

Investment under Uncertainty with Strategic Debt Service

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We integrate the financial architecture into the theory of investment by building on two strands of literature: irreversible investment and debt pricing/capital structure. We extend the real options approach to investment, pioneered by Michael J. Brennan and Eduardo S. Schwartz (1985) and Robert McDonald and Daniel Siegel (1986), to allow for capital structure decisions under strategic debt service.¹ We also draw insights from corporate debt pricing/capital structure literature, which focuses on leverage and security pricing after investment has already been made (Robert C. Merton 1974; Hayne E. Leland 1994). Our paper shows that the *interaction* between financing and investment decisions in the presence of strategic debt service generates new insights and also significant *quantitative* effects on ex ante firm value. We show that stronger equity holders' bargaining power lowers debt capacity, reduces firm value, and discourages growth option exercising.

I. Model Setup and Solution

The firm observes its potential earnings before interests and taxes (EBIT) given by the following geometric Brownian motion (GBM) process:

$$(1) \quad dX_t = \mu X_t dt + \sigma X_t dW_t,$$

where W is a standard Brownian motion. The firm decides when to exercise its growth option by paying the fixed investment cost I , and then collecting the stochastic stream of X . Let $r > 0$ denote the risk-free interest rate. Assume $r > \mu$

for convergence. Let the corporate tax rate be $\tau > 0$. After-tax, all-equity financed firm value $\Pi(x)$ is given by

$$(2) \quad \Pi(x) = \left(\frac{1 - \tau}{r - \mu} \right) x, \quad x \geq 0.$$

Debt has tax benefits. The firm chooses a mixture of equity and (risky) debt to finance the investment cost I at endogenously chosen investment time T_i . The firm takes advantage of the tax benefits, but faces a conflict of interest between equity holders and debt holders after debt is in place (Stewart C. Myers 1977). Assume that the risky debt is perpetual (Leland 1994).

After investing and issuing the risky debt at T_i , equity holders may renege on the contractual debt payments and declare default at their chosen time, when X is sufficiently low. We follow Leland (1994) to assume that the firm's liquidation value is $(1 - \alpha)\Pi(x)$, a fraction $(1 - \alpha)$ of the unlevered after-tax firm value $\Pi(x)$. In Leland (1994), when equity holders do not make promised debt payments, debt holders will liquidate the firm and collect $(1 - \alpha)\Pi(x)$ from liquidation. In our model, when equity holders threaten to default, debt holders may not want to liquidate the firm, and instead renegotiate with equity holders the terms of the debt contract (Ronald W. Anderson and Sundaresan 1996). Intuitively, there are gains to be realized and divided between equity holders and debt holders by renegotiation, when the firm, as a going concern, is worth more than its liquidation value. The division of the surplus generated from avoiding costly liquidation between equity holders

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¹Avinash K. Dixit and Robert S. Pindyck (1994) provide a standard textbook treatment on real options approach toward investment. Andrew B. Abel and Janice C. Eberly (1994) develop a unified framework integrating the neoclassical adjustment cost literature with the literature on

irreversible investment. The insights of this (real options) literature, which were developed in an all-equity financing framework, have been extended to settings with debt financing. Without exception, the extensions have relied on numerical procedures to draw out the relationship between optimal investment and financing decisions. See Antonio S. Mello and John E. Parsons (1992), David C. Mauer and Alexander J. Triantis (1994), Christopher A. Hennessy and Toni M. Whited (2005), and Grzegorz Pawlina (2005) for examples of research on investment with debt financing.

and debt holders depends on their relative bargaining powers. Let η and $(1 - \eta)$ denote their respective bargaining powers. We model the renegotiation between equity holders and debt holders via a Nash bargaining game as in Hua Fan and Sundaresan (2000).

Let $E_0(x)$ denote firm value before investment. The firm chooses the optimal investment threshold x_i , and the optimal coupon policy c to maximize $E_0(x)$, anticipating the possible renegotiation in the future. The following proposition provides the closed-form solutions for various decision rules as functions of structural parameters in the model.

PROPOSITION 1: *The firm’s investment threshold x_i is given by*

$$(3) \quad x_i = \frac{\beta}{\beta - 1} \left(\frac{r - \mu}{1 - \tau} \right) \times \left(1 + \tau \frac{1 - \eta\alpha}{1 - \tau(1 - \eta)} \frac{1}{g} \right)^{-1} I,$$

where $g > 1$ is a constant and is given by

$$(4) \quad g = \left[\frac{\beta}{\beta - \gamma} (1 - \gamma) \right]^{-1/\gamma},$$

and $\beta > 1$ and $\gamma < 0$ are the two roots of $\sigma^2 z^2/2 + (\mu - \sigma^2/2)z - r = 0$. The coupon for debt (in the “normal” region $x \geq x_s$) is given by

$$(5) \quad c = r \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} \times \left(g \frac{1 - \tau(1 - \eta)}{1 - \eta\alpha} + \tau \right)^{-1} I.$$

Equity holders strategically renegotiate with debt holders, whenever $X(t) \leq x_s$, where x_s is the endogenously determined strategic renegotiation threshold and is given by

$$(6) \quad x_s = \frac{x_i}{g} = \frac{\beta}{\beta - 1} \left(\frac{r - \mu}{1 - \tau} \right) \times \left(g + \tau \frac{1 - \eta\alpha}{1 - \tau(1 - \eta)} \right)^{-1} I.$$

The reduced coupon payment in the “renegotiation” region ($x < x_s$) is given by

$$(7) \quad S(x) = (1 - \eta\alpha)(1 - \tau)x, \quad x \leq x_s.$$

Note that the investment threshold x_i , the renegotiation threshold x_s , and the (contractual) coupon payment c are all proportional to the growth option exercising cost I . These results are due to the GBM assumption for the EBIT process (1), perpetual debt, and perpetual renegotiation and investment options, among others. Note that the ratio between the investment threshold x_i and the renegotiation threshold x_s is constant and is larger than unity, $x_i/x_s = g > 1$, where g is given in (4). Moreover, the ratio $x_i/x_s = g$ is independent of the bargaining power η .

An important implication of Proposition 1 is that the inefficiency of costly liquidation (captured by α) directly enters into the determination of the optimal investment threshold x_i , the optimal leverage c , and the debt concessions ($c - S(x)$), even though liquidation merely acts as a credible threat and does not occur in equilibrium.

II. Model Analysis and Predictions

Our model features the interaction between the investment and the financing decisions. Unlike the standard all equity-based real options models, the investment decision in our model is fundamentally tied to the financing friction induced by the strategic renegotiation between debt holders and equity holders in the future. Unlike Leland (1994) and other credit risk/contingent claim structural models, which may be viewed as models for financing of assets in place, our paper studies financing of growth option exercising. The next proposition characterizes the properties of the optimal renegotiation threshold x_s , the coupon payment c , and the optimal investment threshold x_i in our model with respect to equity holders’ bargaining power η .

PROPOSITION 2: *Both the renegotiation threshold x_s given in (6) and the optimal investment threshold x_i given in (3) increase in equity holders’ bargaining power η : $dx_i/d\eta > 0$ and $dx_s/d\eta > 0$. The optimal coupon payment c given in (5) decreases in equity holders’ bargaining power η : $dc/d\eta < 0$.*

When equity holders' bargaining power is stronger, they can extract more out of the surplus from renegotiation, and debt holders anticipate higher reductions of the contractual coupon payments in the renegotiation region. This suggests that the renegotiation threshold x_s increases with η . A higher renegotiation threshold x_s lowers tax benefits, *ceteris paribus*. This implies that debt capacity and the optimal coupon level c decrease with equity holders' bargaining power η , *ceteris paribus*. Hence, incentives to invest decrease in η , and the firm waits longer before exercising its growth option (a higher threshold x_i), when η is higher.

Next, we characterize the effects of equity holders' bargaining power (the value of η) on ex ante firm (equity) value $E_0(x)$, and compare with two natural benchmark settings: one setting with all equity financing and $\tau = 0$, and one setting with all equity financing and taxes ($\tau > 0$). Let x_i^* and $E_0^*(x)$ denote the investment threshold and the ex ante firm value when $\tau = 0$ (the first benchmark). Let x_i^{ae} and $E_0^{ae}(x)$ denote the investment threshold and the ex ante firm value under all equity financing and with taxes (the second benchmark). Taxes weaken the incentives of investment and lower equity values, in that $x_i^{ae} > x_i^*$ and $E_0^{ae}(x) < E_0^*(x)$, for all values of x . In the presence of taxes, issuing debt alleviates the investment distortions induced by taxes. Therefore, the investment threshold x_i under equity/debt financing is lower than x_i^{ae} , in that $x_i < x_i^{ae}$. Moreover, firm value is higher under optimal financing than under equity financing ($E_0(x) > E_0^{ae}(x)$, for all x). The next proposition summarizes the results on investment thresholds and firm values $E_0(x)$ under different financing arrangements.

PROPOSITION 3: *The investment threshold x_i under optimal financing satisfies the inequality $x_i^* < x_i < x_i^{ae}$. Payoffs at different times of exercising the growth option under all three settings are equal. Finally, ex ante firm value $E_0(x)$ satisfies $E_0^*(x) > E_0(x) > E_0^{ae}(x)$, for all x .*

Figure 1 plots ex ante firm value $E_0^*(x)$ under the benchmark setting (all equity financing and $\tau = 0$), firm value $E_0(x)$ under optimal financing (with $\eta = 0.5$ and $\tau > 0$), and $E_0^{ae}(x)$ under all equity financing ($\tau > 0$). The horizontal line shows that the payoffs (at different endoge-

nously chosen investment times) under all three settings are equal, confirming the results in Proposition 3. Our intuition relies on the following observation. First, the present discounted value of receiving a unit payoff contingent on hitting the investment threshold x_i is $\Phi(x; x_i) = (x/x_i)^\beta$ for $x < x_i$. Note that $\Phi(px; px_i) = (x/x_i)^\beta$ for any constant $p > 0$, provided that X follows a GBM process (1). Second, we may show that the gross payoffs, upon exercising the growth option at any given candidate threshold level x_i , are equal to px_i , and proportional to x_i , under all three settings with different financing. Therefore, by optimally exercising the growth option, equity holders *optimally* choose the investment threshold x_i by setting $px_i = \beta I / (\beta - 1)$. The net payoffs upon investment at different investment times under the three settings are thus all equal to $px_i - I = I / (\beta - 1)$. Because ex ante firm value is given by the product of $\Phi(x; x_i) = (x/x_i)^\beta$ and the net payoffs $I / (\beta - 1)$, we have the result that the ordering of the ex ante firm value $E_0(x)$ is determined by the ordering of $(1/x_i)^\beta$. Figure 1 displays the ordering of the investment thresholds $x_i^{ae} > x_i > x_i^*$.

Next, we analyze the impact of strategic renegotiation (measured by the degree of equity holders' bargaining power η) on ex ante firm value $E_0(x)$ before its growth option is exercised. We compare the model's predictions with two previously constructed benchmark settings (under all equity financing): one without taxes and the other with $\tau > 0$. Table 1 reports the effects of equity holders' bargaining power η on ex ante firm value $E_0(x)$ scaled by firm values under comparison benchmark settings. Here, we choose the initial value x_0 to be in the waiting region ($x_0 \leq x_i^*$). The second row shows that taxes substantially lower firm value $E_0(x)$ by 17 percent to 39 percent of $E_0^*(x)$, the corresponding firm value under equity financing with $\tau = 0$. The loss of firm value is greater when equity holders' bargaining power is stronger (a larger η). Intuitively, a higher bargaining power gives equity holders more incentives to engage in ex post opportunistic behavior, and hence lowers ex ante debt capacity and firm value more, *ceteris paribus*. The third row shows that firm value $E_0(x)$ is substantially higher than $E_0^{ae}(x)$, firm value under all equity financing and $\tau > 0$. The gap between $E_0(x)$ and $E_0^{ae}(x)$ measures the net benefits of debt. When equity

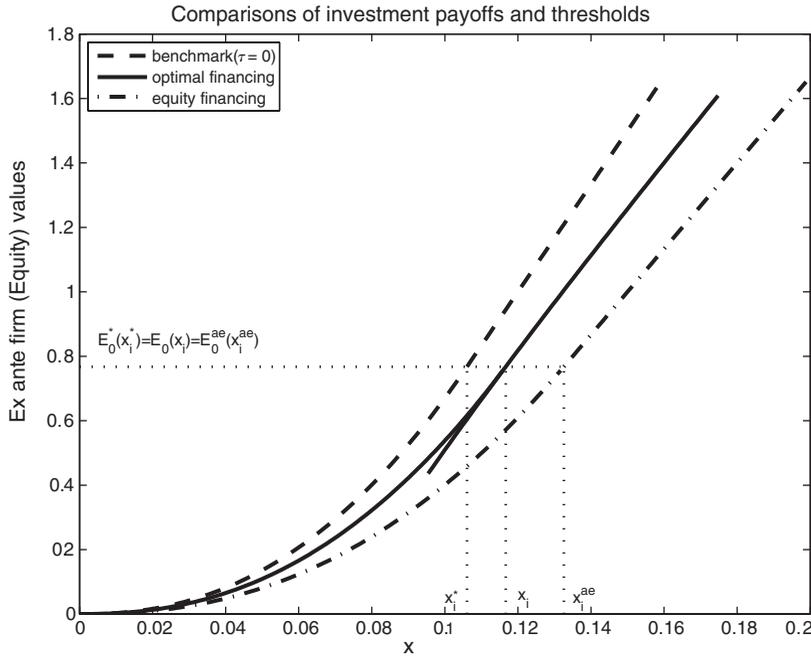


FIGURE 1. EQUITY VALUES $E_0(x)$ AND THE INVESTMENT THRESHOLDS UNDER ALL EQUITY FINANCING (WITH $\tau = 20$ PERCENT), (OPTIMAL) DEBT FINANCING (WITH $\eta = 0.5$ AND $\tau = 20$ PERCENT), AND ALL EQUITY FINANCING (WITH $\tau = 0$)

Notes: The respective investment thresholds are ordered sequentially: $x_i^* < x_i < x_i^{ae}$. Payoffs at (different) exercising thresholds are equal under the three settings, in that $x_i^*/(r - \mu) = V_a(x_i) = (\Pi x_i^{ae})$, as seen from the horizontal dashed line. Equity value $E_0(x)$ under benchmark (with all equity financing and $\tau = 0$) is highest. Equity value $E_0(x)$ under all equity financing (with $\tau > 0$) is lowest. Equity value $E_0(x)$ under (optimal) debt/equity financing lies between the two equity values under equity financing (with $\tau = 0$ and with $\tau > 0$). The (solid) concave curve $V_a(x) - I$ is the payoff under debt financing from exercising, where $V_a(x)$ is the firm value (in the normal region) after investing. Parameter values: $\alpha = 35$ percent, $r = 6$ percent, $\tau = 20$ percent, $\mu = 0$, $\sigma = 20$ percent, and $I = 1$.

holders have no bargaining power, allowing the firm to choose its optimal leverage at the time of investment increases ex ante firm value by 89 percent! There are two effects contributing to this magnitude of value increase. First, the investment threshold under $\eta = 0$ is equal to $x_i = 0.099$, about 80 percent of the all-equity investment threshold $x_i^{ae} = 0.123$. Second, in order to convert the investment threshold ratio x_i/x_i^{ae} into the value ratio, we need to compute the ratio $\Phi(x; x_i)/\Phi(x; x_i^{ae}) = (x_i^{ae}/x_i)^\beta$, where $\beta = 2.86$ for this calibration. (We use Proposition 3, which states the investment payoffs at different investment thresholds are equal across all settings under our analysis.) Intuitively, the option

feature (captured by β) plays an important role in driving up the investment threshold difference about 20 percent to an ex ante firm value difference of about 89 percent. Interestingly, the growth option feature makes the difference in firm value $E_0(x)$ even greater than the difference for the corresponding investment thresholds.

To understand the effect of renegotiation on ex ante firm value $E_0(x)$, we compare our model with the one-growth option setting, where renegotiation is ruled out, as in Sundaresan and Wang (2006). In that paper, the firm chooses optimal capital structure to finance the exercise of the growth option, and the equity holders make the default decision, as in Leland (1994), in that

TABLE 1—THE EFFECTS OF BARGAINING POWER η ON FIRM VALUE $E_0(x)$,
 COMPARED WITH ALL EQUITY-BASED BENCHMARK SETTINGS

η	0	0.2	0.4	0.6	0.8	1.0
$\frac{E_0(x_0)}{E_0^*(x_0)} - 1$	-17%	-23%	-28%	-32%	-36%	-39%
$\frac{E_0(x_0)}{E_0^{ae}(x_0)} - 1$	89%	75%	64%	54%	46%	39%
$\frac{E_0(x_0)}{E_0^l(x_0)} - 1$	29%	19%	12%	5%	-1%	-6%
Investment threshold x_i	0.099	0.101	0.103	0.106	0.108	0.110
Renegotiation threshold x_s	0.073	0.075	0.077	0.079	0.080	0.082
Coupon c	0.113	0.101	0.090	0.080	0.070	0.061

Notes: This table reports $E_i(x_0)$ as a fraction of firm value under three comparison benchmarks: (a) all equity without taxes, (b) all equity with taxes, and (c) optimal debt/equity financing without renegotiation (as in Leland 1994, and Sundaresan and Wang 2006). We choose the initial value to be in the waiting region. For example, any $x_0 \leq x_i^*$ works. Benchmark parameter values: $\alpha = 35$ percent, $r = 6$ percent, $\tau = 25$ percent, $\mu = 0$, $\sigma = 15$ percent, and $I = 1$. Under the benchmark with all equity financing and no taxes, the first-best investment threshold is 0.092. Under the benchmark with all equity financing and $\tau > 0$, the investment threshold is 0.123.

costly liquidation occurs in equilibrium. The fourth row in Table 1 reports firm value $E_0(x)$ as a percentage of firm value $E_0^l(x)$ (without renegotiation) in Sundaresan and Wang (2006). While renegotiation increases firm value by avoiding costly ex post liquidation, renegotiation also induces a cost on ex ante firm value $E_0(x)$ from equity holders' ex post strategic renegotiation, as discussed earlier.

When equity holders' bargaining power η is low, firm value under renegotiation is higher than under costly liquidation. Intuitively, the benefit of avoiding costly liquidation outweighs equity holder's ex post opportunistic behavior. As a result, allowing equity holders to renegotiate with debt holders ex post enhances firm value. For example, firm value is increased by 29 percent by allowing for renegotiation, if equity holders have no bargaining power. When equity holders' bargaining power η is high, however, ex ante firm value $E_0(x)$ under future renegotiation may be lower than ex ante firm value $E_0^l(x)$ under potentially costly liquidation, as in Sundaresan and Wang (2006). Intuitively, the cost of equity holders' ex post opportunistic behavior dominates the benefit of avoiding costly liquidation. For example, when equity holders have all the bargaining power ($\eta = 1$), firm value $E_0(x)$ is 6 percent lower than firm value under costly liquidation, as seen in Table 1.

Finally, the last three rows in Table 1 confirm our comparative statics results reported in Proposition 2. The fifth and the six rows in Table 1 show that both the investment threshold x_i and the renegotiation threshold x_s increase with equity holders' bargaining power η . The last row shows that the coupon payment c decreases with η .

III. Conclusions

We have provided a parsimonious framework to model the role of financial architecture on ex ante growth option exercising decisions and firm value when debt offers tax benefits. A key ingredient in our paper is the ex post bargaining and renegotiation between equity holders and debt holders. We show that stronger equity holders' bargaining power lowers debt capacity, reduces firm value, and delays growth option exercising.

We suggest two important extensions for future work. First, the framework can be used to model Chapter 11 features of the bankruptcy code and the contingent transfer of control rights from borrowers to lenders when the firm files for Chapter 11. The other is to model the financing and default decisions of a firm which optimally exercise growth options *sequentially* over time (Sundaresan and Wang 2006).

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