

# Organizational Capital, Corporate Leadership, and Firm Dynamics.

Wouter Dessein and Andrea Prat  
Columbia University\*

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## Abstract

We argue that economists have studied the role of management from three perspectives: contingency theory (CT), an organization-centric empirical approach (OC), and a leader-centric empirical approach (LC). To reconcile these three perspectives, we augment a standard dynamic firm model with *organizational capital*, an intangible, slow-moving, productive asset that can only be produced with the direct input of the firm's leadership and that is subject to an agency problem. We characterize the steady state of an economy with imperfect governance, and show that it rationalizes key findings of CT, OC, and LC, as well as generating a number of new predictions on performance, management practices, CEO behavior, CEO compensation, and governance.

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# 1 Introduction

A number of empirical studies, exploiting different data sets, employing different methodologies, and covering different countries have found sizeable and persistent performance differences between firms that operate in the same industry and use similar observable input factors (Syverson 2011). For instance, within narrowly specified US manufacturing industries, establishments at the 90th percentile make almost twice as much output with the same input (Syverson 2004).

One possible explanation for this puzzling observation is that the variation in outcomes is due to a variation in management (Gibbons and Henderson 2013). In turn, management comprises both the management practices that firms put in place and the managerial human capital that they employ. This paper is concerned with the question: Where do differences in management practices and managerial capital come from?

Economists have approached this question from three different angles (summarized in Table 1). The first approach, which we shall refer to as *contingency theory (CT)*, is a natural extension of production theory. Both managerial practices and managerial human capital are production factors and the firm should select them optimally given the business environment it faces. Lucas (1978) is the seminal application of CT to managerial human capital. There is a market for managers where supply is given by a distribution of managers of different talent and demand is given by a distribution of firms. In equilibrium, the more talented managers are employed by the firms that need them more.<sup>1</sup> CT encompasses both managerial talent and management practices, and it can take into account synergies with other productive factors: Milgrom and Roberts' (1995) theory of complementarity in organizations develops general techniques to model these synergies. CT yields two powerful testable predictions: (i) At any point in time, if the solution to the production problem is unique, similar firms should adopt similar management practices and should hire similar managerial talent; (ii) If the production problem has multiple solutions, similar firms may adopt different management practices and/or hire different managers, but this variation will not correlate with their overall profitability. In order to get heterogeneous performance, the CT set-up can be augmented with exogenous productivity or demand shocks, so it leads to a steady state distribution of

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<sup>1</sup>As such, this model can be used to explain the allocation of CEOs to companies according to firm size (Tervio (2008) and Gabaix and Landier (2008)) or of managerial talent within and across organizations (Garicano and Rossi-Hansberg 2006).

firm size and productivity (Hopenhayn (1992), Ericson and Pakes (1995)). However, in this case the employment of different managers or the adoption of different management practices is an effect not a cause of differential performance: (i) and (ii) still hold.

While CT has an explicit theoretical foundation, the other two approaches are mainly empirical. We will refer to the second one as the *organization-centric empirical approach (OC)*. Ichniowski et al (1997) pioneered this approach in economics. They undertook a detailed investigation of 17 firms in a narrowly defined industry with homogeneous technology (steel finishing) and documented how lines that employed innovative human resource management practices, like performance pay, team incentives, and flexible assignments, achieved significantly higher performance than lines that did not employ such practices. Bloom and Van Reenen (2007) developed a survey tool to measure managerial practices along multiple dimensions. Their influential paper and subsequent work have documented both a large variation in management practices across firms within the same industry and the ability of that variation to explain differences between firms on various performance measures, including profitability.<sup>2</sup> These results are robust to the inclusion of firm-level fixed effects (Bloom et al 2019) and they survive the inclusion of detailed employee-level information (Bender et al 2018).

In sum, OC has shown that similar firms adopt different management practices and that this difference matters for performance. As this finding is in apparent conflict with CT's prediction that management practices are optimally chosen, economists often react in one of two ways. First, those seemingly similar firms may actually have different unobservable characteristics that make it optimal for them to adopt different practices. Second, those firms simply make mistakes and adopt the wrong practices. What both alternative explanations have in common is they offer little in the way of empirical guidance. For the first explanation, it would be useful to have a sense of what kind of firm-level unobservable characteristics we should try to observe, especially given how much information we already have about those firms. For the second, it would be good to have some kind of microfoundation for those highly consequential errors.

The *leadership-centric empirical approach (LC)* focuses on the role of individual managers. Some firms may perform better because they are run by better CEOs. A growing

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<sup>2</sup>Graham et al. (2016), based on surveys of over 1300 CEOs and CFOs of US companies, obtain similar results with respect to the heterogeneity and effectiveness of corporate culture. We view both management practices and corporate culture as being part of a firm's organizational capital.

literature, employing different data sets and different methodologies, show that the identity of the CEO can account for a significant portion of firm performance (Bertrand 2009). Among others, Johnson et al (1985) analyze the stock price reaction to sudden executive deaths, Bertrand and Schoar (2003) identify a CEO fixed effect, and Bennedsen et al (2007) show that family CEOs have a negative causal effect on firm performance. Kaplan et al (2012) document how CEOs differ on psychological traits and how those differences explain the performance of the firms they manage. Bandiera et al (2016) perform a similar exercise on CEO behavior and show it accounts for up to 30% of performance differences between similar firms, and the association between behavior and performance appears only three years after the CEO is hired.<sup>3</sup> LC can be seen as the parallel of OC, applied to managerial talent rather than managerial practices, which raises the same set of questions: How do we reconcile the observed variation with CT?

It is also natural to ask whether there exists a link between OC and LC.<sup>4</sup> Are leaders and practices two orthogonal factors that influence firm performance through distinct channels, or are they somehow connected? For instance, do CEOs play a role in the adoption of management practices? Or are firms with certain management practices more likely to hire a certain type of CEO?

This paper is an attempt to reconcile these three approaches in one theoretical framework. The starting point is the observation that all firms have a large, indivisible factor of production: the CEO. CEO quality is hard to observe ex ante and even ex post, as it takes time for him or her to affect firm performance. All firms try to hire a “good” CEO: some are lucky, some are not. Firms who end up with a good CEO receive a positive and highly persistent shock to their management practices and overall performance. Firms who, despite their best efforts, end up with a bad CEO endure a negative and persistent shock. The paper microfound a world where firms face this CEO selection problem, analyzes its steady state behavior, and relates it to the three approaches discussed above.

The objective is not to develop a general, realistic model of management and managers,

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<sup>3</sup>The effect of individual leaders on organizational performance has also been documented for middle managers (Lazear, Shaw, and Stanton 2015, Hoffman and Tadelis 2017)

<sup>4</sup>In a sample of firms where both CEO behavior and management practices are measured, Bandiera et al (2016) find significant cross-sectional correlation between the management score and the CEO behavior, controlling for other observables.

but rather to show that some of the essential lessons from CT, OC, and LC can be distilled in a set-up that is extremely close to a standard CT dynamic firm model. There are only two innovations. The first is a standard assumption of corporate governance theory: there is a serious agency problem between the firm's owner and the CEO who runs it, which creates the potential for inefficient behavior on the part of the CEO (Jensen and Meckling, 1973; Tirole 2010). The nature of this problem will be discussed in more detail below.

The second innovation consists of introducing a class of firm-specific assets whose production depends on the CEO. The performance of a firm depends on its *organizational capital*. This concept is meant to encompass any intangible firm asset with four properties: (i) It affects firm performance; (ii) It changes slowly over time; (iii) Being intangible, it is not perfectly observable; (iv) It must be produced at least partly inside the firm with the active participation of the firm's top management.

While conditions (i) through (iii) are familiar, condition (iv) is mostly novel to economists. It is a tenet of an influential stream of management literature that includes Drucker (1967) and Kotter (2001). It is encapsulated in Schein's (2010) assertion that "leadership is the source of the beliefs and values of employees, and shapes the organizational culture of the firm, which ultimately determines its success or failure." In this perspective, some firms end up with leaders who are more capable and/or willing to act in a way that increases the firm's organizational capital. Leadership is a flow that adds or subtracts to the firm's stock of intangible capital (Rahmandad, Reppenning, and Henderson 2018). Note that this view of leadership is much more precise than simply saying that some CEOs generate more profits than others for some unspecified reason. It identifies a particular mechanism, the growth of organizational assets, through which long-term value creation occurs in ways that lead to a wealth of testable implications.

What could organizational capital be in practice? One possible example is the management practices analyzed by Bloom et al. (2007), which arguably affect firm performance as in (i) and are slow-moving as in (ii). In support of the imperfect observability condition in (iii), note that the correlation rate of two independent and almost simultaneous measurements of management scores within the same plant is 45.4% (Bloom et al. 2019). For (iv), Simons (1994a) uses ten years of observational data collected in over 50 US businesses to document how top managers use 'control systems', namely mechanisms for influencing human endeavor

within the company, to maintain or alter patterns in organizational activities; in particular, new CEOs use their first 18 months of their tenure to define and measure critical performance variables (Simons 1994b). There are other possible examples of organizational capital. Our definition includes at least partially constructs such as relational contracts (Baker, Gibbons, and Murphy 2002), corporate culture (Schein 2010), firm-specific human capital (Prescott and Visscher 1980), or firm capabilities (Teece, Pisano, and Shuen 1997).

In the model we develop in this paper, organizational capital depreciates over time, but the CEO can devote her limited attention to increasing it. Alternatively, the CEO can spend her time boosting short term profit. The firm's profit-maximizing board hires a CEO in a competitive market for CEOs and can fire her at any time. Some CEOs are better than others at improving organizational capital. Firms are otherwise identical. They are born randomly and they die if their performance is below a certain threshold. There are no other factors of production or sources of randomness.

This barebone model is completed by agency frictions. If corporate governance was sufficiently strong, bad CEOs could be screened before they are hired or dismissed (or persuaded to reveal their type and resign) soon after being hired. Instead, we assume two forms of governance problems. The firm owners face *ex ante frictions*: when a board hires a CEO they have limited information about the CEO's type, especially if the candidate has never held a CEO's position, namely they have an imperfect CEO screening technology. Moreover, the board is unable or unwilling to use high-powered incentives so low-type CEOs would quit voluntarily. The firm owners also face *ex post frictions*: while cash flow can be measured almost continuously, the immaterial nature of organizational capital makes it harder to monitor. We assume that the board observes the cash flow stream immediately, but they only spot changes in organizational capital with a delay.

In equilibrium, firms would like to dismiss low-type CEOs but the latter hide their type for some time by boosting short-term behavior rather than investing in organizational capital. If the firm is lucky, it gets a good CEO who increases organizational capital and improves long-term performance (and retires at some point). If the firm is unlucky, it gets a bad CEO who depletes organizational capital and hurts long term performance before the firm fires her. This implies that the organizational capital of each firm follows a stochastic process punctuated by endogenous CEO transitions. Both sources of friction above are necessary and sufficient to

generate this equilibrium.

The main technical result of the paper is the characterization of the steady state distribution of firms in this economy. We first show that the measure of firms with a certain organizational capital is described by a recurrence equation. Although that recurrence equation is somewhat non-standard, we show that under certain assumption it has a unique steady state, which we characterize in closed form. At every moment, there coexist firms with different organizational capital, different leadership styles, and different performance, giving rise to stylized OC and LC cross-sectional patterns.

The main substantive result is a set of testable implications that bring together, in one model, some of the key patterns predicted or observed by CT, OC, and LC as well as new implications that bring together the three approaches. On the CT front, our model displays the performance heterogeneity and persistence predicted by Hopenhayn (1992) as well as a power law at the top of the distribution. In OC, our analytical results are consistent with the findings by Bloom and Van Reenen (2007) and others that (changes in) the quality of management practices are associated with (changes in) firm performance. the quality of corporate governance is found to play a key role. Regarding LC, we show that the CEO behavior, type, and tenure are all predictors of firm performance, as found in the CEO literature – with governance quality and the supply of managerial talent determining CEO variables.

Finally, the model predicts a wealth of new cross-sectional and dynamic interactions between CT, OC, and LC concepts: the tenure, behavior, type, and compensation of present and past firm's CEOs predict the current level and growth rate of the firm's organizational capital. As before, we perform comparative statics on governance quality and leadership supply. We also make predictions linking CEO career paths and the dynamics of organizational capital. For instance, a firm that was run in the recent past by a CEO who is currently employed by a larger firm should display an abnormally high growth in organizational capital and performance. Conversely, a firm whose last CEO was short tenured will have lower organizational capital and performance.

Of course, the model we present is not meant to be exclusive. Other factors, besides leadership, may affect the evolution of a firm's organizational capital. Leadership may influence performance through channels that are distinct from organizational capital. Other frictions

1	2	3	4
F1	CT	Seemingly identical firms display persistent performance differences	Hopenhayn (1992), Ericson and Pakes (1995), Syverson (2011)
F2	CT	The right tail of the performance distribution can be approximated by a power law	Gabaix (2009), Luttmer (1995)
F3*	CT	CEOs and firms engage in assortative matching	Gabaix and Landier (2008), Tervio (2008)
F4	OC	Management practices and firm performance display positive cross sectional correlation	Ichniowski, Shaw, and Prennushi (1997), Bloom and Van Reenen (2007)
F5	OC	Changes in management practices cause (and are associated with) changes in firm performance	Bloom, Eifert, Mahajan, McKenzie, Roberts (2013), Bloom, Sadun, and Van Reenen (2016)
F6	OC	Firms with better governance display better management practices	Bloom and Van Reenen (2007)
F7*	LC	A CEO fixed effect is observable in firm performance data	Johnson, Magee, Nagarajan, Newman (1985), Bertrand and Schoar (2003)
F8	LC	CEO variables (personal characteristics, behavior, etc) explain firm performance	Bennedsen, Perez-Gonzalez, Wolfenson (2007), Kaplan, Klebanov, Sorensen (2012), Bandiera, Hansen, Prat, and Sadun (2017)
F9	LC	Better governance improves CEO variables and firm performance	Shleifer and Vishny (1997)
F10	new	Changes in management practices are correlated with current CEO variables and CEO transitions	n.a.
F11	new	Controlling (perfectly) for current management practices / organizational capital, past CEO variables do not explain current firm performance	n.a.
F12*	new	The employment status and compensation level of a CEO depends on the change in management practices and performance of their previous firm	n.a.
F13*	new	The Bertrand-Schoar approach underestimates the size of the causal effect of CEOs on performance	n.a.

**Table 1:** Connection between the findings of the model and selected existing literature.  
Column 1: Finding number (used for reference in the propositions); findings proven in the model extension are denoted with \*.  
Column 2: Type of prediction (CT, OC, LC, or new prediction).  
Column 3: Informal description of the stylized pattern predicted in the model.  
Column 4: Selected references that discuss the patterns described in Column 3.

may affect both CEO selection and organizational capital. The goal of this paper is to see how far we can go with a parsimonious model.

Our paper is structured as follows. The first part of the paper microfounds a dynamic firm model. Section 2 introduces a continuous-time model of an infinitely-lived firm with organizational capital and endogenous CEO transitions. Section 3 characterizes the equilibrium of the model when frictions are sufficiently strong and shows that it gives rise to a stochastic process determining CEO behavior, CEO turn-over, organizational capital, and firm performance (Proposition 1).

Section 4 contains our main technical result: the characterization of the steady state equilibrium of a dynamic economy with a continuum of firms that behave according to the dynamic firm model of the previous section (Proposition 2). Given some assumptions about firm births and deaths, the equilibrium distribution of firms obeys a recurrence equation, whose steady state admits one closed-form solution. For sufficiently high performance levels, the solution satisfies an approximate power law.

Section 5 contains the main substantive results. It explores the testable implications of the steady state characterization and shows that it reconciles key findings of CT (heterogeneity and persistence of firm performance), OC (cross-sectional and longitudinal relationship between management practices and firm performance), and LC (relationship between CEO behavior/type and performance). The section also analyzes the role of corporate governance and presents novel testable implications linking OC and LC variables. The predictions are related to the existing literature (See Table 1).

Section 6 introduces observable heterogeneity in CEO quality. Suppose CEOs can live for more than one period and work for more than one firms. The market for CEOs will then be segmented into untried CEOs, successful CEOs, and failed CEOs. In equilibrium failed CEOs are not re-hired, untried CEOs work for companies with low organizational capital, and successful CEOs receive a compensation premium to lead companies with high organizational capital. The extension leads to additional predictions: a panel regression run on data generated by this model would yield CEO fixed effect coefficients; however, because of the endogenous assignment of CEOs to companies, such coefficient would under-estimate the true effect of individual CEOs on firm performance. The section also yields novel predictions on the dynamic relationship between CEO compensation, CEO career, firm performance, and the growth of

organizational capital. Section 7 briefly concludes.

## 1.1 Literature Review

Our paper is an attempt to reconcile a number of stylized patterns, predicted by other theoretical papers or observed by existing empirical works. Table 1 contains a list of the patterns that hold in our model, together with some of the papers that inspired them. The table will be discussed in detail in Section 5.

The rest of this section relates our general approach to previous work. At least since Hopenhayn (1992) and Erickson and Pakes (1995), economists have emphasized how firm-specific sources of uncertainty can result in firm dynamics and long-term productivity differences between ex ante similar firms in the same industry. We follow Hopenhayn in analyzing the steady state outcome of this dynamic process, but we micro-found one of the possible sources of the idiosyncratic productivity shocks by introducing managerial skill heterogeneity and moral hazard in the building of a firm's organizational capital (management practices, culture,...). As such, we are able to link the distribution of firm productivity to corporate governance and the supply of managerial talent, and make predictions which directly link managerial talent with organizational capital and firm productivity.

Prescott and Visscher (1980) developed a model of "organization capital," defined as firm-specific accumulated information, for instance about the human capital of its employees and the match between employees and jobs. Prescott and Visscher's organization capital satisfies (i), (ii), and (iii) of our definition of organizational capital above. Our model can be thought of Prescott and Visscher with of a role for leadership, namely the addition of property (iv) and corporate governance frictions.

Bloom et al. (2016) consider a dynamic model which – in our language – attempts to reconcile CT with OC. Firms make costly investments in a 'stock of management'.<sup>5</sup> One of the key result of the paper is that the empirical patterns they observe can be rationalized by assuming a heterogeneous initial draw of management quality: firms are born with a random level of management quality, and this continues with them throughout their lives. As in Lucas

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<sup>5</sup>In Bloom et al. (2016), more management is always better. They refer to this perspective as 'Management as a Technology' and contrast this with 'Management as a Design', a setting in which there are no good or bad management practices.

(1978), this initial variation is not explained within the model and – to fit the data – it must be of the same order of magnitude of the observed (endogenous) variation in management practices. We follow Bloom et al. (2016) in thinking of management quality – an example of organizational capital – as a slow-moving asset. However, we differ in that we fully endogenize this asset and in so doing we create a role for corporate leadership. This has two benefits: there is now a three-way link between CT, OC, and LC and the observed variation in organizational capital can now be explained entirely within the model without invoking exogenous differences between firms.

Within LC, Bandiera et al (2016) consider an assignment model where different types of firms are more productive if they are matched to CEOs that choose the right behavior for that firm. In the presence of limited screening and poor governance, some firms may end up with the wrong CEO, thus generating low performance. This paper uses a similar building block, but combines it with organizational capital in a dynamic firm model and studies steady-state properties.

On the theory front, a number of models explore reasons why similar firms may end up on different performance paths. Li, Matouschek and Powell (2017) show how performance differences between (ex ante) identical firms may arise because of path-dependence in (optimal) relational contracts. In Chassang (2010) differences in a firm's success in building efficient relational contracts determines productivity differences. In Ellison and Holden (2014) path dependence in developing efficient rules for employee behavior also result in performance differences. Halac and Prat (2016) assume the presence of a costly but imperfectly observable monitoring technology that must be maintained by top management: some firms end up in persistent low-trust, low-productivity situations. Board et al. (2016) propose a model where firms with higher levels of human capital are better at screening new talent, creating a positive feedback loop. In Powell (2019), firms which earn higher competitive rents have the credibility to adhere to more efficient relational contacts with their employees, creating a positive feedback loop.

None of the above papers has a role for personal leadership. In contrast, in our model path dependence in productivity stems from the effect of the type and behavior of individual CEOs on the accumulation of organizational capital. Our approach is closest to Rahmandad, Repping, and Henderson (forthcoming), who model the firm's capability as an asset whose

rate of change depends on the behavior of the firm's leader: a short-term behavior leads to slower capability accumulation. More broadly, our paper is inspired by models of corporate leadership where leaders have a 'type' that affects their performance (e.g. Van den Steen 2005, Bolton et al. 2012, Hermalin 2013), or they have beliefs that are reflected in the strategy they develop (Van den Steen 2018), or they influence the shared frames that affect performance (Gibbons, LiCalzi, and Warglien 2017).

A recent paper by Besley and Persson (2018) studies organizational culture from a different angle. They analyze the transmission of cultural values in organizations with overlapping generations of managers. They show how organizational culture becomes a natural source of inertia and prevent organizations from responding to shocks in their environment, thus explaining phenomena such as dysfunctional cultures and resistance to change.

Our paper is related to a literature in corporate finance on managerial short-termism (Stein 1989). Most of this literature is focused on how different financial contracts (e.g. short-term versus long-term debt) trade-off a desire for early termination of unprofitable projects with the need to provide adequate incentives for long-term investments (Von Thadden (1995)). In contrast, we study the consequences of heterogeneity in managerial short-termism on the productivity dispersion of ex ante identical firms. Indeed, in our paper, bad managers are able to temporarily mimic the performance of good managers by boosting short-term performance at the expense of long-term investments in organizational capital. Our model further differs from classic models of managerial short-termism in that only bad managers engage in short-term behavior.

Finally, our paper is also loosely linked to a long-standing debate on the role of individual leaders in determining the evolution of institutions (summarized in Jones and Olken 2005, who also measure the causal effect of individual leaders). At one extreme, a certain interpretation of Marxism sees leaders as mere expressions of underlying social phenomena and structures: the latter are the real drivers of historical change with individuals being essentially fungible. At another extreme, traditional historiography often ascribes enormous importance to the behavior of great leaders, who are credited with single-handedly changing the course of history by developing or destroying institutions.

## 2 A Dynamic Model of Firm Performance

We propose a dynamic model of an industry composed of a mass of individual long-lived firms. Each firm is defined by its organizational capital and faces an agency problem.

Specifically, a firm's profit at time  $t$  is a function of the firm's organizational capital  $\Omega_t$ . This organizational capital includes the quality of the firm's management practices and management system, its culture and norms and so on. The firm has a CEO whose responsibility it is to maintain and grow this organizational capital, denoted as behavior  $x = 1$ , but can shirk on this responsibility and engage instead in activities which boost short term performance, denoted as behavior  $x = 0$ .<sup>6</sup>

In the long-term behavior ( $x = 1$ ), the CEO might be building a management system and provides supervision and motivation to workers. In the short-term behavior ( $x = 0$ ), the CEO might instead spend her time boosting productivity immediately. For example, the CEO could be monitoring operations directly as opposed to creating an accountability system, or going on sales pitches as opposed to incentivizing/training sales managers. Central to our analysis is that there are two types of CEOs, good and bad, who differ in their managerial ability to build organizational capital.

Formally, the firm's performance or flow profit at time  $t$  is given by

$$\pi_t = (1 + b(1 - x)) \Omega_t, \quad (1)$$

where  $b \in [0, \bar{b}]$  is a short-term boost to performance, as chosen by a CEO engaging in behavior  $x = 0$ . The firm's organizational capital is an asset that evolves according to:

$$\dot{\Omega}_t = (\theta x - \delta) \Omega_t,$$

where  $\delta$  is the depreciation rate of managerial capital and  $\theta \in \{\theta^L, \theta^H\}$  represents the CEO's managerial skill with  $\theta^H > \theta^L$ .

The model could easily be extended to include other production factors. For instance, one might have a standard formulation in which

$$\pi_t = (1 + b(1 - x)) \Omega_t f(K_t, L_t) - rK_t - wL_t - F, \quad (2)$$

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<sup>6</sup>One key simplifying assumption is that the CEO chooses her behavior once and for all at the beginning of her tenure. The assumption is discussed – together with other limitations of the model – after Proposition 1.

where  $K_t$  is the amount of capital and  $r$  is its unitary cost,  $L_t$  is the amount of labor and  $w$  is its unitary cost, and  $F$  is a fixed cost. With this formulation,  $K_t$  and  $L_t$  would be chosen given the firm's organizational capital. Under standard assumptions, the optimal amount of capital and labor would be increasing in the value of the firm's organizational capital. The results presented in the rest of the paper would continue to hold, with minimal modifications. To keep notation to a minimum we abstract from other factors and use (1).

The owner (or board) maximizes long-term profits

$$\int_0^{\infty} e^{-\rho t} \pi_t dt$$

We assume that behavior 1 is optimal for both CEO types ( $\theta^L$  large enough compared to  $\bar{b}$ ). Hence, if the owner observed the CEO type she would always hire the high type and instruct her to choose  $x = 1$ .

The owner, however, does not observe the CEO type, the CEO's behavior  $x \in \{0, 1\}$ , or the current level of the organizational capital immediately. They are observable with a delay  $R$ . The only variable the owner observes in real time is performance

The board appoints the CEO and she can fire him whenever she wants, but CEOs must retire after time  $T$ . The probability of selecting a high type  $\theta^H$  is given by  $p > 0$ .

CEO's do not care about profits, but maximize tenure. When hired, the CEO chooses a management style and – for simplicity – we assume she cannot change it over time. We will discuss the relevance of this assumption in the next section.

We assume there is a continuum of firms. Each firm's life is governed by the following birth and death process:

**Assumption S1:** *A firm dies whenever its performance is smaller or equal than a certain profit level  $\pi_0$ .*

**Assumption S2:** *At each moment a mass  $B$  of new firms are born as spin-offs of existing firms. The spin-offs are clones of existing transitioning firms (firms who are changing their CEOs) and they inherit the organizational capital level of the firm they originate from.*

At each instant, we assume events occurs in the following order: (i) if optimal or necessary, firms replace their CEO, (ii) firms with performance smaller or equal than  $\pi_0$  die, (iii) a mass  $B$  of new firms are born as spin-offs of existing transitioning firms. The goal of our

analysis is to characterize the steady state equilibrium of an industry with a mass of firms that follow the assumptions above.

## 2.1 Remarks about Modeling Choices

The objective of this paper is to introduce a simple model that gives rise to steady state behavior that replicates the stylized patterns summarized in Table 1. Other modeling choices are possible, but we have discarded them either because they lead to a set-up that is not solvable analytically or because they do not yield some of the desired steady state patterns.

Here, performance is perfectly observed while CEO behavior and organizational capital are completely unknown at least for period  $R$ . More realistically, we could have assumed that some or all of these variables are observed with some noise, perhaps following the continuous-time Brownian motion set-up introduced by Sannikov (2007). Unfortunately, these formulation does not appear to lead to a tractable steady-state characterization.

Alternatively, one could look for a basic discrete-time formulation, where CEOs are in charge for one period and can be bad or good. While this assumption would lead to an even more tractable set-up, this non-microfounded approach would not generate many of the desired steady state patterns. The effect of corporate governance on steady state variables would have to be assumed in an ad-hoc manner. Unless more ad hoc assumptions are added, the model would also be silent on the role of CEO behavior, on the equilibrium relationship between CEO tenure and other variables, and on a number of other equilibrium relationships discussed in Section 5.

The assumptions about the death and birth processes (S1 and S2) are not crucial to the results. One could complicate the model by assuming that death occurs probabilistically at different levels or that birth occurs with a different probability distribution. The analysis would become more complex and probably require a numerical approach. One could simplify the birth process by assuming that a mass of firms are born in every period at a given level. This too would lead to an analytical characterization of the steady state. However, the equilibrium distribution would display an unrealistic spike in correspondence of the birth level.

### 3 CEO Behavior, CEO Turnover and Firm Performance

We first present the results of our simple model, which is based on a number of stark assumptions. At the end of the section, we discuss how robust the results are to modifications of the assumptions.

To gain intuition, suppose all CEOs behave naively. They all choose optimal behavior:  $x = 1$ . Managerial capital growth then equals

$$\dot{\Omega}_t = (\theta - \delta) \Omega_t,$$

and is thus faster for  $\theta^H$  than for  $\theta^L$ . As performance is given by  $\pi_t = \Omega_t$ , the performance growth rate is

$$\frac{\dot{\pi}_t}{\pi_t} = \theta - \delta$$

Note that in the latter case, the low type would immediately be spotted and fired. As we show next, this cannot be an equilibrium, as a low type CEO then has an incentive choose the short-term behavior.

Consider the case where good CEOs choose  $x = 1$ , but bad CEOs choose the short term behavior  $x = 0$ . While this causes organizational capital to depreciate, it allows the bad CEO to mimic the performance of good CEOs for a while. Normalizing  $t$  to 0 at the time of CEO hire, profits at time  $t \in [0, T]$  are given by

$$\pi_t^H = \Omega_t^H = \Omega_0 e^{(\theta^H - \delta)t}$$

for the high type, whereas

$$\pi_t^L = (1 + b) \Omega_t^L = (1 + b) \Omega_0 e^{-\delta t}$$

for the bad type.

As long as  $(1 + \bar{b}) \Omega_t^L \geq \pi_t^H$ , the bad type can mimic the good type by choosing a short-term boost  $b \in [0, \bar{b}]$  so that  $\pi_t^L = \pi_t^H$ . Mimicking becomes unsustainable after a period:

$$K = \frac{\ln(1 + \bar{b})}{\theta^H}.$$

Throughout the analysis, we assume that

$$T > K. \tag{A1}$$

It follows that CEO type is identified for sure after  $K$  periods. That may come before or after the exogenous observational delay  $R$ . So, a bad CEO is fired after a period of  $\bar{t} = \min(K, R)$ . Good CEOs are kept until retirement ( $T > \bar{t}$ ). Clearly, the above behavior is an equilibrium.

The following result holds:<sup>7</sup>

**Proposition 1** *A low-type CEO chooses behavior 0, is fired after a period  $\bar{t} = \min(K, R)$  with  $K = \frac{\ln(1+\bar{b})}{\theta^H}$ , and leaves a firm with a worse management system:*

$$\Omega_{\bar{t}}^L = \Omega_0 e^{-\delta \bar{t}} < \Omega_0.$$

*A high-type CEO chooses behavior 1, serves until retirement, and leaves a firm with a better management system:*

$$\Omega_T^H = \Omega_0 e^{(\theta^H - \delta)T}.$$

To illustrate the proposition, assume that  $\Omega_0 = 1$ ,  $\theta^H = .10$ ,  $\delta = .06$ ,  $\rho = .05$ ,  $\ln(1+\bar{b}) = .20$ ,  $R = 3$ , and  $T = 5$ . We therefore have that

$$\bar{t} = \frac{.20}{.10} = 2$$

so that a bad manager leaves after two years and leaves organizational capital that is  $e^{-(.06)2} = 0.886$  times the capital she found. A good manager retires after 5 years and leaves an organizational capital that is  $e^{(.04)5} = 1.221$  times what she found.

Figure 1 plots the organizational capital and Figure 2 the performance of a firm that hires a bad CEO, followed by another bad CEO, followed by a good CEO, followed by a bad CEO.

[W\*] Every time a bad CEO departs, the model predict a sharp drop in observed performance. These stark jumps can be interpreted at face value as accounting restatements of financial performance (Hennes et al. (2008) document the correlation between financial restatements and CEO turnover) or, more likely, they should be taken with a grain of salt as an artifact of a model where performance is perfectly observable (see the discussion of assumptions below). In a richer model, the drops would probably be replaced by declines.

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<sup>7</sup>It is easy to see this is the only equilibrium where a good CEO chooses  $x = 1$ . There are also (less plausible) equilibria where the CEO always chooses  $x = 0$ , and any upward deviation in performance is interpreted as coming from a bad CEO.

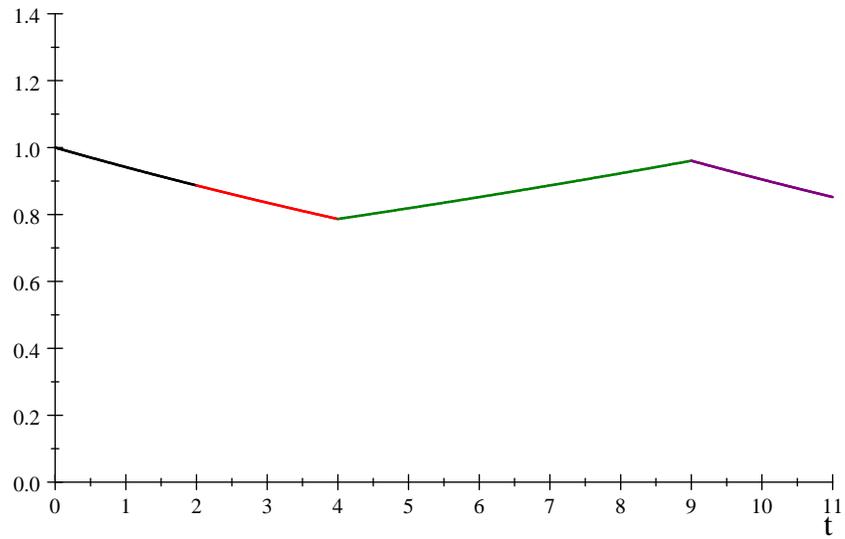


Figure 1: Organizational capital  $\Omega_t$  over time

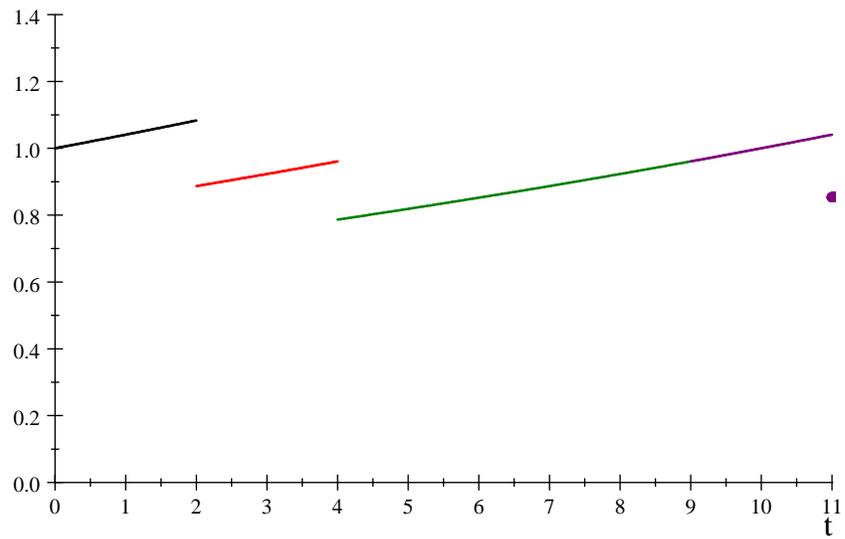


Figure 2: Performance  $\pi_t$  over time

Proposition 1 depends on a number of stark assumptions we have made. As mentioned in the introduction, the results hinge on the presence of a serious agency problem within the company. In a frictionless environment, bad CEOs would either not be hired or leave immediately, in which case CEOs would only be high quality and there would be no leadership heterogeneity. Let us go over the various frictions we have assumed.

First, we posited that the owner is unable to screen CEOs based on their quality  $\theta$ . If the owner had an effective screening technology, she would only hire the good ones. The extension of the model (Section 6) with various quality levels explores the possibility that CEOs can move from one firm to the other, in which case owners can learn something about the CEO's type from the performance of the firm they worked for previously.

Second, we assumed that the CEO receives a flat wage (normalized to zero). If the CEO's contract included a sufficiently strong performance-contingent component, a bad CEO could be incentivized to reveal his type right away.<sup>8</sup> This assumption can be assessed from a pragmatic perspective or a theoretical one. First and foremost, in practice it has been argued that, even in developed market economies such as the US, corporate governance is highly imperfect: the actual incentive schemes that CEOs receive are highly constrained and they do not align the CEO's interest with that of the firm (Bebchuk 2009). From a theoretical perspective, one can also show that enlarging the set of contracts available to the company may not weed out bad CEOs, because the incentive schemes that achieve this goal also increase the rent the firm must concede to all CEOs. This point is explored formally in Appendix II.

Third, we assumed that the owner does not observe organizational capital  $\Omega_t$  directly. Obviously, if she does, she could kick out a bad CEO immediately. One could consider an alternative model where the owner observes a noisy continuous signal of organizational capital and will fire a CEO if enough evidence accumulates. The results would be qualitatively similar to the present model (but the analysis would be more complex – prohibitively so, at least for us, when we move to the aggregate level).

Fourth, we assumed that the owner observes cash flow perfectly. This assumption too could be relaxed. As in the previous point, the resulting model would be much more complex. Having imperfectly observable performance would eliminate the stark negative effect on

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<sup>8</sup>For instance, suppose the CEO is offered a large stock option plan (a share of future profits): then, a bad CEO would rather resign right away in the hope that his replacement is of greater quality. It is possible to think about other schemes that would achieve the same result, like a golden parachute, backloaded compensation, etc.

performance that we currently observe when a bad CEO departs.

Finally, we assumed the CEO cannot change her management style over time. This leads to equilibrium uniqueness. If the CEO were to be able to change her behavior over time, the equilibrium of Proposition 1 would still exist, but other perfect Bayesian equilibria may arise too. The good CEO could signal her type by first playing  $x = 1$ , then plays  $x = 0$  before reverting back to  $x = 1$ . Since it would be sufficient for the good CEO to play  $x = 0$  for an infinitesimal time, separation could occur (almost) immediately and a bad type would be fired (almost) instantly. Those immediate signaling equilibria would mainly be an artifact of our assumption that profits are perfectly observable and predictable. Unfortunately, adding noise to performance renders the analysis unwieldy very quickly. A more tractable way to eliminate signaling equilibria is to assume that the bad type is more productive at the short term behavior, that is  $\bar{b} \in \{\bar{b}^L, \bar{b}^H\}$  and the CEO's type is either  $(\theta^L, \bar{b}^H)$  or  $(\theta^H, \bar{b}^L)$ .<sup>9</sup> Our assumption that CEOs needs to commit to a particular management style once hired achieves the same goal and keeps the model simple.

## 4 Steady-State Distribution of Organizational Capital

Now that we have characterized the equilibrium behavior of an individual firm, we analyze aggregate behavior. Our goal is to characterize the steady state distribution of firms across organizational capital levels.

We proceed in two steps. First, we characterize the steady state in the non-generic case where the primitives of the problem are such that the distribution of firm performance has no drift. Second, we extend it to the case with drift. As most of the analytical complexity is in the first step, the analysis is easier to follow if one abstracts from drift.

### 4.1 Steady State Analysis: Good and Bad CEOs Have Same Absolute Effect

We first perform the analysis under a simplifying assumption:

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<sup>9</sup>Without loss of generality, one could also introduce a third type of manager  $(\theta_L, b_L)$  which is lousy at both behaviors. It suffices then that both other types engage in signalling for a (infinitesimal) short time right after being hired, for such a type to be immediately discovered and fired.

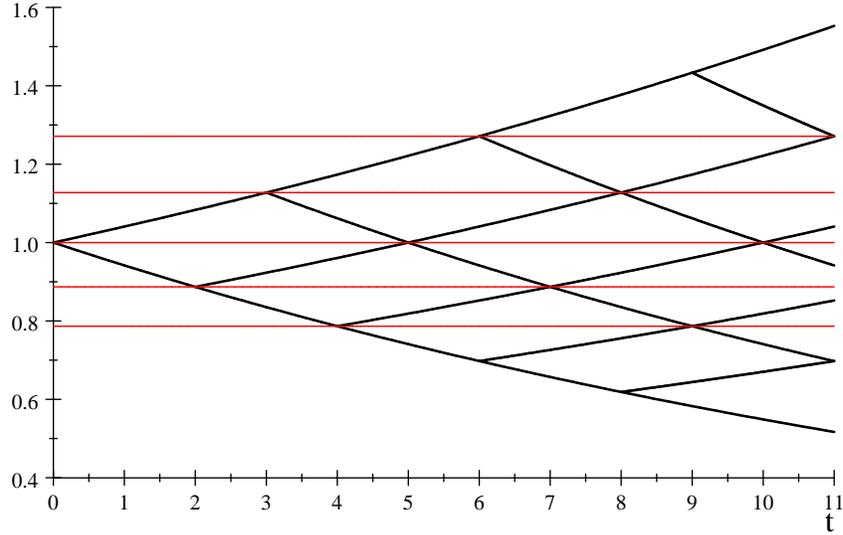


Figure 3: Possible organizational capital paths. Thick black line represents possible paths of organizational capital  $\Omega_t$  over time. Each bifurcation point corresponds to a CEO transition where the firm can either draw a good CEO or a bad CEO. The thin red lines illustrate how under Assumption S3 organization capital is at the same level after one bad CEO and one good CEO.

**Assumption S3:** *The effect of a bad CEO exactly undoes the effect of a good CEO:*

$$\begin{aligned} \Omega_t e^{(\theta^H - \delta)T} e^{-\delta \bar{t}} &= \Omega_t \\ \iff (\theta^H - \delta)T &= \delta \bar{t} \end{aligned}$$

Assumption S3 combined with Proposition 1 implies that all firms will experience transitions at a stable countable number of organizational capital levels. This greatly simplifies the exposition of the results.

The extension of the findings to cases beyond S3 involves a time-dependent rescaling of organizational capital. We will present it in the next section, once the baseline case is understood.

Figure 3 illustrates possible organizational capital paths when  $\Omega_0 = 1$ . Thanks to S3, all CEO transitions occur at a countable number of time-invariant levels.

We begin by defining the distribution of active firms. Let  $\phi : [0, \infty) \times [0, \infty) \rightarrow \mathfrak{R}^+$  be an integrable non-negative function:  $\phi_t(\Omega)$  denotes the measure of firms with organizational

capital  $\Omega$  active at the start of time  $t$ .<sup>10</sup> While we refer to  $\phi$ , and analogous objects, as “distributions,” note that these are not probability distributions because their integral does not equal one: the mass of firms active at time  $t$  is  $\int_0^\infty \phi_t(\Omega) d\Omega$ . We adopt the convention that  $\phi_t(\Omega)$  only includes *existing firms*, and does not include any firms born at time  $t$ .

Let a steady state distribution, if it exists, be denoted with  $\phi(\Omega)$ . This is the object we are trying to characterize.

At any CEO transition, performance  $\pi_t$  and organizational capital  $\Omega_t$  are equal and fully known to the firm. The countable set of organizational capital levels at transition is thus the same as the countable set of performance levels at transitions, which we now define formally. From S1, there is a lowest performance level at CEO transitions, which we denote by  $\pi_0$ . Starting from this lowest performance level, construct a set of *transition* organizational capital levels  $\Pi$  as follows:

$$\Pi = \{ \Omega : \exists j \in \mathbb{N} \text{ such that } \Omega = \Omega_j \equiv \pi_0(1 + \Delta)^j \}$$

where  $\Delta$  is the percentage improvement in organizational capital following a good CEO

$$1 + \Delta \equiv e^{(\theta^H - \delta)T}.$$

Assumption S3 and Proposition 1 imply:

**Lemma 1** *A firm born with organizational capital  $\Omega_j \in \Pi$  can experience a CEO transition only at organizational capital levels in  $\Pi$ . Moreover, for any organizational capital level  $\Omega_j \in \Pi$ , a firm born with organizational capital in  $\Pi$  has a strictly positive probability of transitioning at  $\Omega_j$  at some point during its lifetime.*

Given the function  $\phi$ , we can define  $g : [0, \infty) \times \mathbb{N} \rightarrow \mathfrak{R}^+$  as  $g_t(j) = \phi_t(\Omega_j)$  for all  $\Omega_j \in \Pi$  and all  $t \geq 0$ . The new function  $g$  represents the distribution of transitioning firms over all possible transition organizational capital levels  $\Omega_1, \Omega_2, \Omega_3, \dots$ . The total measure of firms transitioning at  $t$  is

$$G_t = \sum_{j=0}^{\infty} g_t(j).$$

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<sup>10</sup>We focus on atomless distributions because the steady state distribution over organizational capital cannot contain atoms. By Proposition 1, an atom at  $t$  for a given organizational capital would generate at least one atom an instant later, either at a slightly higher level or a at a slightly lower level – thus giving rise to a contradiction.

In steady state, we can interpret  $G_t = G$  as the mass of firms that have a CEO transition per unit of time, in the same way  $B$  equals the mass of firms that are born per unit of time. Similarly, we can interpret  $g_t(j) = g(j)$  as the mass of firms that have a CEO transition at organizational capital level  $j$  per unit of time. Hence,  $B$ ,  $G_t$  and  $g_t(j)$  can best be understood as ‘rates’, as at any instant, a mass zero of firms has a CEO transition.

From assumption S2, a measure  $B$  of new firms are born at each instant and spread across organizational capitals  $\Omega_j \in \Pi$  in proportion to the measure  $g_t(j)$  of existing firms at those levels. Hence, the measure of transitioning and newly-born firms with organizational capital  $\Omega_j$  at time  $t$  equals

$$(1 + B/G_t)g_t(j) \tag{3}$$

Our aim is to characterize  $\phi(\Omega)$ , the steady state distribution of existing firms over all organizational capital levels  $\Omega \in (\pi_0, +\infty]$ . We take the following approach:<sup>11</sup>

1. We first characterize  $g(j)$ , the steady state distribution of existing firms over organizational capital levels  $\Omega_j \in \Pi$  (Proposition 2). Given Proposition 1 and Assumption S2, this is also the steady state distribution of the subset of firms experiencing a CEO transition.
2. The steady distribution of firms over all organizational capital levels,  $\phi(\Omega)$ , then follows mechanically from  $g(j)$  and the equality  $\phi(\Omega_j) = g(j)$  for all  $j \in \mathbb{N}$ .

Consider the subset of firms with organizational capital  $\Omega_j \in \Pi$ . An existing firm that finds itself at organizational capital level  $\Omega_j$  at time  $t$  must belong to one of the following two cases: (i) It is a firm undergoing a CEO transition, and the last CEO was bad. This means that at time  $t - \bar{t}$ , the firm had organizational capital level  $\Omega_{j+1}$ . (ii) It is a firm undergoing a CEO transition, and the last CEO was good. This means that at time  $t - T$ , the firm had organizational capital level  $\Omega_{j-1}$ . Hence, using (3), the measure of existing firms at organizational capital level  $\Omega_j$  at time  $t$  is given by

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<sup>11</sup>We choose the set  $\Pi$  because it is the simplest of its kind. One can of course make other assumptions on the set of performance levels at which firms can transition. It is easy to see that no such set can have a smaller cardinality than (the countably infinite set)  $\Pi$ . In fact, any such set can be expressed as a (possibly infinite) combination of sets of the form  $\Pi$  with different lowest performance levels  $\pi_0$ .

$$g_t(j) = (1-p) \left(1 + \frac{B}{G_{t-\bar{i}}}\right) g_{t-\bar{i}}(j+1) + p \left(1 + \frac{B}{G_{t-T}}\right) g_{t-T}(j-1) \quad (4)$$

The expression in (4) is a *recurrence equation* in two dimensions,  $j$  and  $t$ . However, it is non-standard because: (i) it combines a discrete dimension  $j$  with a continuous dimension  $t$ ; (ii)  $g_t(j)$  depends on two sets of past values of  $g$ , taken at two different times ( $g_{t-T}(j-1)$  and  $g_{t-\bar{i}}(j+1)$ ); and its right-hand side contains two variables that depend on a summation of past variables ( $G_{t-T}$  and  $G_{t-\bar{i}}$ ).

To make progress, we use (4) to create a new recurrence equation that applies to “even CEO transitions”, namely the set of organizational capital levels  $\Omega_0, \Omega_2, \Omega_4$ , etc:

$$\Pi^E = \left\{ \Omega : \exists k \in \mathbb{N} \text{ such that } \Omega = \pi_0(1 + \Delta)^{2k} \right\}$$

We define a new function  $f_t(k) = g_t(2k)$  for  $k = 1, \dots$ . This will help us characterize the steady state. Suppose that the whole system is in a steady state, namely  $\phi_t(\Omega) = \phi(\Omega)$  and  $G_t = G$  for every  $t$ . Then, it is also true that  $f_t(k) = f(k)$  for every  $t$ .

We can show that a necessary condition for steady state is a one-variable difference equation:

**Lemma 2** *In steady state, the following conditions must be satisfied for some  $\gamma > 0$ :*

- *A difference equation:*

$$f(k) = (1 + \gamma) \left( p^2 f(k-1) + 2(1-p) p f(k) + (1-p)^2 f(k+1) \right) \text{ for every } k \geq 1. \quad (5)$$

- *Two boundary conditions:*

$$\begin{aligned} f(0) &= 0 \\ f(1) &= \frac{B}{(1+\gamma)(1-p)^2} \end{aligned}$$

**Proof.** By applying (4) twice, we obtain

$$\begin{aligned} g_t(j) &= \left(1 + \frac{B}{G_{t-T}}\right) p g_{t-T}(j-1) + \left(1 + \frac{B}{G_{t-\bar{i}}}\right) (1-p) g_{t-\bar{i}}(j+1) \\ &= \left(1 + \frac{B}{G_{t-T}}\right) p \left( \left(1 + \frac{B}{G_{t-2T}}\right) p g_{t-2T}(j-2) + \left(1 + \frac{B}{G_{t-\bar{i}-T}}\right) (1-p) g_{t-\bar{i}-T}(j) \right) \\ &\quad + \left(1 + \frac{B}{G_{t-\bar{i}}}\right) (1-p) \left( \left(1 + \frac{B}{G_{t-\bar{i}-T}}\right) p g_{t-\bar{i}-T}(j) + \left(1 + \frac{B}{G_{t-2\bar{i}}}\right) (1-p) g_{t-2\bar{i}}(j+2) \right) \end{aligned}$$

In steady state, it must be that  $g_t(j)$ , and thus  $G_t$  are constant over time. Dropping time subscripts and defining

$$\gamma = \left(1 + \frac{B}{G}\right)^2 - 1,$$

the expression above simplifies to

$$g(j) = (1 + \gamma) \left( p^2 g(j-2) + 2p(1-p)g(j) + (1-p)^2 g(j+2) \right)$$

When we re-write it in terms of  $f$  we obtain the first part of the lemma.

The first boundary condition is by definition. For the second boundary condition, note that the measure of dying firms must equal the measure of new born firms  $B$ . If a firm dies at time  $t$ , this means that at the end of time  $t - \bar{t}$ , this firm had a bad CEO and organizational capital level  $\Omega_1$ . It follows that the total measure firms dying at time  $t$  is given by

$$(1-p) \left(1 + \frac{B}{G_{t-\bar{t}}}\right) g_{t-\bar{t}}(1)$$

Applying (4) to  $g_{t-\bar{t}}(1)$  and taking into account that  $g_{t-2\bar{t}}(0) = 0$ , we can rewrite this as

$$(1-p)^2 \left(1 + \frac{B}{G_{t-\bar{t}}}\right) \left(1 + \frac{B}{G_{t-2\bar{t}}}\right) g_{t-2\bar{t}}(2)$$

Hence, in steady state, we must have that

$$\begin{aligned} (1-p)^2 \left(1 + \frac{B}{G}\right)^2 g(2) &= B \\ \iff (1+\gamma)(1-p)^2 f(1) &= B \end{aligned}$$

■

The difference equation (5) expresses the measure of firms at organizational capital level  $j = 2k$ , as a function of the measure of firms at levels  $2(k+1)$ ,  $2k$ , and  $2(k-1)$ . A firm at level  $\Omega_j$  must belong to one of the following four cases: (i) It was at level  $2(k+1)$  two transitions earlier and got two bad CEOs; (ii) It was at  $2k$  two transitions earlier and got a bad CEO and a good CEO in either order; (iii) It was at  $2(k-1)$  two transitions earlier and got two good CEOs; (iv) It was born in the preceding CEO transition or the one before that from a firm in (i), (ii), or (iii)

Case (iv) yields the term  $(1 + \gamma)$  in equation (5). Recall that  $G = \sum_j g(j)$  is the steady-state measure of transitioning firms. In the proof of Lemma 2 we see that

$$1 + \gamma = \left(1 + \frac{B}{G}\right)^2,$$

namely  $\gamma$  can be interpreted as the expected firm birth rate over two transitions.<sup>12</sup> We will henceforth simply refer to  $\gamma$  as the birth rate. If  $\gamma$  were an exogenous parameter, equation (5) would be a relatively standard second-degree difference equation in  $k$ . Unfortunately, as we discuss below, (5) does not pin down  $\gamma$  which is an endogenous variable. This complicates things considerably.

We assume that  $p < 1/2$ . If the share of good CEOs is larger than 50%, it is easier to see that there is no steady state as the average firm does better and better over time. Instead if most CEOs are bad, individual firms are worsening over time on average and they eventually die: a steady state exists because some firms do well in the medium term and new firms are born.

Recurrence equations are sometimes used to represent heat diffusion processes in discrete time: a solid is subject to heating and cooling sources and we are interested in knowing the steady state temperature of different discrete points of the solid. One may wonder whether our problem corresponds to known diffusion problems. In this perspective, (5) can be loosely interpreted as a discrete version of the heat diffusion process of an imaginary one-dimension rod, with some additional features: (i) The rod begins at zero on the left side and it is unbounded on the right side; (ii) The diffusion parameter is asymmetric (as  $p < \frac{1}{2}$ , heat tends to flow left rather than right); (iii) The rod is heated along its length in a way that increases temperature at every point by rate  $\gamma$  per period; (iv) The total amount of added heat is constant (making  $\gamma$  endogenous) (iv) The left end of the rod is next to a powerful cooling source that keeps the temperature at zero. While this process is reminiscent of well-studied processes (especially in a continuous setting), we are not aware of any existing result that is directly applicable to the discrete case we are studying.

The characterization of  $f(\cdot)$ , proven formally in the Appendix, proceeds in a number of steps. We begin with the solution to the difference equation (5) when we treat the birth rate  $\gamma$  as an exogenous parameter. Together with the boundary conditions on the death threshold and the mass of births, it is easy to show this difference equation has a unique, distinct solution for every possible value of  $\gamma$  provided  $p < 1/2$  (the value of  $B$  just determines a rescaling of the whole distribution).

The proof then shows that this solution for  $f(\cdot)$  has a positive value for all  $k > 0$  if and

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<sup>12</sup>This condition has already been used to derive the second boundary condition. Thus, adding it to the set of conditions in Lemma does not reduce the set of solutions of the difference equation.

only if  $\gamma \in (0, \gamma^*]$  where

$$1 + \gamma^* \equiv \frac{1}{1 - (1 - 2p)^2}$$

In economic terms, a steady state cannot exist if the size of the economy, as proxied by  $G_t = \sum_j g_t(j)$ , is small relative to constant inflow of new firms  $B$ , that is whenever  $1 + \gamma_t \equiv (1 + B/G_t)^2 > 1 + \gamma^*$ . Intuitively, given an exogenous influx of new firms, we expect the economy to keep growing until  $G_t$  is sufficiently large and the (endogenous) number of deaths equal the (exogenous) number of births.

While the difference equation (5) provides a distinct solution to  $f(\cdot)$  for each  $\gamma \leq \gamma^*$ , a simple refinement narrows the set of steady states down to one. Note that in steady state, we must have that  $\lim_{k \rightarrow \infty} f(k) = 0$ .<sup>13</sup> Consider therefore the  $N$ -level version of our problem where we impose the boundary condition  $f_t(k) = 0$  for  $k > N$  with  $N$  a finite positive integer. In this finite version of our problem, organizational capital is bounded above by  $\bar{\Omega} = \Omega_N$ .

**Definition 1** *We say that a steady state is reachable from below if can be the limit of a sequence of steady states of the finite  $N$ -level version of our problem when  $N \rightarrow \infty$ .*

This refinement excludes steady states that cannot be found as the limit of a sequence of steady states of the finite version of our problem when the upper bound goes to infinity. Intuitively, those steady states require a “large mass” of firms with high levels of organizational capital. That is why they cannot be approximated by steady states of problems with an upper bound, no matter how high the upper bound is.<sup>14</sup>

In the appendix, we show that in any finite version of our original problem the birth rate cannot be lower than  $\gamma^*$ . Formally, this is proven by deriving an upper bound to the eigenvalue of the transition matrix. If it were lower than that, then intuitively a steady state could not exist because the mass  $M_t$  of transitioning firms would be decreasing. This proves that a necessary condition for a steady state to be reachable from below is that in equilibrium the birth rate is exactly  $\gamma^*$ .

Once the value of  $1 + \gamma^* \equiv 1 / (1 - (1 - 2p)^2)$  is plugged into the general solution of (5),

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<sup>13</sup>If not, the mass of transitioning firms  $M$  is infinite, which cannot be a steady state.

<sup>14</sup>Consistent with this, our numerical simulations indicate that only the steady state where  $\gamma = \gamma^*$  is reached.

it yields a simple expression for  $f^*(k)$  of the form

$$f^*(k) = 4B \cdot k \left( \frac{p}{1-p} \right)^k$$

**Proposition 2** *In a steady state reachable from below*

$$1 + \gamma^* \equiv (1 + B/G^*)^2 = \frac{1}{1 - (1 - 2p)^2}. \quad (6)$$

*The distribution of firms over transition organizational capital levels  $\Omega_j \in \Pi$ ,  $g(j)$ , is given by*

$$g(2k) = f^*(k) \equiv 4B * k \left( \frac{p}{1-p} \right)^k$$

*for  $j = 2k$  with  $k \in \mathbb{N}$ , and*

$$g(2k - 1) = \sqrt{1 + \gamma^*} [p f^*(k - 1) + (1 - p) f^*(k)]$$

*for  $j = 2k - 1$ , with  $G^* = \sum_j g(j)$ .*

**Proof.** See Appendix. ■

Proposition 2 gives us the steady state distribution  $g(j)$  of firms over all *transition* organizational capital levels  $\Omega_j \in \Pi$  with  $j \in \mathbb{N}$ . The steady state distribution  $\phi(\Omega)$  of firms over all organizational capital levels  $\Omega \in (\pi_0, +\infty]$ , denoted by  $\phi(\Omega)$ , then follows mechanically from the equality  $\phi(\Omega_j) = g(j)$  and the evolution of organizational capital in between two CEO transitions, as determined by Proposition 1.

Let us denote by  $\varphi(k) \equiv f^*(k)/G^*$  the probability that a transitioning firm has organizational capital level  $2k$  for  $k = 1, 2, \dots$ <sup>15</sup> Figure 4 plots  $\varphi(k)$  for  $p = 1/3$  and  $p = 4/9$  (ignoring integer constraints):

To understand the steady state, consider the three forces that affect the performance distribution of firms: over two CEO transitions, a firm at organizational capital level  $2k$  can transition to  $2(k + 1)$ ,  $2k$ , or  $2(k - 1)$  (and on average it drifts downward); low performers disappear when they hit the death threshold; a fixed mass of firms (not a percentage) is born at every moment. The third force offsets the other two forces: if the total mass of firms became too low, the birth rate would go up. If the total mass of firms became too high, the birth rate would go down. This determines a unique steady state, where the outflow of firms through

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<sup>15</sup>Note that from 6,  $G^* = B/(\sqrt{1 + \gamma^*} - 1)$ .

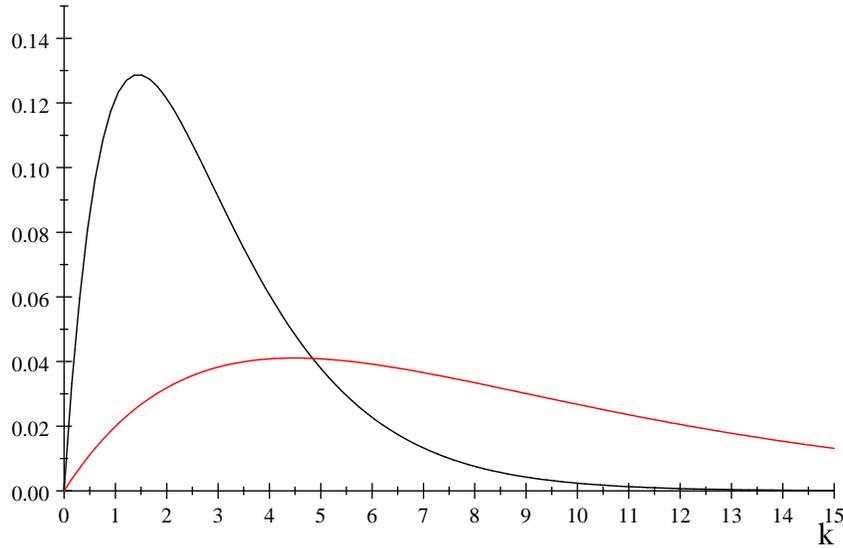


Figure 4: Probability distribution  $\varphi(k) \equiv f^*(k)/G^*$  (ignoring integer constraints) of transitioning firms, and this for  $p = 1/3$  (black line) and  $p = 4/9$  (red line).

death equals the outflow of firms through birth and the organizational capital distribution replicates itself over time.

The steady state distribution has the following properties:

**Corollary 1** *The steady state distribution of (even) transitioning firms,  $f(k)$  :*

- *Is single-peaked, with modal performance level  $k_m = \arg \max_k f^*(k)$  increasing in  $p$ .*
- *Shifts to the right in the sense of first-order stochastic dominance as  $p$  increases:*

$$\frac{d}{dp} F^*(k) < 0 \quad \text{for all } k \geq 1$$

where  $F^*(k)$  is the cumulative probability distribution:

$$F^*(k) \equiv \frac{\sum_{l \leq k} f(l)}{\sum_l f(l)} = 1 - \left( \frac{p}{1-p} \right)^k \left( 1 + k \frac{1-2p}{1-p} \right).$$

- *Follows a power-law at the right tail (top performers): There exists a  $c > 0$  such that for  $k$  large  $f(k) \approx c * \Omega_k^{-\zeta}$  with  $\zeta = \frac{1}{2 \ln(1+\Delta)} * \ln \frac{1-p}{p}$*

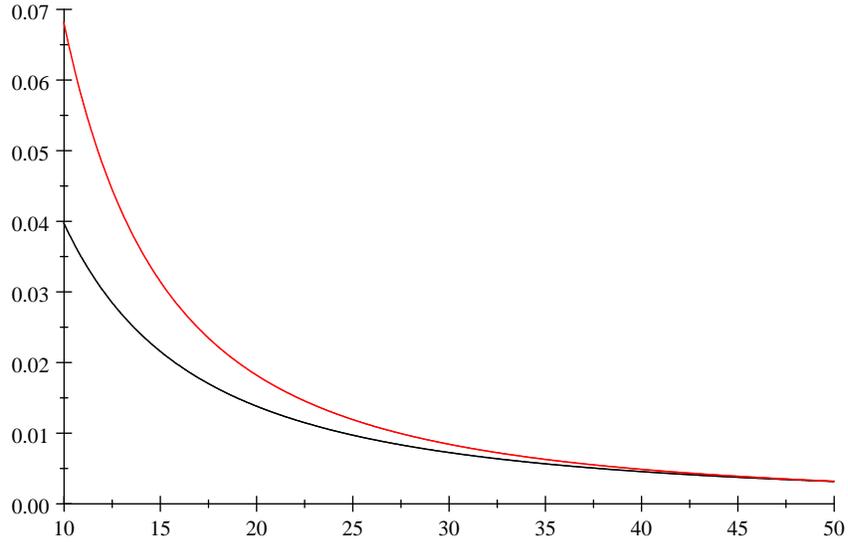


Figure 5: The black curve plots the frequency distribution  $\tilde{\phi}(\Omega_k)$  for  $\Omega_k \geq 10$  (parameters:  $\Omega_0 = 1$ ,  $\Delta = 0.2$  and  $p = 1/3$ ). The red curve plots the asymptotic power-law distribution  $c * \Omega_k^{-\zeta}$  with  $\zeta \approx 1.9$ .

**Proof.** See Appendix. ■

A well-documented empirical regularity on firm dynamics is that the right tail of the firm-size distribution follows a power law (Gabaix 2009, Luttmer 2010). In line with this observation, Proposition 2 implies that the right tail of the distribution of organizational capital follows a power law. Figure 5 illustrates the convergence of the organizational capital distribution to a power law for large levels of organizational capital. The power law approximation is a consequence of the underlying microfoundation, whereby performance follows a Markov chain.

Corollary 1 implies that economies with, on average, higher organizational capital have a performance distribution whose mode is shifted to the right. This result is consistent with the findings by Bloom et al. (2016) on management practices across countries. While this result is intuitive and appealing, we note that steady state models where new firms are born with some exogenous organizational capital level would not deliver this result. Instead, the modal firm in such models cannot have a higher organizational capital in steady state than the organizational capital level at which firms are born, which is inconsistent with the findings

by Bloom et al. (2016).<sup>16</sup>

## 4.2 Steady State: Good and Bad CEOs Have Different Absolute Effects.

The previous analysis was performed under Assumption S3, which states that the positive effect on organizational capital of a good CEO is exactly undone by the negative effect of a bad CEO, as depicted in Figure 6. We now remove this non-generic condition and allow the effect of a good CEO to be greater or smaller than that of a bad CEO. If, for instance, a good CEO has a larger absolute effect, then we have a situation as shown in Figure 7.

The red lines in Figure 7 can be called *neutral transition paths*. Assume without loss of generality that at  $t = 0$ , one of the neutral transition paths goes through  $\Omega_0 = \pi_0 = 1$ . Then, that path is defined by

$$\pi_0(t) = e^{\frac{(\theta^H - \delta)T - \delta \bar{t}}{T + \bar{t}} t}$$

and all other transition paths are defined by  $\Omega_j(t) = \pi_0(t) (1 + \Delta')^j$  where  $j$  is an integer and

$$1 + \Delta' = e^{(\theta^H - \delta)T - \left(\frac{(\theta^H - \delta)T - \delta \bar{t}}{T + \bar{t}}\right)T} = e^{\theta^H \frac{\bar{t}T}{T + \bar{t}}}$$

All firms which experience a CEO transition at time  $t$  have organizational capital  $\Omega = \Omega_j(t)$  for some  $j \in \mathbb{Z}$ . Consider therefore the set of (time-dependent) CEO transition organizational capital levels

$$\Pi(t) = \{\Omega : \exists j \in \mathbb{N} \text{ such that } \Omega = \Omega_j(t) \equiv \pi_0(t) (1 + \Delta')^j\} \quad (7)$$

We are interested in characterizing a steady state economy with a balanced growth path, that is a steady state where all variables grow at a constant rate.<sup>17</sup> In the context of our model, this requires that the performance level  $\pi_0$  at which firms exit is growing at a constant rate as well. It is immediate to see the following:

**Proposition 3** *Suppose  $g(j)$  is the steady state measure of firms with organizational capital  $\Omega_j \in \Pi$  for an environment defined by  $(p, \theta^H, \delta, T, \bar{t})$  with  $(\theta^H - \delta)T = \delta \bar{t}$ . Then at time  $t$ ,*

<sup>16</sup>Formally, if firms are born at organizational capital level  $\Omega_{\bar{k}}$  only and the steady state distribution is unimodal, then the modal organizational capital level cannot be greater than  $\Omega_{\bar{k}}$ . Indeed, suppose it is. Then we must have that  $f^*(\bar{k} + 1) > f^*(\bar{k}) > f^*(\bar{k} - 1)$ . But is easy to see that this also implies that  $f^*(\bar{k} + 2) > f^*(\bar{k} + 1) > f^*(\bar{k})$  and so on, because the difference equation is then always the same for  $k \geq \bar{k} + 2$ .

<sup>17</sup>In neoclassical growth theory, a balanced growth path refers to a steady state where both output and capital grow at a constant rate.

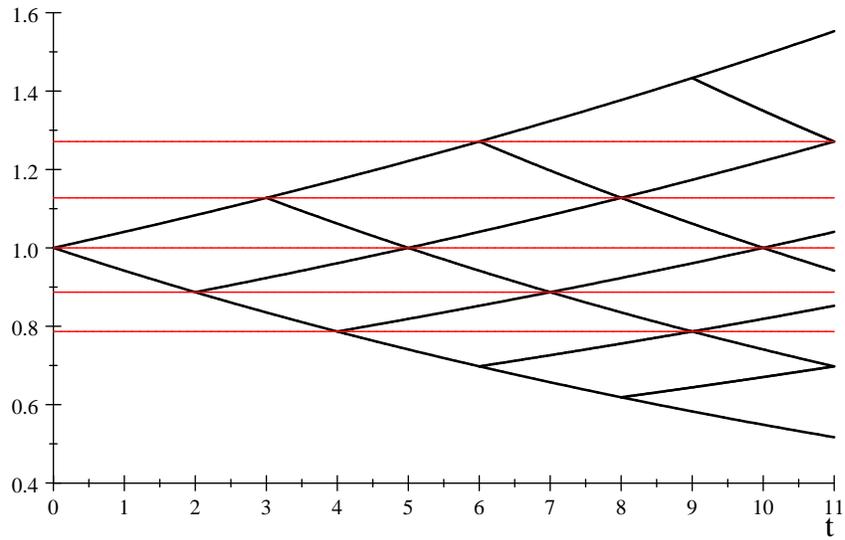


Figure 6: Possible organizational capital paths under Assumption S3 – when  $(\theta^H - \delta)T = \delta\bar{t}$  (same as Figure 3)

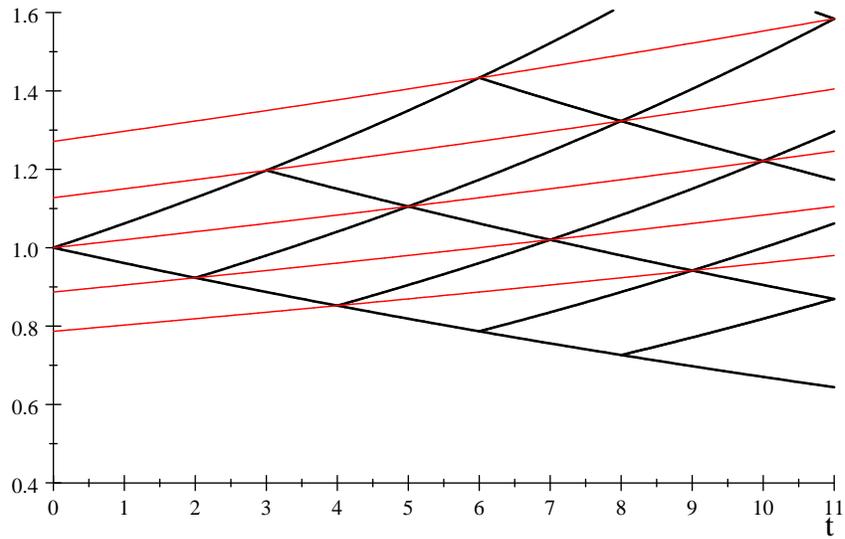


Figure 7: Possible organizational capital paths without Assumption S3. The figure depicts the case with  $(\theta^H - \delta)T > \delta\bar{t}$ .

$g(j)$  is also the steady state measure of firms with organizational capital  $\Omega_j(t) \in \Pi(t)$  for any environment defined by  $(p, \theta^H, \delta', T', \bar{t})$  where firms die whenever they reach  $\pi_0(t)$ .<sup>18</sup> In this steady state:

- All organizational capital levels  $\Omega_j(t) \in \Pi(t)$  as well as total output are increasing at a constant rate.
- Better ex post governance (smaller  $\bar{t}$ ) increases the average CEO behavior/type and growth rate of organizational capital.
- In the limit as ex post agency problems disappear ( $\bar{t}$  goes to zero), firm heterogeneity vanishes as well. Conversely, differences between any two performance levels  $j$  and  $j'$  are increasing in ex post agency problems ( $\bar{t}$ ).

**Proof.** See Appendix. ■

Proposition 3 characterizes a steady state with a balanced growth path. All organizational capital levels  $j = 0, 1, 2, \dots$ , including organizational capital level  $\Omega_0 = \pi_0$  at which firms exit, are growing at a constant rate, given by

$$\frac{(\theta^H - \delta)T - \delta\bar{t}}{T + \bar{t}}$$

An appealing feature of this steady state is that firm heterogeneity disappears as ex post governance becomes perfect. In the limit where bad CEOs are fired immediately ( $\bar{t}$  goes to 0), the difference between any two organizational capital levels  $j$  and  $l > j$  goes to zero as well. Formally, we have that the ratio of two subsequent organizational capital levels is given by

$$\frac{\Omega_{j+1}(t)}{\Omega_j(t)} = 1 + \Delta' = e^{\theta^H \frac{\bar{t}T}{T+\bar{t}}}$$

which is decreasing in  $\bar{t}$  and equals 1 in the limit as  $\bar{t}$  goes to 0. The growth rate of the economy then converges to  $\theta^H - \delta$ , which is exactly the growth rate of the organizational capital of a firm led by a good CEO.

A final comparative static discussed in Proposition 3 regards the impact of better ex post governance (a lower  $\bar{t}$ ) on the average CEO type and average CEO behavior. While the fraction of newly appointed CEOs who are mediocre and behave badly is constant, better ex post governance (a lower tenure for mediocre CEOs) increases the average CEO type.

<sup>18</sup>Maintaining the assumption that  $\theta^H$  and  $\delta'$  are consistent with the conditions in Proposition 1.

## 5 Steady State Predictions

One goal of our simple model was to reconcile key predictions of the three existing approaches, CT, OC, and LC. This section lists the predictions that are consistent with each of the three perspectives. It also generates a number of new testable implications that cross over the three approaches.

The section is therefore divided into four sections: predictions consistent with CT, predictions consistent with OC, predictions consistent with LC, new predictions that cross over multiple approaches.

### 5.1 CT Predictions

Hopenhayn (1992) and Ericson and Pakes (1995) posit that firms are subject to idiosyncratic shocks that affect their performance level. In steady state, we observe persistent performance differences (Gibbons and Henderson 2013). Namely: (i) A cross-section of otherwise identical firms exhibits different performance levels; (ii) The performance difference between any two firms is correlated over time.

Our model makes similar predictions. Let  $\pi_{i,t}$  be the performance of firm  $i$  at time  $t$ . Based on Proposition 2, we immediately see

**Proposition 4** *In steady state: (i) A cross-section of otherwise identical firms exhibits different performance levels ( $Var(\pi_{i,t}) > 0$ ); (ii) The performance difference between any two firms is correlated over time: for any two firms  $i$  and  $j$ , and any  $s > 0$ , we have*

$$Corr(\pi_{i,t} - \pi_{j,t}, \pi_{i,t+s} - \pi_{j,t+s}) > 0$$

This Proposition corresponds to F1 in Table 1. If an econometrician analyzes performance data generated by our model, she would observe persistent cross-sectional differences. This is in line with models like Hopenhayn (1992) and Erickson and Pakes (1995) (except possibly for functional differences in the way (i) and (ii) manifest themselves). However, once organizational and managerial variables are observed, our model makes many more falsifiable prediction that we discuss in the next three subsections.

A well-documented regularity on firm dynamics is further that the right tail of the firm-size distribution follows a power law (Gabaix 2009, Luttner 2010). Building on Hopenhayn

(1992) and Gabaix (1999), Luttner (2007) shows how – given the appropriate assumptions on the entry and exit process– models of firm dynamics with idiosyncratic shocks can generate such power laws. Similarly, as shown in Proposition 1, our model predicts that the right tail of the distribution of organizational capital follows a power law. This corresponds to F2 in Table 1.

## 5.2 OC Predictions

Suppose now that the econometrician observes performance as well as organizational variables. The leading example is Bloom and Van Reenen (2007), where the form of organizational capital observed is the quality of management practices. They document how, after controlling for all observables, (changes in) the quality of management practices explains (changes in) firm performance. Moreover, the quality of management practices is correlated with corporate governance and the availability of managerial human capital.

These predictions are consistent with the relation between performance  $\pi$  and organizational capital  $\Omega$  in our model. Again, take a steady state with a mass of otherwise identical firms. In our model, the quality of governance depends on how well bad CEOs are screened out before they are hired, as captured by  $p$ , and how quickly bad CEOs are fired after they are hired, as captured by  $\bar{t} = \min\{K, R\}$ , where  $R$  is the firm’s monitoring technology and  $K = \ln(1 + \bar{b})/\theta^H$ .  $K$  is increasing in the ability of the CEO to create short term performance (the lower is  $\bar{b}$ , the faster bad CEOs are fired), and decreasing in a measure of managerial human capital,  $\theta^H$ . Note that also  $p$  can be seen as a measure of the supply of managerial human capital (extensive margin).

Define average performance growth,  $E(\Delta\pi)$ , as the average (instantaneous) growth of a randomly selected firm. From Proposition 2, we see:

**Proposition 5** *In steady state:*

(i) *In a cross-section of firms, performance and organizational capital are positively correlated:  $\text{Corr}(\pi_{i,t}, \Omega_{i,t}) > 0$ .*

(ii) *In a cross-section of firms, changes in performance are positively correlated with changes in organizational capital.<sup>19</sup>*

$$\text{Corr}(\pi_{i,t+s} - \pi_{i,t}, \Omega_{i,t+s} - \Omega_{i,t}) \geq 0$$

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<sup>19</sup>If  $s > \bar{t}$ , the correlation is strictly positive.

(iii) Average performance growth is increasing in the quality of ex ante and ex post corporate governance and in the availability of managerial talent:

$$\frac{d}{dp}E(\Delta\pi) > 0, \quad \frac{d}{dt}E(\Delta\pi) < 0, \quad \frac{d}{d\theta^H}E(\Delta\pi) > 0.$$

**Proof.** See Appendix. ■

In steady state, ex ante identical firms have different levels of organizational capital, and this affects their performance. The heterogeneity is due to different leadership styles in the past. The same is true in terms of changes: firms whose last CEO was a good type experience a growth in both their organizational capital and their performance.

Note that in the model the effect of organizational capital on performance is causal. So, if an external intervention such as the one in Bloom et al (2013) were to increase  $\Omega_{i,t}$ , it would also increase performance  $\pi_{i,t+s}$ . Of course, the benefit of the model is that it explains where the heterogeneity in organizational capital comes from and it links it to another set of observables, as discussed below.

Proposition 5 corresponds to F4, F5, and F6 of Table 1.

### 5.3 LC Predictions

The LC approach has studied the effect of CEO variables on firm performance (Bertrand and Schoar 2003, Bennedsen et al 2007, Kaplan et al 2012, Bandiera et al 2016). The CEO variables considered include the identity, the characteristics, and the behavior of the CEO.

The next section, where CEOs will be allowed to work at multiple firms and salaries are endogenous, will generate even more testable predictions on career trajectories and compensation patterns. However, for now, let us focus on the implications of the stylized model considered so far.

Consistent with the core of those observed patterns, our model predicts a connection between CEO variables and performance. In the equilibrium described in Proposition 1, good CEOs behave differently, produce more organizational capital, generate better performance, and stay longer on the job. This in turn leads to a number of cross-sectional patterns:

**Proposition 6** (a) *In steady state, firm  $i$ 's current performance level  $\pi_{i,t}$  is higher when past CEOs: (i) Chose the organization-building behavior rather than the short-term profit boost*

( $x_{i,t-s} = 1$  not 0); (ii) Were of the high type rather than the low type ( $\theta_{i,t-s} = \theta_H$  not  $\theta_L$ ); (iii) Had longer on-the job tenure ( $T$  not  $\bar{t}$ ).

(b) In steady state, in a cross-section of firms, better governance (lower  $\bar{b}$  or lower  $R$ ) weakly increases the average behavior and type of the CEO, the tenure variance among CEOs, and average performance.

Note that predictions (a)(i) and (a)(ii) hold also in a probabilistic sense. If certain categories of CEOs are more likely to be high types and behave well, the firms run by those CEOs will in general have better performance and higher organizational capital. This rationalizes the findings of Bennedsen et al (2007) that family and professional CEOs impact long term performance differently.<sup>20</sup> Finding (a)(iii) is to the best of our knowledge untested but it is an immediate implication of a model where bad CEOs are more likely to be dismissed early. Finding (b) is consistent with classic findings of the literature on international differences in governance (see for instance the influential survey by Shleifer and Vishny 1997). These findings correspond to F8 and F9 of Table 1.

#### 5.4 Predictions Linking OC and LC

As mentioned in the introduction, the OC and LC approaches have mostly operated in a separate manner. Our model suggests a number of testable implications involving OC variables and LC variables. CEOs play a part in growing or destroying organizational capital, which in turn determines performance. So our model predicts a lagged effect of CEO variables on organizational capital. It is immediate to see that:

**Proposition 7** (a) In steady state, the rate of growth of organizational capital  $\Omega_{i,t}$  is greater when the current CEO: (i) Chooses the organization-building behavior rather than the short-term profit boost ( $x_{i,t} = 1$  not 0); (ii) Is of the high type rather than the low type ( $\theta_{i,t} = \theta_H$  not  $\theta_L$ ); (iii) Has longer on-the job tenure ( $T$  not  $\bar{t}$ ).

(b) Firm  $i$ 's current organizational capital  $\Omega_{i,t}$  is higher when past CEOs: (i) Chose the organization-building behavior rather than the short-term profit boost ( $x_{i,t-s} = 1$  not 0); (ii) Were of the high type rather than the low type ( $\theta_{i,t-s} = \theta_H$  not  $\theta_L$ ); (iii) Had longer on-the-job tenure ( $T$  not  $\bar{t}$ ).

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<sup>20</sup>Our model does not capture the feature that family CEOs are often difficult to dismiss. The result would hold a fortiori.

(c) *Controlling for current organizational capital  $\Omega_{i,t}$ , past CEO variables have no predictive value on current firm performance  $\pi_{it}$ .*

Organizational capital is a stock, while CEO behavior is a flow that influences the growth of the stock. Part (a) is an immediate consequence of this: organizational capital grows faster when at least one of the following is true: the CEO behaves better, is a higher type, or has been there for longer (meaning that his type is more likely to be high). Part (b) is the cumulative correspondent of part (a): the current level of organizational capital is predicted by the type, behavior, and tenure of past CEOs. For instance, a firm that has experienced a sequence of short-lived CEOs is predicted to have a lower organizational capital. The first two parts of the proposition correspond to F8 of Table 1.

Part (c), which corresponds to F11 of Table 1, helps distinguish the present model from other stories that give the CEO a productive role. It is reasonable to expect Part (c) to be falsified in reality. For instance, a charismatic CEO may have a direct motivating effect on employees that does not go through the growth of organizational capital. Such a model would create a direct link between CEO type/behavior and performance that would violate Part (c).

In practice it would be interesting to see how much of the CEO effect operates through organizational capital, as postulated in our model, as opposed to other avenues. Part (c) suggests a possible way of disentangling these two sets of channels.

## **6 Model with Endogenous Wages and CEO Quality**

So far we have assumed that CEOs only work once. What happens if a CEO can “prove herself” in one firm and then go to another firm? Which firms will hire better CEOs?

In this section, we first show a general result: if multiple CEO types are available and higher types are scarce, better CEOs will be hired by firms that already have more organizational capital.

We then apply this general result to a situation where CEOs can take a succession of jobs. In equilibrium, rookie CEOs are hired by low-performance firms. If they succeed, they move on to better firms. The salary differential between new and proven CEOs is determined in equilibrium.

## 6.1 The Marginal Value of CEO Quality

Reconsider our baseline model but assume that there are multiple categories of prospective CEOs. CEOs in category  $j$  have a  $p_j$  probability of being type  $\theta_H$  and a  $1 - p_j$  probability of being type  $\theta_L$ . CEO compensation is endogenous. Assume that there is an unlimited number of CEOs of the lower category ( $j = 1$ ), but the total number of CEOs of higher categories is smaller than the total number of firms, so that CEO quality will have to be rationed. In equilibrium all CEOs in category  $j$  earn the same instantaneous wage  $w_j$  (we are maintaining the hypothesis that the only possible form of compensation is a constant per period wage). Which firms will pay to get the most promising CEOs?

We assume that the cash flow boost  $b$  is not only bounded above by  $\bar{b}$ , but it also does not allow the CEO to reach a performance level that is greater than that of a high-quality CEO who chooses  $x = 1$ .<sup>21</sup> Without this additional assumption, a highly impatient firm might ask its CEO to engage in short term profit boosting. Alternatively, one could assume that  $b$  is not profit boosting, but covert borrowing: unbeknown to the board, the CEO borrows funds on behalf of the firm at instantaneous rate  $\rho$  that must be repaid by the firm when the CEO is fired. The result below holds a fortiori in the alternative scenario.

For the rest, the model is unchanged. We can show:

**Proposition 8** *Consider a CEO with  $p'$  and a CEO with  $p'' > p'$ . Let  $\Omega'$  and  $\Omega''$  denote the organizational capital levels of the firms employing the two CEOs respectively. If firms are sufficiently impatient ( $\rho$  is sufficiently high), in steady state  $\Omega' \leq \Omega''$ .*

**Proof.** See Appendix ■

Proposition 8 says that more promising CEOs must be hired by firms with higher organizational capital. A CEO with a higher  $p$  is more likely to protect the firm's organizational capital – something that is more useful when the size of the organizational capital is larger. The key assumption is that the effect of CEO behavior/type on organizational capital is multiplicative:

$$\dot{\Omega}_t = (\theta x - \delta) \Omega_t,$$

To reverse this effect, one must assume that

$$\dot{\Omega}_t = \theta x z(\Omega_t) - \delta \Omega_t,$$

---

<sup>21</sup>Formally, if  $t$  is the time when the CEO was hired and  $s$  is her tenure,  $b_s \in [0, \min(\bar{b}, e^{(\theta^H - \delta)s})]$ .

where  $z(\cdot)$  is a decreasing function. In that case, firms with lower organizational capital may hire more promising CEOs.

The requirement that  $\rho$  is sufficiently high is mainly technical and derives from the inability to characterize the value function of this problem.

Proposition 8 is related in spirit to results on assortative matching between CEOs and firm size (Tervio (2008) and Gabaix and Landier (2008)). There, more capable CEOs are matched with larger firms. Here, CEOs who are more likely to be good are matched with firms with a higher organizational capital. The connection would become direct if we used the production function in (2).

Of course, the fact that more expensive CEOs are more likely to be good types does not eliminate the stochastic element that underpins our organizational capital process. Even an expensive CEO may turn out to be bad and destroy organizational capital. The following section explores such dynamics.

## 6.2 Equilibrium with Proven CEO Quality

Consider now the endogenous allocation of CEO talent. As before, there are two types of CEOs, good and bad, and CEOs that are revealed to be bad can be fired at any time. We maintain S1-S3 above so that a bad CEO exactly undoes the effect of a good CEO on organizational capital. But rather than retiring after a period of time  $T$ , a good CEO may move to a different firm.<sup>22</sup> We assume that the type of a CEO is only partially persistent. A CEO with a low type always remains low. A CEO with a high type becomes a low type with probability  $\kappa$  at the end of a contract term  $T$ .

There are then three categories of CEOs. A *new CEO* denote a CEO who has never worked. We assume that there is never any scarcity of potential new CEOs and a share  $p_L$  of them is of the high type. The type of new CEOs is unobservable. Let a *successful CEO* denote a CEO which has already been hired at least once and completed a period of time  $T$  (which is now the “standard” contract duration). We denote by  $p_H \equiv 1 - \kappa$  the probability that a successful CEO remains a high type. We assume that the type persistence is sufficiently large so that  $p_H > p_L$ . Finally, let a *failed CEO* denote a CEO who was hired and then fired.

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<sup>22</sup>For simplicity, we assume that CEOs can only move to a different firm after their contract term  $T$ . Without loss of generality, their contract may also be renewed at the same firm.

We consider a competitive market for managerial talent, where firms offer CEOs a wage  $w$  based on their performance. The wage  $w$  is fixed for the duration of the contract (or until the CEO is fired). Since there is no scarcity of new CEO's, the wage of new CEO's is set equal to their reservation value, which we normalize to 0. For the same reason, no firm ever hires a failed CEO. Consider now the successful CEOs. In steady state, the fraction of (previously) successful CEOs among all CEO hires is given by

$$\mu = p_L(1 - \mu) + p_H\mu = \frac{p_L}{1 - p_H + p_L}.$$

In line with the intuition developed in Proposition 8, successful CEOs will receive a positive wage  $\bar{w} > 0$  and they will be hired by the share  $\mu$  of most productive firms. In particular, we obtain the following result, proven for the case where  $\rho$  is sufficiently large (firms are sufficiently myopic):

**Proposition 9** *Assume  $\rho$  is sufficiently large. In steady state, there exists a cutoff  $\bar{\pi}$  such that:*

- *Firms with productivity  $\pi_t > \bar{\pi}$  hire only successful CEOs and pay a wage differential  $\bar{w} > 0$ .*
- *Firms with  $\pi_t < \bar{\pi}$  hire only new CEOs.*
- *No firm hires failed CEOs.*
- *Firms at  $\pi_t = \bar{\pi}$  are indifferent between hiring a new CEO or a (more expensive) successful CEO.*
- *Each firm's organizational capital at CEO transition times follows a Markov chain: if the firm is at level  $\Omega_j$ , the probability of going up (down) one level is given by  $p_i$  ( $(1 - p_i)$ ), where*

$$p_i \begin{cases} = p_L & \text{if } \Omega_j < \bar{\pi} \\ \in [p_L, p_H] & \text{if } \Omega_j = \bar{\pi} \\ = p_H & \text{if } \Omega_j > \bar{\pi} \end{cases}$$

**Proof.** We prove by contradiction. Let  $q = (q_l)_{l \in \mathbb{N}}$  be the steady state hiring profile where  $q_l \in \{p_H, p_L\}$  denotes the type of manager hired at organizational capital level  $\Omega_j \in \Pi$ . Assume

that our proposition does not hold. Then there must exist an  $m$  such that  $q(m) = p_H$  but  $q(m + 1) = p_L$ . But by Proposition 8, this is impossible. ■

When  $\Omega_k = \bar{\pi}$ , the firm is indifferent over whether to hire a successful CEO or a new one. This creates (local) equilibrium multiplicity, which is allowed for in the statement of the proposition. For performance levels above and below  $\bar{\pi}$ , the stochastic process is uniquely defined.

### 6.3 Implications of the Endogenous Wage Model

Section 5 discussed the testable implications of the baseline model where CEOs can only work for one period. Let us now examine the additional predictions we can make when CEOs work for multiple periods and wages are endogenous.

In the equilibrium in Proposition 9, CEO careers display certain patterns. Bad CEOs are employed only once: after damaging the organizational capital of one firm, they become unemployable. Good CEOs are employed repeatedly and receive a compensation premium until they underperform. Firms with higher organizational capital hire better CEOs.

**Proposition 10** *In steady state:*

(i) *Firms with better performance and higher organizational capital employ CEOs of a better type (on average), with better behavior (on average), who are paid more.*

(ii) *The current employment status and compensation of a CEO depends on the change in performance and organizational capital of its previous firm.*

(iii) *A fixed-effect regression on data generated by this model returns significant individual coefficients but underestimates the true effect of individual CEOs on performance.*

**Proof.** See Appendix. ■

Prediction (i) (F3 in Table 1) relates to an influential prediction of the CT literature: larger firms should hire better CEOs on average (Tervio (2008) and Gabaix and Landier (2008)). It also has a potential connection with the findings by Bender et al (2016) that firms with better management performance also hire workers – especially highest paid workers – with higher human capital.

Prediction (ii) (F12 in table 1) relates the career path of CEOs to their effect on previous firms they worked for. Past employers of CEOs who currently command higher wages and work

for more productive firms have experienced unusually strong growth in both performance and organizational capital.

Prediction (iii) (F13 in table 1) relates to the estimation of CEO fixed effects developed by Bertrand and Schoar (2003). Consider an econometrician who observes the last  $N$  periods of a random sample of firms and estimates fixed effects for firms and CEOs. As firms with high organizational capital hire better CEOs on average, part of the CEO fixed effect will be attributed to the firm, thus underestimating the true causal effect of CEO on performance. The presence of this selection bias means that the CEO fixed effect estimated by Bertrand and Schoar (2003) is a lower bound to the true fixed effect.

## 7 Conclusions

This paper began by noting that economists have studied the effect of management on firm performance from three distinct perspectives: CT, OC, and LC. The goal of the paper was to develop a parsimonious model that can reconcile key patterns predicted or observed by the three perspectives. The main novel ingredient of the model was organizational capital, a set of productive assets that can only be produced with the direct input of the firm's leadership and is subject to an agency problem. Besides yielding predictions that are consistent with the three perspectives, the model generates novel predictions that combine OC and LC variables.

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## Appendix I: Proof of Proposition 2

**Proposition 2:** *The distribution of firms over transition organizational capital levels  $\Omega_j \in \Pi$ ,  $g(j)$ , is given by*

$$g(2k) = f^*(k) \equiv 4B * k \left( \frac{p}{1-p} \right)^k$$

for  $j = 2k$  with  $k \in \mathbb{N}$ , and

$$g(2k-1) = \sqrt{1+\gamma^*} [pf^*(k-1) + (1-p)f^*(k)]$$

for  $j = 2k-1$ , with  $G^* = \Sigma_j g(j)$ .

For the proof, we proceed in five steps.

**Part 1: Solution of the difference equation** In steady state,  $f(\cdot)$  must satisfy the difference equation (5),

$$f(k) = (1+\gamma) \left( p^2 f(k-1) + 2(1-p)pf(k) + (1-p)^2 f(k+1) \right), \quad (8)$$

with the following boundary conditions:

$$f(0) = 0 \text{ and } f(1) = \frac{B}{(1+\gamma)(1-p)^2}$$

For every value of  $\gamma$ , standard techniques show that the difference equation (5) has at most one solution with non-negative values of  $f(\cdot)$  as follows:

$$f(k) = \frac{A^k - D^k}{C},$$

where

$$\begin{aligned} A &= A(\gamma) = \frac{1}{2} \left( \frac{1-2p(1+\gamma)+2p^2(1+\gamma)}{(1-p)^2(1+\gamma)} + \sqrt{\frac{1-4p(1+\gamma)+4p^2(1+\gamma)}{(1-p)^4(1+\gamma)^2}} \right); \\ D &= D(\gamma) = \frac{1}{2} \left( \frac{1-2p(1+\gamma)+2p^2(1+\gamma)}{(1-p)^2(1+\gamma)} - \sqrt{\frac{1-4p(1+\gamma)+4p^2(1+\gamma)}{(1-p)^4(1+\gamma)^2}} \right); \\ C &= C(\gamma) = \frac{1}{B} (1+\gamma)(1-p)^2 \sqrt{\frac{1-4p(1+\gamma)+4p^2(1+\gamma)}{(1-p)^4(1+\gamma)^2}} \end{aligned}$$

Let

$$\gamma^* \equiv \frac{(1-2p)^2}{1-(1-2p)^2}$$

Consider the term under the three square roots that appears in the expressions of  $A$ ,  $D$ , and  $C$ . When  $\gamma > \gamma^*$ , the term is negative, in which case it can be shown that  $f(k)$  is strictly negative for certain values of  $k$ .<sup>23</sup> When  $\gamma \rightarrow \gamma^*$ , the expression above tends to:

$$\begin{aligned} f^*(k) &= B \left( \frac{1 - (1 - 2p)^2}{(1 - p)^2} \right) k \left( \frac{p}{1 - p} \right)^{k-1} \\ &= 4B * k \left( \frac{p}{1 - p} \right)^k \end{aligned}$$

**Part 2: Impossibility of  $\gamma > \gamma^*$**  We first rule out steady states with  $\gamma > \gamma^*$ . Intuitively, there cannot be a steady state with an excessively large  $\gamma$  because the distribution would keep shifting to the right. This impossibility is shown by proving that in a steady state with  $\gamma > \gamma^*$  there must be negative values of  $f(k)$ .

**Lemma 3** *If  $\gamma > \gamma^*$  there exists  $k > 1$  such that  $f(k) < 0$ .*

**Proof.** Consider the term under the three square roots that appears in the expressions of  $A$ ,  $D$ , and  $C$ . When  $\gamma > \gamma^*$ , the term is negative and  $A$ ,  $B$ , and  $C$  are complex numbers.

Note that we can rewrite

$$f(k) = H \frac{(a + \sqrt{b})^k - (a - \sqrt{b})^k}{\sqrt{b}},$$

where

$$\begin{aligned} H &= \frac{B}{2^k} \left( \frac{1}{(1 - p)^2 (1 + \gamma)} \right)^k \\ a &= 1 - 2p(1 + \gamma) + 2p^2(1 + \gamma) \\ b &= 1 - 4p(1 + \gamma) + 4p^2(1 + \gamma) \end{aligned}$$

Note that  $b < 0$  when  $\gamma > \gamma^*$  and  $\sqrt{b}$  is a complex number.

As  $H$  is a positive real number, the analysis will focus on the sign of:

$$S(k) \equiv \frac{(a + \sqrt{b})^k - (a - \sqrt{b})^k}{\sqrt{b}}$$

---

<sup>23</sup>The fact that the value under the square root is negative is not a problem per se because all terms with an even power drop out. However, for  $k$  large enough,  $f(k) < 0$ .

A feasible solution for  $f(k)$  does not exist when  $\gamma$  is too high, because a very high birth rate leads to explosive growth in the number of firms.

We begin by showing that, although  $\sqrt{b}$  is a complex number,  $S(k)$  is a real number for every  $k$ . To see this note that:

$$S(k) = \frac{a^k - a^k}{\sqrt{b}} + 2\delta_1 \frac{a^{k-1}\sqrt{b}}{\sqrt{b}} + \delta_2 \frac{a^{k-2}(\sqrt{b})^2 - a^{k-2}(\sqrt{b})^2}{\sqrt{b}} + 2\delta_3 \frac{a^{k-3}(\sqrt{b})^3}{\sqrt{b}} + \dots$$

where

$$\delta_j = \frac{k!}{j!(k-j)!}.$$

Therefore

$$S(k) = 2 \left( \delta_1 a^{k-1} + \delta_3 a^{k-3} b + \delta_5 a^{k-5} b^2 + \delta_7 a^{k-7} b^3 + \dots \right),$$

which is a real number.

If  $a < 0$ , we have that  $S(2) = \delta_1 a < 0$ , and the lemma is proven for  $k = 2$ . If  $a = 0$ ,  $S(k) = 0$  for all  $k$ , which is clearly impossible. Therefore, from now on, assume that  $a > 0$  (note that with  $\gamma = \gamma^*$ , we have  $a = 1/2$ , so the case is relevant).

Rewrite the summation as

$$\begin{aligned} \frac{1}{2} S(k) &= \delta_3 a^{k-3} \left( \frac{\delta_1}{\delta_3} a^2 + b \right) + \delta_7 a^{k-7} b^2 \left( \frac{\delta_5}{\delta_7} a^2 + b \right) + \dots \\ &= \sum_{j=0}^k \delta_{4j+3} a^{k-3-4j} b^{2j} \left( \frac{\delta_{1+4j}}{\delta_{3+4j}} a^2 + b \right) \end{aligned}$$

Note that for  $i \geq 3$

$$\frac{\delta_{i-2}}{\delta_i} = \frac{\frac{k!}{(i-2)!(k-i+2)!}}{\frac{k!}{i!(k-i)!}} = \frac{(i-2)!(i-1)i(k-i)!}{(i-2)!(k-i+2)(k-i+1)(k-i)!} = \frac{(i-1)i}{(k-i+2)(k-i+1)}$$

Thus,

$$\frac{1}{2} S(k) = \sum_{j=0}^k \delta_{4j+3} a^{k-3-4j} b^{2j} \left( \frac{(3+4j-1)(3+4j)}{(k-3-4j+2)(k-3-4j+1)} a^2 + b \right)$$

Take any  $k$  that is the square of an integer  $h$ . Consider a series that comprises the first  $h$  elements of  $S(k)$ :

$$\begin{aligned} \frac{1}{2} \tilde{S}(h) &= \sum_{j=0}^h \delta_{4j+3} a^{k-3-4j} b^{2j} \left( \frac{(3+4j-1)(3+4j)}{(k-3-4j+2)(k-3-4j+1)} a^2 + b \right) \\ &= \sum_{j=0}^h \delta_{4j+3} a^{h^2-3-4j} b^{2j} \left( \frac{(3+4j-1)(3+4j)}{(h^2-3-4j+2)(h^2-3-4j+1)} a^2 + b \right) \end{aligned}$$

By construction

$$\lim_{h \rightarrow \infty} \tilde{S}(h) = \lim_{k \rightarrow \infty} S(k)$$

Note that, with  $h$  is sufficiently large,

$$\frac{(3 + 4j - 1)(3 + 4j)}{(h^2 - 3 - 4j + 2)(h^2 - 3 - 4j + 1)} \leq \frac{(3 + 4h - 1)(3 + 4h)}{(h^2 - 3 - 4h + 2)(h^2 - 3 - 4h + 1)} \leq \frac{16}{h^2},$$

and hence

$$\frac{1}{2} \tilde{S}(h) \leq \left( \frac{16}{h^2} a^2 + b \right) \sum_{j=0}^h \delta_{4j+3} a^{h^2-3-4j} b^{2j}.$$

As  $a$  and  $b^2$  are positive, for  $h$  sufficiently large the value of the summation is positive.

Therefore:

$$\lim_{h \rightarrow \infty} \frac{1}{2} \tilde{S}(h) = b \lim_{h \rightarrow \infty} \sum_{j=0}^h \delta_{4j+3} a^{h^2-3-4j} b^{2j} < 0,$$

which completes the proof of the lemma. ■

**Part 3: Impossibility of  $\gamma < \gamma^*$**  We now show that  $\gamma < \gamma^*$  is not consistent with the condition that the steady state is reachable from below.

Consider the  $N$ -level version of our problem where we impose the boundary condition  $f_t(k) = 0$  for  $k > N$  with  $N$  a finite positive integer. In this finite version of our problem, organizational capital is bounded above by  $\Omega_N$ .

Let us denote by  $f_{N,t}(k)$  the mass of firms with organizational capital level  $2k$  at time  $t$  in the finite  $N$ -level version of our problem. If a steady state of this finite version exists where  $f_{N,t}(k) = f_N(k)$  for every  $t$  and  $k$ , then the total mass of transitioning firms will also be constant and the two-period steady state spin-off rate will be given by

$$\gamma_N \equiv (1 + B/G_N)^2 - 1$$

where  $G_N$  is the steady state measure of transitioning firms.

In steady state,  $f_N(\cdot)$  must solve the following recurrence equation for all  $k = 1, 2, \dots, N$ :

$$f_N(k) = (1 + \gamma_N) \left( p^2 f_N(k-1) + 2(1-p) p f_N(k) + (1-p)^2 f_N(k+1) \right). \quad (9)$$

in combination with two boundary conditions:

$$\begin{aligned} f_N(0) &= 0 \\ f_N(1) &= \frac{B}{(1-p)^2(1+\gamma_N)} \\ f_N(N+1) &= 0 \end{aligned}$$

The first two boundary conditions are identical as before. The third boundary condition caps organizational capital  $\Omega_N$ , making it a finite approximation of our original problem. Note that in our original problem,  $\lim_{k \rightarrow \infty} f(k) = 0$ .<sup>24</sup>

The following result holds:

**Lemma 4** *In a steady state of the finite  $N$ -level version of our problem, we must have that*

$$\gamma_N \geq \gamma^* = \frac{(1-2p)^2}{4p(1-p)}$$

**Proof.** Assume  $f_N(\cdot)$  characterizes a steady state of the finite  $N$ -level version of our problem. Define  $\mathbf{f}_N = [f_N(1), f_N(2), \dots, f_N(N)]^T$  and

$$\mathbf{A}_N = \begin{bmatrix} b & c & 0 & \cdots & 0 \\ a & b & c & \cdots & 0 \\ & \cdots & \cdots & \cdots & \\ 0 & \cdots & a & b & c \\ 0 & \cdots & 0 & a & b \end{bmatrix}_{N \times N}$$

with  $a = (1 + \gamma_N)p^2$ ,  $b = 2(1 + \gamma_N)p(1-p)$ ,  $c = (1 + \gamma_N)(1-p)^2$ . Given the recurrence equation (9) and the boundary conditions  $f_N(0) = 0$  and  $f_N(N+1) = 0$ , we must have that

$$\mathbf{f}_N = \mathbf{A}_N \times \mathbf{f}_N$$

Cheng (2003, Theorem 16) states that the eigenvalues of  $\mathbf{A}_N$  are given by

$$b + 2(\text{sign } a)\sqrt{ac} \cos \frac{i\pi}{\pi+1} \text{ for } i = 1, 2, \dots, N.$$

Let  $\lambda$  be the largest real eigenvalue of  $\mathbf{A}_N$ . As the value of a cosine can never be larger than one, this implies that for any  $N$

$$\lambda \leq b + 2\sqrt{ac} = 4(1 + \gamma_N)p(1-p)$$

If  $\lambda < 1$ , there exists no vector  $\mathbf{f} > \mathbf{0}$  such that  $\mathbf{f} = \mathbf{A}_N \times \mathbf{f}$ . Hence, a necessary condition for  $f_N(\cdot)$  to be a steady state is that  $\lambda \geq 1$  or still  $4(1 + \gamma_N)p(1-p) \geq 1$  or still

$$\gamma_N \geq \frac{(1-2p)^2}{4p(1-p)}$$

---

<sup>24</sup>If not, the mass of transitioning firms  $M$  is infinite, which cannot be a steady state.

■

Consider now again our original problem. In the text, we defined a *steady state reachable from below* as a steady state which can be the limit of a sequence of steady states of the finite  $N$ -level version of our problem when  $N \rightarrow \infty$ . Together with the previous lemma this implies:

**Lemma 5** *In a steady state reachable from below, we must have that*

$$\gamma \geq \gamma^* \equiv \frac{(1-2p)^2}{4p(1-p)}$$

We conclude that if a steady state  $f(\cdot)$  exists, then it must be that

$$1 + \gamma \equiv \left(1 + \frac{B}{G}\right)^2 = 1 + \gamma^*$$

so that the total mass of transitioning firms (which excludes new-born firms) is given by

$$\begin{aligned} G^* &= \frac{1}{\sqrt{1+\gamma^*} - 1^*} B \\ &= \frac{1 + \sqrt{1+\gamma^*}}{\gamma^*} B \end{aligned}$$

The linear difference equation (5) then implies that

$$f(k) = f^*(k) \equiv 4B * k \left(\frac{p}{1-p}\right)^k$$

**Part 4: Total measure of transitioning firms.** To conclude, we verify that we have indeed that

$$\sum_{k=1}^{k=\infty} g^*(k) = G^*$$

We have that

$$\sum_{k=1}^{k=\infty} g^*(k) = \sum_{k=1}^{k=\infty} f^*(k) + \sum_{k=1}^{k=\infty} f^{*-}(k)$$

where  $f^*(k) = g(2k)$  is the steady state distribution of firms at even transition and  $f^{*-}(k) = g(2k-1)$  is the steady state distribution of firms at odd-numbered transitions. Considering first firms at even transitions, we have that

$$\sum_{k=1}^{k=\infty} f^*(k) = 4B \sum_{k=1}^{k=\infty} k \left(\frac{p}{1-p}\right)^k = 4B \frac{(1-p)p}{(1-2p)^2} = \frac{B}{\gamma^*}$$

Next consider the odd-transitioning firms. For  $k = 1, \dots$  we have that

$$f^-(k) = (pf(k-1) + (1-p)f(k))\sqrt{1+\gamma^*}$$

and thus

$$\begin{aligned} \sum_{k=1}^{k=\infty} f^-(k) &= \left( p \sum_{k=1}^{k=\infty} f(k-1) + (1-p) \sum_{k=1}^{k=\infty} f(k) \right) \sqrt{1+\gamma^*} \\ &= \left( p \sum_{k=1}^{k=\infty} f(k) + (1-p) \sum_{k=1}^{k=\infty} f(k) \right) \sqrt{1+\gamma^*} \\ &= \sqrt{1+\gamma^*} \sum_{k=1}^{k=\infty} f(k) \end{aligned}$$

We then obtain that

$$\begin{aligned} \sum_{k=1}^{k=\infty} g^*(k) &= \left( 1 + \sqrt{1+\gamma^*} \right) \sum_{k=1}^{k=\infty} f^*(k) \\ &= \left( 1 + \sqrt{1+\gamma^*} \right) \frac{B}{\gamma^*} \\ &= G^* \end{aligned}$$

This concludes the proof of Proposition 2.

## Appendix II: Other Proofs

**Corollary 1** *In steady state, for  $\Omega_k \in \Pi$  large,  $\phi(k)$  approximates a power law: There exists a  $c > 0$  such that for  $\pi_k$  large  $\phi(k) \approx c * \pi_k^{-\zeta}$  with  $\zeta = h * \ln \frac{1-p}{p}$  with  $h \equiv 1/(2 \ln(1 + \Delta))$ .*

**Proof.** From Proposition 2, we have that  $f(k) \sim k \left( \frac{p}{1-p} \right)^k$  for  $\Omega_k \in \Pi$ . Wlog, set  $\Omega_0 = 1$ . Since  $\Omega_k = (1 + \Delta)^{2k}$ , we can rewrite this as

$$f(\Omega_k) \sim h * \ln \Omega_k * \left( \frac{p}{1-p} \right)^{h \ln \Omega_k}$$

where  $h = 1/(2 \ln(1 + \Delta)) > 0$ . Consider now  $\Omega_{k+l} = a\Omega_k \in \Pi$  where  $a = (1 + \Delta)^{2l}$ . Then

$$f(a\Omega_k)/f(\Omega_k) = \ln a\Omega_k / \ln \Omega_k * \left( \frac{p}{1-p} \right)^{h(\ln a\Omega_k - \ln \Omega_k)}$$

from which

$$\lim_{k \rightarrow \infty} f(a\Omega_k)/f(\Omega_k) = \left( \frac{p}{1-p} \right)^l = a^{-\zeta}$$

with

$$\begin{aligned}\zeta &= -\frac{l}{\ln a} * \ln \frac{p}{1-p} \\ &= -\frac{1}{2 \ln(1 + \Delta)} * \ln \frac{p}{1-p}\end{aligned}$$

It follows that for  $\Omega_k$  large,  $f(\Omega_k) \approx c * \Omega_k^{-\zeta}$  for some constant  $c$ . ■

**Proposition 3** *Suppose  $g(j)$  is the steady state measure of firms with organizational capital  $\Omega_j \in \Pi$  for an environment defined by  $(p, \theta^H, \delta, T, \bar{t})$  with  $(\theta^H - \delta)T = \delta\bar{t}$ . Then at time  $t$ ,  $g(j)$  is also the steady state measure of firms with organizational capital  $\Omega_j(t) \in \Pi(t)$  for any environment defined by  $(p, \theta'^H, \delta', T', \bar{t}')$  where firms die whenever they reach  $\pi_0(t)$ .<sup>25</sup> In this steady state:*

- *All organizational capital levels  $\Omega_j(t) \in \Pi(t)$  as well as total output are increasing at a constant rate.*
- *Better ex post governance (smaller  $\bar{t}$ ) increases the average CEO behavior/type and growth rate of organizational capital.*
- *In the limit as ex post agency problems disappear ( $\bar{t}$  goes to zero), firm heterogeneity vanishes as well. Conversely, differences between any two performance levels  $j$  and  $j'$  are increasing in ex post agency problems ( $\bar{t}$ ).*

**Proof.** Compute  $\Omega_{2k}(t)$  for  $k = 1, 2, 3, \dots$  and define level  $k$  as  $\Omega_{2k} = \Omega_{2k}(t)$ . The recurrence equation for organizational capital levels  $k = 1, 2, 3, \dots$  is identical to that analyzed in Proposition 2. Proposition 3 applies to the steady state distribution over ordinal levels  $k = 1, 2, 3, \dots$ . It also applies to time-variant cardinal levels defined by  $\Omega_{2k}(t)$  with  $k = 1, 2, 3, \dots$ . ■

**Proposition 5** *In steady state:*

(i) *In a cross-section of firms, performance and organizational capital are positively correlated:  $\text{Corr}(\pi_{i,t}, \Omega_{i,t}) > 0$ .*

(ii) *In a cross-section of firms, changes in performance are positively correlated with changes in organizational capital.<sup>26</sup>*

$$\text{Corr}(\pi_{i,t+s} - \pi_{i,t}, \Omega_{i,t+s} - \Omega_{i,t}) \geq 0$$

<sup>25</sup>Maintaining the assumption that  $\theta'^H$  and  $\delta'$  are consistent with the conditions in Proposition 1.

<sup>26</sup>If  $s > \bar{t}$ , the correlation is strictly positive.

(iii) Average performance growth is increasing in the quality of ex ante and ex post corporate governance and in the availability of managerial talent:

$$\frac{d}{dp}E(\Delta\pi) > 0, \quad \frac{d}{dt}E(\Delta\pi) < 0, \quad \frac{d}{d\theta^H}E(\Delta\pi) > 0.$$

**Proof.** (i) Immediate.

(ii) Let  $t' \geq t$  be the first CEO transition in  $[t, t+s]$  and let  $t'' \leq t+s$  be last time in  $[t, t+s]$  that a CEO is either revealed bad or revealed good (after being in tenure for  $\bar{t}$ ). Then, the period  $[t, t+s]$  can be subdivided into three subperiods. In  $[t, t']$  (if nonempty),  $\pi_{i,t'} - \pi_{i,t}$  is positive if and only if  $\Omega_{i,t'} - \Omega_{i,t} > 0$  (including the negative jump in  $\pi$  when a bad CEO is fired). In  $[t', t'']$ ,  $\pi_{i,t''} - \pi_{i,t'} = \Omega_{i,t''} - \Omega_{i,t'}$  (including the possible negative performance jump after  $t''$ ). In  $[t'', t+s]$ ,  $\pi_{i,t+s} - \pi_{i,t''}$  is independent of  $\Omega_{i,t+s} - \Omega_{i,t''}$ . The assumption that  $s > \bar{t}$  guarantees that at least one of the first subperiods is nonempty and hence the correlation is strictly positive, as noted in footnote 26.

(iii) Recall that a firm gets a good CEO with probability  $p$  and grows at an instantaneous rate  $\theta^H - \delta$  for  $T$  periods or gets a bad CEO with probability  $1-p$  and grows at rate  $-\delta$  for  $\bar{t}$ . So the average (instantaneous) growth of a randomly selected firm is

$$E(\Delta\pi) = \frac{p(\theta^H - \delta)T - (1-p)\delta\bar{t}}{pT + (1-p)\bar{t}}$$

where  $\bar{t} = \min\{R, \frac{\ln(1+b)}{\theta^H}\}$ . Thus, it is easy to see that an increase in  $p, \bar{t}$ , and  $\theta^H$  produces the effects in (iii). ■

**Proposition 8** Consider a CEO with  $p'$  and a CEO with  $p'' > p'$ . Let  $\Omega'$  and  $\Omega''$  denote the organizational capital levels of the firms employing the two CEOs respectively. If firms are sufficiently impatient ( $\rho$  is sufficiently high), in steady state  $\Omega' \leq \Omega''$ .

**Proof.** Suppose for contradiction that  $\Omega' > \Omega''$ .

Let  $W(p_j)$  represent the expected discounted cost given the instantaneous wage  $w_j$  and the probability of success  $p_j$  of employing a CEO of category  $j$ . Note that  $W(p_j)$  is independent of the organizational capital of the firm that employs the CEO.

Let  $u_k$  denote the steady state expected discounted payoff of a firm at level  $k$  (who does not yet know the quality of its new CEO). The payoff of a firm at level  $k$  who hires a CEO of

category  $p$  is given by

$$\begin{aligned}\tilde{u}_k(p) &= p \left( \int_0^T e^{-\rho t} e^{(\theta^H - \delta)t} \Omega_k dt + e^{-\rho T} u_{k+1} \right) + (1-p) \left( \int_0^{\bar{t}} e^{-\rho t} e^{(\theta^H - \delta)t} \Omega_k dt + e^{-\rho \bar{t}} u_{k-1} \right) \\ &= p \left( \frac{1 - e^{-\rho T} e^{(\theta^H - \delta)T}}{\rho + \delta - \theta^H} \Omega_k + e^{-\rho T} u_{k+1} \right) + (1-p) \left( \frac{1 - e^{-\rho \bar{t}} e^{(\theta^H - \delta)\bar{t}}}{\rho + \delta - \theta^H} \Omega_k + e^{-\rho \bar{t}} u_{k-1} \right) \\ &= v_k + e^{-\rho \bar{t}} z_k(p)\end{aligned}$$

where

$$\begin{aligned}v_k &= \frac{1 - e^{-\rho \bar{t}} e^{(\theta^H - \delta)\bar{t}}}{\rho + \delta - \theta^H} \Omega_k \\ z_k(p) &= p \left( \frac{1 - e^{-\rho(T-\bar{t})} e^{(\theta^H - \delta)(T-\bar{t})}}{\rho + \delta - \theta^H} \Omega_k e^{(\theta^H - \delta)\bar{t}} \right) + e^{-\rho(T-\bar{t})} (p u_{k+1} + (1-p) u'_{k-1}),\end{aligned}$$

where  $u'_{k-1}$  is defined as the expected steady state discounted payoff of a firm who  $T - \bar{t}$  periods ago had organizational capital  $\Omega_{k-1}$ . Note that we necessarily must have that  $u'_{k-1} < u_k$ .

It is optimal for a firm at level  $k$  to employ a CEO with  $p'$  rather than one with  $p''$  if

$$\tilde{u}_k(p') - W(p') \geq \tilde{u}_k(p'') - W(p'')$$

Conversely, it is optimal for a firm at level  $m$  to employ a CEO with  $p''$  rather than one with  $p'$  if

$$\tilde{u}_m(p') - W(p') \leq \tilde{u}_m(p'') - W(p'')$$

Subtracting one condition from the other we obtain

$$\tilde{u}_m(p') - \tilde{u}_k(p') \leq \tilde{u}_m(p'') - \tilde{u}_k(p'')$$

which can be re-written as

$$z_m(p') - z_k(p') \leq z_m(p'') - z_k(p'') \quad (10)$$

Note that

$$\lim_{\rho \rightarrow \infty} (\rho + \delta - \theta^H) z_k(p) = p \Omega_k e^{(\theta^H - \delta)\bar{t}}$$

Thus,

$$\lim_{\rho \rightarrow \infty} z_k(p) = p \frac{\Omega_k}{\rho + \delta - \theta^H} e^{(\theta^H - \delta)\bar{t}}$$

For  $\rho$  large enough, inequality (10) holds if and only if

$$p' \Omega_m - p' \Omega_k \leq p'' \Omega_m - p'' \Omega_k$$

namely  $(p' - p'') (\Omega_m - \Omega_k) \leq 0$ , which is false when  $p' < p''$  and  $\Omega_m > \Omega_k$ . ■

**Proposition 10** *In steady state:*

(i) *Firms with better performance and higher organizational capital employ CEOs of a better type (on average), with better behavior (on average), who are paid more.*

(ii) *The current employment status and compensation of a CEO depends on the change in performance and organizational capital of its previous firm.*

(iii) *A fixed-effect regression on data generated by this model returns significant individual coefficients but underestimates the true effect of individual CEOs on performance.*

**Proof.** Parts (i) and (ii) are immediate consequences of Proposition 9.

For (iii), note that if a CEO is employed by  $n$  firms, she must perform well in the first  $n - 1$  firms and badly in the last one. Let us express performance changes in terms of levels, so the effect of a good CEO is 1 and the effect of a bad one is -1. The *true* fixed effect of a CEO with  $n$  employments is  $\frac{n-2}{n}$ .

Note, however, that a fixed-effect regression would attribute some of the CEO fixed effect to the firm. Consider a panel regression that includes the last  $N$  CEO transitions of every firm. Let  $\bar{k}$  be the performance level corresponding to  $\bar{\pi}$ . All firms whose initial performance level is  $\bar{k} + N$  or higher will only hire CEOs with  $p_H$ . All firms whose initial performance level is  $\bar{k} - N$  or lower will only hire CEOs with  $p_L$ . The average fixed effect difference between firms in the former set and firms in the latter set with

$$p_H - (1 - p_H) - (p_L - (1 - p_L)) = 2(p_H - p_L).$$

As the true fixed-effect of firms is zero, this means that the regression will underestimate the fixed effect of CEOs hired by high-performance firms and overestimate that of CEOs hired by low-performance firms. ■

## Appendix III: Full Agency Problem

We keep the model defined in Section 2 except for the following modifications:

- The agent receives a minimum wage  $w > 0$  while employed. The wage is instantaneous and it is a share of the company's performance when the agent is hired (this assumption is made to abstract from a scale effect). The wage can be thought of as  $w = \bar{w} + \psi$ , where  $\bar{w}$  is a minimum statutory monetary wage and  $\psi$  is a psychological benefit of being a

CEO. As the firm owner must pay  $\bar{w}$  to all CEOs and the firm must always have a CEO, the minimum wage can be omitted when solving the firm-owners dynamic optimization problem.

- The firm owner can also promise a performance bonus to the CEO. The bonus may depend on performance as well as any message that the agent may send.
- The CEO and the firm owner have the same discount rate  $\rho$ .

We say that a contract is a first-best contract if it guarantees that the firm is always run by a good CEO.

**Proposition 11** *There exists a contract that achieves first best. However, for any positive  $w$ , if  $p$  is sufficiently small, the firm will not offer it.*

**Proof.** In order to achieve an efficient outcome, the owner must induce bad CEOs to resign as soon as they are hired – or equivalently, reveal their type truthfully and be fired. Suppose the owner offers a performance bonus  $b$  if a CEO resigns right after being hired. If a bad CEO does not resign at zero, he receives payoff

$$\int_t^{t+\bar{t}} e^{-\rho t} w dt = \frac{1 - e^{-\rho \bar{t}}}{\rho} w \Omega_t.$$

If he resigns (and we assume that any other bad CEO resigns immediately), he instead gets  $b$ . Thus, the minimum cost for the principal (evaluated at the beginning of the relationship) for persuading one bad CEO to resign (which satisfies the incentive constraint) is

$$b = \frac{1 - e^{-\rho \bar{t}}}{\rho} w \Omega_t$$

Note that given a bonus  $b$  at time 0, a good CEO strictly prefers not to resign as her tenure at the firm,  $T$ , is longer than that of a bad CEO,  $\bar{t}$ .

If the owner gets rid of a bad CEO, she still faces a probability  $1 - p$  that the next CEO is bad as well, implying that she would have to pay  $b$  again. Thus, the average cost of guaranteeing that the CEO hired at  $t$  is good for sure is:

$$(1 - p + (1 - p)^2 + \dots) \frac{1 - e^{-\rho \bar{t}}}{\rho} w \Omega_t = \left( \frac{1 - p}{p} \right) \frac{1 - e^{-\rho \bar{t}}}{\rho} w \Omega_t.$$

We now compare the expected value of a firm at  $t$  who chooses to implement the incentive scheme above as compared to one that does not (and therefore behaves like the firm in Proposition 1). With the incentive scheme, all CEOs are good and have a tenure of length  $T$ . At each CEO transition, the firm sustains expected cost  $\frac{1-p}{p} \frac{1-e^{-\rho\bar{t}}}{\rho} w \Omega_t$ . The expected value of the firm is given by

$$\begin{aligned} \tilde{V}_t &= \left( \frac{1}{\rho + \delta - \theta^H} \left( 1 - e^{-(\rho + \delta - \theta^H)T} \right) - \left( \frac{1-p}{p} \right) \frac{1 - e^{-\rho\bar{t}}}{\rho} w \right) \Omega_t + \tilde{V}_{t+T} \\ &= \Omega_t \left( \frac{1}{\rho + \delta - \theta^H} \left( 1 - e^{-(\rho + \delta - \theta^H)T} \right) - \left( \frac{1-p}{p} \right) \frac{1 - e^{-\rho\bar{t}}}{\rho} w \right) \sum_{k=0}^{\infty} e^{-(\rho + \delta - \theta^H)Tk} \\ &= \Omega_t \left( \frac{1}{\rho + \delta - \theta^H} \left( 1 - e^{-(\rho + \delta - \theta^H)T} \right) - \left( \frac{1-p}{p} \right) \frac{1 - e^{-\rho\bar{t}}}{\rho} w \right) \frac{1}{1 - e^{-(\rho + \delta - \theta^H)T}} \end{aligned}$$

Instead, as we know from Proposition 1, the value of a firm that does not offer this incentive scheme is

$$\Omega_t \frac{1}{\rho + \delta - \theta^H} \frac{1 - pe^{-(\rho + \delta - \theta^H)T} - (1-p)e^{-(\rho + \delta - \theta^H)\bar{t}}}{1 - pe^{-(\rho + \delta - \theta^H)T} - (1-p)e^{-(\rho + \delta)\bar{t}}}$$

The owner does not find it in her interest to induce bad CEOs to resign if

$$\begin{aligned} &\left( \frac{1-p}{p} \right) \frac{1 - e^{-\rho\bar{t}}}{\rho} w \\ &\geq \left( \frac{1}{\rho + \delta - \theta^H} - \frac{1}{\rho + \delta - \theta^H} \frac{1 - pe^{-(\rho + \delta - \theta^H)T} - (1-p)e^{-(\rho + \delta - \theta^H)\bar{t}}}{1 - pe^{-(\rho + \delta - \theta^H)T} - (1-p)e^{-(\rho + \delta)\bar{t}}} \right) \left( 1 - e^{-(\rho + \delta - \theta^H)T} \right) \\ &= \frac{1}{\rho + \delta - \theta^H} \frac{(1-p) \left[ e^{-(\rho + \delta - \theta^H)\bar{t}} - e^{-(\rho + \delta)\bar{t}} \right]}{1 - pe^{-(\rho + \delta - \theta^H)T} - (1-p)e^{-(\rho + \delta)\bar{t}}} \left( 1 - e^{-(\rho + \delta - \theta^H)T} \right) \end{aligned}$$

That is

$$w \geq p \frac{\rho}{\rho + \delta - \theta^H} \frac{1 - e^{-(\rho + \delta - \theta^H)T}}{1 - e^{-\rho\bar{t}}} \frac{e^{-(\rho + \delta - \theta^H)\bar{t}} - e^{-(\rho + \delta)\bar{t}}}{1 - pe^{-(\rho + \delta - \theta^H)T} - (1-p)e^{-(\rho + \delta)\bar{t}}}$$

from which we can see the statement of the Proposition. ■

The intuition for this result is that, in order to achieve an efficient outcome, the owner must induce bad CEOs to resign as soon as they are hired – or equivalently, reveal their type truthfully and be fired. As such, the firm must offer the bad CEO an incentive scheme that pays at least as much as what a bad CEO would get by staying at the firm for  $\bar{t}$ . This compensation must be paid to all the bad CEOs that are hired and resign immediately. The latter part grows unboundedly as  $p \rightarrow 0$ .