Appendix to Penman, S., F. Reggiani, S. Richardson, and İ Tuna. 2018. A Framework for Identifying Accounting Characteristics for Asset Pricing Models, with an Evaluation of Book-to-Price. Forthcoming. *European Financial Management*.

The expected return under alternative accounting

Section 2.1 introduced equation (1a):

$$E(R_{t+1}) = \frac{E(Earnings_{t+1})}{P_t} + \frac{E(P_{t+1} - B_{t+1}) - (P_t - B_t)}{P_t}$$

This appendix demonstrates that the second component captures expected earnings growth subsequent to period t+1. It also demonstrates the calculation of the expected return in the four accounting cases in section 2.1: case I with market-to-market accounting; case II with permanent income accounting and no expected earnings growth; case III with expected earnings growth with no relation to risk; and case IV where expected growth is priced as risky. These four cases assume no leverage. Case V then adds leverage to stress the separation of (unlevered) accounting numbers from the effects of leverage. In each case the role of B/P in indicating the expected return is highlighted.

We consider a single firm assumed to be a going-concern. We track accounting numbers (book value of equity, B, and earnings) for three periods, t+1, t+2, and t+3 after the present time, t, but the example generalizes to many future periods. Successive book values are at the end of each period and earnings are a flow variable over periods, observed at the end of each period. Book value and earnings articulate across time periods via the clean surplus relation. In the first three cases, we use a pre-specified required rate of return of 10 percent. In the fourth case, the accounting conveys risk and hence relates to the required rate of return. In all cases, prices obey the inter-temporal no-arbitrage condition.

Dividends make no difference to the premium, P – B, as dividends reduce book value and price dollar-for-dollar under Miller & Modigliani (M&M) assumptions (as will be demonstrated). And dividends do not affect expected returns under finance theory: $P_t + d_t - P_t$ is independent of dividends. However, dividends do affect earnings growth via retention, but that retention does not add to value under M&M: dividends are reinvested at the required return rate (with no effect on investments). Accordingly, the examples are with full payout—all earnings are distributed—just to keep it simple. Consequently, earnings growth introduced into the examples is potentially value-relavant growth. It is easy to show that the demonstrations go through under any payout policy.

In each case, the reader can confirm that the valuations agree with those from the residual income model and the Ohlson-Jeuttner model.

I. $P_t = B_t$ (mark-to-market accounting). Expected return = 10%, no financing leverage

This base case sets $P_t = B_t$: mark-to-market accounting which is applied in all subsequent periods. By no-arbitrage residual income valuation, book return on equity (ROE) equals the required return of 10%, yielding $Earnings_{t+1}$ of 10 on book value of 100 at t. Successive book values satisfy the clean-surplus relation. Prices at all points satisfy the no-arbitrage condition: $E(P_{t+1} + Dividends_t) = P_t \times 1.10$, and so for subsequent periods. As price equals book value at all points, the expected change in premium is zero. The expected return is equal to the forward earnings yield, as in equation (1a). B/P = 1 for any required return, so B/P does not relate to the required return.

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Time	t	t+1	t+2	t+3
Earnings		10	10	10
Dividends		10	10	10
Book value (B)	100	100	100	100
Earnings growth rate[1]			0%	0%
Price (P)	100	100	100	100
Premium (P – B)	0	0	0	0
ΔPremium		0	0	0
Earnings yield		*10%*	*10%*	*10%*
$\Delta Premium/Price_{t-1}$		*0%*	*0%*	*0%*
Expected return		*10%*	*10%*	*10%*
ROE		10%	10%	10%
B/P	1.0			

[1] With zero payout (retention), $Earnings_{t+2}$ = 11and the earnings growth rate = 10%. But this comes only from retention, with no effect on P_t. As $B_{t+1} = P_{t+1}$ = 110 and $Earnings\ yield_{t+1}$ = 10%, there is no effect on the expected premium for t+1 or the inferred expected return.

II. $P_t \neq B_t$ and permanent income accounting: no expected earnings growth and consequently no expected change in premium. Expected return = 10%, no financing leverage

This case introduces a premium over book value with $P_t = 100$ and $B_t = 80$. There is no expected earnings growth in period t+2 and beyond. With book values generated according to the clean surplus relation and prices generated under the no-arbitrage condition, there are expected premiums over book value in future periods but no expected change in the premium. Accordingly, the expected return is equal the forward earnings yield, as in equation (1a). B/P is lower than in case I, but the expected return does not change.

Time	t	t+1	t+2	t+3
Earnings		10	10	10
Dividends		10	10	10
Book value	80	80	80	80
Earnings growth rate			0%	0%
Price	100	100	100	100
Premium	20	20	20	20
ΔPremium		0	0	0
Earnings yield		*10%*	*10%*	*10%*
$\Delta Premium/Price_{t-1}$		*0%*	*0%*	*0%*
Expected return		*10%*	*10%*	*10%*
ROE		12.5%	12.5%	12.5%
B/P	0.80			

III. The case of $P_t \neq B_t$ with expected earnings growth and consequently an expected change in premium but no effect on the expected return. Expected return = 10%, no financing leverage.

With Case II as a starting point, earnings of 10 in t+1 are now reduced by a 0.25 shift from t+1 to t+2, yielding expected earnings growth for t+2 of 5.38%. This is a pure one-period accounting shift unrelated to value, with earnings reverting to 10 in t+3 with a growth rate of -2.68%. The earnings growth induces a change in premium: with prices set by the no-arbitrage condition (and maintaining the same payout as in Case II), the premium of price over book value increases in period t+1 (with positive earnings growth) and decreases in t+2 (with negative growth). As in equation (1a), the expected equity return of 10% is equal to the forward earnings yield *plus* the expected change in the premium relative to beginning-of-period price. However, while equation (1a) yields a calculation of the expected return, there is no connection between the accounting (and the growth it generates) and the required return. There is no effect on B/P.

	|——————|——————|——————|……→			
Time	t	t+1	t+2	t+3
Earnings[1]		9.75	10.275	10
Dividends		9.75	10.275	10
Book value	80	80	80	80
Earnings growth rate			5.385%	-2.676%
Price	100	100.25	100	100
Premium	20	20.25	20	20
ΔPremium		0.25	-0.25	0
Earnings yield		*9.75%*	*10.25%*	*10.0%**
$\Delta Premium/Price_{t-1}$		*0.25%*	*-0.25%*	*0.00%**
Expected return		*10.0%*	*10.0%*	*10.0%*
ROE		12.19%	12.85%	12.5%
B/P	0.8			

[1] Earnings in t+2 = Case II Earnings + Earnings shift from t+1 to t+2 + Earnings from lower dividends (higher retention) in t+1 = 10 + 0.25 + (0.25 ×0.1) = 10.275.

*Rounded

IV. $P_t \neq B_t$ with expected earnings growth and consequently an expected change in premium, but now with an effect on the required return. No financing leverage.

The final case connects earnings growth to risk and demonstrates the P1 and P2 properties in section 2.1.2 under which B/P indicates risk and the required return.

Case II again serves as the base case. In case IV, there is again a reduction of earnings in t+1 to 9.75 (as in Case III), but the forward E/P is maintained at the 10% in case II. Assuming A2 in section 2.1.2 and setting G = 1 and $\lambda = 0.02$, the forecast of cum-dividend earnings growth in t+2 is given by $g_2 = \lambda \frac{B_t}{Earnings_{t+1}} = 16.41\%$. However, the growth adds to risk: Setting $\gamma = 1$ for simplicity (that is, no abnormal growth after t+2 and thus no earnings growth with full payout, as in the example), the required return increases from 10% in case II to 12.81% with the maintained E/P ratio of 10%: By equation (1d) with G = 1,

$$r = \sqrt{\frac{\lambda B_0}{P_0}} = \sqrt{0.02 \times 0.8205} = 12.81\%$$

(the Fama and French special case).With this required return, the forecast of ex-dividend earnings in t+2 is 10.10, 10.452in t+3, and so for subsequent earnings by setting $\gamma = 1$.The A1 valuation is satisfied.

B/P increases from 0.80 in cases II and III to 0.8205. For the same E/P as in Case II, a higher B/P indicates higher earnings growth, as in P1. Further, P2 is demonstrated: For a given E/P, a higher B/P indicates a higher required return. The reader can continue the comparative statics, allowing G, λ, and γ to change.

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Time	t	t+1	t+2	t+3
Earnings		9.75	10.101	10.452
Dividends		9.75	10.101	10.452
Book value	80	80	80	80
Cum-div growth rate			16.41%	16.29%
Earnings growth rate			3.6%	3.47%

Expected return		12.81%	12.81%	12.81%
Price[1]	97.50	100.240	102.979	105.719
Premium	17.50	20.240	22.979	25.719
ΔPremium		2.740	2.739	2.739
Earnings yield		*10.00%*	*10.077%*	*10.150%*
ΔPremium/Price$_{t-1}$		*2.81%*	*2.733%*	*2.660%*
Expected return		*12.81%*	*12.81%*	*12.81%*
ROE		12.19%	12.63%	13.07%
B/P	0.8205			

[1] $P_t = \frac{Earnings_{t+1}}{r} \times \frac{g_2 - (\gamma - 1)}{r - (\gamma - 1)} = \frac{9.75}{0.1281} \times \frac{0.1641 - 0}{0.1281 - 0} = 97.50$. As γ = 0 (abnormal earnings growth after t+2 is zero, this is equivalent to $P_t = \frac{9.75}{0.1281} + \frac{0.35094}{0.1281} = 97.50$, where 0.35094 in abnormal earing growth in t+2 continuing with no change.

V. $P_t \neq B_t$ with expected earnings growth in operations with an effect on the required return. Now with added financing leverage.

This case adds leverage to case IV in the form of a debt to equity swap at market value that leaves unlevered operating activities unchanged: book value for operations is now financed by 40 in equity and 40 in net debt. With no effect operations, the numbers for operating activities remain the same as in case III but, with a borrowing rate of 5%, earnings equal operating income reduced by net interest on the debt. A full payout policy is maintained such that free cash flow (which is the same as in case IV) is split between this dividend and debt service.

The example shows that leverage increases the earnings growth rate in t+2 over the operating income growth rate in accordance with equation (6) and also increases ROE_{t+1} in accordance with equation (7). Leverage increases the expected return in accordance with equation (3), but price is unaffected. Leverage does not affect the equity premium but affects the forward E/P ratio in accordance with equation (4) such that the higher expected return is indicated by the increased (levered) E/P rather than B/P. While the unlevered B/P remains the same as in case IV, the levered B/P decreases in accordance with equation (5).

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Time	t	t+1	t+2	t+3
Operating income (*OI*)		9.75	10.101	10.452
Net interest expense (at 5%)		2.00	2.00	2.00
Earnings		7.75	8.101	8.452
Dividends (*d*)		7.75	8.101	8.452
Payment on net debt (*F*)		2.00	2.00	2.00
Free cash flow (*FCF*)		9.75	10.101	10.452
Net operating assets (*NOA*)	80	80	80	80
Net debt (*ND*)	40	40	40	40
Book value of Equity (*B*)	40	40	40	40
OI growth rate			3.6%	3.47%
Earnings growth rate			4.529%	4.33%
Expected unlevered return		12.81%	12.81%	12.81%

Expected equity return[1]		18.243%	17.995%	17.770%
Unlevered price (P^{NOA})	97.50	100.240	102.979	106.070
Net debt (P^{ND}= *ND*)	40.0	40.0	40.0	40.0
Equity price (*P*)	57.5	60.240	62.979	65.719
Equity premium	17.5	20.240	22.979	25.719
ΔPremium		2.740	2.739	2.740
Earnings yield		*13.478%*	*13.448%*	*13.420%*
ΔPremium/Price$_{t-1}$		*4.765%*	*4.547%*	*4.350%*
Expected return		*18.243%*	*17.995%*	*17.770 %*
Financing leverage	0.6956			
RNOA		12.18%	12.63%	13.07%
ROE		19.38%	20.25%	21.13%
Unlevered B/P	0.8205			
Levered B/P	0.6956			

[1] From equation (3), the expected equity return for t+2 = $12.81\% + \left[\frac{40.0}{57.5} \times (12.81\% - 5\%)\right]$ = 18.243%, and so for subsequent years. This expected return declines over time because of a decline in leverage.