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Product Quality in a Distribution Channel with Inventory Risk

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Abstract. In many industries, product design and manufacturing lead times are sufficiently long that both the quality level of a product and the amount of inventory produced must be determined before a firm knows what the actual demand will be. In this paper, we conduct a theoretical analysis of such a setting. We first consider a centralized channel and characterize the optimal decisions by establishing relationships that must hold between the elasticity of cost of quality and the elasticity of revenue and show that quality and inventory are strategic substitutes. Next, we consider a decentralized channel with a wholesale price contract, in which a manufacturer determines quality and wholesale price, while a retailer determines inventory and retail price. We find that, different from the case without endogenous inventory, product quality can be higher in a decentralized channel compared to a centralized channel, and this is because a wholesale price contract shields the manufacturer from inventory risk. For both centralized and decentralized channels, we find that as demand uncertainty increases, quality decreases, while, different from the case without endogenous quality, inventory can be U-shaped. Interestingly, to mitigate the impact of demand uncertainty on profit, quality can be a more effective lever than inventory in a centralized channel; however, in a decentralized channel, quality is less responsive and inventory is more responsive to demand uncertainty than in a centralized channel.

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1. Introduction

When selling products to consumers, firms must make multiple decisions, including what the quality of the product will be, what quantity to produce, and what price to charge to consumers. Furthermore, these decisions have to be made under demand uncertainty, and in many instances they involve a channel partner. For instance, consider the case for products such as apparel that take time to design, manufacture, and transport but have short selling seasons and uncertain demand (e.g., in apparel, typical "design-to-shelf" lead times are on the order of six to nine months, while selling seasons average just three to four months; Ghemawat and Nueno 2003, Cachon and Swinney 2011). The product has to be designed and produced by the manufacturer before the beginning of the selling season; i.e., the quality of the product and the quantity to be produced have to be decided early and cannot subsequently be adjusted. In such situations, supply–demand mismatch is a central problem, leading to stockouts or unsold inventory. However, the retail price is typically decided as the season unfolds and is therefore responsive to the realization of demand uncertainty. Furthermore, if the manufacturer sells through an independent retailer, then the manufacturer and the retailer interact through a vertical contract (that is often a simple wholesale price contract, or may be a more complicated mechanism such as a quantity discount or a buyback contract).

The above example is representative of many real-world situations. In this paper, we develop a stylized model to study the interplay of quality, inventory, pricing, and vertical channel interactions. We highlight the role played by product quality and its interaction with demand uncertainty and inventory choice, a tension that has been understudied in the channels literature, except for a few notable exceptions (Desai et al. 2004, 2007; Ferguson and Koenigsberg 2007). Quality refers to a product attribute (or a combination of attributes), more of which increases consumer product valuation but also marginal cost. Value-enhancing product quality may include new product features, enhanced performance of existing features, improved product durability by upgrading raw materials or components (e.g., the quality of fabric in an article of clothing), or improved quality of the manufacturing process (e.g., handcrafted items versus mass-produced items). Because quality directly impacts both cost and price, it clearly interacts with inventory risk. (Here, inventory "risk" refers to the fact that because of uncertain demand and inflexible inventory, the selling firm may be left with unsold units, or may fall short of units to sell.) Essentially, we merge a framework with consumer heterogeneity in taste for quality and increasing marginal cost of quality, which is widely used to study quality decisions, with the classic newsvendor framework, which is widely used to study inventory decisions.

For a centralized channel, in which a manufacturer sells directly to consumers, we characterize the optimal inventory, quality, and price, and demonstrate (in the spirit of Dorfman and Steiner 1954) that at the optimal inventory–quality pair, the elasticity of the unit quality cost function equals the reciprocal of the elasticity of the expected revenue function. Using this characterization, we find several important managerial insights about the simultaneous management of quality, inventory, and price. First, quality and inventory are substitutes, because higher quality requires greater marginal cost and thus a greater loss on each unit produced but not sold, and the firm reduces inventory to lower the cost of unsold units. Second, quality decreases with demand uncertainty, because higher uncertainty leads to a larger number of expected unsold units, and to compensate, the firm finds it optimal to reduce the cost of unsold units by investing in lower quality. Third, inventory can be nonmonotonic (specifically, U-shaped) in demand uncertainty. This is an interesting and important result because it is commonly found that with exogenously determined quality and price, inventory is monotonic in demand uncertainty. Nonmonotonicity is driven by endogenously determined quality because greater demand uncertainty can have a direct effect of reducing inventory but also an indirect effect, through reduced quality, of increasing inventory. Fourth, endogenously determined quality can significantly moderate the inventory decision, such that at their optimal values, quality may be more sensitive to demand uncertainty than inventory is. This illustrates that product design is an important managerial lever for dealing with uncertain demand.

We also examine a decentralized channel. A critical component of a decentralized system is the contract between the manufacturer and the retailer. To guide our analysis and make it relevant to practice, we consulted industry practitioners (especially in the apparel industry; details available on request) and the academic literature on contract prevalence. Our investigation revealed that flat wholesale price contracts are widely used in the industry because of their simplicity (even though they are known to be suboptimal from a channel efficiency and coordination point of view). Motivated by this observation, we focus our analysis on the wholesale price contract. Our analysis reveals that the relationship between optimal inventory and quality is expressed by an equation that maintains the same structure as the one in the centralized channel case, but with a different elasticity function: at the optimal quality–inventory pair the elasticity of the unit quality cost function equals the reciprocal of the elasticity of the "inventory-constrained" expected revenue function (as opposed to the elasticity of the expected revenue function in the centralized case). The main driver of this departure from the centralized case is that here the manufacturer does not itself control the inventory level; rather, while determining the quality level, it has to impute the inventory level that the retailer will subsequently choose. We show that quality in this case can be lower than, equal to, or higher than that in a centralized system. The result that quality can be higher is an interesting and important finding because it runs counter to the existing understanding in the literature that decentralization reduces product quality because of double marginalization that lowers the manufacturer's incentive to invest in quality. The insight behind our result is that in a decentralized channel, the manufacturer (who makes the quality decision) faces only an indirect effect of demand uncertainty through the retailer; i.e., the manufacturer is shielded from inventory risk. Therefore, the manufacturer is less sensitive to the cost incurred due to leftover inventory and has an incentive to set quality at a level higher than in the centralized case, where it directly faces demand uncertainty.[1]

Our research contributes to the literature on product quality (Mussa and Rosen 1978, Moorthy 1984, Moorthy and Png 1992, Krishnan and Zhu 2006, Heese and Swaminathan 2006, Netessine and Taylor 2007). Contrary to previous work that shows that decentralization reduces quality (Jeuland and Shugan 1983, Villas-Boas 1998, Economides 1999) unless particular assumptions on demand are made (Xu 2009, Shi et al. 2013), we show that decentralization may lead to higher quality even under a simple demand framework because it influences inventory risk allocation in the channel. Desai et al. (2004, 2007) and Ferguson and Koenigsberg (2007) consider quality and inventory jointly in a multiperiod scenario in which quality deteriorates over time, but they consider quality as exogenous. Desai et al. (2004) considers the choice of channel structure as well and shows that decentralization leads to higher profit for the manufacturer; in our setting, we find that even though decentralization can distort quality upward, it does not lead to higher profit for the manufacturer. Another related paper is that by Carlton and Dana (2008), who examine joint quality and inventory decisions for a monopolist and competing firms; however, they do not consider heterogeneous consumer tastes for quality, a decentralized channel, or issues of channel coordination, as we do. A number of papers show that considering operational aspects can influence marketing decisions related to quality, pricing, returns policies, sales force compensation, etc., in important ways (Iyer et al. 2007; Netessine and Taylor 2007; Shulman et al. 2009; Jerath et al. 2010; Tereyagoglu and Veeraraghavan

2012; Dai and Jerath 2013, 2016), and our paper adds to this literature.

The rest of this paper is organized as follows. In Section 2, we describe our model. In Section 3, we analyze the centralized and decentralized channels and conduct a comparison of the results in the two scenarios. In Section 4, we conclude with a discussion. We provide proofs and additional analyses in the appendix and the online appendix accompanying this paper.

2. Model

In this section, we describe our theoretical model. A firm (the "manufacturer") designs and produces a single seasonal product at a certain quality level. Quality is an attribute that increases consumers' willingness to pay, which allows the firm to charge a higher price, but higher quality also requires a greater manufacturing cost (Mussa and Rosen 1978, Moorthy 1984, Villas-Boas 1998). The product is sold over a single selling season, and before the selling season begins, demand for the product is uncertain. Design and production lead times are sufficiently long that the product must be designed and its inventory level determined well in advance of the start of the season, i.e., before the revelation of demand information, and there is no opportunity to procure additional inventory or adjust the design of the product after the selling season begins. Therefore, the model setup is essentially that of a single-period newsvendor with the added complication of endogenously determined product quality. By contrast to the canonical newsvendor model, however, the retail price of the product is decided after the uncertainty in demand is resolved. This reflects that price is a more flexible decision than quality and inventory, and sellers often change prices as they learn about the true realization of demand; this is referred to as the practice of "responsive pricing" or "price postponement" (Van Mieghem and Dada 1999, Chod and Rudi 2005, Granot and Yin 2008). At the conclusion of the season, excess inventory is salvaged for zero revenue.

We consider two channel structures: (1) the centralized channel in which the manufacturer sells the product directly to consumers and (2) the decentralized channel in which the manufacturer sells the product to consumers through a retailer via a contractual relationship. The objectives of the manufacturer and the retailer are to maximize their respective profits. The following is a summary of the decisions to be made, who makes them, and their timings:

1. Before the selling season (i.e., before demand uncertainty is resolved), the following decisions must be made:

 (a) The product quality level, denoted by θ (always decided by the manufacturer)

 (b) The wholesale price, denoted by w (only in the case of a decentralized channel, decided by the manufacturer and offered to the retailer)

 (c) The inventory level, denoted by q (decided by the manufacturer in a centralized channel and by the retailer in a decentralized channel)

2. During the selling season (i.e., after demand uncertainty is resolved), the retail price, denoted by p, is decided by the manufacturer in a centralized channel and by the retailer in a decentralized channel.

We assume that consumers have heterogeneous valuations for quality. We denote the valuation by v and assume it to be uniformly distributed in the interval $[0,1]$. The net utility from purchasing and consuming a product for a consumer with valuation v is given by $U = v\theta - p$, where θ is the quality level and p is the price; i.e., consumers value higher quality and lower price. The multiplicative relationship between v and θ captures the idea that higher valuation consumers are more responsive to quality changes. The fraction of the consumer population with nonnegative utility for purchasing the product is denoted by $\alpha \le 1$, and this fraction is determined endogenously by the quality and the retail price.

We denote by X the *potential demand* for the product. We assume X to be a random variable with cumulative distribution function F and probability density function f over the support $[0, \bar{X})$, where $\bar{X}$ may be infinite. We denote the mean of X by $\mu \equiv E[X]$. We denote by D the *target demand*, i.e., a portion of the potential demand that consists of consumers with high enough valuations who generate positive utility for purchasing the product. Target demand D is related to X as $D = \alpha X$, where $\alpha \le 1$ as defined earlier. We define the generalized failure rate of the demand distribution as $g(x) \equiv x f(x)/\bar{F}(x)$ and make the following assumptions:

Assumption 1. *$g(x)$ has the following properties:*
(i) *$g'(x) > 0$; i.e., the demand distribution has an increasing generalized failure rate (IGFR).*
(ii) *$g(x)$ satisfies the regularity conditions $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to \bar{X}} g(x) > 1$.*

Note that these properties are satisfied by most common probability distributions used to model uncertain demand (see Lariviere 2006). We denote the unit production cost at the quality level θ by $c(\theta)$ and the elasticity of the production cost (the percentage change in unit production cost with respect to a percentage change in quality) by

$$\epsilon(\theta) \equiv \frac{\theta c'(\theta)}{c(\theta)}, \tag{1}$$

and make the following assumptions about the cost function:

Assumption 2. *The unit cost $c(\theta)$ and its elasticity $\epsilon(\theta)$ have the following properties:*
(i) *$c'(\theta) > 0$, $c''(\theta) > 0$, $\lim_{\theta\to 0} c(\theta) = \lim_{\theta\to 0} c'(\theta) = 0$, and $\lim_{\theta\to 1} c(\theta) > 1$.*
(ii) *$\epsilon(\theta) > 1$ and $\epsilon'(\theta) \ge 0$.*

The first part of the assumption states that $c(\theta)$ is convex increasing and that zero quality can be had at no cost. The assumption $\lim_{\theta\to 1} c(\theta) > 1$ ensures that an excessive quality level reduces the firm's profit to zero or negative. The conditions in Assumption 2(i) together imply that there exists a unique value $\bar{\theta} < 1$ that solves the equation $c(\theta)/\theta = 1$. We restrict the range of feasible θ to the interval $(0, \bar{\theta})$; it can be shown that the optimal inventory is $q = 0$ if θ falls outside of this interval.[2] The assumptions on elasticity in the second part are made to facilitate the analysis, but they are not restrictive. Many representative cost functions satisfy all of the conditions listed in Assumption 2, e.g., the isoelastic unit cost function $c(\theta) = c_0\theta^n$, where $n > 1$ is the elasticity of the cost function, and the constant $c_0 > 1$ is the unit cost coefficient, i.e., the scale of the cost function.

Note that in our model, higher quality leads to greater demand, and quality has a convex increasing cost. Some other aspects of the marketing mix may have similar characteristics; e.g., sales effort by the manufacturer or the retailer would lead to higher demand and can be expected to have a convex increasing cost (Taylor 2002). However, a key difference between quality and sales effort is that quality cost directly influences the marginal cost of the product, which then fundamentally affects the inventory stocking decision, while sales effort cost does not directly affect the marginal cost of the product; therefore, their implications are expected to be different.

3. Analysis and Results

3.1. Retail Pricing Decision

The retail pricing decision is common across the centralized and decentralized scenarios. Therefore, we solve for this decision first and use the results in the analyses of both the centralized and the decentralized channels.

At the time of pricing, the price-setting firm (either the manufacturer in a centralized channel or the retailer in a decentralized channel) has an inventory of q units with quality θ, and the potential demand X has been realized. The firm's revenue is given by

$$R = p\min\{D, q\} = p\min\{\alpha X, q\}.$$

Given the retail price p and the uniformly distributed consumer valuation $v \in [0, 1]$, the fraction of consumers who purchase the product, α, is specified as follows.

Lemma 1. *Given θ and p, the fraction of consumers who purchase the product is $\alpha = 1 - p/\theta$ if $0 \le p/\theta \le 1$ and $\alpha = 0$ if $1 < p/\theta$, i.e., $\alpha = (1 - p/\theta)^+$, where $(\cdot)^+ \equiv \max\{0, \cdot\}$.*

Note that if $0 < \alpha < 1$ (when $0 < p/\theta < 1$), α is linearly decreasing in p and concave increasing in θ—lower price and higher quality attract more purchases, and there is a diminishing return in quality. The firm faces the demand $D = \alpha X$. As a result, firm revenue is equal to

$$R = p\min\{D, q\} = p\min\left\{\left(1 - \frac{p}{\theta}\right)^+ X, q\right\}, \qquad (2)$$

where p is restricted in the interval $[0, \theta]$.

Lemma 2. *Given θ, q, and X, the firm sets the optimal price as $p = (\theta/2)(1 + (1 - 2(q/X))^+)$. This determines the fraction of purchasing consumers as $\alpha = \frac{1}{2}(1 - (1 - 2(q/X))^+)$, and the firm's revenue as*

$$R(\theta, q, X) = \frac{\theta}{2}\left(\min\left\{\frac{X}{2}, q\right\} + \left(1 - 2\frac{q}{X}\right)^+ q\right). \qquad (3)$$

Note from Lemma 2 that the optimal price p is decomposed into two parts: (i) mean-valuation price $\theta/2$, which is independent of potential demand X and inventory q, and (ii) inventory-constrained price $(\theta/2)(1 - 2(q/X))^+$, which decreases in the inventory–demand ratio q/X. For $q \le X/2$, the price is given by $\theta(1 - q/X)$, and the revenue is given by $\theta(1 - q/X)q$. As a direct consequence of this pricing structure, the firm's revenue function (3) is also decomposed into two parts, $\theta/2\min\{X/2, q\}$ and $(\theta/2)q(1 - 2(q/X))^+$. Hence, as the inventory q increases, the price goes down linearly from $p = \theta$ at $q = 0$ (only the highest-valuation customers are served when inventory is scarce) until it reaches $p = \theta/2$ at $q = X/2$ (exactly half of the customer population is served, maximizing the firm's profit), and the revenue increases in a concave manner. For $q > X/2$, the inventory level does not affect the price since reducing the price further hurts the firm's profitability; here the price is given by $\theta/2$, and the revenue is given by $\theta X/4$.

The decisions regarding θ, q, and contractual parameters are made before the demand realization, and will therefore be based on expected revenue, i.e., on $R(\theta, q) \equiv E[R(\theta, q, X)]$, which we now evaluate. To this end, we define, for $y \in (0, \bar{X})$, the following:

$$S_1(y) \equiv \int_0^y \bar{F}(x)\,dx, \quad S_2(y) \equiv y^2 \int_y^{\bar{X}} \frac{1}{x^2}\bar{F}(x)\,dx, \quad \text{and} \quad S(y) \equiv S_1(y) + S_2(y), \qquad (4)$$

where $\bar{F}(x) = 1 - F(x)$. Furthermore, we define the elasticities of $S_j(y)$, $j \in \{1, 2\}$, and $S(y)$ as[3]

$$\eta_j(y) \equiv \frac{yS_j'(y)}{S_j(y)} \quad \text{and} \quad \eta(y) \equiv \frac{yS'(y)}{S(y)}. \qquad (5)$$

Given these definitions, the expected revenue can be expressed as follows:

Lemma 3. *The expected revenue, accounting for the optimal pricing decision as in Lemma 2, is $R(\theta, q) \equiv E[R(\theta, q, X)] = (\theta/4)S(2q)$.*

Building on Lemma 2 and the discussions following it, we can see that the expected revenue has two components. The first component represents the expected revenue associated with mean-valuation price $\theta/2$, which is charged to consumers when there are ample units of inventory. This component is equal to $(\theta/2)E[\min\{X/2, q\}]$. The second component represents the expected revenue associated with the inventory-constrained price $(\theta/2)(1-2(q/X))^+$, which is charged to consumers when the price is inflated due to insufficient units of inventory compared to demand. This component is equal to $(\theta/2)E[(1-2(q/X))^+]$. From Lemma 3, we see that the function $S(\cdot)$ in fact corresponds to the expected revenue, and the functions $S_1(\cdot)$ and $S_2(\cdot)$ correspond to the two component terms of expected revenue, i.e., the inventory-unconstrained expected revenue and the inventory-constrained expected revenue, respectively (see the proof of Lemma 3 for more details).

3.2. Centralized Channel

In a centralized channel, the manufacturer sells directly to consumers, setting quality θ and inventory q before demand is realized, and subsequently the retail price p after demand is realized. Using the expected revenue function found in Lemma 3, we can write the manufacturer's expected profit as

$$\Pi_M^c(\theta, q) = -c(\theta)q + R(\theta, q) = -c(\theta)q + \frac{\theta}{4}S(2q), \quad (6)$$

where the superscript c denotes the centralized channel. Note that this is the expected profit *at the optimal retail price*. The first-order conditions that need to be satisfied at the optimal q and θ are

$$\frac{d\Pi_M^c(\theta, q)}{dq} = -c(\theta) + \frac{\theta}{2}S'(2q), \quad (7)$$

$$\frac{d\Pi_M^c(\theta, q)}{d\theta} = -c'(\theta)q + \frac{1}{4}S(2q). \quad (8)$$

The manufacturer determines θ and q jointly as follows:

Proposition 1. *With quality and inventory decisions and responsive pricing, in a centralized channel, the manufacturer sets the inventory–quality pair (q^c, θ^c) that uniquely solves the system of equations $S'(2q) = 2(c(\theta)/\theta)$ and $S(2q)/(2q) = 2c'(\theta)$, which together imply*

$$\epsilon(\theta^c)\eta(2q^c) = 1. \quad (9)$$

Moreover, q^c and θ^c are strategic substitutes.

Note that since $R(\theta, q) = (\theta/4)S(2q)$,

$$\frac{\partial R/\partial q}{R/q} = \frac{(\theta/2)S'(2q)}{(\theta/4)S(2q)/q} = \frac{(2q)S'(2q)}{S(2q)} = \eta(2q),$$

implying that $\eta(2q)$ appearing in the optimality condition (9) represents the elasticity of the revenue function. From (9) we see that the optimal inventory and quality are determined jointly at a combination where the elasticity of the revenue function with respect to the inventory q coincides with the reciprocal of the elasticity of the unit cost function $c(\theta)$. Note that we state the optimal solution in terms of a relationship between elasticities of revenue and cost of quality, in the spirit of Dorfman and Steiner (1954).

We also find that quality and inventory are strategic substitutes. Intuitively, increasing quality leads to a higher marginal cost of production, which increases the firm's losses on unsold inventory. To compensate for this, the firm produces less. Note that this result is not a priori obvious, since higher quality also increases the selling price (see Lemma 2), which should provide some pressure for higher inventory because of an increase in the cost of a "lost sale"; however, the cost of a lost sale is also a function of the marginal production cost, and the effect of increased cost from higher quality dominates the effect of increased price. As a result, all else equal, higher quality leads to lower inventory.

3.3. Decentralized Channel

In this section, we study the scenario in which the manufacturer sells the product to the consumers through a retailer. We focus on the wholesale price contract given its widespread use in the industry. Under this contract, the manufacturer sells the product to the retailer at a flat wholesale price. First, the manufacturer decides the quality θ and the wholesale price w before X is realized; next, the retailer decides the inventory q before X is realized; and finally, the retailer decides the retail price p after X is realized. The retailer's price decision is identical to the one described in Lemma 2, resulting in the expected revenue function $R(\theta, q) = (\theta/4)S(2q)$ found in Lemma 3. Then, given θ and w, the retailer sets the inventory level q that maximizes the profit

$$\Pi_R^d(\theta, w, q) = -wq + R(\theta, q) = -wq + \frac{\theta}{4}S(2q). \quad (10)$$

The manufacturer's profit function is $\Pi_M^d(\theta, w, q) = (w - c(\theta))q$. Given this, we establish that a wholesale price contract *cannot* coordinate a decentralized channel with quality, inventory, and responsive pricing decisions and uncertain demand. To see the argument behind this, note that the following first-order conditions need to be satisfied at the optimal q and θ for the decentralized case:

$$\frac{d\Pi_R^d(\theta, w, q)}{dq} = -w + \frac{\theta}{2}S'(2q),$$

$$\frac{d\Pi_M^d(\theta, w, q)}{d\theta} = -c'(\theta)q + (w - c(\theta))\frac{dq}{d\theta}.$$

From (7) and (8), we have the first-order conditions that need to be satisfied at the optimal q and θ for the centralized case. Following the approach in Krishnan and Winter (2007) (a similar approach was taken in Iyer 1998), the difference in first-order conditions is

$$\frac{d\Pi_R^d(\theta,w,q)}{dq} = \frac{d\Pi_M^c(\theta,q)}{dq} - (w - c(\theta)), \quad (11)$$

$$\frac{d\Pi_M^d(\theta,w,q)}{d\theta} = \frac{d\Pi_M^c(\theta,q)}{d\theta} + (w - c(\theta))\frac{dq}{d\theta} - \frac{1}{4}S(2q). \quad (12)$$

We can see from the above that both inventory and quality are subject to externalities. An increase in inventory is less valuable to the decentralized retailer because the manufacturer collects its wholesale margin. Therefore, there is a vertical externality that distorts inventory downward in the decentralized channel. Per (11), this externality cannot be eliminated unless w is chosen such that, at the optimal quality, $w = c(\theta)$. Thus, any coordinating wholesale price contract necessarily leaves the manufacturer with zero profit, and the manufacturer will not choose such a wholesale price; i.e., the channel is not coordinated. Furthermore, note that if the externality in (11) is eliminated, then there must be a negative externality in (12) (because $w - c(\theta) = 0$ still leaves the $-S(2q)/4$ term). This is why the channel cannot be coordinated in quality and inventory even if the manufacturer accepts $w = c(\theta)$.

The next lemma specifies the retailer's optimal inventory decision.

Lemma 4. *Given θ and w, the retailer sets $q(\theta,w) = 0$ if $\theta \le w$ and $q(\theta,w) > 0$ that uniquely solves the equation $S'(2q) = 2(w/\theta)$ if $\theta > w$.*

We invert the optimal inventory q as a function of w specified in Lemma 4 into the optimal wholesale price w as a function of q, utilizing the fact that q is monotone in w (this is evident from the condition $S'(2q) = 2(w/\theta)$, since $S''(y) < 0$). The inversion allows us to express the wholesale price as

$$w(\theta,q) = \frac{\theta}{2}S'(2q). \quad (13)$$

Substituting this in (10) yields the retailer's reduced profit

$$\Pi_R^d(\theta,q) = -w(\theta,q)q + \frac{\theta}{4}S(2q) = \frac{\theta}{4}[S(2q) - 2qS'(2q)],$$

and the manufacturer's reduced profit

$$\Pi_M^d(\theta,q) = (w(\theta,q) - c(\theta))q = -c(\theta)q + \frac{\theta}{2}qS'(2q). \quad (14)$$

Note that these expressions are of expected profit at the optimal retail price. Given this, we may derive the manufacturer's optimal wholesale price contract as follows:[4]

Proposition 2. *With quality and inventory decisions and responsive pricing, the manufacturer in a decentralized channel with a wholesale price contract sets the wholesale price–quality pair (w^d, θ^d), where $w^d \equiv \theta^d c'(\theta^d)$, that induces the retailer's choice of inventory level q^d, where the inventory–quality pair (q^d, θ^d) uniquely solves the system of equations $S'(2q)\eta_2(2q) = 2(c(\theta)/\theta)$ and $S'(2q) = 2c'(\theta)$. Together, these imply*

$$\epsilon(\theta^d)\eta_2(2q^d) = 1. \quad (15)$$

Furthermore, q^d and θ^d are strategic substitutes.

Let $R_2(\theta,q) = (\theta/4)S_2(2q)$ be the revenue function associated with inventory-constrained pricing ("inventory-constrained revenue function"). Note that

$$\frac{\partial R_2/\partial q}{R_2/q} = \frac{(\theta/2)S_2'(2q)}{(\theta/4)S_2(2q)/q} = \frac{(2q)S_2'(2q)}{S_2(2q)} = \eta_2(2q),$$

implying that $\eta_2(2q)$ appearing in the equilibrium condition (15) represents the elasticity of the inventory-constrained revenue function. From (15) we see that the equilibrium inventory and quality are determined jointly at a combination where the elasticity of the inventory-constrained revenue function with respect to the inventory q coincides with the reciprocal of the elasticity of the unit cost function $c(\theta)$. Note that, as for the centralized case, we state the optimal solution in terms of a relationship between elasticities of revenue and cost of quality, in the spirit of Dorfman and Steiner (1954).

3.4. Comparison Between Centralized and Decentralized Channel Outcomes: Impact of Demand Uncertainty

In this section, we compare the outcomes in the centralized and decentralized channels. This comparison helps us to derive several interesting insights driven by the impact of demand uncertainty on the channel outcomes.

First, from comparing the results in Propositions 1 and 2, it is clear that the wholesale price contract cannot coordinate the decentralized channel. However, we compare the optimal quality levels in the two cases and find the interesting result that quality can be higher in a decentralized channel because of the impact of demand uncertainty. We state this as a proposition.

Proposition 3. *With quality and inventory decisions and responsive pricing, the optimal quality in a decentralized channel with a wholesale price contract can be higher than, equal to, or lower than the optimal quality in a centralized channel.*

To see the argument behind this, consider (12). An increase in quality is more or less valuable to the decentralized manufacturer than a centralized manufacturer, depending on the net effect of the two externalities, $(w - c(\theta))(dq/d\theta) - \frac{1}{4}S(2q)$. The first term represents that an increase in quality boosts the retail price (all else equal) and hence increases the order quantity of the retailer; this increases the manufacturer's incentives to raise quality. The second term represents that a decentralized manufacturer fails to account for the increase in expected sales revenue resulting from higher quality; this decreases the manufacturer's incentives to raise quality. In other words, when increasing quality, the decentralized manufacturer enjoys a positive externality from *the quantity ordered by the retailer* but fails to account for the positive externality on eventual *sales of the item*. Depending on the problem parameters, one effect or the other will dominate, meaning quality can be higher or lower in the centralized channel compared to the decentralized channel.

In certain instances, the externality on the quantity ordered may exceed the externality on the sales of the item, which means that quality will be higher in the decentralized channel than in the centralized channel. Thus, in contrast to the known results from the quality decision literature (Jeuland and Shugan 1983, Economides 1999), decentralization may actually lead to *increased* product quality even with an extremely simple (uniform) consumer valuation distribution (unlike, e.g., Shi et al. 2013). We emphasize that what drives this difference is demand uncertainty, which distorts the relationship between quality and inventory choices through the newsvendor dynamics. Below, we present an example in which quality is higher in the decentralized channel under certain parametric conditions; specifically, quality under decentralization is higher if and only if distortion in inventory is large enough, i.e., if q^d is sufficiently small, compared to q^c.

Example 1. Suppose that the unit cost function $c(\theta)$ has the isoelastic form $c(\theta) = c_0\theta^n$ with $c_0 > 0$ and $n > 1$. Then $\theta^d > \theta^c$ if and only if $(\bar{F}(2q^d)/\bar{F}(2q^c))\eta_1(2q^c) > (3n-2)/(2n-1)$. Also, $q^c > q^d$.

Next, we turn to the impact of demand uncertainty on the firm's optimal decisions of quality, inventory, and price. We conduct this analysis first for the centralized channel and then for the decentralized channel. Given the complexity of the $S(q)$ and $c(\theta)$ functions, analytical study of the optimal decisions is quite challenging, so we employ a numerical analysis to illustrate several interesting effects. Specifically, we numerically determine the centralized firm's optimal decisions when the cost of quality is given by the isoelastic function $c(\theta) = c_0\theta^n$ and potential demand X follows a gamma distribution with mean μ and standard deviation σ.[5] The free parameters in this analysis are thus n and c_0 (components of the cost function) and μ and σ (components of the demand distribution). We focus on the impact of the standard deviation of demand σ, representing uncertainty in the market size. In Figure 1, we show representative plots for both the centralized and decentralized cases for $\mu = 100$, $c_0 = 2$, and $n = 2$.[6]

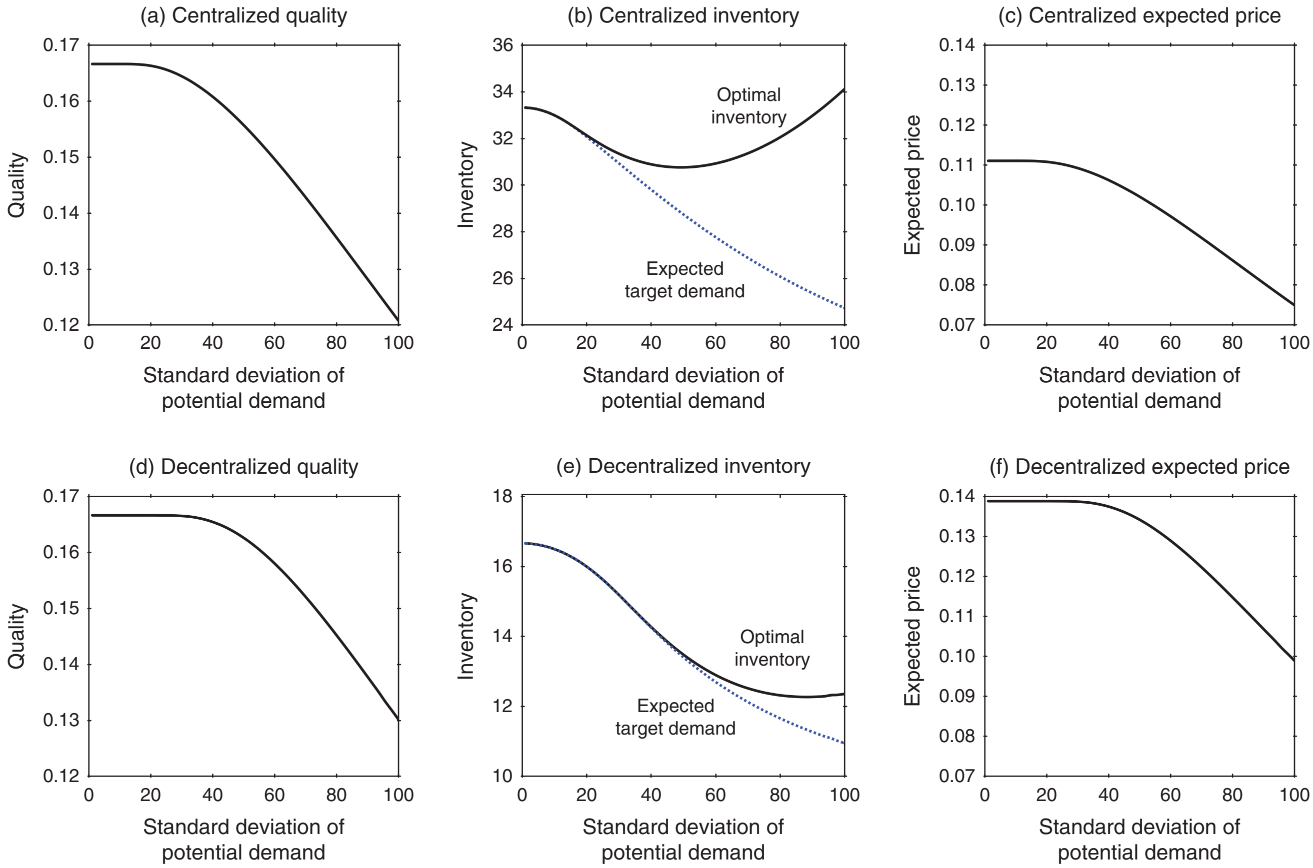
First, we find that quality is decreasing in σ (Figure 1(a)). This is because the benefit of quality (an increase in consumer willingness to pay, and hence the price) is earned on each unit sold, whereas the cost of quality is paid on each unit produced. Since greater uncertainty leads to a larger number of expected unsold units, the centralized firm will find it optimal to invest in a lower quality as demand uncertainty increases to reduce the cost of unsold units.

Next, we consider the impact of σ on inventory. The solid line in Figure 1(b) shows a surprising effect: inventory can be nonmonotonic in σ. Note that, by contrast, it is commonly found in the inventory literature that for any fixed, exogenous quality and price, inventory is monotonic in σ (e.g., as in the canonical newsvendor model); thus, we observe an interesting departure from the typical result. The reason for this difference when quality is endogenous is the presence of counteracting forces influencing the inventory decision: greater demand uncertainty can lead to lower optimal inventory, but also lower optimal quality, which in turn leads to higher optimal inventory (through the aforementioned substitution effect between quality and inventory). For a small σ, the former effect is stronger, while for a large σ, the latter effect dominates. The pattern in Figure 1(b) is obtained numerically for the gamma distribution. However, we have verified this for other distributions as well. In fact, this can be proved analytically for a uniform demand distribution and an isoelastic cost function, as stated below (proof available in the online appendix).

Example 2. Assume X is uniformly distributed on $[\mu - r, \mu + r]$ for $0 \le r \le \mu$ and $c(\theta) = c_0\theta^n$ with $n \ge 2$. Then, $\lim_{r\to 0}(dq^c/dr) < 0$ and $\lim_{r\to\mu}(dq^c/dr) > 0$.

This discussion shows that when quality is endogenous, inventory can vary with uncertainty in nontrivial ways, behaving counter to prevailing wisdom. This suggests that inventory in these cases should be greatest for low uncertainty products (because it is easy to predict demand) and high uncertainty products (because the optimal quality is low, leading to a cheaper product that the firm uses to flood the market). We also observe that the optimal inventory level is always greater than the expected target demand D (plotted using a dashed line alongside the optimal inventory level in Figure 1(b)); hence, it is optimal

Figure 1. (Color online) The Impact of Demand Uncertainty (Standard Deviation of Potential Demand, σ) on Quality, Inventory, and Price for a Centralized Firm (Top Row) and a Decentralized Firm (Bottom Row)



Note. In the example, demand follows a gamma distribution with $\mu = 100$, $c_0 = 2$, and $n = 2$.

to design a product (i.e., choose a quality level) that results in a positive amount of safety stock.

Figures 1(a) and 1(b) also show, however, that quality is more sensitive to demand uncertainty than inventory: over the plotted range of σ, quality changes by 38%, while inventory changes by less than 11%. By contrast, if quality is exogenously fixed at the deterministic optimal level, inventory varies by more than 36% over the same range. While the magnitude of this effect depends on the problem parameters, these observations suggest that in many cases the manufacturer in a centralized channel can use quality as a primary lever to mitigate the impact of demand uncertainty, and inventory as a secondary lever, because quality directly impacts the cost of unsold units and has a first-order effect on the firm's profit. In other words, the efficiency loss due to demand uncertainty may be mitigated more effectively by the product design decision than by the inventory decision. Note that most of the literature on product quality has ignored demand uncertainty, and most of the literature on demand uncertainty and inventory choice has ignored product quality. When the two are combined, quality can emerge as a preferred lever for managing uncertain demand in a centralized channel. Inventory is beneficial in managing uncertainty further, for a given quality level, though it may have a smaller effect that is significantly moderated by the quality decision.

In Figure 1(c), the expected optimal price (note that price is decided after demand is realized) decreases with demand uncertainty. This is primarily because quality (and therefore marginal cost) decreases with demand uncertainty and also because inventory increases for large enough σ; both of these forces lead to a downward pressure on price. By contrast, when quality is exogenously determined, the expected optimal price is not sensitive to demand uncertainty. (It would be displayed in the figure as a straight horizontal line.) In this case also, it is apparent that making quality endogenous and considering demand uncertainty can have a significant impact on the retail price that is charged.

Next, we perform the same analyses for the decentralized case under a wholesale price contract. On comparing Figure 1(d) with 1(a), 1(e) with 1(b), and 1(f) with 1(c), we note that as demand uncertainty increases, quality, inventory, and price follow the same patterns as in the centralized case; however, their levels are different—quality is weakly higher, inventory is lower, and price is higher in the decentralized channel compared to the centralized channel. Interestingly, in this example, we find that, different from the centralized channel case, in a decentralized channel, quality is no longer the primary lever for dealing with demand uncertainty. This is clear from the observation that even though inventory can still be nonmonotonic in σ while quality decreases in σ, the dependence of inventory on σ is stronger than in the centralized case—for the decentralized case, optimal quality varies by approximately 28% over the plotted range, while optimal inventory varies by approximately 36% (Figures 1(d) and 1(e)); for the centralized case, the variations in optimal quality and inventory are approximately 38% and 11% for the same range of σ (Figures 1(a) and 1(b)). Thus, the distortions introduced by an uncoordinated system cause the retailer to overreact to changes in demand uncertainty by adjusting the inventory level too much, compared to a centralized system. The manufacturer reduces quality as in the centralized case, but because of decentralization and that the retailer rather than the manufacturer controls inventory, the impact of the change in quality on the channel outcomes is indirect and therefore reduced. Furthermore, as seen in Figure 1(e), the optimal inventory level is always greater than the expected target demand D (as in the centralized case); hence, it is optimal for the manufacturer to design a product (i.e., choose a quality level) and choose a wholesale price for which it is optimal for the retailer to carry a positive amount of safety stock.

Finally, our analysis also provides the following intuitive results for both the centralized and decentralized cases: (1) If the scale of the demand changes without changing the variance, i.e., μ changes but not σ, then quality is not affected but inventory is; in other words, inventory is the preferred lever for dealing with changes in the level of demand. (2) If the scale of the cost of quality changes, i.e., c_0 changes, all else equal, then quality is affected but inventory is not.

4. Discussion and Conclusions

The quality level of a product is a central product design decision. We consider a situation in which a firm designs and produces a seasonal product with demand uncertainty and with significant lead time between production and sales. In this setting, there can be a mismatch between supply and demand, and we find that this aspect has important implications for quality and inventory choices for centralized and decentralized channels.

In a centralized channel, where the manufacturer makes all decisions, we demonstrate that at the optimal inventory–quality pair, the elasticity of the quality cost function equals the reciprocal of the elasticity of the expected revenue function, and that quality and inventory are strategic substitutes. We find that with endogenous quality, the inventory level can be nonmonotonic (specifically, U-shaped) in demand uncertainty. This is an interesting departure from the typical result that with exogenous quality and price, the inventory level is monotonic in demand uncertainty. Thus, we show that quality decisions can have a significant influence on other decisions such as inventory. In fact, we also show that because quality directly influences the per-unit cost of the product, quality choice can be a primary lever to mitigate the impact of demand uncertainty on profit, and this, in turn, may significantly moderate the use of inventory in the same role.

In a decentralized channel, where the manufacturer determines quality and contractual terms while a retailer chooses inventory and retail price, we show that a simple flat wholesale price contract cannot coordinate the channel. We demonstrate that at the optimal quality–inventory pair, the elasticity of the quality cost function equals the reciprocal of the elasticity of the "inventory-constrained expected revenue function." We find that product quality can be higher in this case compared to a centralized channel. This is an interesting result because previous research has stressed that quality decreases in a decentralized channel because of double marginalization (Villas-Boas 1998). In our case, quality can be higher because of the inclusion of inventory considerations—with a simple wholesale price contract, the manufacturer is shielded from the possibility of having to hold unsold inventory if demand realization is low (because the retailer bears this risk), and therefore the manufacturer does not internalize the cost of excessively high quality on unsold inventory. As in the centralized case, we find that quality decreases in demand uncertainty and inventory can be U-shaped, but, different from the centralized case, the role of quality as a lever for mitigating the effects of demand uncertainty is reduced, while the role of inventory for the same is enhanced.

Robustness to Timing of Pricing. In our analysis, we assume that pricing is responsive, i.e., price is determined after the potential demand has been realized. However, in some cases market size is not known until after the price has been established and the selling season has begun, and the firm may be unable to adjust the price in response to revealed demand information. We analyze this scenario to understand the robustness of our key results. Because of analytical complexity, we conduct a comprehensive numerical analysis. Our analysis shows that the key insights from our main model are reproduced under this alternative pricing sequence. First, we verify that with early pricing, quality can be higher in a decentralized channel than in a centralized channel. Second, we verify that with early pricing, the following can hold for both the centralized and decentralized channel cases: optimal quality and price decrease in demand uncertainty, but optimal inventory is U-shaped. More details are available in the online appendix.

Channel Coordination. In the analysis, we focus on the simple wholesale price contract for a decentralized channel because of its widespread use in the industry. However, advanced contracts such as quantity discount and buyback contracts are also reported to be used in the industry as per our interviews with practitioners.[7] These contracts are known to coordinate the channel in price and inventory (Jeuland and Shugan 1983, Moorthy 1987, Pasternack 1985, Iyer 1998, Lariviere and Porteus 2001, Cachon 2003, Krishnan and Winter 2007); we find that these contracts continue to coordinate even if quality is added to the mix. (In fact, the same result holds for two-part tariff and revenue-sharing contracts.) In other words, quality coordination does not pose additional challenges once inventory and price are coordinated through these contracts. This result occurs because each of these contracts achieves coordination by making the manufacturer profit (and retailer profit) an affine transformation of the centralized system profit; this, in turn, gives the manufacturer the correct incentives to set quality according to the centralized optimal level. Thus, any contract that achieves coordination in this manner (i.e., through affine transformations) will achieve the same outcome. Detailed analyses for the advanced contracts are provided in the online appendix.

To conclude, our research develops an understanding of how product quality, inventory, and price interact in centralized and decentralized channels. Further research can study richer problems of this flavor. For instance, we assume that the retailer has no impact on the quality of the product. However, retailers often impact the final consumer-perceived quality of a product by offering value-added services, and future research could explore the implications of this. Another interesting avenue for future research could be to understand the impact of supply chain management strategies such as quick response (Iyer and Bergen 1997, Cachon and Swinney 2011) on the manufacturer's incentives to invest in product quality and on the channel contracts that are used.

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Appendix

A.1. Preliminary Results with Proofs

Lemma A1. *$S_1(y)$ and $S_2(y)$ defined in (4) for $y \in (0, \bar{X})$ have the following properties*:

(a) *$S_1(y)$ is a concave increasing function with $\lim_{y\to 0} S_1(y) = 0$, $\lim_{y\to\bar{X}} S_1(y) = \mu$, $S_1'(y) = \bar{F}(y) > 0$, $\lim_{y\to 0} S_1'(y) = 1$, $\lim_{y\to\bar{X}} S_1'(y) = 0$, and $S_1''(y) = -f(y) < 0$.*

(b) *$S_2(y)$ is a quasiconcave function peaking at an interior point with $\lim_{y\to 0} S_2(y) = \lim_{y\to\bar{X}} S_2(y) = 0$, $S_2'(y) = -\bar{F}(y) + 2y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx$, $\lim_{y\to 0} S_2'(y) = 1$, $\lim_{y\to\bar{X}} S_2'(y) = 0$, and $S_2''(y) = f(y) - 2\int_y^{\bar{X}}(1/x)f(x)\,dx$.*

(c) *$S_2(y) < y\bar{F}(y)/(1+g(y)) < y\bar{F}(y) < S_1(y) < y$, $S_2'(y) < S_1'(y)$, and $(d/dy)(S_1(y)/S_2(y)) > 0$.*

Proof. The following identities, derived by integration by parts, are extensively used in proving the properties in this lemma:

$$\int_0^y x f(x)\,dx = -y\bar{F}(y) + \int_0^y \bar{F}(x)\,dx, \tag{A.1}$$

$$\int_y^{\bar{X}} \frac{1}{x} f(x)\,dx = \frac{1}{y}\bar{F}(y) - \int_y^{\bar{X}} \frac{1}{x^2}\bar{F}(x)\,dx. \tag{A.2}$$

(a) Differentiating $S_1(y) = \int_0^y \bar{F}(x)\,dx$ yields $S_1'(y) = \bar{F}(y) > 0$ and $S_1''(y) = -f(y) < 0$. The limiting values at the lower and upper bounds are $\lim_{y\to 0} S_1(y) = 0$, $\lim_{y\to\bar{X}} S_1(y) = \int_0^{\bar{X}} \bar{F}(x)\,dx = E[X] = \mu$, $\lim_{y\to 0} S_1'(y) = 1$, and $\lim_{y\to\bar{X}} S_1'(y) = 0$.

(b) Differentiating $S_2(y) = y^2\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx$ and using the identity (A.2), we get $S_2'(y) = -\bar{F}(y) + 2y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx$ and $S_2''(y) = f(y) - (2/y)\bar{F}(y) + 2\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx = f(y) - 2\int_y^{\bar{X}}(1/x)f(x)\,dx$. The limiting values at the lower and upper bounds are evaluated using L'Hôpital's rule: $\lim_{y\to 0} S_2(y) = \lim_{y\to\bar{X}} S_2(y) = \lim_{y\to\bar{X}} S_2'(y) = 0$ and $\lim_{y\to 0} S_2'(y) = 1$. These results together imply that $S_2(y)$ starts from zero at $y = 0$ with a positive slope and converges to zero as $y \to \bar{X}$, suggesting that the global maximum of $S_2(y)$ is found in the interior of the interval $(0, \bar{X})$ at a critical point $\hat{y}$ that satisfies the first-order condition $S_2'(\hat{y}) = 0$. To show that there is exactly one such critical point, note that the condition $S_2'(\hat{y}) = 0$ is equivalent to $\int_{\hat{y}}^{\bar{X}}(1/x^2)\bar{F}(x)\,dx = \bar{F}(\hat{y})/(2\hat{y})$. Substituting this in the expression of $S_2''(y)$ derived above yields $S_2''(\hat{y}) = -(1/\hat{y})\bar{F}(\hat{y})[1 - g(\hat{y})]$, where $g(y) \equiv yf(y)/\bar{F}(y)$ is the generalized failure rate of X. Suppose there are multiple critical points $\hat{y}_1, \hat{y}_2, \hat{y}_3, \ldots$ with $0 < \hat{y}_1 < \hat{y}_2 < \cdots < \bar{X}$ that satisfies the first-order condition $S_2'(\hat{y}_j) = 0$. Then, given continuity of the function $S_2''(y) = -(1/y)\bar{F}(y)[1 - g(y)]$ and the boundary values $\lim_{y\to 0} S_2(y) = \lim_{y\to\bar{X}} S_2(y) = 0$, $\lim_{y\to 0} S_2'(y) = 1 > 0$, and $\lim_{y\to\bar{X}} S_2'(y) = 0$, these critical points should alternate between a local maximizer and a local minimizer, i.e., $S_2''(\hat{y}_1) < 0$, $S_2''(\hat{y}_2) > 0$, $S_2''(\hat{y}_3) < 0, \ldots$. Recall that $g(y)$ is monotone increasing by the IGFR property, requiring $g(\hat{y}_1) < g(\hat{y}_2) < g(\hat{y}_3)$. ... This monotone sequence implies that there is at most one cutoff above which $g(\hat{y}_j) > 1$ and below which $g(\hat{y}_j) < 1$, which in turn implies that the sign of $S_2''(\hat{y}_j)$ changes at most once as j increases. However, this contradicts the earlier assertion that the sign of $S_2''(\hat{y}_j)$ alternates. We therefore conclude that there is exactly one critical point and it is the global maximum satisfying $S_2''(\hat{y}) < 0$. Hence,

$S_2(y)$ is quasiconcave with a unique maximum occurring in the interior.

(c) Since $\bar{F}(x)$ is decreasing, $\bar{F}(y) < \bar{F}(x) < \bar{F}(0) = 1$ for $0 < x < y$. Therefore $y\bar{F}(y) = \int_0^y \bar{F}(y)\,dx < S_1(y) = \int_0^y \bar{F}(x)\,dx < \int_0^y \bar{F}(0)\,dx = y$. To show $S_2(y) = y^2\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx < y\bar{F}(y)/(1+g(y))$, observe that $\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx = (1/y)\bar{F}(y) - \int_y^{\bar{X}}(1/x)f(x)\,dx$ from (A.2) satisfies

$$\int_y^{\bar{X}} \frac{1}{x^2}\bar{F}(x)\,dx = \frac{1}{y}\bar{F}(y) - \int_y^{\bar{X}} \frac{1}{x^2}g(x)\bar{F}(x)\,dx < \frac{1}{y}\bar{F}(y) - g(y)\int_y^{\bar{X}} \frac{1}{x^2}\bar{F}(x)\,dx,$$

where we used the fact that $g(x) > g(y)$ for $x > y$ since $g(x) = xf(x)/\bar{F}(x)$ is increasing by the IGFR property. Collecting the terms on the left- and right-hand sides of the above inequality, we get $\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx < \bar{F}(y)/(y[1+g(y)])$, which implies $S_2(y) = y^2\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx < y\bar{F}(y)/(1+g(y))$. Combining the two results, we have $S_2(y) < y\bar{F}(y)/(1+g(y)) < y\bar{F}(y) < S_1(y) < y$. Moreover, $\bar{F}(x) < \bar{F}(y)$ for $y < x < \bar{X}$ implies

$$S_2'(y) = -\bar{F}(y) + 2y\int_y^{\bar{X}} \frac{1}{x^2}\bar{F}(x)\,dx < -\bar{F}(y) + 2y\int_y^{\bar{X}} \frac{1}{x^2}\bar{F}(y)\,dx = -\bar{F}(y) + 2y\left(\frac{1}{y} - \frac{1}{\bar{X}}\right)\bar{F}(y) < -\bar{F}(y) + 2\bar{F}(y) = \bar{F}(y) = S_1'(y).$$

To prove $(d/dy)(S_1(y)/S_2(y)) > 0$, we first show $\eta_1(y) + g(y) - 1 > 0$, where $\eta_1(y) = yS_1'(y)/S(y)$ is defined in (5). Using the results from parts (a) and (b) and the identity $\int_0^y \bar{F}(x)\,dx = \int_0^y xf(x)\,dx + y\bar{F}(y)$, we may write $\eta_1(y)$ as $\eta_1(y) = 1 - (\int_0^y g(x)\bar{F}(x)\,dx)/(\int_0^y \bar{F}(x)\,dx)$. Since $g(x)$ is increasing by the IGFR property, $g(x) < g(y)$ for $x < y$. Hence, $\int_0^y g(x)\bar{F}(x)\,dx < g(y)\int_0^y \bar{F}(x)\,dx$, which implies $\eta_1(y) > 1 - (g(y)\int_0^y \bar{F}(x)\,dx)/(\int_0^y \bar{F}(x)\,dx) = 1 - g(y)$ and therefore confirms $\eta_1(y) + g(y) - 1 > 0$. We now differentiate $S_1(y)/S_2(y)$ and substitute the expressions from (4) and parts (a) and (b) to get

$$\frac{d}{dy}\left(\frac{S_1(y)}{S_2(y)}\right) = \frac{\bar{F}(y)(\int_0^y \bar{F}(x)\,dx) - [2y(\int_0^y \bar{F}(x)\,dx) - y^2\bar{F}(y)](\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx)}{S_2(y)^2}.$$

Note that since $\bar{F}(x) > \bar{F}(y)$ for $x < y$

$$2y\left(\int_0^y \bar{F}(x)\,dx\right) - y^2\bar{F}(y) > 2y\left(\bar{F}(y)\int_0^y dx\right) - y^2\bar{F}(y) = 2y^2\bar{F}(y) - y^2\bar{F}(y) = y^2\bar{F}(y) > 0.$$

Combining this with the earlier findings $\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx < \bar{F}(y)/(y[1+g(y)])$ and $\eta_1(y) + g(y) - 1 > 0$, we conclude

$$\frac{d}{dy}\left(\frac{S_1(y)}{S_2(y)}\right) > \frac{[1+g(y)]\bar{F}(y)(\int_0^y \bar{F}(x)\,dx) - 2\bar{F}(y)(\int_0^y \bar{F}(x)\,dx) + y\bar{F}(y)^2}{S_2(y)^2[1+g(y)]} = \frac{\bar{F}(y)(\int_0^y \bar{F}(x)\,dx)}{S_2(y)^2[1+g(y)]}[\eta_1(y) + g(y) - 1] > 0. \quad \square$$

Corollary A1. *$S(y) = S_1(y) + S_2(y)$ is a concave increasing function with $y\bar{F}(y) < S(y) < y + y\bar{F}(y)$, $\lim_{y\to 0} S(y) = 0$, $\lim_{y\to\bar{X}} S(y) = \mu$, $S'(y) = 2(\bar{F}(y) - y\int_y^{\bar{X}}(1/x)f(x)\,dx) = 2y\cdot\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx > 0$, $\lim_{y\to 0} S'(y) = 2$, $\lim_{y\to\bar{X}} S'(y) = 0$, and $S''(y) = 2(-(1/y)\bar{F}(y) + \int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx) = -2\int_y^{\bar{X}}(1/x)f(x)\,dx < 0$.*

Lemma A2. *$\eta_1(y)$, $\eta_2(y)$, and $\eta(y)$ defined in (5) for $y \in (0, \bar{X})$ have the following properties:*

(a) $\eta(y) = 2(S_2(y)/S(y))$, $\eta_2(y) = 1 + yS''(y)/S'(y)$, and $\eta_1(y)/\eta(y) = (2-\eta_2(y))/(2-\eta(y))$.

(b) $\eta_2(y) < 1 - g(y) < \eta_1(y)$ and $\eta_2(y) < \eta(y) < \eta_1(y)$.

(c) $\eta_1'(y) < 0$, $\lim_{y\to 0}\eta_1(y) = 1$, $\lim_{y\to\bar{X}}\eta_1(y) = 0$, and $y\eta_1'(y)/\eta_1(y) = 1 - g(y) - \eta_1(y)$.

(d) $\eta_2'(y) < 0$, $\lim_{y\to 0}\eta_2(y) = 1$, $\lim_{y\to\bar{X}}\eta_2(y) = 1 - \lim_{y\to\bar{X}} g(y)$, and $y\eta_2'(y)/\eta_2(y) = -(2/\eta_2(y) - 1)[1 - g(y) - \eta_2(y)]$.

(e) $\eta'(y) < 0$, $\lim_{y\to 0}\eta(y) = 1$, $\lim_{y\to\bar{X}}\eta(y) = 0$, and $y\eta'(y)/\eta(y) = \eta_2(y) - \eta(y)$.

Proof. We use the results from (4), Lemma A1, and Corollary A1 in the following proofs.

(a) From Corollary A1 we see that $S_2(y) = y^2\int_y^{\bar{X}}(1/x^2)\cdot\bar{F}(x)\,dx = (1/2)yS'(y)$. Hence, $S_2'(y) = (1/2)[S'(y) + yS''(y)]$. Using these results, we find $\eta(y) = yS'(y)/S(y) = 2S_2(y)/S(y)$ and $\eta_2(y) = yS_2'(y)/S_2(y) = 1 + yS''(y)/S'(y)$. Moreover, $S_1'(y) = S'(y) - S_2'(y) = S'(y) - (1/2)[S'(y) + yS''(y)] = (1/2)\cdot[S'(y) - yS''(y)]$, and therefore

$$\eta_1(y) = \frac{yS_1'(y)}{S_1(y)} = \frac{yS'(y)}{S(y)}\frac{1 - yS''(y)/S'(y)}{2 - yS'(y)/S(y)} = \eta(y)\frac{2-\eta_2(y)}{2-\eta(y)},$$

from which we get $\eta_1(y)/\eta(y) = (2-\eta_2(y))/(2-\eta(y))$.

(b) The proof for $1 - g(y) < \eta_1(y)$ is found in the proof of part (c) in Lemma A1. To show $\eta_2(y) < 1 - g(y)$, recall from (4) and Lemma A1 that $S_2(y) = y^2\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx$ and $S_2'(y) = -\bar{F}(y) + 2y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx$, and therefore $\eta_2(y) = yS_2'(y)/S_2(y) = 2 - \bar{F}(y)/(y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx) < 1 - g(y)$, where we use the inequality $\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx < \bar{F}(y)/(y[1+g(y)])$ from Lemma A1. Moreover, from Lemma A1, we have $(d/dy)(S_1(y)/S_2(y)) > 0$, which implies $S_2'(y)/S_2(y) < S_1'(y)/S_1(y)$. This in turn implies $S_2'(y)/S_2(y) < (S_1'(y) + S_2'(y))/(S_1(y) + S_2(y)) < S_1'(y)/S_1(y)$. Multiplying each term by y, we conclude $\eta_2(y) < \eta(y) < \eta_1(y)$.

(c) Recall from (4) and Lemma A1 that $S_1(y) = \int_0^y \bar{F}(x)\,dx$ and $S_1'(y) = \bar{F}(y)$, and therefore

$$\eta_1(y) = \frac{yS_1'(y)}{S_1(y)} = \frac{y\bar{F}(y)}{\int_0^y \bar{F}(x)\,dx} = 1 - \frac{\int_0^y xf(x)\,dx}{\int_0^y \bar{F}(x)\,dx} = 1 - \frac{\int_0^y g(x)\bar{F}(x)\,dx}{\int_0^y \bar{F}(x)\,dx},$$

where we use the identity $\int_0^y \bar{F}(x)\,dx = \int_0^y xf(x)\,dx + y\bar{F}(y)$. Differentiating $\eta_1(y)$ yields

$$\eta_1'(y) = \frac{\bar{F}(y)}{\int_0^y \bar{F}(x)\,dx}\left(-\frac{yf(y)}{\bar{F}(y)} + \frac{\int_0^y xf(x)\,dx}{\int_0^y \bar{F}(x)\,dx}\right) = \frac{\eta_1(y)}{y}[1 - g(y) - \eta_1(y)] < 0,$$

where the inequality follows from the result $\eta_1(y) > 1 - g(y)$ in part (b). Rearranging the above, we get $y\eta_1'(y)/\eta_1(y) = 1 - g(y) - \eta_1(y)$. The limiting values at the lower and upper bounds are evaluated using L'Hôpital's rule: $\lim_{y\to 0}\eta_1(y) = 1$ and $\lim_{y\to\bar{X}}\eta_1(y) = 0$.

(d) Recall from part (b) that $\eta_2(y) = 2 - \bar{F}(y)/(y\int_y^{\bar{X}}(1/x^2)\cdot\bar{F}(x)\,dx) < 2$. Differentiating $\eta_2(y)$ yields

$$\begin{aligned}\eta_2'(y) &= \frac{1}{y}\frac{\bar{F}(y)}{y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}\left(1 + \frac{yf(y)}{\bar{F}(y)} - \frac{\bar{F}(y)}{y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}\right)\\ &= \frac{1}{y}[2-\eta_2(y)][1+g(y)-(2-\eta_2(y))]\\ &= \frac{1}{y}[2-\eta_2(y)][g(y)+\eta_2(y)-1] < 0,\end{aligned}$$

where the inequality follows from the earlier observation $\eta_2(y) < 2$ and the result $\eta_2(y) < 1 - g(y)$ in part (b). Rearranging the above, we get $y\eta_2'(y)/\eta_2(y) = -(2/\eta_2(y)-1)[1-g(y)-\eta_2(y)]$. The limiting values at the lower and upper bounds are evaluated using L'Hôpital's rule, as follows. For the lower bound, since

$$\begin{aligned}\lim_{y\to 0}\left(y\int_y^{\bar{X}}\frac{1}{x^2}\bar{F}(x)\,dx\right) &= \lim_{y\to 0}\left(\frac{\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}{1/y}\right)\\ &= \lim_{y\to 0}\left(\frac{-(1/y^2)\bar{F}(y)}{-1/y^2}\right) = \lim_{y\to 0}\bar{F}(y) = 1,\end{aligned}$$

we have

$$\begin{aligned}\lim_{y\to 0}\eta_2(y) &= 2 - \lim_{y\to 0}\frac{\bar{F}(y)}{y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}\\ &= 2 - \frac{\lim_{y\to 0}\bar{F}(y)}{\lim_{y\to 0}(y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx)} = 2 - \frac{1}{1} = 1.\end{aligned}$$

For the upper bound, use the identity $\int_y^{\bar{X}}(1/x)f(x)\,dx = (1/y)\cdot\bar{F}(y) - \int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx$ from (A.2) to derive

$$\begin{aligned}\lim_{y\to\bar{X}}\eta_2(y) &= 2 - \lim_{y\to\bar{X}}\frac{\bar{F}(y)}{y\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}\\ &= 1 - \lim_{y\to\bar{X}}\left(\frac{\int_y^{\bar{X}}(1/x)f(x)\,dx}{\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}\right)\\ &= 1 - \lim_{y\to\bar{X}}\left(\frac{\int_y^{\bar{X}}(1/x^2)g(x)\bar{F}(x)\,dx}{\int_y^{\bar{X}}(1/x^2)\bar{F}(x)\,dx}\right)\\ &= 1 - \lim_{y\to\bar{X}}\left(\frac{-(1/y^2)g(y)\bar{F}(y)}{-(1/y^2)\bar{F}(y)}\right) = 1 - \lim_{y\to\bar{X}}g(y).\end{aligned}$$

(e) Differentiating $\eta(y) = yS'(y)/S(y)$ yields

$$\eta'(y) = \frac{S'(y)}{S(y)}\left(1 - \frac{yS'(y)}{S(y)} + \frac{yS''(y)}{S'(y)}\right) = \frac{\eta(y)}{y}[\eta_2(y) - \eta(y)] < 0,$$

where we used the results $\eta_2(y) = 1 + yS''(y)/S'(y)$ and $\eta_2(y) < \eta(y)$ proved in parts (a) and (b). Rearranging the above, we get $y\eta'(y)/\eta(y) = \eta_2(y) - \eta(y)$. To evaluate the limiting values at the lower and upper bounds, recall from Lemma A1 and Corollary A1 that $\lim_{y\to 0}S_2(y) = \lim_{y\to\bar{X}}S_2(y) = 0$, $\lim_{y\to 0}S_2'(y) = 1$, $\lim_{y\to 0}S(y) = 0$, $\lim_{y\to\bar{X}}S(y) = \mu$, and $\lim_{y\to 0}S'(y) = 2$. Using these results and the identity $\eta(y) = 2(S_2(y)/S(y))$ from part (a), we get $\lim_{y\to 0}\eta(y) = 2\lim_{y\to 0}S_2(y)/S(y) = 2\lim_{y\to 0}S_2'(y)/S'(y) = 2\cdot(1/2) = 1$ and $\lim_{y\to\bar{X}}\eta(y) = 2\lim_{y\to\bar{X}}S_2(y)/S(y) = 2\cdot(0/\mu) = 0$. □

Lemma A3. *$c(\theta)$ and $c(\theta)/\theta$ have the following properties:*

(a) *$c'(\theta) = c(\theta)\epsilon(\theta)/\theta$ and $c''(\theta) = (c(\theta)\epsilon(\theta)/\theta^2)(\epsilon(\theta) - 1 + \theta\epsilon'(\theta)/\epsilon(\theta))$.*

(b) *$(d/d\theta)(c(\theta)/\theta) = (c(\theta)/\theta^2)[\epsilon(\theta) - 1] > 0$ with boundary limits $\lim_{\theta\to 0}c(\theta)/\theta = 0$ and $\lim_{\theta\to\bar{\theta}}c(\theta)/\theta = 1$. Therefore, $c(\theta)/\theta < 1$ for all $\theta \in (0, \bar{\theta})$.*

(c) *$c'(\theta) > c(\theta)/\theta$ and $c''(\theta) > (d/d\theta)(c(\theta)/\theta)$.*

Proof. $c'(\theta) = c(\theta)\epsilon(\theta)/\theta > c(\theta)/\theta$ follows from the definition of $\epsilon(\theta)$ and the condition $\epsilon(\theta) > 1$ in Assumption 2. Moreover,

$$\begin{aligned}c''(\theta) &= \frac{d}{d\theta}\left(\frac{c(\theta)\epsilon(\theta)}{\theta}\right) = \epsilon(\theta)\frac{d}{d\theta}\left(\frac{c(\theta)}{\theta}\right) + \epsilon'(\theta)\frac{c(\theta)}{\theta}\\ &= \epsilon(\theta)\frac{c(\theta)}{\theta^2}[\epsilon(\theta)-1] + \epsilon'(\theta)\frac{c(\theta)}{\theta}\\ &= \frac{c(\theta)}{\theta^2}[\epsilon(\theta)^2 - \epsilon(\theta) + \theta\epsilon'(\theta)]\\ &= \frac{c(\theta)\epsilon(\theta)}{\theta^2}\left(\epsilon(\theta) - 1 + \frac{\theta\epsilon'(\theta)}{\epsilon(\theta)}\right).\end{aligned}$$

Differentiating $c(\theta)/\theta$ yields $(d/d\theta)(c(\theta)/\theta) = (\theta c'(\theta) - c(\theta))/\theta^2 = (c(\theta)/\theta^2)[\epsilon(\theta) - 1] > 0$ since $\epsilon(\theta) > 1$. Moreover, $\lim_{\theta\to 0}c(\theta)/\theta = \lim_{\theta\to 0}c'(\theta) = 0$, where we used L'Hôpital's rule. By the definition of $\bar{\theta}$, $\lim_{\theta\to\bar{\theta}}c(\theta)/\theta = 1$. Finally, since $\epsilon(\theta) > 1$ and $\epsilon'(\theta) \geq 0$ by Assumption 2, we have

$$\begin{aligned}c''(\theta) &= \frac{c(\theta)\epsilon(\theta)}{\theta^2}\left(\epsilon(\theta) - 1 + \frac{\theta\epsilon'(\theta)}{\epsilon(\theta)}\right) > \frac{c(\theta)}{\theta^2}[\epsilon(\theta) - 1]\\ &= \frac{d}{d\theta}\left(\frac{c(\theta)}{\theta}\right). \quad \square\end{aligned}$$

A.2. Proofs of Lemmas and Propositions

Proof of Lemma 1. At price p, only the consumers with utility $U = v\theta - p \geq 0$ purchase the product. Suppose $0 \leq p/\theta \leq 1$. Then only the consumers with valuation at or above $v = p/\theta$ purchase the product. At the cutoff, $U = (p/\theta)\theta - p = 0$. Hence, $\alpha = \int_{p/\theta}^1 dv = 1 - p/\theta$. Next, suppose $1 < p/\theta$. Then no consumer purchases the product since the consumer with the highest valuation $v = 1$ has utility $U = \theta - p < 0$. Hence, $\alpha = 0$. □

Proof of Lemma 2. Recall from (2) that the firm's revenue is given by $R = p\min\{(1 - p/\theta)X, q\}$, where we restrict p to the range $0 \leq p \leq \theta$ since $R = 0$ otherwise. Suppose $X > q$. If $p < (1 - q/X)\theta$, then $R = pq$, so it is optimal for the firm to increase p until the upper bound $(1 - q/X)\theta$ is reached. Since the upper bound is defined as a strict inequality, the maximum does not exist in this range of p. Hence, the maximum should exist in the range $(1 - q/X)\theta \leq p \leq \theta$, where $R = p(1 - p/\theta)X$. Observe that $\partial R/\partial p = (1 - 2(p/\theta))X$ and $\partial^2 R/\partial p^2 = -(2/\theta)X < 0$ with the boundary values $\lim_{p\to(1-q/X)\theta}\partial R/\partial p = 2q - X$ and $\lim_{p\to\theta}\partial R/\partial p = -X < 0$. Combined with the condition $X > q$, these results imply that the maximum is found at $p = \theta/2$ if $q < X < 2q$ and at $p = (1 - q/X)\theta$ if $X \geq 2q$. Next, suppose $X \leq q$. In this case $R = p(1 - p/\theta)X$, and the same analysis as above shows that it is optimal to set $p=\theta/2$. Summarizing all cases, the optimal p is $p=\theta/2$ if $X<2q$ and $p=(1-q/X)\theta$ if $X\geq 2q$. Equivalently, $p=\theta/2(1+(1-2q/X)^+)$. This implies $\alpha=1-p/\theta=(1/2)(1-(1-2q/X)^+)$, using the result from Lemma 1. We now evaluate the revenue function R. Suppose $X<2q$, when the firm sets $p=\theta/2$. These two conditions together imply $p/\theta>1-q/X$ or, equivalently, $(1-p/\theta)X<q$. Thus, from (2) we see that the revenue function becomes $R=p(1-p/\theta)X=(\theta/4)X$. Next, suppose $X\geq 2q$, when the firm sets $p=(1-q/X)\theta$, which implies $(1-p/\theta)X=q$. Thus, from (2) we see that $R=p(1-p/\theta)X=pq=\theta(1-q/X)q$. Combining the two cases, we have $R=(\theta/2)(\min\{X/2,q\}+q(1-2q/X)^+)$. □

Proof of Lemma 3. Taking expectation of $R(\theta,q,X)$ given in (3), we get

$$\begin{aligned}E[R(\theta,q,X)]&=\frac{\theta}{2}E\left[\min\left\{\frac{X}{2},q\right\}\right]+\frac{\theta}{2}qE\left[\left(1-2\frac{q}{X}\right)^+\right]\\&=\frac{\theta}{2}\left(\frac{1}{2}\int_0^{2q}xf(x)\,dx+q\int_{2q}^{\bar X}f(x)\,dx\right)\\&\quad+\frac{\theta}{2}q\int_{2q}^{\bar X}\left(1-2\frac{q}{x}\right)f(x)\,dx\\&=\frac{\theta}{4}\int_0^{2q}\bar F(x)\,dx+\theta q^2\int_{2q}^{\bar X}\frac{1}{x^2}\bar F(x)\,dx=\frac{\theta}{4}S(2q),\end{aligned}$$

where we use the identities $\int_0^y xf(x)\,dx=\int_0^y\bar F(x)\,dx-y\bar F(y)$ and $\int_y^{\bar X}(1/x)f(x)\,dx=(1/y)\bar F(y)-\int_y^{\bar X}(1/x^2)\bar F(x)\,dx$ from (A.2) and the definitions of $S_1(\cdot)$, $S_2(\cdot)$, and $S(\cdot)$ appearing in (4). □

Proof of Proposition 1. Note that $\Pi=-c(\theta)q+(\theta/4)S(2q)$ from (6) is defined for $2q\in(0,\bar X)$ and $\theta\in(0,\bar\theta)$. Differentiating Π with respect to q and θ yields $\partial\Pi/\partial q=-c(\theta)+(\theta/2)S'(2q)$ and $\partial\Pi/\partial\theta=-c'(\theta)q+(1/4)S(2q)$. Using the results from Assumption 2, Lemma A3, and Corollary A1, we see that the limiting values at the lower and upper bounds are $\lim_{q\to 0}(\partial\Pi/\partial q)=-c(\theta)+\theta>0$, $\lim_{q\to\bar X/2}(\partial\Pi/\partial q)=-c(\theta)<0$, $\lim_{\theta\to 0}(\partial\Pi/\partial\theta)=\frac{1}{4}S(2q)>0$, and

$$\begin{aligned}\lim_{\theta\to\bar\theta}\frac{\partial\Pi}{\partial\theta}&=-\left(\lim_{\theta\to\bar\theta}c'(\theta)\right)q+\frac{1}{4}S(2q)<-\left(\lim_{\theta\to\bar\theta}\frac{c(\theta)}{\theta}\right)q+\frac{1}{4}S(2q)\\&=-q+\frac{1}{4}S(2q)<-q+\frac{1}{4}[2q+2q\bar F(2q)]\\&<-q+\frac{1}{4}(4q)=0.\end{aligned}$$

These inequalities imply that a maximizer exists in the interior. Let $(\hat q,\hat\theta)$ be the maximizer, which is found from the first-order conditions $\partial\Pi/\partial q=0$ and $\partial\Pi/\partial\theta=0$ or $S'(2q)=2(c(\theta)/\theta)$ and $S(2q)/(2q)=2c'(\theta)$. Dividing the first equation by the second yields $\epsilon(\hat\theta)\eta(2\hat q)=1$, where $\epsilon(\theta)=\theta c'(\theta)/c(\theta)$ and $\eta(y)=yS'(y)/S(y)$, as defined in (5) and (1). The Hessian H of Π has the determinant equal to $\det H=H_{qq}H_{\theta\theta}-(H_{q\theta})^2=\theta S''(2q)(-c''(\theta)q)-(-c'(\theta)+\frac{1}{2}S'(2q))^2$. Using the identities $c'(\theta)=(c(\theta)\epsilon(\theta))/\theta$ and $c''(\theta)=((c(\theta)\epsilon(\theta))/\theta^2)(\epsilon(\theta)-1+(\theta\epsilon'(\theta))/\epsilon(\theta))$ from Lemma A3 and $y\eta'(y)/\eta(y)=\eta_2(y)-\eta(y)=1+yS''(y)/S'(y)-\eta(y)$ from Lemma A2, we can rewrite $\det H$ as

$$\begin{aligned}\det H=&\frac{c(\theta)\epsilon(\theta)}{\theta}\left(\epsilon(\theta)-1+\frac{\theta\epsilon'(\theta)}{\epsilon(\theta)}\right)\left(1-\eta(2q)-\frac{y\eta'(2q)}{\eta(2q)}\right)\\&\cdot\frac{S'(2q)}{2}-\left(\frac{1}{2}S'(2q)-\frac{c(\theta)\epsilon(\theta)}{\theta}\right)^2.\end{aligned}$$

Evaluating this at the maximizer $(\hat q,\hat\theta)$ using the first-order condition $S'(2\hat q)=2(c(\hat\theta)/\hat\theta)$ derived above yields

$$\begin{aligned}\det\hat H=&\frac{c(\hat\theta)^2\epsilon(\hat\theta)}{\hat\theta^2}\left(\epsilon(\hat\theta)-1+\frac{\hat\theta\epsilon'(\hat\theta)}{\epsilon(\hat\theta)}\right)\left(1-\eta(2\hat q)-\frac{(2\hat q)\eta'(2\hat q)}{\eta(2\hat q)}\right)\\&-\frac{c(\hat\theta)^2}{\hat\theta^2}[\epsilon(\hat\theta)-1]^2.\end{aligned}$$

Since $\epsilon(\theta)>1$, $\epsilon'(\theta)>0$, and $\eta'(y)<0$ by Assumption 2 and Lemma A2, we have

$$\begin{aligned}\det\hat H&>\frac{c(\hat\theta)^2}{\hat\theta^2}\epsilon(\hat\theta)[\epsilon(\hat\theta)-1][1-\eta(2\hat q)]-\frac{c(\hat\theta)^2}{\hat\theta^2}[\epsilon(\hat\theta)-1]^2\\&=\frac{c(\hat\theta)^2}{\hat\theta^2}[\epsilon(\hat\theta)-1][-\epsilon(\hat\theta)\eta(2\hat q)+1]=0,\end{aligned}$$

where we used the optimality condition $\epsilon(\hat\theta)\eta(2\hat q)=1$. Since $\hat H_{qq}<0$, $\hat H_{\theta\theta}<0$, and $\det\hat H>0$, $\hat H$ is negative definite, and thus the interior maximizer is unique. Finally, to prove that $\hat q$ and $\hat\theta$ are substitutes, note that the cross partial $H_{q\theta}$ evaluated at $(\hat q,\hat\theta)$ satisfies $\hat H_{q\theta}=-c'(\hat\theta)+\frac{1}{2}S'(2\hat q)=-(c(\hat\theta)\epsilon(\theta))/\hat\theta+c(\hat\theta)/\hat\theta=-(c(\hat\theta)/\hat\theta)[\epsilon(\hat\theta)-1]<0$. This implies $\partial\hat q/\partial\hat\theta=-\hat H_{q\theta}/\hat H_{qq}<0$, i.e., $\hat q$ and $\hat\theta$ are substitutes. □

Proof of Lemma 4. Let $\Pi=-wq+(\theta/4)S(2q)$ be the retailer profit from (10). Differentiating this with respect to q and using the results from Corollary A1, we find $\partial\Pi/\partial q=-w+(\theta/2)S'(2q)$ and $\partial^2\Pi/\partial q^2=\theta S''(2q)<0$ with the limiting values at the lower and upper bounds $\lim_{q\to 0}(\partial\Pi/\partial q)=-w+(\theta/2)\lim_{y\to 0}S'(y)=-w+(\theta/2)\cdot 2=\theta-w$ and $\lim_{q\to\bar X/2}(\partial\Pi/\partial q)=-w+(\theta/2)\lim_{y\to\bar X}S'(y)=-w<0$. These results imply that the maximum of Π is found at $q=0$ if $\theta\leq w$, while it is found in the interior otherwise, specified by the first-order condition $\partial\Pi/\partial q=0$, which yields the equation $S'(2q)=2(w/\theta)$. □

Proof of Proposition 2. Note that $\Pi=-c(\theta)q+(\theta/2)qS'(2q)$ from (14) is defined for $2q\in(0,\bar X)$ and $\theta\in(0,\bar\theta)$. Differentiating Π with respect to q and applying the identity $\eta_2(y)=1+yS''(y)/S'(y)$ from Lemma A2, we get

$$\begin{aligned}\frac{\partial\Pi}{\partial q}&=-c(\theta)+\frac{\theta}{2}[S'(2q)+2qS''(2q)]\\&=-c(\theta)+\frac{\theta}{2}[S'(2q)-S'(2q)(1-\eta_2(2q))]\\&=-c(\theta)+\frac{\theta}{2}S'(2q)\eta_2(2q).\end{aligned}$$

Moreover, differentiating Π with respect to the variable θ yields $\partial\Pi/\partial\theta=-c'(\theta)q+\frac{1}{2}qS'(2q)$. Using the results from Assumption 2, Lemmas A2 and A3, and Corollary A1, we see that the limiting values at the lower and upper bounds of $\partial\Pi/\partial q$ are $\lim_{q\to 0}(\partial\Pi/\partial q)=-c(\theta)+\theta>0$ and $\lim_{q\to\bar X/2}(\partial\Pi/\partial q)=-c(\theta)<0$, where we use the regularity condition $\lim_{x\to\bar{X}} g(x) > 1$ to prove the inequality in the second line. Moreover, $\lim_{\theta\to 0}(\partial\Pi/\partial\theta) = \frac{1}{2}qS'(2q) > 0$ and

$$\lim_{\theta\to\bar{\theta}} \frac{\partial\Pi}{\partial\theta} = -\Big(\lim_{\theta\to\bar{\theta}} c'(\theta)\Big)q + \frac{1}{2}qS'(2q) < -\Big(\lim_{\theta\to\bar{\theta}} \frac{c(\theta)}{\theta}\Big)q + \frac{1}{2}qS'(2q) = -q + \frac{1}{2}qS'(2q) < q\Big(-1+\frac{1}{2}\cdot 2\Big) = 0,$$

where the last line is proved using the fact that $S'(y)$ has the maximum at $y = 0$ with $S'(0) = 2$ (see Corollary A1). These inequalities imply that a maximizer exists in the interior. Let $(\hat{q}, \hat{\theta})$ be the maximizer, which is found from the first-order conditions $\partial\Pi/\partial q = 0$ and $\partial\Pi/\partial\theta = 0$ or $S'(2q)\eta_2(2q) = 2(c(\theta)/\theta)$ and $S'(2q) = 2c'(\theta)$. Dividing the first equation by the second yields $\epsilon(\hat{\theta})\eta_2(2\hat{q}) = 1$, where $\epsilon(\theta) = \theta c'(\theta)/c(\theta)$ and $\eta_2(y) = yS_2'(y)/S_2(y)$, as defined in (5) and (1). Note that the above condition implies $0 < \eta_2(2\hat{q}) < 1$ since $\epsilon(\hat{\theta}) > 1$ by Assumption 2. Even though $\eta_2(y)$ may assume a negative value for sufficiently large y (see Lemma A2), $\eta_2(2\hat{q}) > 0$ when it is evaluated at the maximizer $(\hat{q}, \hat{\theta})$. We now compute the components of Hessian H of Π. Using the identities $\eta_2(y) = 1 + yS''(y)/S'(y)$ and $y\eta_2'(y)/\eta_2(y) = -(2/\eta_2(y) - 1)[1 - g(y) - \eta_2(y)]$ from Lemma A2, we can write $H_{qq} \equiv \partial^2\Pi/\partial q^2$ as

$$H_{qq} = \theta[S''(2q)\eta_2(2q) + S'(2q)\eta_2'(2q)] = -\theta\frac{S'(2q)}{2q}[2 - 2g(2q) - 2\eta_2(2q) + g(2q)\eta_2(2q)].$$

Evaluating this at $(\hat{q}, \hat{\theta})$ using the first-order condition $S'(2q)\eta_2(2q) = 2(c(\theta)/\theta)$ yields

$$\hat{H}_{qq} = -\frac{c(\hat{\theta})}{\hat{q}}\frac{2 - 2g(2\hat{q}) - 2\eta_2(2\hat{q}) + g(2\hat{q})\eta_2(2\hat{q})}{\eta_2(2\hat{q})} = -\frac{c(\hat{\theta})}{\hat{q}}\Big(2\frac{1-g(2\hat{q})}{\eta_2(2\hat{q})} - 2 + g(2\hat{q})\Big).$$

Since $\eta_2(2\hat{q}) > 0$ as noted above and $\eta_2(2\hat{q}) < 1 - g(2\hat{q})$ as proved in Lemma A2, $(1 - g(2\hat{q}))/(\eta_2(2\hat{q})) > 1$, and therefore $\hat{H}_{qq} < -c(\hat{\theta})(g(2\hat{q})/\hat{q}) < 0$. We can rewrite $\hat{H}_{qq}$ as follows using the optimality condition $\epsilon(\hat{\theta})\eta_2(2\hat{q}) = 1$:

$$\hat{H}_{qq} = -\frac{c(\hat{\theta})}{\hat{q}}(2\epsilon(\hat{\theta})[1 - g(2\hat{q})] - 2 + g(2\hat{q})) = -\frac{c(\hat{\theta})}{\hat{q}}(2[\epsilon(\hat{\theta}) - 1] - [2\epsilon(\hat{\theta}) - 1]g(2\hat{q})). \tag{A.3}$$

Next, evaluating $H_{\theta\theta} \equiv \partial^2\Pi/\partial\theta^2$ yields $H_{\theta\theta} = -c''(\theta)q < 0$. Rewriting this using the relation $c''(\theta) = ((c(\theta)\epsilon(\theta))/\theta^2) \cdot (\epsilon(\theta) - 1 + (\theta\epsilon'(\theta))/\epsilon(\theta))$ from Lemma A3 and evaluating it at $(\hat{q}, \hat{\theta})$, we get

$$\hat{H}_{\theta\theta} = -c''(\hat{\theta})\hat{q} = -\frac{c(\hat{\theta})\epsilon(\hat{\theta})}{\hat{\theta}^2}\Big(\epsilon(\hat{\theta}) - 1 + \frac{\hat{\theta}\epsilon'(\hat{\theta})}{\epsilon(\hat{\theta})}\Big)\hat{q}. \tag{A.4}$$

Next, evaluating $H_{q\theta} \equiv \partial^2\Pi/\partial\theta\partial q = \partial^2\Pi/\partial q\partial\theta$ yields $H_{q\theta} = -c'(\theta) + \frac{1}{2}S'(2q)\eta_2(2q)$. Rewriting this using the relation $c'(\theta) = (c(\theta)\epsilon(\theta))/\theta$ from Lemma A3 and evaluating it at $(\hat{q}, \hat{\theta})$ using the first-order condition $S'(2q)\eta_2(2q) = 2(c(\theta)/\theta)$, we get

$$\hat{H}_{q\theta} = -\frac{c(\hat{\theta})\epsilon(\hat{\theta})}{\hat{\theta}} + \frac{c(\hat{\theta})}{\hat{\theta}} = -\frac{c(\hat{\theta})}{\hat{\theta}}[\epsilon(\hat{\theta}) - 1] < 0, \tag{A.5}$$

where the inequality follows from the condition $\epsilon(\theta) > 1$ found in Assumption 2. Combining (A.3)–(A.5), we compute the determinant of H evaluated at the maximizer $(\hat{q}, \hat{\theta})$ as follows:

$$\det\hat{H} = \frac{c(\hat{\theta})^2\epsilon(\hat{\theta})}{\hat{\theta}^2}(2[\epsilon(\hat{\theta}) - 1] - [2\epsilon(\hat{\theta}) - 1]g(2\hat{q})) \cdot \Big(\epsilon(\hat{\theta}) - 1 + \frac{\hat{\theta}\epsilon'(\hat{\theta})}{\epsilon(\hat{\theta})}\Big) - \frac{c(\hat{\theta})^2}{\hat{\theta}^2}[\epsilon(\hat{\theta}) - 1]^2.$$

Since $\epsilon(\theta) > 1$ and $\epsilon'(\theta) > 0$ by Assumption 2, we have

$$\det\hat{H} > \frac{c(\hat{\theta})^2\epsilon(\hat{\theta})}{\hat{\theta}^2}(2[\epsilon(\hat{\theta}) - 1] - [2\epsilon(\hat{\theta}) - 1]g(2\hat{q}))[\epsilon(\hat{\theta}) - 1] - \frac{c(\hat{\theta})^2}{\hat{\theta}^2}[\epsilon(\hat{\theta}) - 1]^2 = \frac{c(\hat{\theta})^2}{\hat{\theta}^2}[\epsilon(\hat{\theta}) - 1][2\epsilon(\hat{\theta}) - 1]\Big(\frac{1 - g(2\hat{q})}{\eta_2(2\hat{q})} - 1\Big) > 0,$$

where in the last line we used the optimality condition $\epsilon(\hat{\theta})\eta_2(2\hat{q}) = 1$ and the inequality $\eta_2(2\hat{q}) < 1 - g(2\hat{q})$ proved in Lemma A2. Since $\hat{H}_{qq} < 0$, $\hat{H}_{\theta\theta} < 0$, and $\det\hat{H} > 0$, $\hat{H}$ is negative definite, and thus the interior maximizer is unique. To prove that $\hat{q}$ and $\hat{\theta}$ are substitutes, recall from (A.5) that $\hat{H}_{q\theta} < 0$. This implies $\partial\hat{q}/\partial\hat{\theta} = -\hat{H}_{q\theta}/\hat{H}_{qq} < 0$, i.e., $\hat{q}$ and $\hat{\theta}$ are substitutes. The expression for the wholesale price $w = \theta^d c'(\theta^d)$ is obtained by substituting the optimality condition $S'(2q) = 2c'(\theta)$ in (13). □

Proof of Proposition 3. The proof of this proposition is based on the arguments made in the main text of the paper and is omitted here to avoid redundancy. □

Endnotes

[1] In Section 4, we briefly discuss advanced contracts, including the quantity discount contract and the buyback contract, which were also revealed by our investigation to be popular in the industry, and the revenue-sharing contract and two-part tariff contract, which are of theoretical interest. We find that all of these contracts can coordinate the channel in quality, inventory, and price.

[2] Note that the value of one on the right-hand side of the equation $c(\bar{\theta})/\bar{\theta} = 1$ comes from the assumption that consumer valuation is distributed uniformly on $[0, 1]$; a different number will appear if the upper bound of the support is not equal to one.

[3] In Lemma A1, Corollary A1, and Lemma A2, we outline certain properties of $S_j(y)$, $S(y)$, $\eta_j(y)$, and $\eta(y)$ that are instrumental in obtaining the key results.

[4] In Proposition 2 and the subsequent discussion, for the sake of brevity and following the style of Lariviere and Porteus (2001), we assume that the retailer's reservation profit is sufficiently small so that the retailer's participation constraint does not bind under the manufacturer's optimal wholesale price contract.

[5] With shape parameter α and scale parameter β, this corresponds to $\alpha = (\mu/\sigma)^2$ and $\beta = (\sigma^2/\mu)$.

[6] We also considered each of the parameter values $n = \{1.2, 1.5, 2, 8\}$ and demand following uniform, lognormal, normal, and logistic distributions (in addition to gamma). In each case, we assumed $c_0 = 2$ and $\mu = 100$, and we examined 50 different values of σ. Thus, in total, we considered 1,000 problem instances and verified that our qualitative insights hold in each case.

[7] In a quantity discount contract, the manufacturer sells the product to the retailer at a wholesale price that can vary with the amount of

product ordered by the retailer. In a buyback contract, the manufacturer sells inventory to the retailer for a wholesale price and, after demand realization, buys back any unsold units at a prespecified buyback price.

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