Asset Demand Tests of Risk Preferences with Probability Dependent NM Indices

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 \Rightarrow Asset demands are functions of *probabilities*, prices, and income.

 \Rightarrow Asset demands can be generated by 'EU' preferences, where the von Neumann-Morgenstern (NM) index depends on probabilities.

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- ▶ Revealed preference (Polisson, Quah, and Renou, 2017),
- Experimental design (Choi *et al.* 2007).
- ▶ We compare the empirical performances of EU, probability dependent NM, and rank dependent utility (RDU).

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- (a) EU over all slices with the *same* NM index,
- (b) EU on each slice but with different NM indices corresponding to the different distributions defining each slice,
- (c) RDU over all slices with the *same* value function and probability weighting function.

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With $S \ge 2$, a consumption plan is given by $x = (x_1, x_2, \dots, x_S) \in \mathbb{R}^S_+$ and $\Delta(S) = \{\pi \in \mathbb{R}^S_{++} : \sum_s \pi_s = 1\}.$

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(TC) Tradeoff Consistency: For any given $\pi \in \Delta(S)$,

$$\begin{aligned} x_{-s}(a) \sim_{\pi} x'_{-s}(b), \ x_{-s}(c) \sim_{\pi} x'_{-s}(d), \ x''_{-s'}(a) \sim_{\pi} x''_{-s'}(b) \\ \implies \\ x''_{-s'}(c) \sim_{\pi} x''_{-s'}(d), \end{aligned}$$

where $x_{-s}(y)$ denotes x with x_s is replaced by $y \in \mathbb{R}_+$.

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TC is critical for existence of an EU representation on a single slice.

Furthermore, TC can be *modified*/strengthened to hold *across* slices, which is critical for existence of an EU representation on a *set* of slices and with the same NM index on each slice.

Preferences over Contingent Claims – Example

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Preferences over Contingent Claims – Example

Consider the utility function given by

$$U(x;\pi) = -\sum_{s=1}^{3} \pi_s (\exp(-\pi_1 x_s) + \exp(-\pi_2 x_s) + \exp(-\pi_3 x_s))$$
$$= \sum_{s=1}^{3} \pi_s u_{\pi}(x_s),$$

where $u_{\pi}(y) = -(\exp(-\pi_1 y) + \exp(-\pi_2 y) + \exp(-\pi_3 y)).$

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Let $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ be a finite set of prices $p^t = (p_1^t, p_2^t, \dots, p_\ell^t) \gg 0$ and demands $x^t = (x_1^t, x_2^t, \dots, x_\ell^t) \ge 0$ drawn on a consumer.

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Definition: A utility function $U : \mathbb{R}^{\ell}_+ \to \mathbb{R}$ is said to rationalize the data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ if, at every observation $t = 1, 2, \ldots, T$,

$$U(x^t) \geqslant U(x) \text{ for any } x \in \{x \in \mathbb{R}^\ell_+ : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

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Afriat's (1967) Theorem establishes the following equivalence:
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Afriat's (1967) Theorem establishes the following equivalence:

- (1) \mathcal{O} is rationalizable by a locally nonsatiated utility function U,
- (2) \mathcal{O} obeys a no-cycling condition (GARP),
- (3) There exists a solution to a particular system of linear (Afriat) inequalities constructed from \mathcal{O} ,

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- (3) There exists a solution to a particular system of linear (Afriat) inequalities constructed from \mathcal{O} ,
- (4) \mathcal{O} is rationalizable by a utility function U, which is increasing, concave, and continuous.

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Now suppose that an agent is choosing contingent consumption, i.e.,

$$p^{t} = (p_{1}^{t}, p_{2}^{t}, \dots, p_{S}^{t}),$$
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are vectors of state prices and contingent consumption, respectively.

How might one conduct revealed preference tests analogous to Afriat's for different tailor-made models of decision making under risk?

E.g., if we know the probability of state s to be $\pi_s > 0$, how do we test for rationalizability by EU, i.e., that there is an *increasing* and *continuous* function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^{S} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s u(x_s) \text{ for any } x \in B^t,$$

where $B^t = \{x \in \mathbb{R}^S_+ : p^t \cdot x \leqslant p^t \cdot x^t\}$?

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Given \mathcal{O} , define the set $\mathcal{X} = \{x_s^t : (s,t) \in \{1,\ldots,S\} \times \{1,\ldots,T\}\} \cup 0$, and then the finite lattice $\mathcal{L} = \mathcal{X}^S$.

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E.g., suppose that we observe $x^1 = (2, 5)$, $p^1 = (5, 2)$, $x^2 = (6, 1)$, $p^2 = (1, 3)$, $x^3 = (4, 3)$, $p^3 = (3, 4)$, $\pi = (1/2, 1/2)$.

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Then, $\mathcal{X} = \{0, 1, 2, 3, 4, 5, 6\}$, and $\mathcal{L} = \mathcal{X} \times \mathcal{X}$.





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Then, $\mathcal{X} = \{0, 1, 2, 3, 4, 5, 6\}$, and $\mathcal{L} = \mathcal{X} \times \mathcal{X}$.

For EU-rationalizability, it is clearly *necessary* that there are real numbers $\bar{u}(0) < \bar{u}(1) < \cdots < \bar{u}(6)$, such that, at every $t \in \{1, 2, 3\}$,

$$\frac{1}{2}\overline{u}(x_1^t) + \frac{1}{2}\overline{u}(x_2^t) \ge \frac{1}{2}\overline{u}(x_1) + \frac{1}{2}\overline{u}(x_2) \text{ for any } x \in B^t \cap \mathcal{L},$$
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It is also *sufficient* to guarantee EU-rationalizability by an increasing and continuous function $u : \mathbb{R}_+ \to \mathbb{R}$ that extends $\bar{u} : \mathcal{X} \to \mathbb{R}$.

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It is also *sufficient* to guarantee EU-rationalizability by an increasing and continuous function $u : \mathbb{R}_+ \to \mathbb{R}$ that extends $\bar{u} : \mathcal{X} \to \mathbb{R}$.

So we only need to check for EU-rationalizability on a finite lattice, which is a straightforward linear test.

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Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ such that, at every observation t = 1, 2, ..., T,

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Intuition: First we replace \bar{u} with the step function $\hat{u} : \mathbb{R}_+ \to \mathbb{R}$ such that $\hat{u}(y) = \bar{u}(y)$ for all $y \in \mathcal{X}$ and \hat{u} is constant between values of \mathcal{X} . Clearly, \hat{u} rationalizes the data in the sense that

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The only problem is that \hat{u} is neither increasing nor continuous. But it is possible to find another utility function u, arbitrarily close to \hat{u} , that is increasing and continuous which also rationalizes the data.

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$$\mathcal{X} = \{0,1,2,\ldots,6\}$$

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Suppose now that x^t is instead chosen from a compact constraint set $B^t \subset \mathbb{R}^S_+$, so the data set is now $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$.

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Typically, the utility function in particular model of choice under risk or under uncertainty takes the form

$$U(x) = \phi(u(x_1), u(x_2), \dots, u(x_S)),$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is again an increasing and continuous function, and where $\phi : \mathbb{R}^S \to \mathbb{R}$ is an increasing and continuous function that is drawn from the family Φ , which is specific to the model.

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E.g., objective and subjective expected utility, rank dependent utility, disappointment aversion, choice acclimating personal equilibrium, maxmin expected utility, and variational preferences.

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The lattice test can be further extended in two important directions.

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The lattice test can be further extended in two important directions.

(1) We can allow for *probability dependence* in the NM index.

• Given the finite set of probability weights $\{\pi^t\}_{t=1}^T$, we can check for rationalizability by $\phi : \mathbb{R}^S \to \mathbb{R}$ and a finite collection $\{u_\pi\}_{\pi \in \Pi}$, where for each $\pi \in \Pi = \{\pi \in \Delta(S) : \pi = \pi^t \text{ for some } t\}$, the function $u_\pi : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing and continuous.

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 - ▶ We find $U : \mathbb{R}^S_+ \to \mathbb{R}$ such that $U(x^t) \ge U(x)$ for any $x \in B^t(e)$, where $e \in [0, 1)$, and where

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• The largest e at which a data set passes the test is known as the critical cost efficiency index (CCEI).

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We implement an array of tests using data collected from a portfolio choice experiment that extends Choi, Fisman, Gale, and Kariv (2007).

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In each decision problem, every subject was given a budget; income was normalized to one, and state prices were chosen at random.

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We conduct a set of *nonparametric* empirical analyses.

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▶ We check F-GARP (Nishimura, Ok, and Quah, 2017) in order to test for *stochastically monotone* utility maximization, i.e., for a utility function which obeys *first order stochastic dominance*.

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- (5) Basic rationalizability results hold even after controlling for the empirical permissiveness/stringency of the different models ...