# A Macroeconomic Model with Financially Constrained Producers and Intermediaries \*

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#### Abstract

How much capital should financial intermediaries hold? We propose a general equilibrium model with a financial sector that makes risky long-term loans to firms, funded by deposits from savers. Government guarantees create a role for bank capital regulation. The model captures the sharp and persistent drop in macro-economic aggregates and credit provision as well as the sharp change in credit spreads observed during the Great Recession. Policies requiring intermediaries to hold more capital reduce financial fragility, reduce the size of the financial and non-financial sectors, and lower intermediary profits. They redistribute wealth from savers to the owners of banks and non-financial firms. Precrisis capital requirements are close to optimal. Counter-cyclical capital requirements increase welfare.

JEL: G12, G15, F31.

Keywords: financial intermediation, macroprudential policy, credit spread, intermediarybased asset pricing

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# 1 Introduction

The financial crisis and Great Recession of 2007-09 underscored the importance of the financial system for the broader economy. Borrower default rates, bank insolvencies, government bailouts of financial institutions, and credit spreads all spiked while real interest rates were very low. The disruptions in financial intermediation fed back on the real economy. Consumption, investment, and output all fell substantially and persistently.

These events have prompted a vigorous yet unresolved debate among policymakers and academics on whether the economy would be better off with stricter bank capital requirements. The 2017 Minneapolis Plan, reflecting the Federal Reserve's view, proposes raising bank capital requirements to 23.5% of risk-weighted assets, with further increases to 38% for banks that remain systemically important. In their seminal book, Admati and Hellwig (2013) propose raising capital requirements to 25% of assets. Larger equity capital buffers would result in less risk-taking, lower risk of bank failure and concomitant government bailouts, but also in a smaller banking sector that lends less to the real economy, depressing investment and output. Considering this trade-off, Admati and Hellwig argue that "for society, there are in fact significant benefits and essentially no cost from much higher equity requirements." The authors of the Minneapolis Plan agree, writing that their plan "will have paid for itself many times over if it avoids one financial crisis." This argument is not without controversy in the academy (Calomeris, 2013) and heavily contested by the industry.

What is missing in this debate is a quantitative general equilibrium model that embeds a financial sector in a model of the macro-economy, and that can capture infrequent but large financial crises. Our paper fills this void. In the model, banks extend long-term loans to firms who invest and are subject to aggregate and idiosyncratic productivity shocks. Firm default results in losses for their lenders. The high leverage of banks, which far exceeds that of firms, amplifies modest credit losses into financial disasters. Because financial intermediaries are constrained in their ability to raise new debt and new equity, even banks that do not default cut lending to firms. The intermediary sector shrinks substantially and persistently. The reduction in credit supply feeds back on the real economy and depresses investment and output, in a second financial accelerator. The nonlinear behavior of credit spreads reflects this financial distress. The government bails out the creditors of the banks that fail by issuing government

debt, gradually repaid through future taxation. Real interest rates must fall to induce savers to accommodate the reduction in deposits. Banks' reduced ability to absorb aggregate risk in financial crises results in a deterioration of risk sharing and higher macro-economic volatility. The intermediary-driven dynamics arise in equilibrium since all aggregate shocks emerge from the real sector.

The calibrated model matches many features of the data, both in terms of macro-economic quantities and prices. The average credit spread and its volatility are high, as in the data. Faced with a realistic corporate bond rate, firms choose the observed amount of leverage. The non-financial leverage ratio is 38%, close to the U.S. data. The model delivers a 93% leverage ratio for financial firms, a key moment not directly targeted by the calibration, which is close to the data. Debt is attractive to banks for four reasons. First, debt enjoys a tax shield. Second, the government guarantees the liabilities of the bank. This guarantee captures not only deposit insurance but also broader too-big-to-fail guarantees to banks and the rest of the levered financial system.<sup>1</sup> Third, banks face equity adjustment costs which increase the cost of equity relative to debt. Fourth, banks provide a safe asset to patient households with a desire for holding risk-free assets. While the first motive for debt financial and non-financial sector leverage is a key feature of many developed economies and crucial to understanding systemic risk in society. The equilibrium fully takes into account that the cost of bank debt and bank equity changes endogenously with the safety of the financial sector.

Our main exercise is to study macro-prudential policy in this environment. We study increasing the minimum bank equity capital requirement from its pre-crisis level of 7% of assets. Higher capital requirements are successful at reducing financial leverage and the bank failure rate. Corporate debt also becomes safer and loss rates fall. The reduction in firm and bank bankruptcies frees up resources otherwise spent on deadweight losses from bankruptcy. Macroeconomic volatility declines when banks are better capitalized, reflecting the balance of two forces. A smaller banking sector has less risk absorption capacity, raising volatility, but the reduced financial fragility lowers volatility.

<sup>&</sup>lt;sup>1</sup>We use the labels intermediaries and banks interchangeably to mean the entire levered financial sector. That sector also includes broker-dealers and insurance companies, which –like banks– are highly levered, are subject to macro-prudential regulation, often engage in maturity transformation, and enjoy explicit or implicit government guarantees on their liabilities. The issues of systemic risk equally apply to those non-bank institutions. Appendix C.1 provides a detailed definition of our intermediary sector.

While the banking sector becomes safer, it also becomes smaller. Savers' direct holdings of corporate debt do not fully make up for the loss in intermediation capacity, and credit to the real economy shrinks. Firms borrow less, reducing both leverage and investment. The aggregate capital stock and GDP fall. The reduced size of the economy is the main adverse effect from tighter macro-prudential policy.

Tighter macro-prudential policy hurts bank profitability. The cost of debt for banks is low and insensitive to capital regulation thanks to government guarantees on bank liabilities. The cost of bank equity falls when banks are better capitalized, reflecting their improved safety. However, the weighted average cost of capital increases significantly because higher bank equity capital requirements force banks to finance themselves to a larger extent with expensive equity rather than with cheap debt. In equilibrium, banks can only pass through some of the higher cost of funding to firms. Bank profitability falls and so does the franchise value of banking.

Substantially looser capital requirements increase the size of the banking sector and the real economy, but financial fragility increases non-linearly with bank leverage. Financial crises beget larger bailouts and destroy resources that could otherwise be consumed.

To rank economies that differ in capital requirement, we calculate welfare for the two types of households in the model: patient savers and impatient borrowers. Savers invest in both risk-free debt (deposits and government bonds) as well as in corporate debt. Borrowers are the equity holders of the non-financial and financial firms. Tighter capital requirements redistribute wealth from savers to borrowers. A smaller banking sector reduces deposits and thereby the wealth of savers. Borrowers receive higher dividend payments as banks and firms shift their capital structure towards equity. Thus, the owners of banks (and firms) gain from tighter bank regulation. Since savers have a higher marginal value of wealth than borrowers given the difference in patience rates, the welfare losses to savers exceed the gains to borrowers. It is not possible to implement a Pareto-improving tax-and-transfer scheme. An additional \$6 trillion would be necessary to make the economy as well off at a 25% capital requirement as at the baseline 7% requirement. Welfare is maximized at a 6% equity requirement. Capital requirements below 6% reduce welfare as the economy becomes more volatile and both savers' and borrowers' consumption fall due to large deadweight losses from bankruptcies.

An alternative counter-cyclical capital requirement policy that tightens capital requirements in good times and relaxes them in times of financial stress increases aggregate welfare to the tune of 5% of GDP. The policy mitigates the fallout from financial crises, preventing much of the contraction in firm investment that takes place in the baseline economy. It allows for a larger financial sector and improved risk sharing.

Our work is at the intersection of macro-economics, asset pricing, corporate finance, and banking. We contribute to the literature on the role of credit constraints in models of the macro-economy. To help understand and quantify the role of the various frictions in our model, we solve a sequence of simpler problems. The simplest one is a standard heterogeneous-agent macro model where patient savers lend directly to firms owned by impatient borrowers. This is an economy like Kiyotaki and Moore (1997), except that savers cannot directly hold productive capital.<sup>2</sup> Markets are incomplete, and debt is uncontingent, long-term, and enjoys tax benefits.<sup>3</sup> Our solution method recognizes the importance of nonlinear dynamics, as emphasized by Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013). This model features the standard financial accelerator mechanism of Bernanke, Gertler, and Gilchrist (1999a) in that negative aggregate productivity shocks lower the price of firm capital, tightening firm borrowing constraints, and amplifying the contraction. However, the amplification is modest and the simple model fails to generate deep crises.

Next, we introduce the possibility of corporate default. This activates uncertainty shocks which change the cross-sectional dispersion of firm productivity, as in Christiano, Motto, and Rostagno (2014).<sup>4,5</sup> Recessions that combine a TFP decline with an increases in uncertainty are substantially deeper than in the model without default. The credit spread rises significantly,

 $<sup>^{2}</sup>$ The data reveal that a large fraction of households indeed do not participate in risky asset markets; stock market wealth is heavily concentrated. A large literature explores deeper reasons for limited stock market participation including both monetary (one-time and recurring fixed costs) and non-monetary costs, possibly arising from limited investor sophistication, attention, or financial literacy. See Guiso and Sodini (2013) for a review.

<sup>&</sup>lt;sup>3</sup>Debt-like contracts arise in order to reduce the cost of gathering information and to mitigate principal-agent problems. See the costly state verification models in the tradition of Townsend (1979) and Gale and Hellwig (1985), and the work on the information insensitivity of debt by Dang, Gorton, and Holmstrom (2019).

<sup>&</sup>lt;sup>4</sup>The shock is calibrated from firm-level evidence in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018). Jermann and Quadrini (2012) study how financial shocks affect balance sheet variables. One interpretation of our uncertainty shocks is as aggregate misallocation shocks, as in Hsieh and Klenow (2009). Ai, Li, and Yang (2019) study the role of intermediaries in reducing capital misallocation.

<sup>&</sup>lt;sup>5</sup>Most models in the literature feature no default on corporate loans. For example, Cúrdia and Woodford (2016), Goodfriend and McCallum (2007), Meh and Moran (2010), and Christiano, Motto, and Rostagno (2014). A few exceptions are Gertler and Kiyotaki (2010), Angeloni and Faia (2013), Hirakata, Sudo, and Ueda (2013), Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015), and Gete (2020). Those that do feature default employ short-term debt, abstracting from a key source of risk –maturity mismatch–associated with financial intermediation.

but is still only half as large as in the data.

In a third step, we switch on the intermediation sector by increasing the cost to savers for holding corporate debt directly. A large banking literature has micro-founded the comparative advantage of banks in lending by appealing to their superior screening, monitoring, enforcement, modification, and risk management technologies.<sup>6</sup> Much of the prior intermediation literature combines the balance sheets of firms and banks, implicitly assuming that financial intermediaries hold equity claims in productive firms, while in reality, they hold debt-like claims. A key innovation of our paper is to study the quantitative implications of separating the balance sheets of firms and banks.<sup>7</sup> Intermediaries help allocate risk between borrowers and savers, and their net worth is a key state variable. When intermediaries have unlimited liability and can raise new equity from their shareholders costlessly, they internalize the risk they take on and remain well capitalized throughout the cycle. Credit spreads are much higher, however, because they reflect the higher degree of impatience of bank owners, as well as the higher holding costs of corporate debt for savers. The higher cost of borrowing reduces firm leverage and makes the economy safer. This model illustrates that the intermediation sector can play a stabilizing role.

Next, we introduce limited liability for banks coupled with deposit insurance. As Reinhart and Rogoff (2009) and Jorda, Schularick, and Taylor (2015) make clear, financial intermediaries frequently become insolvent. When they do, their creditors (mostly depositors) are typically bailed out by the government. Bailout guarantees create franchise value, which banks are eager to preserve. However, the same guarantees also affect risk taking incentives and creates scope for regulation that limits bank leverage.<sup>8</sup> A default wave on the loan portfolio erodes bank capital, and reduces banks' ability and willingness to extend loans to producers. The cost of credit shoots up. Through this second financial accelerator, corporate investment and output decline. Some banks decide to default. The deadweight losses associated with firm and bank

<sup>&</sup>lt;sup>6</sup>Costly state verification models also justify the existence of financial intermediaries who avoid the duplication of verification costs, as in Williamson (1987), Krasa and Villamil (1992), Diamond (1984). Diamond (2020) shows how financial intermediaries arise endogenously when safe assets are scarce. Intermediaries make risky loans to non-financial firms and transform them into safe assets they issue to households. The size of the financial sector trades off the agency cost of bearing risk and the benefit of safe asset creation.

<sup>&</sup>lt;sup>7</sup>Recent work by Klimenko, Pfeil, Rochet, and Nicolo (2016), Rampini and Viswanathan (2020), and Gale and Gottardi (2020) also models intermediaries separately from producers. The setting is simpler since their focus is theoretical; ours is quantitative. As noted, Diamond (2020) provides a micro-foundation for this balance sheet separation.

<sup>&</sup>lt;sup>8</sup>See Kareken and Wallace (1978), Van den Heuvel (2008), Farhi and Tirole (2012), or Gomes, Grotteria, and Wachter (2019). Others justify the presence of bank leverage or net worth constraints by the ability of banks to divert cash flows, as in Gertler, Kiyotaki, and Queralto (2012).

default reduce resources for all. Limited liability and deposit insurance are a key amplifying force through which financial sector instability arises and feeds back on the real economy.

The final friction, which gets us back to the benchmark model, is equity issuance costs for banks.<sup>9</sup> They reduce bank risk taking ex-ante, since banks hold more equity to save on issuance costs in states of the world where losses are large. But, conditional on being in a financial recession, they make bank recapitalization more costly ex-post. The result is deeper financial crises and higher credit spreads.

Our model also contributes by introducing fiscal policy and the endogenous determination of safe asset rates. In a financial crisis, intermediaries contract the size of their balance sheet, reducing the supply of safe assets. The reduction in the supply of deposits is offset by an increase in government debt due to counter-cyclical fiscal policy and bank bailouts. Demand for safe assets increases due to a precautionary demand. The net effect is lower interest rates in a crisis. The lower cost of debt allows intermediaries to recapitalize more quickly, dampening the effect of the crisis. Under tighter macro-prudential policy, financial crises become shallower. Banks' supply of deposits shrinks by less, the government's supply of T-bills rises by less, and precautionary savings demand is weaker. The net effect is that the safe interest rate falls by less in crises and that its unconditional mean is nearly invariant to the level of bank capital regulation. Banks' weighted cost of capital rises with tighter bank capital requirements because banks must substitute cheap debt with expensive equity. In contrast, Begenau (2020) assumes that savers have an explicit preference for liquid assets in the utility function and that the convenience yield from deposits rises as the amount of deposits in the economy shrinks. As capital requirements tighten, the cost of debt falls in her model. Since bank equity also earns low returns, bank funding becomes cheaper and tighter macro-pru policy welfare improving. This conclusion reverses in a model with a more realistic bank equity risk premium.

Our paper belongs to the literature on quantitative models of bank regulation, including Van den Heuvel (2008), Nguyen (2018), Begenau (2020), Begenau and Landvoigt (2019), Corbae and D'Erasmo (2019), and Davydiuk (2019). Relative to the previous literature, we study a general equilibrium model that features severe financial recessions arising due to the nonlinear interaction of financial constraints in the production and intermediation sectors. This allows us

 $<sup>^{9}</sup>$ Beginning with Myers and Majluf (1984), the corporate finance literature has micro-founded equity issuance costs using asymmetric information of managers over investors.

to quantify the benefit of preventing such financial crises using regulation. A different branch of the normative literature instead studies the interactions between conventional and unconventional monetary policy and financial intermediation.<sup>10</sup> More broadly, our model creates room for macro-prudential regulation due to incomplete markets and borrowing constraints.<sup>11</sup>

Our work also contributes to the intermediary-based asset pricing literature.<sup>12</sup> Our model generates the unconditional credit spread, a puzzle in the asset pricing literature (Chen (2010)). It also generates the observed volatility and counter-cyclicality of that spread, consistent with patterns documented by Krishnamurthy and Muir (2017). The (shadow) stochastic discount factor of the intermediaries, driven by the intermediary net worth dynamics, is volatile and counter-cyclical. As in Santos and Veronesi (2017), all shocks arise in the non-financial sector, yet give rise to rich–endogenous–intermediary dynamics.

Finally, our paper contributes in its solution technique. The model has two exogenous and persistent sources of aggregate risk and five endogenous aggregate state variables which track the wealth distribution. It features default and occasionally binding borrowing constraints in both non-financial and financial sectors. To solve this difficult problem, we provide a nonlinear global solution method. The algorithm, detailed in computational appendix B, solves for a set of nonlinear equations including the Euler, Kuhn-Tucker, and market clearing equations. A judicious choice of state variables and several improvements, such an analytical Jacobian, result in a stable, precise, and reasonably fast algorithm that is portable to different questions.

The rest of the paper is organized as follows. Section 2 discusses the model setup. Section 3 presents the calibration. Section 4 contains the main results. Section 5 uses the model to study various macro-prudential policies. Section 6 concludes. All model derivations, computational details, some details on the calibration, and some additional quantitative results are relegated to the appendix.

<sup>&</sup>lt;sup>10</sup>See Gertler and Karadi (2011), Angeloni and Faia (2013), Drechsler, Savov, and Schnabl (2018, 2017), Cúrdia and Woodford (2016), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), Elenev (2016), and De Fiore, Hoerova, and Uhlig (2017).

<sup>&</sup>lt;sup>11</sup>Papers that have studied the qualitative role of these frictions in determining optimal policy are Lorenzoni (2008), Mendoza (2010), Korinek (2012), Bianchi and Mendoza (2018), Guerrieri and Lorenzoni (2017), and Clerc et. al. (2015).

<sup>&</sup>lt;sup>12</sup>In addition to the work cited above, notable contributions are He and Krishnamurthy (2012, 2013, 2019), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2017), and Moreira and Savov (2016). On the empirical side, He, Kelly, and Manela (2017) develop a risk factor which captures the systematic risk associated with declines in intermediary equity capital, and Adrian, Etula, and Muir (2014) document that intermediary leverage performs well in pricing the cross-section of stock returns.

# 2 The Model

# 2.1 Preferences, Technology, Timing

**Preferences** The model features two groups of households: borrower-entrepreneurs (denoted by superscript B) and savers (denoted by S). Savers are more patient than borrowerentrepreneurs, implying for the discount factors that  $\beta_B < \beta_S$ . All agents have Epstein-Zin preferences over utility streams  $\{u_t^j\}_{t=0}^{\infty}$  with intertemporal elasticity of substitution  $\nu_j$  and risk aversion  $\sigma^j$ 

$$U_t^j = \left\{ (1 - \beta_j) \left( u_t^j \right)^{1 - 1/\nu_j} + \beta_j \left( \mathbf{E}_t \left[ (U_{t+1}^j)^{1 - \sigma_j} \right] \right)^{\frac{1 - 1/\nu_j}{1 - \sigma_j}} \right\}^{\frac{1}{1 - 1/\nu_j}},$$
(1)

for j = B, S. Agents derive utility from consumption of the economy's sole good, such that  $u_t^j = C_t^j$ , for j = B, S. The calibration will specialize (1) to log utility for both households, but the model is readily solved for Epstein-Zin preferences.

**Technology** Non-financial firms, or firms for short, operate the production technology, which turns capital and labor into aggregate output:

$$Y_t = Z_t^A K_t^{1-\alpha} L_t^{\alpha}, \tag{2}$$

where  $K_t$  is capital,  $L_t$  is labor, and  $Z_t^A$  is total factor productivity (TFP). Shocks to  $Z^A$  are the first source of aggregate risk in the model. Borrower-entrepreneurs and savers are endowed with  $\bar{L}^B$  and  $\bar{L}^S$  units of labor, respectively. Both types of households supply their labor endowment inelastically to the firms. In addition to the technology for producing consumption goods, the economy also has access to a technology that turns consumption into capital goods subject to adjustment costs. Firms are funded by long-term corporate debt that they issue to intermediaries and savers, and by equity issued to borrower-entrepreneurs.

Financial intermediaries, or banks for short, are profit-maximizing firms that extend loans to non-financial firms. They fund these loans through deposits that they issue to savers and equity capital that they raise from borrower-entrepreneurs.

Firms and banks face equity issuance costs, an important financial friction described in detail below. Both firms and banks face idiosyncratic shocks. This within-type heterogeneity allows us to capture fractional default. We make assumptions that imply aggregation into a representative firm and a representative bank, allowing us to focus on incomplete risk-sharing between savers, borrowers, firms, and banks.

Savers do not directly hold corporate equity to capture the reality of limited participation in equity markets. However, they invest in risk-free assets (bank and government debt), and risky corporate debt issued by firms. Unlike banks, savers incur holding costs when they buy corporate debt. This cost creates a comparative disadvantage for saver ownership of corporate debt, and provides a role for intermediaries in transforming long-term risky debt into short-term safe debt.

Figure 1 illustrates the balance sheets of the model's agents and their interactions. Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and intermediaries.

**Timing** The timing of agents' decisions at the beginning of period t is as follows:

- 1. Aggregate shocks are realized. Firms choose labor inputs and pay a fixed cost of production.
- 2. Idiosyncratic productivity shocks for firms are realized. Production occurs. Firms with negative profits (low idiosyncratic productivity shocks) default. Banks and savers assume ownership of bankrupt firms.
- 3. Idiosyncratic profit shocks for banks are realized. Individual banks decide whether to declare bankruptcy. The government liquidates bankrupt intermediaries. If intermediary assets are insufficient to cover the amount owed to depositors, the government provides the shortfall (deposit insurance).
- 4. All agents solve their consumption and portfolio choice problems. Markets clear. Households consume.

We now describe the borrower, firm, intermediary, and saver problems in more detail. The full set of Bellman equations and first order conditions is relegated to appendix A.

### 2.2 Borrowers

Borrowers own the equity capital of firms and banks, and receive aggregate dividend payments  $D_t^P$  from producers and  $D_t^I$  from banks, defined below in (10) and (16), respectively.

Borrowers jointly operate an investment technology.<sup>13</sup> In order to create  $X_t$  new capital units, the required input of consumption goods is  $X_t + \Psi(X_t/K_t)K_t$ , with adjustment cost function  $\Psi(\cdot)$  which satisfies  $\Psi''(\cdot) > 0$ ,  $\Psi(\delta_K) = 0$ , and  $\Psi'(\delta_K) = 0$ , and where  $K_t$  is the aggregate capital stock of the economy.

Borrowers inelastically supply their unit of labor  $\overline{L}^B$  and earn wage  $w_t^B$ . Their problem is to choose consumption  $C_t^B$  and investment  $X_t$  to maximize life-time utility  $U_t^B$  in (1), subject to the budget constraint:

$$C_t^B + X_t + \Psi(X_t/K_t)K_t \le (1 - \tau_t^B)w_t^B \bar{L}^B + p_t X_t + D_t^P + D_t^I + G_t^{T,B} + O_t^B.$$
(3)

Borrowers receive after-tax labor income, revenues from the sale of newly produced capital units to firms  $(p_t X_t)$ , dividends from the firms and intermediaries, transfer income from the government  $(G_t^{T,B})$ , and transfer income from bankruptcy proceedings  $(O_t^B)$ . These resources are used to pay for consumption and investment including adjustment costs.

#### 2.3 Firms

#### 2.3.1 Setup

Individual producers maximize the present value of the stream of dividends paid to their shareholders, the borrower-entrepreneurs. They produce using the technology

$$y_t = \omega_t Z_t k_t^{1-\alpha} l_t^{\alpha}$$

where  $\omega_t$  is an idiosyncratic productivity shock with mean one. The  $\omega_t$ -shocks are uncorrelated across firms and time. However, the cross-sectional dispersion of the  $\omega$ -shocks varies over time; specifically,  $\sigma_{\omega,t}$  follows a first-order Markov process. Productivity dispersion is the second

 $<sup>^{13}</sup>$ Equivalently, we can set up a separate investment-good producing firm sector with borrowers as their shareholders.

exogenous source of aggregate risk in the model. We refer to changes in  $\sigma_{\omega,t}$  as uncertainty shocks.

Producers buy and sell capital at price  $p_t$  in a competitive market and can borrow in the corporate debt market. Corporate debt is long-term, modeled as perpetuity bonds. Bond coupon payments decline geometrically,  $\{1, \delta, \delta^2, \ldots\}$ , where  $\delta$  captures the duration of the bond. We define a "face value"  $F = \frac{\theta}{1-\delta}$  as a fixed fraction  $\theta$  of all repayments for each bond issued. Per definition, interest payments are the remainder  $\frac{1-\theta}{1-\delta}$ . Firms issue these bonds to banks and savers in a competitive market at price  $q_t^m$ .

Labor input  $l_t$  is the composite of borrower and saver labor

$$l_t = (l_t^B)^{\gamma_B} (l_t^S)^{\gamma_S},$$

with  $\gamma_B + \gamma_S = 1$ . Further, producers have limited liability and may default for liquidity reasons. The decision problem of producers within each period has the following timing:

- 1. The aggregate productivity shock is realized. Given capital  $k_t$  and outstanding debt  $a_t^P$ , producers choose labor inputs  $l_t^j$ ,  $j \in \{B, S\}$ . Further, producers pay a fixed cost of production.
- 2. Idiosyncratic productivity shocks are realized. Production occurs. Producers that cannot service their debt from current profits default and shut down.
- Failed producers are replaced by new producers such that the total mass of producers remains unchanged. All producers pay a dividend, issue new debt, and buy capital for next period.

The flow profit at stage 2 before taxes is

$$\pi_t = \omega_t Z_t k_t^{1-\alpha} l_t^{\alpha} - \sum_j w_t^j l_t^j - a_t^P - \varsigma k_t, \tag{4}$$

where  $\varsigma$  is the fixed cost that is proportional in capital  $k_t$ . Producers with  $\pi_t < 0$  are in default and shut down. This implies a default threshold

$$\omega_t^* = \frac{a_t^P + \varsigma k_t + \sum_j w_t^j l_t^j}{Z_t k_t^{1-\alpha} l_t^{\alpha}},\tag{5}$$

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such that producers with low idiosyncratic shocks  $\omega_t < \omega_t^*$  default.

Denote producer net worth by  $n_t^P$ . This is the only individual state variable of producers at stage 3. Denote aggregate state variables by  $S_t$ . Each period, producers are expected to pay a fraction  $\phi_0^P$  of their net worth to shareholders as dividend. However, producers can deviate by raising equity  $e_t^P$  at a convex cost  $\Psi^P(e_t^P, n_t^P)$ . We can state the recursive problem as

$$V^{P}(n_{t}^{P}, \mathcal{S}_{t}) = \max_{e_{t}^{P}, k_{t+1}, a_{t+1}^{P}} \phi_{0}^{P} n_{t}^{p} - e_{t}^{P} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} V^{+}(k_{t+1}, a_{t+1}^{P}, \mathcal{S}_{t+1}) \right]$$
(6)

subject to the budget constraint

$$(1 - \phi_0^P)n_t^P + e_t^P - \Psi^P(e_t^P, n_t^P) \ge p_t k_{t+1} - q_t^m a_{t+1}^P, \tag{7}$$

and subject to the leverage constraint

$$\Phi p_t k_{t+1} \ge F a_{t+1}^P. \tag{8}$$

Constraint (8), familiar from Kiyotaki and Moore (1997), limits the total new borrowing of the firm to a fraction  $\Phi$  of its new capital valued at market prices,  $p_t k_{t+1}$ . Rather than limiting the market value of debt, the constraint applies to the book value F. The constraint is a stand-in for real-world debt limits in bank loan contracts or bond covenants.

We characterize the decision problem at stage 1 in the intra-period sequence,  $V^+(k_t, a_t^P, \mathcal{S}_t)$ , in the appendix. Importantly, this function takes into account the possibility of default for liquidity reasons.

The total profit of producers' business,  $\Pi_t^B$ , is given in appendix A.2.2 and subject to a corporate profit tax with rate  $\tau^{\Pi}$ . The profit for tax purposes is defined as sales revenue net of labor expenses, capital depreciation, and interest payments. The fact that interest expenditure is deducted from taxable profit creates a "tax shield" and hence a preference for debt funding.

#### 2.3.2 Aggregation

For producers' cost of issuing equity, we assume

$$\Psi^{P}(e_{t}^{P}, n_{t}^{P}) = \frac{1}{n_{t}^{P}} \frac{\phi_{1}^{P}}{2} (e_{t}^{P})^{2}.$$
(9)

In appendix A.2.2, we show that given the functional form in (9), the individual producer problem has constant returns to scale in net worth  $n_t^P$ . Thus, all producers choose the same amount of capital, labor, debt, and new equity as fraction of their net worth.<sup>14</sup>

At the beginning of each period, a fraction of producers defaults before paying dividends to shareholders and choosing the portfolio for next period. Debt holders take ownership of these bankrupt firms and liquidate them to recover some of the outstanding debt. Bankrupt producers are replaced by newly started firms that borrowers endow with initial equity  $n^0$  per firm. These new producers then solve problem (6) with  $n_t^P = n^0$ .

Denote aggregate net worth of surviving and newly started producers by  $N_t^P$ , and the ratio of new equity over net worth as  $\tilde{e}_t^P = e_t^P/n_t^P$ . This ratio is identical across producers due to scale invariance. Then the aggregate dividend to borrowers is:

$$D_t^P = N_t^P \left(\phi_0^P - \tilde{e}_t^P\right) - F_{\omega,t}(\omega_t^*) n^0.$$
(10)

The dividend has two parts: (i) all firms, both surviving and newly started firms, pay a dividend share  $\phi_0^P - \tilde{e}_t^P$  out of their net worth, and (ii) newly started firms, equal in mass to bankrupt firms  $F_{\omega,t}(\omega_t^*)$ , receive initial equity  $n^0$ .

Aggregate output  $Y_t$ , capital  $K_t$ , and producer debt  $A_t^P$  are functions of aggregate producer net worth  $N_t^P$  and individual producer decisions.

<sup>&</sup>lt;sup>14</sup>Idiosyncratic productivity shocks have permanent effects on producer net worth, so that the model features a non-degenerate firm size distribution in each period. However, the distribution is irrelevant for aggregate outcomes.

## 2.4 Intermediaries

#### 2.4.1 Setup

Intermediaries ("banks") are financial firms that buy long-term risky corporate debt issued by producers and use this debt as collateral to issue short-term debt to savers. They maximize the present discounted value of net dividend payments to their shareholders, the borrowerentrepreneurs.

Similar to producing firms, banks are required to pay a fraction  $\phi_0^I$  of equity as dividend each period, but they can deviate from this target by issuing equity  $e_t^I$  at a convex cost  $\Psi^I(e_t^I)$ . Like firms, banks are subject to idiosyncratic profit shocks  $\epsilon_t^I$ , realized at the time of dividend payouts. The shocks are i.i.d. across banks and time with  $E(\epsilon_t^I) = 0$  and c.d.f.  $F_{\epsilon}$ .<sup>15</sup>

Banks hold a diversified portfolio of corporate debt. At the beginning of each period, banks own  $a_t^I$  bonds and have to repay  $b_t^I$  deposits.<sup>16</sup> The coupon payment on performing loans in the current period is thus  $(1 - F_{\omega,t}(\omega_t^*))a_t^I$ . For firms that default and enter into foreclosure, banks repossess the firms, sell current period's output, pay current period's wages, and sell off the assets, yielding a recovery payoff per bond of

$$M_t = \frac{F_{\omega,t}(\omega_t^*)}{A_t^P} \left[ (1 - \zeta^P) \left( \mathbf{E}_{\omega,t} \left[ \omega \,|\, \omega < \omega_t^* \right] Y_t + \left( (1 - \delta_K) p_t - \varsigma \right) K_t \right) - \sum_j w_t^j \bar{L}^j \right], \qquad (11)$$

where  $A_t^P$ ,  $Y_t$ , and  $K_t$  denote aggregate producer debt, output and capital, respectively, and  $\zeta^P$  is the fraction of firm assets and output lost to lenders in bankruptcy.

Like firms, intermediaries are subject to corporate profit taxes at rate  $\tau^{\Pi}$ . Their profit for tax purposes is the net interest income on their loan business:

$$\Pi_t^I = (1 - \theta) \left( 1 - F_{\omega,t}(\omega_t^*) \right) a_t^I - r_t^f b_{t+1}^I.$$

Banks must pay a deposit insurance fee  $\kappa$  to the government that is proportional to the amount

<sup>&</sup>lt;sup>15</sup>The idiosyncratic shocks to bank profitability capture unmodeled heterogeneity in bank portfolios, such as that resulting from differences in credit quality across banks' loan portfolios or from differences in consumer lending such as mortgages. Technically, the assumption guarantees that there is always a fraction of banks which defaults. The shocks only affect the dividend payout, but have no effect on bank net worth going forward.

<sup>&</sup>lt;sup>16</sup>More generally, banks choose their position in the economy's riskfree asset, positive or negative. For any of the results we report, it is always optimal for banks to hold a negative safe bond position, i.e., to issue safe assets.

of short-term bonds (deposits) they issue.

The net worth after tax of a bank at the beginning of t is:

$$n_t^I = \left[ (1 - F_{\omega,t}(\omega_t^*))(1 - (1 - \theta)\tau^{\Pi} + \delta q_t^m) + M_t \right] a_t^I - b_t^I.$$
(12)

Each intermediary optimally decides on bankruptcy, conditional on  $n_t^I$  and the idiosyncratic profit shock realization  $\epsilon_t^I$ . Bankrupt intermediaries are liquidated by the government, which redeems deposits at par value. Immediately thereafter, shareholders (borrowers) replace all bankrupt intermediaries with new banks that receive initial equity equal to the average equity of non-defaulting banks.

We can state the recursive problem of an individual bank as:

$$V^{I}(n_{t}^{I}, \epsilon_{t}^{I}, \mathcal{S}_{t}) = \max_{a_{t+1}^{I}, b_{t+1}^{I}, e_{t}^{I}} \phi_{0}^{I} n_{t}^{I} - e_{t}^{I} + \epsilon_{t}^{I} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} \max\{V^{I}(n_{t+1}^{I}, \epsilon_{t+1}^{I}, \mathcal{S}_{t+1}), 0\} \right]$$
(13)

subject to the budget constraint:

$$(1 - \phi_0^I)n_t^I + e_t^I - \Psi^I(e_t^I) \ge q_t^m a_{t+1}^I - (q_t^f + \tau^\Pi r_t^f - \kappa)b_{t+1}^I,$$
(14)

and the regulatory constraint:

$$q_t^f b_{t+1}^I \le \xi q_t^m a_{t+1}^I.$$
(15)

Intermediaries discount future payoffs by  $\mathcal{M}_{t,t+1}^B$ , which is the stochastic discount factor of borrowers, their equity holders. The continuation value takes into account the possibility of optimal default, in which case shareholders get zero.

The constraint (15) is a Basel-style regulatory bank capital constraint, and requires that deposits are collateralized by banks' loan portfolio. The parameter  $\xi$  determines how much debt can be issued against each dollar of assets. It is the key macro-prudential policy parameter in the paper. We have chosen to have market prices on the right-hand side of (15) because levered financial intermediaries face regulatory constraints that depend on market prices.<sup>17</sup> Banks' leverage choice is affected by the same tax benefit and cost of distress trade-off faced by firms.

<sup>&</sup>lt;sup>17</sup>Insurance companies face such constraints as part of the Solvency II regime, broker-dealers face value-at-risk constraints, and market prices affect bank regulation through their effect on risk weights. Further, note that bank loans are marked-to-market each period in the model.

Banks enjoy deposit insurance and provide safe assets to patient households, which further increases their desire for leverage.

#### 2.4.2 Aggregation and Government Bailouts

For the equity issuance cost of banks we assume the functional form:

$$\Psi^{I}(e_{t}^{I}) = \frac{\phi_{1}^{I}}{2}(e_{t}^{I})^{2}.$$

Unlike firms', banks' value function is not scale invariant in net worth  $n_t^I$ . However, in appendix A.3.1 we show that under two assumptions, we achieve aggregation to a representative bank with aggregate net worth  $N_t^I$ . First, as can be seen from (13), the bank objective is linear in the idiosyncratic shock, which does not affect net worth going forward. Second, new banks that replace failed banks are seeded with the average equity of non-defaulting banks. The linearity assumption allows us to define a value function  $\tilde{V}^I(N_t^I, \mathcal{S}_t) = V^I(N_t^I, \epsilon_t^I, \mathcal{S}_t) - \epsilon_t^I$ . The bank default rate is

$$F_{\epsilon,t} \equiv F_{\epsilon} \left( -\tilde{V}^{I}(N_{t}^{I}, \mathcal{S}_{t}) \right),$$

implying that banks with low idiosyncratic profit shocks default.

The aggregate net dividend paid by the banking sector is:

$$D_t^I = \phi_0^I N_t^I - e_t^I + (1 - F_{\epsilon,t}) \epsilon_t^{I,+} - F_{\epsilon,t} N_t^I,$$
(16)

where  $\epsilon_t^{I,+} = \mathcal{E}_{\epsilon}(\epsilon | \epsilon \geq -\tilde{V}^I(N_t^I, \mathcal{S}_t))$ , is the expected idiosyncratic profit conditional on not defaulting. The last term represents the cost to shareholders of recapitalizing defaulted banks, from zero net worth post-bailout to the same positive net worth of the non-defaulted banks.

Defaulting intermediaries are liquidated by the government. During the bankruptcy process, a fraction  $\zeta_t^I$  of the asset value of a bank is lost. Hence the aggregate bailout payment of the government is:

$$\text{bailout}_{t} = F_{\epsilon,t} \left[ \zeta_{t}^{I} ((1 - F_{\omega,t}(\omega_{t}^{*}))(1 + \delta q_{t}^{m}) + M_{t}) A_{t}^{I} - N_{t}^{I} - \epsilon_{t}^{I,-} \right].$$
(17)

The conditional expectation,  $\epsilon_t^{I,-} = \mathbf{E}_{\epsilon}(\epsilon \,|\, \epsilon \leq -\tilde{V}^I(N_t^I, \mathcal{S}_t))$ , is the expected idiosyncratic profit

of defaulting intermediaries.

#### 2.5 Savers

Savers can invest in one-period risk free bonds (deposits and government debt) that trade at price  $q_t^f$  as well as corporate loans that trade at price  $q_t^m$ . Savers do not have access to the intermediaries' superior (costless) monitoring technology. Savers can hold corporate debt that does not require screening and monitoring, such as highly rated corporate bonds, without incurring any monitoring cost. A subset of the total supply of corporate debt  $\varphi_0 < A_t^P$  satisfies this requirement. If savers want to expand (or shrink)<sup>18</sup> their holdings of corporate debt away from the amount  $\varphi_0$ , they incur costs:

$$\Psi^{S}(A_{t+1}^{S}) = \frac{\varphi_{1}}{2} \left(\frac{A_{t+1}^{S}}{\varphi_{0}} - 1\right)^{2} \varphi_{0}.$$
(18)

Like borrower-entrepreneurs, savers inelastically supply their unit of labor  $\bar{L}^S$  and earn wage  $w_t^S$ . Entering with wealth  $W_t^S$ , the saver's problem is to choose consumption  $C_t^S$  short-term bonds  $B_{t+1}^S$ , and corporate bonds  $A_{t+1}^S$  to maximize life-time utility  $U_t^S$  in (1), subject to the budget constraint:

$$C_t^S + (q_t^f + \tau^D r_t^f) B_{t+1}^S + q_t^m A_{t+1}^S + \Psi^S(A_{t+1}^S) \le W_t^S + (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + O_t^S,$$
(19)

where saver wealth is defined as the portfolio payoff:

$$W_t^S = \left[ (1 - F_{\omega,t}(\omega_t^*))(1 + \delta q_t^m) + M_t \right] A_t^S + B_t^S.$$

The budget constraint (19) shows that savers use beginning-of-period wealth, after-tax labor income, transfer income from the government  $(G_t^{T,S})$ , and transfer income from bankruptcy proceedings  $(O_t^S)$  to be defined below, to pay for consumption and purchases of bonds. Savers are taxed on deposit income at rate  $\tau^D$ . The deposit interest rate is the yield on risk free bonds,  $r_t^f = 1/q_t^f - 1$ .

<sup>&</sup>lt;sup>18</sup>For tractability and analytical simplicity, we formulate a symmetric cost function that equally penalizes upward and downward deviations from the target. However, our calibrated model features a large enough difference in discount factors  $\beta_S - \beta_B$  such that absent the cost  $\Psi^S(A_{t+1}^S)$ , savers would hold all corporate debt directly. The cost therefore shrinks saver holdings towards  $\varphi_0$ .

### 2.6 Aggregate Bankruptcy Costs

Default of producing firms and intermediaries causes bankruptcy losses. When firms default, a fraction  $\zeta^P$  of their capital value and output is lost to banks, see equation (11). We assume that only a fraction  $\eta^P$  of this total loss from bankruptcy is a deadweight loss to society while the remainder is rebated to the households in proportion to their population shares. Similarly, when banks default, a fraction  $\zeta_t^I$  of their asset value is lost to the government, see equation (17), and only a fraction  $\eta^I$  of this resolution cost is a deadweight loss to society. These are the  $O_t^i$  terms in the budget constraints (3) and (19):

$$O_{t}^{B} + O_{t}^{S} = \zeta^{P} (1 - \eta^{P}) F_{\omega,t}(\omega_{t}^{*}) \left( \mathbb{E}_{\omega,t+1} \left[ \omega \,|\, \omega < \omega_{t+1}^{*} \right] Y_{t} + \left( (1 - \delta_{K}) p_{t} - \varsigma \right) K_{t} \right) + \zeta_{t}^{I} (1 - \eta^{I}) F_{\epsilon,t} \left[ (1 - F_{\omega,t}(\omega_{t}^{*})) (1 + \delta q_{t}^{m}) A_{t}^{I} + M_{t} \right].$$
(20)

These rebates can be interpreted as fire sale losses that represent transfers to buyers of distressed assets. We avoid the strong assumption that all bankruptcy costs are deadweight losses to society.

# 2.7 Government

The actions of the government are determined via fiscal rules: taxation, spending, bailout, and debt issuance policies. Government tax revenues,  $T_t$ , are labor income tax, non-financial and financial profit tax, deposit income tax, and deposit insurance fee receipts:

$$T_{t} = \sum_{j=B,S} \tau_{t}^{j} w_{t}^{j} L_{t}^{j} + \tau^{\Pi} \Pi_{t}^{B} + \tau^{\Pi} \Pi_{t}^{I} + \tau^{D} r_{t}^{f} B_{t+1}^{S} + \kappa B_{t+1}^{I}$$

Government expenditures,  $G_t$  are the sum of exogenous government spending,  $G_t^o$ , transfer spending  $G_t^T$ , and financial sector bailouts:

$$G_t = G_t^o + G_t^{T,B} + G_t^{T,S} + \text{bailout}_t.$$

The government issues one-period risk-free debt. Debt repayments and government expendi-

tures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_t^G + G_t \le q_t^f B_{t+1}^G + T_t$$
(21)

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathbf{E}_t \left[ \mathcal{M}_{t,t+u}^S B_{t+u}^G \right] = 0$$

where  $\mathcal{M}^{S}$  is the SDF of the saver. Because of its unique ability to tax, the government can spread out the cost of financial sector rescue operations over time.

Government policy parameters are  $\Theta_t = \left(\tau_t^i, \tau^{\Pi}, \tau^D, G_t^o, G_t^{T,i}, \xi, \kappa\right)$ . The capital requirement  $\xi$  in equation (15) and the deposit insurance fee  $\kappa$  are macro-prudential policy tools.

#### 2.8 Equilibrium

Given a sequence of aggregate productivity shocks  $\{Z_t^A, \sigma_{\omega,t}\}$ , idiosyncratic productivity shocks  $\{\omega_{t,i}\}_{i\in B}$ , and idiosyncratic intermediary profit shocks  $\{\epsilon_{t,i}\}_{i\in I}$ , and given a government policy  $\Theta_t$ , a competitive equilibrium is an allocation  $\{C_t^B, X_t\}$  for borrower-entrepreneurs,  $\{e_t^P, K_{t+1}, A_{t+1}^P, L_t^j\}$  for producers,  $\{C_t^S, A_{t+1}^S, B_{t+1}^S\}$  for savers,  $\{e_t^I, A_{t+1}^I, B_{t+1}^I\}$  for intermediaries, and a price vector  $\{p_t, q_t^m, q_t^f, w_t^B, w_t^S\}$ , such that given the prices, borrower-entrepreneurs and savers maximize life-time utility, intermediaries maximize shareholder value, the government satisfies its budget constraint, and markets clear. The market clearing conditions are:

Risk-free bonds: 
$$B_{t+1}^G = B_{t+1}^S + B_{t+1}^I$$
 (22)

Loans: 
$$A_{t+1}^P = A_{t+1}^I + A_{t+1}^S$$
 (23)

Capital: 
$$K_{t+1} = (1 - \delta_K)K_t + X_t$$

$$(24)$$

Labor: 
$$L_t^j = \bar{L}^j$$
 for  $j = B, S$  (25)

Consumption:  $Y_t = C_t^B + C_t^S + G_t^o + X_t + K_t \Psi(X_t, K_t) + \Psi^I(e_t^I) + \Psi(e_t^P, N_t^P) + \Psi^S(A_{t+1}^S) + DWL_t$ (26)

The last equation is the economy's resource constraint. It states that total output (GDP) equals the sum of aggregate consumption, discretionary government spending, investment including capital adjustment costs, bank equity adjustment costs, and aggregate resource losses from corporate and intermediary bankruptcies. The  $DWL_t$  term equals:

$$DWL_{t} = \zeta^{P} \eta^{P} F_{\omega,t}(\omega_{t}^{*}) \left( \mathbb{E}_{\omega,t+1} \left[ \omega \mid \omega < \omega_{t+1}^{*} \right] Y_{t} + \left( (1 - \delta_{K}) p_{t} - \varsigma \right) K_{t} \right)$$
$$+ \zeta_{t}^{I} \eta^{I} F_{\epsilon,t} \left[ (1 - F_{\omega,t}(\omega_{t}^{*}))(1 + \delta q_{t}^{m}) A_{t}^{I} + M_{t} \right].$$

#### 2.9 Welfare

In order to compare economies that differ in the policy parameter vector  $\Theta$ , we must take a stance on how to weigh the two households, borrowers and savers. We compute an ex-ante measure of welfare based on compensating variation similar to Alvarez and Jermann (2005). Consider the equilibrium of two different economies k = 0, 1, characterized by policy vectors  $\Theta^0$  and  $\Theta^1$ , and denote expected lifetime utility at time 0 for agent j in economy k by  $\bar{V}^{j,k} = E_0[V_1^j(\cdot; \Theta^k)]$ . Denote the time-0 price of the consumption stream of agent j in economy k by:

$$\bar{P}^{j,k} = \mathcal{E}_0 \left[ \sum_{t=0}^{\infty} \mathcal{M}_{t,t+1}^{j,k} C_{t+1}^{j,k} \right],$$

where  $\mathcal{M}_{t,t+1}^{j,k}$  is the SDF of agent j in economy k. The percentage welfare gain for agent j from living in economy  $\Theta^1$  relative to economy  $\Theta^0$ , in expectation, is:

$$\Delta \bar{V}^j = \frac{\bar{V}^{j,1}}{\bar{V}^{j,0}} - 1.$$

Since the value functions are expressed in consumption units, we can multiply these welfare gains with the time-0 prices of consumption streams in the  $\Theta^0$  economy and add up:

$$\mathcal{W}^{cev} = \Delta \bar{V}^B \bar{P}^{B,0} + \Delta \bar{V}^S \bar{P}^{S,0}.$$

This measure is the minimum one-time wealth transfer in the  $\Theta^0$  economy (the benchmark) required to make agents at least as well off as in the  $\Theta^1$  economy (the alternative). If this number is positive, a transfer scheme can be implemented to make the alternative economy a Pareto improvement. If this number is negative, such a scheme cannot be implemented because it would require a bigger transfer to one agent than the other is willing to give up. We solve the model using projection-based numerical methods and provide a detailed description of the globally nonlinear algorithm in appendix B.

# 3 Calibration

The model is calibrated at annual frequency. A subset of model parameters, listed in Table 1, have direct counterparts in the data. The remaining parameters are calibrated to match target moments from the data within the model. While these parameters are chosen simultaneously to match all targeted moments, Table 2 lists for each parameter the specific moment that is most affected by this parameter. Appendix C.4 conducts a parameter sensitivity analysis of the type suggested by Andrews, Gentzkow, and Shapiro (2017) that clarifies which moments structurally identify which parameters.

Aggregate Productivity Following the macro-economics literature, the TFP process  $Z_t^A$  follows an AR(1) in logs with persistence parameter  $\rho_A$  and innovation volatility  $\sigma^A$ . Because TFP is persistent, it becomes a state variable. We discretize  $Z_t^A$  into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points and the transition probabilities between them to match the volatility and persistence of HP-detrended GDP. GDP is endogenously determined but heavily influenced by TFP. Consistent with the model, our measurement of GDP excludes net exports and government investment. We define the GDP deflator correspondingly. Observed real per capita HP-detrended GDP has a volatility of 2.56% and its persistence is 0.55. The model generates a volatility of 2.55% and a persistence of 0.55.

Idiosyncratic Productivity We calibrate firm-level productivity risk directly to the micro evidence. We normalize the mean of idiosyncratic productivity at  $\mu_{\omega} = 1$ . We let the crosssectional standard deviation of idiosyncratic productivity shocks  $\sigma_{t,\omega}$  follow a 2-state Markov chain, with four parameters. Fluctuations in  $\sigma_{t,\omega}$  govern aggregate corporate credit risk since high levels of  $\sigma_{t,\omega}$  cause a larger left tail of low-productivity firms to default in equilibrium. We refer to periods in the high  $\sigma_{t,\omega}$  state as *high uncertainty* periods. We set  $(\sigma_{L,\omega}, \sigma_{H,\omega}) =$ (0.1, 0.18). The value for  $\sigma_{L,\omega}$  targets the unconditional mean corporate default rate. The model-implied average default rate of 2.2% is similar to the data.<sup>19</sup> The high value,  $\sigma_{H,\omega}$ , is chosen to match the time-series standard deviation of the cross-sectional interquartile range of firm productivity, which is 4.9% according to Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) (their Table 6). The transition probabilities from the low to the high uncertainty state of 9% and from the high to the low state of 20% are taken directly from Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018).<sup>20</sup> The model spends 31% of periods in the high uncertainty regime. Like in Bloom et al., our uncertainty process is independent of the (firstmoment) aggregate TFP shock. About 10% of periods feature both high uncertainty and low TFP realizations. We will refer to those periods as financial recessions or financial crises, since those periods will feature (endogenous) financial fragility in the equilibrium of the model. Using a long time series for the U.S., Reinhart and Rogoff (2009) find the same 10% frequency of financial crises.

**Production** Investment adjustment costs are quadratic. We set the marginal adjustment cost parameter  $\psi = 2$  in order to match the observed volatility of (detrended) log investment of 8.13%. The model generates a value of 9.41%. We set the parameter  $\alpha$  in the Cobb-Douglas production function equal to 0.71, which yields an overall labor income share of 66.20%, the standard value in the business cycle literature. We choose an annual depreciation of capital  $\delta_K$  of 8.25% to match the investment-to-output ratio of 17.74% observed in the data. The fixed cost of production is set to  $\varsigma = 0.004$  targeting a capital-GDP ratio of 224% in the data.

**Population and Labor Income Shares** To pin down the population shares of savers and borrowers (shareholders), we turn to the Survey of Consumer Finance (SCF). For each house-hold, we define the risky share as the ratio of risky assets to financial assets plus net business wealth. Risky assets consist of stocks, mutual funds, and net business wealth.<sup>21</sup> We then cal-

<sup>&</sup>lt;sup>19</sup> We look at two sources of data: corporate loans and corporate bonds. From the Federal Reserve Board of Governors, we obtain delinquency and charge-off rates on Commercial and Industrial loans and Commercial Real Estate loans by U.S. Commercial Banks for the period 1991-2015. The average delinquency rate is 3.1%. The second source of data is Standard & Poors' default rates on publicly-rated corporate bonds for 1981-2014. The average default rate is 1.5%; 0.1% on investment-grade bonds and 4.1% on high-yield bonds. The model is in between these two values.

<sup>&</sup>lt;sup>20</sup>They estimate a two-state Markov chain for the cross-sectional standard deviation of establishment-level productivity using annual data for 1972-2010 from the Census of Manufactures and Annual Survey of Manufactures. We annualize their quarterly transition probability matrix.

 $<sup>^{21}</sup>$ Financial wealth, as measured by the SCF, consists of liquid assets, certificates of deposit, directly held pooled investment funds, stocks, bonds, quasi-liquid assets, savings bonds, whole life insurance, other managed

culate the fraction of households whose risky share is less than one percent. This amounts to 71.1% of SCF households. These are the savers in our model who hold no equity claims ( $\ell^S$ ). The remaining  $\ell^B = 28.9\%$  of households have a nontrivial risky share. The labor income share of savers in the SCF is 64%. The income share of the borrower-entrepreneurs is the remaining 36%.<sup>22</sup> The income shares determine the Cobb-Douglas parameters  $\gamma_B$  and  $\gamma_S$ . Appendix D.5 explores the effect of a lower saver share on the macro-prudential policy experiment.

Corporate Loans and Producer Financial Frictions In the model, a corporate loan is a geometric bond. The issuer of one unit of the bond at time t promises to pay 1 at time t + 1,  $\delta$  at time t + 2,  $\delta^2$  at time t + 3, and so on. Given that the present value of all payments  $(1/(1 - \delta))$  can be thought of as the sum of a principal (share  $\theta$ ) and an interest component (share  $1 - \theta$ ), we define the book value of the debt as  $F = \theta/(1 - \delta)$ . We set  $\delta = 0.937$  and  $\theta = 0.582$  (F = 9.238) to match the observed duration of corporate bonds. Appendix C.5 contains the details. The model's corporate loans have a duration of 6.8 years on average.

We set the maximum LTV ratio parameter  $\Phi = 0.4$ . The LTV constraint limits corporate borrowing as a fraction of the market value of capital. For this value of  $\Phi$ , the model generates a ratio of borrower book debt-to-assets of 37.33% matching the 37% number for the average ratio of loans and debt securities of the nonfinancial corporate and non-financial non-corporate businesses in the Flow of Funds data.

To compute firms' dividend target as fraction of equity,  $\phi_0^P$ , we construct time series for dividends, share repurchases, equity issuances, and book equity aggregating over all publicly traded non-financial firms. Details are provided in Appendix C.2. Over the period 1974–2018, nonfinancial firms paid out 7.8% of their book equity per year as dividends and share repurchases, which is the value we set for  $\phi_0^P$ . For simplicity, we abstract from equity issuance frictions for non-financial firms in the baseline version of our model, and thus set the issuance cost parameter to  $\phi_1^P = 0$ . Appendix D.2 explores the effect of equity issuance costs for producers and appendix D.5 shows that they do not change the macro-prudential policy conclusions.

assets, and other financial assets. Those with zero financial plus net business wealth get assigned a zero risky share. We use all survey waves from 1995 until 2013 and average across them.

 $<sup>^{22}</sup>$ We use SCF weights when computing aggregate income for each group. If we alternatively define savers to be those households with fewer than \$250 in risky assets in 2016 dollars, we get very similar population and income shares. SCF data strongly support the notion that stock market wealth is concentrated. Only 5.4% of SCF households collectively account for 90% of stock ownership.

**Intermediaries** The cross-sectional standard deviation  $\sigma_{\epsilon} = \operatorname{Var}(\epsilon_t^I)^{0.5}$  of intermediary profit shocks governs the default rate of banks in the model. We target the average failure rate of banks, which the model matches at 0.63%.<sup>23</sup>

The intermediary borrowing constraint parameter  $\xi$  can be interpreted as a minimum regulatory equity capital requirement. We set  $\xi = 0.93$  in the baseline calibration, or a 7% equity capital requirement, conforming with the Basel limits.<sup>24</sup> This is the key parameter we vary in or macro-prudential policy experiments.

We set the deposit insurance fee as a fraction of bank liabilities, captured by the parameter  $\kappa$ , to the observed 8.4 basis points.<sup>25</sup>

Bennett and Unal (2015) report resolution costs of 33.18% of assets for the average bank taken over by the FDIC between 1986 and 2004. We set  $\overline{\zeta}^I = .332$  to match this number.<sup>26</sup> Granja, Matvos, and Seru (2017) report a similar resolution cost of 28% for the period 2007–2013 . Bennett and Unal (2015) also shows that total receivership expenses are 36.2% of the resolution cost. This is a good measure for the fraction of losses from bank failure that are deadweight losses to society. Therefore, we set  $\eta^I = 0.362$ .

To determine the dividend target  $\phi_0^I$  of banks, we construct time series of dividends, share repurchases, equity issuances, and book equity, aggregating across all publicly-traded banks. Details are in Appendix C.1. Over the period from 1974 to 2018, banks paid out an average 6.8% of their book equity per year as dividends and share repurchases, which is the value we set for  $\phi_0^I$ .

<sup>26</sup>The resolution cost of failed banks is defined as  $\zeta_t^I = \bar{\zeta}^I \frac{A_t^P}{A_t^I} \mathbb{E} \left[ \frac{A_t^I}{A_t^P} \right]$ . This assumption restates bank resolution costs relative to firm credit rather than relative to bank-intermediated firm credit. This is a technical assumption that avoids us having to keep track of  $A_t^I$  as an additional state variable.

<sup>&</sup>lt;sup>23</sup>We calculate the asset-weighted failure rate of depository institutions in the FDIC data. The sample period has two episodes of elevated bank defaults, the savings and loan crisis in the early 1990s and the financial crisis of 2008-09.

<sup>&</sup>lt;sup>24</sup>Under Basel II and III, corporate loans on banks' balance sheets have a 100% risk weight while corporate bonds have a risk weight that depends on their credit rating. The risk weight on corporate bonds under the standardized approach of Basel II ranges from 20% for AAA to AA- ratings, 50% for A+ to A- ratings, to 100% for BBB+ to B- ratings. As of 2016 year-end, banks held \$7.6 trillion in corporate loans and \$5.1 trillion in corporate bonds. Given the observed 40%-40%-20% split of corporate bonds in the three ratings buckets, the average risk weight for corporate bonds is 48%, and the overall risk weight is 79%. The resulting capital requirement is  $8\%^*79\%=6.32\%$ .

<sup>&</sup>lt;sup>25</sup>To compute this number, we divide the total assessment revenue reported by the FDIC for 2016, \$10 billion, by the total short-term debt of U.S. chartered financial institutions from the Flow of Funds, \$11,849 billion. Banks pay 14.2 cents per \$100 dollar of insured deposits but 8.4 cents per \$100 of insured and uninsured deposits. Since the model has only insured deposits, we use the latter number.

We calibrate the marginal equity issuance cost for intermediaries,  $\phi_1^I = 7$ , using the same data. With this parameter, we target the net payout ratio of the financial sector, defined as dividends plus share repurchases minus equity issuances divided by book equity. A higher equity issuance cost makes issuing external equity more expensive, and raises the net payout ratio. Since banks issue equity on average, the net payout rate is 5.75% in the data, lower than the gross payout ratio of 6.8%. Appendix D.5 explores the effect of equity issuance costs for banks on the macro-prudential policy experiment.

**Saver Holding Costs** To discipline savers' cost for holding corporate bonds, we compute the fraction of corporate liabilities directly held by households in the Flow of Funds. The household share of debt is 13.7% for the sample from 1981-2015 that we use for other financial variables. Details are in appendix C.3. To match this average share in the model, we set the holdings target of savers to  $\varphi_0 = 0.0115$ . The volatility of saver holdings around the target is governed by the parameter  $\varphi_1$ . We set this parameter to 0.14 to match the volatility of the saver share, which is 3.3% in the data.

**Preference Parameters** Preference parameters affect many equilibrium quantities and prices simultaneously, and are harder to pin down directly by data. For simplicity, we assume that both borrowers and savers have log utility:  $\sigma_B = \nu_B = 1$  and  $\sigma_S = \nu_S = 1.^{27}$  The subjective time discount factor of borrowers  $\beta_B = 0.94$  targets the net payout ratio of non-financial firms, defined as dividend payouts plus share repurchases minus equity issuances. A higher  $\beta_B$  leads to lower net payout ratio, as more patient borrowers want to accumulate more wealth. In the data, net payouts for non-financial firms were 6.41% of their book equity per year (see Appendix C.2). The time discount factor of savers disproportionately affects the mean of the short-term interest rate. We set  $\beta_S = 0.982$  to match the observed average real rate of interest of 2.2%.

**Government Parameters** To add quantitative realism to the model, we match both the unconditional average and cyclical properties of discretionary spending, transfer spending, labor income tax revenue, and corporate income tax revenue.

Discretionary and transfer spending are modeled as follows:  $G_t^i/Y_t = G^i \exp\left\{b_i z_t^A\right\}, i = o, T,$ 

<sup>&</sup>lt;sup>27</sup>We have solved the model with Epstein-Zin preferences for a range of risk aversion and EIS parameter choices. Results are qualitatively similar and available upon request.

where  $z_t^A = \log(Z_t^A)$ . The parameters  $G^o$  and  $G^T$  are set to match average discretionary spending to GDP of 17.58% and transfer spending to GDP of 3.18%, respectively, in the 1953-2014 NIPA data.<sup>28</sup> The model produces 17.50% and 3.15%. The parameters  $b_o = -2$  and  $b_T = -20$  are set to match the slope in a regression of log discretionary/transfer spending-to-GDP on GDP growth and a constant. The slopes are -0.89 and -8.88 in the model versus -0.71 and -7.14 in the data.

Similarly, the labor income tax rate is  $\tau_t = \tau \exp \{b_\tau z_t^A\}$ . We set the tax rate  $\tau = 29.3\%$  in order to match observed average income tax revenue to GDP of 17.3%. Appendix C.6 details how labor income tax revenue is computed in the data. The model generates an average of 19.17%. We set  $b_\tau = 4.5$  to match the regression slope of log income tax revenue-to-GDP on GDP growth and a constant. The slope is 0.62 in the model and 0.70 in the data.

We set the corporate tax rate that both financial and non-financial corporations pay to a constant  $\tau^{\Pi} = 20\%$  to match observed corporate tax revenues of 3.41% of GDP. The model generates an average of 3.56%. The tax shields of debt and depreciation substantially reduce the effective tax rate corporations pay, both in the model and in the data.

We set the tax rate on financial income for savers (interest on short-term debt) equal to  $\tau^D = 13.2\%$ . Appendix C.7 contains the details of the calculation.

Government debt to GDP averages 73.08% of GDP in a long simulation of the benchmark model. While it fluctuates meaningfully over prolonged periods of time (standard deviation of 12.41%), the government debt to GDP ratio remains stationary. Appendix C.8 provides details.

# 4 Results

This section studies the behavior of key macro-economic and balance sheet variables. The model captures important features of macro-economic quantities, non-financial corporate and financial balance sheets, and asset prices in normal times and in crises. The model's fit lends credibility to the macro-prudential policy experiment in Section 5.

<sup>&</sup>lt;sup>28</sup>We divide by  $\exp \{b_i/2\sigma_A^2/(1-\rho_A^2)(b_i-1)\}$ , a Jensen correction, to ensure that average spending means match the targets.

#### 4.1 Macro Quantities

We present impulse-response graphs to explore the behavior of macro-economic quantities conditional on the state of the economy. We start off the model in year 0 in the average TFP state and in the low uncertainty state ( $\sigma_{\omega,L}$ ). The five endogenous state variables are at their ergodic averages. In period 1, TFP falls by one standard deviation. In one case (red line), the recession is accompanied by a switch to the high uncertainty state ( $\sigma_{\omega,H}$ ); a financial recession. In the second case, the economy remains in the low uncertainty state; a non-financial recession (blue line). From period 2 onwards, the two exogenous state variables follow their stochastic laws of motion. For comparison, we also show a series that does not undergo any shock in period 1 but where the exogenous states stochastically mean revert from the low-uncertainty state in period 0 (black line). For each of the three scenarios, we simulate 10,000 sample paths of 25 years and average across them. Figure 2 plots the macro-economic quantities. The top left panel is for the productivity level  $Z^A$ . By construction, it falls by the same amount in financial and non-financial recessions; a 2% drop. Productivity then gradually mean reverts over the next decade.

The other three panels show impulse-responses for output, consumption, and investment. In the initial period of the shock, the drop in output is the same when the economy is additionally hit by an uncertainty shock (red line) and when it is not (blue line). This has to be the case because capital is a state variable, labor is supplied inelastically, and productivity is identical. Output remains lower for much longer following a financial recession. The added persistence resembles the slow recovery that typically follows a financial crisis. Financial recessions are associated with a cumulative output loss of 13.1%, double the 6.2% loss in a typical nonfinancial recession.

The bottom right panel shows a 25% drop in investment in financial recessions but only a modest 7% drop in non-financial recessions. Despite the bounce back in period 2, investment remains depressed for five years. The cumulative loss in investment is 34.3% in a financial recession, more than three times the 9.8% loss in a non-financial recession.

The initial drop in aggregate consumption, plotted in the bottom left panel, is only slightly larger than in a non-financial recession. The low rate of return on savings induces savers to consume relatively more in a financial crisis. Consumption remains below the non-financial recession level for the remaining periods, as the capital stock remains depressed. The cumulative consumption loss is 15.8% in financial recessions compared to 5.8% in non-financial recessions. It is these protracted declines in consumption and investment in financial recessions that macro-prudential policy aims to remedy.

In appendix D.3, we include IRF graphs that compare a financial recession to a pure uncertainty shock, which is a switch to  $\sigma_{\omega,H}$  with TFP remaining constant. The combination of both shocks leads to significant amplification, i.e., the financial recession triggered by the combination is much larger than the sum of the effects of each individual shock. This feature of our model is consistent with the empirical finding that uncertainty shocks alone have only moderate negative effects on output and investment, see for example Bachmann and Bayer (2013) and Vavra (2014).

#### 4.2 Balance Sheet Variables

Next, we turn to the key balance sheet variables in Table 3. We report unconditional means and standard deviations from a long simulation of the model (10,000 years), as well as averages conditional on being in a good state (high TFP, low uncertainty, i.e.  $\sigma_{\omega,L}$ ), non-financial recession (low TFP, low uncertainty), and financial recession (low TFP, high uncertainty  $\sigma_{\omega,H}$ ).

**Non-financial Corporate Sector** The first panel focuses on non-financial firms. It reports the market value of assets and the market and book value of liabilities scaled by GDP. Book leverage is defined as book debt over book assets, while market leverage is market value of debt over market value of assets. As mentioned, the low 37% leverage ratio matches the corporate leverage ratio observed in the data. Total credit to non-financial firms, provided by the financial sector and directly by the household sector, amounts to 80.34% of GDP.

Firms default when they become unprofitable. This is more likely when uncertainty  $\sigma_{\omega}$  is high, as the mass of firms with productivity shocks below the threshold  $\omega_t^*$  increases. The model generates not only the observed average amount of credit risk, matching the average corporate default rate of 2.08% and loss rate of 1.07%, but it also generates a substantial increase in credit risk in financial crises. Default and loss rates are seven times higher in financial recessions than in expansions and non-financial recessions (rows 7 and 9). The large increase in credit risk is an important feature of financial recessions.<sup>29</sup>

Since firms face a higher cost of debt in financial recessions (row 22), they reduce their reliance on debt and corporate leverage falls (row 5). The higher cost of debt occurs because the credit spread is elevated (row 23), and despite a reduction in safe interest rates (row 21). The higher credit spread reflects both the higher quantity of credit risk (row 9) and a higher price of credit risk charged by banks. The latter reflects financial fragility, as discussed below. Hence, financial sector fragility feeds back on the real economy and amplifies the initial shock emanating from the real sector, a second financial accelerator. Firms do not pursue the investment projects they would otherwise undertake. Relative to expansions, investment falls by 18% in financial recessions (row 10).

Financial Intermediaries and Credit Spread The second panel of Table 3 focuses on the financial sector. Intermediary leverage is 92.63% on average in market values (row 11). Financial leverage is not targeted in the calibration, yet is close to the data. The average ratio of total intermediary book debt-to-assets in the 1953-2014 data is 91.5%; see Appendix C.1 for the data calculations. The large difference between the leverage ratio of financial and non-financial firms is of crucial importance for understanding how modest-sized credit shocks can lead to financial crises with large real consequences. It is a feature missed by models that consolidate the financial and non-financial sector balance sheets. Several model ingredients contribute to the high financial leverage. Like non-financial firms, financial firms are owned by impatient shareholders, enjoy a tax shield for debt, and suffer deadweight losses from default. Unlike non-financial firms, they benefit from government guarantees (deposit insurance) and they produce safe assets for patient savers, both of which keep down their cost of funding ( 2.23%, row 21). Finally, they face equity issuance costs, which makes issuing debt attractive compared to equity. We explore the role of these various ingredients in Section 4.3.

Banks realize losses on their credit portfolio in financial recessions (row 25), reducing the book value of their assets. The reduction in the book value of assets is further amplified by a reduction in the price of corporate debt (high yields, row 22). A lower market value of bank assets (row 11) in turn tightens the regulatory bank capital constraint. The constraint

 $<sup>^{29}</sup>$ In the 1991 recession which accompanied the Savings & Loans crisis, the delinquency rate spiked at 8.2% and the charge-off rate at 2.2%. For the 2007-09 crisis, the respective numbers are 6.8% and 2.7%. These are far above the unconditional averages of 3.1% and 0.7% cited in footnote 19.

binds in 99.90% of the financial crises compared to 63.39% unconditionally and only 32.37% in expansions (row 13). While banks are also likely to be constrained in non-financial recessions, the reasons for the tightness of the bank capital constraint are fundamentally different. In financial recessions, banks suffer large credit losses and are forced to delever and issue equity. Going forward, banks earn high yields on corporate debt (4.44%) and face low borrowing costs (1.69%), making intermediation very profitable. They would like to expand credit, but are up against their borrowing constraint which prevents raising new debt. They can and do raise outside equity (net payout ratio of -26.29%, line 16), but are held back by the cost of raising equity.

In contrast, in non-financial recessions, banking becomes much less profitable due to the shrinking net interest margin, which averages around zero. These recessions resemble standard TFP-induced recessions in real business cycle models: as productivity and labor income are temporarily low, savers reduce their demand for safe assets in order to smooth consumption. In addition, the supply of government debt goes up due to increased government spending and lower tax revenue (row 18). The risk free rate has to rise to 4.13% to clear the market for short-term debt. At the same time, low productivity reduces corporate loan demand. In response to the drop in profits which depletes equity, banks lower dividend payments. To avoid raising costly equity, they exhaust their borrowing constraint.

Financial crises are accompanied by a shrinking financial sector, both in terms of assets (row 11) and liabilities (deposits, row 17). Bank assets-to-GDP decline more sharply than overall corporate debt, as savers hold a higher share of debt in the form of corporate bonds (row 19). As banks' shadow cost of funds rises in crises, savers' holding disadvantage becomes less severe. Bank book leverage is lower in financial crises (row 13) as the drop in bank debt is larger than the drop in bank assets, even after taking into account the losses on the legacy debt portfolio. The pro-cyclicality of book leverage of banks is consistent with the data, as shown by Adrian, Boyarchenko, and Shin (2015).

The model generates rare financial disasters when a non-trivial fraction of the banking system is insolvent and needs to be bailed out. Bank failures are concentrated in financial recessions—hence the name—when 4.41% of banks are insolvent, seven times as many as in non-financial recessions. Strategic bank default results from the balance of two forces. Bank shareholders try to avoid low intermediary net worth states because they are risk averse and because of the cost of equity issuance. But when net worth is sufficiently low they have an incentive to shift the risk onto the tax payer due to limited liability.

Figure 3 shows the impulse-response functions for assets and liabilities of both non-financial firms and banks. Loan loss rates spike in financial recessions (red line, first panel) and take several more years to return to normal. The high loan losses cause a spike in bank failures (second panel). A credit crunch ensues as both the asset and liability side of corporate balance sheet shrinks. The credit spread spikes in the first period of a financial crisis (top right panel). The increased cost of credit is consistent with a reduced loan demand from firms. The pattern, whereby the credit spread normalizes fairly quickly after the initial spike but the quantity of credit takes a long time to recover, is consistent with the data. The behavior of quantities and prices in financial crises stands in sharp contrast with the much milder changes in non-financial recessions (blue lines).

The model comes close to matching the credit spread, defined as the spread between the long-term corporate bond yield and the short-term deposit rate, a moment not targeted in the calibration. The mean credit spread over the 1953-2015 period is 2.08% per year in the data and 1.89% in the model (row 23).<sup>30</sup> The model's credit spread is also volatile. The macro-finance literature has had difficulty generating a high and volatile credit spread given the small amount of credit risk; e.g., Chen (2010). What is needed to reconcile the two is a high price of credit risk. In our model, the price of credit risk is high in states with low intermediary wealth. In such states, the Lagrange multiplier on the regulatory capital constraint usually binds and increases the intermediary's (shadow) SDF. Appendix D.4 provides more details. While this result is consistent with the recent intermediary-based asset pricing literature, our model features no shocks that directly hit intermediaries. Rather, all aggregate shocks emanate form the non-financial sector.

Our model produces a negative term spread (row 24), which we compute as the difference between the yield on a hypothetical safe long-term bond in zero net supply, priced by savers, and the deposit rate. This riskfree long-term bond has the same duration as corporate bonds, but no credit risk. A negative term spread is common in models with CRRA preferences where

 $<sup>^{30}</sup>$ We define the credit spread in the data as a weighted average of the Moody's Aaa and Baa yields and subtract the one-year constant maturity Treasury rate. To determine the portfolio weights on the Aaa and Baa grade bonds, we use market values of the respective amounts of bonds outstanding from Barclays. The weights are 80% and 20%, respectively.

long-term bonds hedge consumption growth risk. The negative term spread highlights that the pure credit spread generated by the model is actually larger (at 2.25%) than the simple net interest spread between loan and deposit rates we report above.

**Savers** Risk averse savers buy safe assets from banks and the government. They can also directly invest in the debt of non-financial firms, but have a comparative disadvantage relative to the intermediation sector. In financial recessions, banks become more constrained and sell corporate debt to savers. Savers reduce their deposit holdings and buy these bonds at fire-sale prices, selling them back to banks in the next period (first and third panels in bottom row of Figure 3).

The low equilibrium real interest rate in financial crises results from a decrease in the demand for deposit finance from banks (bottom right panel). A substantial reduction in the real interest rate is consistent with the experience in the Great Recession. The reduction in (deposit) interest rates benefits banks, helping them to rebuild their net worth and restoring their ability to lend to the real economy.

Surprisingly, savers' ability to increase their holdings of corporate bonds in crises does not dampen the severity of financial recessions for two reasons. First, banks earn a lower equilibrium credit spread than they would in an economy where savers cannot buy corporate debt. The increase in the saver share of debt prevents an even larger spike in the credit spread. Second, the riskfree interest rate is higher than in the alternative economy. Without the option to reallocate to corporate bonds, the deposit rate would have to fall by more to induce savers to consume instead. However, as savers can substitute their savings towards bond holdings, the drop in the riskfree rate is mitigated. Higher deposit rates make it more difficult for banks to recapitalize. Combined, more banks fail and bank net worth recovers more slowly.

Government debt goes up in both types of recessions, as shown in Figure 3, because of a reduction in tax revenue and an increase in government discretionary and transfer spending. Financial crises bring the additional fiscal burden of financial sector bailouts, but low interest rates help reverse the debt accumulation over the ensuing years. The size of the bailout in financial crises (5.9% of GDP) is consistent with the experience in the Great recession.

### 4.3 Role of Frictions

To better understand the workings of the benchmark model, we solve a sequence of simpler models. Appendix D.1 provides the details. In the first two simplified models, we switch off the intermediary sector by setting saver holding costs of corporate debt to zero. In these two models, intermediaries have zero assets and liabilities.

The simplest model, which we refer to as Model (1) also turns off corporate default. In this model, we eliminate the need for corporations to service their debt before they can raise new equity or debt, and thereby default; we further set the fixed cost of operation  $\varsigma = 0$ . This economy is close to the model in the seminal paper of Kiyotaki and Moore (1997) (KM) with a representative borrower who owns the firms, representative firms who borrow subject to an occasionally-binding collateral constraint, and a representative saver who invests in corporate debt. Innovations relative to KM are the long-term nature of corporate debt, the introduction of fiscal policy and government debt held by the saver, and the nonlinear solution method. This model is a useful benchmark to help understand and quantify the role of various frictions. The first friction we add is corporate default (Model (2);  $\varsigma > 0$ ). This change activates the role of the uncertainty shocks.

Figure 4 compares financial recessions in these two simplified models to the benchmark model. In Model (1), there is no distinction between non-financial and financial recessions, as idiosyncratic productivity shocks wash out in aggregate and a change in their dispersion has no effects. Thus, the drop in investment (top right) is the model's impulse response to a negative TFP shock, with corporate defaults (top left) and associated deadweight losses (bottom right) remaining zero by construction. This model features a standard financial accelerator mechanism (Bernanke, Gertler, and Gilchrist (1999b)), whereby TFP shocks get amplified through the collateral constraint and result in larger fluctuations in real quantities. However, amplification in this model is small, consistent with the prior literature (see for example Cordoba and Ripoll (2004)).

In Model (2), the rise in idiosyncratic productivity dispersion causes a corporate default wave (blue line in top left panel). In combination with a negative TFP shock, this model generates more severe recessions in which credit spreads spike, credit provision shrinks persistently, and investment falls by -19.90% in the first period of a financial recession, compared to a -8.30%

in Model (1). In other words, there is substantial amplification relative to the KM economy of Model (1).

The red line adds intermediation frictions to arrive at the benchmark model, separating the balance sheets of non-financial and financial firms. It does so by introducing the cost for savers of holding corporate debt directly. Furthermore, banks have limited liability coupled with deposit insurance, and face equity issuance costs. Relative to Model (2), our benchmark model features a "double" financial accelerator since intermediation frictions further amplify corporate credit frictions, causing a -25.26% drop in investment and a spike in deadweight losses caused by bank failures.

Table D.1 in Appendix D.1 reports various other moments. It also includes two additional models that have an intermediation sector, but fewer intermediation frictions than the benchmark. Banks in Model (3) of Table D.1 have unlimited liability and can raise outside equity at no cost. Relative to Model (2), this model has a much higher credit spread, which reflects the shadow cost of intermediary capital. The main effect of introducing intermediaries as in Model (3) is to make debt finance for firms more expensive, since it is costly for savers to hold corporate debt directly, and banks need to be compensated for bearing credit risk – they earn an excess return of 0.69% on corporate debt. Simply "adding intermediaries" does not amplify financial crises. When intermediaries fully bear the risk they take on and can costlessly raise equity in crises, their presence actually makes crises less severe since, by making debt finance less attractive for non-financial firms, they reduce overall credit risk in the economy.

Relative to Model (3), Model (4) in Table D.1 introduces limited liability and deposit insurance for banks. Unlike banks in the benchmark model, however, banks in Model (4) can still issue equity without costs. Comparing Models (3) and (4) highlights that the key intermediation friction that amplifies financial crises is the combination of deposit insurance and limited liability for banks, which creates incentives to strategically default in bad times. The increased risk-taking of banks makes financial recessions more severe: investment falls by -22.06% in the first period of a financial recession in Model (4), compared to the -18.78% drop in Model (3). This risk shifting motive for banks creates franchise value. We define the franchise value of intermediation as the market value of banks to shareholders per dollar of equity capital  $V^I/N^I - 1$ . Absent default and equity issuance costs, franchise value is small (7.92% in Model (3)), reflecting the excess returns earned by banks. Deposit insurance boost franchise value to 16.60%.

Column (5) of Table D.1 adds equity issuance costs for banks to Model (4), thereby recovering our benchmark model. Equity issuance costs lead banks to hold more equity and lower leverage. The higher equity buffer reduces the extent to which regulatory capital constraints bind and sharply reduces bank failures. The corporate credit spread rises, lowering firm leverage and debt issuance. Yet financial recessions are more severe, with investment falling by -25.26%. Despite being less levered, banks now face costly equity issuance in crises, causing a sharper initial drop and a slower recovery.

# 5 Macro-prudential Policy

We use our calibrated model to investigate the effects of macro-prudential policy choices. Our main experiment is a variation in the intermediaries' leverage constraint. In the benchmark model, intermediaries must hold 7% equity capital. We explore tighter constraints ( $\xi < .93$ ), as well as looser constraints ( $\xi > .93$ ). We also study a time-varying capital requirement conditional on the uncertainty state. Tables 4 and 5 show the results. Table 5 reports moments in percentage deviation from the benchmark.

# 5.1 Changing maximum intermediary leverage

Effect on lending and intermediation Rows 12 and 13 of Table 4 show that a policy that constrains bank leverage is indeed successful at bringing down that leverage. Banks reduce the size of their assets (row 11) and liabilities (row 18). Savers take up an increasing share of corporate debt (row 20) but overall credit to non-financial firms falls (rows 2 and 3). Intermediary equity capital increases sharply as  $\xi$  is lowered (row 16). With better capitalized banks, financial fragility falls. Intermediary bankruptcies drop rapidly from 0.6% to 0.1% at  $\xi = .91$ , and are eradicated at tighter capital requirements (row 15). At least initially, banks' leverage constraints bind *less* frequently with tighter prudential regulation (row 14). Intermediaries become more cautious when they are better capitalized since equity capital adjustments become effectively more costly (as explained below) and the option to default is farther out-of-the-money. But as capital requirements tighten further and bank equity becomes scarcer, the
constraint starts to bind more frequently again. Tighter regulation leads to a safer intermediary sector, but also to a smaller one.

The increased safety of the financial sector as  $\xi$  falls is also reflected in lower corporate default, loss-given-default, and loss rates on loans to non-financial firms (rows 7-9). Firms choose to reduce leverage (rows 4-5) despite the more stable financing environment because of the higher interest rates they face on corporate debt (row 22). When intermediary capacity shrinks (with lower  $\xi$ ), the reward for providing intermediation services increases. The modest 26bps increase in the credit spread between the benchmark and  $\xi = 0.75$  is the result of large offsetting movements in the expected loss rate, which more than halves (row 9), and in the excess return intermediaries earn on their asset holdings, which doubles (row 25).

How does tighter macro-prudential regulation affect intermediary profitability? We compute a series of metrics reported in rows 17a-d. Intermediary franchise value declines from 17.9% in the benchmark to 7.5% at  $\xi = .75$ . The decline in franchise value can be understood from the decline in bank profitability (row 17b), as measured by the accounting ROE:

AROE = Excess ret. on loans (row 25) × value of loans (row 11) /  $N^{I}$  (row 16).

Tighter regulation requires more bank equity to operate a smaller banking sector. This shrinkage effect dominates the rise in profitability per dollar of loans issued, reflected in the greater excess return, causing a decline in accounting ROE from 24.9% in the benchmark to 8.1% at  $\xi = 75$ . As Admati, DeMarzo, Hellwig, and Pfleiderer (2013) point out, such a calculation may not tell the whole story. The required return on equity will decline as banks are forced to hold more capital because shareholders are exposed to less risk. This force is present in our model. The cum-dividend market value of intermediary equity to borrower-entrepreneurs is given by  $V_t^I$  as defined in (13). Using this market price, we compute the equilibrium expected market return on equity for bank shareholders as:

$$\mathrm{MROE}_t = \frac{\mathrm{E}_t \left[ \max \left\{ V_{t+1}^I - \epsilon_{t+1}^I, 0 \right\} \right]}{V_t^I - d_t^I}.$$

Indeed, we find that the market return on equity declines as regulation tightens and banks become less risky (row 17c). As an aside, the baseline model generates a sizable equity risk premium for banks of 4.6% (6.82%-2.23%) per annum. Finally, we report the weighted average

cost of capital (WACC) for banks in row 17d. It combines the market cost of equity from row 17c with the effective cost of debt for banks,  $r_t^{debt} = (q_t + \tau^{\Pi} r_t^f - \kappa)^{-1} - 1$ . The cost of debt is below the safe rate because of the tax deductability of interest expenses.<sup>31</sup> Total bank value is  $V_t^{bank} = V_t^I + q_t \times \text{Deposits}_t$ . Hence, bank WACC is:

$$WACC_{t} = \frac{V_{t}^{I}}{V_{t}^{bank}} MROE_{t} + \frac{q_{t} \times Deposits_{t}}{V_{t}^{bank}} r_{t}^{debt}$$

The cost of debt does not change much as regulation tightens, while the cost of equity falls. However, the main effect is the change in the composition of funding, which shifts from deposits to equity as equity capital requirements tighten. As a result, the WACC (in real terms) rises by 36% from 2.23% to 3.03% at  $\xi = 75\%$ . The reduced franchise value is a direct result of this sharp rise in WACC. As intermediaries are forced to fund each dollar of loans with a greater proportion of equity, the value created for bank shareholders per dollar of capital invested declines. Banks charge higher spreads in the loan market, but can only partially pass through their higher total funding cost to borrowers in general equilibrium. The findings are consistent with bankers' argument that tighter regulation destroys shareholder value.

Effect on production and macroeconomic volatility A first major adverse effect of tighter macro-prudential policy is that the economy's output (row 30 of Table 5) and the capital stock shrink meaningfully (row 31) relative to the benchmark model. The reduction in output arises because firms are smaller and borrow less from a smaller intermediary sector. Even though GDP shrinks, aggregate consumption increases slightly (row 32) thanks to lower deadweight losses from firm and bank failures (row 29).

A second adverse effect of tighter capital regulation is that it reduces the risk absorption capacity of the intermediary sector. Intermediaries help funnel resources from borrowers to savers, thereby improving risk sharing. The reduced risk-sharing capacity of the intermediaries is reflected in a more volatile ratio of marginal utility of borrowers and savers, a marker of increased market incompleteness (row 39).

Tighter regulation reduces macroeconomic volatility. The effect of making financial recessions less severe outweighs the intermediary's willingness to absorb aggregate risk. Volatility of consumption growth bottoms out around  $\xi = .85$ . For capital requirements higher than 15%,

 $<sup>^{31}</sup>$ In the data, the cost of deposits for banks is typically also below the short-term T-bill rate.

volatility starts to rise again as the risk absorption effect becomes more important. The economy with  $\xi = .95$  experiences more severe financial recessions as the indicated by the higher default rates of firms and banks. The increased fragility raises macro-economic volatility substantially. The subtle pattern in macro-economic volatility underscores the need for a rich structural model.

Figure 5 summarizes the effects of macro-prudential policy on financial fragility (left panel), the size of the economy (middle panel), and macro-economic volatility (right panel). Tightening the capital requirement has two key effects: (1) it shrinks the economy and lowers leverage of firms and banks, reducing macroeconomic volatility and bankruptcy-related losses; and (2), it reduces banks' willingness to absorb aggregate risk, increasing macroeconomic volatility.

Welfare Tighter macro-prudential policy reduces aggregate welfare in the model (row 26). Such policy redistributes wealth from savers, whose value function falls (row 28), to borrowers, whose value function rises (row 27). It reduces the equilibrium supply of safe assets, hurting the savers whose wealth is mostly made up of—now scarcer—safe assets. As debt finance becomes more expensive, firms rely more on equity finance, and a larger share of firm earnings accrues to its shareholders, the borrowers. Maybe surprisingly at first, forcing banks to hold more equity ends up benefiting their shareholders. Borrowers enjoy higher and less volatile consumption (row 33 and 37). Savers also enjoy less volatile consumption (row 38), but have lower average consumption (row 34). The right panel of Figure 6 highlights the redistributive nature of macro-prudential policy.

Because savers are more patient than borrowers, they require higher compensation for the same permanent reduction in consumption. It is not possible to implement a Pareto-improving transfer from the borrowers to the savers that leaves everyone better off. Even a modest tightening of bank equity requirements from 7% to 9% would result in a welfare loss of 3% of GDP. Much larger bank capital requirements of 25%, as advocated by some, would require a massive lump-sum of 37% of GDP (\$6 trillion) to make both borrowers and savers equally well off as in the status quo.

Looser capital requirements have the opposite distributional effect and increase saver wealth at the expense of borrowers. Because the gains now accrue to the agent with the highest marginal value of a dollar, there is scope for Pareto improving redistribution. At the same time, looser capital requirements beget more financial fragility and macro-economic volatility. The higher DWL from firm and bank defaults destroy aggregate consumption. Trading off these two forces, the welfare-maximizing capital requirement is 6% ( $\xi = .94$ ). For looser requirements (5% equity or less), DWLs eventually become so large that consumption of both agents becomes lower than in the benchmark. The increased financial fragility makes both agents worse off.

In sum, capital requirements around 6% maximize welfare. That number is close to the precrisis level and half as large as the 12% capital held by large banks in 2019.<sup>32</sup> We emphasize the redistributive nature of macro-prudential policy. Tighter capital requirements increase consumption and wealth inequality. Policymakers have signaled concern about redistributive implications of monetary policies adopted after the Great Financial Crisis. We show that macro-prudential policy has similar implications.

# 5.2 Counter-cyclical Capital Requirements

The last column of Tables 4 and 5 shows an experiment with counter-cyclical capital requirements. When uncertainty  $\sigma_{\omega,t}$  is low, banks' constraint is tightened ( $\xi = .91$ ) compared to the benchmark, whereas it is loosened ( $\xi = .95$ ) when uncertainty is high. Figure 7 compares financial recessions in the world with counter-cyclical capital requirements to financial recessions in the benchmark economy. The former features shallower recessions in terms of consumption and investment.

The counter-cyclical capital requirement causes a moderate expansion in corporate leverage, leading to slightly higher loan losses and bank defaults, and hence an uptick in DWLs (+1.17%). The modest increase in financial fragility is offset by a greater capital stock and higher GDP, such that aggregate consumption increases slightly. Even though credit risk increases, the credit spread shrinks due to a smaller credit risk premium. Since intermediaries are less constrained in financial crises now, they require less compensation for carrying aggregate risk. Risk sharing among borrowers and savers improves since the intermediary sector shrinks less in crises, as indicated by the lower volatility of the MU ratio (-11.97\%). Macroeconomic volatility decreases meaningfully. A larger financial sector redistributes wealth from borrowers to savers compared

 $<sup>^{32}</sup>$ The Dodd-Frank Act Stress Test 2019 results released in June 2019 reveal that the 18 banks subject to the supervisory stress tests in 2019 had an aggregate common equity tier 1 capital ratio of 12.3% in the fourth quarter of 2018.

to the benchmark: saver consumption and welfare increase, borrower consumption and welfare decrease. Since the experiment makes the more patient savers significantly better off, it allows for Pareto improving wealth transfers. The compensating variation wealth residual is 5.1% of GDP.

## 5.3 Transitions

The previous experiments compared the ergodic distributions of economies with different capital requirements. How does an unanticipated policy change to a tighter or looser capital requirement affect output, consumption, and the welfare of borrowers and savers in the short term? Figure 8 plots the evolution of these variables after a policy change from the benchmark to either a higher ( $\xi = .85$ ) or a lower ( $\xi = .95$ ) capital requirement. In the long run, output, consumption, and agent welfare converge to their ergodic means. In the short run, consumption "overshoots" in both cases. Tightening the capital requirement leads a contraction in GDP as investment drops. But the policy also causes a consumption boom in the short run as the economy transitions to a permanently lower capital stock. First-period value functions, which capture both short-run and long-run effects, show that the temporary consumption boom reduces the welfare loss to the saver but also the welfare gain to the borrower.

## 5.4 Sensitivity of Macro-prudential Policy Experiments

Appendix D.5 finds that the conclusions of our main macro-prudential policy experiment are robust to changes in key model parameters. Neither the presence of the bankruptcy option, nor the equity adjustment costs for either intermediaries or producers, nor the tax shield for banks are crucial for the qualitative macro-pru implications of the model. In every case, borrowers gain from tighter policy and savers loose. The trade-off between less financial fragility and a smaller economy is present in every model variant.

## 5.5 Mortgage Crisis

In addition to corporate credit risk, emphasized by our paper, Jorda, Schularick, and Taylor (2017) show that mortgage credit risk is an equally important source of financial fragility post

WW-II. Appendix D.6 performs an experiment whereby all banks face an unanticipated onetime negative shock to bank profits, standing in for a mortgage crisis. The shock is sized to lead to the same loss in bank net worth as in a typical financial recession. By comparing how the benchmark economy responds compared to an economy without intermediary sector, we show how losses to the intermediary sector resulting from losses on the mortgage book spill over to the corporate sector.

# 6 Conclusion

We provide the first calibrated macro-economic model which features intermediaries who extend long-term defaultable loans to firms producing output and raise deposits from risk averse savers, and in which both banks and firms can default. The model incorporates a rich set of fiscal policy rules, including deposit insurance, and endogenizes the risk-free interest rate.

Like in the standard accelerator model, shocks to the economy affect firm net worth. Since firm borrowing is constrained by net worth, macroeconomic shocks are amplified by tighter borrowing constraints. For realistic firm leverage ratios, and absent corporate default, this traditional accelerator is not very powerful. Introducing corporate default provides substantial amplification. Introducing financial intermediaries who face regulatory capital constraints and financial frictions activates a second financial accelerator. Macroeconomic shocks that lead to binding intermediary borrowing constraints amplify the shocks through their direct effect on intermediaries' net worth and the indirect effect on borrowers to whom the intermediaries lend.

Policies that enforce lower bank leverage reduce financial fragility and macro-economic volatility, but also shrink the size of the intermediation sector and its risk absorption capacity. Output and the capital stock shrink. Looser restrictions trigger financial fragility. A 6% capital requirement, close to the pre-crisis level, maximizes welfare. Counter-cyclical capital requirements centered around the pre-crisis level generate a Pareto improvement. Tighter macro-prudential policy, as has been enacted over the last decade, increases wealth inequality. The incidence of policies designed to limit the riskiness of the financial sector may fall on other sectors of the economy.

There are several fruitful directions for future research: endogenizing the decision to become an equity owner, studying monetary policy in a model with endogenous labor supply and NewKeynesian ingredients, modeling heterogeneity within the financial sector—splitting institutions into levered and unlevered ones—, adding mortgage borrowers to study spillovers from stress in mortgage markets to the corporate sector, incorporating differences in financing behavior of small and large firms, and allowing from misaligned incentives between bank managers and shareholders.

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Figure 1: Overview of Balance Sheets of Model Agents



Figure 2: Financial vs. Non-financial Recessions: Macro Quantities

The graphs show the average path of the economy through a recession episode which starts at time 1. In period 0, the economy is in the average TFP state. The recession is either accompanied by high uncertainty (high  $\sigma_{\omega}$ ), a financial recession plotted in red, or low uncertainty (low  $\sigma_{\omega}$ ), a non-financial recession plotted in blue. From period 2 onwards, the economy evolves according to its regular probability laws. The black line plots the dynamics of the economy absent any shock in period 1. We obtain the three lines via a Monte Carlo simulation of 10,000 paths of 25 periods, and averaging across these paths. Blue line: non-financial recession, Red line: financial recession, Black line: no shocks.



Figure 3: Financial vs. Non-financial Recessions: Balance Sheet Variables

Blue line: non-financial recession, Red line: financial recession, Black line: no shocks.



Figure 4: The Role of Frictions in Generating Financial Crises

**Red line**: responses of various outcome variables to a financial recession in the benchmark economy; **blue line**: responses of same outcome variables to a financial recession in Model (2); **black line**: responses of same outcome variables in Model (1). The underlying shock sequences are identical across experiments.



Figure 5: Effect of tighter capital requirement on size, fragility, and volatility of the economy

The left panel plots the loss rate on the loans held by banks and the failure rate of banks. The middle panel plots output and the ratio of deposits to output. The right panel plots aggregate consumption growth volatility and investment growth volatility. All variables are plotted as a function of the macro-prudential policy parameter  $\xi$ . Each dot represents a different economy where all parameters are the same as in the benchmark, except for  $\xi$ . The benchmark economy has  $\xi = .93$  and is indicated by the large dot.



Figure 6: Welfare Across Macro-Prudential Policy Experiments

The left panel plots the ex-post population-weighted aggregate welfare function  $\mathcal{W}^{pop}$  in green and the ex-ante consumption equivalent variation welfare function  $\mathcal{W}^{cev}$  in red as a function of the macro-prudential policy parameter  $\xi$ . The right panel plots the value function of Borrower (black) and Saver (orange) as a function of the macro-prudential policy parameter  $\xi$ . Each dot represents a different economy where all parameters are the same as in te benchmark, except for  $\xi$ . The benchmark economy has  $\xi = .94$ .



Figure 7: Financial Recessions with Counter-cyclical Capital Requirements

**Blue line:** responses to financial recession in economy with counter-cyclical capital requirements; **Black line:** responses to financial recession in benchmark economy. The underlying shocks in the two cases are identical.

Figure 8: Transition Dynamics After Change in Capital Requirement



Panel A: Transition to  $\xi=85\%$ 

Panel B: Transition to  $\xi=95\%$ 



Par	Description	Value	Source
	Exc	genous Shock	s
$[p_{LL}^{\omega}, p_{HH}^{\omega}]$	transition prob	0.91,  0.8	Bloom et al. (2012)
	Population a	nd Labor Inco	ome Shares
$\ell^i$	pop. shares $\in \{S, B\}$	$71.1,\!28.9\%$	Population shares SCF 95-13
$\gamma^i$	inc. shares $\in \{S, B\}$	$64{,}36\%$	Labor inc. shares SCF 95-13
	Corporate Lo	oans and Inter	rmediation
δ	average life loan pool	0.937	Duration fcn. in App. C.5
$\theta$	principal fraction	0.582	Duration fcn. in App. C.5
$\eta^P$	% bankr. loss is DWL (producers)	0.2	Bris, Welch, and Zhu (2006)
$\eta^{I}$	% bankr. loss is DWL (banks)	36.2	Bennett and Unal (2015)
$\bar{\zeta}^{I}$	% Resolution cost failed banks	33.2	Bennett and Unal (2015)
$\phi_0^I$	target bank dividend	0.068	Avg bank div
$\phi_0^P$	target firm dividend	0.078	Avg nonfin firm div
$\phi_1^P$	firm equity iss. cost	0	Baseline
		Preferences	
$\sigma^B = \sigma^S$	risk aversion B S	1	Log utility
$\nu^B = \nu^S$	IES B S	1	Log utility
	(	Government	
$ au^D$	interest rate income tax rate	13.2%	tax code; see text
$\kappa$	deposit insurance fee	0.00084	Deposit insurance revenues/bank assets
ξ	max. intermediary leverage	0.93	Basel II reg. capital charge for CI loans bonds

# Table 1: Pre-Set Parameters

$\mathbf{Par}$	Description	Value	Target	Model	Data
			Exogenous Shocks		
$\rho_A$	persistence TFP	0.4	AC(1) HP-detr GDP 53-14	0.55	0.55
$\sigma_A$	innov. vol. TFP	2.3%	Vol HP-detr GDP 53-14	2.55%	2.56%
$\sigma_{\omega,L}$	low uncertainty	0.1	Avg. corporate default rate	2.08%	2%
$\sigma_{\omega,H}$	high uncertainty	0.18	Avg. IQR firm-level prod (Bloom et al. (2012))	5.00%	4.9%
			Production		
ψ	marginal adjustment cost	2	Vol. log investment 53-14	9.41%	8.13%
σ	labor share in prod. fct.	0.71	Labor share of output	66.20%	2/3
$\delta_K$	capital depreciation rate	8.25	Investment-to-output ratio, 53-14	17.74%	17.90%
ν	capital fixed cost	0.004	Capital-to-GDP ratio 53-14	215%	224%
		Corpor	ate Loans and Intermediation		
$\zeta^P$	Losses on defaulting loans	0.6	Corporate loan and bond severities 81-15	51.78%	51.4%
Φ	maximum LTV ratio	0.4	FoF non-fin sector leverage 85-14	37.33%	37%
$\sigma_\epsilon$	cross-sect. dispersion $\epsilon_t^I$	1.9%	FDIC failure rate of deposit. inst., mean	0.63%	0.50%
$\phi_1^I$	bank equity issuance cost	2	Bank net payout rate	5.98%	5.75%
$\varphi_0$	Saver holdings target	0.0115	Avg corp debt holdings outside lev fin sector	13.93%	13.70%
$\varphi_1$	Saver holdings adj cost	0.14	Vol of corp debt holdings outside lev fin sector	3.42%	3.3%
			Preferences		
$\beta^B$	time discount factor B	0.94	Corporate net payout rate	6.90%	6.41%
$\beta^{S}$	time discount factor S	0.982	Mean risk-free rate 76-14	2.23%	2.20%
			Government Policy		
$G^o$	discr. spending	17.2%	BEA discr. spending to GDP 53-14	17.50	17.58%
$G^{T}$	transfer spending	2.52%	BEA transfer spending to GDP 53-14	3.15%	3.18%
τ	labor income tax rate	29.3%	BEA pers. tax rev. to GDP 53-14	19.17%	17.30%
$\tau^{\Pi}$	corporate tax rate	20%	BEA corp. tax rev. to GDP 53-14	3.56%	3.41%
$b_o$	cyclicality discr. spending	-2	slope log discr. sp./GDP on GDP growth	-0.89	-0.75
$b_T$	cyclicality transfer spending	-20	slope log transfer sp./GDP on GDP growth	-8.88	-7.26
$b_{ au}$	cyclicality lab. inc. tax	4.5	slope labor tax/GDP on GDP growth	0.62	0.70

Table 2: Calibrated Parameters

	Uncon	ditional	Expansions	Non-fin Rec.	Fin Rec.
	mean	stdev	mean	mean	mean
			Firm	s	
1. Mkt val of capital / Y (in %)	215.15	3.97	216.88	217.85	210.86
2. Mkt val of corp debt / Y (in %)	82.85	3.96	84.60	82.98	78.88
3. Book val corp debt / Y (in %)	80.34	3.37	80.92	81.60	78.96
4. Market corp leverage (in $\%$ )	38.50	1.52	39.01	38.08	37.40
5. Book corp leverage (in $\%$ )	37.33	1.28	37.70	37.08	36.52
6. % leverage constr binds (in %)	1.63	12.66	0.00	0.59	4.79
7. Default rate (in %)	2.08	2.00	0.74	0.76	5.08
8. Loss-given-default rate (in $\%$ )	51.78	1.87	52.67	51.23	49.99
9. Loss Rate (in $\%$ )	1.07	1.02	0.39	0.39	2.54
10. Investment / Y (in $\%$ )	17.74	1.24	18.73	17.15	15.39
			Bank	s	
11. Mkt val assets / Y (in %)	71.31	4.42	73.32	70.47	66.19
12. Mkt fin leverage (in $\%$ )	92.63	0.63	92.30	92.83	92.99
13. Book fin leverage (in $\%$ )	97.62	1.88	98.87	98.28	94.44
14. % leverage constr binds	63.39	48.18	32.37	84.69	99.90
15. Bankruptcies (in $\%$ )	0.63	4.58	0.04	0.51	4.41
16. Net payout rate (in $\%$ )	5.98	24.57	17.02	2.95	-26.29
			Saver	'S	
17. Deposits / Y (in %)	67.52	4.37	69.34	68.11	62.59
18. Government Debt / Y (in $\%$ )	73.08	12.41	69.88	82.79	80.19
19. Corp Debt Share S	13.93	3.42	13.35	15.07	16.16
			Price	S	
20. Tobin's q	1.00	0.01	1.01	0.99	0.98
21. Risk-free rate (in $\%$ )	2.23	1.59	2.45	4.13	1.69
22. Corporate bond rate (in $\%$ )	4.12	0.19	3.97	4.26	4.44
23. Credit spread (in $\%$ )	1.89	1.56	1.52	0.13	2.75
24. Term spread (in $\%$ )	-0.36	1.37	-0.57	-1.97	0.15
25. Excess ret corp. bonds (in $\%$ )	0.84	2.06	1.80	-0.30	-1.20

Table 3: Balance Sheet Variables and Prices

	Bonch $(\xi = 0.0)$	¢ – 76	¢ — 80	с 1 0	ć — 01	с — ОК	¢ _ [ 01 05]
	Delicit $(\zeta = .30)$	$\zeta = \cdot i \partial$	ς = .ου	$\zeta = .00$	$\zeta = .91$	$\zeta = .30$	$\zeta = \{.31, .30\}$
			ň	orrowers			
1. Mkt value capital/ $Y$	215.2	211.6	213.0	214.0	215.0	215.8	215.6
2. Mkt value corp debt/ $Y$	82.8	60.2	67.0	73.6	81.2	84.5	84.3
3. Book val corp $debt/Y$	80.3	59.7	66.1	72.2	78.9	81.5	81.6
4. Market corp leverage	38.5	28.5	31.5	34.4	37.8	39.2	39.1
5. Book corp leverage	37.3	28.2	31.0	33.7	36.7	37.8	37.8
6. $\%$ producer constr binds	1.6	0.0	0.0	0.0	0.9	3.0	1.8
7. Default rate	2.08	1.47	1.64	1.81	2.03	2.12	2.11
8. Loss-given-default rate	51.8	37.2	42.5	46.7	50.9	52.4	52.3
9. Loss Rate	1.07	0.52	0.67	0.83	1.02	1.10	1.09
10. Investment $/$ Y	17.7	17.4	17.6	17.6	17.7	17.8	17.8
			Inte	rmediari	es		
11.Mkt val assets / Y	71.3	47.0	54.3	61.4	69.5	73.5	72.4
12. Mkt fin leverage	92.6	75.0	80.0	84.9	90.6	94.8	92.2
13. Book fin leverage	97.6	77.3	82.9	88.4	95.3	100.4	97.3
14. % intermed constr binds	63.4	98.9	94.9	86.8	63.0	81.5	91.7
15. Bankruptcies	0.63	0.00	0.00	0.00	0.10	2.69	0.77
16. Wealth I / Y	5.4	12.4	11.4	9.7	6.7	3.9	5.8
17a. Franchise Value	17.9	7.5	7.4	6.2	10.2	25.5	15.5
17b. Bank accounting ROE	24.9	8.1	9.9	12.8	19.8	34.8	23.2
17c. Bank market ROE	6.82	6.46	6.49	6.55	6.70	7.09	6.82
17d. WACC for bank	2.23	3.03	2.80	2.56	2.31	2.15	2.25
				Savers			
18. Deposits/GDP	67.5	36.0	44.4	53.3	64.4	71.2	68.2
19. Government debt/GDP	73.1	70.4	70.8	71.1	71.7	78.4	73.5
20. Corp Debt Share S	13.9	21.9	19.0	16.6	14.4	13.2	14.1
		_		Prices		-	
21. Risk-free rate	2.23	2.19	2.20	2.21	2.22	2.23	2.23
22. Corporate bond rate	4.12	4.34	4.28	4.23	4.14	4.07	4.10
23. Credit spread	1.89	2.15	2.08	2.02	1.92	1.84	1.87
24. Term spread	-0.36	-0.33	-0.33	-0.34	-0.36	-0.36	-0.37
25. Excess return on corp. bonds	0.84	1.66	1.44	1.22	0.91	0.75	0.79

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Table ∉

	Bench ( $\xi = .93$ )	$\xi = .75$	$\xi = .80$	$\xi = .85$	$\xi = .91$	$\xi = .95$	$\xi = \{.91, .95\}$
		-		Velfare			
26. Aggr. welfare $\mathcal{W}^{cev}$		-37.05	-26.14	-15.46	-3.31	1.51	5.08
27. Value function, B	0.260	3.57	2.72	1.71	0.40	-0.74	-0.21
28. Value function, S	0.375	-2.60	-1.87	-1.13	-0.24	0.23	0.30
29. DWL/GDP	0.741	-37.63	-30.29	-22.02	-9.75	30.05	1.17
			Size of	the Eco	nomy		
30. GDP	0.987	-0.67	-0.42	-0.21	-0.03	0.11	0.09
31. Capital stock	2.123	-2.31	-1.44	-0.74	-0.11	0.40	0.31
32. Aggr. Consumption	0.632	0.22	0.22	0.19	0.10	-0.32	0.02
33. Consumption, B	0.258	4.14	3.15	2.03	0.50	-0.77	-0.17
34. Consumption, S	0.374	-2.48	-1.80	-1.08	-0.17	-0.01	0.14
		-	Λ	olatility			
35. Investment gr	10.76	-22.17	-19.70	-16.94	-13.41	31.26	-13.37
36. Consumption gr	1.83	-1.56	-3.79	-5.20	-4.50	17.37	-3.81
37. Consumption gr, B	2.81	-6.91	-6.81	-6.83	-6.28	17.71	-7.04
38. Consumption gr, S	2.46	5.89	3.05	1.19	0.04	5.28	-17.27
39. $\log (MU B / MU S)$	0.04	7.38	6.54	5.24	0.95	3.12	-11.97
Numbers in column 1	are for the bench	mark mod	lel. in leve	els. Numb	oers in co	lumns 2-8	are percentage

Table 5: Macroprudential Policy: Macro and Welfare

ñ0 D 2 • changes relative to the benchmark.

# A Model Appendix

# A.1 Borrower-entrepreneur

#### A.1.1 Optimization Problem

Let  $S_t = (Z_t^A, \sigma_{\omega,t}, K_t, A_t^P, N_t^I, W_t^S, B_t^G)$  be the vector aggregate state variables.

Then the representative borrower problem solves the Bellman equation:

$$V^{B}(\mathcal{S}_{t}) = \max_{\{C_{t}^{B}, X_{t}\}} \left\{ (1 - \beta_{B}) \left(C_{t}^{B}\right)^{1 - 1/\nu} + \beta_{B} \mathbb{E}_{t} \left[ \left(V^{B}(\mathcal{S}_{t+1})\right)^{1 - \sigma_{B}} \right]^{\frac{1 - 1/\nu}{1 - \sigma_{B}}} \right\}^{\frac{1}{1 - 1/\nu}}$$

subject to the budget constraint:

$$C_t^B = D_t^P + D_t^I + (1 - \tau_t^B) w_t^B \bar{L}^B + G_t^{T,B} + p_t X_t - X_t - \Psi(X_t, K_t).$$
(27)

Let the capital adjustment cost function be given by:

$$\Psi(X_t, K_t) = \frac{\psi}{2} \left(\frac{X_t}{K_t} - \delta_K\right)^2 K_t.$$

Its partial derivative w.r.t. investment is:

$$\Psi_X(X_t, K_t) = \psi\left(\frac{X_t}{K_t} - \delta_K\right).$$
(28)

Denote the value function as  $V_t^B = V^B(\mathcal{S}_t)$ . Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathcal{E}_t \left[ \left( V_{t+1}^B \right)^{1-\sigma_B} \right]^{\frac{1}{1-\sigma_B}}$$

#### A.1.2 First-order Conditions

**Investment** The FOC for investment  $X_t$  is:

$$[1 + \Psi_X(X_t, K_t) - p_t] \frac{(1 - \beta_B)(U_t^B)^{1 - 1/\nu}(V_t^B)^{1/\nu}}{C_t^B} = 0,$$

which simplifies to

$$1 + \Psi_X(X_t, K_t) = p_t.$$
<sup>(29)</sup>

#### A.1.3 SDF

We can define the stochastic discount factor (SDF) from t to t + 1 of borrowers:

$$\mathcal{M}_{t,t+1}^{B} = \beta_B \left(\frac{C_{t+1}^{B}}{C_t^{B}}\right)^{-1/\nu_B} \left(\frac{V_{t+1}^{B}}{CE_t^{B}}\right)^{1/\nu_B - \sigma_B}.$$
(30)

Electronic copy available at: https://ssrn.com/abstract=2748230

# A.2 Producer

#### A.2.1 Technology

The exogenous law of motion for the TFP level  $Z_t^A$  is (lower case letters denote logs):

$$\log Z_t^A = (1 - \rho_A) z^A + \rho_A \log Z_{t-1}^A + \epsilon_t^A \quad \epsilon_t^A \sim iid \,\mathcal{N}(0, \sigma^A)$$

Denote  $\mu_{ZA} = e^{z^A + \frac{(\sigma^A)^2}{2(1-\rho_A^2)}}.$ 

Idiosyncratic productivity of borrower-entrepreneur i at date t is denoted by

 $\omega_{i,t} \sim i.i.d. \operatorname{Gamma}(\gamma_{0,t}, \gamma_{1,t}),$ 

where the parameters  $\gamma_{0,t}$  and  $\gamma_{1,t}$  are chosen such that

$$E(\omega_{i,t}) = 1,$$
  
$$Var(\omega_{i,t}) = \sigma_{\omega,t}^{2},$$

#### A.2.2 Individual Firm Problem

To simplify notation, we suppress subscripts i for individual producers. We can state the recursive producer problem as:

$$V(n_t^P, \mathcal{S}_t) = \max_{e_t^P, k_{t+1}, a_{t+1}^P} \phi_0^P n_t^P - e_t^P + \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^B V^+(k_{t+1}, a_{t+1}^P, \mathcal{S}_{t+1}) \right]$$

subject to the budget constraint:

$$(1 - \phi_0)n_t^P + e_t^P - \Psi^P(e_t^P, n_t^P) \ge p_t k_{t+1} - q_t^m a_{t+1}^P$$

and subject to the leverage constraint:

$$\Phi p_t k_{t+1} \ge F a_{t+1}.$$

The decision problem at stage 1 in the intra-period sequence of events is characterized by the continuation value function  $V^+(k_t^P, a_t^P, \mathcal{S}_t)$ . To state the problem that gives rise to this function, recall the definition of the flow profit of producers:

$$\pi_t = \omega_t Z_t k_t^{1-\alpha} l_t^{\alpha} - \sum_j w_t^j l_t^j - a_t^P - \varsigma k_t, \tag{31}$$

which depends on idiosyncratic productivity  $\omega_t$ . Producers with  $\pi_t < 0$  are in default and shut down. This implies a default threshold:

$$\omega_t^* = \frac{a_t^P + \varsigma k_t + \sum_j w_t^j l_t^j}{Z_t k_t^{1-\alpha} l_t^{\alpha}}$$

such that producers with low idiosyncratic productivity  $\omega_t < \omega_t^*$  default.

We can now state the stage-1 value function:

$$V^{+}(k_{t}, a_{t}^{P}, \mathcal{S}_{t}) = \max_{l_{t}^{j}} \left(1 - F_{\omega, t}(\omega_{t}^{*})\right) \mathbb{E}_{\omega, t}\left[V^{P}(n_{t}^{P}, \mathcal{S}_{t}) \mid \omega_{t} > \omega_{t}^{*}\right]$$
(32)

subject to the transition law for net worth:

$$n_t^P = (1 - \tau^{\Pi})\pi_t + (1 - (1 - \tau^{\Pi})\delta_K)p_t k_t - \delta q_t^m a_t^P,$$
(33)

where  $\tau^{\Pi}$  is the corporate profit tax rate.

Scale Invariance We assume the functional form for the equity issuance cost:

$$\Psi(e_t^P, n_t^P) = \frac{1}{n_t^P} \frac{\phi_1^P}{2} (e_t^P)^2.$$

Given this cost function, the producer problem is homogeneous of degree one in net worth  $n_t^P$ . We can thus define the scaled variables  $\tilde{e}_t^P = e_t^P/n_t^P$ ,  $\tilde{a}_{t+1}^P = a_{t+1}^P/n_t^P$ ,  $\tilde{k}_{t+1} = k_{t+1}/n_t^P$ ,  $\tilde{l}_{t+1}^j = l_{t+1}^j/n_t^P$ , and the value function  $v(S_t)$  such that:

$$V^P(n_t^P, \mathcal{S}_t) = n_t^P v^P(\mathcal{S}_t).$$

Using this value function, denote the solution to the stage-1 producer problem in (32) as

$$\tilde{l}^{j}(\tilde{k}_{t}, \tilde{a}_{t}^{P}, \mathcal{S}_{t}) = \operatorname*{argmax}_{\tilde{l}_{t}^{j}} v^{P}(\mathcal{S}_{t}) \left(1 - F_{\omega, t}(\omega_{t}^{*})\right) \operatorname{E}_{\omega, t}\left[\tilde{n}_{t}^{P} \mid \omega_{t} > \omega_{t}^{*}\right],$$

where  $\tilde{n}_t^P = n_t^P/n_{t-1}^P$  is the growth rate of individual producer net worth.

Composite labor input is:

$$\tilde{l}(\tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t) = \tilde{l}^B(\tilde{k}_t, \tilde{a}P_t, \mathcal{S}_t)^{\gamma_B} \tilde{l}^S(\tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t)^{\gamma_S}.$$

Substituting (31) into (33) and substituting in optimal labor demands  $\tilde{l}^{j}(\tilde{k}_{t}, \tilde{a}_{t}^{P}, \mathcal{S}_{t})$ , we can write the growth rate of net worth,  $\tilde{n}_{t}^{P} = n_{t}^{P}/n_{t-1}^{P}$ , for some realization of the idiosyncratic shock  $\omega'$  and given capital and debt  $(\tilde{k}_{t}, \tilde{a}_{t}^{P})$  as:

$$\Pi(\omega', \tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t) = (1 - \tau^{\Pi}) \omega' Z_t^A \tilde{k}_t^{1-\alpha} \tilde{l}(\tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t)^{\alpha} - (1 - \tau^{\Pi}) \sum_j w_t^j \tilde{l}^j(\tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t) + \left( (1 - (1 - \tau^{\Pi}) \delta_K) p_t - (1 - \tau^{\Pi}) \varsigma \right) \tilde{k}_t - \left( 1 - \tau^{\Pi} + \delta q_t^m \right) \tilde{a}_t^P.$$
(34)

Thus, the growth rate next period, conditional on not defaulting is

$$\mathbf{E}_{\omega,t+1}\left[\Pi(\omega_{t+1},\tilde{k}_{t+1},\tilde{a}_{t+1}^{P},\mathcal{S}_{t+1}) \,|\, \omega_{t+1} > \omega_{t+1}^{*}\right] = \Pi(\omega_{t+1}^{+},\tilde{k}_{t+1},\tilde{a}_{t+1}^{P},\mathcal{S}_{t+1}),$$

where

$$\omega_{t+1}^{+} = \mathcal{E}_{\omega,t+1} \left[ \omega \,|\, \omega > \omega_{t+1}^{*} \right].$$

Using the definition of  $\Pi(\omega', \tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t)$  in (34), we can write the scaled stage-3 problem as:

$$v^{P}(\mathcal{S}_{t}) = \max_{\tilde{e}_{t}^{P}, \tilde{k}_{t+1}, \tilde{a}_{t+1}^{P}} \phi_{0} - \tilde{e}_{t}^{P} + E_{t} \left[ \mathcal{M}_{t,t+1}^{B} v^{P}(\mathcal{S}_{t+1}) \left( 1 - F_{\omega,t+1}(\omega_{t+1}^{*}) \right) \Pi(\omega_{t+1}^{+}, \tilde{k}_{t+1}, \tilde{a}_{t+1}^{P}, \mathcal{S}_{t+1}) \right],$$
(35)

subject to the budget constraint

$$1 - \phi_0^P + \tilde{e}_t^P - \frac{\phi_1^P}{2} (\tilde{e}_t^P)^2 = p_t \tilde{k}_{t+1} - q_t^m \tilde{a}_{t+1}^P,$$
(36)

and the leverage constraint

$$\Phi p_t \tilde{k}_{t+1} \ge F \tilde{a}_{t+1}^P. \tag{37}$$

#### A.2.3 First-order Conditions

**Definitions** We define the marginal product of labor input j as:

$$\mathrm{MPL}_t^j = \alpha \gamma_j Z_t^A \frac{\tilde{l}_t}{\tilde{l}_t^j} \left(\frac{\tilde{k}_t}{\tilde{l}_t}\right)^{1-\alpha},$$

and the aggregate marginal product of capital as:

$$MPK_t = (1 - \alpha)Z_t^A \left(\frac{\tilde{k}_t}{\tilde{l}_t}\right)^{-\alpha}$$

**Labor Demand** The FOC for labor inputs in the stage-1 problem given by (32) is:

$$(1 - \tau^{\Pi}) \left(1 - F_{\omega,t}(\omega_t^*)\right) \left(\omega_{t+1}^+ \mathrm{MPL}_t^j - w_t^j\right) = f_{\omega}(\omega_t^*) \Pi \left(\omega_t^*, \tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t\right) \frac{\partial \omega_t^*}{\partial \tilde{l}_t^j},$$
(38)

where the partial derivative of the optimal default threshold with respect to labor input j is:

$$\frac{\partial \omega_t^*}{\partial \tilde{l}_t^j} = \frac{1}{Z_t^A \tilde{k}_t^{1-\alpha} \tilde{l}_t^{\alpha}} \left( w_t^j - \mathrm{MPL}_t^j \omega_t^* \right).$$

Note that  $\Pi\left(\omega_t^*, \tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t\right)$  on the right-hand side is the net worth growth rate at the realization of  $\omega_t$  equal to the default threshold. Condition (38) implicitly defines labor demand functions  $l^j(\tilde{k}_t, \tilde{a}_t^P, \mathcal{S}_t)$ .

Marginal Value of Net Worth and Equity Issuance Now we turn to the stage-3 problem. We denote the Lagrange multiplier on the budget constraint by  $\nu_t^P$ , and the Lagrange multiplier on the leverage constraint by  $\lambda_t^P$ . The marginal value of net worth is:

$$v^P(\mathcal{S}_t) = \phi_0^P + (1 - \phi_0^P)\nu_t^P.$$

The FOC for equity issuance is:

$$\nu_t^P (1 - \phi_1^P \tilde{e}_t^P) = 1.$$

Combining, we get:

$$v^{P}(\mathcal{S}_{t}) = \phi_{0}^{P} + \frac{1 - \phi_{0}^{P}}{1 - \phi_{1}^{P}\tilde{e}_{t}^{P}}.$$

**Debt** The FOC for new debt is:

$$\nu_t^P q_t^m = F \lambda_t^P + \mathcal{E}_t \left\{ \mathcal{M}_{t,t+1}^B v^P(\mathcal{S}_{t+1}) \left[ \left( 1 - F_{\omega,t+1}(\omega_{t+1}^*) \right) \left( 1 - \tau^\Pi + \delta q_{t+1}^m \right) + f_{\omega,t+1}(\omega_{t+1}^*) \Pi \left( \omega_{t+1}^* \right) \frac{\partial \omega_{t+1}^*}{\partial \tilde{a}_{t+1}^P} \right] \right\}$$

The derivative of the default threshold with respect to debt is:

$$\frac{\partial \omega_{t+1}^*}{\partial \tilde{a}_{t+1}^P} = \frac{1}{Z_{t+1}^A \tilde{k}_{t+1}^{1-\alpha} \tilde{l}_{t+1}^\alpha}.$$

**Capital** The FOC for capital is:

$$\nu_{t}^{P} p_{t} = \Phi p_{t} \lambda_{t}^{P} + \mathcal{E}_{t} \left\{ \mathcal{M}_{t,t+1}^{B} v^{P}(\mathcal{S}_{t+1}) \times \left[ \left( 1 - F_{\omega,t+1}(\omega_{t+1}^{*}) \right) \left( (1 - \tau^{\Pi}) \mathrm{MPK}_{t+1} \omega_{t+1}^{+} + (1 - \tilde{\delta}_{K}) p_{t+1} - (1 - \tau^{\Pi}) \varsigma \right) - f_{\omega,t+1}(\omega_{t+1}^{*}) \Pi \left( \omega_{t+1}^{*} \right) \frac{\partial \omega_{t+1}^{*}}{\partial \tilde{k}_{t+1}} \right] \right\}$$

The derivative of the default threshold with respect to capital is:

$$\frac{\partial \omega_{t+1}^*}{\partial \tilde{k}_{t+1}} = \frac{1}{Z_{t+1} \tilde{k}_{t+1}^{1-\alpha} \tilde{l}_{t+1}^{\alpha}} \left(\varsigma - \mathrm{MPK}_{t+1} \omega_{t+1}^*\right).$$

#### A.2.4 Producer SDF and Euler Equations

Since the producer problem is scale invariant, we can construct the stochastic discount factor of a representative producer

$$\mathcal{M}_{t,t+1}^{P} = \mathcal{M}_{t,t+1}^{B} (1 - \phi_{1}^{P} \tilde{e}_{t}^{P}) \left(\phi_{0} + \frac{1 - \phi_{0}}{1 - \phi_{1} \tilde{e}_{t+1}^{P}}\right).$$

Using this SDF, and  $\tilde{\lambda}_t^P = \lambda_t^P / \nu_t^P$ , we get the following more succinct Euler equations for debt:

$$q_{t}^{m} = F\tilde{\lambda}_{t}^{P} + E_{t} \left[ \mathcal{M}_{t,t+1}^{P} \left( \left( 1 - F_{\omega,t+1}(\omega_{t+1}^{*}) \right) \left( 1 - \tau^{\Pi} + \delta q_{t+1}^{m} \right) + \frac{f_{\omega,t+1}(\omega_{t+1}^{*})\Pi\left(\omega_{t+1}^{*}\right)}{Z_{t+1}^{A}\tilde{k}_{t+1}^{1-\alpha}\tilde{l}_{t+1}^{\alpha}} \right) \right], \quad (39)$$

and for capital:

$$p_{t}(1 - \Phi \tilde{\lambda}_{t}^{P}) = E_{t} \left[ \mathcal{M}_{t,t+1}^{P} \left( \left( 1 - F_{\omega,t+1}(\omega_{t+1}^{*}) \right) \left( (1 - \tau^{\Pi}) \mathrm{MPK}_{t+1} \omega_{t+1}^{+} + (1 - \delta_{K}) p_{t+1} - (1 - \tau^{\Pi}) \varsigma \right) - \frac{f_{\omega,t+1}(\omega_{t+1}^{*}) \Pi \left( \omega_{t+1}^{*} \right)}{Z_{t+1}^{A} \tilde{k}_{t+1}^{1 - \alpha} \tilde{l}_{t+1}^{\alpha}} \left( \varsigma - \mathrm{MPK}_{t+1} \omega_{t+1}^{*} \right) \right) \right].$$

$$(40)$$

#### A.2.5 Aggregate Producer Net Worth

At the beginning of each period, a fraction of producers defaults before paying dividends to shareholders and choosing the portfolio for next period. Debt holders take ownership of these bankrupt firms and liquidate them to recover some of the outstanding debt.

Bankrupt producers are immediately replaced by newly started firms that borrowers endow with initial equity  $n^0$  per firm. Then all producers, including newly started ones, solve the identical optimization problem in (35).

Denote aggregate net worth of producers at stage 3, i.e. when producers solve their decision problem for next period, by  $N_t^P$ . Then the average net worth of surviving producers in t + 1 is recursively defined as

$$N_{t+1}^{P+} = \underbrace{\Pi(\omega_{t+1}^+, \tilde{k}_{t+1}, \tilde{a}_{t+1}^P, \mathcal{S}_{t+1})}_{\text{growth rate to } t+1} \underbrace{\left(1 - \phi_0 + \tilde{e}_t^P - \frac{\phi_1^I}{2} (\tilde{e}_t^P)^2\right) N_t^P}_{\text{net worth after payout/issuance in } t}.$$

where  $\Pi(\omega_{t+1}^+, \tilde{k}_{t+1}, \tilde{a}_{t+1}^P, \mathcal{S}_{t+1})$  is the growth rate of net worth of non-defaulting producers as defined in (34).

Aggregate net worth of producers thus follows the recursion:

$$N_{t+1}^P = \left(1 - F_{\omega,t+1}(\omega_{t+1}^*)\right) N_{t+1}^{P+} + F_{\omega,t+1}(\omega_{t+1}^*) n^0.$$
(41)

Given this expression of aggregate producer net worth, we can recover all aggregate producer choices, i.e.  $A_{t+1}^P = \tilde{a}_{t+1}^P N_t^P$ ,  $K_{t+1} = \tilde{k}_{t+1} N_t^P$ , and so forth.

### A.3 Intermediaries

#### A.3.1 Aggregation

Three assumptions we make are sufficient to obtain aggregation to a representative intermediary. These assumptions are: (i) that the intermediary objective is linear in the idiosyncratic profit shock  $\epsilon_{t,i}$ , (ii) that idiosyncratic profit shocks only affect the contemporaneous payout (but not net worth), and (iii) that defaulting intermediaries are replaced by new intermediaries with equity equal to that of non-defaulting intermediaries.

Denote by  $n_{t,i}^I$  the beginning-of-period net worth of intermediary *i* which did not default. Further denote by  $S_t = (Z_t^A, \sigma_{\omega,t}, K_t, A_t^P, N_t^I, W_t^S, B_t^G)$  all aggregate state variables exogenous to the individual intermediary problem, including aggregate intermediary net worth  $N_t^I$ .

We can define the optimization problem of the non-defaulting intermediary with profit shock realization  $\epsilon_{t,i}$  recursively as:

$$V^{I}(n_{t,i}^{I},\epsilon_{t,i},\mathcal{S}_{t}) = \max_{e_{t,i}^{I},b_{t+1,i}^{I},a_{t+1,i}^{I}} \phi_{0}^{I}n_{t}^{I} - e_{t,i}^{I} + \epsilon_{t,i} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} \max\left\{ V^{I}(n_{t+1,i}^{I},\epsilon_{t+1,i},\mathcal{S}_{t+1}),0\right\} \right], \quad (42)$$

subject to the budget constraint (14), the regulatory capital constraint (15), and the definition of net worth (12). Since the objective function is linear (assumption (i)) in the profit shock  $\epsilon_{t,i}$ , we can equivalently define a value function  $\tilde{V}^{I}(n_{t,i}^{I}, \mathcal{S}_{t}) = V^{I}(n_{t,i}^{I}, \epsilon_{t,i}, \mathcal{S}_{t}) - \epsilon_{t,i}$ , and write the objective as:

$$\tilde{V}^{I}(n_{t,i}^{I}, \mathcal{S}_{t}) = \max_{e_{t,i}^{I}, b_{t+1,i}^{I}, a_{t+1,i}^{I}} \phi_{0}^{I} n_{t}^{I} - e_{t,i}^{I} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} \max\left\{ \tilde{V}^{I}(n_{t+1,i}^{I}, \mathcal{S}_{t+1}) + \epsilon_{t+1,i}, 0 \right\} \right],$$
(43)

subject to the same set of constraints. Conditional on the same state variables  $(n_{t,i}^{I}, \mathcal{S}_{t})$ , the objective functions in (42) and (43) imply the same optimal choices  $(e_{t,i}^{I}, b_{t+1,i}^{I}, a_{t+1,i}^{I})$ , independent of the realization of the current profit shock  $\epsilon_{t,i}$ . Thus conjecturing that all non-defaulting banks start the period with identical wealth  $n_{t,i} = N_t^{I}$ , these banks will also have identical wealth at the beginning of the next period,  $N_{t+1}^{I}$ , since idiosyncratic shocks do not affect next-period net worth directly (assumption (ii)). Hence absent default, all banks have identical wealth  $N_t^{I}$ .

What about defaulting banks? By construction, the realization of the profit shock is irrelevant for banks that defaulted and were reserved with initial capital. Here we assume that equity holders (borrower households) seed all newly started banks with identical capital =  $n_t^{I,0}$ . Therefore, all banks newly started to replace defaulting banks are identical and solve the problem

$$\tilde{V}^{I}(n_{t}^{I,0},\mathcal{S}_{t}) = \max_{e_{t}^{I,0},b_{t+1}^{I,0},a_{t+1}^{I,0}} \phi_{0}^{I}n_{t}^{I,0} - e_{t}^{I,0} + \mathcal{E}_{t}\left[\mathcal{M}_{t,t+1}^{B}\max\left\{\tilde{V}^{I}(n_{t+1}^{I,1},\mathcal{S}_{t+1}) + \epsilon_{t+1,i},0\right\}\right],\tag{44}$$

again subject to the same set of constraints. Clearly, if  $n_t^{I,0} = N_t^I$ , which is assumption (iii), then the new banks will choose the same portfolio  $(e_t^{I,0}, b_t^{I,0}, a_{t+1}^{I,0}) = (e_t^I, b_{t+1}^I, a_{t+1}^I)$  as the non-defaulting banks. This means that new banks replacing defaulted banks will also have the same wealth at the beginning of next period,  $n_{t+1}^{I,1} = N_{t+1}^I$ .

Together, this means that all banks have the same beginning-of-period wealth  $N_t^I$ , and we can solve the problem of a representative bank that chooses  $(e_t^I, B_{t+1}^I, A_{t+1}^I)$ .

#### A.3.2 Optimization problem

 $N_t^I$  is the net worth of all intermediaries after firm and intermediary bankruptcies and recapitalization of defaulting intermediaries by borrowers.

At the end of each period, all intermediaries face the following optimization problem over dividend payout and portfolio composition (see equation (13) in the main text):

$$\tilde{V}^{I}(N_{t}^{I}, \mathcal{S}_{t}) = \max_{e_{t}^{I}, B_{t+1}^{I}, A_{t+1}^{I}} \phi_{0}^{I} N_{t}^{I} - e_{t}^{I} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} \max\left\{ \tilde{V}^{I}(N_{t+1}^{I}, \mathcal{S}_{t+1}) + \epsilon_{t+1}^{I}, 0 \right\} \right]$$
(45)

subject to:

$$(1 - \phi_0^I)N_t^I + e_t^I - \Psi^I(e_t^I) \ge q_t^m A_{t+1}^I - (q_t^f + \tau^\Pi r_t^f - \kappa)B_{t+1}^I,$$
(46)

$$N_{t+1}^{I} = \left[ \left( \tilde{M}_{t+1} + (1 - F_{\omega,t+1}(\omega_{t+1}^{*}))\delta q_{t+1}^{m} \right) A_{t+1}^{I} - B_{t+1}^{I} \right],$$
(47)

$$q_t^f B_{t+1}^I \ge -\xi q_t^m A_{t+1}^I, \tag{48}$$

$$A_{t+1}^I \ge 0,\tag{49}$$

$$\mathcal{S}_{t+1}^I = h(\mathcal{S}_t^I). \tag{50}$$

For the evolution of intermediary wealth in (47), we have defined the total after-tax payoff per unit of the bond

$$\tilde{M}_{t+1} = \left(1 - (1 - \theta)\tau^{\Pi}\right) \left(1 - F_{\omega,t+1}(\omega_{t+1}^*)\right) + M_{t+1},$$

where  $M_{t+1}$  is the recovery value per dollar of debt of bankrupt borrower firms, as defined in (11).

Since the idiosyncratic bank profit shocks are independent of the aggregate state of the economy, an individual bank's probability of continuing (i.e. not defaulting) conditional on the aggregate state, but before realization of the idiosyncratic shock is:

$$\operatorname{Prob}\left(\tilde{V}^{I}(N_{t+1}^{I},\mathcal{S}_{t+1}) + \epsilon_{t+1}^{I} > 0\right) = \operatorname{Prob}\left(\epsilon_{t+1}^{I} > -\tilde{V}^{I}(N_{t+1}^{I},\mathcal{S}_{t+1})\right) = 1 - F_{\epsilon}\left(-\tilde{V}^{I}(N_{t+1}^{I},\mathcal{S}_{t+1})\right).$$

By the law of large numbers,  $1 - F_{\epsilon} \left( -\tilde{V}^{I}(N_{t+1}^{I}, \mathcal{S}_{t+1}) \right)$  is also the aggregate survival rate of intermediaries, and  $F_{\epsilon} \left( -\tilde{V}^{I}(N_{t+1}^{I}, \mathcal{S}_{t+1}) \right)$  is the bank default rate. To ease exposition, we adopt the notation:

$$F_{\epsilon,t} = F_{\epsilon} \left( \tilde{V}^{I}(N_{t}^{I}, \mathcal{S}_{t}) \right).$$

Hence we can express the intermediary problem as:

$$\tilde{V}_{t}^{I}(N_{t}^{I}, \mathcal{S}_{t}) = \max_{e_{t}^{I}, B_{t+1}^{I}, A_{t+1}^{I}} \phi_{0}^{I} N_{t}^{I} - e_{t}^{I} + \mathcal{E}_{t} \left\{ \mathcal{M}_{t,t+1}^{B} \left[ (1 - F_{\epsilon,t}) \left( \tilde{V}^{I}(N_{t+1}^{I}, \mathcal{S}_{t+1}) + \epsilon_{t+1}^{I,+} \right) \right] \right\}.$$
(51)

The conditional expectation,  $\epsilon_t^{I,+} = \mathbf{E}_{\epsilon}(\epsilon | \epsilon > \tilde{V}^I(N_{t+1}^I, \mathcal{S}_{t+1}))$ , is the expected idiosyncratic profit conditional on not defaulting.

#### A.3.3 First-order conditions

**Lagrange Multipliers and Derivatives** Before taking first-order conditions, we attach Lagrange multipliers  $\nu_t^I$  to the budget constraint (46),  $\lambda_t^I$  to the leverage constraint (48), and  $\mu_t^I$  to the no-shorting constraint (49).

We compute the derivative of the expression inside the expectation term of (51) with respect to net worth  $N_{t+1}$ . To do so, we write the expression as:

$$(1 - F_{\epsilon,t})\left(\tilde{V}^{I}(N_{t}^{I}, \mathcal{S}_{t}) + \epsilon_{t}^{I,+}\right) = (1 - F_{\epsilon,t})\tilde{V}_{t}^{I}(N_{t}^{I}, \mathcal{S}_{t}^{I}) + \int_{-\tilde{V}_{t}^{I}(N_{t}^{I}, \mathcal{S}_{t}^{I})}^{\infty} \epsilon dF_{\epsilon}(\epsilon)$$

Differentiating with respect to  $N_t^I$  gives, by application of Leibniz' rule:

$$(1 - F_{\epsilon,t})\tilde{V}_{N,t}^{I} + \tilde{V}_{t}^{I}f_{\epsilon,t}\tilde{V}_{N,t}^{I} - \tilde{V}_{t}^{I}f_{\epsilon,t}\tilde{V}_{N,t}^{I} = (1 - F_{\epsilon,t})\tilde{V}_{N,t}^{I},$$

$$(52)$$

where  $f_{\epsilon,t}$  is the p.d.f. of  $\epsilon_t^I$  evaluated at  $-\tilde{V}^I(N_{t+1}^I, \mathcal{S}_{t+1})$ , and

$$\tilde{V}_{N,t}^{I} = \frac{\partial \tilde{V}^{I}(N_{t}^{I}, \mathcal{S}_{t})}{\partial N_{t}^{I}}.$$

Equity Issuance We can differentiate the objective function with respect to  $e_t^I$ :

$$\nu_t^I (1 - \phi_1^I e_t^I) = 1.$$

**Loans** The FOC for loans  $A_{t+1}^I$ , using equations (51) and (52), is:

$$q_t^m \nu_t^I = \xi q_t^m \lambda_t^I + \mu_t^I + \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^B \tilde{V}_{N,t+1}^I \left( 1 - F_{\epsilon,t+1} \right) \left( \tilde{M}_{t+1} + \left( 1 - F_{\omega,t+1}(\omega_{t+1}^*) \right) \delta q_{t+1}^m \right) \right].$$

**Deposits** The FOC for deposits, again using the result in (52), is:

$$(q_t^f + \tau^{\Pi} r_t^f - \kappa) \nu_t^I = q_t^f \lambda_t^I + \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^B \tilde{V}_{N,t+1}^I \left( 1 - F_{\epsilon,t+1} \right) \right].$$

#### A.3.4 Marginal value of wealth and SDF

First take the envelope condition:

$$\tilde{V}_{N,t}^{I} = \phi_0^{I} + (1 - \phi_0^{I})\nu_t^{I}$$

Combining this with the FOC for equity issuance above to eliminate  $\nu_t^I$  yields:

$$\tilde{V}_{N,t}^{I} = \phi_0^{I} + \frac{1 - \phi_0^{I}}{1 - \phi_1^{I} e_t^{I}}.$$
(53)

We can define a stochastic discount factor for intermediaries as

$$\mathcal{M}_{t,t+1}^{I} = \mathcal{M}_{t,t+1}^{B} \left( 1 - \phi_{1}^{I} e_{t}^{I} \right) \left( \phi_{0}^{I} + \frac{1 - \phi_{0}^{I}}{1 - \phi_{1}^{I} e_{t+1}^{I}} \right) \left( 1 - F_{\epsilon,t+1} \right).$$
(54)

#### A.3.5 Euler Equations

Using the definition of the SDF  $\mathcal{M}_{t,t+1}^{I}$  above, we can write the FOC for new loans and deposits more compactly as:

$$q_t^m = \xi \tilde{\lambda}_t^I q_t^m + \tilde{\mu}_t^I + \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^I \left( \tilde{M}_{t+1} + (1 - F_{\omega,t+1}(\omega_{t+1}^*)) \delta q_{t+1}^m \right) \right],$$
(55)

$$q_t^f + \tau^{\Pi} r_t^f - \kappa = \tilde{\lambda}_t^I q_t^f + \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^I \right].$$
(56)

where  $\tilde{\lambda}_t^I = \lambda_t^I / \nu_t^I$  and  $\tilde{\mu}_t^I = \mu_t^I / \nu_t^I$ .

## A.4 Savers

#### A.4.1 Statement of stationary problem

The problem of the representative saver is:

$$V^{S}(W_{t}^{S}, \mathcal{S}_{t}) = \max_{\{C_{t}^{S}, B_{t+1}^{S}, A_{t+1}^{S}\}} \left\{ (1 - \beta_{S}) \left[C_{t}^{S}\right]^{1 - 1/\nu} + \beta_{S} \mathbb{E}_{t} \left[ \left(V^{S}(W_{t+1}^{S}, \mathcal{S}_{t+1})\right)^{1 - \sigma_{S}} \right]^{\frac{1 - 1/\nu}{1 - \sigma_{S}}} \right\}^{\frac{1}{1 - 1/\nu}}$$

subject to

$$C_t^S = (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + W_t^S - q_t^f B_{t+1}^S - q_t^m A_{t+1}^S - \Psi^S(A_{t+1}^S),$$
(57)

$$W_t^S = \left[ (1 - F_{\omega,t}(\omega_t^*))(1 + \delta q_t^m) + M_t \right] A_t^S + B_t^S,$$
(58)

$$A_{t+1}^S \ge 0,\tag{59}$$

$$\mathcal{S}_{t+1} = h(\mathcal{S}_t). \tag{60}$$

Denote the value function and the marginal value of wealth as:

$$V_t^S \equiv V_t^S(W_t^S, \mathcal{S}_t),$$
$$V_{W,t}^S \equiv \frac{\partial V_t^S(W_t^S, \mathcal{S}_t)}{\partial W_t^S}.$$

Denote the certainty equivalent of future utility as:

$$CE_t^S = \mathcal{E}_t \left[ \left( V_{t+1}^S \right)^{1-\sigma_S} \right]^{\frac{1}{1-\sigma_S}}.$$

The monitoring cost function is

$$\Psi^{S}(A_{t+1}^{S}) = \frac{\varphi_{1}}{2} \left(\frac{A_{t+1}^{S}}{\varphi_{0}} - 1\right)^{2} \varphi_{0},$$

such that the marginal cost is:

$$(\Psi^S)'(A_{t+1}^S) = \varphi_1\left(\frac{A_{t+1}^S}{\varphi_0} - 1\right).$$

#### A.4.2 First-order conditions

**Short-term Bonds** The FOC for the short-term bond position  $B_{t+1}^S$  is:

$$q_t^f (C_t^S)^{-1/\nu} (1 - \beta_S) (V_t^S)^{1/\nu} = \beta_S \mathcal{E}_t [(V_{t+1}^S)^{-\sigma_S} V_{W,t+1}^S] (CE_t^S)^{\sigma_S - 1/\nu} (V_t^S)^{1/\nu}.$$
(61)

**Corporate Bonds** The FOC for the long-term corporate bond position  $A_{t+1}^S$  is:

$$(q_t^m + (\Psi^S)'(A_{t+1}^S))(C_t^S)^{-1/\nu}(1 - \beta_S)(V_t^S)^{1/\nu} = \mu_t^S + \beta_S \mathbb{E}_t[(V_{t+1}^S)^{-\sigma_S} V_{W,t+1}^S ((1 - F_{\omega,t}(\omega_t^*))(1 + \delta q_t^m) + M_t)](CE_t^S)^{\sigma_S - 1/\nu} (V_t^S)^{1/\nu}.$$
(62)

where  $\mu_t^S$  is the Lagrange multiplier on the no-shorting constraint (59).

### A.4.3 Marginal Values of State Variables and SDF

The marginal value of saver wealth is:

$$V_{W,t}^S = (C_t^S)^{-1/\nu} (1 - \beta_S) (V_t^S)^{1/\nu},$$
(63)

Defining the SDF in the same fashion as we did for borrowers, we get:

$$\mathcal{M}_{t,t+1}^S = \beta_S \left(\frac{V_{t+1}^S}{CE_t^S}\right)^{1/\nu_S - \sigma_S} \left(\frac{C_{t+1}^S}{C_t^S}\right)^{-1/\nu_S}.$$

#### A.4.4 Euler Equations

Combining the first-order conditions (61) and (62) with the marginal value of wealth, and the SDF, we get the Euler equations for the short-term bonds and corporate bonds, respectively:

$$q_t^f = \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^S \right] \tag{64}$$

$$q_t^m + (\Psi^S)'(A_{t+1}^S) = \tilde{\mu}_t^S + \mathcal{E}_t \left[ \mathcal{M}_{t,t+1}^S \left( M_{t+1} + (1 - F_{\omega,t+1}(\omega_{t+1}^*))(1 + \delta q_{t+1}^m) \right) \right]$$
(65)

where  $\tilde{\mu}_t^S = \mu_t^S / V_{W,t}^S$ .

# A.5 Equilibrium

The optimality conditions describing the problem are (27), (29), (38), (39), and (40) for borrowers, (46), (55), and (56) for intermediaries, and (57), (64), and (65) for savers. We add complementary slackness conditions for the constraints (A.2.2) for producer firms, (48) and (49) for intermediaries, and (59) for savers. Together with the market clearing conditions (22), (23), (24), and (25) these equations fully characterize the economy.

# **B** Computational Method

The equilibrium of dynamic stochastic general equilibrium models is usually characterized recursively. If a stationary Markov equilibrium exists, there is a minimal set of state variables that summarizes the economy at any given point in time. Equilibrium can then be characterized using two types of functions: transition functions map today's state into probability distributions of tomorrow's state, and policy functions determine agents' decisions and prices given the current state. Brumm, Kryczka, and Kubler (2018) analyze theoretical existence properties in this class of models and discuss the literature. Perturbation-based solution methods find local approximations to these functions around the "deterministic steady-state". For applications in finance, there are often two problems with local solution methods. First, portfolio restrictions such as leverage constraints may be occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will therefore inaccurately capture nonlinear dynamics when constraints go from slack to binding. Guerrieri and Iacoviello (2015) propose a solution using local methods. Secondly, the portfolio allocation of agents across assets with different risk profiles is generally indeterminate at the non-stochastic steady state. This means that it is generally impossible to solve for equilibrium dynamics using local methods since the point around which to perturb the system is not known.

Global projection methods (Judd (1998)) avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space. Additional advantages of global nonlinear methods are greater flexibility in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs.

# **B.1** Solution Procedure

The projection-based solution approach used in this paper has three main steps:

- Step 1. **Define approximating basis for the policy and transition functions.** To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our general solution framework provides an object-oriented MATLAB library that allows approximation of arbitrary multivariate functions using linear interpolation, splines, or polynomials. For the model in this paper, splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation.
- Step 2. Iteratively solve for the unknown functions. Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate.
- Step 3. Simulate the model for many periods using approximated functions. Verify that the simulated time path stays within the bounds of the state space for which policy and transition functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

We now provide a more detailed description for each step.

**Step 1** The state space consists of

- two exogenous state variables  $[Z_t^A, \sigma_{\omega,t}]$ , and
- five endogenous state variables  $[K_t, L_t^P, N_t^I, W_t^S, B_t^G]$ .

The state variable  $L_t^P$  is aggregate leverage of producers and defined as

$$L_t^P = \frac{q_t^m A_t^P}{p_t K_t}.$$
(66)

As usual, there are many different possible state variables that encode the same history of aggregate states. We choose this specific set of variables because policy functions turn out to be well-behaved when based on these variables.

We first discretize  $Z_t^A$  into a  $N^{Z_A}$ -state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points  $\{Z_j^A\}_{j=1}^{N^{Z_A}}$  and the  $N^{Z_A} \times N^{Z_A}$  Markov transition matrix  $\Pi_{Z^A}$  between them to match the volatility and persistence of HP-detrended GDP. The dispersion of idiosyncratic productivity shocks  $\sigma_{\omega,t}$  can take on two realizations  $\{\sigma_{\omega,L}, \sigma_{\omega,H}\}$  as described in the calibration section. The 2 x 2 Markov transition matrix between these states is given by  $\Pi_{\sigma_{\omega}}$ . We assume independence between both exogenous shocks. Denote the set of the  $N^x = 2N^{Z_A}$  values the exogenous state variables can take on as  $S_x = \{Z_j^A\}_{j=1}^{N^Z_A} \times \{\sigma_{\omega,L}, \sigma_{\omega,H}\}$ , and the associated Markov transition matrix  $\Pi_x = \Pi_{Z^A} \otimes \Pi_{\sigma_{\omega}}$ .

One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints and market clearing conditions, conditional on any four other state variables. We eliminate saver wealth  $W_t^S$ , which can be computed as

$$W_t^S = \Omega_A(\omega_t^*)(1 + \delta q_t^m)A_t^P + M_t - N_t^I + B_t^G.$$

Our solution algorithm requires approximation of continuous functions of the endogenous state variables. Define the "true" endogenous state space of the model as follows: if each endogenous state variable  $S_t \in \{K_t, A_t^P, N_t^I, B_t^G\}$  can take on values in a continuous and convex subset of the reals, characterized by constant state boundaries,  $[\bar{S}_l, \bar{S}_u]$ , then the endogenous state space  $S_n = [\bar{K}_l, \bar{K}_u] \times [\bar{L}_l^P, \bar{L}_u^P] \times [\bar{N}_l^I, \bar{N}_u^I] \times [\bar{B}_l^G, \bar{B}_u^G]$ . The total state space is the set  $S = S_x \times S_n$ .

To approximate any function  $f: S \to \mathcal{R}$ , we form an univariate grid of (not necessarily equidistant) strictly increasing points for each endogenous state variables, i.e., we choose  $\{K_j\}_{j=1}^{N_K}, \{L_k^P\}_{k=1}^{N_L}, \{N_m^I\}_{m=1}^{N_I}$ , and  $\{B_n^G\}_{n=1}^{N_G}$ . These grid points are chosen to ensure that each grid covers the ergodic distribution of the economy in its dimension, and to minimize computational errors, with more details on the choice provided below. Denote the set of all endogenous-state grid points as  $\hat{S}_n = \{K_j\}_{j=1}^{N_K} \times \{L_k^P\}_{k=1}^{N_P} \times \{N_m^I\}_{m=1}^{N_I} \times \{B_n^G\}_{n=1}^{N_G}$ , and the total discretized state space as  $\hat{S} = S_x \times \hat{S}_n$ . This discretized state space has  $N^S = N^x \cdot N^K \cdot N^P \cdot N^I \cdot N^G$  total points, where each point is a 5 x 1 vector as there are 5 distinct state variables. We can now approximate the smooth function f if we know its values  $\{f_j\}_{j=1}^{N_S}$  at each point  $\hat{s} \in \hat{S}$ , i.e.  $f_j = f(\hat{s}_j)$  by multivariate linear interpolation.

Our solution method requires approximation of three sets of functions defined on the domain of the state variables. The first set of unknown functions  $C_P : S \to P \subseteq \mathbb{R}^{N^C}$ , with  $N^C$  being the number of policy variables, determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents' choice variables, and the
Lagrange multipliers on the portfolio constraints. Specifically, the 15 policy functions are bond prices  $q^m(\mathcal{S})$ ,  $q^f(\mathcal{S})$ , investment  $X(\mathcal{S})$ , consumption  $c^B(\mathcal{S})$ ,  $c^S(\mathcal{S})$ , non-financial firm equity issuance  $e^P(\mathcal{S})$ , bank equity issuance  $e^I(\mathcal{S})$ , wages  $w^B(\mathcal{S})$ ,  $w^S(\mathcal{S})$ , the choice of loans and corporate bonds of banks and savers  $A^I(\mathcal{S})$  and  $A^S(\mathcal{S})$ , the Lagrange multipliers for the bank leverage constraint  $\lambda^I(\mathcal{S})$  and no-shorting constraint  $\mu^I(\mathcal{S})$ , the multiplier for firms' leverage constraint  $\lambda^P(\mathcal{S})$ , and finally the multiplier on the savers' no-shorting constraint  $\mu^S(\mathcal{S})$ . There is an equal number of these unknown functions and nonlinear functional equations, to be listed under step 2 below.

The second set of functions  $C_T : S \times S_x \to S_n$  determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous shocks. There is one transition function for each endogenous state variable, corresponding to the transition law for each state variable, also to be listed below in step 2.

The third set are forecasting functions  $C_F : S \to F \subseteq \mathbb{R}^{N^F}$ , where  $N^F$  is the number of forecasting variables. They map the state into the set of variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with the policy functions, but include additional functions. In particular, the forecasting functions for our model are the bond price  $q^m(S)$ , investment X(S), consumption  $c^B(S)$ ,  $c^S(S)$ , bank equity issuance  $e^I(S)$ , the value functions of households  $V^S(S)$ ,  $V^B(S)$ , and banks  $V^I(S)$ , and the wage bill  $w(S) = w^B(S) + w^S(S)$ .

**Step 2** Given an initial guess  $C^0 = \{C_P^0, C_T^0, C_F^0\}$ , the algorithm to compute the equilibrium takes the following steps.

- A. Initialize the algorithm by setting the current iterate  $\mathcal{C}^m = \{\mathcal{C}^m_P, \mathcal{C}^m_T, \mathcal{C}^m_F\} = \{\mathcal{C}^0_P, \mathcal{C}^0_T, \mathcal{C}^0_F\}.$
- B. Compute forecasting values. For each point in the discretized state space,  $s_j \in \hat{S}$ ,  $j = 1, \ldots, N^S$ , perform the steps:
  - i. Evaluate the transition functions at  $s_j$  combined with each possible realization of the exogenous shocks  $x_i \in S_x$  to get  $s'_j(x_i) = C_T^m(s_j, x_i)$  for  $i = 1, ..., N^x$ , which are the values of the endogenous state variables given the current state  $s_j$  and for each possible future realization of the exogenous state.
  - ii. Evaluate the forecasting functions at these future state variable realizations to get  $f_{i,j}^0 = C_F^m\left(s'_j(x_i), x_i\right)$ .

The end result is a  $N^x \times N^S$  matrix  $\mathscr{F}^m$ , with each entry being a vector

$$f_{i,j}^{m} = [q_{i,j}^{m}, C_{i,j}^{B}, C_{i,j}^{S}, e_{i,j}^{I}, V_{i,j}^{B}, V_{i,j}^{S}, V_{i,j}^{I}, X_{i,j}, w_{i,j}]$$
(F)

of the next-period realization of the forecasting functions for current state  $s_j$  and future exogenous state  $x_i$ .

C. Solve system of nonlinear equations. At each point in the discretized state space,  $s_j \in \hat{S}$ ,  $j = 1, \ldots, N^S$ , solve the system of nonlinear equations that characterize equilibrium in the equally many "policy" variables, given the forecasting matrix  $\mathscr{F}^m$  from step B. This amounts to solving a system of 15 equations in 15 unknowns

$$\hat{P}_{j} = [\hat{q}_{j}^{m}, \hat{q}_{j}^{f}, \hat{X}_{j}, \hat{c}_{j}^{B}, \hat{c}_{j}^{S}, \hat{e}_{j}^{P}, \hat{e}_{j}^{I}, \hat{A}_{j}^{I}, \hat{A}_{j}^{S}, \hat{w}_{j}^{B}, \hat{w}_{j}^{S}, \hat{\lambda}_{j}^{I}, \hat{\mu}_{j}^{I}, \hat{\lambda}_{j}^{P}, \hat{\mu}_{j}^{S}]$$
(P)

at each  $s_i$ . The equations are

$$\hat{q}_{j}^{m} = \hat{\lambda}_{j}^{P}F + \mathbf{E}_{s_{i,j}^{\prime}|s_{j}} \left\{ \hat{\mathcal{M}}_{i,j}^{P} \left[ \left(1 - F_{\omega,i}(\omega_{i,j}^{*})\right) \left(1 - (1 - \theta)\tau^{\Pi} + \delta q_{i,j}^{m}\right) + \frac{f_{\omega,i}(\omega_{i,j}^{*})\Pi\left(\omega_{i,j}^{*}\right)}{Z_{i}^{A}\hat{k}_{j}^{1-\alpha}\hat{l}_{j}^{\alpha}} \right] \right\}$$
(E1)

$$\hat{p}_{j}(1-\Phi\hat{\lambda}_{j}^{P}) = \mathbb{E}_{s_{i,j}^{\prime}|s_{j}} \left[ \hat{\mathcal{M}}_{i,j}^{P} \left\{ p_{i,j} \left( 1-F_{\omega,i}(\omega_{i,j}^{*}) \right) \left( (1-\tau^{\Pi}) \mathbf{M} \hat{\mathbf{P}} \mathbf{K}_{i,j} \omega_{i,j}^{+} + (1-\delta_{K}) p_{i,j} - (1-\tau^{\Pi}) \varsigma \right) \right\}$$

$$f_{\omega,i}(\omega_{i,j}^{*}) \Pi \left( \omega_{i,j}^{*} \right) \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}$$

$$-\frac{J\omega_{i}(\omega_{i,j})\Pi(\omega_{i,j})}{Z_{i}^{A}\hat{k}_{j}^{1-\alpha}\hat{l}_{j}^{\alpha}}\left(\varsigma - M\hat{P}K_{i,j}\omega_{i,j}^{*}\right)\right\}$$
(E2)

$$(1 - \tau_{\Pi}) \left( 1 - F_{\omega,i}(\hat{\omega}_j^*) \right) \left( \hat{\omega}_j^+ \hat{\mathrm{MPL}}_j^B - \hat{w}_j^B \right) = f_{\omega,j}(\hat{\omega}_j^*) \Pi \left( \hat{\omega}_j^*, \hat{\tilde{k}}_j, \hat{\tilde{a}}_j^P, \mathcal{S}_t \right) \frac{\partial \hat{\omega}_j^*}{\partial \hat{l}_j^B}$$
(E3)

$$(1 - \tau_{\Pi}) \left( 1 - F_{\omega,i}(\hat{\omega}_j^*) \right) \left( \hat{\omega}_j^+ \hat{\mathrm{MPL}}_j^S - \hat{w}_j^S \right) = f_{\omega,j}(\hat{\omega}_j^*) \Pi \left( \hat{\omega}_j^*, \hat{\tilde{k}}_j, \hat{\tilde{a}}_j^P, \mathcal{S}_t \right) \frac{\partial \hat{\omega}_j^*}{\partial \hat{\tilde{l}}_j^S}$$
(E4)

$$\hat{q}_j^f + \tau^\Pi \hat{r}_j^f - \kappa = \hat{q}_j^f \hat{\lambda}_j^I + \mathcal{E}_{s'_{i,j}|s_j} \left[ \hat{\mathcal{M}}_{i,j}^I \right]$$
(E5)

$$\hat{q}_{j}^{m} = \xi \hat{\lambda}_{j}^{I} \hat{q}_{j}^{m} + \hat{\mu}_{j}^{I} + \mathcal{E}_{s_{i,j}^{\prime}|s_{j}} \left[ \hat{\mathcal{M}}_{i,j}^{I} \left( M_{i,j} + \delta q_{i,j}^{m} (1 - F_{\omega,i}(\omega_{i,j}^{*})) \right) \right]$$
(E6)

$$\hat{q}_j^f = \hat{\mu}_j^S + \mathcal{E}_{s'_{i,j}|s_j} \left[ \hat{\mathcal{M}}_{i,j}^S \right]$$
(E7)

$$\hat{q}_{j}^{m} + (\Psi^{S})'(\hat{A}_{j}^{S}) = \hat{\mu}_{j}^{S} + \mathbb{E}_{s_{i,j}'|s_{j}} \left[ \hat{\mathcal{M}}_{i,j}^{S} \left( M_{i,j} + (1 - F_{\omega,i}(\omega_{i,j}^{*}))(1 + \delta q_{i,j}^{m}) \right) \right]$$
(E8)

$$\left(\Phi\hat{p}_{j}\tilde{k}_{j} - F\hat{\tilde{a}}_{j}^{P}\right)\hat{\lambda}_{j}^{P} = 0 \tag{E9}$$

$$\left(\xi\hat{q}_j^m\hat{A}_j^I - \hat{q}_j^f\hat{B}_j^I\right)\hat{\lambda}_j^I = 0 \tag{E10}$$

$$A_j^I \hat{\mu}_j^I = 0 \tag{E11}$$

$$A_j^S \hat{\mu}_j^S = 0 \tag{E12}$$

$$\ddot{B}_j^S = B_j^G + \ddot{B}_j^I \tag{E13}$$

$$\hat{A}_j^P = \hat{A}_j^S + \hat{A}_j^I \tag{E14}$$

$$\hat{c}_{j}^{B} = \hat{D}_{j}^{P} + \hat{D}_{j}^{I} + (1 - \tau_{t}^{B})\hat{w}_{j}^{B}\bar{L}^{B} + \hat{G}_{j}^{T,B} + \hat{p}_{j}\hat{X}_{j} - \hat{X}_{j} - \Psi(\hat{X}_{j}, K_{j}).$$
(E15)

(E1) and (E2) are the Euler equations for borrower-entrepreneurs from (39) and (40). (E3) and (E4) are the intratemporal optimality conditions for labor demand by borrower-entrepreneurs from (38). (E5) and (E6) are the Euler equations for banks from (56) and (55). (E7) and (E8) are the savers' Euler equations for short-term and corporate bonds, (64) and (65). (E9) and (E10) are the leverage constraints (8) and (48) for borrowers and banks, respectively. (E11) and (E12) are the no-shorting constraints (49) and (59) for banks and savers, respectively. (E13) and (E14) are the market clearing condition for riskfree debt and corporate bonds respectively, (22) and (23). Finally, E(15) is the borrower's budget constraint, (3).

Expectations are computed as weighted sums, with the weights being the probabilities of transitioning to exogenous state  $x_i$ , conditional on state  $s_j$ . Hats ( $\hat{\cdot}$ ) in (E1) – E(15) indicate variables that are direct functions of the vector of unknowns (P). These are effectively the choice variables for the nonlinear equation solver that finds the solution to the system (E1) – (E15) at each point  $s_j$ . All variables in the expectation terms with subscript  $_{i,j}$  are direct functions of the forecasting variables (F).

These values are *fixed* numbers when the system is solved, as they we pre-computed in step B. For example, the stochastic discount factors  $\hat{\mathcal{M}}_{i,j}^h$ , h = B, I, S, depend on both the solution and the forecasting vector, e.g. for savers

$$\hat{\mathcal{M}}_{i,j}^{S} = \beta_{S} \left( \frac{V_{i,j}^{S}}{CE_{j}^{S}} \right)^{1/\nu_{S} - \sigma_{S}} \left( \frac{c_{i,j}^{S}}{\hat{c}_{j}^{S}} \right)^{-1/\nu_{S}},$$

since they depend on future consumption and indirect utility, but also current consumption. To compute the expectation of the right-hand side of equation (E7) at point  $s_j$ , we first look up the corresponding column j in the matrix containing the forecasting values that we computed in step B,  $\mathscr{F}^m$ . This column contains the  $N^x$  vectors, one for each possible realization of the exogenous state, of the forecasting values defined in (F). From these vectors, we need saver consumption  $c_{i,j}^S$  and the saver value function  $V_{i,j}^S$ . Further, we need current consumption  $\hat{c}_j^S$ , which is a policy variable chosen by the nonlinear equation solver. Denoting the probability of moving from current exogenous state  $x_j$  to state  $x_i$  as  $\pi_{i,j}$ , we compute the certainty equivalent

$$CE_{j}^{S} = \left[\sum_{x_{i} \mid x_{j}} \pi_{i,j} (V_{i,j}^{S})^{1-\sigma_{S}}\right]^{\frac{1}{1-\sigma_{S}}},$$

and then complete expectation of the RHS of (E7)

$$\mathbf{E}_{s_{i,j}'|s_j}\left[\hat{\mathcal{M}}_{i,j}^S\right] = \sum_{x_i \mid x_j} \pi_{i,j} \beta_S \left(\frac{V_{i,j}^S}{CE_j^S}\right)^{1/\nu_S - \sigma_S} \left(\frac{c_{i,j}^S}{\hat{c}_j^S}\right)^{-1/\nu_S}$$

The mapping of solution and forecasting vectors (P) and (F) into the other expressions in equations (E1) – E(15) follows the same principles and is based on the definitions in model appendix A. For example, the borrower default threshold is a function of current wages and state variables based on (5)

$$\hat{\omega}_j^* = \frac{\hat{w}_j^B l^B + \hat{w}_j^S l^S + a_j^P + \varsigma k_j}{Z_i^A (k_j)^{1-\alpha} l^\alpha}$$

and the capital price is a linear function of investment from the first-order condition (29)

$$\hat{p}_j = 1 + \psi \left( \frac{\hat{X}_j}{k_j} - \delta_K \right).$$

The system (E1) - (E15) implicitly uses the budget constraints of non-financial and financial firms, savers and the government to compute several variables as direct function of the state and policy variables.

Note that we could exploit the linearity of the budget constraint in (E15) to eliminate one more policy variable,  $\hat{c}_j^B$ , from the system analytically. However, in our experience the algorithm is more robust when we explicitly include consumption of all agents as policy variables, and ensure that these variables stay strictly positive (as required with power utility) when solving the system. To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton's method, using policy functions  $C_P^m$  as initial guess. More on these issues in subsection B.2 below.

The final output of this step is a  $N^S \times 15$  matrix  $\mathscr{P}^{m+1}$ , where each row is the solution vector  $\hat{P}_i$  that solves the system (E1) – E(15) at point  $s_i$ .

D. Update forecasting, transition and policy functions. Given the policy matrix  $\mathscr{P}^{m+1}$  from step B, update the policy function directly to get  $\mathcal{C}_P^{m+1}$ . All forecasting functions with the exception of the value functions are also equivalent to policy functions. Value functions are updated based on the recursive definitions

$$\hat{V}_{j}^{S} = \left\{ (1 - \beta_{S}) \left[ \hat{c}_{j}^{S} \right]^{1 - 1/\nu} + \beta_{S} \mathbb{E}_{s_{i,j}' \mid s_{j}} \left[ \left( V_{i,j}^{S} \right)^{1 - \sigma_{S}} \right]^{\frac{1 - 1/\nu}{1 - \sigma_{S}}} \right\}^{\frac{1}{1 - 1/\nu}}$$
(V1)

$$\hat{V}_{j}^{B} = \left\{ (1 - \beta_{B}) \left[ \hat{c}_{j}^{B} \right]^{1 - 1/\nu} + \beta_{B} \mathbb{E}_{s_{i,j}^{\prime} | s_{j}} \left[ \left( V_{i,j}^{B} \right)^{1 - \sigma_{B}} \right]^{\frac{1 - 1/\nu}{1 - \sigma_{B}}} \right\}^{\frac{1}{1 - 1/\nu}}$$
(V2)

$$\hat{V}_{j}^{I} = \phi_{0}^{I} N_{j}^{I} - \hat{e}_{j}^{I} + \mathbf{E}_{s_{i,j}^{\prime}|s_{j}} \left[ \hat{\mathcal{M}}_{i,j}^{B} (1 - F_{\epsilon,i,j}) \left( V_{i,j}^{I} + \epsilon_{i,j}^{I,+} \right) \right],$$
(V3)

using the same notation as defined above under step C. Note that each value function combines current solutions from  $\mathscr{P}^{m+1}$  (step C) for consumption and equity issuance with forecasting values from  $\mathscr{F}^m$  (step B). Using these updated value functions, we get  $\hat{\mathcal{C}}_F^{m+1}$ .

Finally, update transition functions for the endogenous state variables using the following laws of motion, for current state  $s_i$  and future exogenous state  $x_i$  as defined above:

$$K_{i,j} = (1 - \delta_K)K_j + \hat{X}_j \tag{T1}$$

$$L_{i,j}^P = \frac{q_{i,j}A_j^P}{p_{i,j}K_j} \tag{T2}$$

$$N_{i,j}^{I} = \left(M_{i,j} + \delta q_{i,j}^{m} (1 - F_{\omega,i}(\omega_{i,j}^{*}))\right) \hat{A}_{j}^{I} - \hat{B}_{j}^{I}$$
(T3)

$$B_{i,j}^{G} = \frac{1}{\hat{q}_{j}^{f}} \left( B_{j}^{G} + \hat{G}_{j} - \hat{T}_{j} \right).$$
(T4)

(T1) is simply the law of motion for aggregate capital, and (T2) follows from the definition of producer leverage in (66). (T3) is the law of motion for bank net worth (47). (T2) and (T3) combine inputs from old forecasting functions  $\mathscr{F}^m$  and new policy solutions  $\mathscr{P}^{m+1}$ . (T4) is the government budget constraint (21). Updating according to (T1) – (T4) gives the next set of functions  $\hat{\mathcal{C}}_T^{m+1}$ .

E. Check convergence. Compute distance measures  $\Delta_F = ||\mathcal{C}_F^{m+1} - \mathcal{C}_F^m||$  and  $\Delta_T = ||\mathcal{C}_T^{m+1} - \mathcal{C}_TF^m||$ . If  $\Delta_F < \text{Tol}_F$  and  $\Delta_T < \text{Tol}_T$ , stop and use  $\mathcal{C}^{m+1}$  as approximate solution. Otherwise reset policy functions to the next iterate i.e.  $\mathscr{P}^m \to \mathscr{P}^{m+1}$  and reset forecasting and transition functions to a convex combination of their previous and updated values i.e.  $\mathcal{C}^m \to \mathcal{C}^{m+1} = D \times \mathcal{C}^m + (1-D) \times \hat{\mathcal{C}}^{m+1}$ , where D is a dampening parameter set to a value between 0 and 1 to reduce oscillation in function values in successive iterations. Next, go to step B.

**Step 3** Using the numerical solution  $C^* = C^{m+1}$  from step 2, we simulate the economy for  $\overline{T} = T_{ini} + T$  period. Since the exogenous shocks follow a discrete-time Markov chain with transition matrix  $\Pi_x$ , we can simulate the chain given any initial state  $x_0$  using  $\overline{T} - 1$  uniform random numbers based on standard techniques (we fix the seed of the random number generator to preserve comparability across

experiments). Using the simulated path  $\{x_t\}_{t=1}^{\bar{T}}$ , we can simulate the associated path of the endogenous state variables given initial state  $s_0 = [x_0, K_0, L_0^P, N_0^I, W_0^S, B_0^G]$  by evaluating the transition functions

$$[K_{t+1}, L_{t+1}^P, N_{t+1}^I, W_{t+1}^S, B_{t+1}^G] = \mathcal{C}_T^*(s_t, x_{t+1}),$$

to obtain a complete simulated path of model state variables  $\{s_t\}_{t=1}^{\bar{T}}$ . To remove any effect of the initial conditions, we discard the first  $T_{ini}$  points. We then also evaluate the policy and forecasting functions along the simulated sample path to obtain a complete sample path  $\{s_t, P_t, f_t\}_{t=1}^{\bar{T}}$ .

To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions, and restart the procedure at step 1. Secondly, we compute relative errors for all equations of the system (E1) – E(15) and the transition functions (T1) – (T4) along the simulated path. For equations involving expectations (such as (E1)), this requires evaluating the transition and forecasting function as in step 2B at the current state  $s_t$ . For each equation, we divide both sides by a sensibly chosen endogenous quantity to yield "relative" errors; e.g., for (E1) we compute

$$1 - \frac{1}{\hat{q}_j^m} \left( \hat{\lambda}_j^P F + \mathbf{E}_{s'_{i,j}|s_j} \left\{ \hat{\mathcal{M}}_{i,j}^P \left[ \left( 1 - F_{\omega,i}(\omega_{i,j}^*) \right) \left( 1 - (1 - \theta)\tau^{\Pi} + \delta q_{i,j}^m \right) + \frac{f_{\omega,i}(\omega_{i,j}^*) \Pi \left( \omega_{i,j}^* \right)}{Z_i^A \hat{k}_j^{1-\alpha} \hat{l}_j^{\alpha}} \right] \right\} \right),$$

using the same notation as in step 2B. These errors are small by construction when calculated at the points of the discretized state grid  $\hat{S}$ , since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points, at which the functions  $C^*$  are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

## **B.2** Implementation

Solving the system of equations. We solve system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB's fsolve). This nonlinear equation solver uses a variant of Newton's method to find a "zero" of the system. We employ several simple modifications of the system (E1) - E(15) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solver are notoriously bad at dealing with complementary slackness conditions associated with constraint, such as (E9) - E(12). Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose. For example, consider the bank's Euler Equation for risk-free bonds and the Kuhn-Tucker condition for its leverage constraint:

$$\hat{q}_{j}^{f}(1-\hat{\lambda}_{j}^{I})+\tau^{\Pi}\hat{r}_{j}^{f}-\kappa=\mathbf{E}_{s_{i,j}^{\prime}|s_{j}}\left[\hat{\mathcal{M}}_{i,j}^{I}\right]$$
$$\left(\xi\hat{q}_{j}^{m}\hat{A}_{j}^{I}-\hat{q}_{j}^{f}\hat{B}_{j}^{I}\right)\hat{\lambda}_{j}^{I}=0$$

Now define an auxiliary variable  $h_j \in \mathcal{R}$  and two functions of this variable, such that  $\hat{\lambda}_j^{I,+} = \max\{0, h_j\}^3$  and  $\hat{\lambda}_j^{I,-} = \max\{0, -h_j\}^3$ . Clearly, if  $h_j < 0$ , then  $\hat{\lambda}_j^{I,+} = 0$  and  $\hat{\lambda}_j^{I,-} > 0$ , and vice

versa for  $h_j > 0$ . Using these definitions, the two equations above can be transformed to:

$$\hat{q}_{j}^{f}(1-\hat{\lambda}_{j}^{I,+}) + \tau^{\Pi}\hat{r}_{j}^{f} - \kappa = \mathbf{E}_{s_{i,j}^{\prime}|s_{j}}\left[\hat{\mathcal{M}}_{i,j}^{I}\right]$$
(K1)

$$\xi \hat{q}_{j}^{m} \hat{A}_{j}^{I} - \hat{q}_{j}^{f} \hat{B}_{j}^{I} - \hat{\lambda}_{j}^{I,-} = 0 \tag{K2}$$

The solution variable for the nonlinear equation solver corresponding to the multiplier is  $h_j$ . The solver can choose positive  $h_j$  to make the constraint binding  $(\hat{\lambda}_j^{I,-} = 0)$ , in which case  $\hat{\lambda}_j^{I,+}$  takes on the value of the Lagrange multiplier. Or the solver can choose negative  $h_j$  to make the constraint non-binding  $(\hat{\lambda}_j^{I,+} = 0)$ , in which case  $\hat{\lambda}_j^{I,-}$  can take on any value that makes (K2) hold.

Similarly, certain solution variables are restricted to positive values due to the economic structure of the problem. For example, with power utility consumption must be positive. To avoid that the solver tries out negative consumption values (and thus utility becomes ill-defined), we use  $\log(\hat{c}_j^n)$ , n = B, S, as solution variable for the solver. This means the solver can make consumption arbitrarily small, but not negative.

The nonlinear equation solver needs to compute the Jacobian of the system at each step. Numerical central-difference (forward-difference) approximation of the Jacobian can be inaccurate and is computationally costly because it requires 2N+1 (N+1) evaluations of the system, with N being the number of variables, whereas analytically computed Jacobians are exact and require only one evaluation. We follow Elenev (2016) in "pre-computing" all forecasting functions in step 2B of the algorithm, so that we can calculate the Jacobian of the system analytically. To do so, we employ the Symbolic Math Toolbox in MATLAB, passing the analytic Jacobian to fsolve at the beginning of step 2C. This greatly speeds up calculations.

**Grid configuration.** Recall that one endogenous state variable can be eliminated because of the adding-up property of budget constraints in combination with market clearing. We choose to eliminate saver wealth  $W^S$ . The grid points in each state dimension are as follows

- $Z^A$ : We discretize  $Z_t^A$  into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points  $\{Z_j^A\}_{j=1}^5$  and the 5×5 Markov transition matrix  $\Pi_{Z^A}$  between them to match the volatility and persistence of HP-detrended GDP. This yields the possible realizations: [0.957, 0.978, 1.000, 1.022, 1.045].
- $\sigma_{\omega}$ : [0.1, 0.18] (see calibration)
- *K*: [1.60, 1.75, 1.84, 1.98, 2.05, 2.10, 2.26, 2.40]
- $L^P$ : [0.23, 0.30, 0.33, 0.35, 0.37, 0.39, 0.40, 0.41, 0.42, 0.43, 0.44, 0.46, 0.47, 0.48, 0.49, 0.5, 0.55]
- $N^I$ :

 $\begin{bmatrix} -0.040, -0.030, -0.020, -0.010, 0.000, 0.005, 0.010, 0.015, 0.020, 0.025, 0.030, 0.035, 0.040, \dots \\ \dots 0.045, 0.050, 0.055, 0.060, 0.065, 0.070, 0.080, 0.090, 0.100, 0.120, 0.140, 0.160, 0.200, 0.270 \end{bmatrix}$ 

•  $B^G$ : [0.183, 0.467, 0.750, 1.033, 1.317, 1.400, 1.700]

The total state space grid has 257,040 points. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding. Our policy functions are particularly nonlinear in bank net worth  $N^{I}$ , since the status of the bank leverage constraint (binding or not binding) depends predominantly on this state variable. To achieve acceptable accuracy, we have to specify a very dense grid for  $N^{I}$ , as can be seen above. Also note that the lower end of the  $N^{I}$  grid includes some negative values. Negative realizations of  $N^{I}$ can occur in severe financial crisis episodes. Recall that  $N^{I}$  is the beginning-of-period net worth of all banks. Depending on the realization of their idiosyncratic payout shock, banks decide whether or not to default. Thus the model contains two reasons why banks may not default despite initial negative net worth: (i) positive idiosyncratic shocks, and (ii) positive franchise value. The lower bound of  $N^{I}$ needs to be low enough such that bank net worth is not artificially truncated during crises, but it must not be so low that, given such low initial net worth, banks cannot be recapitalized to get back to positive net worth. Thus the "right" lower bound depends on the strength of the equity issuance cost and other parameters. Finding the right value for the lower bound is a matter of experimentation.

**Generating an initial guess and iteration scheme.** To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic "steady-state" of the model under the assumption that the bank leverage constraint is binding and government debt/GDP is 60%. We then initialize all functions to their steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, the steady state delivers a good enough initial guess.

In case the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these "failed" points using as initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually find solutions at all points.

To further speed up computation time, we run the initial 100 iterations with a coarser state space grid (19,500 points total). After these iterations, the algorithm is usually close to convergence; however, the accuracy during simulation would be too low. Therefore, we initialize the finer (final) solution grid using the policy, forecasting, and transition function obtained after 100 coarse grid iterations. We then run the algorithm for at most 30 more iterations on the fine grid.

To determine convergence, we check absolute errors in the value functions of households and banks, (V1) - V(3). Out of all functions we approximate during the solution procedure, these exhibit the slowest convergence. We stop the solution algorithm when the maximum absolute difference between two iterations, and for all three functions and all points in the state space, falls below 1e-3 and the mean distance falls below 1e-4. For appropriately chosen grid boundaries, the algorithm will converge within the final 30 iterations.

In some cases, our grid boundaries are wider than necessary, in the sense that the simulated economy never visits the areas near the boundary on its equilibrium path. Local convergence in those areas is usually very slow, but not relevant for the equilibrium path of the economy. If the algorithm has not achieved convergence after the 30 additional iterations on the fine grid, we nonetheless stop the procedure and simulate the economy. If the resulting simulation produces low relative errors (see step 3 of the solution procedure), we accept the solution. After the 130 iterations described above, our simulated model economies either achieve acceptable accuracy in relative errors, or if not, the cause is a badly configured state grid. In the latter case, we need to improve the grid and restart the solution procedure. Additional iterations, beyond 100 on the coarse and 30 on the fine grid, do not change any statistics of the simulated equilibrium path for any of the simulations we report.

We implement the algorithm in MATLAB and run the code on a high-performance computing

(HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 28 CPU cores of a single HPC node. From computing the initial guess and analytic Jacobian to simulating the solved model, the total running time for the benchmark calibration is about 1 hour and 30 minutes.

**Simulation.** To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 500 periods. The starting point of the simulation is the ergodic mean of the state variables. As described in detail above, we verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse response function (IRF) graphs, we simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution non-parametrically. We fix the initial exogenous shock realization to mean productivity  $(Z^A = 1)$  and low uncertainty  $(\sigma_{\omega,low})$ . The "impulse" in the second period is either only a bad productivity shock  $(Z^A = 0.978)$  for non-financial recessions, or both low  $Z^A$  and a high uncertainty shock  $(\sigma_{\omega,hi})$  for financial recessions. For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition.

## **B.3** Evaluating the solution

Equation errors. Our main measure to assess the accuracy of the solution are relative equation errors calculated as described in step 3 of the solution procedure. Table B.1 reports the median error, the 95<sup>th</sup> percentile of the error distribution, the 99<sup>th</sup>, and 100<sup>th</sup> percentiles during the 10,000 period simulation of the model. Median and 75th percentile errors are small for all equations. Maximum errors are on the order of 2% for equations (E5) – (E6). These errors are caused by a suboptimal approximation of the bank's Lagrange multiplier  $\lambda^{I}$  in rarely occurring states. It is possible to reduce these errors by placing more grid points in those areas of the state space. In our experience, adding points to eliminate the tail errors has little to no effect on any of the results we report. Since it increases computation times nonetheless, we chose the current grid configuration.

**Policy function plots.** We further visually inspect policy functions to gauge whether the approximated functions have the smoothness and monotonicity properties implied by our choices of utility and adjustment cost functions. Such plots also allow us to see the effect of binding constraints on prices and quantities. For example, figure B.1 shows investment by firms and the Lagrange multiplier on the bank's leverage constraint. It is obvious from the graphs that a binding intermediary constraint restricts investment. The intermediary constraint becomes binding for low values of intermediary net worth. Further note the interaction with borrower-entrepreneur net worth: holding fixed intermediary net worth, the constraint is more likely to become biding for low borrower wealth.

**State space histogram plots.** We also create histogram plots for the endogenous state variables, overlaid with the placement of grid points. These types of plots allow us to check that the simulated path of the economy does not violate the state grid boundaries. It further helps us to determine where to place grid points. Histogram plots for the benchmark economy are in figure B.2.

Eq	uation	Percentile							
		50th	75th	95th	99th	Max			
E1	(39)	0.0009	0.0012	0.0020	0.0068	0.0177			
E2	(40)	0.0003	0.0004	0.0006	0.0024	0.0062			
E3	(38), B	0.0002	0.0003	0.0003	0.0003	0.0003			
E4	(38), S	0.0002	0.0003	0.0003	0.0003	0.0003			
E5	(56)	0.0019	0.0038	0.0095	0.0125	0.0183			
E6	(55)	0.0015	0.0034	0.0095	0.0129	0.0190			
${ m E7}$	(64)	0.0001	0.0002	0.0004	0.0007	0.0014			
$\mathbf{E8}$	(65)	0.0002	0.0005	0.0015	0.0021	0.0032			
E9	(8)	0.0039	0.0046	0.0053	0.0057	0.0083			
E10	(48)	0.0003	0.0003	0.0004	0.0005	0.0018			
E11	(49)	0.0000	0.0000	0.0000	0.0000	0.0003			
E12	(59)	0.0004	0.0005	0.0006	0.0008	0.0010			
E13	(22)	0.0000	0.0000	0.0000	0.0001	0.0002			
E14	(23)	0.0000	0.0000	0.0000	0.0000	0.0000			
E15	( <b>3</b> )	0.0015	0.0024	0.0044	0.0098	0.0261			
T1		0.0000	0.0000	0.0000	0.0000	0.0000			
T2	(66)	0.0003	0.0004	0.0005	0.0009	0.0041			
T3	(47)	0.0004	0.0005	0.0008	0.0014	0.0113			
T4	(21)	0.0002	0.0003	0.0003	0.0005	0.0023			

Table B.1: Computational Errors

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the benchmark model. Each row contains errors for the respective equation of the nonlinear system (E1) – (E15) listed in step 2 of the solution procedure, and the transition equations for the state variables (T1) – (T4). The table's second column contains corresponding equation numbers in the main text and appendix A.



Figure B.1: Plot of optimal investment and Lagrange multiplier on bank leverage constraint

The left panel plots investment by borrower-entrepreneurs as function of borrower-entrepreneur wealth  $W^B$  and bank net worth  $N^I$ . The right panel plots the Lagrange multiplier on the bank leverage constraint for the same state variables. Both plots are for the benchmark economy. The other state variables are fixed to the following values:  $Z^A = 1$ ,  $\sigma_{\omega} = \bar{\sigma}_{\omega,L}$ ,  $K^B = 2.3$ ,  $B^G = 0.5$ .



Figure B.2: Histogram plots of endogenous state variables

The plots show histograms for capital and borrower-entrepreneur wealth in the top row, and intermediary net worth and and government debt in the bottom row, for the 10,000 period simulation of the benchmark economy. The vertical lines indicate the values of grid points.

# C Calibration Appendix

#### C.1 Bank Payouts and Leverage

**Dividend Payout and Equity Issuance** We construct the equity issuance and payout series using Compustat and CRSP data. Following Baron (2020), we define banks as firms engaging in "depository credit intermediation" (NAICS codes beginning with 5221). For the period before NAICS were introduced, we consider firms with SIC codes between 6020 and 6036. We discard all bank-quarters in which a bank's assets or equity grew by more than 20% to avoid attributing equity-financed M&A to repurchases or issuances. Baron (2020) uses a lower cutoff of 10%. The 20% cutoff is more robust for our calculation, since it is less likely to produce false positives, particularly in the immediate aftermath of the crisis when the largest banks grew rapidly.

Further following Baron (2020), we construct time series of dividends, share repurchases, and equity issuances as percent of book equity aggregating across all publicly traded banks. Because a bank may both issue and repurchase equity in the same year, we examine monthly changes in split- and stock-dividend-adjusted shares outstanding across all of a bank's share classes using CRSP data. Repurchases are negative changes in shares outstanding, multiplied by the end-of-month adjusted price. Issuances are positive changes. This procedure could still produce downwards-biased estimates of both if a company issues and repurchases shares within the same month, but this is less of a concern than at lower frequencies. We construct dividends as Compustat's dividends-per-share multiplied by shares outstanding, aggregate across all banks in a given year, and divide by aggregate bank book equity at the end of the previous year.

To ensure comprehensive coverage, we restrict the sample to 1974 through 2018. Over this period, banks paid out an average of 6.8% of their book equity per year as dividends plus share repurchases, and issued 4.8% as new equity. In the data, banks raise equity in part to finance trend balance sheet expansion, which averages 3.7% in real terms. Since our model is stationary, we target issuance net of asset growth. This yields a net payout ratio (dividends + repurchases - issuances + real asset growth rate), which is 5.8% on average and procyclical with a volatility of 6.0%. The gross payout ratio of 6.8% directly pins down  $\phi_0^I$ , and we target a net payout ratio if 5.8% by setting  $\phi_1^I = 7$ .

**Measuring Intermediary Sector Leverage** Our notion of the intermediary sector is the levered financial sector. We take book values of assets and liabilities of these sectors from the Financial Accounts of the United States (formerly Flow of Funds). We subtract holding and funding company equity investments in subsidiaries from those subsidiaries' liabilities. Table C.1 reports the assets, liabilities, and leverage of each sector as of 2014, as well as the average leverage from 1953 to 2014. We find that the average leverage ratio of the levered financial sector was 91.5%. This is our calibration target.

Krishnamurthy and Vissing-Jorgensen (2015) identify a similar group of financial institutions as net suppliers of safe, liquid assets. Their financial sector includes money market mutual funds (who do not perform maturity transformation) and equity REITS (who operate physical assets) but excludes life insurance companies (which are highly levered). The financial sector definition of Krishnamurthy and Vissing-Jorgensen (2015) suggests a similar ratio of 90.9%. As an aside, we note that Krishnamurthy and Vissing-Jorgensen (2015) report lower total assets and liabilities than in our reconstruction of their procedure because they net out positions within the financial sector by instrument while we do not.

			Dec 2014		Avg 53-14
Table	Sector	Assets	Liabilities	Leverage	Leverage
L.111	U.SChartered Depository Institutions	\$ 13,647	\$ 12,161	0.891	0.921
L.112	Foreign Banking Offices in U.S.	2,093	2,086	0.996	1.065
L.113	Banks in U.SAffiliated Areas	\$ 92	\$ 88	0.953	1.080
L.114	Credit Unions	1,066	958	0.899	0.916
	Subtotal: Banks	\$ 16,898	\$ 15,292	0.905	0.928
L.125	Government-Sponsored Enterprises (GSEs)	\$ 6,400	6,387	0.998	0.971
L.126	Agency- and GSE-Backed Mortgage Pools	1,649	1,649	1.000	1.000
L.127	Issuers of Asset-Backed Securities (ABS)	\$ 1,424	1,424	1.000	1.003
L.129.m	Mortgage Real Estate Investment Trusts	568	\$ 483	0.851	0.955
L.128	Finance Companies	1,501	1,376	0.916	0.873
L.130	Security Brokers and Dealers	3,255	1,345	0.413	0.808
L.131	Holding Companies	4,391	2,103	0.479	0.441
L.132	Funding Corporations	1,305	1,305	1.000	1.000
	Subtotal: Other Liquidity Providers	\$ 20,492	\$ 16,070	0.784	0.872
L.116	Life Insurance Companies	\$ 6,520	\$ 5,817	0.892	0.932
	Total	\$ 43,910	\$ 37,179	0.847	0.915
L.121	Money-Market Mutual Funds	\$ 2,725	\$ 2,725	1.000	1.000
L.129.e	Equity Real Estate Investment Trusts	157	539	3.427	2.577
	Total (K-VJ Definition)	\$ 40,271	33,549	0.833	0.909

## Table C.1: Balance Sheet Variables and Prices

## C.2 Non-financial Corporate Payouts and Leverage

**Dividend Payout and Equity Issuance** We compute a time series of aggregate payouts by non-financial firms in a similar way as for banks. Our goal is to calculate aggregate annual payouts – dividends and share repurchases – by public non-financial firms as a fraction of the previous year's aggregate book equity. As is standard in the literature, we use all Compustat firms except regulated utilities (SIC codes 4000-4999) and financial services firms (SIC codes 6000-6999).

Like we do for banks, we use the 1974-2018 sample and discard firm-quarters in which equity grew by more than 20% to exclude equity-financed M&A. Share repurchases are monthly declines in shares outstanding. On average, from 1974 to 2018 the non-financial sector paid out 7.8% of its equity value in dividends and repurchases and issued 3.1% of new equity. Firms issue equity in part to finance growth. Unlike for banks, non-financial firms' assets/GDP remain stable, however the data still features real growth. Because our model is stationary, we calculate issuance as net of real GDP growth, which averages 1.7% in our sample. This yields a net payout ratio = repurchases + dividends - issuance + real GDP growth. Its mean is 6.4%, and it is pro-cyclical with a volatility of 3.1%. The gross payout ratio of 7.8% directly pins down  $\phi_0$ , and we target a net payout ratio if 6.4% by setting  $\beta_B = 0.94$ .

**Measuring Non-financial Leverage** We define the non-financial sector as the aggregate of the nonfinancial corporate and nonfinancial noncorporate business sectors in the Financial Accounts of the United States (formerly Flow of Funds). We construct leverage as the ratio of loans plus debt securities to non-financial assets using the following FRED identifiers:

$$Leverage = \frac{TCMILBSNNCB + NNBDILNECL + OLALBSNNB + NNBTML}{TTAABSNNCB + TTAABSNNB}$$

Non-financial leverage steadily increases until the mid-1980s. To ensure we're calculating the average of a stationary series, we start our sample in 1985. From then until 2015, leverage is acyclical with a mean of 36.9% and a volatility of 3.4%.

### C.3 Measuring the Household Share

In our model, non-financial firm debt can be held either by the levered financial sector or by households directly. The empirical counterpart to non-financial firm debt are domestic corporate bonds and loans. The counterpart to household holdings includes debt both held by households as well as debt held by non-levered (e.g. pass-through) intermediaries. To construct this series, we turn to the Financial Accounts of the United States (formerly Flow of Funds).

There are two empirical challenges. First, foreign investors, who don't have a counterpart in our model, own an appreciable share of corporate bonds. Second, reported household holdings are of "corporate and foreign bonds" include asset-backed securities, bonds issued by financial firms, and bonds issued by foreign entities. To deal with these challenges, we reconstruct aggregates from Table L.213 as follows, using market values (codes beginning with LM) over book values (codes beginning with FL) where available. An example of this calculation for December 2014 is presented in Table C.2.

We begin with total corporate and foreign bonds in the economy, issued by either non-financial firms, the levered financial sector (including ABS), or rest of the world. When these assets are held by a financial firm, the FoF breaks down the holding into ABS and other, allowing us to exclude these holdings from the calculation. We then use the liability portion of the table to compute the fraction of remaining corporate and foreign bonds that were issued by the non-financial sector and *assume* that all investors hold non-financial sector issued corporate bonds in the same proportion as the total.

This allows us to compute holdings of non-financial sector issued corporate bonds by each of the FoF sectors. We classify these sectors into (1) rest of the world, (2) "levered financial sector" consistent with our computation of financial sector leverage in Section C.1, and (3) a broadly defined "household sector," which includes FoF Household Sector, as well as pension funds (private, federal, state & local) and pass-through investment funds (MMFs, mutual funds, closed-end funds, and ETFs).

Finally, we compute the household share of non-financial firm debt as the amount of non-financial sector corporate bonds held by the broadly defined household sector divided by the sum of total non-financial sector corporate bonds held by (1) households and (2) levered financial sector, as well as (3) loans taken out by nonfinancial corporate and noncorporate business. According to Table L.216, there are small amounts – e.g. 24.3 billion in Dec 2014 – of loans to non-financial firms held by the household sector. We do not include these in the numerator of our saver share because it is difficult for households to own whole loans, so these are likely held by some financial intermediaries e.g. hedge funds, that the FoF treats as part of the household sector.

### C.4 Parameter Sensitivity Analysis

In a complex, non-linear structural general equilibrium model like ours, it is often difficult to see precisely which features of the data drive the ultimate results. This appendix follows the approach advocated by Andrews, Gentzkow, and Shapiro (2017) to report how key moments are affected by changes in the model's key parameters, in the hope of improving the transparency of the results. Structural identification of parameters and sensitivity of results are two sides of the same coin.

Consider a generic vector of moments **m** which depends on a generic parameter vector  $\theta$ . Let  $\iota_i$  be a selector vector of the same length as  $\theta$  taking a value of 1 in the *i*'th position and zero elsewhere. Denote the parameter choices in the benchmark calibration by a superscript *b*. For each parameter  $\theta_i$ , we solve the model once for  $\theta^b \circ e^{\iota_i \varepsilon}$  and once for  $\theta^b \circ e^{-\iota_i \varepsilon}$ . We then report the symmetric finite difference:

$$\frac{\log\left(\mathbf{m}(\theta^{\mathbf{b}} \circ \mathbf{e}^{\iota_{\mathbf{i}}\varepsilon})\right) - \log\left(\mathbf{m}(\theta^{\mathbf{b}} \circ \mathbf{e}^{\iota_{\mathbf{i}}\varepsilon})\right)}{2\varepsilon}$$

We set  $\varepsilon = .01$ , or 1% of the benchmark parameter value. The resulting quantities are elasticities of moments to structural parameters.

We report the sensitivity of the first 16 target moments in Table 2 to the 16 parameters in the first 16 rows of that table. Those are all parameters calibrated inside the model except for fiscal policy parameters which are identified almost directly by their corresponding fiscal policy target moment. Each panel of Figure C.1 lists the same 16 moments and shows the elasticity of the moments to one of the 16 parameters. For consistency, we report percentage changes, which are unit-free, in every moment. The movements in the bank bankruptcy rate in response to multiple parameters appear to be large, but they are only large relative to a small baseline bank bankruptcy rate of 0.63% per year.

Some parameters are identified mainly by their target moment. When more resources seized from a defaulting producer are lost to the lender (higher  $\zeta^P$ ), loss severities on corporate loans increase. Raising the savers' target holdings of corporate bonds  $\varphi_0$  increases their share held by savers, while making it costlier for savers to deviate from that target (higher  $\varphi_1$ ) reduces the share's volatility.

Others affect the results in more complex ways. Many parameters that make the economy riskier decrease bank net payouts, because banks choose to retain earnings out of precaution. These include higher capital adjustment costs  $\psi$ , which make Tobin's q more volatile and higher persistence  $\rho_A$  and volatility  $\sigma_A$  of TFP shocks. This effect can be seen best in the  $\phi_1^I$  panel. Making it costlier for banks to raise equity mechanically decreases issuance and hence increases net payouts in crises. However,

		Example: Decem	ber 2014
FoF Data Code	Description	Value from FoF	Scaled
FL893163005	Total Corporate and Foreign Bonds	12,097,540	
	Less Identifiable Holdings of ABS and MBS	100.000	
LM763063605	U.Schartered depository institutions	138,820	
LM473063605	Credit Unions	10,952	
LM513063605	Property-Casualty Insurance Companies	98,840	
LM543063675	Life Insurance Companies	465,741	
FL403063605	Government-Sponsored Enterprises	79,819	
LM263063603	Rest of the World	$405,\!130$	
	Subtotal	1,199,302	
	Remaining Corporate and Foreign Bonds	10,898,238	
FL103163003	Bonds Issued by the Non-Fin Corp Sector	4,554,756	
	Fraction Issued by Non-Fin Corp Sector		41.8%
	Broadly Defined Household Sector		
LM153063005	Households	1,627,149	680,043
LM573063005	Private Pension Funds	698,600	291,969
LM343063005	Federal Pension Funds	12,086	5,051
LM223063045	State & Local Pension Funds	579,039	242,001
LM653063005	Mutual Funds	1,823,312	762,026
LM553063003	Closed-End Funds	75,783	$31,\!672$
LM563063003	Exchange-Traded Funds	231,911	96,924
	Subtotal	,	$2,\!109,\!686$
	Levered Financial Sector		
LM763063095	U.Schartered depository institutions	386.950	161,720
LM753063005	Foreign banking offices in U.S.	190,787	79,737
LM743063005	Banks in U.Saffiliated areas	5,367	2,243
LM473063095	Credit unions	194	81
LM513063095	Property-casualty insurance companies	358.528	149,841
LM543063095	Life insurance companies	1.966.899	822,036
FL633063005	Money Market Funds	68,588	28,665
FL403063095	Government-sponsored enterprises	5,970	2,495
LM613063003	Finance companies	68.839	28,770
FL643063005	REITs	38,922	16,267
FL663063005	Brokers and dealers	114.534	47.868
LM733063003	Holding companies	29.179	12.195
FL503063005	Funding companies	77.891	32.553
1 2000000000	Subtotal	,	1,384,472
FL144123005	Loans Issued by Nonfinancial Business		6,940,354
	Household Share		20.22%

Table	C.2:	Measu	iring	the	Hous	sehold	Share	of N	Non-H	Finan	icial	Firm	Debt
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banks anticipate this and retain more earnings ex-ante, which both decreases net payouts and increases their net worth. Both effects serve to lower the ratio.

Banks also decrease their payouts when the volatility of idiosyncratic bank shocks  $\sigma_{\epsilon}$  decreases because in part because the decrease makes the economy riskier – corporate defaults and volatility of investment are higher. As the financial sector becomes more efficient at sharing risk within itself, it take on more systemic risk in the aggregate, making it more vulnerable in the aggregate.

Lastly, an important driver of intermediation in our model is the difference in discount rates between borrowers (high) and savers (low). When this difference increases e.g. when the discount rate of borrowers  $-\log \beta_B$  goes up, so does firm leverage, making it substantially riskier with more corporate defaults and more volatile investment. Inversely, when the difference decreases e.g. when the discount rate of savers  $-\log \beta_S$  moves closer to that of the borrowers, there is less debt and lower risk. In Appendix D.5, we check if our results are robust to changes in  $\beta_B$  and  $\beta_S$  that would decrease the intermediation motive. They are.

#### C.5 Long-term Corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time t + 1,  $\delta$  at time t + 2,  $\delta^2$  at time t + 3, and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by  $F = \frac{\theta}{1-\delta}$ , a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for  $\delta$  and  $\theta$ .

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices (Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively) we obtain a time series of monthly market values, durations (the sensitivity of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices (C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively), we obtain a time series of option-adjusted spreads over the Treasury yield curve.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC c of 5.5% and WAM T of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate corresponding to that period's WAM to get a time series of bond yields  $r_t$ . Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^{c}(r_{t}) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_{t})^{i/2}} + \frac{1}{(1+r_{t})^{T}}$$

We can write the steady-state price of a geometric bond with parameter  $\delta$  as

$$P^G(r_t) = \frac{1}{1+r_t} \left[ 1 + \delta P^G(r_t) \right]$$

Solving for  $P^G(y_t)$ , we get

$$P^G(r_t) = \frac{1}{1 + r_t - \delta}$$

The calibration determines how many units X of the geometric bond with parameter  $\delta$  one needs to sell to hedge one unit of plain vanilla bond  $P^c$  against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta, X} \sum_{t=1997.1}^{2015.12} \left[ P^c(r_t) - X P^G(r_t; \delta) \right]^2$$

We estimate  $\delta = 0.937$  and X = 12.9, yielding an average pricing error of only 0.41%. This value for  $\delta$  implies a time series of durations  $D_t = -\frac{1}{P_t^G} \frac{dP_t^G}{dr_t}$  with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it  $D_t$  years from now. The principal of this loan is just the price of the corresponding  $D_t$  maturity zero-coupon bond  $\frac{1}{(1+r_t)^{D_t}}$ 

We set the "principal" F of one unit of the geometric bond to be some fraction  $\theta$  of the undiscounted sum of all its cash flows  $\frac{\theta}{1-\delta}$ , where

$$\theta = \frac{1}{N} \sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get  $\theta = 0.582$  and F = 9.18.

## C.6 Measuring Labor Income Tax Revenue

We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures to the rest of the world rather than being consumed domestically. Since the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption back to domestic agents.

### C.7 Taxation of Savers' Financial Income

Savers earn financial income from two sources. First, they earn interest on their private lending i.e. deposits in the financial intermediaries. This income is ultimately a claim on the capital rents in the economy and should be taxed at the same rate  $\tau_K$  as borrowers' and intermediaries' net income.

Second, they earn interest on their public lending i.e. government bonds. In the data, Treasury coupons are taxed at the household's marginal tax rate,  $\tau$  in the model. However, the tax revenue

collected by the government from interest income on its own bonds is substantially lower than  $\tau B_t^G$  because (a) Treasury coupons are exempt from state and local taxes, and (b) more than half of privately owned Treasury debt is held by foreigners, who also do not pay federal income taxes.

In the model, there is one tax rate  $\tau_D$  at which all of the saver's interest income is taxed. We choose  $\tau_D$  to satisfy

$$\tau_D(\hat{B}^I + \hat{B}^G) = \tau^K(\hat{B}^I - \hat{B}^I_{\text{pension}}) + \tau \frac{\hat{\tau}^{\text{federal}}}{\hat{\tau}^{\text{total}}}(\hat{B}^G - \hat{B}^G_{\text{foreign}} - \hat{B}^I_{\text{pension}})$$

where hats denote quantities in the data. Specifically, the revenue from taxes collected at rate  $\tau_D$  on all private safe debt and government debt must equal the sum of tax revenues collected on taxable private safe debt (private safe debt not held in tax-advantaged pension funds) at rate  $\tau_K$ , and tax revenues collected on taxable public debt (Treasury debt not held by foreigners, the Fed, or pension funds) taxed at rate  $\tau \frac{\hat{\tau}^{\text{federal}}}{\hat{\tau}^{\text{total}}}$ .

We measure all quantities at December 31, 2014. Private debt stocks are taken from the Financial Accounts of the United States. Treasury debt stocks are taken from the Treasury Bulletin. Federal and total personal tax revenues are taken from the BEA's National Income and Product Accounts. There is approximately \$13 trillion each outstanding of private and public debt. Almost all private debt is taxable, but only \$4 trillion of public debt is. Federal taxes constitute approximately 80% of all personal income tax revenue. Using the calibration for  $\tau_K$  and  $\tau$ , we get

$$\tau_D \approx \frac{20\% \times \$13T + 29.5\% \times 0.8 \times \$4T}{\$13T + \$13T}$$

or  $\tau_D = 13.4\%$  precisely.

#### C.8 Stationarity of Government Debt

In our numerical work, we guarantee the stationarity of government debt by gradually lowering the tax rate when debt falls (profligacy) and raising it when debt rises (austerity). The labor income tax rate  $\tau_t^B = \tau_t^S = \tau_t$  is the target rate  $\tau$  multiplied by the term  $\exp b_\tau z_t^A$  in order to capture the cyclicality of labor tax revenue. To enforce stationarity, we set the target tax rate  $\tau$  as follows:

$$\tau = \tau_0 \left( \frac{B_t^G - \underline{B}^G}{B_{ss}^G - \underline{B}^G} \right)^{b_G}$$

We set  $\underline{B}^G = -0.4$  to allow debt (as a fraction of steady state output normalized to 1) to be in the [-0.4, 0] region without getting a complex number for the tax rate. In the final results, this is not necessary since  $B_t^G$  never drops below 0.5. But having a well-defined tax rate in negative  $B_t^G$  regions helps in the early computation stages. So,  $\underline{B}^G$  is a purely technical parameter. Given its value, we then set  $b^G$  to target the volatility of government debt to GDP. In the data (53-14), it is 10.96%. In the model, it is 12.41%.



#### Figure C.1: Parameter Sensitivity Analysis

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Electronic copy available at: https://ssrn.com/abstract=2748230

# **D** Results Appendix

## D.1 Role of Frictions

This section describes the four simplified models we solve as special cases of our benchmark model to understand the role of different frictions. As discussed in Section 4.3, Models (1) and (2) remove the banking sector. In these models, savers hold all corporate debt directly, and banks have zeros assets and liabilities. Models (3) and (4) are economies with an intermediation sector, but different assumptions regarding bank default and equity issuance costs.

Models (2), (3), and (4) are parametric special cases of our benchmark model. The first parameter that is different in these three models is the cost of equity issuance for banks,  $\phi_1^I$ , which is 7 in the benchmark, but we set  $\phi_1^I = 0$  in models (2) – (4). Thus, banks can freely raise new equity from borrower-entrepreneurs in these economies. The second change in models (2) and (3) is that banks do not have the option to strategically default. This is implemented through an extended version of the bank problem, in which the managers of the bank incur a default penalty  $\rho$  if the bank defaults, such that bank optimization problem in (13) becomes

$$V^{I}(n_{t}^{I}, \epsilon_{t}^{I}, \mathcal{S}_{t}) = \max_{a_{t+1}^{I}, b_{t+1}^{I}, e_{t}^{I}} \phi_{0}^{I} n_{t}^{I} - e_{t}^{I} + \epsilon_{t}^{I} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} \max\{V^{I}(n_{t+1}^{I}, \epsilon_{t+1}^{I}, \mathcal{S}_{t+1}), -\rho\} \right],$$
(13)

subject to the same set of constraints as before. The benchmark is simply a special case with  $\rho = 0$ . In models (2) and (3), we set  $\rho = 100$ , such that banks default with probability zero for any aggregate state realization (any large enough number will yield the same result).

Model (2) further eliminates the saver adjustment cost for holding corporate bonds by setting  $\varphi_1 = 0$ , as opposed to 0.14 in the benchmark. Absent any cost, the large difference in discount factors between borrower-entrepreneurs and savers guarantees that savers hold all corporate debt in equilibrium for any realization of the aggregate state, and banks effectively vanish.

Model (1) has the same parameters as Models (2) – (4), i.e.  $\phi_1^I = \varphi_1 = 0$  and  $\rho = 100$ . Further, it assumes that non-financial firms cannot default and are not subject to the fixed cost of production  $\varsigma k_t$  by setting  $\varsigma = 0$ . Abstracting from corporate default greatly simplifies the firm problem. The value function in (6) becomes

$$V^{P}(\hat{n}_{t}^{P}, \mathcal{S}_{t}) = \max_{e_{t}^{P}, k_{t+1}, a_{t+1}^{P}} \phi_{0}^{P} n_{t}^{p} - e_{t}^{P} + \mathcal{E}_{t} \left[ \mathcal{M}_{t,t+1}^{B} V^{P}(\hat{n}_{t+1}^{P}, \mathcal{S}_{t+1}) \right]$$
(6')

subject to the same budget and leverage constraints, (7) and (8), and the transition law for net worth:

$$\hat{n}_{t+1}^{P} = (1 - \tau^{\Pi}) \left( \omega_{t+1} Z_{t+1} k_{t+1}^{1-\alpha} \hat{l}_{t+1}^{\alpha} - \sum_{j} w_{t+1}^{j} \hat{l}_{t+1}^{j} - a_{t+1}^{P} \right) + (1 - (1 - \tau^{\Pi}) \delta_{K}) p_{t+1} k_{t+1} - \delta q_{t+1}^{m} a_{t+1}^{P}.$$
(67)

Labor demand  $\hat{l}_{t+1}^{j}$ , for j = B, S in (67) reflects the firm's optimal choices at the beginning of t+1, solving the problem

$$\hat{l}_t^j = l^j(k_t, a_t^P, \mathcal{S}_t) = \operatorname*{argmax}_{l_t^j} \operatorname{E}_{\omega} \left[ V^P(n_t^P, \mathcal{S}_t) \right],$$
(68)

with

$$n_t^P = (1 - \tau^{\Pi}) \left( \omega_t Z_t k_t^{1 - \alpha} l_t^{\alpha} - \sum_j w_t^j l_t^j - a_t^P \right) + (1 - (1 - \tau^{\Pi}) \delta_K) p_t k_t - \delta q_t^m a_t^P.$$

as in (33). As in the benchmark model, the value function  $V^P(n_t^P, \mathcal{S}_t)$  is linearly homogeneous in net worth  $n_t^P$ . Thus, in Model (1) only the mean of the idiosyncratic shocks matters, which is  $E_{\omega,t}[\omega_t] = 1$ . Equivalently, the  $\omega$ -shocks play no role in Model (1).

Table D.1 reports the most important moments for Model (1) - (4) and compares them to the benchmark model. Model (1) features a greater capital stock, higher GDP and consumption, and more debt than the more frictional Models (2) - (5). Since firms do not need to fear suboptimal liquidity default in future periods, they choose higher leverage and their constraint binds in 71.0% of periods. Savers hold 100% of corporate debt, and the "credit spread" between corporate bonds and government debt is negative at -0.37%. Because there is no credit risk, it arises from a negative term spread. Like many other models without nominal frictions, long-term bonds in our model hedge long-run consumption risk, thus commanding a negative premium. Because the economy in Model (1) is larger and not subject to disruptive financial recessions, average consumption is higher and consumption growth volatility is lower for both households.

Model (2) adds corporate default risk as in the benchmark model. As a result, the corporate default rate is 2.4% on average, and firms slightly reduce leverage to avoid future states in which they are forced to default. The unconditional credit spread turns positive, overcoming the negative term spread, but still is only half as large as in the benchmark model and the data. Because debt is now defaultable, its price is lower and the market value of debt and market leverage falls. While the economy in Model (2) is still larger than in the benchmark in terms of capital and GDP, average consumption is only 0.43% higher than in the benchmark. This is because Model (2) features sharp financial recessions, during which investment plummets and many firms default. Model (2) demonstrates that corporate credit risk, independent of intermediation frictions, contributes substantially to our model's ability to generate large financial recessions. During times of high credit risk, triggered by high  $\sigma_{\omega}$ , the price of credit for firms rises sharply, forcing them to cut back on borrowing and investment.

Model (3) activates the intermediation sector by turning on saver's adjustment cost for holding corporate debt. In response to this change, the average fraction of corporate debt held by savers shrinks from 100% to 13.2%. The term spread remains negative, but the credit spread rises from 0.95% to 1.82%, despite a reduction in corporate debt and defaults. The rise in the credit spread is due to an excess return of 0.69% that banks earn on corporate debt. Since in equilibrium both banks and savers are marginal in the market for corporate debt, this excess return also reflects the marginal holding cost for savers. The excess return earned by banks is compensation for their levered exposure to credit risk: in Model (3), banks are at a binding capital requirement 100.0%. Thus, they hold risky assets and fund 93.0% of these holdings with riskfree debt, implying large losses in bank equity during times of high corporate defaults. Since debt is more expensive for firms in Model (3) relative to Model (2), firms cut back on leverage, and financial recessions are slightly less severe than in Model (2). Model (3) showcases that simply adding intermediaries to the model does not necessarily amplify financial crises: since intermediaries fully bear the risk they take on and can costlessly raise equity in crises, their presence does not make crises more severe. Rather, by making debt finance less attractive for non-financial firms, they reduce overall credit risk in the economy.

Model (4) allows banks to default strategically, with their debt insured by the government. This bankruptcy option significantly changes bank risk taking, with the average default rate of banks rising from 0.00% to 0.96%. Banks are effectively insured against the worst possible payoffs of their portfolios: in the first period of a financial recession, 15.84% of banks fail in Model (3). Financial recessions are more severe and consumption growth volatility is higher as a consequence of greater bank risk taking.

Moving from Model (4) to the benchmark, the only change is an increase in the equity issuance cost for banks from zero to 7. This change has two main effects. First, it reduces bank risk taking ex-ante, since banks hold more equity to save on issuance costs in states of the world where losses are large, but not large enough to make bankruptcy optimal. Second, conditional on being in a financial recession, the positive issuance costs make bank recapitalization more costly and thus amplify intermediary frictions. The net effect of less risk taking ex-ante, but greater amplification ex-post are slightly deeper financial recessions. The issuance costs further increase the excess return banks earn on their assets, raising the cost of debt finance for firms and leading to slightly lower firm leverage. As bank intermediation becomes more expensive with rising  $\phi_1^I$ , savers increase their share of the corporate debt market. Thus, savers' ability to expand their holdings when bank finance becomes costlier mitigates the impact of higher equity issuance costs for banks.

#### D.2 Equity Issuance Costs for Producers

The last column of Table D.1 explores the effect of activating equity issuance costs for producers, i.e. setting  $\phi_1^P > 0$ . With a marginal cost parameter  $\phi_1^P = 1$ , the net payout rate of firms becomes 7.17%, higher than the 6.90% rate in the benchmark, since the cost reduces equity issuance by firms. Imposing this additional financial friction on firms shrinks the size of the economy, causing lower investment, a roughly 1% smaller capital stock and -0.45% lower GDP. Since this cost raises the cost of equity finance relative to debt, firms choose higher leverage and default risk increases slightly. The intermediation sector is larger, but has lower book leverage.

Financial recessions are milder in the model with firm equity issuance, with investment falling less (-20.72% versus -25.26% in the benchmark) and bank failures only rising to 3.81%, compared to 11.20% in the benchmark. General equilibrium effects in the corporate loan market cause this differential response. When issuing equity is more costly for firms, they become more constrained in crises and their demand for debt falls by more. Equivalently, firms supply fewer corporate bonds at any price, resulting in a smaller drop in the price of corporate bonds. This in turn only leads to a smaller decline in the value of intermediary assets and fewer bank defaults.

With fewer banks defaulting, the required bailout financing by the government is lower, and as a result the riskfree rate drops by more. Thanks to the greater fall in the riskfree rate, banks earn higher spreads and recover more quickly, further reducing the fraction of banks that default. In the end, when fewer banks default, credit supplied by banks suffers less, and the drop in investment is smaller despite the fact that firms face greater costs for equity issuance.

This chain of general equilibrium effects demonstrates that the interaction of firm equity issuance and intermediation frictions adds significant complexity to the model's behavior in crises. We leave a full analysis of this interaction, in particular when combined with deeper firm heterogeneity, for future work. For the purposes of our macro-prudential policy experiments in this paper, we verified that our results are very similar if equity issuance costs for firms are present, see section D.5.

## D.3 Pure Uncertainty Shock

Figure D.1 compares the dynamics of important macro-economic aggregates and balance sheet variables in a financial recession (red lines) to the effect of a pure second-moment shock. The IRF plots are generated as explained in the main text. The red line in the plots of Figures D.1 and D.2 is identical to the red line in Figures 2 and 3 in the main text, as both are caused by the same combination of a low TFP realization and an increase in  $\sigma_{\omega}$  in period 1. The blue line in Figures D.1 and D.2 show dynamics after the economy is hit only by the increase in  $\sigma_{\omega}$ , with stable TFP. The plots show that this pure uncertainty shock has much smaller negative effects on output, consumption and investment than the combination that causes a financial recession. This feature of our model is consistent with the empirical finding that uncertainty shocks alone have at most moderate negative effects on output

	(1)	(2)	(3)	(4)	Bench	$\phi_1^P = 1$
	Borrowers					
1. Mkt value capital/ Y	225.5	223.9	215.5	215.4	215.2	212.8
2. Mkt value corp debt/ Y	117.9	100.7	85.1	84.6	82.8	83.7
3. Book val corp debt/ Y	89.6	88.7	82.1	81.7	80.3	81.4
4. Market corp leverage	52.3	45.0	39.5	39.3	38.5	39.3
5. Book corp leverage	39.7	39.6	38.1	37.9	37.3	38.2
6. $\%$ producer constr binds	71.0	39.3	4.0	3.1	1.6	7.2
7. Default rate	-	2.42	2.16	2.14	2.08	2.11
8. Loss-given-default rate	-	56.45	52.84	52.62	51.78	53.21
9. Loss Rate	-	1.35	1.13	1.12	1.07	1.10
10. Investment / Y	18.59	18.46	17.77	17.76	17.74	17.55
			Intern	nediarie	s	I
11. Mkt val assets / Y	-	-	73.9	73.2	71.3	72.3
12. Mkt fin leverage	-	-	93.0	93.0	92.6	92.6
13. Book fin leverage	-	-	98.6	98.5	97.6	97.3
14. % intermed constr binds	-	-	100.0	100.0	63.4	54.8
15. Bankruptcies	-	-	0.00	0.96	0.63	0.54
16. Wealth I	-	-	5.18	5.16	5.34	5.43
17. Franchise Value	-	-	7.92	16.60	17.86	16.25
			Sa	vers	1	I
18. Deposits/GDP	-	-	70.2	69.6	67.5	68.4
19. Government debt/GDP	75.1	79.5	71.8	74.2	73.1	72.5
20. Corp Debt Share S	100.0	100.0	13.2	13.4	13.9	13.7
			Pr	rices	1	I
21. Risk-free rate	2.28	2.24	2.24	2.26	2.23	2.22
22. Corporate bond rate	1.91	3.19	4.06	4.07	4.12	4.14
23. Credit spread	-0.37	0.95	1.82	1.81	1.89	1.92
24. Term spread	-0.42	-0.37	-0.38	-0.39	-0.36	-0.35
25. Excess return on corp. bonds	-0.33	-0.41	0.69	0.70	0.84	0.83
			Crisis I	$\mathbf{RF} \ (t =$	1)	L
26. Investment	-8.30	-19.90	-18.78	-22.06	-25.26	-20.72
27. Consumption	-1.76	-0.78	-0.97	-2.22	-1.37	-1.34
28. Bankruptcies I	-	-	0.00	15.84	11.20	3.81
		Size,	Volatil	ity, & V	Velfare	I
29. GDP	1.94	1.65	0.07	0.05	0.00	-0.45
30. Consumption	1.74	0.43	0.11	-0.02	0.00	-0.12
31. Consumption gr vol	-14.79	-6.02	-2.11	3.82	0.00	1.61
32. Value Function, B	0.88	-1.80	-0.16	-0.21	0.00	0.18
33. Value Function, S	3.84	2.88	0.22	0.14	0.00	-0.22

Table D.1: Role of Frictions

All numbers are in percent. The last panel reports percentage changes relative to benchmark. Columns (1) and (2) are economies without intermediaries. Column (1) further turns off liquidity defaults for producer firms. Columns (3) and (4) activate portfolio costs of savers, resulting in emergence of a banking sector. In column (3), banks have unlimited liability and can raise outside equity costlessly. In column (4), banks have limited liability, but can still issue equity without cost. The fifth column is the benchmark model that makes issuance costly for banks. The last column changes equity issuance costs for non-financial firms.

and investment, see for example Bachmann and Bayer (2013) or Vavra (2014).



Figure D.1: Financial Recession vs. Uncertainty Shock: Macro Quantities

Blue line: uncertainty shock, red line: financial recession, black line: no shock

A closer look at the balance sheet variables in Figure D.2 reveals that the fundamental difference between both types of shocks lies in the response of intermediaries. The losses suffered on loans during a financial crisis are only marginally larger than those from the uncertainty shock. However, corporate borrowing shrinks by less after the uncertainty shock. Producers hardly reduce debt despite a temporarily smaller capital stock, effectively increasing leverage. Banks reduce assets and liabilities on impact, but their contraction is almost completely offset by savers' expansion. The spike in the bank failure rate is only 2%, compared to over 20% in financial recessions. After the initial period, financial intermediary variables quickly return to their long-run levels.

Why are financial recessions so much worse despite identical losses from borrower defaults for banks? The dynamics of the corporate bond price are the key amplifying force. This price drops sharply in financial recessions, causing large market value losses for intermediaries. The stronger financial accelerator caused by the combined effect of TFP and uncertainty shocks in a financial recessions means that intermediary net worth falls only half as much in a pure uncertainty shock episode. As a result, intermediaries are not forced to shrink as much as they are in a financial recession. Continuity in lending to borrower-entrepreneurs prevents a sharp reduction in investment despite intermediary losses on loans.

Another key difference between financial recessions and uncertainty shocks in our model is the response of consumption. During a pure uncertainty shock, producers cut investment by 15%, yet output hardly drops. Further, unlike in financial recessions, deadweight costs of bank failures are small during uncertainty episodes. Consequently, consumption rises as investment falls. The increase in aggregate consumption reflects a rise in consumption of savers (borrower consumption falls also in



Figure D.2: Financial Recession vs. Uncertainty Shock: Balance Sheets and Prices

Blue line: uncertainty shock, red line: financial recession, black line: no shock

uncertainty episodes). Savers have to shed their holdings of deposits as the banking sector shrinks; to clear the market for short-term bonds, the riskfree rate needs to fall more sharply than in financial recessions.

We conclude that only the combination of TFP and uncertainty shock activates the intermediarybased financial accelerator that generates financial recessions akin to the data.

## D.4 Credit Spread and Risk Premium

One important quantitative success of the model is its ability to generate a high unconditional credit spread while matching the observed amount of default risk. The credit spread is also highly volatile (1.56% standard deviation) and higher in financial recessions than in expansions. The rise in the credit spread in financial recessions to 2.75% reflects not only the increase in the quantity default risk but also an increase in the price of credit risk. The model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical "shadow SDF" for the intermediary sector.

The intermediary SDF is given by:

$$\mathcal{M}_{t,t+1}^{I} = \mathcal{M}_{t,t+1}^{B} \left( 1 - \phi_{1}^{I} e_{t}^{I} \right) \left( \phi_{0}^{I} + \frac{1 - \phi_{0}^{I}}{1 - \phi_{1}^{I} e_{t+1}^{I}} \right) \left( 1 - F_{\epsilon,t+1} \right),$$

where  $\mathcal{M}_{t,t+1}^B$  is the borrower SDF,  $F_{\epsilon,t+1}$  is the probability of intermediary failure in t+1, and

 $\phi_0^I + \frac{1-\phi_0^I}{1-\phi_1^I e_t^I}$  is the marginal value of wealth to intermediaries in t.

Figure D.3 shows the histogram of the intermediary wealth share plotted against two different measures of credit risk compensation earned by intermediaries. The solid red line plots the credit spread, the difference between the yield  $r_t^m$  on corporate bonds and the risk-free rate. We compute the bond yield as  $r_t^m = \log\left(\frac{1}{q_t^m} + \delta\right)$ . This is a simple way of transforming the price of the long-term bond into a yield; however, note that this definition assumes a default-free payment stream  $(1, \delta, \delta^2, \ldots)$  occurring in the future. Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary's wealth share is low. Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and a credit risk premium.



Figure D.3: The Credit Spread and the Financial Intermediary Wealth Share

Solid line: credit spread; dashed line: expected excess return

To shed further light on the source of the high credit spread, we compute the expected excess return (EER) on corporate loans earned by the intermediary. The EER consists both of the credit risk premium, defined as the (negative) covariance of the intermediary's stochastic discount factor with the corporate bond's excess return, and an additional component that reflects the tightness of the intermediary's leverage constraint. This component arises because the marginal agent in the market for risk-free debt is the saver household, while corporate bonds are priced by both the constrained intermediary and the saver. The market risk free rate is lower than the "shadow" risk free rate implied by the intermediary SDF. Given log preferences, most of the action in the EER comes from the constraint tightness component rather than the covariance with the intermediary SDF. When intermediary wealth is relatively high, the leverage constraint is not binding and the EER is approximately zero. Low levels of intermediary wealth result from credit losses, and the lowest levels occur during financial crises. At these times, credit risk increases and the intermediary becomes constrained. In the worst crisis episodes when intermediary wealth reaches zero or drops below zero, the EER reaches 4 percent.

## D.5 Sensitivity of Macro-prudential Policy

In this appendix we study how sensitive the macro-prudential policy conclusions are to specific model ingredients/parameter constellations. In each experiment, we compare the effects of relaxing bank capital requirements by two percentage points versus tightening them by two percentage points, around the benchmark model. In other words, we study a four percentage point relaxation from  $\xi = .91$  to  $\xi = .95$ . The first column of Table D.2, Panels A and B, reports the results from this particular relaxation for the benchmark model. Firm loss rates and bank bankruptcies both increase, the size of the banking sector and the economy as a whole increase, investment volatility falls modestly while consumption growth volatility rises modestly, and savers gain while borrowers lose. All these results are in line with our discussion in the main text.

#### Table D.2: Sensitivity of Macro-prudential Policy Experiment

			<b>NI TE GLI L</b>							
	Bench	No Bankruptcy	No Tax Shield	$\phi_{1}^{I} = 0$	$\phi_1^{I} = 8$					
		Financial Fragility								
Loss rate	+0.08%	+0.10%	+0.09%	+0.07%	+0.07%					
Bankruptcies	+2.60%	0.00%	+1.60%	+2.99%	+2.50%					
		Size o	f the Economy							
GDP	+0.15%	+0.13%	+0.09%	+0.14%	+0.13%					
Deposits / GDP	+10.51%	+10.72%	+11.56%	+10.21%	+9.82%					
		Mac	cro Volatility							
Investment	+0.36%	+0.02%	+0.09%	+0.25%	+0.38%					
Consumption	+0.20%	+0.01%	+0.06%	+0.19%	+0.18%					
log MU ratio	+0.08%	-0.04%	-0.12%	+0.21%	+0.06%					
			Welfare							
Borrower	-1.13%	-0.70%	-0.78%	-1.15%	-1.06%					
Saver	+0.48%	+0.62%	+0.51%	+0.46%	+0.46%					
Aggr. $\mathcal{W}^{cev}$	+4.75%	+9.40%	+6.63%	+4.44%	+4.84%					

#### Panel A: Bank-specific Parameters

Panel B:	Other	Parameters
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	Bench	Higher $\beta_B$	Lower $\beta_S$	Firm Iss. Cost	60% Savers	No Saver Hold.					
	Financial Fragility										
Loss rate	+0.08%	+0.10%	+0.09%	+0.04%	+0.08%	+0.11%					
Bankruptcies	+2.60%	+2.33%	+2.43%	+2.68%	+2.12%	0.00%					
			Size	e of the Economy							
GDP	+0.15%	+0.09%	+0.10%	+0.27%	+0.22%	+0.21%					
Deposits / GDP	+10.51%	+11.59%	+11.18%	+8.78%	+10.38%	+12.36%					
			Ν	Iacro Volatility							
Investment	+0.36%	+0.31%	+0.30%	+0.34%	+0.26%	-0.27%					
Consumption	+0.20%	+0.16%	+0.16%	+0.21%	+0.19%	-0.05%					
log MU ratio	+0.08%	-0.07%	-0.06%	+0.18%	+0.44%	-0.60%					
	Welfare										
Borrower	-1.13%	-1.17%	-1.16%	-1.10%	-1.16%	-0.75%					
Saver	+0.48%	+0.48%	+0.49%	+0.44%	+1.67%	+0.47%					
Aggr. $\mathcal{W}^{cev}$	+4.75%	+4.68%	+4.19%	+4.17%	+7.41%	+5.77%					

The other columns of Table D.2, Panel A, study the same change in macro-prudential policy in a model without bankruptcy option (column 2), without tax shield for banks (column 3), without bank equity issuance costs (column 4), and with higher bank equity issuance costs (column 5). Panel B

adds sensitivity analyses in a model with more patient borrowers (column 1,  $\beta_B$  increases by 0.15%), and less patient savers (column 2,  $\beta_S$  decreases by 0.15%). The latter two changes decrease the wedge between the patience of borrowers and savers and reduce the need for intermediation services. Column 4 studies the effect of the same experiment in an economy where equity issuance of non-financial firms is costly ( $\phi_1^P = 1$ ). Column 5 of Panel B shows the effect of lowering capital requirements in an economy, in which a higher share of the population are savers and in which they collectively earn a higher share of the labor income – 58% and 24%, respectively. Finally, Column 6 of Panel B conducts the experiment in a fully intermediated economy i.e. one, in which savers cannot hold corporate loans.

The main finding is that the welfare changes from macro-prudential policy are robust to these parameter variations. In all experiments, borrower welfare decreases and saver welfare increases in response to the four percentage point increase in maximum allowable financial sector leverage from 91% to 95%. In all cases, we see more fragility in the form of higher corporate loss rates. When banks are allowed to default, we also see higher bank bankruptcies in almost all cases. The exception is the economy with no saver holdings of corporate debt. When crises occur in this economy, banks must continue to own corporate debt even as they experience losses. As a result, expected excess returns on loans are much higher, and the opportunity to earn these returns discourages banks from defaulting. Even as capital requirements are relaxed, the frequency of bank bankruptcies remains virtually zero. The basic trade-off between a larger economy and more financial fragility is also present in every model. The quantitative slope of that trade-off depends on the model details. The largest difference to the benchmark arises in the model with no bankruptcy option for banks. We know from the discussion in Section 4.3 that limited liability and deposit insurance are the key drivers of bank risk taking. Hence, in the model where banks cannot default and these forces are not present, macro-prudential policy has much smaller effects.

In economies where bank bankruptcies are non-zero, macro-economic volatility also responds similarly across all parameter variations to changes in  $\xi$ . Risk sharing tends to worsen (MU vol rises) as macro-prudential policy is relaxed, reflecting the financial sector's greater risk taking and worse financial recessions. The effects are strongest in the economy with equity issuance frictions for non-financial firms, indicating further amplification from the interaction of this friction with intermediary risk taking. Absent an increase in bank bankruptcies (No Bankruptcy and No Saver Holdings economies), a relaxation of capital requirements increases the financial sector's ability to share risk.

## D.6 Mortgage Crisis Experiment

This appendix performs an exercise to study the fall-out from a "mortgage crisis" for banks and firms. In the model, the idiosyncratic bank loss/profit shocks stand in for banks' heterogeneous exposure to other loan losses, such as (subprime residential) mortgages. We engineer a large, unanticipated shock to idiosyncratic bank losses for all banks and trace out its effects on the real economy. The size of the "mortgage crisis" shock is chosen to generate the same-size destruction of intermediary wealth as in the average financial recession we focus on in the paper. Note that this is a one-time shock, and this shock is orthogonal to both aggregate TFP and aggregate uncertainty (the dispersion of TFP). In the real world, the mortgage crisis may well have coincided with a decline in TFP and/or a rise in uncertainty. Even so, our model generates spillovers form the financial sector to the real economy. To illustrate the amplification, we compare the same "mortgage crisis" experiment in our benchmark model and in a simpler model where we shut down two key financial frictions, limited liability and equity issuance costs. (The new section 4.3 discusses a sequence of simpler models that lead up to our benchmark model, including this simpler model –called Model (4)– with banks but with frictionless recapitalization and unlimited liability and hence no bank default).

The figure below shows that the response to the mortgage shock is more severe in our benchmark model than in this simpler Model (4) without the key financial frictions. Consumption, output, and especially investment fall more in the benchmark. The larger decline in investment is consistent with the higher corporate loan rate. The latter reflects both higher safe interest rates and a higher spread of corporate loans over equal-maturity Treasury bonds (bottom right panel). The higher safe rates in turn reflect the new government bond supply due to bank bailouts. Savers require a higher rate to be absorb these bonds rather than consume. Since uncertainty remains low and TFP remains average in this experiment, precautionary demand for bonds is not particularly high, nor are investment opportunities in corporate loans bad. The mortgage crisis spills over to the corporate sector through the equilibrium safe and risky lending rates. Those in turn depend in quantitatively important ways on the financial frictions in the model.



