Strategic Bank Liability Structure Under Capital Requirements

Suresh Sundaresan* Zhenyu Wang†‡

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Abstract

Banks strategically choose and dynamically restructure deposits and non-deposit debt in response to the minimum requirements on total capital and tangible equity. We derive the optimal strategic liability structure and show that it minimizes the protection for deposits, conditional on capital requirements. Although given any liability structure, regulators can set capital requirements high enough to mitigate the incentive for risk substitution, the strategic response to the capital requirements always preserves this incentive. Banks reduce leverage but increase the proportion of non-deposit debt if regulations raise the capital requirements.

Keywords:
liability structure, capital requirement, bank leverage,
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*Columbia University, Graduate School of Business, ms122@columbia.edu

†Indiana University, Kelley School of Business, zw25@indiana.edu

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1 Introduction

Bank liability structure, which consists of deposits, non-deposit debt, and equity, has drawn attention from law makers and economists because of the recent crisis in banking industry. After the global financial crises during 2007–2009, regulators around the world have gradually rolled out new rules on bank liability structure.[1] Regulators and academics have been grappling with the questions about both the level and the composition of capital that banks should hold. While the desired level of bank capital has been controversial, there are debates on the amount of non-deposit debt a bank should hold along with deposits and equity.[2] Bank capital requirements often distinguish non-deposit debt from deposits, and they do not always treat deposits and non-deposits alike. Since the global financial crisis of 2007–2009, regulators have raised the capital requirements to mitigate bank risk-taking.[3]

An important issue that the academic literature has left unaddressed is how banks' optimal liability structure may respond, ex ante, to the capital requirements set by the resolution authorities, such as the OCC and state banking commissions in the United States, and the resolution authorities in other countries. Regulators typically set minimum capital requirements. Typically, some regulators have the authority to close and liquidate a bank if the bank cannot recapitalize to maintain its capital above a threshold required by the regulation.

The goal of this paper is to develop and solve a dynamic structural continuous-time model of banks that strategically choose the composition of deposits and non-deposit debt to maximize the total bank value of shareholders in response to the capital requirements. We call this value-maximizing choice *the strategic bank liability structure*. Given the sweeping changes in bank capital regulation and the debate over capital requirements, we wish to understand how the regulatory changes influence the mix of deposit and non-deposit liabilities. We do this in a model that unifies different types of capital requirements with risk-based pricing of deposit insurance and costly liquidation of undercapitalized banks that are unable to recapitalize.

[1]In the U.S., the Dodd-Frank Act of 2010 brought sweeping regulatory reforms ranging from deposit insurance to equity capital adequacy. Worldwide, bank regulators agreed on Basel III in 2011 to control the risks in bank liabilities.

[2]While Admati and Hellwig (2013) argue for lowering bank leverage to a level similar to the leverage of non-financial firms, Tarullo (2013) argues that holding non-deposit debt can improve the safety and resolution of banks, consistent with some academic papers (e.g., Flannery and Serescu, 1996). However, some scholars have opined the opposite (e.g., Gorton and Santomero, 1990).

[3]Basel III raises the capital requirement for all banks and imposes additional requirements for large banks that are regarded as systemically important. Both European and U.S. regulators have laid out new capital requirements for banks in accordance with Basel III.

Our analytical solution to the model reveals a salient feature of the strategic liability structure: the bank chooses deposits and non-deposit debt to optimize the advantage of debt[4] and minimizes the protection for the insurer of deposits. This choice of liability structure makes the risk of breaching the requirements coincide with the optimal default decision for the equity holders. Our comparative static analysis shows that this strategic bank liability structure is the key to the understanding of bank responses to regulations.

Banks are different from other firms as they take deposits. Deposits are different from other forms of debt because deposits are liquid and banks provide banking services through deposit accounts. Banks pay little or no interest on deposits and earn fees from the banking services. The regulation on bank entry into the deposits markets implies that banks' cost of attracting deposits is lower than the cost of non-deposit debt. Another important feature of banks is that a large part of deposits in commercial banks are under deposit insurance and the insurance premium depends on the risk exposure of the deposits.[5]

Most importantly, regulators require each bank to maintain capital above certain levels relative to assets. Regulators have the authority to close and liquidate a bank if the bank cannot recapitalize to maintain its capital above the minimum requirements. In a unified model of these bank regulations, we show that given the capital requirements, a bank chooses its liability structure, ex ante, in such a way to leave it indifferent between being liquidated by the regulatory authorities or being able to default on its debt obligations to maximize the market value equity.

This property of the strategic liability structure has an intuitive economic rationale. Deposits are cheaper than non-deposit debt as a financing source. Therefore, banks should generally prefer deposits to non-deposit debt when balancing the debt advantage with the risk of breaching capital requirements. However, as long as endogenous default (a default optimal for equity holders) does not happen before the breach of capital requirements, non-deposit debt does not affect the regulatory risk. The bank should therefore issue as much non-deposit debt as possible for availing of the debt advantage, while avoiding endogenous default to happen when bank capital is still above the requirements. Hence, the optimal level of non-deposit debt sets the endogenous default and the breach of capital requirement concurrent.

[4]The main advantage of debt financing is the benefit of tax deductibility of interest costs. This benefit is especially important for banks because of their high leverage. Debt financing can also have other advantages, such as the benefit to control ownership and the benefit to retain profits.

[5]In an earlier version of the paper, we also present a model in which deposits are uninsured and depositors can run. In that model, a bank is closed when depositors run at a threshold of asset value.

A commonly-stated purpose of capital requirements is to mitigate the problem of risk-taking by banks. This problem arises from the incentives for risk substitution: once debt is issued, shareholders may transfer value to themselves by shifting to riskier assets. We find an important property of the strategic bank liability structure is to preserve the incentives for risk substitution. Indeed, regulators can mitigate the incentives by raising the capital requirement if the bank liability structure does not optimally respond to regulation. However, if a bank strategically chooses liability structure to maximize the total bank value of shareholders, there are always incentives for risk substitution as if deposits and non-deposit debt are both unprotected. The reason is that banks can undo the protection by setting the endogenous default boundary at the same level as the regulatory boundary. This finding suggests that banks can adjust the relative proportion of deposits and non-deposit debt to reduce the effectiveness of capital requirements.

The comparative static analysis of the strategic choice of bank liability structure delivers some nontrivial theoretical implications that are unique to banks. The analysis shows that if regulators raise capital requirements, the non-deposit ratio of a bank goes up, opposite to the change in the deposit ratio. The analysis also shows that the non-deposit debt ratio is more sensitive to the profitability of deposits than the deposit ratio, leading to a higher credit spread. There have been regulations on deposit interest[6] and bank entry that are intended to improve the stability of banks by making deposits profitable, but the regulation may increase the credit risk of banks if banks respond strategically. Moreover, the analysis shows that deposit insurance subsidy leads to higher credit spreads of banks. Each of these implications need to be understood through the strategic adjustment of liability structure. These implications are absent in models that ignore the endogenous choice between deposits and non-deposit debt.

The strategic bank liability structure in our model accounts for the feedback between the risk-based deposit insurance premium and liability structure.[7] While a bank's choice of liability structure affects the insurance premium that the bank has to pay, insurance premium also affects the bank's choice of liability structure. Our model is the first in the literature to incorporate this feedback channel, which is crucial in assessing the regulatory

[6]The Banking Act of 1933, known as the Glass-Steagall Act, prohibits banks from paying interest on demand deposits and authorizes the Federal Reserve to impose ceilings on interest paid on time deposits. The restrictions on deposit interests were removed by the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Depository Institutions Act of 1983, the latter of which is the Garn-St. Germain Act.

[7]The Dodd-Frank Act requires the FDIC to reform its deposit insurance policy, reducing or eliminating the deposit insurance subsidy, because the subsidy may incentivize banks to use leverage. See Federal Deposit Insurance Corporation (2011).

policies pertaining to deposit insurance. Several papers, eminently Merton (1977) and Ronn and Verma (1986), have derived risk-based deposit insurance policy for given bank capital structure. Duffie et al. (2003) value the FDIC deposit insurance for given leverage and given bankruptcy risk. Our model extends their work to incorporate the endogenous determination of regulatory closure risk when bank liability structure optimally adjusts for risk-based insurance premium.

We first derive all the results from a model in which banks set the strategic liability structure only at the initial time but are able to dynamically issue equity to postpone default or meet capital requirements. This model is intuitive to understand and allows us to derive all the results analytically. After gaining insights from this intuitive model, we extend it to incorporate banks' dynamic restructuring decisions. We demonstrate that the findings from the model without dynamic restructuring continue to hold after incorporating dynamic restructuring. In the model without dynamic restructuring, we focus on the total capital requirement, which treats non-deposit debt as a capital to protect deposits. In the model with dynamic restructuring, we consider the tangible equity requirement, which mandates banks to keep a minimum level of tangible equity relative to the assets. The latter requirement is either the tier-1 capital requirement or leverage ratio requirement, which we will explain in the model with dynamic restructuring.

Our model of strategic bank liability structure advances the literature that attempts to apply the structural continuous-time models of Merton (1974, 1977) and Leland (1994) to banks. Those models study the tradeoff between equity and one type of debt, not the composition of different types of debt. Rochet (2008) studies the endogenous default of banks while setting aside the choice of liability structure. Harding et al. (2009) directly apply Leland's model to banks by treating deposits as non-deposit debt without paying attention to deposit insurance or bank regulation. Auh and Sundaresan (2020) investigate a firm's composition of repo and unsecured debt in financing, but the firm does not have the properties of banks, such as capital requirements, deposit profitability, or deposit insurance. These special features of banks are central to our analysis. Our model of banks with dynamic restructuring differs from Subramanian and Yang's (2020) model, which assumes that the mix of deposits and non-deposit debt is *exogenously* given and fixed.[8] By contrast, our model characterizes banks' *endogenous* choice between the two types of liabilities.

The road map for the rest of the paper is as follows. Section 2 develops and analytically

[8] In section 4.3, we provided a detailed discussion of the difference between our model and Subramanian and Yang's (2020).

solves the model of the strategic bank liability structure under the assumption that the bank sets up the optimal liability structure only once. Section 3 provides comparative static analysis to show how the strategic bank liability structure responds to changes in regulations. Section 4 extends our model to allow banks to dynamically restructure their liabilities. Section 5 concludes and discusses further applications and extensions of our model.

2 Bank Liability Structure

While banks share some common characteristics with non-bank firms, they differ in that banks take liquid deposits and provide banking services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The business of taking deposits and providing banking services is under heavy regulation in most countries. A large part of deposits at commercial banks in the U.S. is FDIC-insured, for which FDIC charges risk-based insurance premiums and imposes regulations on depository institutions. The model of FDIC insurance has gained popularity outside the U.S., and many countries and regions offer deposit insurance.[9] Governments also impose regulations on both the opening of new banks and the closing of existing banks.

Deposits and the associated banking services, deposit insurance, and the regulations on opening and closing banks distinguish banks from other non-bank corporations and make the financial decisions of banks different from the decision of other non-bank firms. Both non-bank firms and banks have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Firms operate in a market with at least two major frictions: tax advantage of debt and bankruptcy costs. These frictions are crucial for the choice of firm capital structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed in the structural model by Leland (1994). Banks face these frictions too, but they need to incorporate additional considerations, such as payment services to deposit accounts, deposit insurance, and minimum capital requirements, in determining their liability structure. We provide a careful analysis of the structure in Section 2.1 and discuss deposit insurance and bank regulation in Section 2.2.

Following Leland (1994), we first use a time-independent framework in this section and assume that a bank makes a capital structure decision at the beginning and then

[9] The International Association of Deposit Insurers (IADI), formed on May 6, 2002, works to enhance the effectiveness of deposit insurance systems by promoting guidance and international cooperation. At the end of 2014, the IADI represents 79 deposit insurers from 76 countries and areas.

extend it to allow dynamic restructuring in Section 4. The time-independent framework can be understood as an infinite-horizon Markov stationary environment. Leland (1994, page 1215) explains its advantage as follows: "Time independence permits the derivation of closed-form solutions for risky debt value, given capital structure. These results extend those of Merton (1974) and Black and Cox (1976) to include taxes, bankruptcy costs, and protective covenants (if any). They are then used to derive closed-form solutions for optimal capital structure."

2.1 Bank Assets and Liabilities

A typical bank owns a portfolio of assets that generate cash flows. Most assets in banks are risky. Since our focus is on liability structure, we assume that the value of the risky asset portfolio, denoted by V, follows a stochastic process. Following Merton (1974) and Leland (1994), we assume the stochastic process is a geometric Brownian motion:

$$dV = (r - \delta)V dt + \sigma V dW, \tag{1}$$

where r is the risk-free interest rate, δ is the rate of cash flow, σ is the volatility of asset value, and W is a Wiener process in the risk-neutral probability measure.

The instantaneous cash flow of the assets is δV. In a non-bank firm, δV is the total earnings, but in a bank, δV represents the earnings from bank assets such as loans, but it does not include the profits from serving deposits. The cash flow δV follows a geometric Brownian motion with the same volatility σ, which summarizes the risk of the asset portfolio. One may alternatively assume that the asset cash flow follows a geometric Brownian motion with volatility σ and then show that the asset value follows the stochastic process in equation (1).

We focus on the strategic liability structure, which is characterized by the ratios of deposits, non-deposit debt, and equity to assets. While the strategic choice of assets along with the choice of liability is an interesting research topic, the strategic liability structure should be optimal relative to the optimal asset portfolio. Therefore, the formula of the strategic liability structure in our model is likely to hold in a model that optimizes the assets and liabilities simultaneously. For this reason, our model can be used to shed light on the incentives for risk substitution, as we will show later in Proposition 3.

Following Merton (1974) and Leland (1994), we assume that investors have full information about the asset value. In reality, active investors use all available information to assess bank asset value and cash flows albeit only accounting values of assets are directly observable in quarterly filings. The full-information assumption sets aside the disparity between accounting value and intrinsic value. We interpret V as the fair accounting value. If the assets are of the same risk category, we can interpret V as the value of risk-weighted assets.[10]

When banks take deposits and provide banking services to depositors, deposits serve as an important source of funds for banks to finance their assets. Let D denote the deposits that a bank takes. Deposits are safe if they are under an insurance that guarantees depositors to be paid in full. We assume all deposits are under insurance to focus on the tradeoff between insured deposits and other forms of debt. Deposit insurance requires the bank to pay premiums, which we will discuss in the next subsection. Insured deposits work like protected perpetual debt in the capital structure. A protected debt captures the effects of withdrawals and roll-over of short-term debt in capital structure.[11]

Banks charge fees for banking services such as money transfers, overdrafts, etc. and pay little or no interest on deposits. Depositors accept a lower interest rate than the risk-free rate because they value instantaneous liquidity and banking services. Let η be the bank's profit earned from providing banking services on each dollar of deposits. We use η to characterize the profitability of the bank's deposit business. The profits of deposits can be attributed to some local monopoly rents that banks enjoy because of barriers of entry. We do not explicitly analyze the market for deposits but instead focus on the effects of deposit profitability on bank's choice of liability structure. The deposit profitability η plays an important role in bank liability structure. It represents depositors' sacrifice for the liquidity and services provided by the bank. This sacrifice is a distinctive character of deposits. A bank's net cost of serving deposits, excluding deposit insurance premium, is $C_D = (r-\eta)D$. If we include insurance premium, denoted by I, the cost of serving deposits is $C_D + I$.

Non-deposit debt is another important funding source for banks. There is no banking service associated with non-deposit debt. A bank pays interest on non-deposit debt until the regulator closes the bank or the equity holders default on debt. The holders of non-

[10] Also following Leland (1994), we assume that V is the after-tax value of assets, and thus δV is the after-tax cash flow. Alternatively, one may start the model with the before-tax value of the assets as in Goldstein et al. (2001).

[11] Leland (1994, page 1234) explains this well: "An alternative contractual arrangement approximating this case would be a continuously renewable line of credit, in which the borrowing amount and interest rate are fixed at inception. At each instant the debt will be extended (rolled over at a fixed interest rate) if and only if the firm has sufficient asset value to repay the loan's principal; otherwise bankruptcy occurs. Thus the roll-over process proxies for a positive net-worth requirement. With this latter interpretation, the differences between the unprotected debt and protected debt analyzed *may capture many of the differences between long-term debt and (rolled over) short-term financing*."

deposit debt have a *lower priority* than depositors in claiming a bank's liquidation value. The lower priority of non-deposit debt can potentially protect deposits, and thus some non-deposit debt qualifies for being Tier 2 capital in bank regulations. Non-deposit debt comes with a cost: its interest rate contains a credit premium to compensate the debt holders for bearing the risk of bank closure or default. We solve the *endogenous* credit spread in the model; the spread depends on both the asset risk and the liability structure. Let s be the credit spread, B be the value of non-deposit debt, and C_B be the interest cost. These variables are related by $C_B = (r + s)B$.

Equity holders garner all the residual value and earnings of the bank after paying the contractual obligations on deposits and non-deposit debt. The first slice of the value that equity holders lay claim to is the asset value exceeding the value of deposits and non-deposit debt: $T = V - (D + B)$. This slice is referred to as tangible equity. This is the value that equity holders would receive if the bank liquidates its assets at fair value, without incurring liquidation costs, and pays off deposits and non-deposit debt at their par values. A larger tangible equity means a smaller loss for debt holders after liquidation. Hence, regulators regard tangible equity as bank capital of the highest quality—Tier 1 capital.[12]

Since equity holders receive the net earnings of the bank, the present value of the net earnings is the bank's charter value. The net earnings contain the benefits of financial leverage. Following the literature, we assume that the benefit of debt is proportional to interest costs: $\tau(C_D + C_B)$ where $\tau \in (0, 1)$, because the most important benefit of debt is the tax deductibility of interest costs. We simply interpret τ as a tax rate. The tax rules in the U.S. also allow banks to deduct the insurance premiums paid to the FDIC.[13] Therefore, the total benefit of financial leverage is $\tau(I + C_D + C_B)$. The dividend available to be paid to equity holders is the asset cash flow plus the benefits of leverage and minus the insurance and interest costs: $\delta V - (1 - \tau)(I + C_D + C_B)$.

The total value generated for the original bank shareholders by the liability structure is the combined value of deposits, non-deposit debt and equity. So, $F = D + B + E$ is the total bank value of shareholders. Since equity value depends on its dividend, it depends on the liability structure. The triplet (I, C_D, C_B) therefore determines the liability structure of

[12]It is useful to point out that the tangible equity of a bank can be negative both in theory and in practice. For example, the U.S. operation of Deutsche Bank reported a total asset value of $355 billion and a negative $5.68 billion Tier 1 capital in its December 2011 filing as a bank holding company.

[13]However, some recent proposed changes in tax law (IRC Section 162(r)) intend to phase out the deductibility of insurance premiums paid by large bank holding companies with at least $50 billion assets. We can adjust our model to reduce or exclude tax deductibility of deposit insurance premium, but we assume that deposit insurance premiums are deductible, consistent with the current tax law for typical commercial banks.

a bank and thus determines the total bank value. We provide a closed-form formula of the total bank value in the Online Appendix. Without loss of generality, we ignore the costs of structuring liabilities here, but we will later incorporate such costs in the model of banks that dynamically restructure their liabilities.

Our model of liability structure distinguishes banks from non-bank firms but is consistent with the structural model of firms. If we set $C_D = I = 0$ but keep $C_B > 0$, the model is equivalent to Leland (1994) for non-bank firms that issue equity and corporate bonds but do not take deposits. Our model extends Leland's model to banks and offers a consistent framework for understanding the similarities and differences between banks and non-bank firms. Most importantly, our framework allows banks to strategically adjust the mix of deposits and non-deposit debt in response to regulations; this is the unique feature of our model.

2.2 Bank Regulation and Deposit Insurance

Without deposit insurance, deposits bring a bank the risk that depositors may run, a major challenge commonly faced by banks. As experienced in the crises of the U.S. banking history and theorized in the academic literature, depositors may run from a bank if they think the bank has difficulty in repaying their deposits promptly upon their demand. Diamond and Dybvig (1983) pioneered the bank run literature by showing that bank runs can emerge as an equilibrium. Allen and Gale (1998) and others have extended the literature significantly. The establishment of the FDIC is to deter bank runs by guaranteeing deposits to be paid in full. Therefore, insured deposits are exposed to no risk of bank run in our model.

A bank's charter authority, which is typically either the bank's state banking commission or the Office of the Comptroller of the Currency (OCC), is the regulator that has the authority to close and liquidate the bank. The charter authority closes a bank if the bank is insolvent or if the bank's capital is too low to recapitalize. Regulators categorize a bank as critically under-capitalized when the total capital that protects deposits drops to a fraction of the assets.[14] We model this capital requirement and closure policy exactly as they are in the U.S. bank regulation. The total capital that protects deposits consists of tangible equity and non-deposit debt. This is equivalent to $T + B = V - D$ in our model. The capital requirement is the minimum capital for a bank to operate, as modeled in Rochet (2008).

[14] The FDIC categorizes a bank as critically under-capitalized when the total capital drops to 2% of its asset value. The total capital includes non-deposit debt that protects deposits. For a review of the rules about categorizing banks as critically under-capitalized, we refer readers to Shibut et al. (2003). The capital requirement in our model mirrors the policy of the FDIC.

The charter authority shuts down a bank when the bank cannot recapitalize to keep capital above the minimum requirement. Regulations typically specify a capital requirement as a ratio to the asset value, denoted by β in our model. If the capital requirement is two percent, as in the closure policy of FDIC, then $\beta = 2\%$. Let V_b be the asset value when the charter authority closes the bank. Then $V_b - D = \beta V_b$, which implies $V_b = D/(1-\beta)$. Therefore, the regulator closes the bank when the bank asset value drops to V_b. We call V_b the *regulatory total capital boundary*. This traditional form of capital requirement requires banks to maintain the total capital ratio above certain level. The bank regulation developed after the 2008 financial crisis requires each bank to maintain additional tangible equity as conservation buffer. We will extend our model to incorporate alternative capital requirements in Section 4.2.

The FDIC functions as the insurer of a bank's deposits and as the receiver of the bank when it is closed by its regulator. As a receiver, the FDIC liquidates the assets of a closed bank in its best effort to pay back the bank's creditors. Suppose the liquidation cost is αV_b, proportional to the asset value V_b of the closed bank.[15] As an insurer, the FDIC pays D to the depositors of the closed bank. The insurance corporation loses $D - (1-\alpha)V_b$ if $(1-\alpha)V_b < D$. Thus, the loss function of the insurer is $[D - (1-\alpha)V_b]^+$, where $[x]^+ = x$ if $x \geq 0$ and $[x]^+ = 0$ if $x < 0$. Since $V_b = D/(1-\beta)$, the loss function is positive if $\beta < \alpha$, in which case the FDIC expects to suffer a loss after a bank closure. In practice, the FDIC expects a chance of loss because the liquidation cost is uncertain. To keep our analysis tractable, we assume $\beta < \alpha$ so that the FDIC expects a loss after a bank closure. Otherwise, there is no reason for the insurer to charge any insurance premium. After the FDIC liquidates the bank assets, it pays the depositors first and, if there is money left, the debt holders next. The payoff to the debt holders is therefore $[(1-\alpha)V_b - D]^+$. It follows that the debt value at the regulatory closure is $B(V_b) = [(1-\alpha)V_b - D]^+$.

To cover its potential loss, the FDIC charges insurance premiums. In 2006, Congress passed reforms that permit the FDIC to charge risk-based premiums. To determine deposit insurance assessment, the FDIC places each insured depository institution into one of four risk categories each quarter, depending primarily on the institution's capital level and supervisory evaluation. A riskier bank pays a higher insurance premium than a safer bank.

[15] The cost of liquidation by the FDIC may be different from the costs of liquidation through bankruptcy courts. Since the FDIC does not go through the lengthy procedure of bankruptcy, there is a belief that the FDIC liquidation cost is smaller than the typical bankruptcy cost in the private sector. Title II of the Dodd-Frank Act reflects the belief as it authorizes the FDIC to receive and liquidate the failed large financial institutions to avoid lengthy and costly bankruptcy procedures.

In principle, the insurer should assess insurance premiums based on the deposits under insurance.[16] If the assessment rate is g, the deposit insurance premium paid by a bank is $I = gD$. Since April 2011, the assessment rate may also depend on the credit rating of the bank and on the level of non-deposit debt that protects deposits.[17]

The fair insurance premium should make the insurance contract worth zero to each party of the insurance contract. It should depend on the deposit ratio and the risk exposure of the deposits. We derive, in the Online Appendix, a closed-form formula of the fair assessment rate, which is shown below:

$$g^\circ = \frac{r[\alpha - \beta]^+}{1-\beta} \cdot \left[\frac{D/V}{1-\beta}\right]^\lambda \left(1 - \left[\frac{D/V}{1-\beta}\right]^\lambda\right)^{-1}, \tag{2}$$

where λ is the positive solution to the quadratic equation: $\sigma^2\lambda(1+\lambda)/2 - (r-\delta)\lambda - r = 0$. If the cash flow of the assets is zero, i.e., $\delta = 0$, we have $\lambda = 2r/\sigma^2$, which is proportional to r and inversely proportional to σ^2.

The fair insurance premium for deposits D is $I^\circ = g^\circ D$. Equation (2) indicates that the fair assessment rate g° increases with D. Therefore, if deposits expand, the insurance premium increases not only because of the expansion of deposits but also because of the rise in the assessment rate. The rate is increasing with D because an expansion of deposits exposes the FDIC to a bigger risk. If $\beta < \alpha$, the fair premium I° is positive. It converges to zero as β rises to α. If $\beta \geq \alpha$, the fair premium is zero because the closed bank will have enough assets to pay for the deposits.

The FDIC, however, may not charge fair insurance premium. Duffie et al. (2003) argue that the FDIC does not charge enough insurance premium to cover its risk exposure.[18] A low premium provides subsidized insurance to banks. When the premium is subsidized, it can alter the bank value and affect its choice of liability structure. We use ω to denote the insurance subsidy as a fraction of the fair insurance. To allow a subsidized insurance premium, we assume that the assessment rate of deposit insurance is $g = (1-\omega)g^\circ$ and the insurance premium is $I = gD$. It is natural to expect that insurance subsidy increases bank value because the bank pays a reduced insurance premium for enjoying the full risk-free value of deposits. It is, however, not obvious how the subsidy affects bank liability

[16] There have long been concerns that banks switch deposits to non-deposit debt temporally at quarter-ends to lower the deposits on bank books. To deter such switch, the Dodd-Frank Act (Section 331) required the FDIC to modify the assessment base as the difference between the risk-weighted assets and the tangible equity.

[17] For more details, see Federal Deposit Insurance Corporation (2011). Garnett (2020) provides a history of risk-based premiums at the FDIC.

[18] They also suggest that a lower premium may be necessary to compensate the banks for the costs of reporting requirements and following regulations.

structure.

The equity holders default on the bank's debt obligations and liquidate assets if the equity value reaches zero because this default strategy maximizes equity value and is thus optimal for equity holders. Before the equity value reaches zero, it is optimal for equity holders to issue new equity at market price to recapitalize. When the equity value reaches zero, the bank cannot recapitalize to keep the bank running and thus default on the debt. This is referred to as *endogenous default*. The endogenous default boundary is the asset value, denoted by V_d, that gives zero equity value. In the Online Appendix, we derive the formula of the endogenous default boundary for banks, which is shown below:

$$V_d = (1-\tau)\frac{\lambda}{1+\lambda}\frac{I + C_D + C_B}{r}, \tag{3}$$

given insurance premium I, the deposit liability C_D, and the non-deposit debt liability C_B. The formula indicates that the endogenous boundary is an increasing function of each liability.

A bank ceases to operate and its assets go to liquidation either when the regulator closes the bank or when the equity value reaches zero, whichever happens earlier. Then, the boundary for the bank to liquidate assets is $V_l = \max\{V_b, V_d\}$. We call V_l the liquidation boundary of the bank. Since the claims of the depositors take priority to the claims of non-deposit debt holders, the value of non-deposit debt at the liquidation boundary is $B(V_l) = [(1-\alpha)V_l - D]^+$.

2.3 Strategic Liability Structure

Table 1 summarizes the exogenous parameters discussed in the preceding sections, as well as the assumptions on their range. Deposit business generates profit ηD with $0 < \eta < r$. Deposits and non-deposit debt bring the benefit of leverage: $\tau(I+C_D+C_B)$ with $0 < \tau < 1$. Liquidation results in a cost of cost αV_l with $0 < \alpha < 1$. Bank's total capital has to meet a minimum requirement: $V \geq V_l = D/(1-\beta)$ with $0 \leq \beta < \alpha$. The FDIC may subsidize the insurance premium $g = (1-\omega)g^\circ$ with $0 \leq \omega < 1$. These assumptions are not only realistic but also the requisite mathematical conditions for valuation and optimization.

The strategic choice of liability structure $(D^*/V, B^*/V)$ maximizes the total value of the original equity holders. A bank can adjust either assets or non-deposit debt, or both, to achieve a desired liability structure, even if the bank accepts deposits passively from depositors. A value-maximizing bank accounts for the cost of deposit insurance when trading off the debt advantage and deposit profitability with the capital requirements and liquidation risk. The bank in our setting is fully aware that any decision pertaining to the liability structure has a consequence on the insurance premium because it pays risk-based insurance premium $I = gD$, where $g = (1-\omega)g^{\circ}$ is the assessment rate and g° depends on D and other parameters of the bank as described in equation (2). The bank cares about both the increase in the premium caused directly by the expansion of deposits as well as the increase in premium caused indirectly by the rise of assessment rate. The bank should therefore be mindful of this channel in its choice of liability structure. The relation between risk-based insurance premium and liability structure captures the feedback channel from the insurance cost to the bank and vice versa.

Our first proposition about bank strategic liability structure is that a bank must hold some non-deposit debt along with deposits. The detailed derivation of this proposition is provided in the Online Appendix.

Proposition 1. *Bank can increase its value by increasing non-deposit debt if the current non-deposit debt ratio is so low such that the endogenous default boundary is lower than the regulatory total capital boundary. That is, the liability structure with $V_d < V_b$ is suboptimal.*

The most important implication of this proposition is that the non-deposit debt in a strategic liability structure must be positive. It is suboptimal for a bank to have only deposits and equity in its liability structure. Non-deposit debt is necessary for a bank that strategically chooses its liability structure to maximize value. The bank must hold at least sufficient non-deposit debt so that $V_d \geq V_b$. Song and Thakor (2007) argue that banks issue non-deposit debt to match the risk of loans in relationship banking. The strategic bank liability structure in our model offers a different perspective for banks to issue non-deposit debt.

If asset volatility σ and liquidation cost α are both very high and the capital requirement β is very close to 0, the risk-based assessment rate can be so high that it cancels out the profits of deposits. This unusual and unrealistic situation will make deposits more expensive than non-deposit debt. To keep our formulation succinct for the rest of this section, we assume that the parameters exclude this situation. That is, we assume that the asset volatility σ, liquidation cost α, and capital requirement β are realistic so that the risk-based assessment rate is small relative to the service income. More precisely, we assume that the insurance subsidy ω and the capital requirement β are high enough so that

$$g < \frac{\eta}{1+\lambda}. \tag{4}$$

This assumption means that the assessment rate g should not be so high that wipes out the value of η in the provision of liquid deposits. We call this *the regularity assumption*. For all the practical asset volatility and liquidation cost used in our comparative static analysis in Section 3, we actually find that the regularity assumption holds for each capital requirement $\beta \in [0, \alpha)$.

The next proposition, derived in the Online Appendix, provides a complete characterization of the strategic liability structure.

Proposition 2. *A bank's strategic liability structure sets the deposit ratio and the non-deposit debt ratio so that the endogenous default boundary coincides with the regulatory total capital boundary: $V_b^*/V = V_d^*/V = \pi^{1/\lambda}$, where π is the state price of regulatory total capital boundary and it is related to the parameters by*

$$\pi = \frac{1}{1+\lambda} \cdot \frac{\eta(1-\tau)\lambda(1-\beta) + r\tau(1+\lambda)}{\eta(1-\tau)\lambda(1-\beta) + r\tau(1+\lambda) + r(1-\tau)\lambda[(1-\omega)\alpha + \omega\beta]}. \tag{5}$$

The deposit ratio and the non-deposit debt ratio are related to the parameters by:

$$D^*/V = (1-\beta)\pi^{1/\lambda}, \quad B^*/V = \left(\frac{\tau}{1-\tau} + \beta + (1-\beta)\frac{\eta}{r}\frac{\lambda}{1+\lambda} + \frac{1-\pi}{\lambda}\right)\pi^{1/\lambda}. \tag{6}$$

The credit spread and assessment rate are related to the parameters by:

$$s^* = \frac{r\pi}{1-\pi}, \quad g^* = (1-\omega)\frac{\alpha-\beta}{1-\beta} \cdot \frac{r\pi}{1-\pi}. \tag{7}$$

The strategic liability structure maximizes the total value by balancing the benefits and costs. The benefits are the deposit service income and the debt advantage. The costs are the risk-based premium for deposit insurance and the risk of costly liquidation. The proposition shows that the strategic bank liability structure depends on the following parameters: r, σ, δ, η, α, β, ω, and τ. The parameters σ, δ, η and α are likely to be heterogeneous among banks, whereas the parameters β, ω, and τ mainly depend on regulations and government policies.

If we set both the profit of serving deposits (η) and the subsidy of deposit insurance (ω) to zero, the strategic liability structure reduces to the optimal firm capital structure derived by Leland (1994). In this sense, Leland's model of capital structure of a non-bank firm is a special case of our model of bank liability structure. Nesting the model of non-bank firms as a special case is useful for comparing banks with non-bank firms and for examining the special properties that make the banks different from non-bank firms.

The ratio of deposits to assets, D^*/V, and the ratio of non-deposit debt to assets, B^*/V, in Proposition 2 determine the optimal bank liability structure. Regulators and investors monitor these ratios closely. They measure the leverage of a bank by its tangible equity ratio: $T^*/V = 1-(D^*+B^*)/V$, for which we can have a closed-form formula from equation (6). A lower tangible equity ratio means a higher leverage. Proposition 2 suggests that a value-maximizing bank strategically determines these financial ratios.

It is important to note that banks strategically use non-deposit debt to set the endogenous default boundary relative to the asset value. The regulatory total capital boundary determines the risk that the regulator closes the bank. The strategic bank liability structure in Proposition 2 sets the endogenous default boundary V_d to be same as the regulatory total capital boundary V_b. Recall that the endogenous default boundary is the optimal point for equity holders to default, in the absence of total capital requirement. Therefore, the strategic liability structure balances deposits and non-deposit debt so that the regulatory total capital boundary is optimal for equity holders. The coincidence of the two boundaries implies that the capital is exactly the minimum required by regulation at the time the equity holders choose to default optimally. This strategy leaves no extra capital to protect the deposit insurer. Therefore, the strategic choice of the liability structure minimizes the protection for the deposit insurer.

We can intuitively understand the strategic response as follows. Deposits bring profits from banking services, in addition to the tax advantage, and the extra cost of taking deposits is the insurance premium. By contrast, non-deposit debt brings only the tax advantage, while it bears the credit premium. If the assessment rate of deposit insurance is lower than the credit spread, the bank should issue as much non-deposit debt as possible to avail of the tax advantage but should avoid issuing too much non-deposit debt to move the endogenous default boundary above the regulatory total capital boundary. Therefore, the bank should use non-deposit debt to set the two boundaries exactly equal.

An important issue related to the choice of capital structure is the incentives for risk substitution. There are such incentives if the market value of equity is an increasing, convex function of asset value. In this case, equity resembles a call option on asset value. The call option value is higher for a riskier underlying asset. The literature has pointed out that corporate debt may create incentives to substitute assets with higher risk (e.g., Green, 1984, and Harris and Raviv, 1991). Leland (1994, 1998) shows that the unprotected debt in a highly leveraged firm has such incentives because the market value of equity is an increasing, convex function of asset value. The literature suggests that such incentives may arise in banks as well when deposits are insured (e.g., Pennacchi, 2006, and Schneidar and Tornell, 2004). The incentives for risk substitution is important for banks because banks use high leverage.[19] Leland (1998) and Toft and Prucyk (1997) show that protection (covenant) for debt can mitigate the incentives for risk substitution. Because capital requirements are expected to protect deposits, they are analogous to covenants that pro-

[19] A large literature on bank loans pays attention to the incentives for risk substitution. For example, Gorton and Kahn (2000) consider this incentives in the design of bank loans.

tect firm debt. One may therefore expect capital requirements to mitigate the incentives for risk substitution.

The next proposition presents two implications about the incentives of risk substitution in the context of bank liability structure.

Proposition 3. *(1) For given non-deposit debt obligation C_B, there are $\bar{D}$ and $\bar{\beta}$ with $\bar{D} > 0$ and $0 < \bar{\beta} < \alpha$ so that for any given deposit ratio $D/V \in (\bar{D}/V, +\infty)$ and any capital requirement $\beta \in (\bar{\beta}, \alpha)$, the market value of bank equity is an increasing, concave function of asset value. (2) However, if a bank strategically chooses liability structure to maximize bank value, the market value of bank equity is always an increasing, convex function of asset value for all β.*

The proposition is derived in the Online Appendix. The first part of the proposition says that the regulator can raise the capital requirement β high enough, but below the bankruptcy cost α, to remove the incentives for risk substitution, given a fixed composition of non-deposit debt and deposits in bank liability structure. The lower bound $\bar{D}$ is given in the Online Appendix that derives the proposition. The lower bound specifies the minimum deposits in the balance sheet such that the incentives for risk substitution can be removed by raising the capital requirement. The reason is that capital requirement protects deposits but does not protect non-deposit debt. If the bank holds too few deposits, protecting merely deposits does not have enough impact on the value function of bank equity. These mirror the results of Leland (1998) and Toft and Prucyk (1997), who show that protection (covenant) of debt can mitigate the incentives for risk substitution.

The second part of the proposition shows that a bank's strategic liability structure preserves the incentives for risk substitution by keeping the market value of equity an increasing, convex function of asset value. When a bank sets the endogenous default boundary at the same level as the regulatory total capital boundary to optimize the total bank value, the entire debt, which includes deposits and non-deposit debt, is essentially unprotected. As a result, the equity value with unprotected debt is increasing and convex as a function of asset value. This consequence has not been noted in the existing theories of bank capital structure that abstract from the strategic choice between deposits and non-deposit debt.[20]

[20]Proposition 3 does not imply that capital requirements lead to higher or lower incentives for risk shifting. The proposition only states that the incentives for risk shifting is preserved, instead of being removed, if banks use the strategic liability structure. Since asset risk is fixed, our model does not speak about the effects of capital requirements on credit portfolios, which are the subject in most empirical studies of capital requirements (for examples, see Aiyar et al., 2014 and Jimenez et al., 2016). A theory related to this issue is perhaps in the model of Della Seta et al (2020), which shows that short-term debt may engender greater risk-taking propensity.

3 Comparative Static Analysis

The strategic bank liability structure in our model depends on several factors. Debt advantage (τ), asset volatility (σ), and liquidation cost (α) are well-known factors in the capital decision in all firms. The unique factors for banks are the minimum total capital requirement (β), the profitability of deposits (η), and the deposit insurance with potential subsidy (ω). We focus on the effects of the three banking factors (η, β, and ω). We first examine the effects of deposits service because it distinguishes bank liabilities from non-bank firm's liabilities.

3.1 Effects of Banking Factors

To examine the effects of deposits, we imagine a comparable non-bank firm that holds the same assets as a bank but does not take deposits. The firm issues non-deposit debt and equity as in the model of Leland (1994). The parameters (η, β, ω) do not affect the non-bank firm's capital structure. One may regard the firm as a bank that constrains deposits to zero. Without deposits on its balance sheet, the firm liquidates when it endogenously defaults. There is no regulatory capital requirement.

Let $\bar{B}$ be the optimal non-deposit debt issued by the comparable non-bank firm that holds the same assets as the bank. The tangible equity in the optimal firm capital structure is $\bar{T} = V - \bar{B}$ because there are no deposits. The proposition below compares the strategic liability structure of a bank that takes deposits with the optimal capital structure of the non-bank firm that holds the same assets as the bank but does not take deposits. All propositions in this subsection are derived in the Online Appendix.

Proposition 4. *The strategic bank liability structure has higher leverage, higher default risk, and a higher credit spread than the optimal capital structure of a non-bank firm that holds the same assets. That is,*

$$\frac{T^*}{V} < \frac{\bar{T}}{V}, \quad \frac{V_d^*}{V} > \frac{\bar{V}_d}{V}, \quad s^* > \bar{s}, \tag{8}$$

where $\bar{V}_d$ is the endogenous default boundary and $\bar{s}$ is the associated credit spread of the debt in the optimal capital structure of the non-bank firm.

The most striking prediction of the above proposition is that a bank uses higher leverage than a comparable non-bank firm, even if they hold the same assets. It is well known that bank leverage is higher than firm leverage, but some academic papers attribute the high bank leverage to principal agent problems in banks and argue for reducing bank leverage to a level similar to the leverage of non-bank firms. In our model, banks maximize the total bank value. So, there is no principal agent problem in our model. Yet, the optimal leverage of banks is still much higher than the optimal leverage of non-bank firms, after we control their assets to be same. Since the business of serving deposits is the difference between the bank and non-bank firm in the above proposition, it is the reason for higher bank leverage.

As explained in our model, the strategic liability structure internalizes the capital requirement to make the regulatory boundary optimal for equity holders. We therefore first consider the effects of a change in the minimum capital requirement on bank liability structure.

Proposition 5. *In response to tighter capital requirements, a bank chooses a lower deposit ratio but a higher non-deposit debt ratio. The overall leverage is lower because the drop in deposit ratio is larger than the rise in non-deposit debt ratio. The associated assessment rate and credit spread are lower if the capital requirement is higher. The assessment rate is more sensitive to the capital requirement than the credit spread. The inequalities below summarize these effects of capital requirement:*

$$-\frac{\partial(D^*/V)}{\partial\beta} > \frac{\partial(B^*/V)}{\partial\beta} > 0, \quad \frac{\partial T^*}{\partial\beta} > 0, \quad \frac{\partial g^*}{\partial\beta} < \frac{\partial s^*}{\partial\beta} < 0\,. \tag{9}$$

According to this proposition, tightening the minimum capital requirement leads the deposit ratio and non-deposit debt ratio to be adjusted in opposite directions. Most importantly, a bank uses more non-deposit ratio relative to assets if the capital requirement is higher. This proposition makes predictions not only about the direction of the effects but also about the relative sensitivity of the financial ratios to the capital requirement. The first chain of inequalities in the proposition says that the drop in the deposit ratio is larger than the rise in the non-deposit debt ratio. It follows that $\partial(T^*/V)/\partial\beta > 0$, which means that a tighter capital requirement leads to a lower leverage, as expected. However, the different effects on deposits and non-deposit debt will not be discovered in models that do not distinguish deposits and non-deposit debt.

As mentioned earlier, deposits are profitable because of banks' market power under the regulation on the entry to the market. We therefore examine the effects of deposit profitability on bank liability structure.

Proposition 6. *The strategic liability structure has a higher deposit ratio and a higher non-deposit debt ratio in a bank if its deposit business is more profitable. The associated assessment rate and credit spread are both higher. The non-deposit debt ratio is more sensitive to deposit profitability than the deposit ratio, and the credit spread is more sensitive than the assessment rate. The inequalities below summarize these effects of deposit profitability:*

$$\frac{\partial(B^*/V)}{\partial\eta} > \frac{\partial(D^*/V)}{\partial\eta} > 0, \quad \frac{\partial s^*}{\partial\eta} > \frac{\partial g^*}{\partial\eta} > 0\,. \tag{10}$$

The first chain of inequalities in (10) shows that the deposit and non-deposit debt ratios are both increasing functions of deposit profitability. It immediately follows that the tangible equity ratio is a decreasing function of deposit profitability: $\partial(T^*/V)/\partial\eta < 0$. This means banks use higher leverage if deposits are more profitable.

Given that deposit profitability is a benefit of bank leverage, the positive effect on deposit ratio is not surprising. However, a nontrivial result in Proposition 6 is that the non-deposit debt ratio is more sensitive to deposit profitability than the deposit ratio. The positive and relatively larger effect on non-deposit debt ratio runs counter to the intuition that a bank should mainly increase the deposit ratio if deposit profitability increases. Such intuition overlooks the strategic response of bank liability structure. If a bank raises its deposit ratio, it also needs to raise the non-deposit debt ratio to keep the regulatory total capital boundary and the endogenous default boundary at the same level so that the bank value is maximized.

Another nontrivial result of our model is that both the assessment rate and the credit spread have positive relations to deposit profitability, as shown in the second chain of inequalities in Proposition 6. Before 1980, the U.S. bank regulation prohibited banks from paying interest on deposits and limited bank competition for deposits, based on the belief that making deposit business more profitable would reduce bank failures. This line of thought ignores the strategic response of banks to the changes in deposit profitability. Proposition 6 shows that a bank's strategic response is to raise both the deposit and non-deposit debt ratios when deposit service is more profitable. This response raises the regulatory total capital boundary. Then, the risks to FDIC and non-deposit debt holders are higher, which leads to higher assessment rate and wider credit spread, respectively.

The next proposition concerns how a change in insurance subsidy affects the strategic bank liability structure.

Proposition 7. *A bank chooses higher deposit and non-deposit debt ratios if the insurance subsidy is larger. The deposit ratio is more sensitive to the insurance subsidy than the non-deposit debt ratio. The associated credit spread is higher if the insurance subsidy is larger, but the assessment rate is lower. The inequalities below summarize these effects of insurance subsidy:*

$$\frac{\partial(D^*/V)}{\partial\omega} > \frac{\partial(B^*/V)}{\partial\omega} > 0, \quad \frac{\partial s^*}{\partial\omega} > 0, \quad \frac{\partial g^*}{\partial\omega} < 0\,. \tag{11}$$

It is expected that a bank chooses a higher deposit ratio if the insurance is more subsidized, but it is less obvious why the bank also chooses a higher non-deposit debt ratio. To understand this, we need to recall that a higher deposit ratio implies a higher regulatory total capital boundary. For the bank liability structure to be optimal, the non-deposit debt ratio needs to be higher so that the endogenous default boundary matches the regulatory total capital boundary. This channel of the insurance subsidy effect would not be revealed if the model does not consider the endogenous choice between deposits and non-deposit debt.

While Proposition 7 predicts that the insurance subsidy lowers the insurance rate, as expected, it also predicts that the insurance subsidy leads to higher credit spread. The effect on credit spread is nontrivial. To understand it, we again need to consider the strategic response of banks to regulation. A larger insurance subsidy causes banks to choose not only a higher deposit ratio but also a higher non-deposit debt ratio. The combination of two higher ratios leads to a higher credit spread.

Most of the predictions in these propositions are unique to our model. The predictions are not only qualitative but also quantitative. To better understand the quantitative nature of the predictions, we will present numerical examples in the next subsection to illustrate these theoretical predictions.

3.2 Numerical Illustration

3.2.1 Effects of Including Deposits in Liabilities

In Table 2, we present an numerical example to illustrate the comparison between the strategic bank liability structure and the optimal firm capital structure of a non-bank firm. For the special parameters about banking in this example, the total capital requirement is assumed to be $\beta = 2\%$, the deposit profitability is assumed to be $\eta = 3\%$, and the subsidy in the insurance premium is assumed to be $\omega = 10\%$. For the other general parameters, the asset volatility is assumed to be $\sigma = 5\%$, the cash-flow rate of the assets is assumed to be $\delta = 8\%$, the liquidation cost is assumed to be $\alpha = 27\%$, the benefit of debt (tax rate) is assumed to be $\tau = 15\%$, and the risk-free interest rate is assumed to be $r = 5\%$. These parameters are only for illustration, although they are on average consistent with the observed historical data during 1984–2013. More detailed information about the historical averages can be found in the Online Appendix.

The example demonstrates some distinctive properties of the strategic bank liability structure. The first property to notice is that the bank takes a significant amount of deposits. The deposit ratio is 45.30%. Another noticeable property of the strategic bank liability structure is the magnitude of non-deposit debt. The non-deposit debt ratio is 46.49%, almost same as the deposit ratio. Another interesting property is the high leverage. The strategic bank liability structure has a tangible equity ratio as low as 8.23%.

The financial ratios in this example are broadly comparable to the average liability structure of the FDIC-insured commercial banks and savings institutions. Based on the data of the FDIC, the deposits are 45% of those banks' assets on average during 1984–2013. The other debt is 47% of the bank assets on average. The equity is 8% of the bank assets on average.

The last column of Table 2 presents the optimal capital structure of a comparable non-bank firm. Such a firm uses much lower leverage than the bank. The tangible equity ratio of the firm is nearly 40% of the assets, much higher than the 8% tangible equity ratio of the bank. The firm issues only slightly more non-deposit debt than the bank. The liquidation boundary of the firm is 35.28% of the asset value, lower than the liquidation boundary of the bank, which is 46.20% of the bank assets.

As noted in Proposition 2, the strategic bank liability structure sets non-deposit debt to a level such that the regulatory total capital boundary is optimal for the bank equity holders. The numerical example shows that this strategy leads the bank to higher leverage and higher liquidation risk than the comparable firm. Higher liquidation risk leads to a higher credit spread for the bank. In Table 2, the credit spread for the bank is 227 basis points whereas the credit spread for the firm is only 102 basis points.

While the numerical example assumes that the bank and the comparable non-bank firm hold the same assets, banks generally do not hold the same assets as firms. In the Online Appendix, we show that the volatility of the assets held by the non-bank firms is much higher than the volatility of assets held by banks. If we increase the asset volatility from 5% to 30%, the tangible equity ratio of the firm will be even higher. The association between low bank asset volatility and high bank leverage suggests that low volatility is part of the strategic choices by banks. As we have pointed out earlier, if a bank optimizes its assets and liabilities simultaneously, the strategic liability structure should still maximize total bank value, given any portfolio of assets.

3.2.2 Effects of Total Capital Requirement

The responses of bank liability structure to the changes in minimum total capital requirement are undoubtedly an important issue. Figure 2 provides an illustration of these effects. As we vary β from 0% to 25%, the deposit and non-deposit ratios respond in opposite ways. The deposit ratio does down from 45% to 33%, but the non-deposit debt ratio goes up from 46% to 51%. Overall, the tangible equity ratio rises from 8% to 15%. The reduction in leverage is smaller than the drop in the deposit ratio because of the increase in the non-deposit debt ratio. The reduction in the deposit ratio leads to a lower assessment rate, and the reduction in leverage leads to a smaller credit spread. Panel B of the figure shows the magnitude of these effects, which are predicted by Proposition 5.

The positive effect on the non-deposit debt ratio deserves special attention. This is a result from setting the regulatory total capital boundary to be the same as the endogenous default boundary. If the bank keeps its liability structure fixed, an increase in β will push up the regulatory total capital boundary above the endogenous default boundary. The bank needs to increase the non-deposit debt ratio to bring the endogenous default boundary up to the same level of the regulatory total capital boundary.

The regulatory boundary actually moves in the opposite direction of the total capital requirement. When β is higher, the drop in deposits is so large that it counteracts the direct effect of β on the regulatory total capital boundary. To understand the inverse relation between the total boundary and the capital requirement, we must not ignore the strategic response of the bank. Figure 3 illustrates how such a response complicates the relation. In the figure, we first optimize the liability structure of the bank for $\beta = 2\%$, and then we let β change. If we do not strategically adjust the liability structure for β, the total capital boundary, $V_b/V = (D/V)/(1-\beta)$, should be an increasing function of β (shown as the dashed line in the figure) for the fixed D/V. However, when the strategic liability structure adjusts to the change in β, the relation between the total capital boundary and the capital requirement is completely different; the total capital boundary drops from 46% to 45% of the asset value as β moves up from 0% to 25%. The total capital boundary $V_b^*/V = (D^*/V)/(1-\beta)$ is a complicated function of β because the bank optimally reduces the deposit ratio D^*/V in response to the increase in β.

Empirically testing the effects of capital requirements on bank liability structure is a challenging task. Changes in bank capital requirements are endogenous because regulators increase the requirements usually when they perceive that banks have taken excessive risks. An empirical design needs to isolate the effects of capital requirement changes from the effects of asset risk changes. Most empirical studies of capital requirements actually examine the effects on bank assets and credit portfolios (e.g., Aiyar et al., 2014, and Jimenez et al., 2016, among other), instead of the effects on bank liability structure. An exception is the recent study by Passmore and Temesvary (2022), who empirically find that investors' demand for safe assets causes a negative relationship between banks' capitalization and short-term funding. This empirical finding mirrors the negative effect of capital requirement on deposit ratio in our model.

3.2.3 Effects of Deposit Profitability

Figure 4 illustrates the effects of deposit profitability. The leverage goes up when deposits are more profitable. As η changes from 1% to 3.5%, the tangible equity ratio decreases from about 36% to less than 5% (dotted line in panel A). Both the deposit ratio and the non-deposit debt ratio go up. So, an improvement in deposit profitability does not have a substitution effect between the deposit ratio and the non-deposit debt ratio. They both go up because a higher deposit ratio raises the regulatory total capital boundary, giving more room for non-deposit debt. Thus, the strategic liability structure has a higher non-deposit debt ratio, as well as a higher deposit ratio, if deposits are more profitable.

The effects of deposit profitability on bank leverage are similar to the idea of DeAngelo and Stulz (2015), who suggest that the profitability of deposits is responsible for banks' high leverage. However, the deposits in their paper are uninsured. Nor do they incorporate regulatory capital requirements. Another distinction of our analysis from DeAngelo and Stulz's is that their banks do not issue non-deposit debt.

As predicted by Proposition 6, Panel A of Figure 4 shows that the non-deposit debt ratio rises faster than the deposit ratio. This will be difficult to understand if we ignore the strategic combination of deposits and non-deposit debt.

Panel B of Figure 4 illustrates that both the assessment rate and the credit spread have positive relations to deposit profitability. As we have pointed out earlier, the U.S. bank regulation once prohibited banks from paying interest on deposits and limited bank competition for deposits. The belief was that making more profitable deposits would reduce bank failures. This belief ignores the strategic response of banks. Figure 4 shows that a bank's strategic response to higher deposit profitability is to raise both the deposit and non-deposit debt ratios. This response raises the regulatory total capital boundary and leads to a higher assessment rate and wider credit spread. Also, note that the credit spread in the figure rises faster than the assessment rate, as predicted by Proposition 6. These complicated consequences underscore the importance of banks' strategic choice of bankruptcy risk.

A bank's profitability of deposits reflects the bank's market power. Then, a higher value of η implies a greater market power for a bank in the deposits market as they can issue deposits at a lower cost. The second chain of inequalities (10) and panel B of Figure 4 mean that a bank with more market power in the deposits market has a wider credit spread on non-deposit debt. The reason is that a bank uses more far more deposits than non-deposit debt, as shown in the first chain of inequalities of (10) and panel A of Figure

4. Then, each dollar of non-deposit debt shoulders more bankruptcy risk, because deposits are stable, protected debt. Our theoretical finding that a bank with greater market power relies more on stable funding is supported by the empirical work of Li, Loutskina and Strahan (2019).

3.2.4 Effects of Insurance Subsidy

Figure 5 illustrates Proposition 7. We examine various sizes for subsidy, ranging from zero to 40%. While the deposit ratio goes up from 45% to 47%, the non-deposit debt ratio moves up from 46% to 47% (panel A). Not surprisingly, insurance subsidy encourages bank leverage, but the figure suggests that the effect of insurance subsidy on leverage is modest. As the subsidy increases from zero to 40%, the tangible equity ratio drops from about 9% to about 5%. It is worth noting that the tangible equity ratio is still lower than 10% even if the insurance subsidy is zero. This suggests that insurance subsidy is not a major factor in bank leverage. Panel B of Figure 5 shows that bank credit spread goes up if the FDIC subsidizes the banks more in deposit insurance. This nontrivial theoretical implication is counter-intuitive if one does not distinguish non-deposit debt from deposits, but understandable if we recognize the adjustment in liability structure.

4 Dynamic Restructuring

In the analysis of bank liability structure, we have so far assumed that a bank chooses its liability structure only once and does not restructure later. In reality, banks adjust their liability structure over time. We extend our model to incorporate dynamic restructuring. In section 4.1, we consider dynamic restructuring subject to a minimum requirement on the total capital that protects deposits, the same capital requirement examined in the previous sections. In section 4.2, we add an additional capital requirement: the minimum tangible common equity requirement, which has become more stringent after the 2008 global financial crisis. Our model of dynamic restructuring is along the lines of Goldstein's et al (2001) model of capital structure of a non-bank firm but incorporates deposits and capital requirements.

4.1 Restructuring Under Total Capital Requirements

We continue to assume that the bank asset value follows a geometric Brownian motion (1) as in Section 2.1, but we let the bank restructure its liabilities if the asset value is up by a factor of H_0, where $H_0 > 1$. We refer to H_0 as the endogenous hurdle for restructuring. The time of first restructuring is $t_1^h = \inf\{t \geq 0 : V_t \geq H_0 V_0\}$. Let t_i^h be the time of the i^{th} restructuring, then the time of next restructuring is $t_{i+1}^h = \inf\{t \geq t_i^h : V_t \geq H_i V_i\}$, where H_i

is the new hurdle and V_i denotes the asset value at time t_i^h. The hurdles $\{H_i\}_{i=1,2,\cdots}$ are part of the restructuring decision of the bank. This approach to model dynamic restructuring follows Goldstein et al (2001).

After each restructuring, the asset value may drop to a level that either causes the regulator to close the bank or causes the equity holders to default bank liabilities. Let D_i be the deposits at restructuring time t_i^h and D_0 be the deposits at $t = 0$. The regulatory total capital boundary is $V_{bi} = D_i/(1-\beta)$ for the period from the i^{th} restructuring to the next restructuring. The ratio of the boundary to the initial asset value during this restructuring period is

$$L_{bi} = \frac{V_{bi}}{V_i} = \frac{D_i/V_i}{1-\beta}. \tag{12}$$

The endogenous default boundary will be discussed later in more detail. In general, suppose the bank is liquidated when the asset value reaches L_iV_i, where $L_i \in (0,1)$, in the period following the i^{th} restructuring. Then, the liquidation time is $t_i^l = \inf\{t \in [t_i^h, t_{i+1}^h) : V_t \le L_iV_i\}$. The liquidation cost is αL_iV_i, where L_iV_i is the asset value at liquidation.

At any restructuring time t_i^h, the risk-neutral probability (or state price) for asset value V_t to reach the hurdle of the next restructuring before liquidation can be shown to be

$$\pi_{hi} = \frac{H_i^{\lambda}}{1-(H_i/L_i)^{\lambda-\lambda'}} + \frac{H_i^{\lambda'}}{1-(L_i/H_i)^{\lambda-\lambda'}}. \tag{13}$$

In the above formula, λ and λ' are, respectively, the positive and negative roots of equation $\sigma^2\lambda(1+\lambda)/2 - (r-\delta)\lambda - r = 0$. The risk-neutral probability (or state price) for V_t to reach the liquidation boundary before the next restructuring can be shown to be

$$\pi_{li} = \frac{L_i^{\lambda}}{1-(L_i/H_i)^{\lambda-\lambda'}} + \frac{L_i^{\lambda'}}{1-(H_i/L_i)^{\lambda-\lambda'}}. \tag{14}$$

The derivation of these probabilities is similar to those in Goldstein et al (2001).

Let D_i, B_i, and E_i be the deposits, non-deposit debt, and equity value, respectively, at the i^{th} restructuring time t_i^h, where $i = 1, 2, \cdots$. We also allow $i = 0$ so that D_0, B_0, and E_0 are the deposits, non-deposit debt, and equity value at $t = 0$. Immediately after the i^{th} restructuring, the interests paid to depositors are $C_{D_i} = (r-\eta)D_i$. The coupon paid to non-deposit debt holders during the same time is $C_{B_i} = (r+s_i)B_i$, where s_i is the credit spread, which will be endogenously determined along with the price of non-deposit debt in our model.

The bank chooses the threshold H_i to be above 1 when restructuring is costly. Recent literature examines the effects of committing to restructure at a threshold (see DeMarzo and He, 2020) on the capital structure of non-bank firms. Bezoni et al. (2021) and Dangl and Zechner (2021) show that the effects are insignificant if there are costs for restructuring. In view of these studies, we follow Goldstein et al. (2001) to assume that there are costs for issuing non-deposit debt and taking deposits. The costs are $\theta_B B_i$ and $\theta_D D_i$, respectively, where $\theta_B \in (0,1)$ and $\theta_D \in [0,1)$. In general, we expect $\theta_D < \theta_B$, but our model works as well if $\theta_D \geq \theta_B$.

When a bank chooses its initial liability structure or restructures its liabilities, it maximizes the total value of the current bank shareholders. If the cost of issuing new debt is zero, the total bank value is $F_i = D_i + B_i + E_i$ as we have assumed in Section 2.3. Since the cost of issuing new debt is nonzero here, the total bank value is $F_i = (1-\theta_D)D_i + (1-\theta_B)B_i + E_i$. The choice variables of the bank in maximizing the initial value F_0 are the sequence of deposit liability $\{C_{D_i}\}_{i=0,1,\cdots}$, non-deposit debt liability $\{C_{B_i}\}_{i=0,1,\cdots}$, and the hurdles $\{H_i\}_{i=0,1,\cdots}$ for restructuring.

In the model with dynamic restructuring, we preserve the assumption that the FDIC insurance premium is risk-based, which depends on deposits and the regulatory total capital boundary V_{bi}. Since deposits vary over time across the restructuring periods, the deposit insurance premium varies over time. The risk-based insurance assessment rate also changes. Let g_i be the assessment rate for the period from the i^{th} restructuring to the next restructuring. The risk-based insurance premium during this period is $I_i = g_i D_i$.

At each time of restructuring, the bank calls back the pre-existing non-deposit debt. Therefore, the non-deposit debt issued from the i^{th} restructuring is valued as B_i at time t^h_{i+1}, if the bank has not liquidated by then. If liquidation occurs before t^h_{i+1}, the non-deposit debt value at liquidation is the residual value after subtracting the deposits: $[(1-\alpha)L_iV_i - D_i]^+$. Given coupon C_{B_i} of non-deposit debt, we show in the Online Appendix that the value of non-deposit debt issued from the i^{th} restructuring is

$$B_i = \frac{C_{B_i}}{r} \cdot \frac{1-\pi_{hi}-\pi_{li}}{1-\pi_{hi}} + [(1-\alpha)L_iV_i - D_i]^+ \frac{\pi_{li}}{1-\pi_{hi}}. \tag{15}$$

The equity value at each restructuring time is the residual value beyond the pre-exiting deposits after calling back the pre-existing non-deposit debt. Therefore, the equity issued from the i^{th} restructuring is valued as $F_{i+1} - (D_i + B_i)$ at time t^h_{i+1}, if the bank has not been liquidated by then. If liquidation occurs before t^h_{i+1}, the equity value is zero. In the Online Appendix, we show that the equity value issued from the i^{th} restructuring is

$$E_i = V_i - (1-\tau)\frac{I_i + C_{D_i} + C_{B_i}}{r}(1-\pi_{hi}-\pi_{li}) + [F_{i+1} - (D_i+B_i) - H_iV_i]\pi_{hi} - L_iV_i\pi_{li}. \tag{16}$$

The above debt and equity values assume that the liquidation boundary L_iV_i is given. The endogenous default boundary is the liquidation boundary that maximizes the equity value. We use V_{di} to denote the endogenous default boundary during $[t^h_i, t^h_{i+1})$ and let $L_{di} =$

V_{di}/V_i. Since the bank can be liquidated either by the shareholders or by the regulator, the liquidation boundary during $[t_i^h, t_{i+1}^h)$ is $L_iV_i = \max\{V_{bi}, V_{di}\}$, where $L_i = \max\{L_{bi}, L_{di}\}$.

The pricing equations (13)–(16) allow us to derive a closed-form formula of the total bank value of the shareholders:

$$\begin{aligned}\frac{F_i}{V_i} = \frac{1}{1-H\pi_{hi}}\bigg\{&1 + (1-\theta_D-\pi_{hi})\frac{D_i}{V_i} + (1-\theta_B-\pi_{hi})\frac{B_i}{V_i} \\ &- (1-\tau)(1-\pi_{hi}-\pi_{li})\frac{I_i + C_{Di} + C_{Bi}}{rV_i} - H_i\pi_{hi} - L_i\pi_{li}\bigg\}.\end{aligned} \tag{17}$$

We provide the details of the formula in the Online Appendix. The bank maximizes the total value by choosing the sequence of $\{C_{D_i}, C_{B_i}, H_i\}_{i=0,1,2,\cdots}$.

We can analytically show that the result of Proposition 1 continues to hold in the model with dynamic restructuring.

Proposition 8. *Suppose $\theta_B < \tau(1-\pi_{hi})$. Bank shareholders can increase the total bank value by issuing more non-deposit debt if the current non-deposit debt ratio is so low such that the endogenous default boundary is lower than the regulatory total capital boundary. More precisely, the liability structure with $L_{di} < L_{bi}$ is suboptimal.*

This proposition again predicts that a strategic bank liability structure must hold at least some non-deposit debt so that the endogenous default boundary matches up with the regulatory boundary. That is, we must have $L_{di}^* \geq L_{bi}^*$ in a strategic bank liability structure. The condition $\theta_B < \tau(1-\pi_{hi})$ means that the cost of issuing new non-deposit debt should not be so big that it wipes out the expected value of the debt benefit. Obviously, if the cost is too big, issuing new debt may not increase the total bank value. We derive the proposition in the Online Appendix.

It is tedious to show that $L_{di}^* = L_{bi}^*$ analytically, given the complexity of the model of dynamic restructuring. We find this property always holds in our numerical solutions of the strategic liability structure. We numerically solve the dynamic optimization problem for the bank. Table 3 shows the strategic liability structure at time zero for a set of parameters similar to those we have used earlier. In addition, we need to set the value of θ_D and θ_B, the costs of taking new deposits and issuing new non-deposit debt. We choose $\theta_D = 0.005$ and $\theta_B = 0.01$ for illustration. The restructuring hurdle H_i is an endogenous variable determined by optimization. For the purpose of comparison, the table also presents the numerical solution of the strategic liability structure without dynamic restructuring for the same parameters.

An important result in Table 3 is the equality of default and regulatory boundaries relative to the initial asset value: $L_{d0}^* = L_{b0}^* = 42.01\%$. It confirms that the salient feature of matching boundaries derived from the simpler model in Section 2 remains when the model incorporates dynamic restructuring. The boundary for the bank with restructuring is lower than the boundary for the bank without restructuring. So, restructuring delays bankruptcy.

Goldstein's el al (2001) find that a non-bank firm that restructures dynamically issues less debt at the beginning time because it has the flexibility to issue more later. This is not necessarily true for the non-deposit debt in banks. A new finding unique to our model is that restructuring causes a bank to have higher non-deposit debt ratio. The non-deposit debt ratio is 46.23% in the bank with restructuring, in contrast to 45.18% in the bank without restructuring. The reason is that the bank has the flexibility to reduce the deposit ratio. The overall leverage is lower for the bank with dynamic structuring. In Table 3, the tangible equity ratio is 12.59% in the bank with restructuring, in contrast to 12.22% with no restructuring.

The differential effects of restructuring on deposits and non-deposit debt would be missed if we did not distinguish deposits from non-deposit debt. The differential effects are important for understanding the higher credit spread and the lower deposit insurance assessment rate in banks that dynamically restructure their liabilities.

The numerical solutions in Table 3 demonstrate that dynamic restructuring does not qualitatively change the results we presented in Sections 2 and 3. The quantitative difference between the two columns in the table is rather small. Although the quantitative difference depends on the parameters, the qualitative properties are similar.

We have shown earlier that the strategic liability structure, without restructuring, always has the incentive for risk substitution. Dynamic restructuring does not change this property of the strategic liability structure. Once the bank sets its liability structure strategically, the market value of equity is an increasing, convex function of the asset value. Figure 1 shows the market value of equity as a function of asset value around the initial value $V_0 = 100$. We choose the liability structure in the figure to be optimal for $V_0 = 100$, but we keep it same when the asset value V deviates from V_0. The figure clearly shows that the curve of the market equity value is increasing and convex.

4.2 Restructuring Under Tangible Equity Requirements

An important capital regulation in Basel II and III requires banks to maintain a minimum tangible common equity (TCE) relative to its assets. Tangible equity is also referred to as tier-1 capital, which excludes the "good-will value" derived from the continuing operation of the bank. The capital requirement is a minimum ratio of tangible equity to bank assets.

This ratio is also referred to as tier-1 ratio. In this section, we add this type of capital requirement as an additional capital requirement.

The tangible equity of a bank is the value of equity that the shareholder would receive in the event of an immediate bank liquidation. In our model, the tangible equity at time t is $T_t = V_t - D_t - B_t$. This excludes the value of future profits earned from serving deposits and the value of future benefits from holding debt. The ratio of tangible equity to assets, T_t/V_t, measures the bank's ability to absorb loses. It is used by regulators to determine whether the bank is solvent. Regulations require banks to maintain a tangible equity ratio above a specified level. Let $\gamma \in (0,1)$ be the required minimum tangible equity ratio for the bank.

After the i^{th} restructuring, the time for the tangible ratio to reach the minimum ratio γ is $t_i^c = \inf\{t \geq t_i : T_t \leq \gamma V_t\}$. Since $T_t \leq \gamma V_t$ is equivalent to $V_t \leq (D_i + B_t)/(1-\gamma)$, the time to hit the minimum ratio is $t_i^c = \inf\{t \geq t_i : V_t \leq (D_i + B_t)/(1-\gamma)\}$. Let $V_{ci} = V_{t_i^c}$. We refer to it as the *regulatory tangible equity boundary*, which is another regulatory boundary, to distinguish it from the regulatory boundary V_{bi} imposed by the total capital requirement. The ratio of the tangible equity boundary to assets is $L_{ci} = V_{ci}/V_i$. Since non-deposit debt value is nonnegative, it is obvious that $V_{ci} \geq V_{bi}$.

Similar to the previous section, let π_{hi} be the risk-neutral probability (state price) that the next restructuring happens before the tangible equity ratio drops to γ. Let π_{li} be the risk-neutral probability that the tangible equity ratio drops to γ before the next restructuring.

The values of debt and equity have similar formulae as in the previous subsection, except that the lower boundary in the valuation formulas is $L_i = \max\{L_{bi}, L_{ci}, L_{di}\}$. Notice that L_{ci} endogenously depends on the value of non-deposit debt. If the bank issues more non-deposit debt, the tangible boundary tends to be higher.

We can show that a liability structure with $L_{di} < L_{ci}$ is suboptimal, but we use a numerical solution to show that we actually have $L_{di} = L_{ci}$ in a strategic liability structure. Table 4 presents the numerical solution to the strategic liability structure for a set of parameters similar to the previous illustrations. The new parameter in this table is the tangible equity ratio γ. We choose $\gamma = 4.5\%$, $\gamma = 7\%$, and $\gamma = 9.5\%$ because in Basel III, the basic ratio is 4.5%, the counter-cyclical capital buffer is 2.5%, and the capital surcharge on systemically important banks is additional 2.5%. The basic tangible equity ratio and counter-cyclical buffer add up to the 7% tangible equity ratio requirement for all banks. With the capital surcharge, systemically important banks face the tangible equity ratio requirement of at least 9.5%. To show the effects of the minimum tangible equity ratio requirement, in the table we also include the numerical results for the bank without tangible equity ratio requirements.

The first important observation from Table 4 is that the endogenous default boundary and tangible equity boundary are always exactly same in each strategic liability structure. They are both 41.76% if the minimum tangible equity ratio is 4.5%. For a higher minimum tangible equity ratio, the two boundaries are lower, appearing counterintuitive. This is similar to what we have observed earlier in the effects of the total capital requirement: if the regulator raises the required capital ratio, the optimal adjustment of bank liability structure lowers the default boundary. We also note that a higher tangible equity requirement leads to a lower credit spread for non-deposit debt, which might explain the increased use of non-deposit debt with a higher tangible equity requirement.

As expected, the tangible equity ratio in the strategic liability structure is higher if the minimum tangible equity ratio is higher. It is not surprising that a bank reduces its overall leverage in response to a raise in the tangible equity requirement. However, Table 4 shows nontrivial consequence: the non-deposit debt ratio is higher if the minimum tangible equity ratio is higher. A bank will reduce the overall leverage by cutting deposit ratio, instead of non-deposit debt ratio. This may not be a desired consequence because banks are supposed to provide financial intermediation by serving deposits. This consequence would have been overlooked if we ignore the strategic allocation between deposits and non-deposit debt.

4.3 Distinctions from Other Models with Dynamic Restructuring

Following Goldstein et al (2000), a large body of corporate finance literature uses corporate capital structure models with dynamic restructuring. This literature focuses on the choice of corporate debt and equity. Our model extends Goldstein's model by considering the optimal combination of deposits and non-deposit debt, in addition to equity. The distinction between deposits and non-deposit debt in the liability structure is particularly relevant for banks as apposed to non-bank corporate entities. The dynamic optimization problem is also subject to capital requirements, which are imposed on all depository institutions. The problem to optimize the mix of deposits with other debt subject to capital requirements is unique to banks and more complex than Goldstein's model for non-bank firms, as we have shown.

Subramanian and Yang (2020, simply SY henceforth) introduce a model of bank capital structure with dynamic restructuring, but our model with dynamic restructuring bears important distinctions from SY's model. The most important distinction is that the banks in our model allocate liabilities optimally between deposits and non-deposit debt. By contrast, SY's model assumes that bank liabilities are fixed, exogenously-given combinations of deposits and debt. The endogenous choice between deposits and non-deposit debt is ignored. SY's model examines the optimal choice only between debt and equity, as in Goldstein's model. The distinction between deposits and non-deposit debt is central to modeling bank liability structure as serving deposits is unique to banks. This distinction allows us to discover the differential responses of deposit and non-deposit ratios to changes of capital requirements.

In their valuation of bank debt, Subramanian and Yang (2020) in fact treat the entire bank debt as non-deposit debt. They assume that the entire debt holders will become equity holders when the bank is in financial distress, as made clear in the formulation of boundary conditions in their equation (12). This is clearly not how deposits are treated upon financial distress: depositors enjoy priority relative to non-deposit creditors and should be paid back first as much as possible. Assuming the entire debt to become equity is equivalent to assuming the entire debt to be non-deposit debt. In this sense, their valuation model is a model of a non-bank firm. However, they interpret their debt malleably. When they examine the effect of deposit insurance, they assume that the bank debt contains an exogenously-fixed proportion of deposits. This approach abstracts from the differential effects of regulation on deposits and non-deposit debt.

Another important distinction of our model is that capital regulations are constraints, under which banks make capital decisions. In reality, capital regulations are specified as minimum capital ratios that restrict bank capital decisions, but the decisions are made by the private owners of the banks. The thrust of our model is to derive the endogenous composition of deposits and non-deposit debt under the constraints imposed by bank regulations. The composition of deposits and non-deposit debt and the minimum capital ratios are the issues that are unique to banks. Subramanian and Yang (2020) consider the role of regulators in directly choosing bank liability structure to maximize some social objective function. We abstract from this consideration.

5 Conclusion

We provide a framework for characterizing the strategic bank liability structure. Such a framework can be useful in understanding the responses of banks to changes in regulation. A regulation typically attempts to solve a problem in some part of bank liability structure or asset structure (see Santos, 2001). Arguments for a proposed change in regulation often implicitly assume that the other parts remain unchanged after the regulatory change, ignoring the endogenous responses of banks that strategically adjust other parts of their liability structure. The approach developed in our model is useful for evaluation of regulatory changes because it takes into account banks' strategic responses.

We have shown that banks respond to capital requirements by strategically choosing their liability structure, while taking into account deposit insurance. The value maximization principle implies that a bank acts in the interest of its claim-holders. Our focus on value maximization sets aside the principal-agent problems such as management's conflict of interests with some stake holders, although these problems may play roles in banks' choices of liability structures (Admati at al., 2018). Value-maximizing banks do not maximize social welfare, such as reducing systemic risks or expanding banking services for the economy. While social welfare implications of bank liability structure are unquestionably important, a good understanding of banks' strategic choice of liability structure is a necessary step for a proper social welfare analysis of bank capital regulations. Moreover, a general equilibrium treatment is essential to properly understand the welfare implications.

Assuming that banks set their liability structure only once, we provide a closed-form solution of the strategic bank liability structure that explicitly incorporates an array of factors important for banks. The special factors for banks are capital requirements, deposit profitability, and deposit insurance. These factors are absent in the classic models of non-bak (corporate) capital structure. We then extend the analysis to banks that dynamically restructure their liabilities.

We discover a salient feature of the strategic bank liability structure. A value-maximizing bank uses as much non-deposit debt as possible to take advantage of debt benefits but not so much to offer extra protection for the deposit insurer. This choice makes the risk of bankruptcy optimal for equity holders. Our comparative static analysis shows that this salient feature is a key element in understanding how various factors affect a bank's liability structure as well as its credit risk. This salient feature has important implications to bank regulation and the principal-agent problems in banks. Although regulators can set the capital requirement high enough, given any liability structure, to mitigate the incentives for risk substitution, the strategic liability structure always preserves the incentives for risk substitution.

A direct, careful empirical test of our model is the recent study by Gambacorta et al. (2021), who provide empirical evidence that banks strategically adjust liability structure in response to changes in government tax policy, as predicted by our model. Recall that the tax advantage of leverage is a factor of bank liability structure in our model; and the tax effects presented by Gambacorta et al lend a strong empirical support to our model. Direct empirical evidence of the model's predictions on the effects of other factors (especially regulations) should be interesting future research.

Our model of strategic bank liability structure provides tools for studying additional issues in banking. An important component in bank capital structure is bank's liquidity reserves. Hugonnier and Morellec (2017) use our model to examine the liquidity reserve in banks. Our model is also suitable for studying the effects of tax policy changes on banks. Our model can also be applied to examine the roles of contingent capital, bail-in debt, and bailout in bank strategic choices of capital structures. Berger at al. (2019) and Lambrecht and Tse (2020) are such applications. Using our framework, Vissers (2020) explores the effects of jumps in asset value.

References

Admati, A., P. DeMarzo, M. Hellwig, and P. Pfleiderer, 2018, The leverage ratchet effects, *Journal of Finance*, 73, 145–198.

Admati, A. and M. Hellwig, 2013, *The Bankers'New Clothes*, Princeton University Press.

Aiyar, S., C. Calomiris, J. Hooley, Y. Korniyenko, and T. Wieladek, 2014, The international transmission of bank capital requirements: evidence from the UK, *Journal of Financial Economics,* 113, 368–382.

Auh, J. and S. Sundaresan, 2020, Repo Priority Right and the Bankruptcy Code, *Critical Finance Review*, 9, 77–114.

Baxter, N., 1967, Leverage, risk of ruin, and the cost of capital, *Journal of Finance*, 22, 395—403.

Benzoni, L., L. Garloppi, R. Goldstein, J. Hugonnier, and C. Ying, 2020, Optimal debt dynamics, issuance costs, and commitment, Working paper, University of Minnesota.

Berger, A., C. Himmelberg, R. Roman, and S. Tsyplakov, 2019, Bank bailouts, bail-ins, or no regulatory intervention? a dynamic model and empirical tests of optimal regulation, Working paper, University of South Carolina.

Black F. and J. Cox, 1976, Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, *Journal of Finance,* 31, 351–367.

Dangl, T. and J. Zechner, 2021, Debt maturity and the dynamics of leverage, *Review of Financial Studies*, 34, 5796–5840.

DeAngelo, H. and R. Stulz, 2015, Liquid-claim production, risk management, and bank capital structure: why high leverage is optimal for banks, *Journal of Financial Economics*, 116, 219–236.

Della Seta, M., E. Morellec, and F. Zucchi, 2020, Short-Term Debt and Incentives for Risk-Taking, *Journal of Financial Economics*, 137, 173–209.

DeMarzo, P. and Z. He, 2020, Leverage Dynamics without Commitment, Working paper, Stanford University and University of Chicago.

Diamond, D. and P. Dybvig, 1983, Bank runs, deposit insurance and liquidity, *Journal of Political Economy*, 91, 401–419.

Duffie, D., R. Jarrow, A. Purnanandam, and W. Yang, 2003, Market pricing of deposit insurance, *Journal of Financial Services Research*, 24, 93–119.

Federal Deposit Insurance Corporation, 2011, Assessments, large bank pricing: final rule, Federal Register, Vol. 76, No. 38.

Flannery, M. and S. Sorescu, 1996, Evidence of bank market discipline, *Journal of Finance*, 51, 1347–1377.

Gambacorta, L., G. Ricotti, S. Sundaresan, Z. Wang, 2016, Tax effects on bank liability structure, Working paper, Bank for International Settlements.

Garnett, E., L. Henry, D. Hoople, A. Mihalik, 2020, A History of Risk-Based Premiums at the FDIC, FDIC Staff Study.

Goldstein, R., N. Ju, H. Leland, 2001, An EBIT-based model of dynamic capital structure", *Journal of Business*, 74, 483–512.

Gorton, G. and J.,Kahn, 2000, The Design of Bank Loan Contracts, *Review of Financial Studies*, 13, 331–364.

Gorton, G. and A. Santomero, 1990, Market discipline and subordinated debt: note, *Journal of Money Credit and Banking*, 22, 119–128.

Green, R., 1984, Investment incentives, debt, and warrants. *Journal of Financial Economics*, 13, 115–136.

Harding, J., X. Liang, and S. Ross, 2007, The optimal capital structure of banks: balancing deposit insurance, capital requirements and tax-advantaged debt, Working Paper, University of Connecticut.

Harris, M. and A. Raviv, 1991, The theory of capital structure, *Journal of Finance*. 46, 297–355.

Hugonnier, J. and E. Morellec, 2017, Bank Capital, Liquidity Reserves, and Insolvency Risk, *Journal of Financial Economics*, 125, 266–285.

Jimenez, G., S. Ongena, J. Peydro, and J. Saurina, 2017, Macroprudential policy, countercyclical bank capital buffers and credit supply: Evidence from the Spanish dynamic provisioning experiments, *Journal of Political Economy*, 125, 2126–2177.

Lambrecht, B. and A. Tse, 2020, Liquidation, Bailout, and Bail-in: Insolvency Resolution Mechanisms and Bank Lending, Working Paper, Cambridge.

Leland, H., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance*, 49, 1213–1252.

Leland, H., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance*, 53, 1213–1243.

Li, L., E. Loutskina, P. Strahan, 2019, Deposit market power, funding stability and long-term credit, Working paper, NBER.

Merton, R., 1974, On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, 29, 449–470.

Merton, R., 1977, An analytic derivation of the cost of deposit insurance and loan guarantees: an application of modern option pricing theory, *Journal of Banking and Finance*, 1, 3–11.

Modigliani, F. and M. Miller, 1963, Corporate income taxes and the cost of capital: a correction, *American Economic Review*, 53, 3, 433–443.

Passmore, W. and J. Temesvary, 2022, How investor demands for safety influence bank capital and liquidity trade-offs, *Journal of Financial Stability*, forthcoming.

Pennacchi, G., 2006, Deposit insurance, bank regulation, and financial system risks, *Journal of Monetary Economics*, 53, 1–30.

Rochet, J., 2008, *Why Are There So Many Banking Crises? The Politics and Policy of Bank Regulation*, Princeton University Press, New Jersey.

Ronn, E. and A. Verma, 1986, Pricing risk-adjusted deposit insurance: an option-based model, *Journal of Finance*, 41, 871–896.

Santos, J., 2001, Bank capital regulation in contemporary banking theory: a review of the literature, *Financial Markets, Institutions & Instruments*, 10, 41–84.

Schneider, M. and A. Tornell, 2004, Balance sheet effects, bailout guarantees and financial crises, *Review of Economic Studies*, 71, 883–913.

Shibut, L., T. Critchfield, and S. Bohn, 2003, Differentiating among critically undercapitalized banks and thrifts, *FDIC Banking Review*, 15, No. 2, 1–38.

Song, F. and A. Thakor, 2007, Relationship banking, fragility, and the asset-liability matching problem, *Review of Financial Studies*, 20, 2129–2177.

Subramanian, A. and B. Yang, 2020, Dynamic prudential regulation, *Management Science*, 66, 3183–3210.

Tarullo, D., 2013, Toward building a more effective resolution regime: progress and challenges: progress and challenges, Federal Reserve Board and Federal Reserve Bank of Richmond Conference: Planning for the Orderly Resolution of a Global Systemically Important Bank.

Toft, K. and B. Prucyk, 2012, Options on leveraged equity: theory and empirical tests, *Journal of Finance*, 52, 1151–1180.

Vissers, S., 2020, Bank capital structure and tail risk, SSRN Working Paper.

Parameter	Notation	Range
Asset volatility	σ	$(0, \infty)$
Asset cash flow	δ	$[0, \infty)$
Risk-free interest rate	r	$(0, \infty)$
Deposit profitability	η	$(0, r)$
Debt advantage	τ	$(0, 1)$
Liquidation cost	α	$(0, 1)$
Capital requirement	β	$[0, \alpha)$
Insurance subsidy	ω	$[0, 1]$

Table 1: Exogenous parameters of the model. The range of each parameter is the range allowed in the model.

Endogenous variable	Bank		Firm	
Ratio of deposit to assets	D/V	45.28		
Ratio of non-deposit debt to assets	B/V	46.49	$\bar{B}/V$	60.03
Ratio of tangible equity to assets	T/V	8.24	$\bar{T}/V$	39.97
Ratio of default boundary to assets	V_d/V	46.20	$\bar{V}_d/V$	35.28
Credit spread	s	2.27	$\bar{s}$	1.02
Assessment rate	g	0.53		

Table 2: Comparison of the strategic bank liability structure and the optimal capital structure of a comparable firm that does not take deposits. The values for the endogenous variables are reported in percentage points. The exogenous parameters are $\eta = 0.03$, $\sigma = 0.05$, $\alpha = 0.27$, $\beta = 0.02$, $\omega = 0.1$, $\tau = 0.15$, $\delta = 8\%$, and $r = 5\%$.

Endogenous variable		With dynamic restructuring	No dynamic restructuring
Ratio of deposit to assets	D_0^*/V_0	41.17	42.61
Ratio of non-deposit debt to assets	B_0^*/V_0	46.23	45.18
Ratio of tangible equity to assets	T_0^*/V_0	12.59	12.22
Credit spread of nondeposit debt	s_0^*	2.69	2.64
Assessment rate of deposit insurance	g_0^*	0.57	0.61
Hurdle for restructuring	H_0^*	114.66	
Total capital boundary / assets	L_{b0}^*	42.01	43.48
Endogenous default boundary / assets	L_{d0}^*	42.01	43.48

Table 3: Strategic bank liability structure under total capital requirement. The values for the endogenous variables are reported in percentage points. The exogenous parameters are $\eta = 0.03$, $\beta = 0.02$, $\omega = 0.1$, $\sigma = 0.09$, $\alpha = 0.27$, $\tau = 0.15$, $\delta = 8\%$, $r = 5\%$, $\theta_D = 0.005$, and $\theta_B = 0.01$. The first column of numbers is for the bank with dynamic restructuring, and the last column is for the bank that does not restructure.

Endogenous variable		Minimum tangible equity ratio (γ) NA	4.5%	7.0%	9.5%
Ratio of deposit to assets	D_0^*/V_0	41.17	39.81	38.42	37.01
Ratio of non-deposit debt to assets	B_0^*/V_0	46.23	46.67	47.06	47.41
Ratio of tangible equity to assets	T_0^*/V_0	12.59	13.52	14.52	15.58
Credit spread of non-deposit debt	s_0^*	2.69	2.65	2.60	2.54
Assessment rate of deposit insurance	g_0^*	0.57	0.53	0.50	0.47
Hurdle for restructuring	H_0^*	114.66	115.03	115.38	115.73
Endogenous default boundary / assets	L_{d0}^*	42.01	41.69	41.31	40.89
Tangible equity boundary / assets	L_{c0}^*		41.69	41.31	40.89
Total capital boundary / assets	L_{b0}^*	42.01	40.62	39.26	37.76

Table 4: Dynamic strategic bank liability structure under tangible equity requirement. The values for the endogenous variables are reported in percentage points. The exogenous parameters are $\eta = 0.03$, $\beta = 0.02$, $\omega = 0.1$, $\sigma = 0.09$, $\alpha = 0.27$, $\tau = 0.15$, $\delta = 8\%$, $r = 5\%$, $\theta_D = 0.005$, and $\theta_B = 0.01$. The last three columns of numbers are for the bank with the alternative minimum tangible equity ratio requirements, and the first column is for the bank without such requirements.

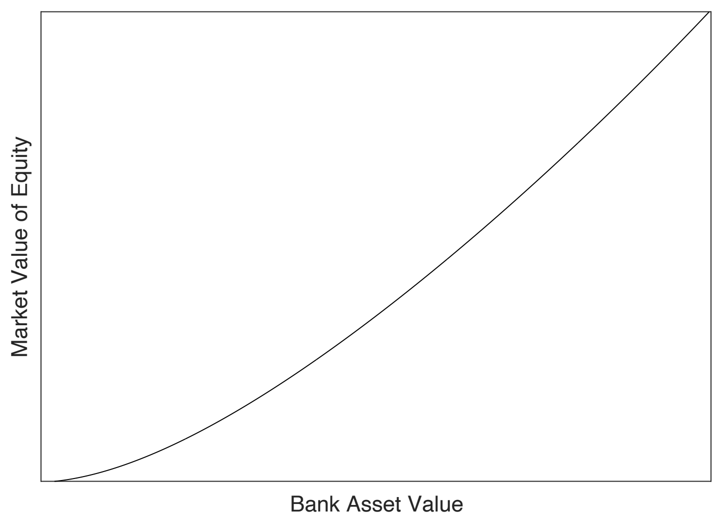

Figure 1: Incentive for risk substitution. We first calculate the strategic liability structure for asset value $V_0 = 100$. Under this liability structure, we plot the market value of equity for asset values deviating from V_0. The parameters are $\eta = 0.03$, $\beta = 0.02$, $\omega = 0.1$, $\sigma = 0.09$, $\alpha = 0.27$, $\tau = 0.15$, $\delta = 8\%$, $r = 5\%$, and $\theta = 0.01$.

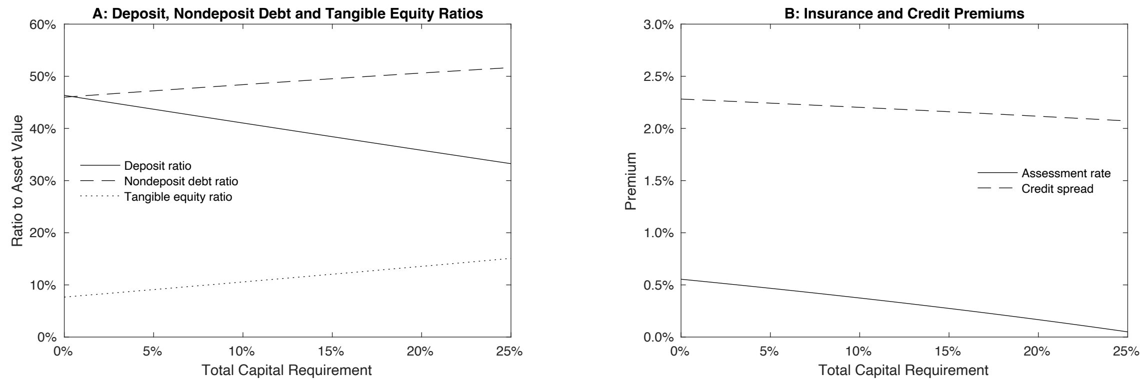

Figure 2: Effects of total capital requirement on the strategic bank liability structure. When β varies, the other parameters are fixed at the values set in Table 2. In penal A, the solid line is the ratio of deposits to assets, the dashed line is the ratio of non-deposit debt to assets, and the dotted line is the ratio of tangible equity to assets. In penal B, the solid line is the assessment rate of deposit insurance, and the dashed line is the credit spread of non-deposit debt.

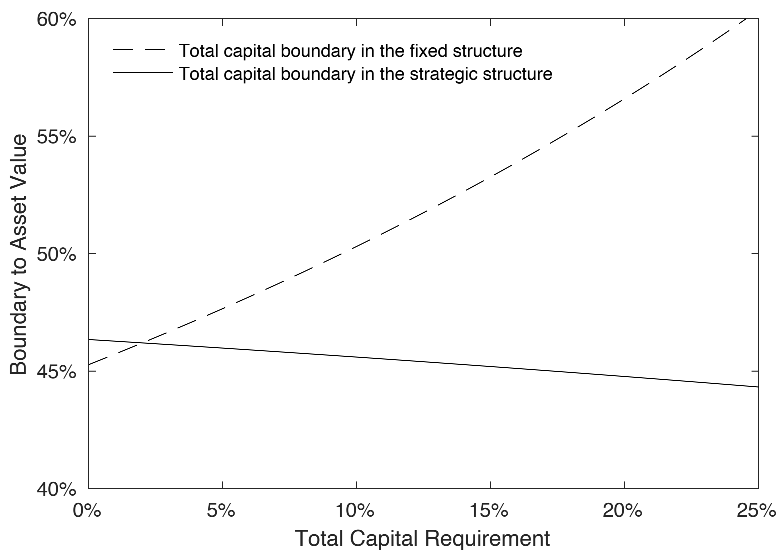

Figure 3: The regulatory total capital boundary of bank liability structure. We first calculate the strategic liability structure for the parameters in Table 2. When we vary β in a range from 0% to 25%, we keep the liability structure fixed and plot the regulatory total capital boundary V_b/V for each value of β in the fixed liability structure. Then, we re-optimized for each changed β and plot the total capital boundary V_b^*/V in the re-optimized liability structure.

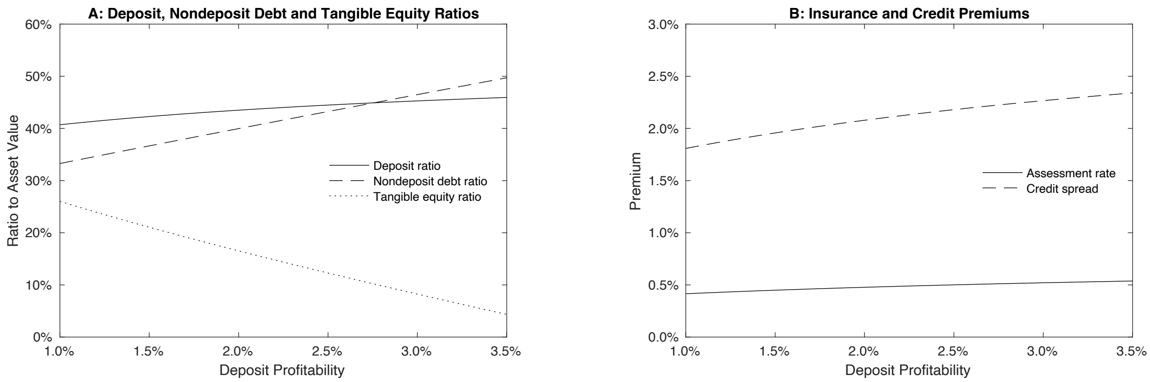

Figure 4: Effects of deposit profitability on the strategic bank liability structure. When η varies, the other parameters are fixed at the values set in Table 2. In penal A, the solid line is the ratio of deposits to assets, the dashed line is the ratio of non-deposit debt to assets, and the dotted line is the ratio of tangible equity to assets. In penal B, the solid line is the assessment rate of deposit insurance, and the dashed line is credit spread of non-deposit debt.

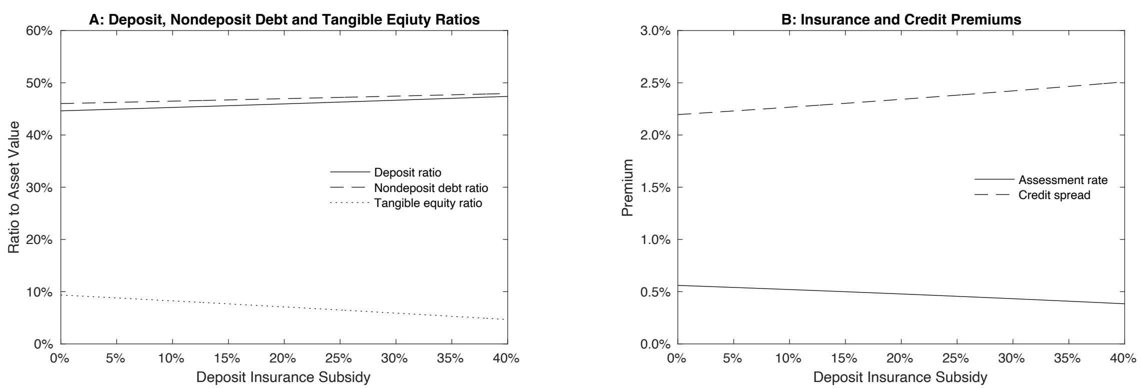

Figure 5: Effects of insurance subsidy on the strategic bank liability structure. When ω varies, the other parameters are fixed at the values set in Table 2. In penal A, the solid line is the ratio of deposits to assets, the dashed line is the ratio of non-deposit debt to assets, and the dotted line is the ratio of tangible equity to assets. In penal B, the solid line is the assessment rate of deposit insurance, and the dashed line is the credit spread of non-deposit debt.

Strategic Bank Liability Structure Under Capital Requirements

Management Science

Suresh Sundaresan
Zhenyu Wang

A Online Appendix

A.1 Derivation of Risk-Based Insurance Premium

As we have discussed earlier, the regulator closes the bank when asset value drops to $V_b = D/(1-\beta)$. The relation between deposits D and the interest cost C_D on serving deposits is $D = C_D/(r-\eta)$. Also notice that the regulatory closure boundary is $V_b = D/(1-\beta)$, which implies $V_b = C_D/[(r-\eta)(1-\beta)]$. The state price of regulatory closure is the value of a security that pays $1 if the closure occurs and pays nothing otherwise. It follows from Merton (1974) that the state price is $[V_b/V]^\lambda$, where λ is the positive root of $\sigma^2\lambda(1+\lambda)/2-(r-\delta)\lambda-r=0$. The above quadratic equation also has a negative root, which is denoted by λ'.

The risk-based insurance premium sets the expected present value of the insurance premium paid to the insurance corporation equal to the expected present value of the insurance obligations at regulatory bank closure. The expectations are under the risk-neutral measure To derive equation (2), let Q be the value of deposit insurance to banks. Its pricing equation is

$$\frac{1}{2}\sigma^2V^2Q'' + (r-\delta)VQ' - rQ - I^\circ = 0\,, \tag{A1}$$

where Q' and Q'' denote the first and second partial derivatives of Q with respect to V. The general solution to the equation is $Q(V) = -I^\circ/r + a_1V^{-\lambda} + a_2V^{-\lambda'}$, where a_1 and a_2 can be any constants. The boundary conditions of the value of the insurance product are $\lim_{V\to\infty} Q = -I^\circ/r$ and $Q(V_b) = [D-(1-\alpha)V_b]^+$. The first boundary condition implies $a_2 = 0$, and the second boundary condition implies $-I^\circ/r + a_1V_b^{-\lambda} = [D-(1-\alpha)V_b]^+$. Therefore, we have $Q(V) = -(1-P_b)I/r + [D-(1-\alpha)V_b]^+P_b$, where $P_b = [V_b/V]^\lambda$. The insurance premium I° is fair if and only if $Q(V) = 0$. It follows that $(1-P_b)I^\circ = r[D-(1-\alpha)V_b]^+P_c$. We obtain equation (2) by substituting $V_b = D/(1-\beta)$ and factoring D out.

The assessment rate, as well as the premium that the bank pays, depends on the deposit ratio and the bank's risk profile. The fair assessment rate is related to the deposit liability C_D/V by

$$g^\circ = \frac{r(\alpha-\beta)}{1-\beta} \cdot \left[\frac{C_D/V}{(r-\eta)(1-\beta)}\right]^\lambda \left(1 - \left[\frac{C_D/V}{(r-\eta)(1-\beta)}\right]^\lambda\right)^{-1}. \tag{A2}$$

The subsidized assessment rate is $g = (1-\omega)g^\circ$.

A.2 Derivation of Bank Value

Bank value depends on (C_D, C_B), the obligatory payments for liabilities as shown in the following lemma.

Lemma 1. *Given liabilities (C_D, C_B), the endogenous default boundary relative to assets is*

$$\frac{V_d}{V} = (1-\tau)\frac{\lambda}{1+\lambda}\frac{I + C_D + C_B}{rV}, \tag{A3}$$

where $I = gD$ is the risk-based insurance premium with the assessment rate g given by equation (A2). The total value of bank shareholders relative to the assets is

$$\begin{aligned}\frac{F}{V} = 1 &+ \left(\frac{\tau r + (1-\tau)\eta}{r-\eta}\frac{C_D}{rV} + \tau\frac{C_B}{rV} - (1-\tau)\frac{I}{rV}\right)\left(1 - \left[\frac{V_l}{V}\right]^\lambda\right) \\ &- \left(\min\left\{\alpha\frac{V_l}{V}, \frac{V_l}{V} - \frac{C_D/V}{r-\eta}\right\}\right)\left[\frac{V_l}{V}\right]^\lambda,\end{aligned} \tag{A4}$$

where $V_l = \max\{V_b, V_d\}$ is the liquidation boundary. The credit spread of non-deposit debt is

$$s = r\frac{\{1 - r[(1-\alpha)V_l - C_D/(r-\eta)]^+/C_B\}[V_l/V]^\lambda}{1 - \{1 - r[(1-\alpha)V_l - C_D/(r-\eta)]^+/C_B\}[V_l/V]^\lambda}. \tag{A5}$$

The first term on the right-hand side of equation (A4) is the asset value normalized to 1 as we express the total value relative to the assets of the bank. The next term reflects the value of banking service income and debt advantage after subtracting the insurance premium in states where the bank is solvent. The last term reflects the loss of value in states where the bank is liquidated.

Equation (A4) shows the roles of banking service, deposit insurance, and regulatory closure in bank valuation. Evidently, income from banking service (η) increases bank value. While deposit insurance increases bank value because it protects the value of deposits, the insurance premium (I) reduces bank value. The equation shows another cost of deposits: deposits raise the regulatory closure boundary because of capital requirement. The bank incurs liquidation cost when its tangible equity drops to the minimum capital requirement as the regulator closes the bank.

To derive Lemma 1, we first derive the value of non-deposit debt. The pricing equation of non-deposit debt before liquidation is

$$\frac{1}{2}\sigma^2 V^2 B'' + (r-\delta)VB' - rB + C_B = 0\,, \tag{A6}$$

where B' and B'' are the first and second partial derivatives of B with respect to V. The general solution to pricing equation (A6) is $B = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + C_B/r$, where a_1 and a_2 can be any constants. Let V_l be the liquidation boundary. There are two boundary conditions: $B = [(1-\alpha)V_l - D]^+$ at $V = V_l$ and $\lim B < \infty$ when $V \to \infty$. These boundary conditions imply $a_2 = 0$ and $a_1 = \{[(1-\alpha)V_l - D]^+ - C_B/r\}V_l^\lambda$, which give:

$$B = \frac{C_B}{r}\left(1 - \left[\frac{V_l}{V}\right]^\lambda\right) + [(1-\alpha)V_l - D]^+ \left[\frac{V_l}{V}\right]^\lambda. \tag{A7}$$

The relation between the credit spread and the interest cost C_B on serving non-deposit debt is $s = C_B/B - r$. Substituting equation (A7), we obtain equation (A5).

The pricing equation of equity value E before liquidation is

$$\frac{1}{2}\sigma^2 V^2 E'' + (r-\delta)VE' - rE + \delta V - (1-\tau)(I + C_D + C_B) = 0\,, \tag{A8}$$

where E' and E'' are the first and second partial derivatives of E with respect to V. The general solution to pricing equation (A8) is

$$E = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + V - (1-\tau)(I + C_D + C_B)/r, \tag{A9}$$

where a_1 and a_2 are arbitrary constants. There are two boundary conditions: $E = 0$ at $V = V_l$ and $\lim E < \infty$ when $V \to \infty$. These boundary conditions imply $a_2 = 0$ and $a_1 = -[V_l - (1-\tau)(I + C_D + C_B)/r]V_l^\lambda$, which give the equity value:

$$E = V - (1-\tau)\frac{I + C_D + C_B}{r}\left(1 - \left[\frac{V_l}{V}\right]^\lambda\right) - V_l\left[\frac{V_l}{V}\right]^\lambda. \tag{A10}$$

To derive the endogenous default boundary in equation (A3), we show that V_l maximizes the equity value when $V_l = V_d$. Differentiation of equation (A10) with respect to V_l leads to

$$\frac{\partial E}{\partial V_l} = \frac{1+\lambda}{V_l}\left[\frac{V_l}{V}\right]^\lambda \left((1-\tau)\frac{\lambda}{1+\lambda}\frac{I + C_D + C_B}{r} - V_l\right) = \frac{1+\lambda}{V_l}\left[\frac{V_l}{V}\right]^\lambda (V_d - V_l)\,. \tag{A11}$$

The above is positive for all V if $V_l < V_d$ and negative if $V_l > V_d$. It follows that $V_l = V_d$ maximizes the equity value. Therefore, it is optimal for equity holders to default if V drops to V_d. Equation (A11) shows that the equity value becomes zero if and only if $V_l = V_d$.

The total value of the original bank shareholders is $F = D + B + E$. We obtain equation (A4) by substituting equations (A7) and (A10).

A.3 Derivation of Proposition 1

The pair $(C_D/V, C_B/V)$ determines the liability structure of a bank. In the strategic liability structure, the pair $(C_D/V, C_B/V)$ maximizes the total value of bank shareholders in equation (A4) subject to the risk-based premium of deposit insurance in equation (A2).

The bank value relative to assets depends on the ratios of various cash flows, instead of the levels of the cash flows. We therefore derive the optimal ratios of cash flows. This approach simplifies the derivation of optimal liability structure. We introduce the following ratios: $x = C_B/C_D$, $y = C_D/(rV)$, and $h = I/C_D$. We also introduce the following two notations in equation because these quantities appear frequently in the derivation: $\iota = \eta/(r-\eta)$ and $\theta = (1-\tau)\lambda/(1+\lambda)$. To express the boundaries in term of the ratios of cash flows, we define $v_b = rV_b/C_D$, $v_d = rV_d/C_D$, and $v_l = rV_l/C_D$ to simplify the boundaries to $V_b/V = yv_b$, $V_d/V = yv_d$, and $V_l/V = yv_l$. By Lemma 1, we have

$$v_b = (1+\iota)/(1-\beta), \quad v_d = \theta(1+h+x). \tag{A12}$$

Notice that $V_b/V < V_d/V$ if and only if $v_b < v_d$. Then, $v_l = \max\{v_b, v_d\}$.

Equation (A2) and $\beta < \alpha$ imply that the assessment rate g is a function of y:

$$g(y) = (1-\omega)(\alpha-\beta)v_b\frac{(yv_b)^\lambda}{1-(yv_b)^\lambda}. \tag{A13}$$

The definition of h implies that it is a function of y: $h(y) = g(y)/(r-\eta)$, following from equation (A13). The derivative of h with respect to y is $h'_y = (\lambda/y)h/[1-(yv_b)^\lambda]$.

Let $f = F/V$. By Lemma 1, we have

$$f(x,y) = 1 + y[\iota - (1-\tau)h + \tau(1+x)][1-(yv_l)^\lambda] - y\min\{\alpha v_l, v_l - (1+\iota)\}(yv_l)^\lambda. \tag{A14}$$

Given the exogenous parameters, $f(x,y)$ is a continuous function of x and y. To help visualize the function, we show an example of the function in panel A of Figure A1. Choosing $(C_D/V, C_B/V)$ to maximize bank value F is equivalent to choosing the duplet (x,y) to maximize f. Once we obtain the optimal (x,y), the optimal $(C_D/V, C_B/V)$ can be obtained as $C_D/V = yr$ and $C_B/V = x(C_D/V)$.

The function $f(x,y)$ is continuously differentiable at least on the points (x,y) such that $v_b \neq v_d$ or $(1-\alpha)v_d \neq 1+\iota$. We first show that the liability structure with $v_d < v_b$ does not maximize bank value. In this case,

$$f(x,y) = 1 + y[\iota - (1-\tau)h + \tau(1+x)][1-(yv_b)^\lambda] - y[(1+\iota)\beta/(1-\beta)](yv_b)^\lambda. \tag{A15}$$

Since v_b is independent of x, the first derivative of f with respect to x is

$$f'_x(x,y) = \tau[1-(yv_b)^\lambda]y. \tag{A16}$$

This is always positive under the regularity assumption (1). It then implies that increasing

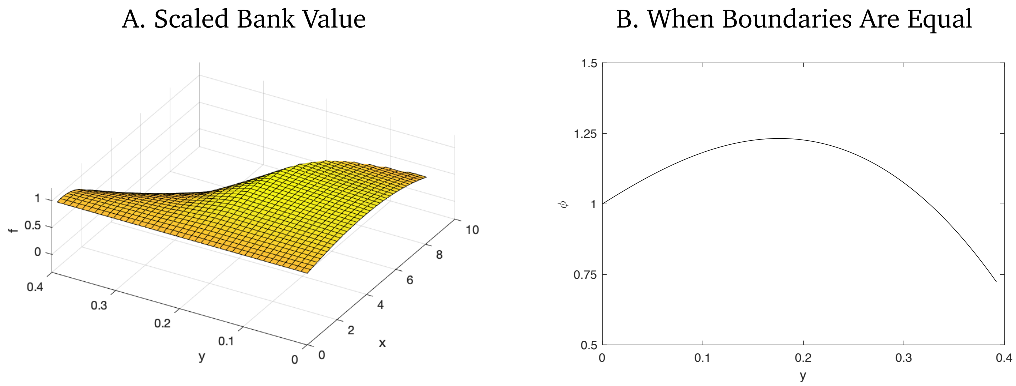

Figure A1: Plots of functions $f(x, y)$ and $\phi(y)$ with the parameters used in Table 2. Panel is the plot of $f(x, y)$, which is defined by equation (A14). Panel B is the plot of function $\phi(y)$, which is defined in Section A.4.

x will increase f. Particularly, this means $x = 0$ is not optimal. That is, zero non-deposit debt is not an optimal liability structure; adding some non-deposit debt can increase the bank value. It follows that a strategic liability structure must have $v_d \geq v_b$.

A.4 Derivation of Proposition 2

In Proposition 1, we have shown that a liability structure with $v_b > v_d$ is not optimal and that increasing x to raise v_d enlarges the total bank value. Next, we will show that a liability structure with $v_b < v_d$ is not optimal either and decreasing x to lower v_d enlarges the total bank value. We do not consider the liability structure with $v_d > (1 + \iota)/(1 - \alpha)$. It is intuitive to understand why such a structure is suboptimal. In that case, the bank's asset value at default is enough to pay back deposits in full. This choice of default boundary eliminates the deposit insurer's risk even though the bank still pays insurance premium.

In the case of $v_b < v_d < (1 + \iota)/(1 - \alpha)$, it follows from equation (A14) that the total bank value scaled by assets is

$$f = 1 + y[\iota - (1 - \tau)h + \tau(1 + x)][1 - (yv_d)^\lambda] - y\left[v_d - (1 + \iota)\right](yv_d)^\lambda. \tag{A17}$$

The partial derivative of f with respect to x is

$$f'_x(x, y) = y\Big[\tau\left[1 - (yv_d)^\lambda\right] - \frac{\lambda x}{1 + h + x}(yv_d)^\lambda\Big], \tag{A18}$$

and the partial derivative of f with respect to y is

$$\begin{aligned} f'_y = &\ \tau(1+h+x) + \iota - h + (1+h)\big[1-(yv_d)^\lambda\big] \\ &- \ [(\tau+\lambda)(1+h+x) - \lambda(1+h)]\,(yv_d)^\lambda \\ &- \ \left\{(1-\tau)\big[1-(yv_d)^\lambda\big] + \frac{\lambda x}{1+h+x}(yv_d)^\lambda\right\} y h'_y. \end{aligned} \tag{A19}$$

Equations (A18) and (A19) imply

$$y f'_y - \left[\tau y(1+x+h) + \frac{\lambda h}{1-(yv_b)^\lambda}\right] f'_x = y\left[\iota - (1+\lambda)h + (1+h)(yv_b)^\lambda\right]. \tag{A20}$$

The regularity assumption (4) implies $(1+\lambda)h < \iota$. Then, the right-hand side of equation (A20) is positive. It thus is impossible to have $f'_x = 0$ and $f'_y = 0$, which are the first-order condition of a maximum bank value. Therefore, no liability structure in the case of $v_b < v_d < (1+\iota)/(1-\alpha)$ maximizes the bank value. If y is chosen so that $f'_y = 0$ in this case, then equation (A20) implies $f'_x < 0$, which implies that lowering x increases the bank value. Lowering x means reducing the non-deposit debt relative to deposits.

What we have shown implies that the optimal x^* and y^* must satisfy $v_d^* = v_b$ and thus $v_l^* = v_b$. Then, the state price of bankruptcy in the strategic liability structure must be: $(y^* v_b)^\lambda$. We denote it by π. Then, $v_b = \pi^{1/\lambda}/y^*$. It follows that

$$y^* = \pi^{1/\lambda}/[(1+\iota)/(1-\beta)]$$

$$h^* = (1+\iota)[1-(1-\alpha)/(1-\beta)]^+\pi/(1-\pi)$$

$$x^* = (1+\iota)\{1/[\theta(1-\beta)] - (1-\omega)[1-(1-\alpha)/(1-\beta)]^+\pi/(1-\pi)\} - 1.$$

Let $x_b(y) = v_b/\theta - [1+h(y)]$ and $\phi(y) = f(x_b(y), y)$. Function $\phi(y)$ is the scaled bank value on the points of (x, y) such that $v_d = v_b$. An example of $\phi(y)$ is presented in panel B of Figure A1. It follows from equation (A14) that

$$\phi(y) = 1 + y\big\{[\iota - h + \tau v_b/\theta][1-(yv_b)^\lambda] - \beta(yv_b)^\lambda\big\}. \tag{A21}$$

This function is differentiable in y, and its derivative is

$$\phi'_y(y) = [\iota - h + \tau v_b/\theta]\big[1-(1+\lambda)(yv_b)^\lambda\big] - \beta(1+\lambda)(yv_b)^\lambda - y h'_y\big[1-(yv_b)^\lambda\big]. \tag{A22}$$

With equation (A13), the above equation can be written as

$$\phi'_y(y) = \iota + \tau v_b/\theta - [\iota + (\tau/\theta + (1-\omega)(1-\beta) + \beta)v_b](1+\lambda)(yv_b)^\lambda. \tag{A23}$$

If (x, y) achieves a maximum, y must maximize $\phi(y)$, and thus $\phi'_y(y) = 0$. Setting equation (A23) to zero, we obtain

$$\pi = \frac{1}{1+\lambda} \cdot \frac{\iota + (\tau/\theta)v_b}{\iota + [\tau/\theta + (1-\omega)(1-\beta) + \beta]v_b}. \tag{A24}$$

Substituting equation (A12) into equation (A24), we obtain

$$\pi = \frac{1}{1+\lambda} \cdot \frac{\tau(1+\lambda) + \lambda(1-\tau)(1-\beta)(\eta/r)}{\tau(1+\lambda) + (1-\tau)\lambda\alpha + \lambda(1-\tau)[(1-\beta)(\eta/r) - \omega(\alpha-\beta)]}. \tag{A25}$$

The above equation is equivalent to equation (5).

We obtain (C_D^*, C_B^*) by substituting the original variables into the formulas for x^*, y^*, and h^* and using $C_D^*/V = y^* r$ and $C_B^*/V = x^*(C_D^*/V)$:

$$C_D^*/V = \frac{r-\eta}{r}(1-\beta)\pi^{1/\lambda}, \tag{A26}$$

$$C_B^*/V = \left[\left(\frac{\tau}{1-\tau} + \beta + (1-\beta)\frac{\eta}{r}\frac{\lambda}{1+\lambda}\right)\frac{1}{1-\pi} + \frac{1}{\lambda}\right] r\pi^{1/\lambda}. \tag{A27}$$

Then, we obtain deposit ratio and debt ratio in equation (6) by replacing C_D^* and C_B^* by those in equations (A7). The credit spread, insurance assessment rate, and return on assets in equation (7). follow immediately after applying the deposits, non-deposit debt, closure boundary, and default boundary in the strategic liability structure. Applying the above optimal liability structure to the bank value function (A4), we obtain the maximum bank value relative to assets as

$$\frac{F^*}{V} = 1 + \left[\frac{\tau}{1-\tau} + (1-\beta)\frac{\eta}{r}\frac{\lambda}{1+\lambda}\right]\pi^{1/\lambda}. \tag{A28}$$

We derive the optimal bank value F^* in equation (A28) under the assumption that bank takes at least some deposits. To conclude that F^* in equation (A28) is the maximum bank value, we need to show that the bank achieves a smaller value than F/V if it does not take deposits. If the bank does not take deposits ($D = 0$, or equivalently, $C_D = 0$), the liquidation boundary is the endogenous default boundary: $V_l = V_d$ and the endogenous default boundary in Lemma 1 becomes

$$\frac{V_d}{V} = (1-\tau)\frac{\lambda}{1+\lambda}\frac{C_B}{rV},$$

and the bank value function in the lemma becomes

$$\frac{F}{V} = 1 + \tau\frac{C_B}{rV}\left(1 - \left[\frac{V_d}{V}\right]^{\lambda}\right) - \alpha\frac{V_d}{V}\left[\frac{V_d}{V}\right]^{\lambda}. \tag{A29}$$

The last two equations are identical to the endogenous default boundary and the firm value in Leland (1994). The debt liability that maximizes the non-bank firm value has been derived by Leland. We present the solution of the optimal capital structure of the firm in the next lemma. This solution is equivalent the solution derived by Leland (1994) except it uses our notations. It is also a special case of the strategic bank liability structure in which the profit of serving deposits (η) and the subsidy of deposit insurance (ω) are both zero.

Lemma 2. *Given assets V, the non-deposit debt value in the optimal capital structure of the firm is*

$$\frac{\bar{B}}{V} = \left(1 + \frac{1-\bar{\pi}}{\lambda}\right)\bar{\pi}^{1/\lambda}, \tag{A30}$$

where $\bar{\pi}$ is the state price of the endogenous default given by

$$\bar{\pi} = \frac{\tau}{\tau(1+\lambda) + (1-\tau)\lambda\alpha}. \tag{A31}$$

The endogenous default boundary in the optimal structure is $\bar{V}_d/V = \bar{\pi}^{1/\lambda}$, the credit spread is

$$\bar{s} = \frac{r\{1+\lambda+[\tau+(1-\tau)\alpha]\lambda\}\bar{\pi}}{1+\lambda+\{1+\lambda+[\tau+(1-\tau)\alpha]\lambda\}\bar{\pi}}, \tag{A32}$$

and the maximum firm value is

$$\bar{F} = V\left\{1 + \frac{\tau}{1-\tau}\bar{\pi}^{1/\lambda}\right\}. \tag{A33}$$

To show that bank value F^* in equation (A28) is larger than the firm value $\bar{F}$ in equation (A33), we first show $\pi > \bar{\pi}$. Equation (A25) implies

$$\pi > \frac{1}{1+\lambda} \cdot \frac{\tau(1+\lambda) + \lambda(1-\tau)(1-\beta)(\eta/r)}{\tau(1+\lambda) + (1-\tau)\lambda\alpha + \lambda(1-\tau)(1-\beta)(\eta/r)}.$$

The algebraic inequality $(a+y)/(a+b+y) > a/(a+b)$, which holds for any positive a, b, and y, implies

$$\frac{\tau(1+\lambda) + \lambda(1-\tau)(1-\beta)(\eta/r)}{\tau(1+\lambda) + (1-\tau)\lambda\alpha + \lambda(1-\tau)(1-\beta)(\eta/r)} > \frac{\tau(1+\lambda)}{\tau(1+\lambda) + (1-\tau)\lambda\alpha}.$$

We thus have $\pi > \bar{\pi}$, in view of equation (A31). Then, it follows from equations (A28) and (A33) that

$$\frac{F^*}{V} > 1 + \left[\frac{\tau}{1-\tau} + (1-\beta)\frac{\eta}{r}\frac{\lambda}{1+\lambda}\right]\bar{\pi}^{1/\lambda} > 1 + \frac{\tau}{1-\tau}\bar{\pi}^{1/\lambda} = \frac{\bar{F}}{V}. \tag{A34}$$

Since equations (A31) and (A33) can be obtained by setting $\eta = 0$ and $\omega = 0$ in equations (A25) and (A28). Therefore, the difference between the maximum bank value F/V and maximum firm $\bar{F}/V$ is value added by the profit of serving deposits (η) and the subsidy of deposit insurance (ω). Inequality (A34) shows that a liability structure without deposits is not optimal for the bank. Therefore, F^* is the maximum bank value.

A.5 Derivation of Proposition 3

The minimum deposits $\bar{D}$ stated in Proposition 3 is

$$\bar{D} = (1-\tau)\frac{C_B}{r}\left[\frac{1}{1-\alpha} - (1-\tau)\frac{r-\eta}{r}\right]^{-1}. \tag{A35}$$

Noticing $I = (1-\omega)g^\circ D$, we use equation (2) to obtain

$$\begin{aligned}(1-\tau)\frac{I+C_D+C_B}{r} - V_b = (1-\tau)\frac{C_B}{r}\\ + \left[(1-\tau)\left((1-\omega)\frac{\alpha-\beta}{1-\beta}\frac{[V_b/V]^\lambda}{1-[V_b/V)]^\lambda} + \frac{r-\eta}{r}\right) - \frac{1}{1-\beta}\right]D.\end{aligned}$$

If β increases toward to α, the above equation converges to

$$\lim_{\beta\to\alpha}\left((1-\tau)\frac{I+C_D+C_B}{r}-V_b\right)=(1-\tau)\frac{C_B}{r}-\left(\frac{1}{1-\alpha}-(1-\tau)\frac{r-\eta}{r}\right)D.$$

For $D>\bar{D}$, the above limit is negative, in view of equation (A35). Therefore, there exists $\beta_1\in(0,\alpha)$ such that

$$(1-\tau)\frac{I+C_D+C_B}{r}-V_b<0$$

for all $\beta\in(\beta_1,\alpha)$.

Using $I=(1-\omega)g^\circ D$ and equations (2) and (A3), we obtain

$$V_b-V_d=\left[\frac{1}{1-\beta}-(1-\tau)\frac{\lambda}{1+\lambda}\left((1-\omega)\frac{\alpha-\beta}{1-\beta}\frac{P_b}{1-P_b}+\frac{r-\eta}{r}\right)\right]D-(1-\tau)\frac{\lambda}{1+\lambda}\frac{C_B}{r}.$$

When β increases toward α, the above equation converges to

$$\lim_{\beta\to\alpha}(V_b-V_d)=\left[\frac{1}{1-\alpha}-(1-\tau)\frac{\lambda}{1+\lambda}\frac{r-\eta}{r}\right]D-(1-\tau)\frac{\lambda}{1+\lambda}\frac{C_B}{r}.$$

For $D>\bar{D}$, the above limit is positive because

$$\lim_{\beta\to\alpha}(V_b-V_d)>(1-\tau)\frac{1}{1+\lambda}\frac{C_B}{r}.$$

Therefore, there exists $\beta_2\in(0,\alpha)$ such that $V_b>V_d$ for all $\beta\in(\beta_2,\alpha)$.

Let $\bar{\beta}=\max\{\beta_1,\beta_2\}$. Then, for any $\beta\in(\bar{\beta},\alpha)$, we have $V_l=V_b>V_d$ and $(1-\tau)(I+C_D+C_B)/r<V_l$. By equation (A10), the first derivative of the market value of equity with respect to asset value is

$$\frac{\partial E}{\partial V}=1-\left((1-\tau)\frac{I+C_D+C_B}{r}-V_l\right)\frac{\lambda}{V}\left[\frac{V_l}{V}\right]^{\lambda}>0,$$

which implies that E is an increasing function of asset value. The second derivative is

$$\frac{\partial^2 E}{\partial V^2}=\left((1-\tau)\frac{I+C_D+C_B}{r}-V_l\right)\frac{\lambda(1+\lambda)}{V^2}\left[\frac{V_l}{V}\right]^{\lambda}<0,$$

which implies that E is a concave function of asset value.

If the bank chooses C_D/V and C_B/V strategically to maximize value, we have $V_l=V_d=V_b$ as stated in Proposition 2. Then, the first derivative of the market value of equity in the strategic liability structure is

$$\frac{\partial E}{\partial V}=1-\left[\frac{V_l}{V}\right]^{1+\lambda}>0,$$

which implies that E is an increasing function of asset value. The second derivative of the market value of equity in the strategic liability structure is

$$\frac{\partial^2 E}{\partial V^2}=\frac{1+\lambda}{V}\left[\frac{V_l}{V}\right]^{1+\lambda}>0,$$

which implies that E is a convex function of asset value.

A.6 Derivation of Propositions 4–7

In view of $\pi > \bar{\pi}$ as we have proved in Appendix A.4, Proposition 4 follows immediately from Proposition 2 and Lemma 2.

From equation (5), we directly obtain

$$\frac{\partial \pi}{\partial \eta} = \frac{r\lambda}{1+\lambda} \cdot \frac{r(1-\tau)^2(1-\beta)[(1-\omega)\alpha+\omega\beta]\lambda}{\{r\tau(1+\lambda)+\eta(1-\tau)(1-\beta)\lambda+r(1-\tau)[(1-\omega)\alpha+\omega\beta]\lambda\}^2} > 0 \tag{A36}$$

$$\frac{\partial \pi}{\partial \beta} = -\frac{r\lambda}{1+\lambda} \cdot \frac{(1-\tau)\{r\omega(1-\tau)(1+\lambda)+\eta[\omega+(1-\omega)\alpha]\lambda\}}{\{r\tau(1+\lambda)+\eta(1-\tau)(1-\beta)\lambda+r(1-\tau)[(1-\omega)\alpha+\omega\beta]\lambda\}^2} < 0 \tag{A37}$$

$$\frac{\partial \pi}{\partial \omega} = \frac{r\lambda}{1+\lambda} \cdot \frac{(1-\tau)(\alpha-\beta)[r\tau(1+\lambda)+\eta(1-\beta)\lambda]}{\{r\tau(1+\lambda)+\eta(1-\tau)(1-\beta)\lambda+r(1-\tau)[(1-\omega)\alpha+\omega\beta]\lambda\}^2} > 0. \tag{A38}$$

It follows from the above equations that

$$\frac{\partial \pi^{1/\lambda}}{\partial \eta} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \eta} > 0 \tag{A39}$$

$$\frac{\partial \pi^{1/\lambda}}{\partial \beta} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \beta} < 0 \tag{A40}$$

$$\frac{\partial \pi^{1/\lambda}}{\partial \omega} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \omega} > 0. \tag{A41}$$

Then, we obtain Propositions 6–7 from Proposition 2 by using the partial derivatives of π and $\pi^{1/\lambda}$ that we have just derived.

A.7 Derivation of Dynamic Restructuring and Proposition 8

The liability C_{Di} to serve deposits D_i in this restructuring period is $C_{Di} = (r-\eta)D_i$. The insurance premium I_i in this restructuring period is $I_i = g_i D_i$, where g_i is the risk-adjusted assessment rate for this period.

After the i^{th} restructuring and before the asset value V_t reaches $H_i V_i$ or $L_i V_i$, the pricing equation of non-deposit debt B_t is

$$\frac{1}{2}\sigma^2 V_t^2 \partial_{V_t}^2 B_t + (r-\delta)V_t \partial_{V_t} B_t - rB_t + C_{Bi} = 0, \tag{A42}$$

given the liability C_{Bi} to serve non-deposit debt. There are two boundary conditions for the differential equation. If the asset value V_t reaches $H_i V_i$ before reaching $L_i V_i$, the non-deposit debt is B_i. If the asset value V_t reaches $L_i V_i$ before reaching $H_i V_i$, the non-deposit debt value is $\big[(1-\alpha)L_i V_i - D_i\big]^+$. Solving B_t from the differential equation and boundary conditions and then set time t to t_i^h, we obtain

$$B_i = \frac{C_B}{r}(1-\pi_{hi}-\pi_{li}) + B_i\pi_{hi} + [(1-\alpha)L_i V_i - D_i]^+\pi_{li}. \tag{A43}$$

Moving the term $B_i\pi_{hi}$ to the left side, we obtain equation (15) for the value of non-deposit debt B_i.

After the i^{th} restructuring and before the asset value V_t reaches $H_i V_i$ or $L_i V_i$, the pricing equation of equity E_t is

$$\frac{1}{2}\sigma^2 V_t^2 \partial_{V_t}^2 E_t + (r-\delta) V_t \partial_{V_t} E_t - r E_t + \delta V_t - (1-\tau)(I_i + C_{Di} + C_{Bi}) = 0. \tag{A44}$$

There two boundary conditions. If the asset value V_t reaches $H_i V_i$ before reaching $L_i V_i$, the equity value is $F_{i+1} - D_i - B_i$. If the asset value V_t reaches $L_i V_i$ before reaching $H_i V_i$, the equity value is zero. We solve the above differential equation and the boundary conditions to obtain the value of equity E_t. Its value at $t = t_i^h$ is equation (16).

Substituting the valuation functions (15) and (16) into the total value of the bank shareholders $F_i = (1-\theta_D) D_i + (1-\theta_B) B_i + E_i$, we obtain

$$\begin{aligned} F_i &= V_i + (1-\theta_D - \pi_{hi}) D_i + (1-\theta_B - \pi_{hi}) B_i \\ &\quad - (1-\tau)(1-\pi_h - \pi_l)\frac{I_i + C_{Di} + C_{Bi}}{r} - H_i V_i \pi_{hi} - L_i V_i \pi_l + F_{i+1}\pi_{hi}. \end{aligned} \tag{A45}$$

Dividing the above equation by V_i, we obtain

$$\begin{aligned} \frac{F_i}{V_i} &= 1 + (1-\theta_D - \pi_{hi})\frac{D_i}{V_i} + (1-\theta_B - \pi_{hi})\frac{B_i}{V_i} \\ &\quad - (1-\tau)(1-\pi_{hi} - \pi_{li})\frac{I_i + C_{Di} + C_{Bi}}{r V_i} - H_i \pi_{hi} - L_i \pi_l + \frac{F_{i+1}}{V_{i+1}} H_i \pi_{hi}. \end{aligned} \tag{A46}$$

where we have used $V_{i+1} = H_i V_i$ in the last term.

At different restructuring time t_i^h, the only difference in optimization problem is the level of asset value V_i. Then, the ratio F_i/V_i should be identical for all i in the strategic liability structure. This is the scaling property described by Goldstein et al (2001). Then, we obtain equation (17) from equation (A46).

Given any liability structure, let $f = F_i/V_i$, $x = C_{Bi}/C_{Di}$, $y = C_{Di}/(rV_i)$, and $z = H_i$. By equation (17), f is the following function of (x, y, z):

$$\begin{aligned} f(x,y,z) &= \frac{1}{1 - z\pi_{hi}}\Big\{1 + (1+\iota) y (1-\theta_D - \pi_{hi}) \\ &\quad - (1-\tau)(1-\pi_{hi} - \pi_{li})(1 + h_i + x) y - z\pi_{hi} - L_i \pi_{li} \\ &\quad + (1-\theta_B - \pi_{hi})\Big[\frac{1-\pi_{hi} - \pi_{li}}{1-\pi_{hi}} xy + [(1-\alpha)L_i - (1+\iota)y]^+ \frac{\pi_{li}}{1-\pi_{hi}}\Big]\Big\}, \end{aligned} \tag{A47}$$

where π_{hi}, π_{li}, and h_i are functions defined below:

$$\pi_{hi} = \frac{z^{\lambda}}{1 - (z/L_i)^{\lambda - \lambda'}} + \frac{z^{\lambda'}}{1 - (L_i/z)^{\lambda - \lambda'}} \tag{A48}$$

$$\pi_{li} = \frac{L_i^{\lambda}}{1 - (L_i/z)^{\lambda - \lambda'}} + \frac{L_i^{\lambda'}}{1 - (z/L_i)^{\lambda - \lambda'}} \tag{A49}$$

$$h_i = (1-\omega)(1+\iota)\frac{\alpha - \beta}{1-\beta}\frac{L_{bi}^{\lambda}}{1 - L_{bi}^{\lambda}} y. \tag{A50}$$

The liability structure maximizes the total bank value if and only if (x, y, z) of the liability structure maximizes $f(x, y, z)$. We can solve this maximization problem numerically.

Suppose $L_{di} < L_{ib}$. Then,

$$L_i = L_{bi} = \frac{D_i/V_i}{1-\beta} < \frac{D_i/V_i}{1-\alpha} = \frac{1+\iota}{1-\alpha} y. \tag{A51}$$

It follows that

$$\begin{aligned} f(x,y,z) = \frac{1}{1-z\pi_{hi}} \Big\{ & 1 + (1+\iota)y(1-\theta_D-\pi_{hi}) \\ & - (1-\tau)(1-\pi_{hi}-\pi_{li})(1+h_i+x)y - z\pi_{hi} - L_i\pi_{li} \\ & + (1-\theta_B-\pi_{hi})\Big[\frac{1-\pi_{hi}-\pi_{li}}{1-\pi_{hi}} xy\Big]\Big\}. \end{aligned} \tag{A52}$$

Since π_{hi}, π_{li} and h_i are independent of x, the partial derivative of f with respect to x is

$$f'_x(x,y,z) = \frac{1-\pi_{hi}-\pi_{li}}{(1-z\pi_{hi})(1-\pi_{hi})}[\tau(1-\pi_{hi}) - \theta_B]y. \tag{A53}$$

If $\theta_B < \tau(1-\pi_{hi})$, we have $f'_x(x, y, z) > 0$, which means (x, y, z) of the liability structure is not optimal. Moreover, increasing x increases f and thus also increases F_i. This completes the proof of Proposition 8.

A.8 Explaining the Choice of Parameters

Our model inherits the advantages of structural models that coherently connect the risk of debt and equity to the risk of assets. The risk is asset volatility (σ), which is one of an important parameter that affects bank leverage and liability structure. Since asset volatility is not directly observable, people typically infer it from accounting data and security prices. Moody's KMV provides estimates of asset volatilities for many companies across a wide range of industries.

We present the average, median, and the 10- and 90-percentiles of Moody's estimates of bank asset volatilities in Figure A2. As a comparison, we also present Moody's estimates of manufacturing-firm asset volatilities. The figure shows a difference between the assets held by banks and those owned by manufacturing firms: bank assets have much lower volatility. The average bank asset volatility is around 10% (panel A), whereas it is $40 \sim 50\%$ for manufacturing firms (panel B). While bank asset volatility fluctuates over time, the median stays closely to 5% during 2001–2012. The 90 percentile of bank asset volatilities is well below 15% for 2001–2007, and it stays below 25% even for the period of 2007–2012. In view of these facts, we set σ below 10% in our numerical illustrations. We have experimented a wide range of σ and find our results qualitatively same.

Another parameter of bank assets is its rate of cash flows (δ). If the assets are only commercial loans and consumer loans, the cash flows are interest and principal payments

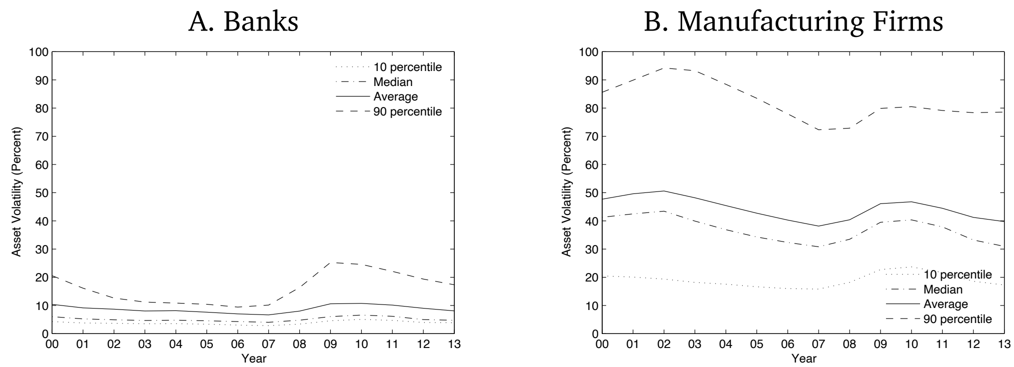

Figure A2: Plots of the average, median, and 10- and 90-percentiles of asset volatilities of banks (panel A) and manufacturing firms (panel B) from 2000 to 2013. Moody's KMV Investor Service provided the estimates of asset volatilities.

of the loans. In all numerical illustrations, we set the cash-flow rate to $\delta = 8\%$, which is the average mortgage rate in the U.S. during 1984–2013. Correspondingly, we set the risk-free rate to the average federal funds rate during the same period; this leads to $r = 5\%$. We choose this period because we would like to make our numbers broadly comparable to the aggregate balance sheet data of FDIC-insured commercial banks and savings institutions. The FDIC made the balance sheet data for this period available. We obtain both the mortgage rate and federal funds rate data from Table H.15 published by the Federal Reserve.

Since the major debt advantage is the tax deductibility of interest expenses, corporate tax rate is an important parameter in determining liability structure. The statutory corporate tax rate in the U.S. ranges up to 35% until 2018. The U.S. Department of Treasury (2007) reports that the effective marginal tax rate on investment in business varies substantially by business sectors. The academic literature suggests that the effective corporate tax rate is around 10% for non-financial firms (Graham, 2000) but can be more important for banks (Heckemeyer and Mooij, 2013). In all numerical illustrations, we set $\tau = 15\%$. However, we have also investigated a wide range of debt advantage and found qualitatively similar results.

Liquidation cost is an important countering force to the tax advantage of debt, but measuring the cost has always been a challenge. A well-known reference is the study of Altman (1984), which examines a sample of 19 industrial firms that went bankrupt over the period of 1970–1978. The estimated liquidation cost is 19.7% of a firm's asset value just before its bankruptcy. Bris, Welch and Zhu (2006), however, show that liquidation cost varies across firms and ranges between 0% and 20% of firm assets. Banks incurred higher liquidation costs. Based on 791 FDIC-regulated commercial banks failed during 1982–1988 (the Savings and Loan Crisis), James (1991) estimates that the liquidation cost is on average 30% of a failed bank's assets. Based on 325 insured depository institutions failed during 2008–2010 (the Great Recession), Flannery (2011) estimates that the liquidation cost is on average 27% of a failed bank's assets. In view of these estimates, we choose $\alpha = 27\%$ in all numerical illustrations.

Deposit profitability is an important factor in bank liability structure. The competitive market for deposits should determine the profitability. In a perfect competitive market with free entry, the deposit profitability should just cover the insurance premium if deposits are under insurance. However, new entry of banks into the market is under regulation by the charter authorities. Founders of a new bank have to show their integrity and ability to manage the bank. Most importantly, the regulators demand evidence of need for a new bank before granting a charter. Peltzman (1965) describe the barriers to new commercial banks. Jayaratne and Strahan (1998) examine the effects of entry restrictions on bank efficiency.

Without free entry, deposits can be profitable because the market power enjoyed by the bank (De Nicolo and Rurk Ariss, 2010). The profitability should depend on the deposits and the bank. Thus, parameter η may differ across banks and should be a function of D. We abstract from the market competition of deposits or the demand function of deposits, $D(\eta)$, to keep the model tractable and to focus on the choice of liability structure. We take η as given but allow banks to have different η. We choose $\eta = 3\%$ for all numerical illustrations, except that we let η vary in the illustration of its effects.

The choices of other important exogenous parameters about banks are as follows. A state banking regulatory agency closes a bank when it cannot meet its obligations to depositors. When a bank's total capital is less than 2% of its assets, the FDIC classifies it as "critically undercapitalized," and the charter authority typically closes the bank if it cannot recapitalize. In view of these institutional arrangements, we set $\beta = 0.02$ all illustrations. However, in the illustration of the effects of capital requirement, we examine a range: $\beta \in (0\%, 25\%)$, motivated by the calls for higher capital requirement after the recent financial crisis. For the insurance subsidy ω, we set $\omega = 10\%$ based on the estimate by Duffie et al. (2003), except that we examine a wide range in the illustration of its effects.

Although we choose the parameters to match historical data broadly, readers should not regard our numerical examples as calibrations of the historical average of the U.S.

bank liability structures. There are at least three complicating factors that make a calibration exercise difficult. First, non-deposit debt issued by banks consists of many kinds of debt, ranging from short-term funding (federal funds and repos, etc.) to long-term debt (senior secured debts, subordinated debts etc.). Some banks even have convertible debts or preferred equity. Second, the FDIC assessment rate adjusts for bank liability structure since 2011. It is not completely risk-based in the earlier years. The insurance subsidy changed over time as the FDIC changed the rules. Third, bank regulation in the U.S. has changed dramatically through the history. Therefore, the numerical examples in the previous subsection only serve as illustration of our theory.

References for Online Appendix

Altman, E., 1984, A further empirical investigation of the bankruptcy cost question, *Journal of Finance*, 39, 1067–1089.

Bris, A., I. Welch, and N. Zhu, 2006, The costs of bankruptcy: chapter 7 liquidation versus chapter 11 reorganization, *Journal of Finance*, 61, 1253–1303.

De Nicolo, G., and R. Turk Ariss, 2010, Bank market power and risk: theory and measurement, “Paolo Baffi” Centre Research Paper Series No. 2010–73.

Duffie, D., R. Jarrow, A. Purnanandam, and W. Yang, 2003, Market pricing of deposit insurance, *Journal of Financial Services Research*, 24, 93–119.

Flannery, M., 2011, Data-driven deposit insurance assessments, Working paper, University of Florida.

Heckemeyer, J., and R. Mooij, 2013, Taxation and corporate debt: are banks any different? IMF Working Paper 221.

James, C., 1991, The losses realized in bank failures, *Journal of Finance*, 46, 1223–1242.

Jayaratne, J., and P. Strahan, 1998, *Journal of Law and Economics*, 41, 239–274.

Merton, R., 1974, On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, 29, 449–470.

Peltzman, S., 1965, Entry in Commercial Banking, *Journal of Law and Economics,* 8, 11–50.

U.S. Department of the Treasury, 2007, Treasury conference on business taxation and global competitiveness, Background paper.