

New-Keynesian Macroeconomics and the Term Structure*

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JEL Classification: E31, E32, E43, E52, G12

Keywords: Monetary Policy, Inflation Target, Term Structure of Interest Rates, Phillips Curve

*We benefited from the comments of two referees, the editor, Pok-sang Lam, Glenn Rudebusch, Adrian Pagan and seminar participants at Columbia University, Carleton University, the Korea Development Institute, the University of Navarra, the University of Rochester (Finance Department), Tilburg University, the National Bank of Belgium, the Singapore Management University, the 2004 Meeting of the Society for Economic Dynamics in Florence, the 2005 Econometric Society World Congress in London, the 2005 European Finance Association Annual Meeting in Moscow and the 2006 EC2 Conference in Rotterdam.

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Abstract

This article complements the structural New-Keynesian macro framework with a no-arbitrage affine term structure model. Whereas our methodology is general, we focus on an extended macro-model with unobservable processes for the inflation target and the natural rate of output which are filtered from macro and term structure data. We find that term structure information helps generate large and significant parameters governing the monetary policy transmission mechanism. Our model also delivers strong contemporaneous responses of the entire term structure to various macroeconomic shocks. The inflation target shock dominates the variation in the “level factor” whereas monetary policy shocks dominate the variation in the “slope and curvature factors”.

1 Introduction

Structural New-Keynesian models, featuring dynamic aggregate supply (AS), aggregate demand (IS) and monetary policy equations are becoming pervasive in macroeconomic analysis. In this article we complement this structural macroeconomic framework with a no-arbitrage term structure model.

Our analysis overcomes three deficiencies in previous work on New-Keynesian macro models. First, the parsimony of such models implies very limited information sets for both the monetary authority and the private sector. The critical variables in most macro models are the output gap, expected inflation and a short-term interest rate. It is well known, however, that monetary policy is conducted in a data-rich environment. Consequently, lags of inflation, the output gap and the short-term interest rate do not suffice to adequately forecast their future behavior. Recent research by Bernanke and Boivin (2003) and Bernanke, Boivin, and Elias (2005) collapses multiple observable time series into a small number of factors and embeds them in standard vector autoregressive (VAR) analyses. Instead, we use term structure data. Under the null of the Expectations Hypothesis, term spreads embed all relevant information about future interest rates. Additionally, a host of studies have shown that term spreads are very good predictors of future economic activity (see, for instance, Harvey (1988), Estrella and Mishkin (1998), Ang, Piazzesi, and Wei (2006)) and of future inflation (Mishkin (1990) or Stock and Watson (2003)). In our proposed model, the conditional expectations of inflation and the detrended output are a function of the past realizations of macro variables and of unobserved components which are extracted from term structure data through a no-arbitrage pricing model.

Second, the additional information from the term structure model transforms a version of a New-Keynesian model with a number of unobservable variables into a very tractable linear model which can be efficiently estimated by maximum likelihood or the general method of moments (GMM). Hence, the term structure information

helps recover important structural parameters, such as those describing the monetary transmission mechanism, in an econometrically convenient manner.

Third, incorporating term structure information leads to a simple VAR in macro variables and term spreads but the reduced-form model for the macro variables is a dynamically rich process with both autoregressive and moving average components. This is important because one disadvantage of most structural New-Keynesian models is the absence of sufficient endogenous persistence. We generate additional channels of persistence by introducing unobservable variables in the macro model and then identify their dynamics using the arbitrage-free term structure model and term spreads.

The approach set forth in this paper also contributes to the term structure literature. In this literature it is common to have latent factors drive most of the dynamics of the term structure of interest rates. These factors are often interpreted ex-post as level, slope and curvature factors. A classic example of this approach is Dai and Singleton (2000), who construct an arbitrage-free three factor model of the term structure.¹ While the Dai and Singleton (2000) model provides a satisfactory fit of the data, it remains silent about the economic forces behind the latent factors. In contrast, we construct a no-arbitrage term structure model where all the factors have a clear economic meaning. Apart from inflation, detrended output and the short term interest rate, we introduce two unobservable variables in the underlying macro model. While there are many possible implementations, our main application here introduces a time-varying inflation target and the natural rate of output. Consequently, we construct a 5 factor affine term structure model that obeys New-Keynesian structural relations.

Our main empirical findings are as follows. First, the model matches the persistence displayed by the three macro variables despite being nested in a parsimonious VAR(1) for macro variables and term spreads. Second, in contrast to previous maxi-

¹Other examples include Knez, Litterman, and Scheinkman (1994) and Pearson and Sun (1994).

mum likelihood (MLE) or GMM estimations of the standard New-Keynesian model, we obtain large and significant estimates of the Phillips curve and real interest rate response parameters. Third, our model exhibits strong contemporaneous responses of the entire term structure to the various structural shocks in the model.

Our article is part of a rapidly growing literature exploring the relation between the term structure and macro economic dynamics. Kozicki and Tinsley (2001) and Ang and Piazzesi (2003) were among the first to incorporate macroeconomic factors in a term structure model to improve its fit. Evans and Marshall (2003) use a VAR framework to trace the effect of macroeconomic shocks on the yield curve whereas Dewachter and Lyrio (2006) assign macroeconomic interpretations to standard term structure factors. Our paper differs from these articles in that all the macro variables obey a set of structural macro relations. This facilitates a meaningful economic interpretation of the term structure dynamics, relating them to macroeconomic shocks or “deep” parameters characterizing the behavior of the private sector or the monetary authority.

Two related studies are Hördahl, Tristani, and Vestin (2006) and Rudebusch and Wu (2008), who also append a term structure model to a New-Keynesian macro model. Our modelling approach is quite different however. First, our pricing kernel is consistent with the IS equation, whereas in these two papers, it is exogenously determined. Because standard linearized New-Keynesian models display constant prices of risk, this implies that our model’s term premiums do not vary through time by construction. While there is some evidence of time-variation in term premiums, we find it useful to examine how incorporating term structure data in a familiar setting affects standard structural parameters and macro dynamics. Imposing the restriction that the Expectations Hypothesis accounts for most of the variation in long rates appears reasonable too. Second, these two articles add a somewhat arbitrary lag structure to the supply and demand equations, whereas we analyze a standard opti-

mizing sticky price model with endogenous persistence. While this modelling choice may adversely affect our ability to fit the data dynamics, it generates a parsimonious state space representation for the macro-economic and term structure variables, with a clear structural interpretation.

Another related article is Wu (2006). He formulates and calibrates a structural macro model with adjustment costs for pricing and only two shocks (a technology shock and a monetary policy shock). Wu (2006) then gauges the fit of the model relative to the dynamics implied by an auxiliary standard term structure model based exclusively on unobservables. Instead of following this indirect approach, we estimate a structural macroeconomic model which directly implies an affine term structure model with five observable and interpretable factors.

The remainder of the paper is organized as follows. Section 2 describes the structural macroeconomic model, whereas section 3 outlines how to combine the macro model with an affine term structure model. Section 4 discusses the data and the estimation methodology employed. Section 5 analyzes the macroeconomic implications while section 6 studies the term structure implications of our model. Section 7 concludes.

2 A New-Keynesian Macro Model with Unobservable State Variables

We present a standard New-Keynesian model featuring AS, IS and monetary policy equations with two additions. First, we assume the existence of a natural rate of output which follows a persistent stochastic process. Second, the inflation target is assumed to vary through time according to a persistent linear process. The monetary authorities react to the output gap which is the deviation of output from the natural rate of output. We allow for endogenous persistence in the AS, IS and monetary policy

equations. In what follows, we describe each equation in turn and describe the model solution. In Bekaert, Cho, and Moreno (2005) we describe the microfoundations of the AS and IS equations. Related theoretical derivations can be found in Clarida, Galí, and Gertler (1999) or Woodford (2003).

2.1 The IS Equation

A standard intertemporal IS equation is usually derived from the first-order conditions for a representative agent with power utility as in the original Lucas (1978) economy. Standard estimation approaches have experienced difficulty pinning down the risk aversion parameter, which is at the same time an important parameter underlying the monetary transmission mechanism. Another disconcerting feature implied by a standard IS equation is that it typically fails to match the well-documented persistence of output. We derive an alternative IS equation from a utility maximizing framework with external habit formation similar to Fuhrer (2000). In particular, we assume that the representative agent maximizes:

$$E_t \sum_{s=t}^{\infty} \psi^{s-t} U(C_s; F_s) = E_t \sum_{s=t}^{\infty} \psi^{s-t} \left[\frac{F_s C_s^{1-\sigma} - 1}{1-\sigma} \right] \quad (1)$$

where C_t is the composite index of consumption, F_t represents an aggregate demand shifting factor; ψ denotes the time discount factor and σ is the inverse of the intertemporal elasticity of substitution. We specify F_t as follows:

$$F_t = H_t G_t \quad (2)$$

where H_t is an external habit level, that is, the agent takes H_t as exogenously given, even though it may depend on past consumption. G_t is an exogenous aggregate demand shock that can also be interpreted as a preference shock. Following Fuhrer (2000), we assume that $H_t = C_{t-1}^\eta$ where η measures the degree of habit dependence

on the past consumption level. It is this assumption that delivers endogenous output persistence.

Imposing the resource constraint ($C_t = Y_t$, with Y_t output) and assuming log-normality, the Euler equation for the interest rate yields a Fuhrer-type IS equation:

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1 - \mu)y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \quad (3)$$

where y_t is detrended log output and i_t is the short term interest rate. The parameter ϕ measures the response of detrended output to the real interest rate; $\phi = \frac{1}{\sigma + \eta}$ and $\mu = \sigma \phi$. The IS shock, $\epsilon_{IS,t} = \phi \ln G_t$, is assumed to be independently and identically distributed with homoskedastic variance σ_{IS}^2 .

2.2 The AS Equation (Phillips Curve)

Building on the Calvo (1983) pricing framework with monopolistic competition in the intermediate good markets, a forward-looking AS equation can be derived, linking inflation to future expected inflation and the real marginal cost. By assuming that the fraction of price-setters which does not adjust prices optimally, indexes their prices to past inflation, we obtain endogenous persistence in the AS equation. Moreover, we follow Woodford (2003) assuming the real marginal cost to be proportional to the output gap. Consequently, we obtain a standard New-Keynesian aggregate supply (AS) curve relating inflation to the output gap:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta)\pi_{t-1} + \kappa(y_t - y_t^n) + \epsilon_t^{AS} \quad (4)$$

where π_t is inflation, y_t^n is the natural rate of detrended output that would arise in the case of perfectly flexible prices and $y_t - y_t^n$ is the output gap;² ϵ_t^{AS} is an exoge-

²The output gap is measured as the percentage deviation of detrended output with respect to the natural rate of output. Both detrended output and the natural rate of output are measured

nous supply shock, assumed to be independently and identically distributed with homoskedastic variance σ_{AS}^2 . The parameter κ captures the short-run tradeoff between inflation and the output gap and $(1 - \delta)$ characterizes the endogenous persistence of inflation.

In practice, structural estimates of the Phillips curve based on output gap measures seem less successful than those based on marginal cost (see Galí and Gertler (1999)). However, whereas in most studies an exogenously detrended output variable serves as the output gap measure in the AS equation, our output gap measure is endogenous and filtered through macro and term structure information. We let the natural rate follow an AR(1) process:

$$y_t^n = \lambda y_{t-1}^n + \epsilon_{y^n,t} \tag{5}$$

where $\epsilon_{y^n,t}$ can be interpreted as a negative markup shock with standard deviation σ_{y^n} .³

2.3 The Monetary Policy Rule

We assume that the monetary authority specifies the nominal interest rate target, i_t^* , as in the forward-looking Taylor rule proposed by Clarida, Galí, and Gertler (1999):

$$i_t^* = [\bar{i}_t + \beta(E_t \pi_{t+1} - \pi_t^*) + \gamma(y_t - y_t^n)] \tag{6}$$

as percentage deviations with respect to a linear trend. Therefore, the means of the output gap, detrended output and the natural rate of output are 0.

³In a previous version of the paper (Bekaert, Cho, and Moreno (2005)), we used the utility function in (1) and a simple production technology to derive an AS equation that provided an explicit link between the marginal cost and the current and past output gap, and also yielded a process for the natural rate of output endogenously. While some of the versions of the structural model converged, it proved very difficult to estimate.

where π_t^* is a time-varying inflation target and \bar{i}_t is the desired level of the nominal interest rate that would prevail when $E_t\pi_{t+1} = \pi_t^*$ and $y_t = y_t^n$. We assume that \bar{i}_t is constant.⁴ Note that β measures the long-run response of the interest rate to expected inflation, a typical measure of the Fed's stance against inflation.

We further assume that the monetary authority sets the short term interest rate as a weighted average of the interest rate target and a lag of the short term interest rate to capture the tendency by central banks to smooth interest rate changes:

$$i_t = \rho i_{t-1} + (1 - \rho) i_t^* + \epsilon_{MP,t} \quad (7)$$

where ρ is the smoothing parameter and $\epsilon_{MP,t}$ is an exogenous monetary policy shock, assumed to be *i.i.d.* with standard deviation, σ_{MP} . The resulting monetary policy rule for the interest rate is given by:

$$i_t = \alpha_{MP} + \rho i_{t-1} + (1 - \rho) [\beta (E_t\pi_{t+1} - \pi_t^*) + \gamma (y_t - y_t^n)] + \epsilon_{MP,t} \quad (8)$$

where $\alpha_{MP} = (1 - \rho)\bar{i}$.

2.4 Inflation Target π_t^*

We close our model by specifying a stochastic process for the inflation target, π_t^* . Little is known about how the monetary authority sets the inflation target. Hördahl, Tristani, and Vestin (2006) assume an AR(1) process for the inflation target. Gürkaynak, Sack, and Swanson (2005) specify it as a weighted average of the past inflation target and past inflation rates. While their specification is empirically supported to some extent, we deem it plausible that the Central Bank takes the long-run inflation expectations of the private sector into account in a forward-looking manner. Therefore,

⁴We estimated alternative model specifications with a time-varying \bar{i}_t in Bekaert, Cho, and Moreno (2005) and the main results in the article are not altered.

we define π_t^{LR} as the conditional expected value of a weighted average of all future inflation rates.

$$\pi_t^{LR} = (1 - d) \sum_{j=0}^{\infty} d^j E_t \pi_{t+j} \quad (9)$$

with $0 \leq d \leq 1$. This equation can be succinctly written as:

$$\pi_t^{LR} = dE_t \pi_{t+1}^{LR} + (1 - d)\pi_t \quad (10)$$

When d equals 0, π_t^{LR} collapses to current inflation, when d approaches 1, long-run inflation approaches unconditional expected inflation. We assume that the monetary authority anchors its inflation target around π_t^{LR} , but smooths target changes, so that:

$$\pi_t^* = \omega \pi_{t-1}^* + (1 - \omega) \pi_t^{LR} + \epsilon_{\pi^*,t}. \quad (11)$$

We view $\epsilon_{\pi^*,t}$ as an exogenous shift in the policy stance regarding the long term rate of inflation or the target, and assume it to be *i.i.d.* with standard deviation σ_{π^*} . Substituting out π_t^{LR} in equation (11) using equation (10), we obtain:

$$\pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_t + \epsilon_{\pi^*,t} \quad (12)$$

where $\varphi_1 = \frac{d}{1+d\omega}$, $\varphi_2 = \frac{\omega}{1+d\omega}$ and $\varphi_3 = 1 - \varphi_1 - \varphi_2$. In Bekaert, Cho, and Moreno (2005) we also estimated a backward-looking inflation target process similar to that of Gürkaynak, Sack, and Swanson (2005), producing results qualitatively similar to the ones we present below.

2.5 The Full Model

Bringing together all the equations, we have a five variable system with three observed and two unobserved macro factors:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \kappa (y_t - y_t^n) + \epsilon_t^{AS} \quad (13)$$

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (i_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \quad (14)$$

$$i_t = \alpha_{MP} + \rho i_{t-1} + (1 - \rho) [\beta (E_t \pi_{t+1} - \pi_t^*) + \gamma (y_t - y_t^n)] + \epsilon_{MP,t} \quad (15)$$

$$y_t^n = \lambda y_{t-1}^n + \epsilon_{y^n,t} \quad (16)$$

$$\pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_t + \epsilon_{\pi^*,t}. \quad (17)$$

Our macroeconomic model can be expressed in matrix form as:

$$Bx_t = \alpha + AE_t x_{t+1} + Jx_{t-1} + C\epsilon_t \quad (18)$$

where $x_t = [\pi_t \ y_t \ i_t \ y_t^n \ \pi_t^*]'$ and $\epsilon_t = [\epsilon_{AS,t} \ \epsilon_{IS,t} \ \epsilon_{MP,t} \ \epsilon_{y^n,t} \ \epsilon_{\pi^*,t}]'$. α is a 5×1 vector of constants and B , A , J and C are appropriately defined 5×5 matrices. The Rational Expectations (RE) equilibrium can be written as a first-order VAR:

$$x_t = c + \Omega x_{t-1} + \Gamma \epsilon_t. \quad (19)$$

Hence, the implied model dynamics are a simple VAR subject to a set of non-linear restrictions. Note that Ω cannot be solved analytically in general. We solve for Ω numerically using the QZ method (see Klein (2000) and Cho and Moreno (2008)). Once Ω is solved for, Γ and c follow straightforwardly.

As the Appendix shows, the reduced-form representation of the vector of observable macro variables is very similar to a VARMA(3,2) process. By adding unobservables, we potentially deliver more realistic joint dynamics for inflation, the output

gap and the interest rate, and overcome the lack of persistence implied by previous studies.

3 Incorporating Term Structure Information

We derive the term structure model implicit in the IS curve that we presented in section 2. This effort results in an easily estimable linear system in observable macro variables and term spreads.

3.1 Affine Term Structure Models with New-Keynesian Factor Dynamics

Affine term structure models require linear state variable dynamics and a linear pricing kernel process with conditionally normal shocks (see Duffie and Kan (1996)). For the state variable dynamics implied by the New-Keynesian model in equation (19) to fall in the affine class, we assume that the shocks are conditionally normally distributed, $\epsilon_t \sim N(0, D_{t-1})$. The pricing kernel process M_{t+1} prices all securities such that:

$$E_t[M_{t+1}R_{t+1}] = 1. \quad (20)$$

In particular, for an n -period bond, $R_{t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}$ with $P_{n,t}$ the time t price of an n -period zero-coupon bond. If $M_{t+1} > 0$ for all t , the resulting returns satisfy the no-arbitrage condition (Harrison and Kreps (1979)). In affine models, the log of the pricing kernel is modelled as a conditionally linear process. Consider, for instance:

$$m_{t+1} = \ln(M_{t+1}) = -i_t - \frac{1}{2}\Lambda_t' D_t \Lambda_t - \Lambda_t' \epsilon_{t+1}. \quad (21)$$

Here $\Lambda_t = \Lambda_0 + \Lambda_1 x_t$, where Λ_0 is a 5×1 vector and Λ_1 is a 5×5 matrix. First, setting $D_t = D$, we obtain a Gaussian price of risk model. Dai and Singleton (2002)

study such a model and claim that it accounts for the deviations of the Expectations Hypothesis (EH) observed in U.S. term structure data. An alternative model sets $\Lambda_t = \Lambda$ and $\epsilon_t \sim N(0, D_{t-1})$ with $D_t = D_0 + D_1 \text{diag}(x_t)$, where $\text{diag}(x_t)$ is the diagonal matrix with the vector x_t on its diagonal. This model introduces heteroskedasticity of the square-root form and has a long tradition in finance (see Cox, Ingersoll, and Ross (1985)). Finally, setting $\Lambda_t = \Lambda_0$ and $D_t = D$ results in a homoskedastic model.

All three of these models imply an affine term structure. That is, log bond prices, $p_{n,t}$ are an affine function of the state variables. The maturity-dependent coefficients follow recursive equations. The three models have different implications for the behavior of term spreads and holding period returns. First, the homoskedastic model implies that the EH holds: there may be a term premium but it does not vary through time. Both the Gaussian prices of risk model and the square root model imply time-varying term premiums. Second, our model includes inflation as a state variable and the real pricing kernel (the kernel that prices bonds perfectly indexed against inflation) and inflation are correlated. It is this correlation that determines the inflation risk premium. If the covariance term is constant, the risk premium is constant over time and this will be true in a homoskedastic model.

The kernel model implied by the IS curve derived above fits in the homoskedastic class. It is possible to modify the pricing framework into one of the two other models, but we defer this to future work. Bekaert, Hodrick, and Marshall (2001) show that a model with minimal variation in the term premium suffices to match the evidence regarding the Expectations Hypothesis for the US. Moreover, the current practice of using linear regressions to infer the properties of term premiums almost surely leads to the over-estimation of their variability (see Bekaert, Wei, and Xing (2007) for simulation evidence).

3.2 The Term Structure Model Implied by the Macro Model

Because our derivation of the IS curve assumed a particular preference structure, the pricing kernel is given by the intertemporal consumption marginal rate of substitution of the model. That is:

$$m_{t+1} = \ln \psi - \sigma y_{t+1} + (\sigma + \eta)y_t - \eta y_{t-1} + (g_{t+1} - g_t) - \pi_{t+1}. \quad (22)$$

The no-arbitrage condition holds by construction. In a log-normal model, pricing a one period bond implies

$$E_t[m_{t+1}] + 0.5V_t[m_{t+1}] = -i_t. \quad (23)$$

For our particular model, (21) holds with Λ , a vector of prices of risk entirely restricted by the structural parameters,

$$\Lambda'_t = \Lambda' = [1 \ \sigma \ 0 \ 0 \ 0]\Gamma - [0 \ (\sigma + \eta) \ 0 \ 0 \ 0]. \quad (24)$$

Logarithmic bond prices and hence, bond yields ($y_t = \frac{-\ln(P_{t,n})}{n}$), are an affine function of the state variables:

$$y_{n,t} = -\frac{a_n}{n} - \frac{b'_n}{n}x_t. \quad (25)$$

The recursive formulas for a_n and b_n can be constructed using equations (19)- (22).⁵

Term spreads are also an affine function of the state variables:

$$sp_{n,t} = -\frac{a_n}{n} - \left(\frac{b'_n}{n} + e_3\right)'x_t \quad (26)$$

where $sp_{n,t} \equiv y_{n,t} - i_t$ is the spread between the n period yield and the short rate.

This model provides a particular convenient form for the joint dynamics of the macro

⁵They are: $a_n = a_{n-1} + b'_{n-1}c + 0.5b'_{n-1}\Gamma D\Gamma' b_{n-1} - \Lambda' D\Gamma' b_{n-1}$ and $b'_n = -e'_3 + b'_{n-1}\Omega$, respectively.

variables and the term spreads. Let $z_t = [\pi_t \ y_t \ i_t \ sp_{n_1,t} \ sp_{n_2,t}]'$, where n_1 and n_2 refer to two different yield maturities for the long-term bond in the spread. Then

$$x_t = c + \Omega x_{t-1} + \Gamma \epsilon_t \quad (27)$$

$$z_t = A_z + B_z x_t \quad (28)$$

where

$$A_z = \begin{bmatrix} 0_{3 \times 1} \\ -\frac{a_{n_1}}{n_1} \\ -\frac{a_{n_2}}{n_2} \end{bmatrix}, B_z = \begin{bmatrix} I_3 & 0_{3 \times 2} \\ -(\frac{b_{n_1}}{n_1} + e_3)' \\ -(\frac{b_{n_2}}{n_2} + e_3)' \end{bmatrix}.$$

Using $x_t = B_z^{-1}(z_t - A_z)$, we find:

$$z_t = a_z + \Omega_z z_{t-1} + \Gamma_z \epsilon_t \quad (29)$$

where

$$\begin{aligned} \Omega_z &= B_z \Omega B_z^{-1} \\ \Gamma_z &= B_z \Gamma \\ a_z &= B_z c + (I - B_z \Omega B_z^{-1}) A_z \end{aligned}$$

In other words, the macro variables and the term spreads follow a first-order VAR with complex cross-equation restrictions. This feature also differentiates our method from previous work in the literature. Because equation (29) consists of observed macro and term spread data, we can directly estimate it employing exactly two term spreads. Once the model is estimated, we can back out the natural rate of output and the inflation target using equation (28).

4 Data and Estimation

In this section, we first describe the data used in the estimation of our macro-finance model. Then we present the general estimation methodology employed.

4.1 Data Description

The sample period is from the first quarter of 1961 to the fourth quarter of 2003. We measure inflation with the CPI (collected from the Bureau of Labor Statistics) but check robustness using the GDP deflator, from the National Income and Product Accounts (NIPA). Detrended output is measured as the output deviation from a linear trend. We also estimated the model with quadratically detrended output, producing qualitatively similar results. The results are by and large similar across alternative detrending methods. Output is real GDP from NIPA. We use the 3-month T-bill rate, taken from the Federal Reserve of St. Louis database, as the short-term interest rate. Finally, our analysis uses term-structure data at the one, three, five and ten year maturities from the CRSP database.⁶ We use the three and five year spreads directly in the estimation and use additional term spreads to test the model ex-post. Consequently, we do not use measurement error in the estimation and do not rely on a Kalman filter to extract the unobservables from the data.

4.2 Estimation Methodology

Because our macro-finance model implies a first-order VAR on z_t with complex cross-equation, non-linear restrictions, we first verify that the BIC criterion indeed selects a first-order VAR among unconstrained VARs of lag-lengths 1 through 5. We perform the estimation on de-meaned data, $\bar{z}_t = z_t - \hat{E}z_t$ with $\hat{E}z_t$ the sample mean of z_t .

⁶The ten year zero-coupon yield was constructed splicing two series. We use the McCulloch and Kwon series up to the 3rd quarter of 1987; from the 4th quarter of 1987 to the end of the sample, we use the ten year zero-coupon yield estimated using the method of Svensson (2004). We thank Refet Gürkaynak for kindly providing this second part of the series.

The structural parameters to be estimated are therefore $\theta = (\delta \ \kappa \ \sigma \ \eta \ \rho \ \beta \ \gamma \ \lambda \ \omega \ d \ \sigma_{AS} \ \sigma_{IS} \ \sigma_{MP} \ \sigma_{y^n} \ \sigma_{\pi^*})$. Assuming normal errors, it is straightforward to write down the likelihood function for this problem and produce Full Information Likelihood Estimates (FIML) estimates. However, to accommodate possible deviations from the strong normality and homoskedasticity assumptions underlying maximum likelihood, we use a two-step GMM estimation procedure based on Hansen (1982). To do so, re-write the model in the following form:

$$\bar{z}_t = \Omega_z \bar{z}_{t-1} + \Gamma_z \epsilon_t = \Omega_z \bar{z}_{t-1} + \Gamma_z \Sigma u_t \quad (30)$$

where $u_t = \Sigma^{-1} \epsilon_t \sim (0, I_5)$ and $\Sigma = \text{diag}([\sigma_{AS} \ \sigma_{IS} \ \sigma_{MP} \ \sigma_{y^n} \ \sigma_{\pi^*}]')$, that is $\Sigma^2 = D$.

To construct the moment conditions, consider the following vector valued processes:

$$h_{1,t} = u_t \otimes \bar{z}_{t-1} \quad (31)$$

$$h_{2,t} = \text{vech}(u_t u_t' - I_5) \quad (32)$$

$$h_t = [h_{1,t}' \ h_{2,t}']' \quad (33)$$

where *vech* represents an operator stacking the elements on or below the principle diagonal of a matrix. The model imposes $E[h_t] = 0$. The 25 $h_{1,t}$ moment conditions capture the feedback parameters; the 15 $h_{2,t}$ moment conditions capture the structure imposed by the model on the variance-covariance matrix of the innovations. Rather than using an initial identity matrix as the weighting matrix, which may give rise to poor first-stage estimates, we use a weighting matrix implied by the model under normality. That is under the null of the model, the weighting matrix must be:

$$W = (E[h_t h_t'])^{-1}. \quad (34)$$

Using normality and the error structure implied by the model, it is then straightforward to show that the optimal weighting matrix is given by:

$$\hat{W} = \begin{bmatrix} I \otimes \frac{1}{T} \sum_{t=1}^T \bar{z}_{t-1} \bar{z}'_{t-1} & 0_{25 \times 15} \\ 0_{15 \times 25} & I_{15} + \text{vech}(I_5) \text{vech}(I_5)' \end{bmatrix}^{-1}. \quad (35)$$

This weighting matrix does not depend on the parameters. Then we minimize the standard GMM objective function:

$$Q = \left(\hat{E}[h_t] \right)' \hat{W} \left(\hat{E}[h_t] \right) \quad (36)$$

where $\hat{E}[h_t] = \frac{1}{T} \sum_{t=1}^T h_t$. This gives rise to estimates that are quite close to what would be obtained with maximum likelihood. Given these estimates, we produce a second-stage weighting matrix allowing for heteroskedasticity and 5 Newey-West (Newey and West (1987)) lags in constructing the variance covariance matrix of the orthogonality conditions. We iterate this system until convergence. This estimation proved overall rather robust with parameter estimates varying little after the first round.

5 Macroeconomic Implications

5.1 Structural Parameter Estimates

In order to assess the fit of the model, we first comment on the standard GMM test of the over-identifying restrictions, which follows a χ^2 distribution with 25 degrees of freedom because there are 40 moment conditions but only 15 parameters. We find that the test fails to reject the model at the 5% level when 5 Newey-West lags are used in the construction of the weighting matrix (the p-value is 26.3%). While the model is not rejected either with 4 Newey-West lags (the p-value is 9.4%), it is rejected when

only 3 Newey-West lags are used (the p-value is 1.3%). This in itself suggests that the orthogonality conditions still display substantial persistence. Nevertheless, the model fits the autocorrelograms of the data series very well (not reported).

The second to fourth columns in table 1 show the parameter estimates of the model and their GMM and bootstrap standard errors.⁷ The bootstrap analysis will prove useful in generating standard errors for a number of model-implied statistics. A first important finding is the size and significance of κ , the Phillips curve parameter. As Galí and Gertler (1999) point out, previous studies fail to obtain reasonable and significant estimates of κ with quarterly data. Galí and Gertler (1999) do obtain larger and significant estimates using a measure for marginal cost replacing the output gap. Our estimates of κ , using the output gap and term spreads are even larger than those obtained by Galí and Gertler (1999). Using the (larger) standard error from the bootstrap, κ remains statistically significantly different from zero. We estimate the forward-looking parameter in the AS equation to be close to 0.61, consistent with previous studies.

When structural models are estimated with techniques such as GMM or MLE, they often give rise to large estimates of σ rendering the IS equation a rather ineffective channel of monetary policy transmission. Two examples are Ireland (2001) and Cho and Moreno (2006). Lucas (2003) argues that the curvature parameter in the representative agent's utility function consistent with most macro and public finance models should be between 1 and 4. While the Lucas' statement does not strictly apply to models with habit persistence, in our multiplicative habit model σ still represents

⁷We bootstrap from the 172 observations on the vector of structural standard errors (ϵ_t) with replacement and re-create a sample of artificial data using the estimated parameter matrices (Ω , Γ) and historical initial values. For each replication, we create a sample of 672 observations, discard the first 500 and retain the last 172 observations to create a sample of length equal to the data sample. We then re-estimate the model with these artificial data and replicate this process 1,000 times to create the small sample distributions of the parameters. Since we identified some serial correlation in the residuals, we also perform a block bootstrap using the Künsch's rule with a block length of 7, as explained in Hall, Horowitz, and Jing (1995). The implied small-sample distributions are very similar to the ones presented in the paper.

local risk aversion. Our estimation yields a small (slightly larger than 3) and significant estimate of σ , although its significance is more marginal using bootstrapped standard errors. Smets and Wouters (2003) and Lubik and Schorfheide (2004) find small estimates of σ using Bayesian estimation techniques. Rotemberg and Woodford (1998) and Boivin and Giannoni (2006) also find small estimates of σ but they modify the estimation procedure towards fitting particular impulse responses. Our model exhibits large habit persistence effects, as the habit persistence parameter, η , is close to 4. Other studies have also found an important role for habit persistence (Fuhrer (2000), Boldrin, Christiano, and Fisher (2001)).⁸ In summary, the parameter estimates for the AS and IS equations imply that our model delivers large economic effects of monetary policy on inflation and output.

Why do we obtain large and significant estimates of κ and ϕ ? Two channels seem to be at work: First, expectations are based on both observable and unobservable macro variables. Therefore, an important variable in the AS equation, such as expected inflation, is directly affected by the inflation target. As a result, changes in the inflation target shift the AS curve. As we show below in variance decompositions, the inflation target shock contributes significantly to variation in the inflation rate. Similarly, the natural rate shock significantly contributes to the dynamics of detrended output. Second, our measure of the output gap is different from the usual detrended output and contains additional valuable information extracted from the term structure. For instance, the first order autocorrelation of the implied output gap is 0.92, which is smaller than 0.96, the first order autocorrelation of linearly detrended output. The Phillips curve coefficients found in previous studies reflect the

⁸Our habit persistence parameter is not directly comparable to that derived by Fuhrer (2000). There is however a linear relationship between them: $\eta = (\sigma - 1)h$, where h is the Fuhrer (2000) habit persistence parameter. Our implied h is close to 2, larger than in previous studies, and violating a theoretical bound in Fuhrer (2000). This implies that $\mu < 0.5$, as in Boivin and Giannoni (2006), for instance. Imposing $h \leq 1$ significantly worsens the fit of the empirical model.

weak link between detrended output and inflation in the data and the large difference in persistence between these two variables. In our model, even though κ is rather large, the relationship between inflation and the output gap is still not strongly positive because the inflation target also moves the AS-curve. In sum, the presence of both the inflation target and the natural rate of output in the AS equation implies a significantly positive conditional co-movement between the output gap and inflation, even though the unconditional correlation between them remains low as it is in the data. The unobservables are also critical in fitting the relative persistence of the output gap and inflation. Similarly, the ϕ parameter still fits the dependence of detrended output on the real interest rate, but the real interest rate is now an implicit function of all the state variables, including the natural rate of output.

The estimates of the policy rule parameters are similar to those found in the literature. The estimated long-run response to expected inflation is larger than 1. The response to the output gap is always close to 0 and insignificantly different from 0. Finally, the smoothing parameter, ρ , is estimated to be 0.72, similar to previous studies.

The two unobservables are quite persistent, but clearly stationary processes. The natural rate of output's persistence is close to 0.96, while the weight on the past inflation target in the inflation target equation is 0.88. Furthermore, the weight on current inflation in the construction of the long-run inflation target ($1-d$) is close to 0.15. Finally, the five shock standard deviations are significant, with the monetary policy shock standard deviation larger than the others. There has been some evidence pointing towards a structural break in the β parameter (see, for instance, Clarida, Galí, and Gertler (1999) or Lubik and Schorfheide (2004)). The large estimate of σ_{MP} may reflect the absence of such a break in our model.

5.2 Output Gap and Inflation Target

One important feature of our analysis is that we can extract two economically important unobservable variables from the observable macro and term structure variables. The output gap is of special interest to the monetary authority, as it plays a crucial role in the monetary transmission mechanism of most macro models. Smets and Wouters (2003) and Laubach and Williams (2003) also extract the natural rate of output for the European and US economies from theoretical and empirical models respectively. An important difference between our work and theirs is that we use term structure information to filter out the natural rate, whereas they back it out of pure macro models through Kalman filter techniques. The dynamics of the inflation target are particularly important for the private sector, as the Federal Reserve has never announced targets for inflation and knowledge of the inflation target would be useful for both real and financial investment decisions.

The top panel in figure 1 shows the evolution of the output gap implied by the model. Several facts are worth noting. Before 1980, the output gap stayed above zero for most of the time. A positive output gap is typically interpreted as a proxy for excess demand. A popular view is that a high output gap made inflation rise through the second half of the 70s. Our output gap graph is consistent with that view. However, right before 1980, the output gap becomes negative. The aggressive monetary policy response to the high inflation rate is responsible for this sharp decline. After this, the output gap remains negative for most of the time up to 1995. This negative output gap was mainly caused by a surge in the natural rate of output, which remains above trend well into the mid-90s. Finally, the output gap grows during the mid-1990s and starts to fall around 2000, coinciding with the latest recession in our sample.

The bottom panel in figure 1 analogously presents the natural rate of output implied by the model. Note that, first, there is a steady upward trend in the natural

rate throughout the 60s. While it is possible that the natural rate did increase during that period, we think that the linear filtering of output overstates this growth. Second, the natural rate falls around 1973 and the late 70s. While the natural rate is exogenous in our setting, this may reflect the side-effects of the productivity slow-down brought about by oil price increases. Third, the natural rate stayed high throughout most of the 80s. Fourth, the natural rate did fall coinciding with the recession of the early 80s, but it remained above trend during the rest of the 80s. In the early nineties it fell below trend and has stayed close to trend since the mid nineties.

Figure 2 focusses on the inflation target. The top panel shows the filtered inflation target.⁹ The bottom panel shows the CPI inflation series for comparison. Three well differentiated sections can be identified along the sample. In the first one, the inflation target grows steadily up to the early 80s. Private sector expectations seem to have built up through the 60s and 70s contributing to the progressive increase in inflation. In the second one, the inflation target remains high for about 5 years. Finally, since the mid-eighties, the inflation target declines and remains low for the rest of the sample, tracking inflation closely.¹⁰

5.3 Implied Macro Dynamics

In this section, we characterize the dynamics implied by the structural model using standard impulse response and variance decomposition analysis. Figure 3 shows the

⁹Because we estimate the model with demeaned data, we add the mean of inflation back to the actual inflation target. This procedure is consistent with our model, where the mean of the inflation target coincides with that of inflation.

¹⁰Notice that the inflation target turns negative at the end of the sample. This occurs because the implied regression coefficient of the inflation target on the short-term interest rate is positive (1.58) and the interest rate rapidly declined during the last years of our sample. However, we also computed a 95% confidence interval for the implied inflation target process, using the bootstrapped parameter estimates and conditioning on the observed macro and term structure information. The inflation target is not significantly different from 0 at the 5% level at the end of the sample.

impulse response functions of the five macro variables to (one standard deviation) structural shocks. The AS shock is a negative technology or supply shock which decreases the productivity of firms. A typical example of an AS shock is an oil shock, as it raises marginal costs overall. As expected, the AS shock pushes inflation almost 2 percentage points above its steady state, but it soon returns to its original level, given the highly forward-looking nature of our AS equation. The monetary authority increases the interest rate following the supply shock. Because of the strong reaction of the Fed to the AS shock (the Taylor principle holds), the real rate increases and output exhibits a hump-shaped decline for several quarters. The inflation target initially increases after the AS shock but then decreases and stays below steady state due to the decline in inflation.

Our IS shock is a demand shock, which can also be interpreted as a preference shock (see Woodford (2003)). Consistent with economic intuition and the results in the empirical VARs of Evans and Marshall (2003), the IS shock increases output, inflation, the interest rate and the inflation target for several quarters.

The monetary policy shock reflects shifts to the interest rate unexplained by the state of the economy. Given our strong monetary transmission mechanism and, analogous to the results obtained in the structural model of Christiano and Eichenbaum (2005), a contractionary monetary policy shock yields a decline of both output and inflation. The inflation target also declines, reinforcing the contractionary effect of the monetary policy shock on inflation and output. The interest rate increases following the monetary policy shock, but after three quarters it undershoots its steady-state level. This undershooting is related to the strong endogenous decrease of output and inflation to the monetary policy shock. As we show below, this reaction of the short-term interest rate to the monetary policy shock has implications for the reaction of the entire term structure to the monetary policy shock.

A standard microeconomic mechanism for our natural rate shock is an increase

in the number of firms, which decreases the wedge between prices and the marginal cost (a negative markup shock) and increases output. In other words, a natural rate shock shifts the AS curve down and, not surprisingly, we see that an expansive natural rate shock increases output and lowers inflation. Through the monetary policy rule, the interest rate follows initially a similar path to inflation, decreasing substantially. Eventually, inflation rises above steady state again and so does the interest rate, both overshooting their steady state during several periods. As a result, the inflation target, which partially reflects expected inflation, rises above steady-state almost immediately. Notice how output converges towards its natural level after 10 quarters following the natural rate shock and moves in parallel with it from then onwards.

An expansionary inflation target shock is an exogenous shift in the preferences of the Fed regarding its monetary policy goal. Because the inflation target is a long-term policy objective, a positive inflation target shock is akin to a persistent expansionary monetary policy shock. As a result, output and inflation exhibit a strong hump-shaped increase in response to the target shock. This response is larger than that estimated by Rudebusch and Wu (2008) and Diebold, Rudebusch, and Aruoba (2006), who explicitly model a level factor instead of the inflation target. Notice that in our setup, the strong response of inflation to a target shock is due to the relation between the inflation target, inflation expectations and inflation.

Figure 4 shows the variance decompositions at different horizons for the five macro variables in terms of the five structural shocks. The variance decompositions show the contribution of each macroeconomic shock to the overall forecast variance of each of the variables at different horizons. Inflation is mostly explained by the AS shock at short horizons. However, at medium and long-run horizons inflation dynamics are mostly driven by the monetary policy shock and the inflation target shock. Short-run output dynamics are mostly due to the IS and monetary policy shocks. The natural rate shock has a growing influence on output dynamics as the time horizon advances,

reflecting the fact that in the long-run output tends to its natural level. Interest rate dynamics are dominated by the monetary policy shock at short horizons whereas in the long-run the inflation target shock has more influence. Given that the monetary authority is responsible for both the monetary policy and inflation target shocks, our results reveal monetary policy to be a key driver of macro dynamics. Smets and Wouters (2003) also find monetary policy shocks to play a key role in explaining macro dynamics in the Euro area.

6 Term Structure Implications

6.1 Model Fit for Yields

Our model represents a five factor term structure model with three observed and two unobserved variables. Dai and Singleton (2000) claim that a model with three latent factors provides an adequate fit with the data. To investigate how well our model fits the complete term structure, we examine its fit with respect to the yields (one year and ten year) not used in the estimation. The difference between the actual and model-predicted yields can be viewed as measurement error and should not be too variable if the model fits the data well. We find that the measurement error for the one and ten-year yields is 45 and 54 basis points (annualized) respectively. While this is significantly different from zero, the fit is reasonable given the parsimonious structural nature of our model.

We also compare the fit of our model to that of a pure macro model in the three observable macro variables. We generate yields under the null of the Expectations Hypothesis, using a VAR(3) for the three variables. Such a VAR can be shown to fit the macro data very well. In figure 5, the left-hand-side graphs plot the one year and ten year yields and their predicted values from the structural model, the right hand side depicts the yields implied by the VAR(3) macro model. The fit of the VAR(3)

is visibly worse for the ten year yield, but similar for the one year yield. Indeed, the implied measurement error standard deviations of the one and ten year yields are 45 and 169 basis points, respectively.

6.2 Structural Term Structure Factors

It is standard to label the three factors that are necessary to fit term structure dynamics as the level, the slope and the curvature factors. We measure the level as the equally weighted average of the three month rate, one year and five year yields, the slope as the five year to three month spread and the curvature as the sum of the three month rate and five year rate minus twice the one year rate. Figure 6 shows the impulse responses of the level, slope and curvature factors to the five structural shocks. The AS shock initially raises the level, but then the level factor undershoots its steady state for several quarters. As explained in section 5.3, the interest rate undershooting is related to the strong endogenous response of the monetary policy authority to inflation, which ends up lowering inflation below its steady-state for some quarters. The Expectation Hypothesis implies that the initial rise in yields is strongest at short maturities. Consequently, it is no surprise that the AS shock initially lowers the slope, and then, when the level effect turns negative, raises the slope above its steady-state. The curvature effect follows the slope effect closely.

As in Evans and Marshall (2003), the IS or demand shock raises the level factor for several years. The IS shock also lowers the slope and curvature during some quarters. These two responses are again related to the hump-shaped response of the short-term interest rate to the IS shock and the fact that the Expectation Hypothesis holds in our setup. Essentially, a positive IS shock causes a flattening upward shift in the yield curve. The monetary policy shock initially raises the level factor, but then it produces a strong hump-shaped negative response on the level. This is again related to the undershooting of the short-term interest rate after a monetary policy shock. The slope

initially decreases after the monetary policy shock but then it increases during several quarters. The initial slope decline happens because a monetary policy shock naturally shifts up the short-end of the yield curve, while it lowers the medium and long part of the yield curve, through its effect on inflationary expectations. The subsequent slope increase arises because the short rate undershoots after a few quarters. Finally, the curvature of the yield curve increases for ten quarters after the monetary policy shock.

The natural rate shock, which is essentially a positive supply shock, not surprisingly induces an initial decline in the level of the yield curve. After four quarters, the level exhibits a persistent increase, mimicking the response of the short-term rate to the natural rate shock. Both the slope and the curvature factors increase after the natural rate shock for ten quarters. As figure 3 shows, the natural rate shock raises the future expected short-term rates whereas it lowers the current short rate. Since the Expectations Hypothesis holds in our setup, that implies that the spread increases. Figure 8 below corroborates this intuition.

Finally, the inflation target shock has a very pronounced positive effect on the level of the yield curve. This effect has to do with the strong persistent hump-shaped response of the interest rate to the target shock. It also makes the slope initially increase, but after three or four quarters the response becomes negative. Thus, the target shock ends up having a stronger positive effect on short term rates than on long rates.¹¹ Finally, the curvature declines after the inflation target shock during several periods.

To complement the impulse response functions, figure 7 shows the variance decompositions of the three factors at different time horizons. The inflation target shock

¹¹Rudebusch and Wu (2008) obtain the opposite reaction of the spread to their level shock. This is probably related to the fact that they incorporate a time-varying risk premium in the term structure whereas we maintain the Expectation Hypothesis throughout.

explains more than 50% of the variation in the level of the term structure at all time horizons and over 75% at short horizons. Ang, Bekaert, and Wei (2008) also find that inflation factors account for a large part of the variation of nominal yields at both short and long horizons. After the fifth quarter, the monetary policy shock explains around 25% of the level dynamics. In the short-run, the IS shock explains around 15%, whereas in the long-run, it is the natural rate shock which explains around 15% of the variation in the level factor.

The monetary policy shock is the dominant factor behind the slope dynamics at all horizons, as it primarily affects the short end of the yield curve. This fact is especially evident at short horizons, where almost 90% of the slope variance is explained by the monetary policy shock. The inflation target shock, which has a dominant effect at the long end of the yield curve gains importance at longer horizons. The IS shock and the natural rate shock explain each around 10% of the slope dynamics at virtually all horizons.

The variance decomposition of the curvature factor yields similar results to the slope factor, with the monetary policy shock being the dominant factor again, explaining around 60% of the curvature factor dynamics at all horizons. Finally, it is worthwhile noting that the AS shock influence on the dynamics of the term structure is overall very small.

An implication of our study is that the inflation target shock drives much of time-variation of the level, as figure 7 shows, whereas the monetary policy shock drives both the slope and curvature factors. These results are consistent with the those in Rudebusch and Wu (2008), where the unobservable variables are directly labeled level and slope.

6.3 “Endogenous” Excess Sensitivity

Our model can shed light on an empirical regularity that has received much attention in recent work, the excess sensitivity of long term interest rates. Gürkaynak, Sack, and Swanson (2005) show a particularly intriguing empirical failure of standard structural models: they fail to generate significant responses of forward interest rates to any macroeconomic and monetary policy shocks. However, in the data, US long-term forward interest rates react considerably to surprises in macroeconomics data releases and monetary policy announcements. They use a model with a slow-moving inflation target to better match these empirical facts. We now show that our model yields a strong contemporaneous response of the term structure to several macro shocks.

Figure 8 shows the contemporaneous responses of the entire term structure to our five structural shocks. The AS shock shifts the short end of the yield curve but has virtually no effect on yields of maturities beyond ten quarters. Our model predicts a long-lasting response of bond yields to the IS shock and the shocks to the unobservable macro variables. The IS shock produces an upward shift in the entire term structure, but affects more strongly the yields of maturities close to one year, leading to a hump-shaped response. The IS and natural rate shocks have inverse but symmetric effects on the term structure. While the IS shock shifts the term structure upwards, the natural rate shifts the term structure down for maturities up to 5 years. This is to be expected as the IS shock is a demand shock, whereas the natural rate shock is essentially a supply shock.

The monetary policy shock shifts the short end of the curve upward but it has a negative, if small, contemporaneous effect on yields of maturities of five quarters and higher. Gürkaynak, Sack, and Swanson (2005) also show this pattern, but in their exercise, the monetary policy shock starts having a negative effect on bond rates at a longer maturities. Our result is again due to the interest rate undershooting in response to the monetary policy shock. Since the Expectations Hypothesis holds,

future expected decreases in short-term rates imply declines of medium and long-term rates. Finally, the inflation target shock produces a very persistent, strong and hump-shaped positive response of the entire term structure. As agents perceive a change in the monetary authority's stance, they adjust their inflation expectations upwards so that interest rates increase at all maturities.

Note that the sensitivity of long rates to the inflation target, the natural rate and the IS shocks remains very strong even at maturities of 10 years. Ellingsen and Söderstrom (2004) and Gürkaynak, Sack, and Swanson (2005) show that their structural macro models can explain the sensitivity of long-rates to structural macro shocks. While their models use several lags of the macro variables in the AS, IS and inflation target equations to generate additional persistence, our model can account for the variability of the long rates with a parsimonious VAR(1) specification. More importantly, whereas Ellingsen and Söderstrom (2004) stress the importance of the monetary policy shock, in our model the IS shock and the shocks to the unobservable macro variables are much more important in explaining the sensitivity of the long rates than the monetary policy shock is.

7 Conclusions

The first contribution of our paper is to use a no-arbitrage term structure model to help identify a standard New-Keynesian macro model with additional unobservable factors. Whereas there are many possible implementations of our framework, in this article we introduce the natural rate of output and time-varying inflation target in an otherwise standard model.

From a macroeconomic perspective, our contribution is that we use term structure information to help identify structural macroeconomic and monetary policy parameters and at the same time match the persistence of the key macro variables. From a

finance perspective, our contribution is that we derive a no-arbitrage tractable term structure model where all the factors obey New-Keynesian structural relations.

Our key findings are as follows. First, our structural estimation identifies a large Phillips curve parameter and a large response of output to the real interest rate. Second, the inflation target shock accounts for most of variation in the level factor whereas the monetary policy shocks dominate the variation in slope and curvature factors.

There are a number of avenues for future work. First, the finance literature has stressed the importance of stochastic risk aversion in helping to explain salient features of asset returns (see Campbell and Cochrane (1999) and Bekaert, Engstrom, and Grenadier (2006)). Dai and Singleton (2002) show how time-varying prices of risk play a critical role in explaining deviations of the Expectations Hypothesis for the U.S. term structure. However, their model has no structural interpretation. Piazzesi and Swanson (2004) find risk premiums in federal funds futures rates which appear counter-cyclical. A follow-up paper will explore the effect of stochastic risk aversion on our findings. Second, Diebold, Rudebusch, and Aruoba (2006) find that macro factors have strong effects on future movements in interest rates and that the reverse effect is much weaker. Our model actually fits this pattern of the data, but we defer a further analysis of the structural origin of these interactions to future work.

Appendix

We now derive the VARMA(3,2) type representation of the three observable macro variables implied by the five state variable dynamics. For simplicity, we work with a demeaned system.

Let $x_{1,t} = [\pi_t \ y_t \ i_t]'$ and $x_{2,t} = [y_t^n \ \pi_t^*]'$. Let $v_t = \Gamma\epsilon_t$ be the vector of the five reduced-form errors. Using the lag operator L , the reduced-form model can be decomposed as $x_{1,t} = \Omega_{11}Lx_{1,t} + \Omega_{12}Lx_{2,t} + v_{1,t}$ and $x_{2,t} = \Omega_{21}Lx_{1,t} + \Omega_{22}Lx_{2,t} + v_{2,t}$. Note that both $v_{1,t}$ and $v_{2,t}$ are functions of all five structural shocks. The task is then to substitute out $x_{2,t}$ in the first equation in terms of $x_{1,t}$ and $v_{2,t}$ as $x_{2,t} = (I_2 - \Omega_{22}L)^{-1}(\Omega_{21}Lx_{1,t} + v_{2,t})$, where $(I_2 - \Omega_{22}L)^{-1}$ can be expressed as:

$$\begin{aligned} (I_2 - \Omega_{22}L)^{-1} &= (1 - \omega_{nn}L)(1 - \omega_{pp}L) - \omega_{np}\omega_{pn}L^2)^{-1} \left(\begin{bmatrix} (1 - \omega_{pp}L) & \omega_{np}L \\ \omega_{pn}L & (1 - \omega_{nn}L) \end{bmatrix} \right) \\ &= d(L)^{-1}(I_2 - BL) \end{aligned}$$

where $\Omega_{22} = \begin{bmatrix} \omega_{nn} & \omega_{np} \\ \omega_{pn} & \omega_{pp} \end{bmatrix}$, $B = \begin{bmatrix} \omega_{pp} & -\omega_{np} \\ -\omega_{pn} & \omega_{nn} \end{bmatrix}$, $d(L) = 1 - b_1L + b_2L^2$, $b_1 = \omega_{pp} + \omega_{nn}$, $b_2 = \omega_{pp}\omega_{nn} - \omega_{pn}\omega_{np}$. Therefore $x_{1,t}$ can be expressed as a linear function of its lags and the reduced-form residual vectors as:

$$x_{1,t} = \Phi_1x_{1,t-1} + \Phi_2x_{1,t-2} + \Phi_3x_{1,t-3} + \Psi_0\epsilon_t + \Psi_1\epsilon_{t-1} + \Psi_2\epsilon_{t-2},$$

where $\Phi_1 = b_1I_3 + \Omega_{11}$, $\Phi_2 = -b_2I_3 - b_1\Omega_{11} + \Omega_{12}\Omega_{21}$, $\Phi_3 = b_2\Omega_{11} - \Omega_{12}B\Omega_{21}$, $\Psi_0 = [I_3 \ 0_{3 \times 2}]\Gamma$, $\Psi_1 = [-b_1I_3 \ \Omega_{12}]\Gamma$, $\Psi_2 = [b_2I_3 \ -\Omega_{12}B]\Gamma$.

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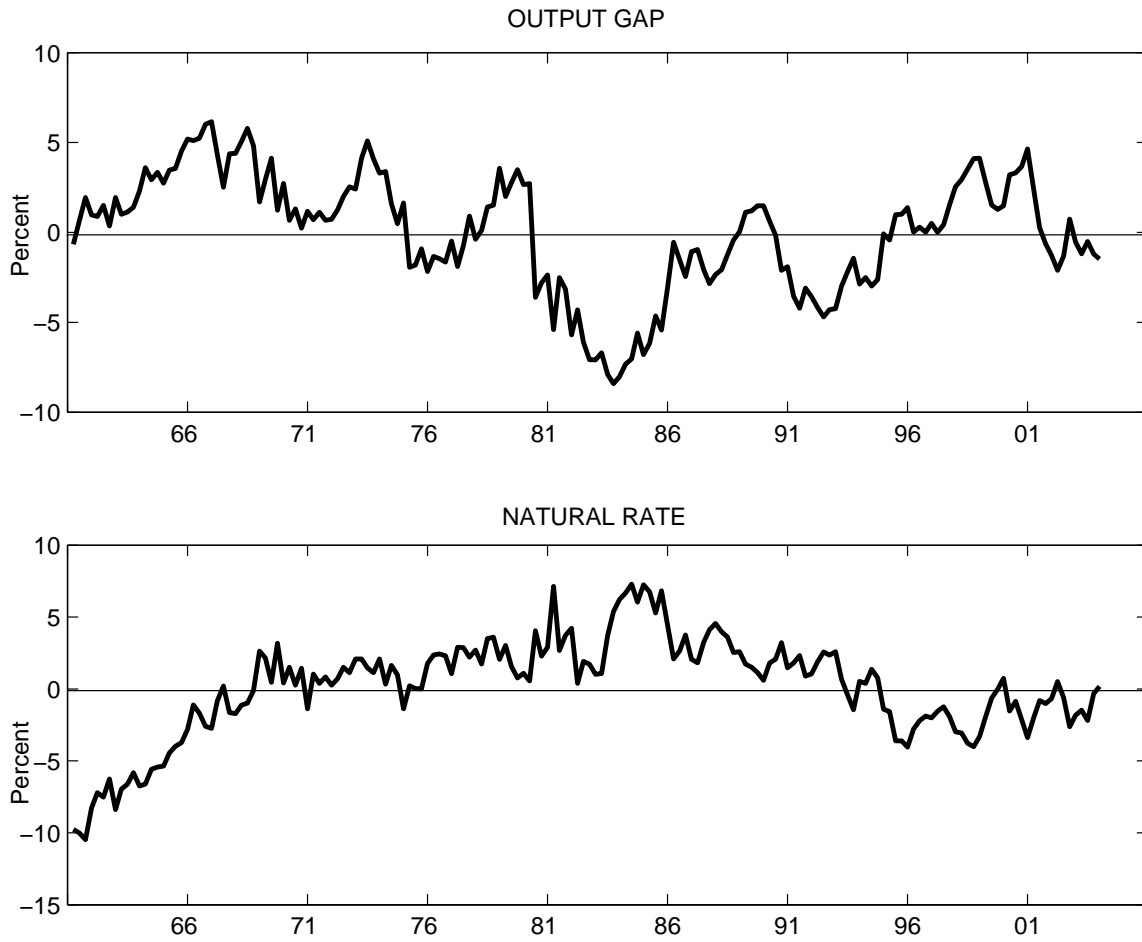
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Table 1: **GMM Estimates of the Structural Parameters**

Parameter	Estimate	Std Error	
		GMM	Bootstrap
δ	0.611	(0.010)	(0.031)
κ	0.064	(0.007)	(0.022)
σ	3.156	(0.466)	(1.632)
η	4.294	(0.470)	(1.383)
ρ	0.723	(0.028)	(0.083)
β	1.525	(0.148)	(0.251)
γ	0.001	(0.047)	(0.020)
λ	0.958	(0.006)	(0.026)
ω	0.877	(0.013)	(0.031)
d	0.866	(0.014)	(0.041)
σ_{AS}	1.249	(0.053)	(0.123)
σ_{IS}	0.671	(0.033)	(0.055)
σ_{MP}	2.177	(0.119)	(0.287)
σ_{y^n}	1.380	(0.115)	(0.817)
σ_{π^*}	0.730	(0.059)	(0.723)
Implied		Std Error	
Parameter	Estimate	GMM	Bootstrap
μ	0.424	(0.013)	(0.026)
ϕ	0.134	(0.017)	(0.029)
φ_1	0.500	(0.003)	(0.007)
φ_2	0.492	(0.003)	(0.005)
φ_3	0.008	(0.002)	(0.003)

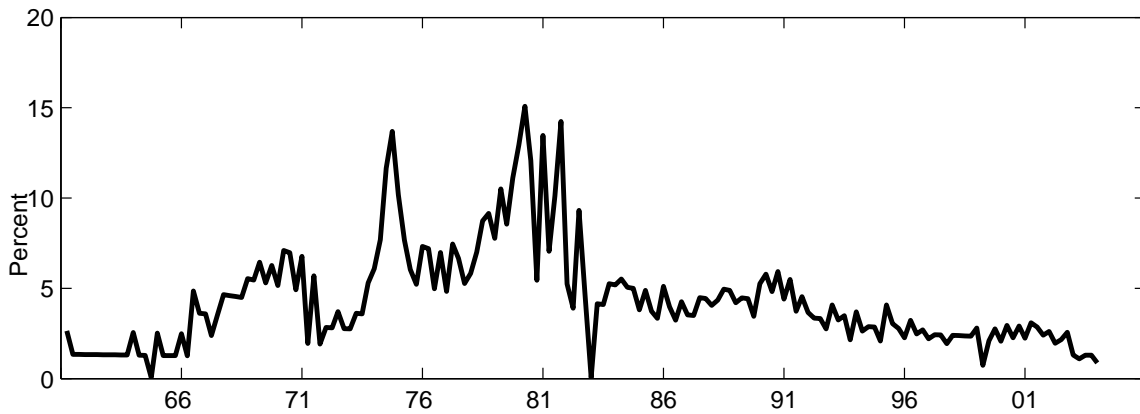
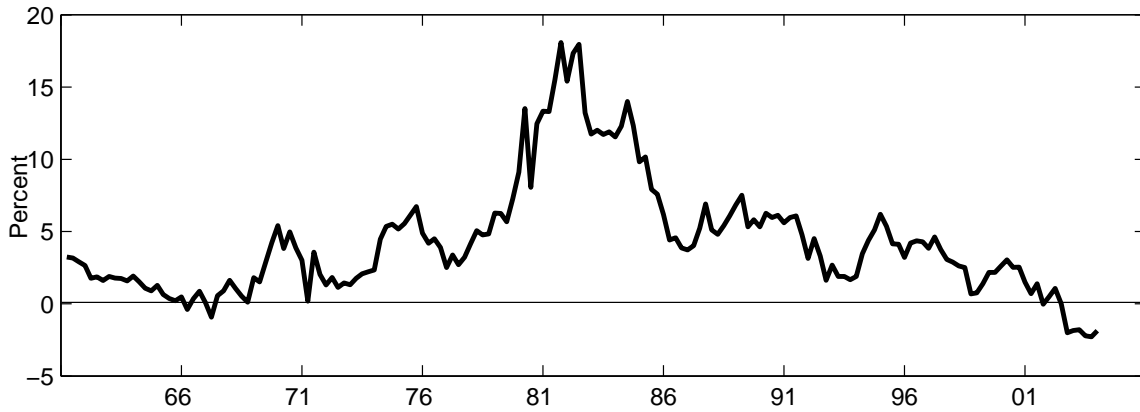
The second column reports the parameter estimates for our macro-finance model. The third and fourth columns list the GMM standard errors of the structural parameters and those obtained through the bootstrap procedure described in the text.

Figure 1: Output Gap and Natural Rate of Output



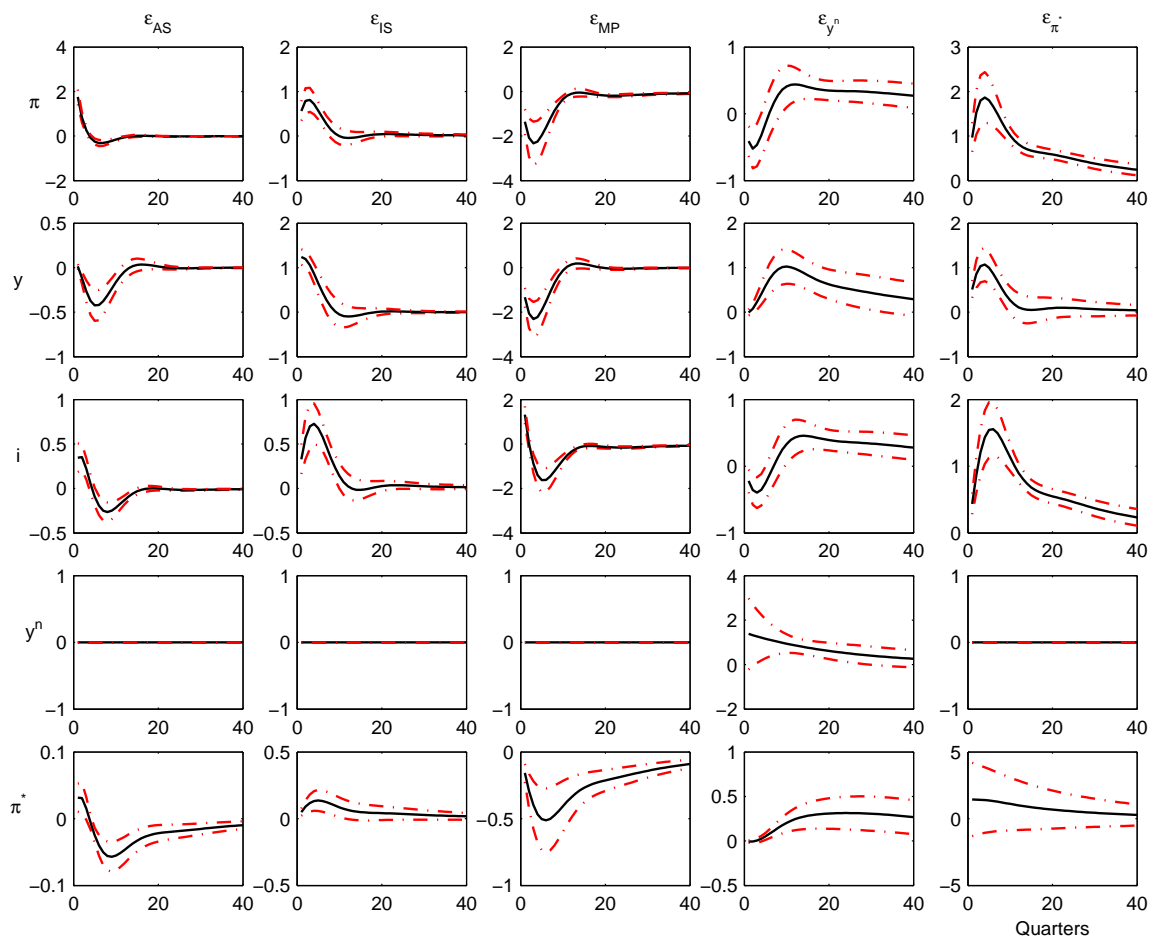
The top panel shows the output gap implied by the New-Keynesian model for our sample period: 1961:1Q-2003:4Q. The bottom panel shows the filtered natural rate of output.

Figure 2: Inflation Target and Inflation



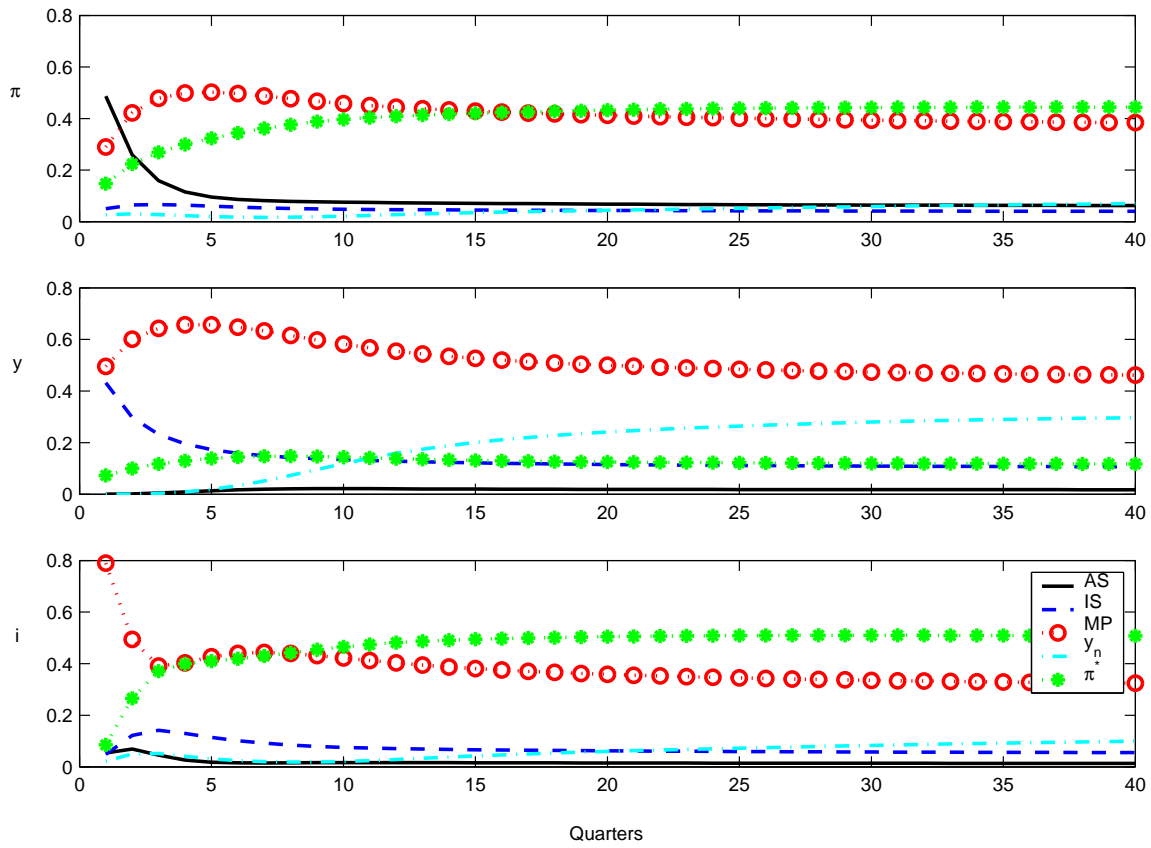
The top panel shows the inflation target implied by the New-Keynesian model for our sample period: 1961:1Q-2003:4Q. The bottom panel shows the CPI inflation rate.

Figure 3: Impulse Response Functions of Macro Variables



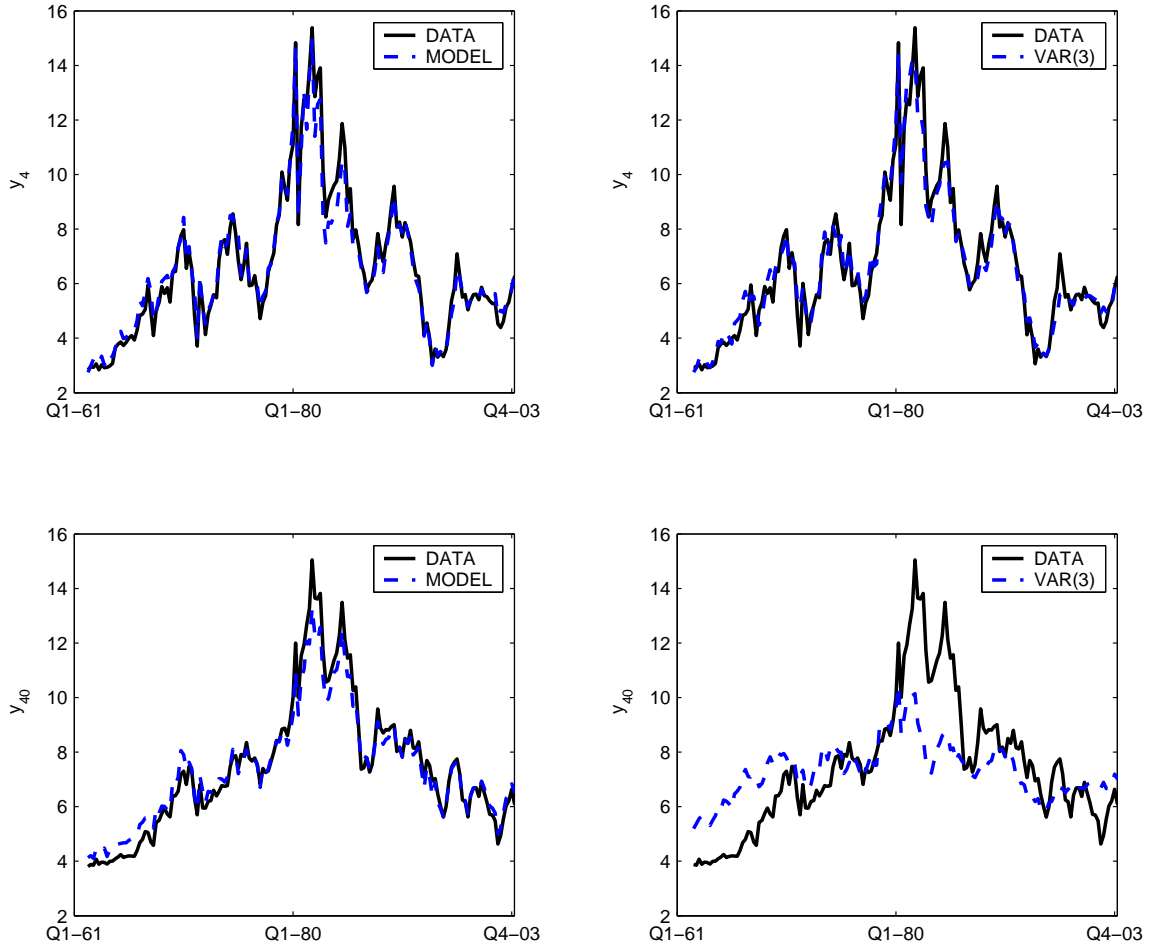
This figure shows the impulse response functions (in percentage deviations from steady state) of the five macro variables to the structural shocks. 95% confidence intervals appear in dashed lines and were constructed using the bootstrap procedure described in the text.

Figure 4: Variance Decompositions for the Macro Variables



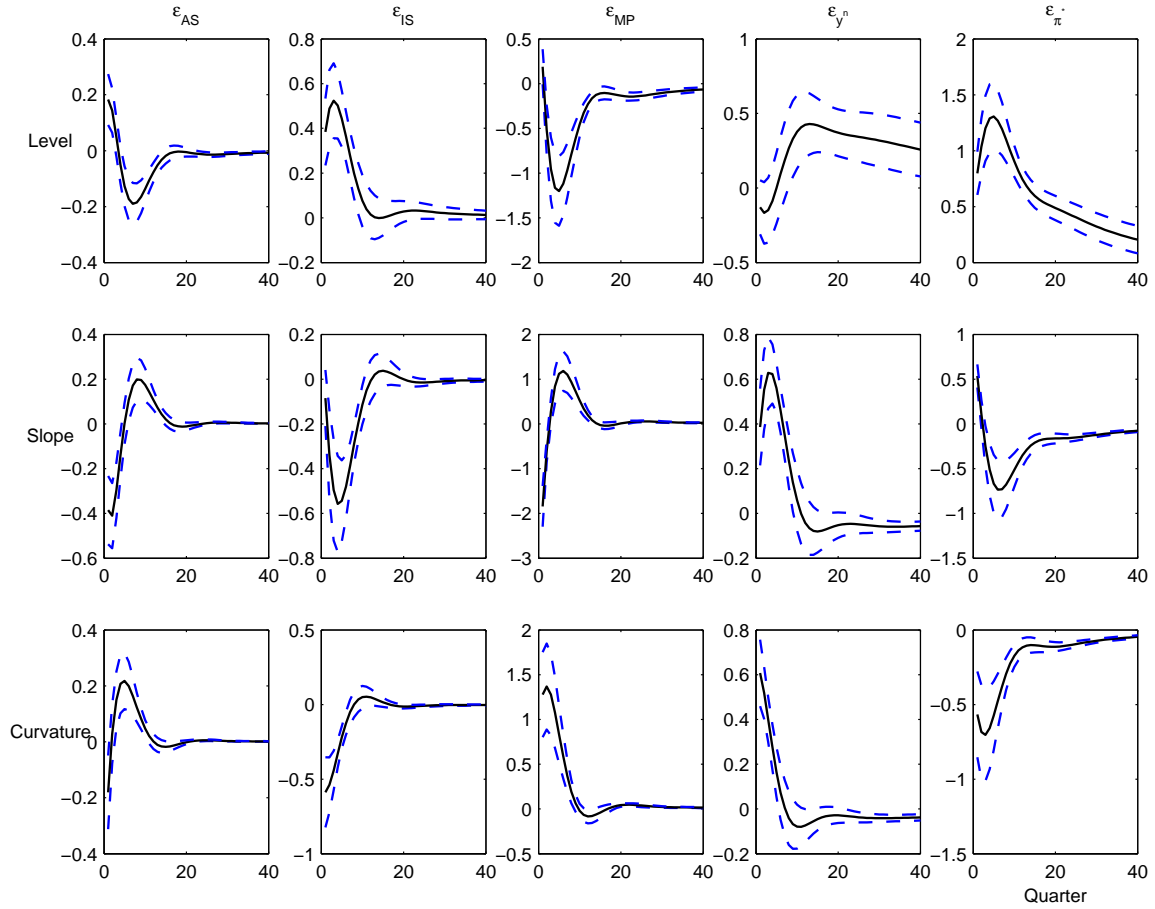
This figure shows the variance decomposition at different time horizons for the macro variables in terms of the five structural macro shocks. The variance decomposition of a variable at quarter h represents the percentage of the h -step forecast variance explained by each shock.

Figure 5: Fit for Yields



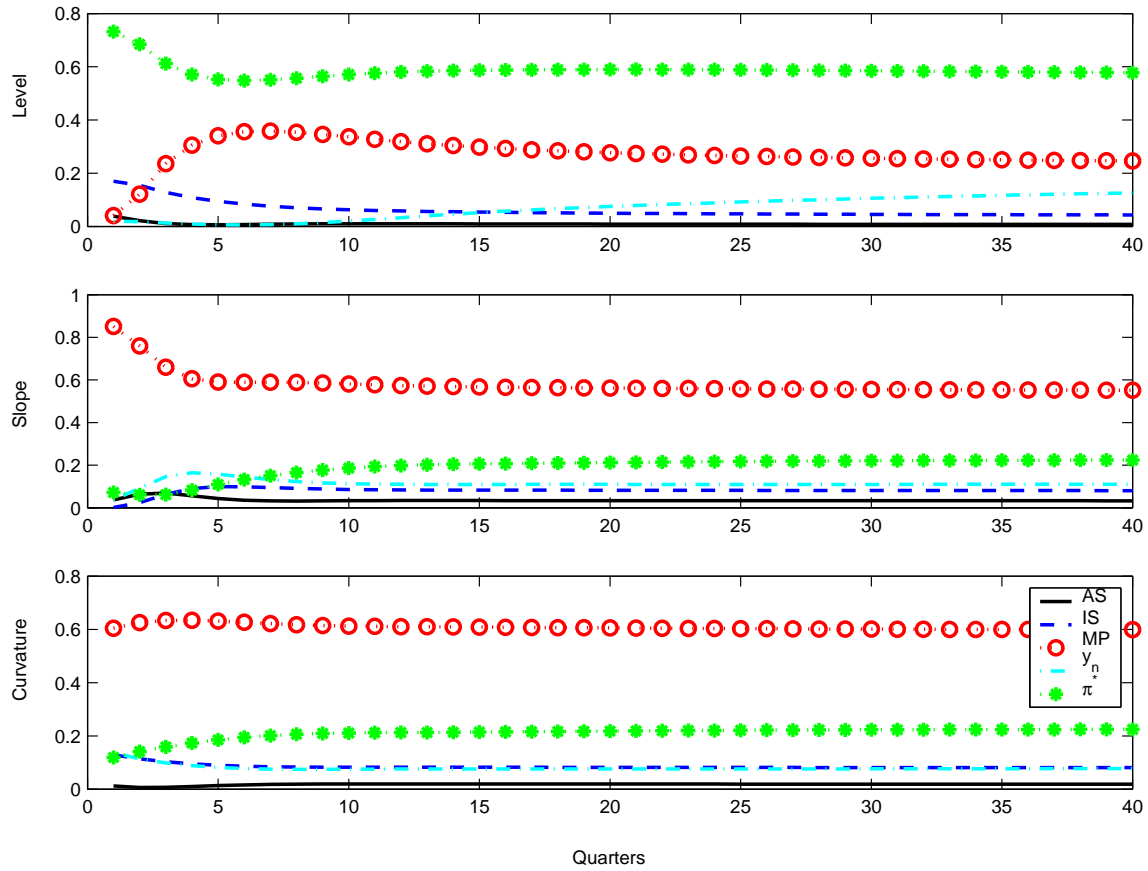
This figure graphs the fit of the 1 (y_4) and 10 year yields (y_{40}) implied by our macro-finance model (on the left) and those implied by a VAR(3) (on the right) for our sample period: 1961:1Q-2003:4Q. Each graph also plots the actual time series (in bold).

Figure 6: Impulse Responses of Term Structure Factors to Macro Shocks



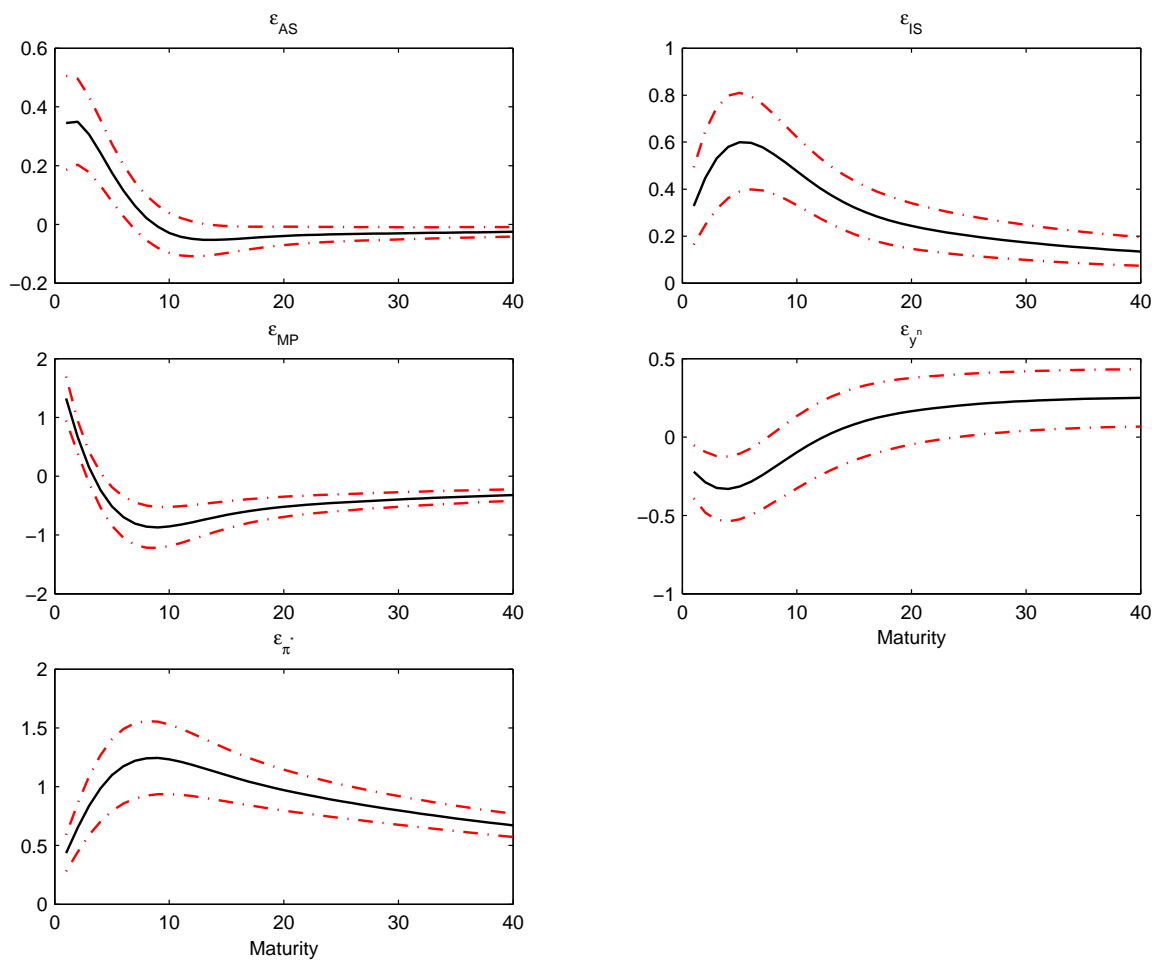
This figure shows the impulse response functions (in percentage deviations from steady state) of the three term structure factors - level, slope and curvature - to the structural shocks. 95% confidence intervals appear in dashed lines.

Figure 7: Variance Decompositions for the Term Structure Factors



This figure shows the variance decomposition of the term structure factors in terms of the five structural macro shocks. The variance decomposition of a variable at quarter h represents the percentage of the h -step forecast variance explained by each shock.

Figure 8: Contemporaneous Responses of Yields to Macro Shocks



This figure shows the contemporaneous responses of yields of different maturities to the five structural macro shocks. 95% confidence bands appear in dashed lines and were constructed using the bootstrap procedure described in the text.