HOUSEHOLD DEBT AND UNEMPLOYMENT*

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January 31, 2017

Abstract

We use a labor-search model to explain why the worst employment slumps often follow expansions of household debt. We find that households protected by limited liability suffer from a household-debt-overhang problem that leads them to require high wages to work. Firms respond by posting high wages but few vacancies. This *vacancy-posting effect* implies that high household debt leads to high unemployment. Even though households borrow from banks via bilaterally optimal contracts, the equilibrium level of household debt is inefficiently high due to a *household-debt externality.* We analyze the role that a financial regulator can play in mitigating this externality.

Keywords: Household debt, limited liability, employment, financial regulation

JEL Classification Numbers: D14, G21, G28, J63, E24

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*We gratefully acknowledge helpful comments from Steven Ambler, Ulf Axelson, Juliane Begenau, Jonathan Berk, Nittai Bergman, Asaf Bernstein, V.V. Chari, Alex Edmans, Fred Malherbe, Vincent Glode, Radha Gopalan, Kyle Herkenhoff, Anastasia Kartasheva, Amir Kermani, Deborah Lucas, Asaf Manela, Holger Mueller, Christian Opp, Alessandro Previtero, Adriano Rampini, Felipe Severino, Kelly Shue, Ngoc-Khanh Tran, Randy Wright and seminar participants at Berkeley Haas, the Bank of England, BI Oslo (Norway), the 2016 Cambridge Corporate Finance Theory Symposium, the 2015 Canadian Economic Association, the 2015 CFF Conference on Bank Stability and Regulation in Gothenburg (Sweden), the 2015 European Summer Symposium in Financial Markets at Gerzensee, the Federal Reserve Bank of Cleveland, the Federal Reserve Bank of Philadelphia, the Federal Reserve Bank of St Louis, the 2015 IDC Summer Finance Conference, the IMF, the 2015 Labor and Finance Group conference at Vanderbilt, the 2015 LBS Summer Symposium, the 2015 Midwest Macro conference, the 2015 SED Meetings, the 2016 SFS Cavalcade, the Sixth Duke–UNC Corporate Finance Conference, the Toulouse School of Economics, and Washington University in St Louis. We alone are responsible for any remaining errors.

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This article is forthcoming in the *Journal of Finance*.

1 Introduction

Personal bankruptcy is pervasive in the US—about one in ten Americans will declare bankruptcy in his lifetime.[1] Under the US bankruptcy code, households are protected by limited liability; they can discharge their debt and still keep a substantial amount of their assets. Such limited-liability protection distorts the incentives of indebted households, just as it distorts the incentives of levered firms in corporate finance. In this paper, we investigate how this distortion can affect the labor market. In particular, we ask the following questions. How does limited-liability debt distort household labor supply and how does this affect aggregate employment in equilibrium? Further, do households take on too much limited-liability debt? And should a regulator intervene to mitigate its distortions?

Model preview. To address these questions, we develop a two-date general equilibrium model of household borrowing and the labor market. At the first date, households borrow from banks. At the second date, firms post vacancies and households and firms are randomly matched in a decentralized labor market à la Diamond–Mortensen–Pissaridis. Once matched, firms and households negotiate wages bilaterally. Then households work or do not. If households work, firms produce output and pay wages. Households use these wages to repay banks. If households do not work, firms do not produce output and do not pay wages. Thus, households cannot repay banks, so they default.

Results preview. Our first main result is that limited-liability debt on households' balance sheets leads to a *debt-overhang problem*, which results in households requiring relatively high wages to work. The reason is that households' wages net of debt repayments must compensate them for the cost of working.

Our second main result is that high levels of household debt lead firms to post relatively few vacancies, which, in turn, leads to low employment. This is a result of the household-debt-overhang problem. Because firms must pay indebted workers high wages, firms have high labor costs and few firms can afford to post vacancies. This *vacancy-posting effect* implies that high household debt leads to high unemployment.

Our third main result is that households take on excessive debt in equilibrium, even though they borrow from banks via bilaterally optimal contracts. This is due to a *household-debt externality* that works through the vacancy-posting effect. Specifically, when a household takes debt onto its balance sheet, it decreases the likelihood that households are employed, as implied by the vacancy-posting effect. Since unemployed households are likely to default on their debts, this increases the default rate on all loans, including other banks' loans to other households. In other words, when households take

[1] See Stavins (2000).

on debt, they do not take into account the negative effect that their borrowing has on other agents in the economy through the labor market. Thus, there is scope for a financial regulator to intervene to mitigate this externality.

Our fourth main result is that banks' beliefs about future employment are self-fulfilling, which generates multiple equilibria. If banks believe that the rate of employment will be low—so household default risk is high—banks require high face values of debt to offset this risk. Households thus have high debt, so employment is indeed low due to the vacancy-posting effect. In contrast, if banks believe that employment will be high—so household default risk is low—banks require low face values of debt, and employment is indeed high. Thus, there is another reason for regulatory intervention: to prevent the economy from ending up in the "bad" equilibrium with high debt and low employment.

We also show that households optimally finance themselves with debt contracts. This is because they want to minimize the repayments to banks when they have the most opportunity to get rents, i.e. when they have high wages. Within the class of repayments schedules that are (weakly) increasing in wages, the repayment schedule that minimizes repayments when wages are high is the one that increases most slowly with wages, i.e. debt. Further, we argue in an extension that repayments must indeed be increasing, since otherwise they can be manipulated by households and firms to lower repayments to banks (see Subsection 4.4).

We also the following three extensions, which generate further results and empirical content. (i) We include aggregate productivity shocks and discuss how household debt may contribute to sticky wages. (ii) We include household collateral and discuss how low collateral values, e.g. low house prices, may exacerbate the vacancy-posting effect. And (iii) we include default penalties and discuss how they may attenuate the vacancy-posting effect.

Policy. Our model is stylized, but may still cast light on two contemporary policy questions: Should household debt be limited? And should the personal bankruptcy code be more forgiving? Our model suggests that limiting household debt ex ante may be a good thing. In our model, caps on household debt can prevent the economy from ending up in the "bad" equilibrium with high debt and low employment. In contrast, making the bankruptcy code more debtor-friendly and limiting the liability of households ex post could be a bad thing. In our model, decreasing default penalties can tighten households' incentive constraints and exacerbate the vacancy-posting effect (see Subsection 4.3).

Empirical content. Our prediction that limited-liability household debt leads to a decrease in labor supply finds support in a number of recent empirical papers. Bernstein (2015) shows that instrumented household financial distress causes a decline in labor supply of between 2.3 and 6.3 percent, an effect that he calculates may have accounted for over twenty percent of the decline in employment between 2008 and 2010, i.e. almost two million fewer US jobs. Further, Herkenhoff (2012) finds a spike in the employment rate of households when their debt expires, suggesting that when households discharge their debt, the household debt-overhang distortion is mitigated, thereby increasing the employment rate. Our results are also in line with the findings of Dobbie and Goldsmith-Pinkham (2015), who find that limited recourse for mortgage debt—i.e. household limited liability—leads to a decrease in the employment rate.[2]

Our model captures the following stylized facts at the macroeconomic level: (i) high household leverage causes severe employment slumps (Mian and Sufi (2010), Mian and Sufi (2014b)), (ii) wages are rigid, especially downwardly[3] (Bewley (1999), Daly and Hobijn (2015)), and (iii) negative shocks to household collateral values (house prices) contribute to labor market slumps (Mian and Sufi (2014b)).

After we present the baseline analysis in Section 3, we discuss empirical evidence in support of each the key assumptions underlying our main mechanism (Subsection 3.5).

Related literature. A number of papers explore how the household credit market interacts with the labor market via the aggregate demand channel (see, e.g., Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2015), Mian and Sufi (2011, 2014b), Midrigan and Philippon (2011), and Mishkin (1978, 1978)). Our paper is complementary to this work in that we explore how the household credit market interacts with the labor market via distortions in labor supply. Mulligan (2009, 2010) also explores how household debt distorts labor supply. He studies the costs and benefits of employment-contingent mortgage write-downs, focusing on the tradeoff between preventing foreclosures and distorting labor supply. In these and other existing models of household debt overhang, the debt overhang works on the extensive margin, insofar as indebted households are reluctant to apply for jobs at prevailing wages. In our model, in contrast, the debt-overhang works on the intensive margin, insofar as households require high wages to exert effort. This leads to lower employment because fewer firms post vacancies in anticipation of a high wage bill (and not because households do not enter the labor market).

Bethune, Rocheteau, and Rupert (2015) and Kehoe, Midrigan, and Pastorino (2016)

[2]This paper finds seemingly contradictory evidence for homestead exemptions—it finds that these lead to increases in the employment rate. We think this may be because homestead exemptions, which essentially protect home equity from credit card and auto loans, are likely to cause households to discharge their debt sooner, thereby reducing household leverage and mitigating the vacancy-posting effect. This contrasts with mortgage default, which is likely be be delayed, because it is typically associated with deadweight losses, perhaps due to foreclosure, relationship-specific investment, costs of relocation, or other personal difficulties.

[3]Note that in our model the wages of new hires are rigid. While some work has suggested that wages for new hires are relatively flexible (e.g. Pissarides (2009)), Gertler, Huckfeldt, and Trigari (2014) suggest that these findings are mainly due to compositional effects, and that the wages of new hires are indeed rigid.

also incorporate household debt in search models of the labor market. Further, Kehoe, Midrigan, and Pastorino (2016) find another channel by which household borrowing can lead to a reduction in firm vacancy posting. In their model, households require high wages when their current borrowing constraints are tight, since the tightening of constraints increases their discount rate, leading them to demand high wages today. In our model, in contrast, households require high wages because of the distortion of limited liability on debt that is already in place. Herkenhoff (2013) also analyzes how household borrowing constraints distort labor market outcomes, with a focus on credit card debt. He investigates a channel by which household credit can worsen employment slumps: if households can borrow on their credit cards while unemployed, they hold out for high-wage jobs. This is because access to credit allows them to smooth consumption, making unemployment less costly. Whereas the distortion in Herkenhoff's model results from households taking on *more* debt when they are unemployed, the distortion in our model results from households *discharging* debt when they are unemployed.

In Section 4.3, we show that an increase in household debt induces the same distortion as an increase in unemployment insurance, amplifying the vacancy-posting effect. Acemoglu and Shimer (1999) have emphasized how unemployment insurance can distort labor market search, leading to decreased employment. We show that, given household limited liability, household debt induces a similar distortion. Nonetheless, the effect we characterize is likely to be even more severe than that induced by unemployment insurance, in light of the size of transfers to defaulting households (see Subsection 3.5). Additionally, the household-debt externality suggests that by levering up too much, households are effectively "over-insuring" employment risk.

Layout. In Section 2, we present the model. In Section 3, we present our main results. In Section 4, we analyze extensions. Section 5 is the Conclusion. The Appendix contains all the proofs.

2 Model

This section describes the model. There are two dates, Date 0 and Date 1, and three types of players, households, banks, and firms. Banks lend to households at Date 0 and firms employ households at Date 1. Thus, "households" are "borrowers" at Date 0 and "workers" at Date 1.

2.1 Players: Preferences and Actions

Households. There is a unit continuum of penniless households. Each has linear utility over consumption at Date 1 and requires the fixed amount B of liquidity at

Date 0, i.e. it maximizes its expected Date-1 payoff subject to the constraint that it meets its liquidity need at Date 0. This liquidity need creates a reason for households to borrow at Date 0; it could represent the need to smooth consumption or to make a fixed investment. At Date 1, each household may be matched with a firm, in which case it must work to generate output. It has cost of working c. This implies that firms have to incentivize households to work, which will give rise to the incentive constraint that determines wages.

Firms. There is a large continuum of competitive, profit-maximizing firms. At Date 1, each can pay the cost k to post a vacancy and to attract a household/worker. If a firm is matched with a household, it produces output y if the household works. Otherwise it produces nothing. We assume that $y > c + k$, so the benefits, y, of production are greater than the costs c and k of working and posting vacancies.

Banks. There is a large continuum of deep-pocketed, profit-maximizing banks. They lend to households/borrowers at Date 0 and discount the future at rate zero.

2.2 Labor Market

We model the labor market with a one-shot version of a random search model. The number of households is fixed (with unit mass) and the number of firms is determined by endogenous entry. The ratio of searching households to firms posting vacancies is the *queue length* q. We assume that households and firms are matched via a homogeneous matching function: households are matched with firms with probability $\alpha(q)$ and firms are matched with households with probability $q\alpha(q)$.

2.3 Contracts

Labor contracts. After a firm and a household are matched, they negotiate a labor contract, which constitutes the wage w that the firm pays the household when output equals y (the wage is zero when output equals zero, since firms cannot pay more than they have). The contract is determined to split the surplus between the firm and the household, which we model via a simple random-proposer bargaining protocol: with probability half, the firm makes a take-it-or-leave-it offer to the household; with probability half, the household makes a take-it-or-leave-it offer to the firm.[4]

[4] Note that this bargaining protocol is equivalent to Nash bargaining under the equilibrium (debt) contract. We use this non-cooperative protocol because it allows us to formalize the game for all contracts off the equilibrium path. By assuming that firms and households are equally likely to propose the labor contract, we are effectively giving them equal bargaining power. In our model, this allows us to focus on distortions arising from household debt and not from search and bargaining, since it implies that the so-called Hosios condition is satisfied (Hosios (1990)), which guarantees that the search market equilibrium is constrained efficient. The Hosios condition says that firms' bargaining power equals the elasticity of their matching

Financial contracts. Each household borrows B from a bank at Date 0. In exchange the household makes the repayment $R(w)$ when it receives wage w, where R is a function determined optimally, i.e. it maximizes the household's expected utility subject to the constraints that banks break-even and households are protected by limited-liability, $R(w) \leq w$. For now, we also assume that R is (weakly) increasing. This assumption is common in the literature (see, e.g., Brennan and Kraus (1987), Harris and Raviv (1989), Nachman and Noe (1994)), but it precludes some contracts that are optimal in some contexts (see, e.g., Innes (1990)). However, in Subsection 4.4, we extend the model to show that our results are robust to relaxing this assumption.

Note that banks and households are all "small" (they are indexed by continua), so they take the employment probability $\alpha(q)$ as given when they negotiate these lending contracts.

2.4 Timing

The sequence of moves is as follows. At Date 0, each household negotiates a lending contract R with a bank. At Date 1, firms post vacancies and they are randomly matched with households according to the matching technology described above. Next, firms and households negotiate wages, and households work or do not. Finally, output is realized and contracts are settled.

2.5 Equilibrium Definition

We look for the subgame perfect equilibria of the game described above. This constitutes: (i) the lending contract R, (ii) the labor contract w given R—i.e. the wages w_h when the household proposes and w_f when the firm proposes—(iii) the households' decisions to work or not given w and R, and (iv) firms' entry decisions which determine the queue length q such that all of (i)–(iv) above are chosen optimally given players' beliefs, and these beliefs are consistent.

See Lemma 1 below for an expression of the equilibrium contracts as the solution to an optimization program.

probability. Given Assumption 1 below on the matching function, this says that

$$\text{elasticity of } q\alpha \equiv \frac{q(q\alpha)'}{q\alpha} = \frac{\left(a\sqrt{q}\right)'}{a/\sqrt{q}} = \frac{1}{2} \equiv \text{ bargaining power.}$$

2.6 Assumptions

We make several assumptions on parameters. We make these assumptions to solve the model in closed form, but they are not essential for our qualitative results (in fact we do not use them until Subsection 3.2).

Assumption 1. *The matching probability α takes the following functional form:*

$$\alpha(q) = \frac{a}{\sqrt{q}}, \tag{1}$$

where a is a positive constant.[5]

The next assumption guarantees that the matching probabilities are between zero and one in equilibrium.

Assumption 2.

$$a^2\left(y + c + \sqrt{(y-c)^2 - \frac{8kB}{a^2}}\right) < 4k < y + c - \sqrt{(y-c)^2 - \frac{8kB}{a^2}}. \tag{2}$$

In Appendix 5, we demonstrate that these bounds are sufficient to ensure that $\alpha, q\alpha \in [0,1]$.

Finally, we assume that a household's Date-0 liquidity need is not too large.

Assumption 3.

$$B < \frac{a^2(y-c)^2}{8k}. \tag{3}$$

This ensures that the equilibrium face value of household debt exists (cf. equation (17)).

3 Results

We now present the main analysis of our model. We solve for the optimal labor and lending contracts as well as the equilibrium entry of firms. We show that (i) the optimal contract is debt, but that (ii) there is a household-debt-overhang problem that leads indebted households to require high wages. In equilibrium, this leads to (iii) the vacancy-posting effect, by which high levels of household debt lead to low employment. However, (iv) households do not take into account the effect of their debt on aggregate employment, i.e. there is a household-debt externality. Finally we show that (v) there are multiple self-fulfilling equilibrium outcomes.

[5]This probability satisfies the properties induced by standard matching functions in the literature, namely the probability α that a household matches with a firm is decreasing and convex in the queue length, whereas the probability $q\alpha$ that a firm matches with a household is increasing and concave in the queue length.

3.1 Optimal Contracts

We first solve for the optimal labor contract between a firm and a household and the optimal lending contract between a household and a bank. Recall that a labor contract is determined by the random-proposer bargaining protocol—firms and households each make take-it-or-leave-it offers with probability half. When the firm proposes, it maximizes its payoff subject to the constraint that the household is willing to work at cost c, i.e. it proposes the wage w_f to solve the program to

$$\text{maximize} \quad y - w \tag{4}$$

subject to the household's incentive constraint

$$w - R(w) - c \geq 0. \tag{5}$$

Note that the household's repayment R appears only on the left-hand side of its incentive constraint. This is because $R(0) = 0$ due to limited liability. When the household proposes, it maximizes its payoff subject to the constraint that the firm is willing to participate and pay the wage, i.e. it proposes the wage w_h to solve the program to

$$\text{maximize} \quad w - R(w) - c \tag{6}$$

subject to the firm's individual rationality constraint

$$y - w \geq 0. \tag{7}$$

The optimal lending contract is determined anticipating that the labor contracts w_f and w_h solve the problems above. At Date 0, a household makes a bank a take-it-or-leave-it offer to determine the repayment $R(w)$. The household and the bank anticipate that the household will be employed with probability α. Thus, it will get wage w_f with probability $\alpha/2$, wage w_h with probability $\alpha/2$, and wage zero with probability $1 - \alpha$ (when it is unemployed). We can now set up this contracting problem as an optimization program. Recall that, since all players are small, they do not take into account the effect of their actions on aggregate employment. Thus, optimal contracts are determined taking the employment rate α as given.

LEMMA 1. *Given an employment rate α, an optimal lending contract R solves the following program to maximize the household's expected payoff,*

$$\text{maximize} \quad \alpha \left(\frac{1}{2}\big(w_f - R(w_f) - c\big) + \frac{1}{2}\big(w_h - R(w_h) - c\big) \right), \tag{8}$$

subject to the following constraints:

- *the wage w_f maximizes firms' payoff subject to the constraint that it is incentive compatible for the household to work, given the repayment R,*

$$w_f \in \arg\max \left\{ y - w \,\middle|\, w - R(w) - c \geq 0 \right\}, \tag{9}$$

- *the wage w_h maximizes household's payoff subject to the constraint that it is individually rational for the firm to participate, given the repayment R,*

$$w_h \in \arg\max \left\{ w - R(w) - c \,\middle|\, y - w \geq 0 \right\}, \tag{10}$$

- *banks break even, given the employment rate α,*

$$\alpha \mathbb{E}\left[R(w)\right] \geq B, \tag{11}$$

- *households have limited liability, $R(w) \leq w$,*
- *R is weakly increasing.*

Our first main result is that the optimal lending contract can be implemented with defaultable debt. We denote the face value of a representative household's debt by F.

Proposition 1. *Given an employment rate α, defaultable debt with face value $F := B/\alpha$ is an optimal lending contract. I.e. $R(w) = \min\{B/\alpha, w\}$ is a solution to the program in Lemma 1.*

To see why debt is optimal, think of an arbitrary contract in which the household's debt repayment is weakly increasing in its wage. Now, observe that the household always gets a zero net payoff (the wage minus the debt repayment minus the cost of effort) when the firm proposes the wage, since in this case the firm pushes the household to its participation constraint. Thus, the household chooses the lending contract R to minimize its repayment when it proposes the wage, since this is the only opportunity for the household to get rent. Given that the wage is relatively high when the household proposes, the household chooses the lending contract to minimize the repayment $R(w)$ when the wage w is high. To make the bank break even in expectation, the household must then increase the repayment $R(w)$ when the wage w is low (which occurs when the firm proposes). Since R must be monotonic, the contract that minimizes the repayment for high wages and maximizes the repayment for low wages is the flat contract, i.e. debt.

Now, given the face value of debt F, the firm proposes the wage w_f to make the household's incentive constraint bind, and the household proposes the wage w_h to make the firm's individual rationality constraint bind.

Proposition 2. *The equilibrium wages are $w_f = F + c$ and $w_h = y$. Thus, the average wage is*

$$\bar{w} := \mathbb{E}\left[w\right] = \frac{y + F + c}{2}. \tag{12}$$

Observe that the expected wage is increasing in the face value of debt F. This is because the more indebted the household is, the more of its wage goes to the bank, and as a result the more the firm has to compensate it for working. This finding that wages are increasing in household debt is the key to the vacancy-posting effect, which we turn to next.

3.2 Firm Vacancy Posting

We now solve for the queue length q and employment rate $\alpha(q)$, which are determined by firms' willingness to post vacancies. Recall that firms are matched with households with probability $q\alpha(q)$. If they are matched, they get $y - \bar{w}$ on average, so their expected payoff from posting vacancies is $q\alpha(q)(y - \bar{w})$. Since they must pay the cost k to post vacancies, firms post vacancies whenever

$$q\alpha(q)\big(y - \bar{w}\big) \geq k. \tag{13}$$

We can now solve for q by substituting in for $\bar{w}$ from Corollary 2 and observing that the inequality must bind in equilibrium since firms compete away all the rent from posting vacancies.

Proposition 3. *Given the face value of debt F, the queue length and employment rate are*

$$q = \left(\frac{2k}{a(y - F - c)}\right)^2 \tag{14}$$

and

$$\alpha(q) = \frac{a^2(y - F - c)}{2k}, \tag{15}$$

as long as $\alpha(q)$ is between zero and one.

This proposition leads us immediately to the vacancy-posting effect, by which firms post fewer vacancies when the level of household debt is high, leading to low employment.

Corollary 1. *The employment rate α is decreasing in the level of household debt F.*

This vacancy-posting effect works through the effect of household debt on wages. Recall that increasing the level of household debt F increases the average wage $\bar{w}$ (by Corollary 2). Thus, the higher is F the higher is a firm's wage bill and the lower is its profit. As a result, fewer firms can afford to enter and post vacancies.

3.3 Household Debt and Unemployment in Equilibrium

We have characterized the face value of household debt F as a function of the employment rate α (Proposition 1), and we have characterized the employment rate α as a function of the face value of household debt F (Proposition 3). We now solve for the equilibrium of the model by finding the face value of debt that makes these findings consistent with each other. In other words, the face value of debt is determined as a fixed point: $F(\alpha(F)) = F$. Specifically, the household offers the bank the face value F so that the bank breaks even, i.e.

$$\alpha F = B, \tag{16}$$

where α is determined in equilibrium as a function of F. Substituting in for α from Proposition 3, we have that

$$\frac{a^2}{2k}\big(y - F - c\big)F = B. \tag{17}$$

This is a quadratic equation in F which has two solutions, i.e. the model has two equilibria, corresponding to different levels of household debt and different employment rates.[6]

Proposition 4. *Define*

$$d := \frac{2Bk}{a^2}. \tag{18}$$

There are two equilibria. There is an equilibrium with a low face value of debt

$$F_- = \frac{y - c - \sqrt{(y-c)^2 - 4d}}{2} \tag{19}$$

and a high employment rate

$$\alpha_- = \frac{a^2}{2k}\big(y - F_- - c\big). \tag{20}$$

There is also an equilibrium with a high face value of debt

$$F_+ = \frac{y - c + \sqrt{(y-c)^2 - 4d}}{2} \tag{21}$$

and a low employment rate

$$\alpha_+ = \frac{a^2}{2k}\big(y - F_+ - c\big). \tag{22}$$

[6] There is an analogous result in Rocheteau's (1999) model of financing government expenditure. In that model, if the government has to finance expenditure B with a payroll tax F on α employed households, then the government's balanced-budget constraint is $\alpha F = B$. This is the analog of the bank's break-even constraint in our model, which generates multiplicity.

There are multiple equilibria because banks' beliefs about future employment are self-fulfilling. When banks believe that the rate of employment will be high—making household default unlikely—banks demand low face values of debt and employment is indeed high. Likewise, when banks believe that the rate of employment will be low—making household default likely—banks demand high face values of debt and unemployment is indeed high.

3.4 The Constrained-Efficient Outcome

We define the constrained-efficient outcome as the queue length q and the employment rate $\alpha(q)$ that maximize total surplus given the search friction, i.e. they maximize the total output minus the total costs of working and vacancy posting. Recall that there is a unit of households. Thus, α is the number of firm-household matches and $1/q$ is the number of firms that pay k to enter. Therefore, the constrained-efficient outcome must maximize the output αy minus the costs of working αc and the costs of posting vacancies k/q, i.e. it solves the program to

$$\text{maximize } \alpha(q)\big(y - c\big) - \frac{k}{q}. \tag{23}$$

LEMMA 2. *The constrained-efficient queue length and employment rate are given by*

$$q_{\text{CE}} = \left(\frac{2k}{a(y-c)}\right)^2 \tag{24}$$

and

$$\alpha_{\text{CE}} = \frac{a^2(y-c)}{2k}. \tag{25}$$

We now ask whether the equilibrium outcome in Proposition 4 is constrained efficient. The next proposition says that the answer is no.

PROPOSITION 5. *Employment is too low even in the high-employment equilibrium: the employment rate in the high-employment equilibrium in Proposition 4 is lower than the employment rate in the constrained-efficient outcome in Lemma 2, i.e.*

$$\alpha_- < \alpha_{\text{CE}}. \tag{26}$$

The equilibrium outcome is not constrained efficient due to a *household-debt externality* that works as follows. When banks lend to households, they take the employment rate α as given. However, bank lending decreases the employment rate via the vacancy-posting effect (Corollary 1). This increases the default rate on all loans—including other banks' loans to other households—since unemployed households default on their debts.

In other words, when banks lend to households, they do not take into account the negative effect that their lending has on other banks and households through labor market externalities.

Given this externality, there is scope for a regulator to intervene in labor and credit markets to improve efficiency.

PROPOSITION 6. *A regulator can implement the constrained-efficient outcome by regulating wages and household debt. If the regulator sets*

$$w_{\text{CE}} = \frac{y+c}{2}, \tag{27}$$

then the constrained-efficient outcome is achieved as long as household debt is not too high. Specifically, the regulator must set

$$F \leq F_{\text{CE}} = \frac{y-c}{2}. \tag{28}$$

The intuition for this result is as follows. In equilibrium, the employment rate $\alpha(q)$ is determined by firms' entry condition: firms continue to post vacancies as long as the cost of posting is less than their expected profit from posting given the wage w, so

$$k = q\alpha(q)\big(y - w\big). \tag{29}$$

We find that $q = q_{\text{CE}}$ exactly when $w = w_{\text{CE}}$. In other words, setting w_{CE} implements the constrained-efficient outcome. However, it must be incentive compatible for the household to work. That is, it must be true that

$$w_{\text{CE}} - F \geq c. \tag{30}$$

This incentive compatibility constraint gives the upper bound on F in the proposition above. It implies that a regulator may not be able to implement the constrained-efficient outcome by intervening in the labor market alone, even though the household-debt externality works through wages. Indeed, a regulator may need to cap and/or write down household debt to stimulate the labor market.

COROLLARY 2. *Household debt is too high in equilibrium in the following two senses.*

(i) The level of household debt in the high-debt equilibrium in Proposition 4 is higher than the upper bound on the level of household debt in the constrained-efficient outcome in Proposition 6, i.e. $F_+ > F_{\text{CE}}$. Thus, the regulator cannot implement the constrained-efficient outcome even if it can intervene in the labor market and set wages.

(ii) *If wages are determined bilaterally by firms and households given household debt F as in Proposition 2, then decreasing F brings the economy closer to the constrained-efficient outcome (it increases the objective function in equation (23)).*

To the extent that capping household debt occurs via regulations imposed on the banks that lend to households, this proposition implies that the central bank, in its regulatory role, can affect employment through prudential bank regulations. This provides the central bank with a new way to target employment as an alternative to monetary policy.

3.5 Discussion of Assumptions

In this subsection, we discuss the empirical support for the microeconomic ingredients that drive our main results.

The mechanism behind the vacancy-posting effect relies on the following four ingredients: (i) households default when they are unemployed; (ii) households are protected by limited liability; (iii) households take their limited liability protection into account; and (iv) firms internalize this household preference distortion when posting vacancies. Each of these ingredients has empirical support in the literature, some of which we discuss below.

As for (i), Geradi, Herkenoff, Ohanian, and Willen (2013) find that individual unemployment is the strongest predictor of default. Similarly, Herkenhoff (2012) finds that unemployment (and not negative equity) is the primary reason for household default, implying that households default mainly when they fail to find employment. As for (ii), household limited liability in the event of default is salient in the US, where debtors can dissolve debt obligations by filing for personal bankruptcy (see Dobbie and Goldsmith-Pinkham (2015) and Mahoney (2015), for example). As for (iii), Mahoney (2015) establishes that households do indeed take into account limited liability—they use the protection afforded by it as informal insurance. Further, Melzer (forthcoming) demonstrates that limited liability in the form of asset exemptions in mortgage default leads to distortions in households' investment decisions. Households with negative equity cut back substantially on home improvements, but continue to invest in durable assets that can be retained in the event of default.

Finally, consider (iv). Work on the effects of unemployment insurance provides evidence that firms do respond to household preference distortions when posting vacancies. Notably, Hagedorn, Manovskii, and Mitman (2015) exploit variation in unemployment insurance policies across US states to show that increasing unemployment insurance causes firms to post fewer vacancies. They estimate that cuts to unemployment insurance created about 1.8 million jobs in the US in 2014 due to increased job creation by firms. As we show in Subsection 4.3, in our model unemployment insurance has the same distortionary effect as household debt does. This is because household debt is effectively a "tax" for finding employment—households repay their debts out of their wages—whereas unemployment insurance is a subsidy for *not* finding employment. The labor market distortions resulting from household leverage are likely to be even more important than those resulting from employment insurance. This is because personal bankruptcy results in more effective transfers than all state unemployment insurance programs combined (Lefgren, McIntyre, and Miller (2010)). Moreover, household limited liability is not limited to debt that is discharged in bankruptcy; in fact, bankruptcies constitute only about one-sixth of household defaults (Herkenhoff (2012)).

4 Extensions

In this section, we consider four extensions. In each of the first three extensions, we add a realistic ingredient to the model in reduced-form to generate new results. Specifically, we take Date-0 debt contracts as given, and add aggregate productivity shocks at Date 1 in the first extension, household collateral at Date 1 in the second extension, and default penalties/unemployment insurance at Date 1 in the third extension. In the fourth extension, we enrich the model and show that households and firms can effectively collude to manipulate non-monotonic financial contracts. This justifies our restriction to monotonically increasing financial contracts in the baseline model.

4.1 Aggregate Shocks and Wage Dynamics

We now discuss the effects of changes in firm output y on employment and wages. We argue that household debt may be a source of sticky wages, and discuss the complementarities between our household-debt-externality channel of unemployment and the aggregate demand channel.

We now include two possible aggregate states: a boom in which firm output is y_H and a recession in which firm output is $y_L < y_H$. Thus, given household debt with face value F, Proposition 2 gives the labor market outcomes in the boom and recession states. In particular, we have the equations for the wages

$$w_H = \frac{y_H + F + c}{2} \quad \text{and} \quad w_L = \frac{y_L + F + c}{2}. \tag{31}$$

The following proposition says that the fluctuation of wages across macroeconomic states decreases as household debt increases, suggesting that high levels of household debt represent a potential source of wage rigidity (see Bewley (1999)).

Proposition 7. *The percentage change of wages across macroeconomic states,*

$$\frac{w_H - w_L}{w_H} = \frac{y_H - y_L}{y_H + F + c} \tag{32}$$

is decreasing in the level of household debt F.

Now turn to the employment rates. We see that

$$\alpha_H = \frac{a^2}{2k}\big(y_H - F - c\big) \quad \text{and} \quad \alpha_L = \frac{a^2}{2k}\big(y_L - F - c\big), \tag{33}$$

suggesting that high levels of household debt may decrease employment in booms and, more importantly, amplify employment slumps in recessions. Thus, while our channel of unemployment, based on the impact of household debt on the labor market, is novel, it is complementary to channels based on varying aggregate output. In particular, when aggregate demand decreases, firm revenues decrease. In our model, this corresponds to a decrease in y. This shock to y has a more severe effect on the labor market when households are more highly levered (F is higher). This is consistent with evidence in studies of the aggregate demand channel, such as Mian and Sufi (2014a).

4.2 The Inclusion of Collateral

Next we show how our results are affected by the inclusion of collateral on household balance sheets. We argue that depressed collateral values may amplify the vacancy-posting effect.

Suppose households have collateral in place with value h. If $h \geq F$, a household can always repay its debt by liquidating its collateral, even if it is unemployed. In contrast, if $h < F$, a household defaults on its debt and gets zero if it is unemployed. Thus, it prefers to work at wage w as long as

$$w - F - c + h \geq \max\{h - F, 0\}. \tag{34}$$

Thus, Proposition 2 gives the wage

$$w = \frac{y + c + \max\{F - h, 0\}}{2}. \tag{35}$$

Proposition 8. *Whenever collateral values are low, $h < F$, limited liability leads to a distortion in households' incentives, which induces high wages and low employment via the vacancy-posting effect.*

In contrast, whenever collateral values are high, $h \geq F$, limited liability does not lead to a distortion in households' incentives.

This extension yields the additional empirical prediction that the vacancy-posting effect should be strongest when collateral values are low (or liquidation discounts are high), i.e. when $h < F$. This explains why the connection between household debt and unemployment is strongest in economic downturns, which are periods during which assets values are depressed and asset illiquidity is low. This was the case for housing during the Great Recession when household collateral values were low due to the fall in house prices. This is consistent with evidence in Mian and Sufi (2014b).

4.3 Default Penalties and Unemployment Insurance

Next we extend our model to include default penalties. We show that default penalties attenuate the vacancy-posting effect and therefore may help to boost employment. We also discuss the role of unemployment insurance, which is analogous to a negative default penalty.

We now assume that a household that defaults on its debt suffers a penalty d. Thus, it prefers to work at wage w as long as

$$w - F - c \geq -d. \tag{36}$$

Thus, Proposition 2 gives the wage

$$w = \frac{y + c + F - d}{2}. \tag{37}$$

PROPOSITION 9. *Increasing the default penalty d decreases wages and increases employment; i.e. default penalties attenuate the vacancy-posting effect.*

This result may help to test our model empirically, since there is significant cross-state variation in default penalties.[7] Notably, Dobbie and Goldsmith-Pinkham (2015) find that the post-crisis employment slump was deeper in states with limited recourse for mortgage debt, consistent with our finding that higher default penalties mitigate the vacancy-posting effect.

Observe that a negative default penalty exacerbates the vacancy-posting effect. This can be interpreted as unemployment insurance. Denoting the transfer to unemployed households by UI, the household's IC reads

$$w - F - c \geq \mathrm{UI} \tag{38}$$

[7] In particular, asset exemption laws, which specify the types and levels of assets that can be seized in bankruptcy, vary across states. According to Mahoney (2015), "Kansas, for example, allows households to exempt an unlimited amount of home equity and up to $40,000 in vehicle equity. Neighboring Nebraska allows households to keep no more than $12,500 in home equity or take a $5,000 wildcard exemption that can be used for any type of asset" (p. 711).

so, by Proposition 2,

$$w = \frac{y + c + F + \mathrm{UI}}{2}. \tag{39}$$

Thus, an increase in household debt induces the same distortion as an increase in unemployment insurance, amplifying the vacancy-posting effect. The literature has established that unemployment insurance can distort labor market search, decreasing employment (Acemoglu and Shimer (1999)). We show that, given household limited liability, household debt induces the same distortion—*more* household leverage corresponds to *more* insurance, in contrast to other models in the literature (see, e.g., Rampini and Viswanathan (2015)). Further, given the size of transfers to defaulting households discussed in Subsection 3.5, the negative effects of household debt for the labor market are likely to be even larger than those of unemployment insurance. Additionally, the household-debt externality suggests that by levering up too much, households are effectively "over-insuring" employment risk.

4.4 Non-increasing Financial Contracts

We now show that our restriction to increasing financial contracts is not overly restrictive, because non-increasing contracts are subject to manipulation by the household and the firm.

We extend the model to include a continuous effort choice. As above, if a household works, it incurs fixed cost c to produce output y for the firm. But now we also allow the household to work more to produce more. This extra work leads to zero-NPV production in the sense that the marginal cost of household effort equals its marginal productivity: for every amount of extra output z, the household bears the cost of effort z. Our main result of this subsection is that if financial contracts are decreasing in wages, then the household always manipulates its wage upward to minimize the repayment. In other words, decreasing repayments are effectively not implementable.

Proposition 10. *Suppose that the financial contract R is decreasing on some region, i.e. $R(w_H) < R(w_L)$ for some $w_H > w_L$. The household never makes the repayment $R(w_H)$.*

The intuition behind this result is that if the financial contract has a repayment that is decreasing in the wage, then the household and the firm can collude to decrease the repayment: the firms pays the household a higher wage for more effort and hence more output. Even though the household's effort does not create NPV, it allows the firm to pay a higher wage and hence the household to make a lower repayment to the bank. This is akin to the argument in Innes (1990) for why entrepreneurs' financial contracts must be increasing: if repayments were decreasing in output, entrepreneurs could secretly borrow, report higher output, make low repayments, and then repay their secret debt. Our mechanism implements the same contractual manipulation via only the labor contract. Our argument does not rely on secret borrowing and thus it is immune from the criticism that repayments of secret debts may be difficult to enforce.

5 Conclusion

This paper examines the effect of household credit on the labor market. We find that debt on household balance sheets leads to a debt-overhang problem, which results in households requiring relatively high wages to work. The reason is that households' wages net of debt repayments must compensate them for the cost of working. This result is established in a setting in which debt is the optimal contract with which households finance current liquidity needs. Firms respond to households' distorted preferences by posting high wages but few vacancies. This vacancy-posting effect explains why high levels of household debt precede unemployment slumps. Further, we show that households fail to internalize this negative effect that they have on the labor market. This household-debt externality leads to excessive household debt in equilibrium. A financial regulator can intervene to mitigate this externality by capping household debt. Thus, a central bank can target unemployment in its role as a financial regulator.

Appendix

Sufficiency of Bounds in Assumption 2

Here we show the sufficiency of the bounds stated in Assumption 2 for the matching probabilities to be well-defined. In order for the matching probabilities to be between zero and one it must be that

$$a^2 < q < \frac{1}{a^2}. \tag{A.1}$$

We can substitute the equilibrium q from Proposition 3 into this expression to get

$$a^2(y - F - c) < 2k < (y - F - c). \tag{A.2}$$

Plugging in for for the smallest F from Proposition 4, i.e., F_{-}, in the left-hand side of the equation and for the largest F from Proposition 4, i.e., F_{+}, in the right-hand side of the equation we obtain sufficient conditions for the inequality above to hold, namely

$$a^2\left(y - c + \sqrt{(y-c)^2 - \frac{8Bk}{a^2}}\right) < 4k < y - c - \sqrt{(y-c)^2 - \frac{8Bk}{a^2}}, \tag{A.3}$$

which is the condition in Assumption 2. □

Proof of Lemma 1

The result follows immediately from backward induction. The program just says that wages are determined optimally given R and R is determined optimally in anticipation of the wages. The only subtly is that households and banks take the employment probability α as given even though firms post vacancies contingent on financial contracts. This is because we have assumed that banks and households are indexed by continua and are therefore too small to affect α individually. □

Proof of Proposition 1 and Proposition 2

The proof has four main steps. In Step 1, we show that the wage is lower when the firm proposes than when the household proposes, $w_f \leq w_h$, which implies that $R(w_f) \leq R(w_h)$, by monotonicity. In Step 2, we show that for any financial contract R the household's IC binds when the firm proposes, $w_f - R(w_f) - c = 0$, so the household gets surplus rent only when it proposes. In Step 3, we show that repayments to the bank are the same when the firm proposes and the household proposes, so the optimal financial contract is implementable with debt with face value $F := R(w_f) = R(w_h)$. In Step 4, we find the optimal wage and face value of debt.

Before we start the main steps of the proof, we note that we can restrict attention to contracts in which the household always works, i.e. its IC is satisfied for both $w = w_f$ and $w = w_h$, since output and repayments are all zero if the IC is violated.

Step 1: $w_f \leq w_h$. To see this, suppose (in anticipation of a contradiction) that $w_f > w_h$ in equilibrium. But then the firm can deviate and offer $w'_f = w_h$ and get profit $y - w'_f = y - w_h > y - w_f$ (since the IC is necessarily satisfied when $w = w_h$). This is a contradiction to the supposition that the firm offers $w_f > w_h$. We conclude that $w_f \leq w_h$.

Step 2: $w_f - R(w_f) - c = 0$. Intuitively, this says that when the firm makes the offer it pushes the household to its binding IC. The subtlety is to prove that it holds for all admissible financial contracts R. Recall equation (9), which says that the firm chooses the smallest wage that satisfies the household's IC:

$$w_f \in \arg\max \left\{ y - w \,\middle|\, w - R(w) - c \geq 0 \right\}. \tag{A.4}$$

We must prove that the constraint binds (which requires a bit of work since R may be discontinuous). We now show that if

$$\hat{w}_f := \inf \left\{ w \,\middle|\, w - R(w) - c \geq 0 \right\} \tag{A.5}$$

then $\hat{w}_f - R(\hat{w}_f) - c = 0$, so the infimum above is attained. Recall that R is increasing by assumption, so

$$\lim_{\varepsilon \to 0^+} R(\hat{w}_f - \varepsilon) \leq R(\hat{w}_f) \leq \lim_{\varepsilon \to 0^+} R(\hat{w}_f + \varepsilon) \tag{A.6}$$

(note that the inequalities bind when R is continuous). Now we have that

$$\lim_{\varepsilon \to 0^+} \hat{w}_f - \varepsilon - R(\hat{w}_f - \varepsilon) - c \geq \hat{w}_f - R(\hat{w}_f) - c \geq \lim_{\varepsilon \to 0^+} \hat{w}_f + \varepsilon - R(\hat{w}_f + \varepsilon) - c. \tag{A.7}$$

(This follows since $\hat{w}_f + \varepsilon$ is continuous in ε, so $\lim_{\varepsilon \to 0^+} \hat{w}_f - \varepsilon = \lim_{\varepsilon \to 0^+} \hat{w}_f + \varepsilon = \hat{w}_f$.) We now proceed by contradiction to show that it cannot be that either $\hat{w}_f - R(\hat{w}_f) - c > 0$ or $\hat{w}_f - R(\hat{w}_f) - c < 0$, so equality must hold.

Suppose $\hat{w}_f - R(\hat{w}_f) - c > 0$. Now, by equation (A.7) there is an $\varepsilon > 0$ such that

$$\hat{w}_f - \varepsilon - R(\hat{w}_f - \varepsilon) - c \geq 0, \tag{A.8}$$

which says that the wage $w_f = \hat{w}_f - \varepsilon < \hat{w}_f$ satisfies the IC, contradicting the definition of $\hat{w}_f$ as the infimum in equation (A.5).

Suppose $\hat{w}_f - R(\hat{w}_f) - c < 0$. Now, by the monotone convergence theorem, there is a decreasing sequence that satisfies that IC and converges to the infimum in equation

(A.5), i.e. $\hat{w}_f = \lim_{n\to\infty} \hat{w}_f + \varepsilon_n$ where $\varepsilon_n > 0$, $\varepsilon_n \to 0$, and $\hat{w}_f + \varepsilon_n$ satisfies the IC:

$$\hat{w}_f + \varepsilon_n - R(\hat{w}_f + \varepsilon_n) - c \geq 0 \tag{A.9}$$

Now, from equation (A.7), we know that for n sufficiently large (i.e. ε_n small and positive), we have that

$$w + \varepsilon_n - R(\hat{w}_f + \varepsilon_n) - c < 0. \tag{A.10}$$

This contradicts the supposition the IC is satisfied for the sequence $\hat{w}_f + \varepsilon_n$.

Thus, the IC binds at $\hat{w}_f$. This is thus the smallest wage satisfying the IC, it is the optimal wage for the firm to propose, $w_f = \hat{w}_f$.

Step 3: $R(w_f) = R(w_h) =: F$. Step 2 above says that the household's IC binds whenever the firm proposes the wage and, thus, the household gets zero utility whenever the firm proposes. Hence, the household maximizes its utility when it proposes the wage. I.e. its optimization problem is now to

$$\text{maximize} \quad w_h - R(w_h) \tag{A.11}$$

subject to

$$R(w_h) \geq R(w_f), \tag{A.12}$$

$$w_h \leq y, \tag{A.13}$$

$$\alpha \left(\frac{1}{2} R(w_h) + \frac{1}{2} R(w_f) \right) \geq B. \tag{A.14}$$

Since the objective is decreasing in $R(w_h)$, and $R(w_f)$ enters only in the constraints, the monotonicity constraint in equation (A.12) binds. I.e. $R(w_f) = R(w_h)$. In other words, the repayment is independent of the wage. We label this number F.

Step 4: Wage and face value. Given Step 3 above, we can rewrite the problem as the program to

$$\text{maximize} \quad w_h - F \tag{A.15}$$

subject to

$$w_h \leq y, \tag{A.16}$$

$$\alpha F \geq B. \tag{A.17}$$

This is maximized when the constraints bind, so $w_h = y$ and $F = B/\alpha$.

Summary. To sum up: The optimal financial contract is $R(w_f) = R(w_h) \equiv F = B/\alpha$ and the corresponding labor contracts are $w_f = F + c$ and $w_h = y$. □

Proof of Proposition 3

The result follows from substituting in for the functional form of the matching probability α from Assumption 1 into the vacancy-posting condition in equation (13). This gives

$$a\sqrt{q}(y-\bar{w}) \geq k. \tag{A.18}$$

Recalling that firms continue to post vacancies to compete away profits and that $\bar{w} = (y+F+c)/2$ from Proposition 2, we have

$$a\sqrt{q}\left(y-\frac{y+F+c}{2}\right) = k. \tag{A.19}$$

Rearranging gives the expressions in the proposition. □

Proof of Proposition 4

The result follows directly from the fixed-point condition $F(\alpha(F)) = F$ summarized in equation (17). The expressions for F_- and F_+ are the solutions of this quadratic equation and the corresponding employment levels α_- and α_+ follow from substituting the expressions for F_- and F_+ into the expression for α in Proposition 3. Assumption 3 (that B is not too large) ensures that both of the roots F_- and F_+ are real. □

Proof of Lemma 2

The result follows simply from substituting into the objective function in equation (23) for the functional form of α in Assumption 1. Thus, we must solve the program to

$$\text{maximize} \quad \frac{a}{\sqrt{q}}\big(y-c\big) - \frac{k}{q}. \tag{A.20}$$

The following first-order condition gives the global maximum q_{CE}:

$$-\frac{1}{2}a(y-c)q_{\text{CE}}^{-3/2} + kq_{\text{CE}}^{-2} = 0. \tag{A.21}$$

Solving for q_{CE} and substituting into α gives the expressions in the lemma. □

Proof of Proposition 5

The result follows from comparing α_{CE} from Lemma 2 with α_- from Proposition 4. We have that $\alpha_{\text{CE}} > \alpha_-$ whenever

$$\frac{a^2}{2k}(y-c) > \frac{a^2}{2k}\big(y-F_- - c\big) \tag{A.22}$$

which is always satisfied since $F_{-} > 0$. $\square$

Proof of Proposition 6

To see that setting the wage equal to w_{CE} implements the constrained-efficient level of vacancy posting conditional on households working, substitute w_{CE} from equation (27) into firms' vacancy posting condition in equation (29), noting as before that firms continue to post vacancies until this inequality binds. I.e. we have that

$$q\alpha(y - w_{\text{CE}}) = q\alpha\left(y - \frac{y+c}{2}\right) = k \tag{A.23}$$

which, with $\alpha(q) = a/\sqrt{q}$ gives

$$q = \left(\frac{2k}{a(y-c)}\right)^2 \equiv q_{\text{CE}}. \tag{A.24}$$

Thus, setting the wage equal to w_{CE} implements the constrained efficient outcome as long as it induces the household to work, i.e. as long as the household's IC is satisfied. This is the case as long as

$$w_{\text{CE}} - F - c \geq 0 \tag{A.25}$$

or as long as

$$F \leq \frac{y-c}{2} \equiv F_{\text{CE}}, \tag{A.26}$$

as stated in the proposition. $\square$

Proof of Proposition 7

The result follows immediately from the argument in the text and Proposition 3. $\square$

Proof of Proposition 8

The result follows immediately from the argument in the text and Proposition 3. $\square$

Proof of Proposition 9

The result follows immediately from the argument in the text and Proposition 3. $\square$

Proof of Proposition 10

Suppose (in anticipation of a contradiction) that the household gets the wage w_L and makes the repayment $R(w_L)$ in equilibrium. In this case, the payoffs to the household and firm are as follows:

$$\text{household payoff} = w_L - R(w_L) - c - z, \tag{A.27}$$

$$\text{firm payoff} = y + z - w_L, \tag{A.28}$$

where z is the effort exerted by the household (on top of the initial cost of working c). Now consider the deviation in which the household exerts effort

$$z' = z + w_H - w_L + \frac{R(w_L) - R(w_H)}{2} \tag{A.29}$$

and the firm pays the wage w_H. In this case, the payoff to the household is

$$\begin{aligned}
\text{household payoff}' &= w_H - R(w_H) - c - z' && \text{(A.30)}\\
&= w_H - R(w_H) - c - z - w_H + w_L - \frac{R(w_L) - R(w_H)}{2} && \text{(A.31)}\\
&= w_L - \frac{R(w_L) + R(w_H)}{2} - c - z && \text{(A.32)}\\
&> w_L - R(w_L) - c - z && \text{(A.33)}
\end{aligned}$$

since $R(w_L) > R(w_H)$ by assumption. Thus the deviation is strictly profitable for the household. The firm's payoff is

$$\begin{aligned}
\text{firm payoff}' &= y + z' - w_H, && \text{(A.34)}\\
&= y + z + w_H - w_L + \frac{R(w_L) - R(w_H)}{2} - w_H && \text{(A.35)}\\
&= y + z - w_L + \frac{R(w_L) - R(w_H)}{2} && \text{(A.36)}\\
&> y + z - w_L, && \text{(A.37)}
\end{aligned}$$

since $R(w_L) > R(w_H)$ by assumption. Thus the deviation is strictly profitable for the firm.

This contradicts the supposition that the household makes the repayment $R(w_L)$ in equilibrium. Thus the household never makes the repayment $R(w_L)$ as desired. □

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