around its optimum.

Figure D.5 provides the results. Evidently, 11 of the 12 internally estimated parameters are well informed by the joint information contained in the targeted moments. The exception is the intercept in the reservation wage, r_0 , for which the minimum distance is relatively flatter compared to other parameters. For the main objective of this paper, we believe that this is a somewhat minor issue. The reason is that the impact of the minimum wage on both inequality and employment is essentially invariant to the particular value of this parameter, as we highlight further in Appendix E.4.

The second exercise plots how an individual parameter moves its particularly informative moment. Figure D.6 plots those parameters that are particularly informed by a single moment against its chosen moment as the parameter varies around its estimated value (between 25 and 300 percent of the estimated value), holding the other parameters fixed at their estimated values. Reassuringly, each parameter distinctly moves its particularly informative moment, suggesting that these parameters are well informed by our choice of targets. In the interest of space, we focus this exercise primarily on those parameters that are particularly informed by a single moment. Nevertheless, for reference we also show in Figure D.7 how the overall EN and EE rates move as we change the intercept in the separation rate, δ_0 , and the intercept in relative search efficiency, ϕ_0 , respectively. Note, though, that these two moments are not targeted in estimation, as we jointly target the EN (EE) rate *by decile* of AKM worker fixed effect deciles for δ_0 (ϕ_0) and δ_1 (ϕ_1). These two parameters move the overall mobility rates in the expected direction.

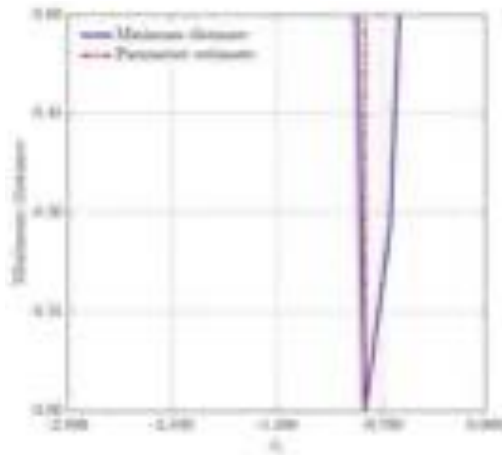
Figure D.4. Minimum distance, model



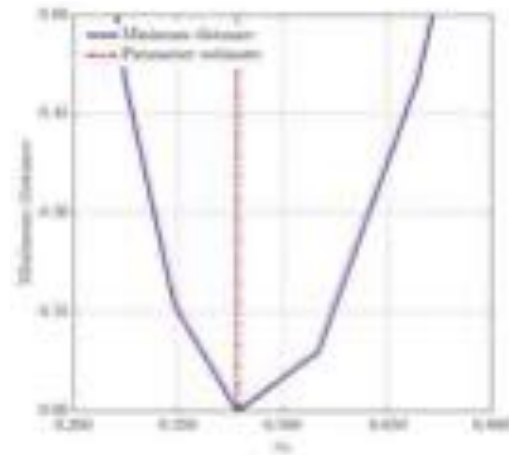
Notes: Impact on minimum distance objective function of varying one parameter at a time, holding all other parameters fixed at their estimated values. Source: Model.

Figure D.5. Minimum distance, model (cont'd)

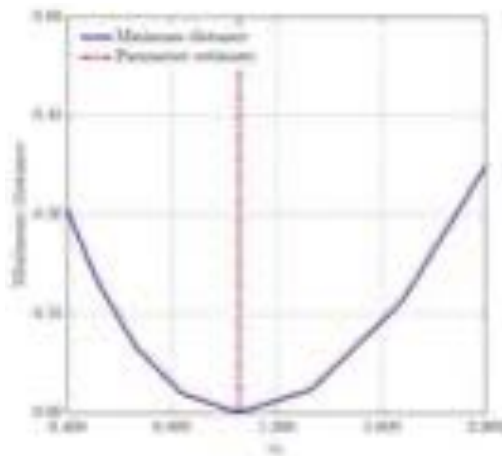
A. δ_1



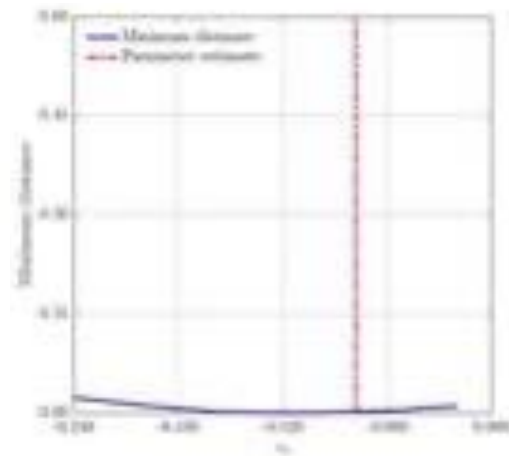
B. ϕ_0



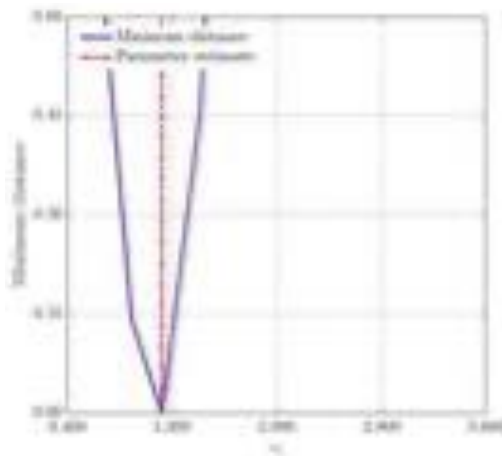
C. ϕ_1



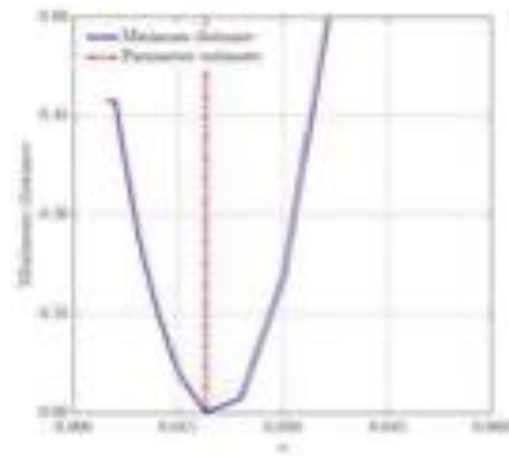
D. r_0



E. r_1

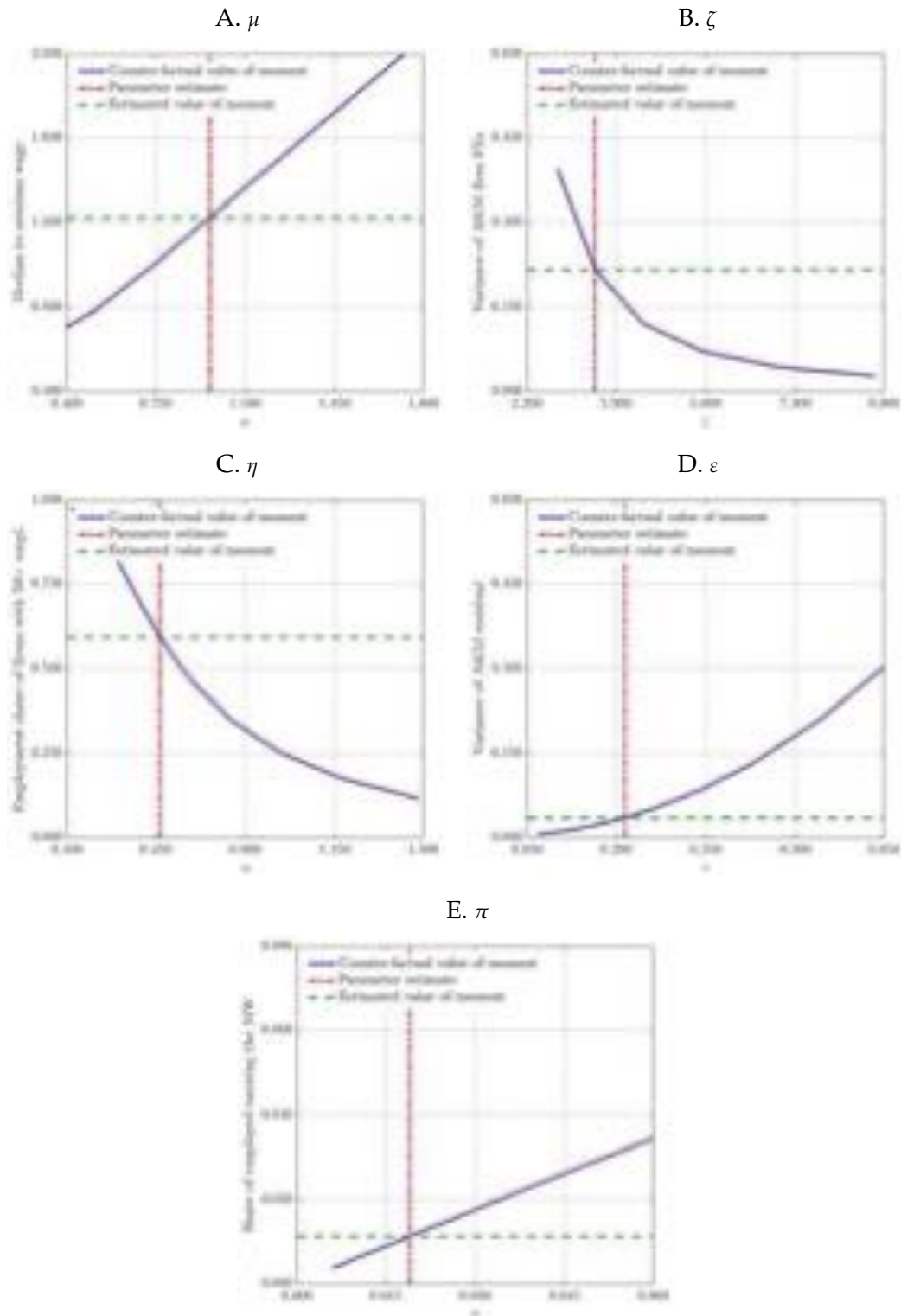


F. π



Notes: Impact on minimum distance objective function of varying one parameter at a time, holding all other parameters fixed at their estimated values. Source: Model.

Figure D.6. Targeted moments versus parameters, model

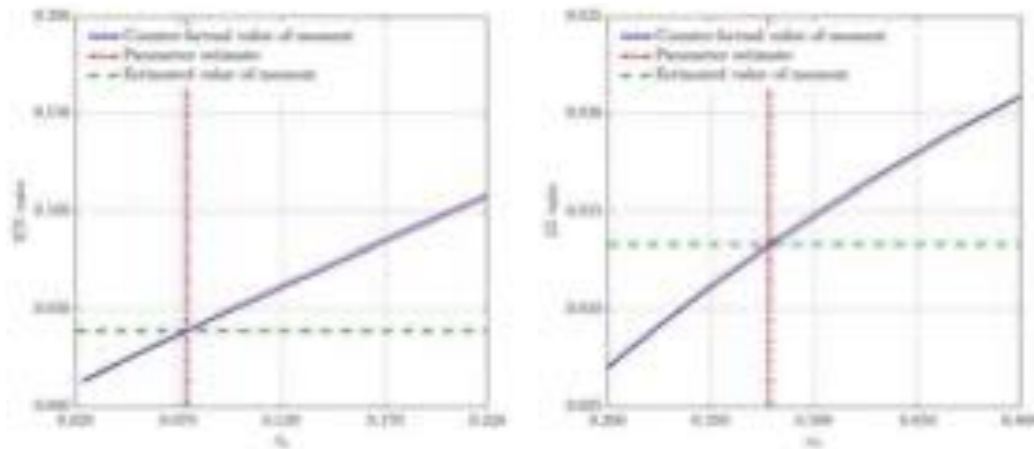


Notes: Impact on an associated target moment of varying one parameter at a time, holding all other parameters fixed at their estimated values.
Source: Model.

Figure D.7. Additional moments versus parameters, model

A. δ_0

B. ϕ_0

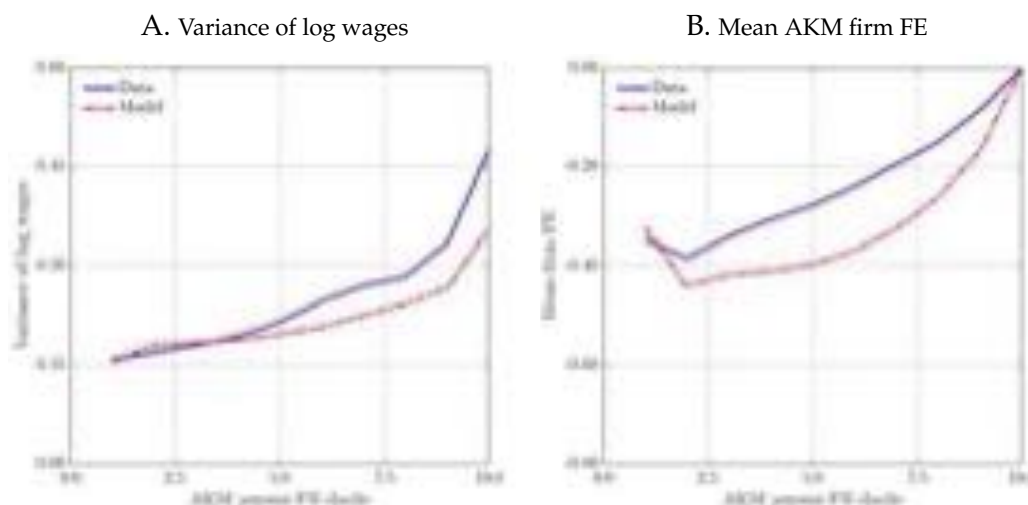


Notes: Impact on an associated target moment of varying one parameter at a time, holding all other parameters fixed at their estimated values.
Source: Model.

D.4 Additional model validation

Figure D.8 contrasts with the data some additional predictions of the model that were not explicitly targeted. Panel A shows that dispersion in (log) pay is larger among high paid workers, in both the model and data. This pattern is driven by the fact that the top deciles have greater dispersion in underlying worker ability, a , as well as greater dispersion in pay conditional on ability among high skilled workers. The latter is, in turn, due to the fact that the minimum wage does not constrain pay at the top of the ability distribution. Panel B plots the average AKM firm fixed effect by decile of AKM worker fixed effects. Higher paid workers work for higher paying firms, in both the model and data. The reason is that more skilled workers climb the job ladder faster and fall off it less frequently. At the bottom of the worker pay distribution, however, the pattern is reversed, because the minimum wage makes matches between the lowest skilled workers and the lowest productivity firms unviable.

Figure D.8. Model validation across AKM person FE deciles, model versus data



Notes: Figure shows the estimated impact of a 44.4 log point increase in the productivity adjusted real minimum wage. Workers are binned by decile of AKM worker fixed effect. Firms are binned by (employment-unweighted) AKM firm fixed effect decile. AKM regression is estimated on model-simulated monthly data aggregated to the annual level following a sample selection and variable construction methodology identical to the one in the data. Panel A shows the impact on the variance of log wages. Panel B shows the impact on the mean of AKM firm fixed effects. Source: Model and RAIS, 1994–2014.

D.5 Structure of the unbalanced panel

A salient feature of the formal-sector RAIS data is that both workers and firms do not appear in a balanced panel. Our estimated model rationalizes this through stochastic worker separation rates into nonemployment and stochastic job findings rates from nonemployment. Our estimation procedure relies on an indirect inference logic, and the AKM wage equation, which we use as an auxiliary model, is estimated on finite samples—both in the data and on the model-simulated data. Therefore, it is interesting to know to what extent panel structure in the data is replicated by simulations from our estimated model.

Table D.1 compares worker and firm survival rates in the data for two periods, the estimation period 1994–1998 (panel A) and the final period 2010–2014 (panel B), as well as in the simulated data from our estimated model (panel C), which we fit to a separate set of moments from the estimation period 1994–1998. Specifically, we consider two concepts of worker or firm survival rates. First, we compute the survival rates of a cohort of workers or firms observed in the first year of the five-year time window. We report the share of that cohort of workers or firms that survives for each number of consecutive years, including the first one, during this time window in the columns labeled “Cohort”. Second, we compute the share of all workers or firms that is observed for each of one, two, three, four, or five years in the data in the columns labeled “Pooled”. The two statistics are related but distinct because—in the real world as in our simulated model—workers may be observed for the first time after the first year of our data time window.

A few points are worth noting about the results in Table D.1. Cohort survival rates are concentrated around five years—the complete panel—both for workers and for firms, as well as in the two data periods and in the model. In comparison, pooled survival rates are more spread out. The model matches very well the empirical worker cohort survival rates. At the same time, the model overpredicts the cohort survival rates of firms. Vis-a-vis the data, the model also underpredicts the pooled survival rates of workers but does a very good job at capturing that of firms. The model fit is not perfect, which may be not surprising given that there were no free parameters that could have been used to jointly match these survival rate profiles. At the same time, the parsimonious model does an adequate job at capturing key features of the empirical panel structure, which gives us confidence in our indirect inference procedure.

Table D.1. Cohort survival rates and pooled survival shares, data and model

<i>Panel A. Data, 1994–1998</i>				
Number of years	Workers		Firms	
	Cohort	Pooled	Cohort	Pooled
1	2.0%	4.5%	3.0%	2.8%
2	3.6%	8.6%	5.2%	5.7%
3	5.1%	12.4%	6.6%	7.4%
4	8.8%	20.2%	8.4%	11.5%
5	80.4%	54.3%	76.7%	72.7%
 <i>Panel B. Data, 2010–2014</i>				
Number of years	Workers		Firms	
	Cohort	Pooled	Cohort	Pooled
1	1.6%	3.2%	2.6%	1.8%
2	1.9%	6.2%	4.4%	4.2%
3	3.6%	9.2%	5.3%	6.0%
4	5.9%	16.3%	7.0%	8.6%
5	87.0%	65.1%	80.7%	79.5%
 <i>Panel C. Model</i>				
Number of years	Workers		Firms	
	Cohort	Pooled	Cohort	Pooled
1	1.0%	7.8%	0.0%	1.3%
2	2.7%	14.2%	0.3%	4.7%
3	5.6%	20.0%	1.4%	9.4%
4	11.2%	23.9%	4.7%	15.3%
5	79.4%	34.2%	93.7%	69.4%

Notes: Table compares worker and firm survival rates in the data for two periods, the estimation period 1994–1998 (panel A) and the final period 2010–2014 (panel B), as well as in the simulated data from our estimated model (panel C), which was fit to data from 1994 to 1998. Two concepts of worker or firm survival rates are presented: the share of the starting year’s cohort of workers or firms that survives for each number of consecutive years (columns “Cohort”) and the share of all workers or firms that is observed for each number of years in the data (columns “Pooled”). *Source:* RAIS, 1994–1998 and 2010–2014, and model.

E Results Appendix

This section provides additional details on the simulated impact of the minimum wage, building on the material presented in Section 6, including subsections on a model-based AKM wage decomposition (Appendix E.1), the impact of the minimum wage on sorting (Appendix E.2), further results on heterogeneity in effects on disemployment and firm size (Appendix E.3), and the dependence of minimum wage effects on model parameters (Appendix E.4).

E.1 AKM decomposition

Table E.1 summarizes the impact of the minimum wage on earnings inequality as viewed through the lens of the AKM decomposition. The increase in the minimum wage accounts for about a third of the fall in the overall variance of earnings over this period in Brazil. It accounts for roughly half of the compression in the variance in AKM person fixed effects and 10 percent of the fall in the variance of the AKM firm effects. We caution, however, that *all* of the fall in inequality in the model is due to changes in firms' wage and vacancy policies.

Table E.1. Impact of minimum wage on AKM decomposition, model versus data

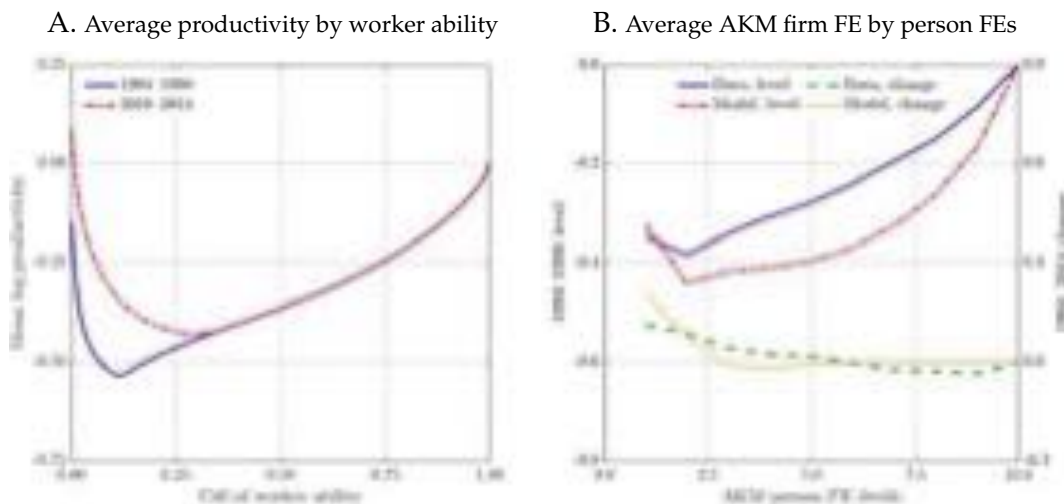
	1994–1998		2010–2014		Change		
	Data	Model	Data	Model	Data	Model	Due to MW
Variance of log wages	0.704	0.600	0.444	0.514	-0.259	-0.086	33.1%
Variance of AKM person FEs	0.332	0.259	0.252	0.221	-0.080	-0.037	46.7%
Variance of AKM firm FEs	0.217	0.195	0.090	0.183	-0.127	-0.012	9.6%
Variance of AKM residual	0.032	0.035	0.020	0.036	-0.012	0.001	-7.9%
2×covariance AKM person-firm FEs	0.123	0.111	0.082	0.074	-0.041	-0.037	91.2%

Notes: Men ages 18–54. Estimated impact of a 44.4 log point increase in the minimum wage in the model as well as the raw data. Model and data sample selection and variable construction are identical. Source: Model and RAIS 1994–2014.

E.2 Empirical support for the impact of the minimum wage on sorting

Figure E.1 illustrates this change in sorting in the model and provides empirical support for it. Panel A plots average firm productivity by worker ability, highlighting that average productivity rises, particularly among the lowest-ability workers. The reason is that matches between low-ability workers and low-productivity firms become unviable when the minimum wage is raised. Panel B of the figure provides reduced-form support consistent with this prediction. Specifically, it plots average AKM firm fixed effect by decile of AKM worker fixed effects in the model and data. For completeness, we replicate the level in the initial period from Figure D.8. As the minimum wage is increased, the average AKM firm fixed effect rises disproportionately among the lowest AKM worker fixed effects workers.

Figure E.1. Reallocation of lower-ability workers toward higher-productivity firms, model versus data

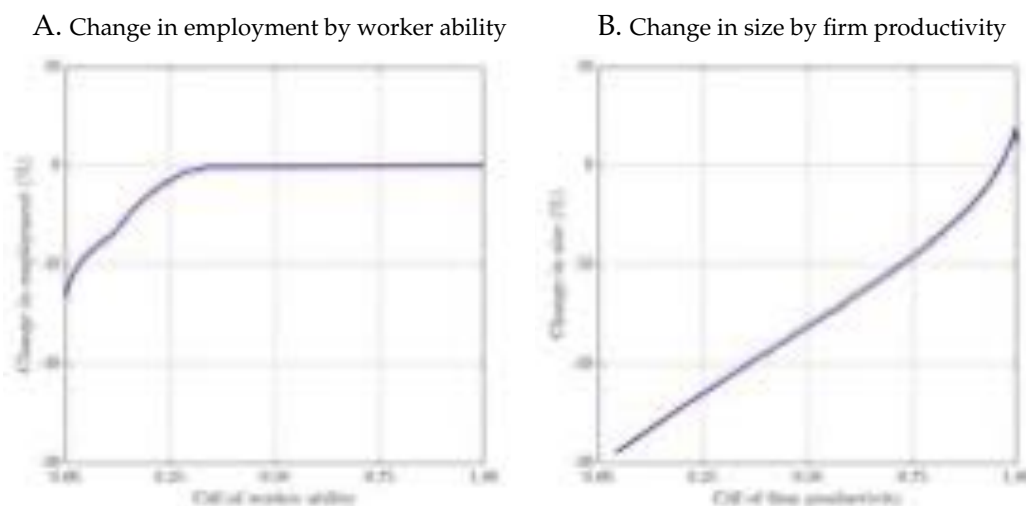


Notes: Men ages 18–54. Estimated impact of a 44.4 log point increase in the productivity adjusted real minimum wage. Panel A shows average log firm productivity by worker ability, $\int_z \log z dG(z|a, s)$, in market for workers with $s(a) > 0$ (the vast majority of workers). Panel B shows average AKM firm fixed effect by decile of AKM worker fixed effects. AKM regression is estimated on model-simulated monthly data aggregated to the annual level following an identical sample selection and variable construction methodology as in the data. Source: Model and RAIS 1994–2014.

E.3 Further results on heterogeneity in effects on disemployment and firm size

Figure E.2 sheds light on the heterogeneous effects of the minimum wage on employment and firm sizes. Panel A shows that, among the lowest-ability workers, employment falls by over 13 percent, while employment is essentially unaffected among workers above the bottom third of the ability distribution. Panel B focuses again on a group of workers most affected by the minimum wage—specifically, the first percentile of worker ability. Firms near the bottom of the firm productivity distribution shrink by almost 30 percent, while firms in the top 5 percent of the productivity distribution in fact expand in response to an increase in the minimum wage, for reasons that we analyze further below.

Figure E.2. Impact of minimum wage on aggregate outcomes, model

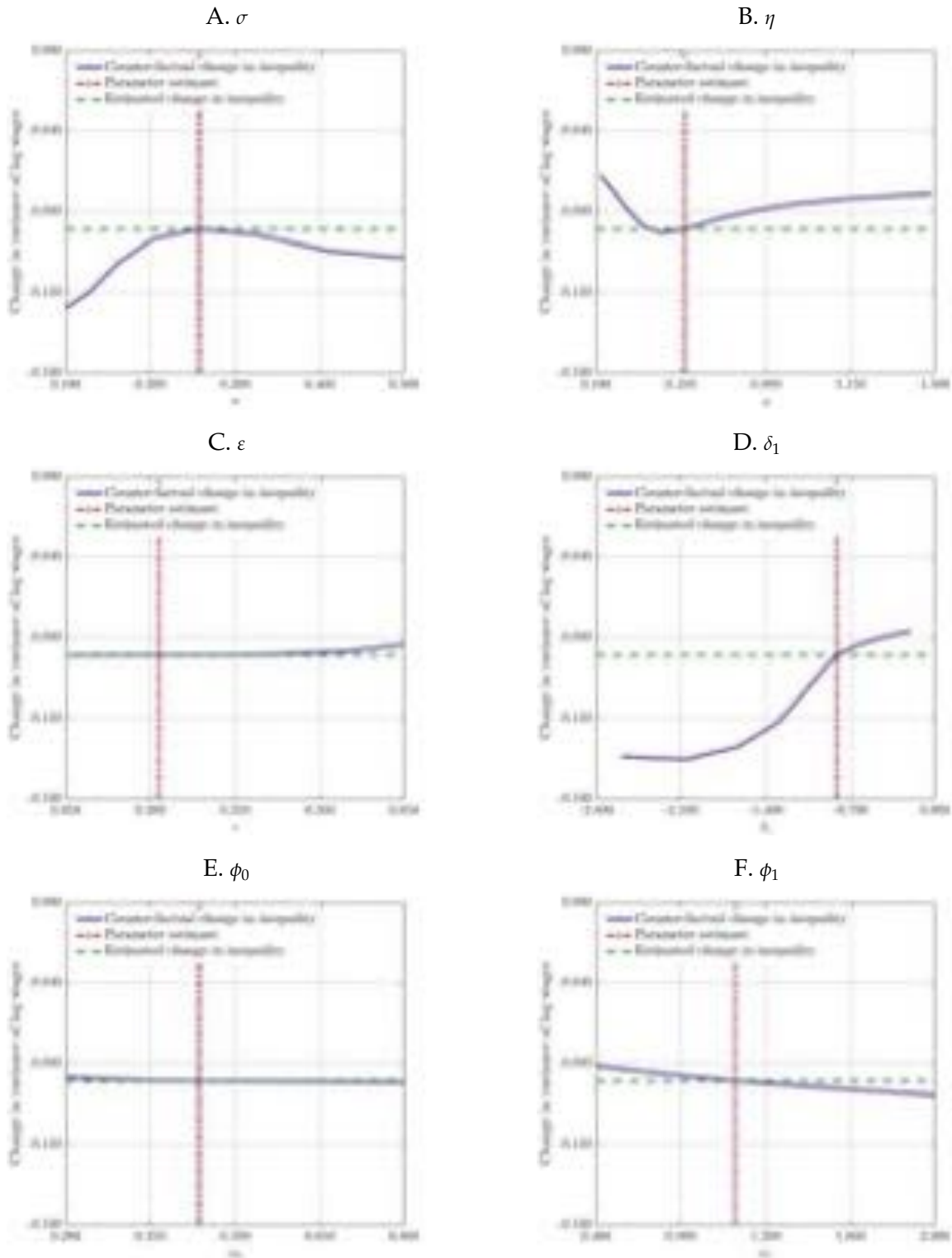


Notes: Impact of a 44.4 log point increase in the minimum wage in the estimated model. Panel A shows the log change in employment rate by worker ability in market with positive search efficiency, $s(a) > 0$ (the vast majority of workers). Panel B shows the log change in employment by firms ranked by employment-unweighted productivity in the first percentile of the worker ability distribution among workers with $s(a) > 0$. Source: Model.

E.4 Dependence of estimated minimum wage effects on model parameters

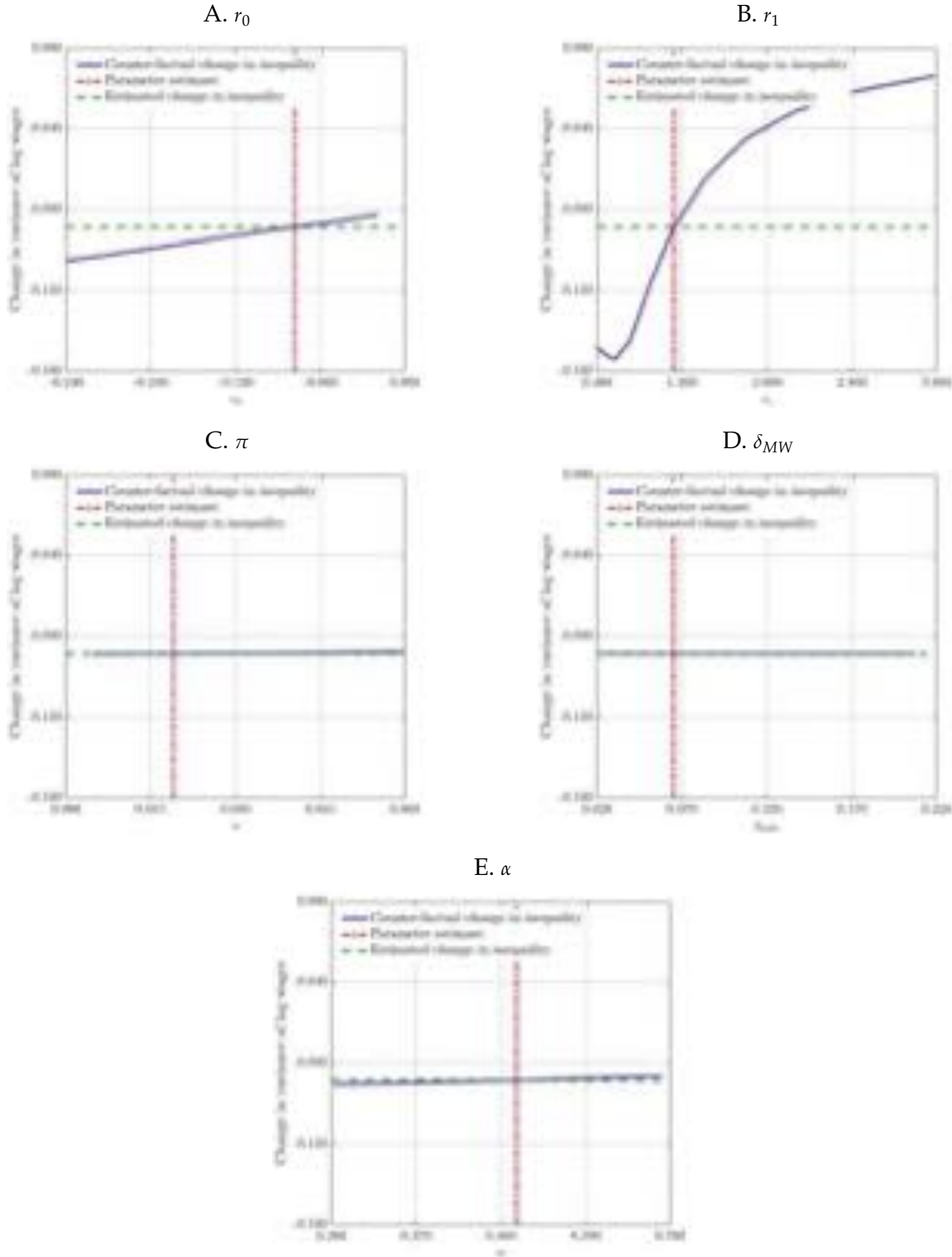
Figures E.3–E.4 conduct the same exercise as in Section 6.6 across the remaining nine internally estimated structural parameters of the model, the calibrated separation rate of minimum wage workers (δ_{MW}) and the preset elasticity of the matching function (α). Most of these parameters have at most a modest effect on the impact of a rise in the minimum wage on inequality. The main exception is the slope of the reservation wage, r_1 . Intuitively, a higher reservation wage leaves less scope for the minimum wage to impact markets, as the reservation increasingly becomes the binding constraints across worker ability markets. Recall from Appendix D.3 that the parameter the model primarily struggles to inform well based on the available data is r_0 . This parameter, however, is not critical in terms of driving the estimated impact of the minimum wage on inequality. Finally, δ_{MW} and α have no meaningful effect on the impact of the minimum wage on inequality.

Figure E.3. Change in the variance of log wages across model parameters



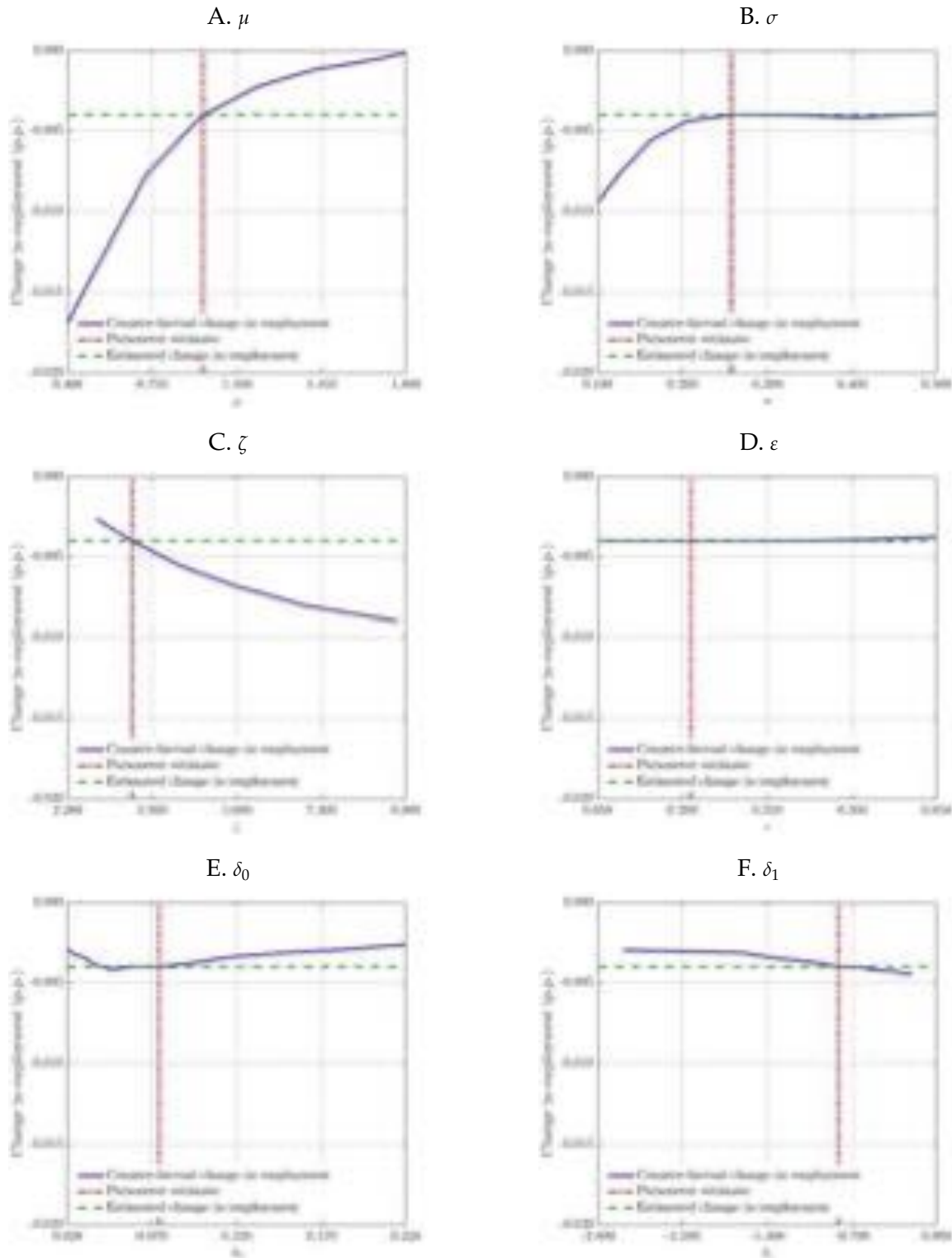
Notes: Estimated impact of a 44.4 log point increase in the minimum wage on the variance of log wages as the parameter in question varies between 25 percent and 300 percent of its estimated value, holding all other parameters fixed at their estimated values. Source: Model.

Figure E.4. Change in the variance of log wages across model parameters, continued



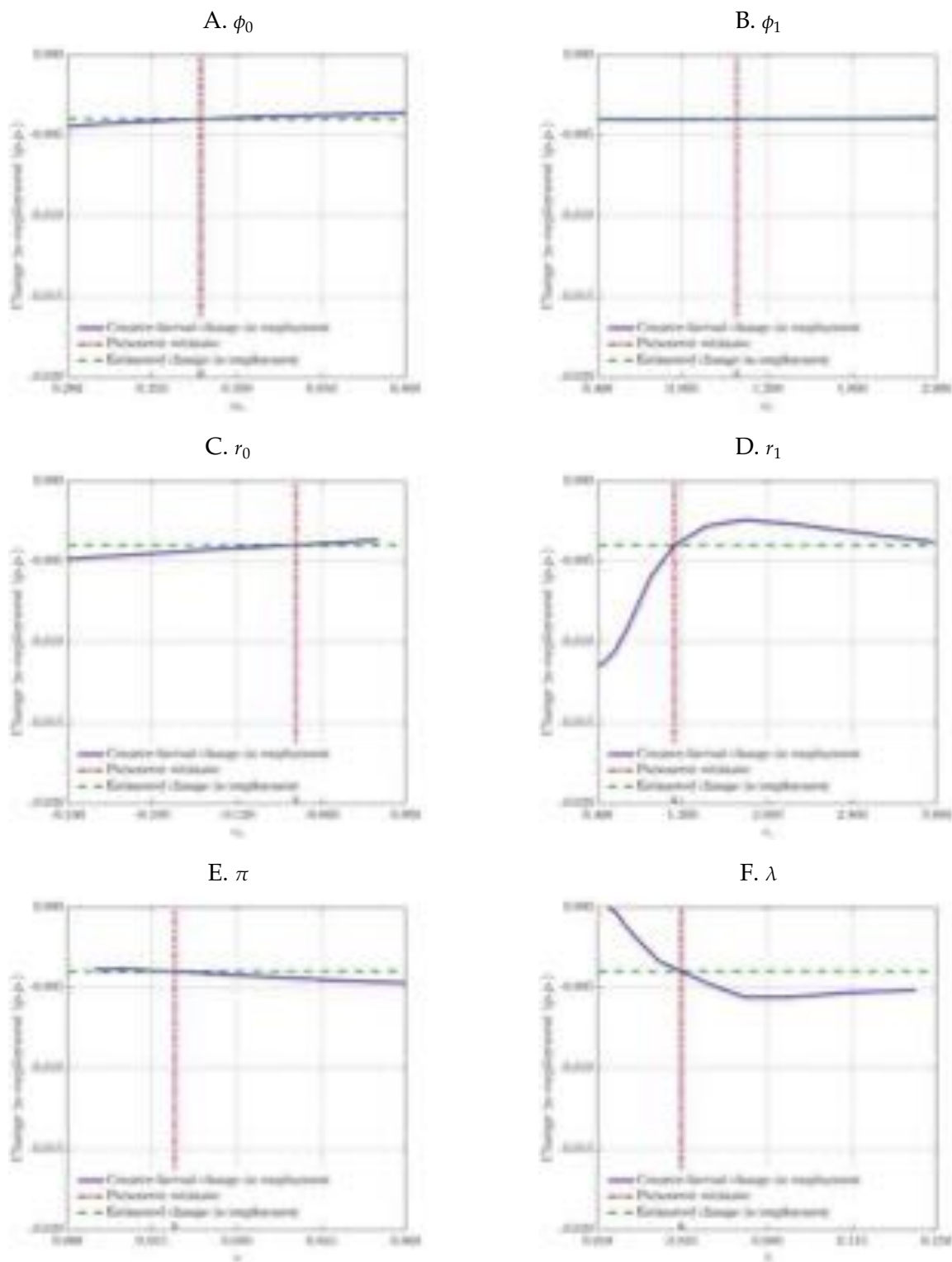
Figures E.5–E.7 conduct the same exercise as in Section 6.5 across the remaining 11 internally estimated structural parameters of the model, as well as for the calibrated job finding rate λ and separation rate of minimum wage workers, δ_{MW} . As for the impact of the minimum wage on inequality, the key parameters determining the employment effect of the minimum wage are the mean of worker ability (μ), the tail index of the firm productivity distribution (ζ), the slope of the reservation wage (r_1), and to a lesser extent the job finding rate (λ). The larger μ is, the less binding is the minimum wage initially and the smaller is the effect of an increase in the minimum wage on employment (as well as inequality—recall Figure 12). A larger ζ (i.e., a thinner tail of the firm productivity distribution) raises the disemployment effect of the minimum wage, as there is a larger number of low productive firms that are heavily exposed to the minimum wage and fewer high productive firms to pick up the employment slack. The faster the reservation wage rises in ability—the larger is r_1 —the less the minimum wage binds and hence the smaller is the disemployment effect of a rise in the minimum wage. A lower λ is associated with a smaller disemployment effect of the minimum wage. The remaining parameters have at most a modest effect on the impact of a rise in the minimum wage on employment.

Figure E.5. Change in the employment rate across model parameters



Notes: Estimated impact of a 44.4 log point increase in the minimum wage on aggregate employment as the parameter in question varies between 25 percent and 300 percent of its estimated value, holding all other parameters fixed at their estimated values. Source: Model.

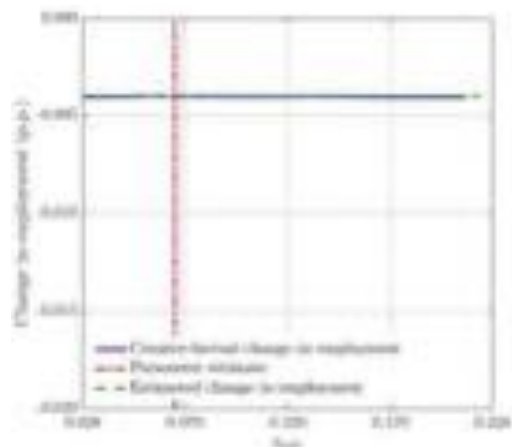
Figure E.6. Change in the employment rate across model parameters, continued



Notes: Estimated impact of a 44.4 log point increase in the minimum wage on aggregate employment as the parameter in question varies between 25 percent and 300 percent of its estimated value, holding all other parameters fixed at their estimated values. Source: Model.

Figure E.7. Change in the employment rate across model parameters, continued

A. δ_{MW}



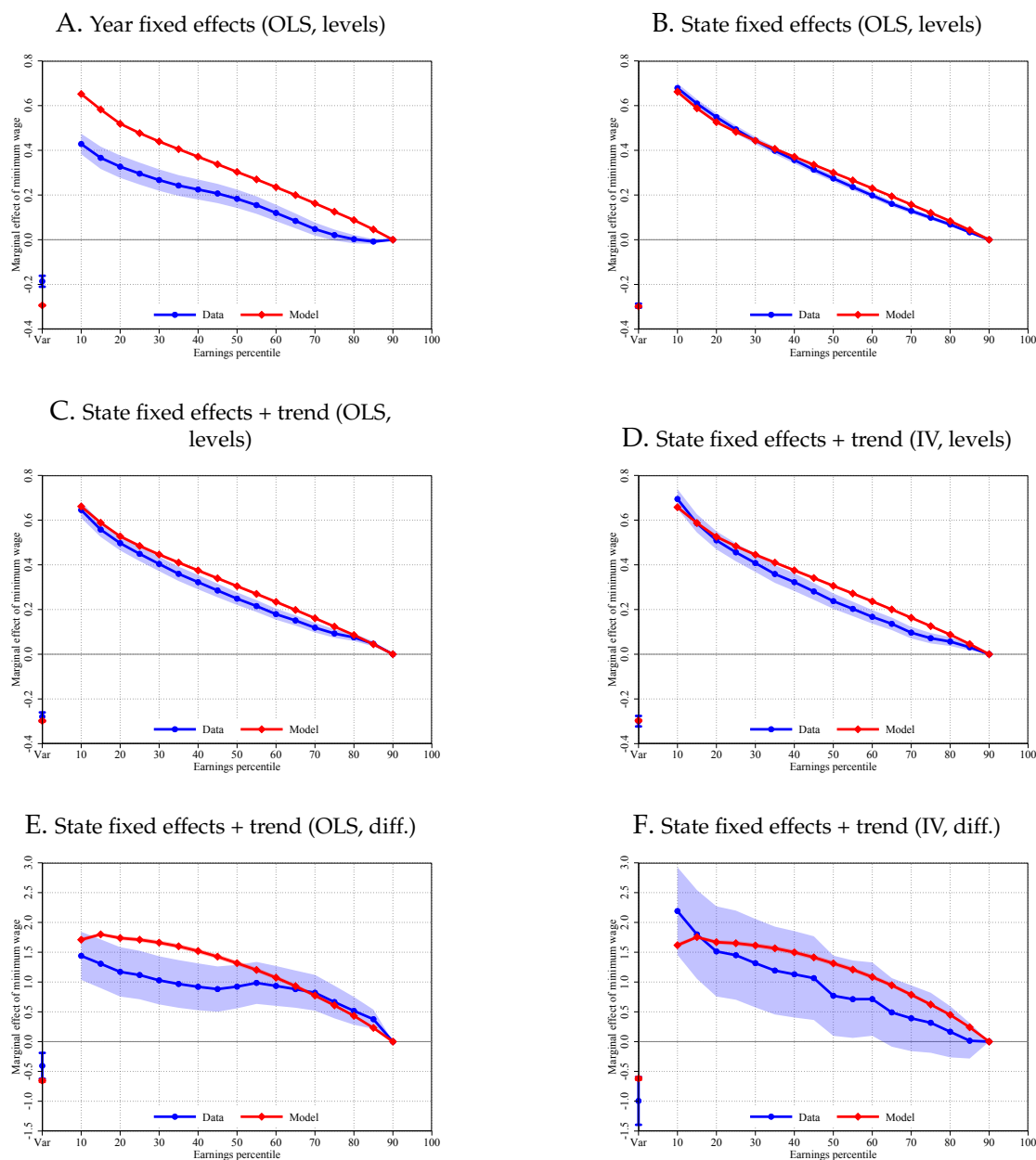
Notes: Estimated impact of a 44.4 log point increase in the minimum wage on aggregate employment as the parameter in question varies between 25 percent and 300 percent of its estimated value, holding all other parameters fixed at their estimated values. *Source:* Model.

F Model Validation Appendix

This appendix provides further details on the model validation presented in Section 7, including subsections on the comparison of spillover effects between the model and the data (Appendix F.1), comparative statics of the distributional effects of the minimum wage with respect to local economic conditions (Appendix F.2), and comparative statics of the employment effects of the minimum wage with respect to local economic conditions (Appendix F.3).

F.1 Comparing estimated spillover effects between model and data

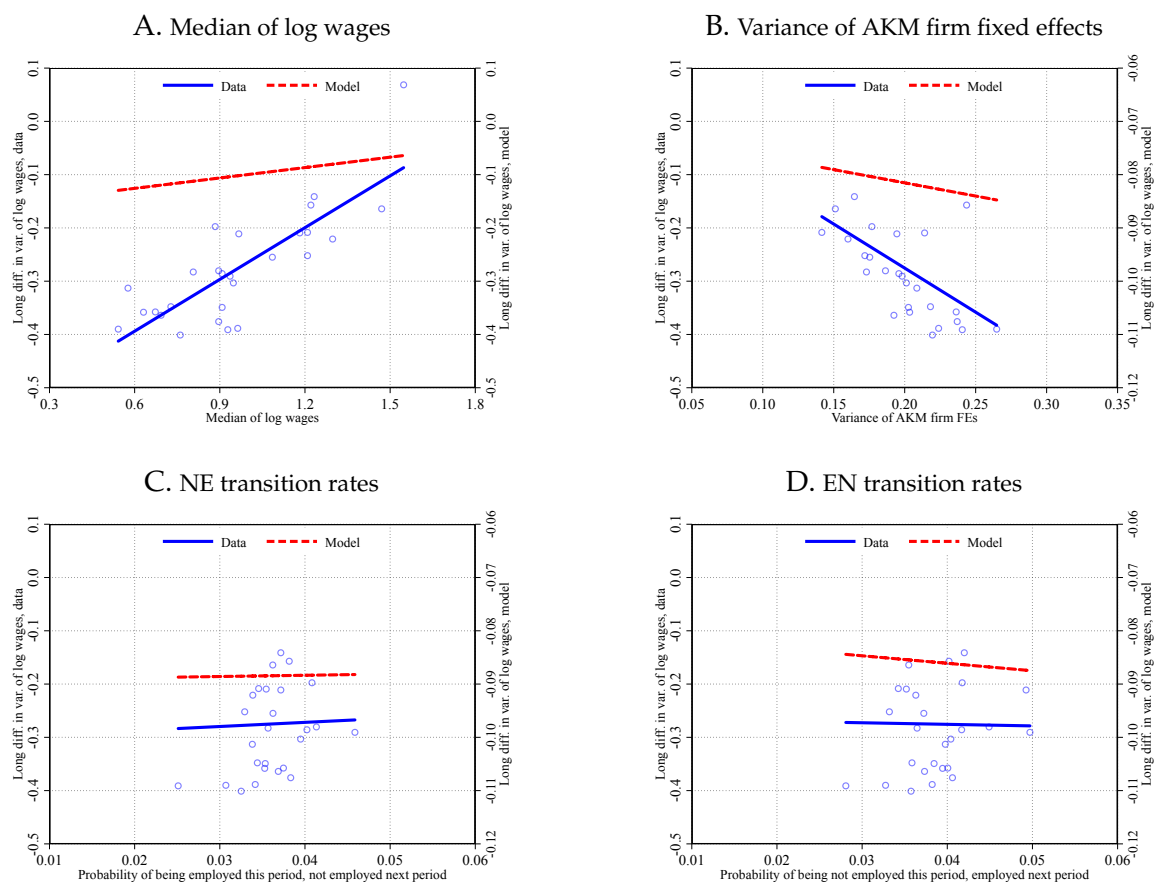
Figure F.1. Estimated minimum wage effects under alternative specifications, model versus data



Notes: Figure plots estimates of the marginal effects from equation (3) based on the regression framework in equation (2) estimated across Brazil's 27 states. In each panel, results from two separate estimates are shown, those based on the RAIS data (blue line and error bar or shaded area) and those based on model-simulated data (red line and error bar or shaded area). Each panel uses one of six different state-year controls: year fixed effects (panel A), state fixed effects (panel B), state fixed effects in addition to state-specific quadratic time trends (panel C)—all estimated using OLS in levels—and the latter specification estimated using an IV strategy in levels (panel D). The final two panels show results from a specification with state fixed effects in addition to state-specific quadratic time trends estimated in differences (panel E), and the latter specification estimated using an IV strategy in differences (panel F). The IV strategy instruments the generalized p -Kaitz index and its square using an instrument set that consists of the log real statutory minimum wage, its square, and the log real statutory minimum wage interacted with the mean of the log real median wage for the state over the full sample period. The estimated marginal effect of the minimum wage on the variance of log earnings ("Var" on the x-axis) and on wages between the 10th and the 90th percentiles of the wage distribution ("10" to "90" on the x-axis) relative to a base wage $p = 90$ are shown. In each panel, the two error bars and two shaded areas represent 99 percent confidence intervals based on regular standard errors. *Source:* RAIS, 1985–2014, and model.

F.2 Additional comparative statics of the distributional effects of minimum wage with respect to local economic conditions

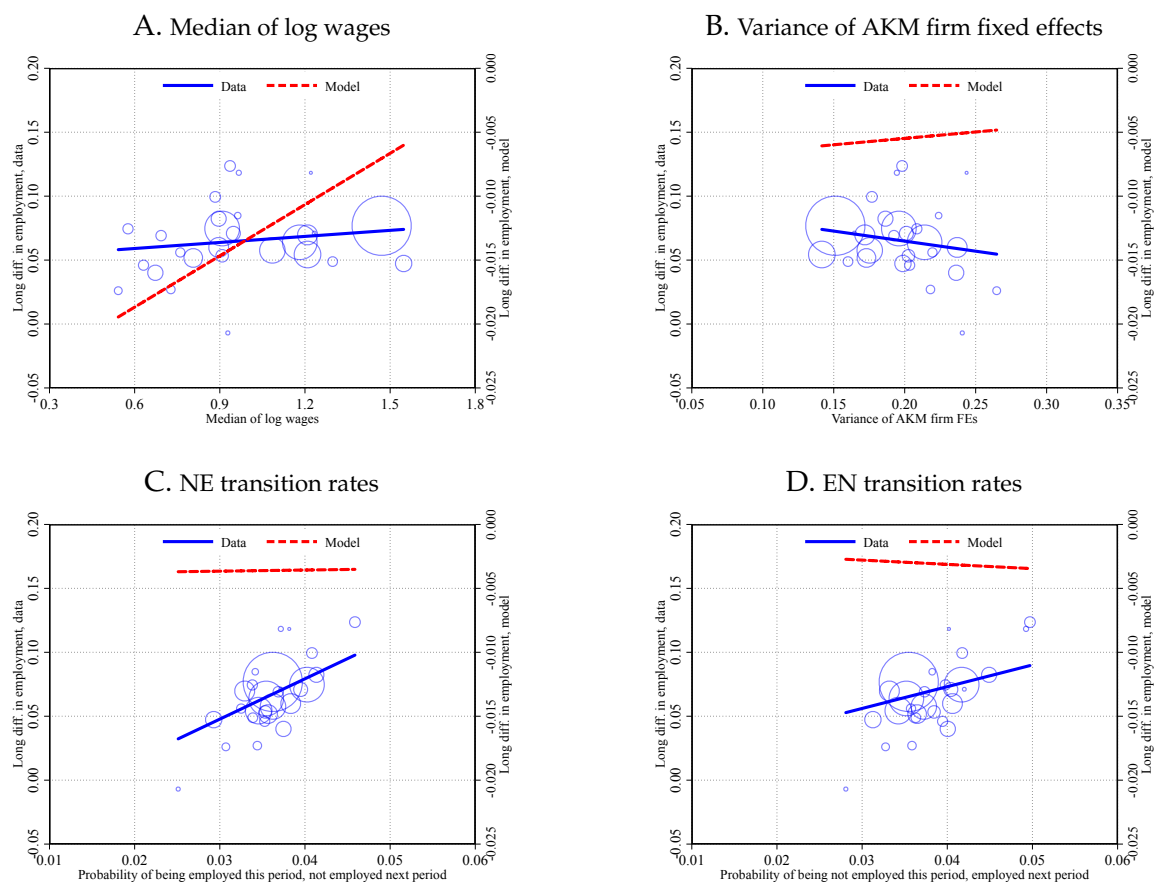
Figure F.2. Changes in inequality across observable labor market characteristics, model vs. data (unweighted)



Notes: Figure plots long differences between 1996 and 2012 in wage inequality—measured by the variance of log wages—across states in the data (blue solid line) and across simulations in the model (red dashed line). Variables on the x-axis are the median of log wages (panel A), the variance of AKM firm fixed effects (panel B) the probability of a transition from nonemployment into employment (panel C), and the probability of a transition from employment into nonemployment (panel D). The median of log wages is computed in 1996, while transition probabilities are computed in a balanced monthly panel from 1994 to 1998. States are represented as hollow circles, with their area proportional to their sample population in 1996. Linear best fit line is shown for the data (blue solid line) and the model (red dashed line), using OLS regression of inequality measures on the respective variable on the x-axis. Data points and line of best fit are equally weighted across states both in the data and in the model. *Source:* RAIS, 1994–1998 and 2012.

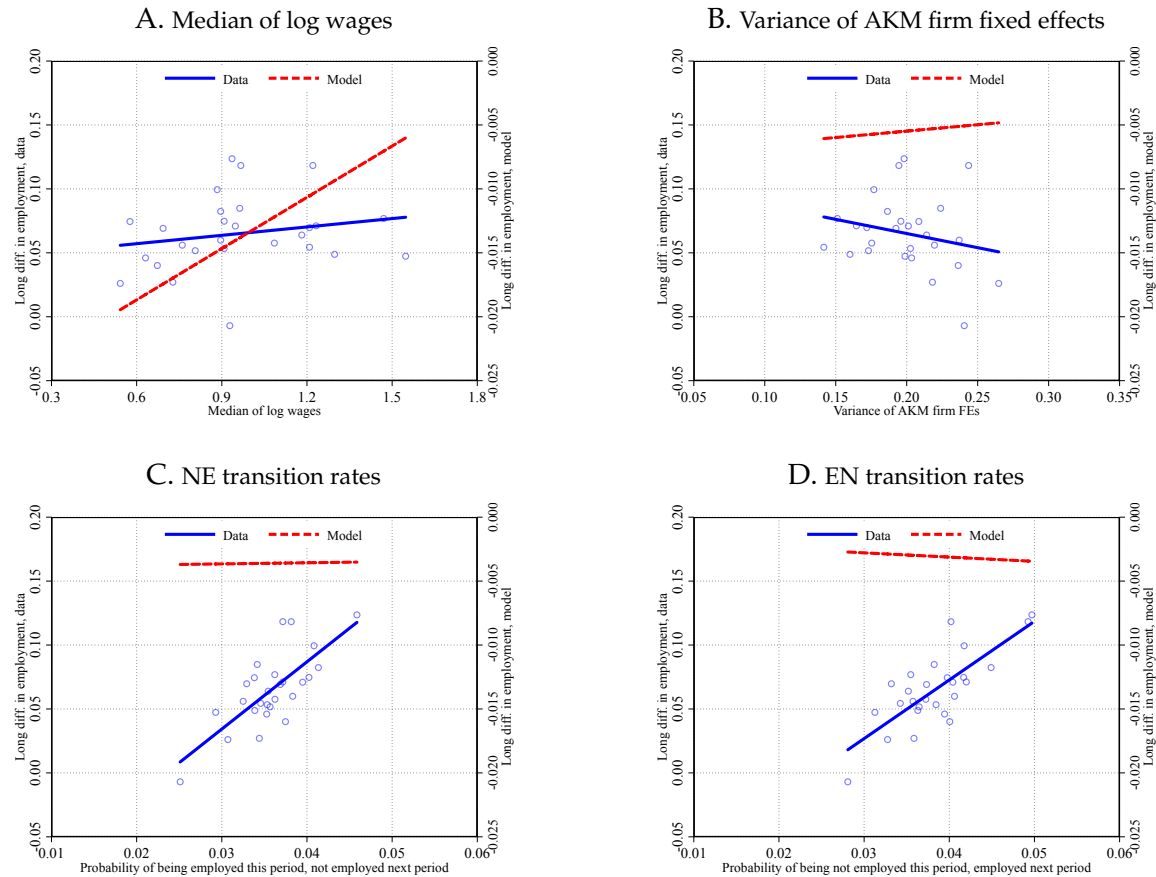
F.3 Comparative statics of the employment effects of minimum wage with respect to local economic conditions

Figure F.3. Changes in employment rates across observable labor market characteristics, model vs. data (weighted)



Notes: Figure plots long differences between 1996 and 2012 in wage inequality—measured by the share of workers that are employed at a given point in time—across states in the data (blue solid line) and across simulations in the model (red dashed line). Variables on the x-axis are the median of log wages (panel A), the variance of AKM firm fixed effects (panel B), the probability of a transition from nonemployment into employment (panel C), and the probability of a transition from employment into nonemployment (panel D). The median of log wages is computed in 1996, while transition probabilities are computed in a balanced monthly panel from 1994 to 1998. States are represented as hollow circles, with their area proportional to their sample population in 1996. Linear best fit line is shown for the data (blue solid line) and the model (red dashed line), using OLS regression of inequality measures on the respective variable on the x-axis. Data points and line of best fit are population-weighted across states in the data and unweighted in the model. *Source:* RAIS, 1994–1998 and 2012.

Figure F.4. Changes in employment rates across observable labor market characteristics, model vs. data (unweighted)



Notes: Figure plots long differences between 1996 and 2012 in wage inequality—measured by the share of workers that are employed at a given point in time—across states in the data (blue solid line) and across simulations in the model (red dashed line). Variables on the x-axis are the median of log wages (panel A), the variance of AKM firm fixed effects (panel B) the probability of a transition from nonemployment into employment (panel C), and the probability of a transition from employment into nonemployment (panel D). The median of log wages is computed in 1996, while transition probabilities are computed in a balanced monthly panel from 1994 to 1998. States are represented as hollow circles, with their area proportional to their sample population in 1996. Linear best fit line is shown for the data (blue solid line) and the model (red dashed line), using OLS regression of inequality measures on the respective variable on the x-axis. Data points and line of best fit are equally weighted across states both in the data and in the model. *Source:* RAIS, 1994–1998 and 2012.