A Structural Model of Bank Balance Sheet Synergies and the Transmission of Central Bank Policies

William Diamond
Wharton

Zhengyang Jiang
Northwestern Kellogg

Yiming Ma
Columbia Business School

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Abstract

This paper estimates a structural model of unconventional monetary policy transmission through bank balance sheets using cross-sectional instruments for loan and deposit demand. We estimate the demand for banking at a branch-specific level from the response of a bank's quantities at one branch to interest rate changes caused by demand shocks at other branches. Depositors are considerably less sensitive to interest rates that corporate or mortgage borrowers. We use our demand estimates to infer a bank's marginal cost of borrowing and lending and apply a novel procedure to estimate how these costs depend on the composition of a bank's entire balance sheet. We use our estimated model for a counterfactual quantitative easing, in which a $4.76 trillion supply of bank reserves causes a 15 basis point increase in the yield on reserves. The increase in reserves crowds out bank lending by $555.9 billion, which implies that liquid reserves and illiquid loans are substitutes rather than complements for banks. We also find a modest $15.4 billion increase in bank deposits due to their inelastic demand.

1 Introduction

In the aftermath of the Covid-19 pandemic, the expansion of the Quantitative Easing (QE) program has nearly doubled the outstanding volume of central bank reserves from \$1.72 trillion in February to \$3.22 trillion at the end of May 2020. In the US, QE was first introduced as one of the most important policy responses by the Federal Reserve to the 2008 financial crisis. It involves the purchase of Treasury debt and mortgage-backed securities, financed by the issuance of central bank reserves. Importantly, these reserves are safe and liquid assets which can only be held on the balance sheets of US banks. What is the impact of these large increase in reserve supply on the banking system? How do they impact the prices and quantities of deposits and loans offered by banks? Do they crowd out the need for banks to invest in non-reserve assets, or allow banks to actually increase their lending to the economy?

Central bank policy interventions like QE are usually responses to macroeconomic conditions. The behavior of the banking system after a policy intervention may therefore be caused by economic fundamentals and not necessarily the policy itself. This paper develops and estimates a structural model of the US banking system to analyze the effects of such central bank policy interventions. The model has two important features. First, it captures the demand for banks' deposits, mortgages, and loans in an imperfectly competitive framework. Second, it accounts for the fact that banks supply these products jointly. For example, the cost of providing illiquid mortgages or loans may depend on the available quantity of liquid bank reserves. As result, the supply side of the model tells us how central bank interventions impact banks' cost of borrowing and lending, while the demand side tells us how these cost changes pass through to quantities of deposits, mortgages, and loans in equilibrium.

In our model, the passthrough of reserve supply increases to the banking system depends on two key quantities. The first is how an increase in a bank's holding of reserves impacts its cost of providing deposits, loans, and mortgages. Theoretically, reserve holdings could either increase or decrease a bank's cost of lending and providing deposits. One effect of reserves is to mitigate the mismatch between banks' liquid liabilities (i.e., deposits) and their illiquid asset holdings (i.e., mortgages and loans), so that reserve holdings would reduce the cost of lending. Conversely, if a bank has a scarce quantity of funding that it can invest in either reserves or other assets, reserve holdings may crowd out bank lending. The second key quantity is how changes in lending or borrowing costs pass through to equilibrium interest rates and quantities, which is determined by the elasticity of demand banks face in deposit, loan, and mortgage markets. With our estimated model, we show that a $4.76 trillion increase in the supply of bank reserves crowds out bank lending by $555.9 billion, which implies that liquid reserves and illiquid loans are substitutes rather than complements for banks. We also find a modest $15.4 increase in bank deposits.

Our model features an imperfectly competitive market for bank deposits, mortgages, and corporate loans provided by profit-maximizing banks. The demand side is modeled using logit demand systems common in the industrial organization literature (Berry, 1994). Consumers view deposits from different banks (and similarly mortgages and loans) as differentiated products, and gradually shift away from a bank's deposits as the deposit rate decreases. We estimate the elasticity of demand by observing how consumers respond to exogenous supply shocks to interest rates. Our internal-capital-market based identification strategy relies crucially on the use of branch-level data. Because banks reallocate resources across their network of branches, a demand shock to a bank's branch in one region causes indirect supply shocks to branches of the same bank in other regions. We use increases in borrowing demand after natural disasters to construct a supply shock across bank branch networks following Cortés and Strahan (2017) to estimate our demand systems.

Our demand estimates show that the demand for bank deposits is considerably less interest-rate sensitive than the demand for mortgages or corporate loans. For comparison, if all banks in a market raise their deposit rates by 10 basis points, this will only cause a 1.3 % in overall deposit quantities. Such a rate increase would cause a 4.4 % decrease in mortgage quantities and a 22.8 % decrease in corporate loan quantities. This result is consistent with the intuition that mortgage borrowers, and particularly larger firms obtaining commercial loans, are extremely sensitive to small rate differences and maximize profits aggressively. Retail depositors, however, are "sleepy" and less sensitive to changes in their deposit rates. Therefore, as a bank's balance sheet cost changes, its deposit quantities will remain relatively sticky while its lending might vary significantly with the rate offered to borrowers.

We next infer banks' cost of providing deposits, loans, and mortgages. Banks inherently resemble multi-product firms, where the firm that provides deposits invariably also provides loans. The joint provision of deposits and loans suggests that a bank's cost of lending depends on its ability to raise deposit financing. A large theoretical literature provides explanations for this so-called asset-liability synergy (Diamond and Rajan, 2000; Kashyap et al., 2002; Hanson et al., 2015; Diamond, 2019). Our paper is the first to quantitatively estimate these synergies using data, and to quantify the important role they play in the transmission mechanism of central bank policies.

A key methodological advance in this paper is to show how the synergies on balance sheets can be identified by jointly using two separate sources of cross-sectional variation in loan demand and deposit demand. If a bank's cost of issuing deposits only depended on the quantity of deposits produced, we could identify its deposit supply curve using only deposit demand shocks. However, if a bank adjusts both its deposit and loan quantities when loan demand changes, it is impossible to tell whether its cost of deposit issuance is sensitive to only the existing stock of deposits or also to the amount of loans on balance sheet. By observing a bank' response to both a deposit demand shock, which disproportionately impacts deposit quantities, and a loan demand shock, which disproportionately impacts loan quantities, we can infer precisely how a bank's cost of supplying deposits, mortgages, and loans depends on their existing volumes on bank balance sheets. We use a Bartik-style instrument for deposit demand and our previous-mentioned disaster instrument for loan demand. Despite the complexity of banks' cost synergies, our approach reduces simply to using several instrumental variable regressions jointly. Another benefit of our identification strategy is that we also learn how a bank's cost of producing deposits and loans depends on its holding of liquid securities such as bank reserves, which is crucial for understanding how the banking system responds to increase in bank reserves due to QE.

Our cost function estimates imply that increasing a bank's reserve holdings crowds out mortgage and corporate lending and crowds in deposit issuance. We find that mortgages and corporate loans are nearly perfect substitutes for banks to hold and that a bank's cost of lending is reduced when it has more deposit financing. Using data from 2007, we find that a $1 trillion increase in bank reserves distributed across all banks leads to a 1.1 basis point increase in mortgage costs, a 1.49 basis point increase in loan costs, a 2.3 basis point reduction in deposit costs, and a 3.73 basis point increase in the required return on reserves. To map these cost changes to the equilibrium impact of QE on the banking system, we present a counterfactual analysis using both our estimated cost function and demand systems.

With our estimated model, we infer the effects of an increased supply of bank reserves on the banking system. We proportionately increase banks' holdings of liquid securities and allow them to change their interest rates on deposits, mortgages, and loans. We then use our demand elasticity estimates for deposits, mortgages, and loans to infer how the quantities of these goods respond to changes in interest rates. We find that a $ 4.25 trillion increase in the supply of reserves pushes up the reserves rate by 15 basis points. This is comparable to the size of the spread between the interest rate on excess reserves and the federal funds rate during the QE period, suggesting that our results have the right order of magnitude.

At the same time, we find that banks' holdings of reserves and other liquid assets are a substitute and not a compliment for bank loans, while deposit and mortgage markets are minimally impacted by banks' holding of liquid reserves. An additional $ 4.25 trillion in the supply of reserves induces a pass-through of 6.19 basis points in deposit rates, 3.86 basis points in mortgage rates, and 5.20 basis points in corporate loan rates. These rate changes imply a $15.4 billion increase in deposits, a $6.1 billion decrease in mortgages, and a $555.9 billion decrease in corporate loans. The pass-through of QE to deposit and mortgage quantities through the reserve supply channel we analyze is small, but the crowding-out effect reduces corporate loan quantities by 13 % of the size of the reserve supply increase. This result implies that one more dollar in reserves takes up of $0.13 in bank balance sheet capacity, which would have been available for lending to the real economy. Our findings provide important considerations for designing the scale and timing of QE, especially since of the main goals is to stimulate lending to the non-financial sector.

Literature Review

We provide the first structural model to quantify synergies between illiquid loans, liquid securities and liabilities on bank balance sheets. The interaction between different components of bank balance sheets has been studied by the seminal theory literature (e.g. Diamond and Rajan (2000) and Kashyap, Rajan, and Stein (2002)). More recently, Hanson, Shleifer, Stein, and Vishny (2015) show that commercial banks create money-like claims from illiquid fixed-income assets by relying on costly equity capital and deposit insurance. Diamond (2019) shows how banks optimally issue riskless deposits backed the least risky portfolio: a diversified pool of non-financial firm debt. Empirical studies are rare because balance sheet components co-move for many different reasons in the time series. Our framework can identify and quantify balance sheet synergies by first tracing out the demand systems for loans, deposits, and mortgages. Then, we estimate a simple cost function to capture the synergies between these components. Importantly, we only rely on cross-sectional instruments to identify the structural parameters in both steps.

The interplay between various bank assets and liabilities is crucial for determining the passthrough of unconventional monetary policy through the banking sector. There is a large literature on the effects of Quantitative Easing. Our work is most closely related to Rodnyansky and Darmouni (2017) and Chakraborty et al. (2020), who also focus on transmission through bank balance sheets. The use of cross-sectional instruments to identify the demand system and cost function allows us to avoid using variation in the time series, where it is difficult to differentiate the effect of policy from movements in the underlying real economy.

Our estimates show that that the passthrough of unconventional monetary policy through the banking system is heavily influenced by the demand elasticities of deposits, loans, and mortgages.

We thereby complement a growing literature on the transmission of conventional monetary policy in the presence of imperfect competition. On the liability side, Drechsler, Savov, and Schnabl (2017) examine the effect of deposit competition on the passthrough of the Fed funds rate, Li, Ma, and Zhao (2019) juxtapose the impact of the Fed funds rate against that of Treasury supply, Jiang, Krishnamurthy, and Lustig (2019) study the global spillover of the US monetary policy through the supply of safe assets, and Xiao (2020) focuses on the transmission through money market funds. On the asset side, Scharfstein and Sunderam (2016) zoom in on the effect of imperfect competition in mortgage markets. Like Wang, Whited, Wu, and Xiao (2020), we also jointly consider the effect of imperfect competition affecting both the asset and liability side of bank balance sheet. Our focus, however, is on the transmission of unconventional monetary policy through affecting the availability of liquid assets on bank balance sheets. The eventual impact on illiquid loans and mortgages as well as the deposit base also requires understanding the synergies between different balance sheet components,. which is made possible through our cost function estimation.

More generally, we contribute to a growing literature on structural estimation in banking. Some are based on a BLP framework to the demand for deposits and loans (Egan, Hortaçsu, and Matvos, 2017; Buchak, 2018; Wang, Whited, Wu, and Xiao, 2020; Xiao, 2020; Buchak, Matvos, Piskorski, and Seru, 2018), while others use revealed preferences to estimate structural parameters (Akkus, Cookson, and Hortacsu, 2016; Schwert, 2018; Craig and Ma, 2018). Our key innovation is the use of cross-sectional geographical instruments at the bank branch-level, which have previously been established by the reduced-form literature (e.g. Cortés and Strahan (2017)), to provide credible identification of our demand systems. Our estimation of the synergies between liquid assets, illiquid assets, and bank deposit funding is also unlike existing work. In this regard, the use of cross-sectional instruments to estimate structural parameters and conduct counterfactual analysis relates to the use of cross-sectional variation to identify aggregate shocks in the macroeconomics literature (see Nakamura and Steinsson (2018) for a review).

2 A Model of Banks Balance Sheets

The purpose of our model is to quantify how the banking system responds to policy interventions, such as an increase in reserve supply caused by QE, and other external shocks. Because policy interventions tend to respond to macroeconomic conditions, it is difficult to observe exogenous policy experiments in the data; what appears in the data like the response to a policy may be caused by the macroeconomic conditions that induced policymakers to act. This endogeneity problem suggests that using a structural model, which itself can be estimated using other sources credible exogenous variation, is an attractive alternative approach to policy analysis. In the context of our model, which we introduce in Subsection 2.1, Subsection 2.2 shows that the effect of a policy like QE that increases the supply of bank reserves depends on two things: the slopes of the demand curves banks face and the "balance sheet costs" banks face in supplying deposits, loans, and mortgages. These demand curves and supply costs can then be estimated using cross-sectional instrumental variables that avoid the endogeneity problems of using actual policy interventions as a source of identification.

2.1 Model Set-Up

We consider a set of banks indexed by m that operated in a set of markets indexed by n at each time t. Each bank m chooses market-specific rates $R_{P,nmt}$, where P corresponds to D, M, and L, for its deposits, mortgages and corporate loans in market n at time t. These markets are imperfectly competitive, and bank m faces demand curves that determine its quantities $Q_{P,nmt}(R_{P,nmt}, \omega_t)$ of deposits (D), mortgages (M), and loans (L) in market n at time t. These demand curves depend on the bank's own chosen rates as well as a vector ω_t of variables the bank does not choose, such as competitors' rates and exogenous shocks. In addition, bank m chooses its quantity $Q_{S,mt}$ of liquid securities at time t that trade in a competitive market paying an interest rate $R_{S,t}$.

In period $t+1$, bank m makes a payout to its equity holders of

$$\begin{aligned}
&\Pi_{m,t+1} = \qquad (1)\\
&\sum_n Q_{L,nmt}(1+R_{L,nmt}) + \sum_n Q_{M,nmt}(1+R_{M,nmt}) + Q_{S,mt}(1+R_{S,t}) - \sum_n Q_{D,nmt}(1+R_{D,nmt})\\
-&\left(\sum_n Q_{L,nm,t+1} + \sum_n Q_{M,nm,t+1} + Q_{S,m,t+1} - \sum_n Q_{D,nm,t+1}\right) - C(\Theta_{mt}),
\end{aligned}$$

where $C(\Theta_{mt})$ is a "balance sheet cost" incurred by the bank at time $t+1$ that depends on the composition of the bank's balance sheet at time t. Specifically, the argument Θ_{mt} is a vector that contains bank m's balance sheet items $Q_{D,nmt}, Q_{M,nmt}, Q_{L,nmt}, Q_{S,mt}$ for all markets n, as well as the exogenous shocks ω_t. The cost $C(\Theta_{mt})$ is a reduced-form function that accounts for the cost synergies between the various borrowing and lending businesses of a bank. For example, having more liquid assets on balance sheets may reduce the cost of fire-sales in the event of a bank-run and render bank runs less likely to begin with (Diamond and Dybvig, 1983). The use of demandable deposits may also serve as a commitment device in reducing fire sales (Diamond and Rajan, 2000). We do not need to take a stand on the specific source of the cost ex ante. Instead, our framework can uncover how the overall costs vary with the relative magnitudes of various balance sheet components. For instance, if the marginal cost of providing loans in market n drops with the bank's supply of deposits in market n', $\partial^2 C/(\partial Q_{D,nmt}\partial Q_{L,n'mt})$ would be negative.

The bank's equity holder has a pricing kernel $\Lambda_{t,t+j}$ and maximizes the present value of its payouts

$$\max_{(R_{D,nmt}, R_{M,nmt}, R_{L,nmt}, Q_{mt})} \sum_{j=0}^{\infty} \mathbb{E}_t[\Lambda_{t,t+j}\Pi_{m,t+j}] \qquad (2)$$

subject to equation 1. Note that each rate chosen at time $t+j$ only impacts $\Pi_{m,t+j}$ and $\Pi_{m,t+j+1}$ The first-order conditions for the bank's problem are [1]

$$\frac{\partial Q_{D,nmt}}{\partial R_{D,nmt}} = \frac{1}{1+R_t^{D,m}}\left(\frac{\partial Q_{D,nmt}}{\partial R_{D,nmt}}(1+R_{D,nmt})+Q_{D,nmt}+\frac{\partial Q_{D,nmt}}{\partial R_{D,nmt}}\frac{\partial C(\Theta_{mt})}{\partial Q_{D,nmt}}\right), \quad (3)$$

$$\frac{\partial Q_{L,nmt}}{\partial R_{L,nmt}} = \frac{1}{1+R_t^{L,m}}\left(\frac{\partial Q_{L,nmt}}{\partial R_{L,nmt}}(1+R_{L,nmt})+Q_{L,nmt}-\frac{\partial Q_{L,nmt}}{\partial R_{L,nmt}}\frac{\partial C(\Theta_{mt})}{\partial Q_{L,nmt}}\right), \quad (4)$$

$$\frac{\partial Q_{M,nmt}}{\partial R_{M,nmt}} = \frac{1}{1+R_t^{M,m}}\left(\frac{\partial Q_{M,nmt}}{\partial R_{M,nmt}}(1+R_{M,nmt})+Q_{M,nmt}-\frac{\partial Q_{M,nmt}}{\partial R_{M,nmt}}\frac{\partial C(\Theta_{mt})}{\partial Q_{M,nmt}}\right), \quad (5)$$

$$1 = \frac{1}{1+R_t^{S,m}}\left((1+R_{S,t})-\frac{\partial C(\Theta_{mt})}{\partial Q_{S,mt}}\right). \quad (6)$$

2.2 Responses to External Shocks

To illustrate how an increased supply of bank reserves would impact the banks in our model, we compute a comparative static where our bank m's liquid security holdings $Q_{S,mt}$ exogenously increases. The bank continues to choose its deposit, loan, and mortgage rates optimally so first order conditions above still hold. To compute this comparative static, our FOCs can be simplified to give

$$R_t^{D,m} - R_{D,nmt} - \frac{Q_{D,nmt}}{\partial Q_{D,nmt}/\partial R_{D,nmt}} = \frac{\partial C(Q_{D,nmt},\ldots)}{\partial Q_{D,nmt}} \quad (7)$$

$$R_t^{P,m} - R_{P,nmt} - \frac{Q_{P,nmt}}{\partial Q_{P,nmt}/\partial R_{P,nmt}} = -\frac{\partial C(Q_{P,nmt},\ldots)}{\partial Q_{P,nmt}} \quad (8)$$

where P indexes mortgages M or loans L, which have identical FOCs. On the right hand side, we have the marginal cost of borrowing or lending, which may depend on the bank's entire balance sheet. If we parametrize the left hand side by the bank's market level quantity (which implies an interest rate by inverting the demand curve) and add one unit of securities $Q_{S,mt}$ to the bank's balance sheet, we

[1] For simplicity, we assume that the riskiness of a bank's entire deposit base is the same (and respectively all of its mortgages and all of its loans). This allows us to define bank-asset-specific discount rates ($R_t^{D,m}$,$R_t^{M,m}$,$R_t^{L,m}$,$R_t^{Q,m}$ in each first order condition implied by the pricing kernel $\Lambda_{t,t+j}$.

must have that

$$\begin{aligned}
\frac{\partial Q_{D,nmt}}{\partial Q_{S,mt}} \cdot \frac{\partial}{\partial Q_{D,nmt}}(R_t^{D,m} - R_{D,nmt} - \frac{Q_{D,nmt}}{\partial Q_{D,nmt}/\partial R_{D,nmt}}) &= \frac{\partial^2 C(Q_{D,nmt},\ldots)}{\partial Q_{D,nmt}\partial Q_{mt}} \cdot \frac{\partial Q_{mt}}{\partial Q_{S,mt}} \\
\frac{\partial Q_{M,nmt}}{\partial Q_{S,mt}} \cdot \frac{\partial}{\partial Q_{M,nmt}}(R_t^{D,m} - R_{D,nmt} - \frac{Q_{D,nmt}}{\partial Q_{M,nmt}/\partial R_{D,nmt}}) &= -\frac{\partial^2 C(Q_{M,nmt},\ldots)}{\partial Q_{M,nmt}\partial Q_{mt}} \cdot \frac{\partial Q_{mt}}{\partial Q_{S,mt}} \\
\frac{\partial Q_{L,nmt}}{\partial Q_{S,mt}} \cdot \frac{\partial}{\partial Q_{L,nmt}}(R_t^{D,m} - R_{D,nmt} - \frac{Q_{D,nmt}}{\partial Q_{L,nmt}/\partial R_{D,nmt}}) &= -\frac{\partial^2 C(Q_{L,nmt},\ldots)}{\partial Q_{L,nmt}\partial Q_{mt}} \cdot \frac{\partial Q_{mt}}{\partial Q_{S,mt}}
\end{aligned}$$

where $\frac{\partial Q_{D,nmt}}{\partial Q_{S,mt}}, \frac{\partial Q_{M,nmt}}{\partial Q_{S,mt}}, \frac{\partial Q_{D,nmt}}{\partial Q_{S,mt}}$ are the response of each individual bank branch quantity to the reserve increase and $\frac{\partial Q_{mt}}{\partial Q_{S,mt}}$ is a vector of how all of the bank's balance sheet quantities respond to the reserve increase. By construction, the term representing securities satisfies $\frac{\partial Q_{S,mt}}{\partial Q_{S,mt}} = 1$, and the remainder of this vector is determined by solving this system of equations.

This system of equations determines how all of a bank's borrowing and lending quantities $Q_{P,nmt}$, $P = D, M, L$ change if reserves are added to its balance sheet. On the left hand side is a term determined only by the demand curve a bank faces in an individual market. We estimate this term with an industrial organization style demand system.[2] On the right hand is an expression reflecting how a bank's marginal cost of borrowing or lending in a market changes with the composition of its entire balance sheet. We therefore need to estimate the *cost synergies* between the different components of a bank's balance sheet (e.g., the synergy between borrowing from depositors and lending to homeowners or firms, a central concept in banking theory). We develop and apply a novel econometric approach to estimating these cost synergies that requires two separate instrumental variables for the demand for a bank's services. Together, our estimates of the demand for a bank's services and its cost of providing them allows us to compute the aggregate effect of an increased supply of bank reserves—the policy we intend to analyze.

[2]This section considers a single bank in isolation, while our full model allows for competition between banks. Thus, we need to estimate a *demand system* across all banks rather than just a *demand curve* faced by an individual bank.

3 Demand Systems

This section estimates the demand systems for deposits, mortgages, and loans. Subsection 3.1 introduces the logit demand system curves and their estimation strategy. Subsection 3.2 and 3.3 explain the data and instruments we use. The estimation results on demand elasticities, size of outside options, and implied mark-ups are shown in Subsection 3.4.

3.1 Estimation Strategy

3.1.1 Demand Curves

Depositors in each market n at time t have a total supply of funds $F_{D,nt}$ that they choose how to invest. They can either invest in deposits at each bank m which has branches in the market or can invest in an unobserved outside option 0. This outside option allows for the possibility for consumers to substitute between deposits and other savings vehicles such as money market fund shares that are not in our data. An observed quantity $Q_{D,nmt}$ of deposits are invested in bank m's branches in market n in time t. In addition, an unobserved quantity $Q_{D,n0t}$ is invested in the outside option.

Similarly, borrowers of loans and mortgages have a total funding needs of $F_{M,nt}$ and $F_{L,nt}$, respectively. They can either borrow from banks or resort to the outside option, which includes borrowing from non-banks or not obtaining funding altogether. $Q_{M,nmt}$ and $Q_{L,nmt}$ denote the observed quantities of mortgages and loans borrowed from bank m in market n in time t, while $Q_{M,n0t}$ and $Q_{L,n0t}$ denote the unobserved quantity of the respective outside option.

Preferences of depositors (firm borrowers or mortgage borrowers) follow a standard logit demand system (Berry, 1994)

$$u_{P,jnmt} \quad = \quad \alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt} + \varepsilon_{P,jnmt},$$

where P corresponds to deposits D, mortgages M, or loans L. The utility for customer j investing in (borrowing from) bank m is made up of four components. The first is the interest rate paid on deposits (charged on mortgages or loans) $R_{P,nmt}$, times the customer's preference for the interest rate α_P. Notice that depositors prefer a higher interest rate while borrowers prefer a lower cost of funding so that α_D is positive and α_M and α_L are negative. Customer utility is also affected by the desirability of its deposits (mortgages or loans), which depends on a vector of observed characteristics X_{nmt} and their preferences β_P and unobservable characteristics $\delta_{P,nmt}$. Finally. the error term $\varepsilon_{P,jnmt}$ is assumed to be i.i.d. and follow a standard logit distribution. We normalize outside options to 0 without loss of generality since only differences in utility across the choices available to a customer impact her decisions.

Under the assumptions of logit demand systems, the quantity of deposits invested in (quantity of mortgages and loans borrowed from) branches of bank m in market n at time t satisfies

$$Q_{P,nmt} = F_{P,nt} \frac{\exp(\alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt})}{1 + \sum_{m'} \exp(\alpha_P R_{P,nm't} + X_{P,nm't}\beta_P + \delta_{P,nm't})}. \tag{9}$$

Since the denominator is common across all banks in market nn at time t, this demand system implies

$$\log Q_{P,nmt} = \zeta_{P,nt} + \alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt}.$$

This linear specification with a market-time specific constant $\zeta_{P,nt}$ allows us to transparently estimate α_P and β_P using market-time fixed effects, which pin down the price disutility parameters required for the demand side of our model. Nevertheless, directly regressing log market shares $Q_{P,nmt}$ on interest rates $R_{P,nmt}$ and observable characteristics $X_{P,nmt}$ may yield biased estimates of the price disutility parameter because a bank with high quality banking services $\delta_{P,nmt}$ may rationally pay a lower deposit rate on deposits or charge a higher rate on loans or mortgages than a bank with low quality banking services. This implies that $R_{P,nmt}$ may likely be correlated with $\delta_{P,nmt}$. However, if we have an instrumental variable $z_{P,nmt}$ that only affects a bank's choice of interest rates but is uncorrelated with its unobserved quality characteristics $\delta_{P,nmt}$, the model can be consistently estimated using two-stage least squares. That is, we can obtain the price disutility parameters α_P and β_P by running the following two-stage least squares regression

$$R_{P,nmt} = \gamma_{P,nt} + \gamma_P z_{P,nmt} + X_{P,mt}\gamma_P + e_{P,nmt}, \quad (10)$$

$$\log Q_{P,nmt} = \zeta_{P,nt} + E_{P,nt}\delta_{P,nmt} + \alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt} - E_{P,nt}\delta_{P,nmt}, \quad (11)$$

where P corresponds to deposits D, mortgages M, or loans L. Note that the residual of the second stage regression is $\delta_{P,nmt} - E_{P,nt}\delta_{P,nmt}$ rather than $\delta_{P,nmt}$. While we assume that unobserved product quality $\delta_{P,nmt}$ is uncorrelated with our instrument, its market-year specific mean $E_{P,nt}\delta_{P,nmt}$ need not be everywhere zero. Some markets may have unobservably better banking services provided than others, and this will impact the size of the market-time fixed effect $\zeta_{P,nt} + E_{P,nt}\delta_{P,nmt}$.

3.1.2 Market Size

Our two-stage least squares procedure, where market-time-specific means are differenced out through market-time fixed effects, relies on observing how the difference in two bank's log-quantities responded to the difference in their interest rates. It does not tell us how the overall quantity of deposits in a deposit market would respond if every bank in the market raised its interest rates. Similarly, we cannot tell how the overall quantity of mortgages would change if every bank raised its mortgage rates. This section develops a novel approach to estimating how the overall quantity in a market changes with an aggregate change in rates, which is the final piece of information needed to complete the estimation of our demand systems.

For loans, we obtain the outside option size by directly multiplying the number of potential borrowers by the average loan size. We count the number of firms in the Dealscan database that did not borrow in a given year and state as potential borrowers of that year. The average loan size is linearly projected from the existing loans in that year with state fixed effect to account for state-level heterogeneity in the size of loans. The underlying assumption is that potential borrowers would have on average obtained a loan of the same size as the existing ones in the market that year.

For deposits and mortgages, we do not observe an analogous population of those who choose not to take out a mortgage or to hold bank deposits. The overall size of the market is therefore unobserved. This leaves our demand system not entirely identified based on the price disutility parameters obtained in Subsection 3.1.1 alone.

We use $\mathcal{Q}_{P,nt}$ in a different font to denote the total quantity of deposit, mortgage or loan in a market n:

$$\mathcal{Q}_{P,nt} = \sum_m Q_{P,nmt}.$$

Summing equation 9 across all branches in a market, we have

$$\mathcal{Q}_{P,nt} = \mathcal{F}_{P,nt} \frac{\sum_m \exp(\alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt})}{1 + \sum_{m'} \exp(\alpha_P R_{P,nm't} + X_{P,nm't}\beta_P + \delta_{P,nm't})}. \tag{12}$$

We define $\delta_{P,nt} = \log(\sum_m \exp(\alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt}))$, which can be interpreted as the desirability of a "composite deposit" or "composite mortgage" representing all banks operating in a market. Then, $\mathcal{Q}_{P,nt} = \mathcal{F}_{P,nt} \frac{\exp(\delta_{P,nt})}{1+\exp(\delta_{P,nt})}$, and using a log-linear approximation, $\log \mathcal{Q}_{P,nt} \approx \log \mathcal{F}_{P,nt} + \beta_{P,o}\delta_{P,nt}$, we can observe how $\log \mathcal{Q}_{P,nt}$ changes with the value of $\delta_{P,nt}$ to learn the value of $\beta_{P,o}$. $\beta_{D,o}$ quantifies the sensitivity of total deposit quantities to changes in the overall desirability of deposits, whereas $\beta_{M,o}$ quantifies the sensitivity of total mortgage quantities to changes in the overall desirability of mortgages.

We apply an instrumental variables approach to consistently estimate parameter $\beta_{P,o}$. From our estimation of the price disutility parameters in Equations 10 and 11, we can observe all terms in the expression for $\delta_{P,nt}$ except the mean of $\delta_{P,nmt}$. We therefore decompose into an "observable" and

”unobservable” desirability component $\delta_{P,nt} = \delta^o_{P,nt} + \delta^u_{P,nt}$, where

$$\delta^u_{P,nt} = \frac{1}{N_{nt}} \sum_{m=1}^{N_{nt}} \delta_{P,nmt}. \tag{13}$$

$$\delta^o_{P,nt} = \log\left(\sum_{m'} \exp(\alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt} - \delta^u_{P,nt})\right). \tag{14}$$

If we have an instrumental variable $z_{P,nt}$ that is uncorrelated with $(\log F_{P,nt} + \beta_{P,o}\delta^u_{P,nt})$ conditional on a vector $\chi_{P,nt}$ of controls, we can estimate $\beta_{P,o}$ by two-stage least squares as[3]

$$\delta^o_{P,nt} = \rho_{P,t} + \theta_P z_{P,nt} + \chi_{P,nt}\theta_P + \varepsilon^o_{P,nt}, \tag{15}$$

$$\log Q_{P,nt} = \alpha_{P,t} + \beta_{P,o}\delta^o_{P,nt} + \chi_{P,nt}\rho_P + \eta_{P,nt}. \tag{16}$$

To construct this market-year level instrument, we take our market-bank-time level instrumental variable we used previously $z_{P,mnt}$, and construct a market-year level measure of exposure to it:

$$z_{P,nt} = \sum_m \frac{Q_{P,nmt}}{Q_{P,nt}} z_{P,nmt},$$

which measures how exposed a region is to indirect rate changes coming through internal capital markets. The identifying assumption is that these indirect shocks through banks' internal capital markets are uncorrelated with the log-size of each market ($log(F_{P.nt})$) and with the average unobservable quality ($\delta^u_{P,nt}$). In the appendix, we show that together with our previous estimates of the rate sensitivity coefficient α_P at an individual bank, the aggregate quantity's sensitivity to rate changes $\beta_{P,o}$ yields the following expressions for banks' demand curves [4]

$$\frac{\partial \log Q_{P,nmt}}{\partial R_{P,nmt}} = \alpha_P + \alpha_P(\beta_{P,o} - 1)\frac{Q_{P,nmt}}{Q_{P,nt}}. \tag{17}$$

[3]Recall that under our log-linear approximation, we have $logQ_{P,nt} = logF_{P,nt} + \beta_{P,o}\delta^o_{P,nt} + \beta_{P,o}\delta^u_{P,nt}$.

[4]We provide additional expressions for how a bank's quantities depend on all banks' chosen rates in Appendix 8.4.1. We also discuss some details of how we implemented our construction of $\delta^o_{D,nt}$ in the presence of missing data in some markets in in Appendix 8.2.

3.1.3 Mark-ups

After estimating the demand systems for deposits, mortgages, and loans, we proceed to infer bank's mark-ups in these markets. Mark-up estimates are not only interesting on their own. They also allow us to infer the marginal costs of producing deposits, loans and mortgages, which are essential for estimating the cost function parameters in Section 4.

To express mark-ups, we can simply rewrite the FOC (8) in Subsection 2.2. For deposits, we have

$$\nu_{D,nmt} = -\frac{Q_{D,nmt}}{\partial Q_{D,nmt}/\partial R_{D,nmt}} \quad = \quad R_{D,nmt} - R_t^{D,m} + \frac{\partial C(\Theta_{mt})}{\partial Q_{D,nmt}} \tag{18}$$

and for mortgages $P = M$ or loans $P = L$, we have

$$\nu_{P,nmt} = -\frac{Q_{P,nmt}}{\partial Q_{P,nmt}/\partial R_{P,nmt}} \quad = \quad R_{P,nmt} - R_t^{P,m} - \frac{\partial C(\Theta_{mt})}{\partial Q_{P,nmt}} \tag{19}$$

which is the spread between the interest rate and the rate at which the bank would make zero profit, accouting both for its discount rate $R_t^{P,m}$ and for the balance sheet cost $\frac{\partial C(\Theta_{mt})}{\partial Q_{P,nmt}}$ of holding additional assets.

We expect deposit mark-ups to be negative because market power allows banks to offer depositors a lower return than they would have obtained in competitive markets. Loan and mortgage mark-ups should be positive because market power raises the cost of funding relative to a competitive benchmark. Security markets are competitive so that mark-ups are absent, i.e., $\nu_{S,mt} = 0$.

3.2 Data

3.2.1 Deposits

County-level deposit volumes are obtained from the FDIC, which covers the universe of US bank branches at an annual frequency from June 2001 to June 2018. We exclude branches reporting consolidated deposits with another location, non-deposit accepting locations, and belonging to a foreign bank. We define each county-year as a deposit market and sum branch-level deposits at the bank-county-year level. Our sample is from 2001 to 2017.

County-level deposit rates are obtained from RateWatch, which collects weekly branch-level deposit rates by product. Data coverage varies by product, especially in the earlier years. To maximize the sample size, we focus on the most commonly available savings account type, which is the 10K money market account. We collapse the data at the bank-county-year level from June 2001 to June 2018 to match with the reporting of the branch-level deposit volumes from the FDIC.

The branch-level identifier in Ratewatch (accountnumber) is matched to the branch-level identifier in the FDIC data (uninumbr) using the mapping file developed by Bord (2017).[5]

3.2.2 Mortgages

We use data on mortgage originations made available under the Home Mortgage Disclosure Act (HMDA). The data available to us is at the annual frequency and includes information on the lender, loan size, location of the property, loan type, and loan purpose. Any depository institution with a home office or branch in a Central Business Statistical Area (CBSA) is required to report data under HMDA if it has made or refinanced a home purchase loan and if it has assets above $30 million. As explained by Cortés and Strahan (2017), the bulk of residential mortgage lending activity is likely to

[5]Special thanks to Vitaly Bord for sharing the mapping file with us.

be reported under this criterion.[6] We define each county-year as a mortgage market and sum mortgage loan volumes at the bank-county-year level. Our sample is from 2001 to 2017.

County-level mortgage rates are obtained from RateWatch, which collects weekly branch-level mortgages rates by product. Data coverage varies by product, especially in the earlier years. To maximize the sample size, we focus on the most commonly available mortgage loan product, which is the 15-year Fixed Rate Mortgage. We collapse data at the bank-county-year level from 2001 to 2018 to match with the reporting of the mortgage volume data from the HMDA.

We first merge bank-level identifiers in HMDA to the FDIC bank-level identifiers using the mapping file developed by Bob Avery.[7] Then, the branch-level identifier in the FDIC data (uninumbr) is merged with the branch-level identifier in Ratewatch (accountnumber) using the mapping file developed by Bord (2017).[8]

3.2.3 Loans

We use data on syndicated loans from Thomson Reuters Dealscan database. We select all loans originated by US banks and sum loan volumes at the bank-state-year level, where the location of the borrower is given in Dealscan. We define loan markets at the state-year level instead of the county-year level because firm borrowers tend to be less geographically confined than individual depositors. Similarly, we collapse loan spreads at the bank-state-year level. Our sample is from 2001 to 2017.

We build on the mapping file used in Chakraborty et al. (2018) to hand-match lenders in Dealscan to Call Report bank identifiers (RSSD).[9]

[6] Any non-depository institution with at least 10% of its loan portfolio composed of home purchase loans must also report HMDA data if its asset size is above $ million. These institutions are not included in our sample given our focus on deposit-taking commercial banks.
[7] The version we used is available here https://sites.google.com/site/neilbhutta/data.
[8] Special thanks to Vitaly Bord for sharing the mapping file with us.
[9] Special thanks to Indraneel Chakraborty, Itay Goldstein, and Andrew MacKinlay for sharing the mapping file with us.

3.2.4 Bank Characteristics

We use Call Reports to obtain bank-level characteristics as control variables. Specifically, we calculate the ratio of insured deposits as insured deposits over total liabilities and the ratio of loan loss provision as loan loss provisions over total loans. We collapse the data at the bank-year level from 2001 to 2017.

3.3 Instruments

The Spatial Hazard Events and Losses Database for the United States (SHELDUS) records information on the location, time, and damage brought about by natural disasters in the US . We include all reported disasters in the database and calculate the total property losses for each county-year from 2001 to 2018.

Our instrument z_{nmt} is constructed following Cortés and Strahan (2017). To identify the demand curve coefficients, the instrument has to be a supply shock rather than a demand shock to banks.

For deposits and mortgages, z_{nmt} is defined at the bank-county-year level and measures for branches of bank m in county n, the property losses from natural disasters accrued to the bank's branches in all other counties n':

$$z_{nmt} = \frac{1}{N^u_{mt}} \log \left(\sum_{n'} damage_{n't} \cdot \frac{Q_{D,n'mt}}{\sum_{n_0} Q_{D,n_0mt}} \right),$$

where N^u_{mt} is the number of branches of bank m that are not affected by natural disasters, and $damage_{n't}$ is the property loss in county n'. Following Cortés and Strahan (2017), we scale $damage_{n't}$ by the fraction of deposits belonging to branches of bank m in county n , $\frac{Q_{D,n'mt}}{\sum_{n_0} Q_{D,n_0mt}}$, and take logs after summing the scaled damage losses. The former adjustment captures the portion of the demand shock in county n absorbed by branches of bank m, while the latter ensures that the largest shocks (e.g. Hurricane Katrina) do not drive the overall result.

The rationale behind our instrument is that property losses from natural disasters create loan demand shocks in the regions they affect so that funds are allocated away from branches in county n to branches in affected counties n' through banks' internal capital market. Property losses to bank m's branches in regions n' therefore constitute a supply shock to bank m's branches in county n, which allows us to trace out the demand curve for deposits and mortgages. In all specifications, we include the log property damage to that county to account for direct effects of disaster losses on demand.

The exclusion restriction requires that natural disasters affect branches in other counties only through increased demand at branches in shocked counties. For example, natural disasters cannot directly influence local demand for deposits and mortgages in unaffected counties. One possible alternative mechanism is through loan losses by the damage itself, which could affect deposit and loan rates through increased credit risk of bank assets. To this end, we include the ratio of share loan loss provision as control variable in all specifications.

For commercial loans, we use the same instrument constructed at the bank-state-year level instead of the bank-county-year level.

3.4 Estimation Results

Table 2 reports the first-stage and second-stage results for the price disutility estimation for deposits, mortgages, and loans as in Equations 10 and 11. For all specifications, we include the ratio of loan loss provisions over total loans to remove any direct effects of natural disasters on the credit risk of bank assets. Since our deposit volume is a stock measure, whereas the issuances of mortgages and loans are flow measures, we include the lagged deposit market share to account for persistence in the stock of deposits and the share of insured deposits to capture differences in the deposit base.

The price disutility parameters reported in the first row of Panel (b) are positive for deposits and negative for mortgages and loans. These signs are consistent with downward-sloping demand curves.

Since the deposit rates are paid by the bank, raising deposit rate increases a bank's market share. In contrast, mortgage, and loan rates are paid by borrowers, so a bank can improve its market share by offering lower mortgage and loan rates. Quantitatively, the coefficients imply that when an infinitely small bank raises its deposit rate in one county by 10 basis points, its deposit volume will increase by 4.6%.[10] When the same bank lowers its mortgage and loan rates in one market by 10 basis points, its mortgage and loan volumes increase by 55.7% and 51.9%, respectively. The price disutility of deposits is an order of magnitude smaller than that for mortgages and loans, consistent with banks having much more market power in retail deposit markets than in mortgage and loan markets.

The outside option size can be directly obtained from the loans data, which we report in Table 4. For deposits and mortgages, we proceed to estimate the sensitivity of market-level quantities $Q_{P,nt}$ to the market-level desirability parameter $\delta^o_{P,nt}$ as in Equations 15 and 16. We include the average age, income, the share of residents with a college degree, log population, growth of house prices, log property damage due to natural disasters, and lagged quantities as county-level control variables.

Panel b in Table 3 reports the sensitivity of market-level quantities $Q_{P,nt}$ to the market-level desirability parameter $\delta^o_{P,nt}$ to be 0.29 for deposits and 0.08 for mortgages. Hence, when all banks in a county raise their deposit rates by 10 basis points, the deposit quantity in that county increases by

$$\frac{\partial \log Q_{D,nt}}{\partial R_{D,nt}} = \frac{\partial \log Q_{D,nt}}{\partial \delta^o_{D,nt}} \frac{\partial \delta^o_{D,nt}}{\partial R_{D,nt}} = 0.29 \times 4.6\% = 1.3\%;$$

where 4.6% is the increase in the aggregate desirability of deposits relative to the outside option at the county-level. Similarly, when all banks in a county lower their mortgage rates by 10 basis points, the aggregate desirability of mortgages increases by 55.7% in that county relative to the outside option, and hence the mortgage quantity increases by $0.08 \times 55.7\% = 4.4\%$.

For loans, we report the outside option size at the state-year level in Table 4. On average, the

[10] The magnitude of the price disutility parameters can be interpreted for an infinitely small bank because the interest rates of that bank will have a negligible impact on the observed desirability of the aggregate deposits at the county level, and hence the share of bank deposits relative to the outside option at the county level.

implied β_o is 0.44. When all banks in a state lower their loan rates by 10 basis points, the aggregate desirability of loans in that state increases by 51.9% relative to their outside options, and hence the loan quantity in that county increases by $0.44 \times 51.9\% = 22.8\%$. Notice that the demand elasticity of loans is much higher than that of mortgages because although the sensitivity of their observed desirability to changes in interest rate is similar, the outside option of loans responds much more to changes in observed desirability than in the case of mortgages. Deposits have a low sensitivity along both dimensions which leads to a highly inelastic deposit demand curve.

In absolute terms, if all banks raise their deposit rates by 10 basis points based on 2007 levels, the aggregate deposit volume will increase by \$62.7 billion. if all banks lowered their mortgage rates by 10 basis points, the aggregate mortgage volume will increase by \$91.6 billion, and if all banks lowered their loan rates by 10 basis points, the aggregate loan volume would increase by \$1.03 trillion.

The behavior of an actual bank, due to its heterogeneity in bank characteristics and size, is different from that of a very small bank. Still, we can ask how an average bank's balance sheet quantities will change if it adjusts its deposit, mortgage, or loan rate. By Equation (17), the response of an average bank's deposit quantity in a given county is 4.3%, or 12.1 million dollars, with respect to a 10 basis points increase in deposit rate . Similarly, the response of an average bank's mortgage quantity in a given county is 55.0%, or 4.1 million dollars, with respect to a 10 basis points decrease in the mortgage rate. The average response in the average bank's loan quantity in a given state is 51.7%, or 2.1 billion dollars, with respect to a 10 basis points decrease in the loan rate.

Table 5 reports the summary statistics of the implied mark-ups, defined as the spread between the actual rate and the hypothetical competitive rate that incorporates liquidity cost. The average deposit mark-up is 2.50%, consistent with banks having high market power in deposit markets. In comparison, the average mortgage and loan mark-ups are 0.19% and 0.60%, which reflect more competitive lending markets.

4 Cost Function

This section specifies and estimates the bank's cost function for producing deposits, mortgages, and loans. We first use our estimated demand system to infer a bank's marginal cost in each market from the interest rate it chooses in that market. To identify the effects of a policy intervention like QE that impacts the composition of bank balance sheets, we need to know how these marginal costs change as bank balance sheets adjust. We begin with a reduced form analysis of how the quantities and marginal costs of these balance sheet components respond to cross-sectional instrumental variables that shock the demand for the bank's services. We then estimate the bank's cost function by choosing its parameters to be consistent with these reduced form natural experiments.

4.1 Cost Function Specification

We begin by specifying the bank's cost function and showing how it can be estimated using cross-sectional natural experiments. We assume that the bank's cost function for bank m at time t takes the form

$$\begin{aligned} C(\Theta_{mt}) &= H(Q_{D,mt}, Q_{M,mt}, Q_{L,mt}, Q_{S,mt}) \\ &+ \sum_{n}(Q_{M,nmt}\varepsilon^{Q}_{M,nmt} + Q_{L,nmt}\varepsilon^{Q}_{L,nmt} + Q_{D,nmt}\varepsilon^{Q}_{D,nmt}) + Q_{S,mt}\varepsilon^{S}_{mt}. \end{aligned} \tag{20}$$

This includes a term $H(Q_{D,mt}, Q_{M,mt}, Q_{L,mt}, Q_{S,mt})$ that can depend on the bank-level quantities of deposits, mortgages, loans, and securities. This allows, for example for the bank's holding of securities to impact its cost of mortgage lending, but not in a manner that depends on the specific mortgage market. In addition, the cost function features shocks to the cost of borrowing or lending in individual markets (given by each of the ε_{nmt} variables). These market-specific shocks are assumed to be linear in the bank's market-specific quantities. As shown above, the response of our model to external shocks depends entirely on the second derivatives of the bank's cost function, which are due only to the function H. Our cost function is therefore flexible enough to match the data with the ε_{nmt} shocks while ensuring that the cost synergies between a bank's borrowing, lending, and security holdings are the same across all branches.

To model the synergies between the bank's assets and liabilities, in a manner that is both flexible and yet restrictive enough to be identified from data, we assume the following functional form for H

$$\begin{aligned} H(Q_{D,mt}, Q_{M,mt}, Q_{L,mt}, Q_{S,mt}) &= \mu_D Q_{D,mt} + \mu_M Q_{M,mt} + \mu_L Q_{L,mt} + \mu_Q Q_{S,mt} \\ &+ \frac{1}{2}(K_1 \mathcal{E}_{mt}^2 + K_2 \mathcal{I}_{mt}^2 + K_3 Q_{D,mt}^2 + 2K_4 \mathcal{I}_{mt} Q_{D,mt} + 2K_5 \mathcal{E}_{mt} D_t), \end{aligned}$$

where $\mathcal{E}_{mt} = Q_{M,mt} + Q_{L,mt} + Q_{S,mt} - Q_{D,mt}$ and $\mathcal{I}_{mt} = Q_{S,mt} + \omega_M Q_{M,mt} + \omega_L Q_{L,mt}$. The term $\mathcal{E}_{mt}$ can loosely be interpreted as the bank's "equity" and measures the cost of expanding the size of the bank's balance sheet with non-deposit funding. This is because it equals the gap between the value of the assets we observe on the bank's balance sheet and its deposit financing.[11] The term $\mathcal{I}_{mt}$ we interpret as a measure of the "liquidity" of a bank's assets, where the coefficients ω_M and ω_L quantify how much less liquid mortgages and loans are than bank reserves.

This cost function has two key features. First, it is quadratic in all bank-level quantities, which implies that a bank's marginal costs of borrowing and lending are linear in the quantities on the bank's balance sheet. This will allow us to use linear instrumental-variable regressions as a straightforward tool for estimating its parameters. Second, the quadratic component of the cost function has 7 unknown parameters (ω_M,ω_L,K_1,K_2,K_3,K_4,K_5). As we show below, this is precisely the number of parameters that can be estimated by observing how our bank responds to two different cross-sectional instrumental variables.

[11] This term is not a perfect measure of a bank's equity capital since it ignores wholesale funding and other non-deposit debt financing as well as assets held on the bank's balance sheet that are not included in $Q_{S,mt}$. $Q_{S,mt}$ is a measure only of liquid securities such as reserves and treasuries held by a bank and does not include, for example, mortgage-backed securities.

4.2 Estimation Strategy

Differentiating Eq. (20) implies that the marginal costs of deposit, mortgage and loan for bank m in market n at time t is

$$\frac{\partial C}{\partial Q_{D,nmt}} = \mu_D - K_1\mathcal{E}_{mt} + K_3 Q_{D,mt} + K_4\mathcal{I}_{mt} + K_5(\mathcal{E}_{mt} - Q_{D,mt}) + \varepsilon^D_{nmt} \tag{21}$$

$$\frac{\partial C}{\partial Q_{M,nmt}} = \mu_M + K_1\mathcal{E}_{mt} + K_2\mathcal{I}_{mt}\omega_M + K_4 Q_{D,mt}\omega_M + K_5 Q_{D,mt} + \varepsilon^M_{nmt} \tag{22}$$

$$\frac{\partial C}{\partial Q_{L,nmt}} = \mu_L + K_1\mathcal{E}_{mt} + K_2\mathcal{I}_{mt}\omega_L + K_4 Q_{D,mt}\omega_L + K_5 Q_{D,mt} + \varepsilon^L_{nmt} \tag{23}$$

$$\frac{\partial C}{\partial Q_{S,mt}} = \mu_S + K_1\mathcal{E}_{mt} + K_2\mathcal{I}_{mt} + K_4 Q_{D,mt} + K_5 Q_{D,mt} + \varepsilon^S_{mt} \tag{24}$$

Recall that our markup estimate allowed us to recover $\partial C/\partial X_{nmt} - R_t^{m,X}$—a term that combines the discount rate for the balance sheet item of type X together with the marginal cost. If we replace the left hand sides of each of equations 21 to 24 with this observable counterpart, the right hand sides would change only in their intercept μ_X, since the discount rate does not depend on the composition of the bank's balance sheet. Averaging these equations across the markets n in which the bank operates yields

$$\frac{1}{N_{mt}}\sum_n \left(\frac{\partial C}{\partial Q_{D,nmt}} - R_t\right) = \mu^*_D - K_1\mathcal{E}_{mt} + K_3 Q_{D,mt} + K_4\mathcal{I}_{mt} + K_5(\mathcal{E}_{mt} - Q_{D,mt}) + \varepsilon^D_{mt} \tag{25}$$

$$\frac{1}{N_{mt}}\sum_n \left(\frac{\partial C}{\partial Q_{M,nmt}} + R_t^{M,m}\right) = \mu^*_M + K_1\mathcal{E}_{mt} + K_2\mathcal{I}_{mt}\omega_M + K_4 Q_{D,mt}\omega_M + K_5 Q_{D,mt} + \varepsilon^M_{mt} \tag{26}$$

$$\frac{1}{N_{mt}}\sum_n \left(\frac{\partial C}{\partial Q_{L,nmt}} + R_t^{L,m}\right) = \mu^*_L + K_1\mathcal{E}_{mt} + K_2\mathcal{I}_{mt}\omega_L + K_4 Q_{D,mt}\omega_L + K_5 Q_{D,mt} + \varepsilon^L_{mt} \tag{27}$$

where each intercept μ is now some other constant μ^* due to the change in the left hand side.

To estimate the parameters in these equations, we need to see how the marginal costs on the left hand side of each equation respond to changes in the bank balance sheet quantities on the right hand side. Because banks may face unobservable shocks to their cost of borrowing or lending (and may choose to adjust their quantities in response to these shocks), we require exogenous variation in the quantities on the right hand side of each equation that is uncorrelated with the cost shocks ε^X_{mt}. Further complicating the problem, there are multiple endogenous variables on the right hand side of each equation. If we see how a bank's marginal cost of mortgage lending responds to an increase in both its deposit quantities and its mortgage quantities, we are unable to tell how each of these two quantity changes individually impacted the bank's marginal cost. To overcome this problem, we use two cross-sectional instrumental variables z^1_{mt}, z^2_{mt} that are both assumed to be uncorrelated with the cost shocks ε_{mt}.

We regress all of the marginal costs and quantities in these equations on our two instruments z^i_{mt} (indexed by $i = 1, 2$). For deposits, mortgages, loans, and securities, the response of the marginal cost to an instrument must equal the response of the quantity times the associated cost function parameter.[12] We can show that the regression coefficients solve a system of 8 equations, which identify the 7 parameters of our cost function. For the specific equations and details on how we average two of our equations to obtain a just-identified system, we refer the reader to Appendix 8.3.

While this cost function estimation procedure necessarily relies on a simultaneous system of equations, it builds directly on our reduced-form instrumental variable analysis. Our procedure matches the causal effects we estimated of how changes in a bank's balance sheet quantities impact its marginal costs of borrowing and lending.[13] Our approach is an application of using multiple instrumental variables to estimate models with multiple endogenous variables (e.g., a bank's quantities of deposits, mortgages, loans, and securities).

[12]Notice that there is no cross-sectional variation in the return on securities. Hence, the sensitivity of securities' marginal costs to the instrument is zero.

[13]To resolve the overidentification problem, we average two of our equations to obtain a just identified system as shown in Appendix 8.3.

4.3 Data

Data for the cost function is at the bank level. Specifically, mortgage, deposit, and loan costs are obtained from interest rates and the mark-up estimates in Subsection 3.4. They are averaged at the bank level and merged to the respective bank-level volumes from Call Reports. Mortgages loans are mapped to residential loans and commercial loans make up the remainder of loans from Call Reports. we further include bank-level securities from Call Reports, which is the sum of cash, Fed funds, Treasury securities, and agency securities. Finally, we normalize all volume variables by the number of counties in which the bank operates to align with the definition of the bank-level instruments.

4.4 Instruments

We require two instruments, z^1_{mt} and z^2_{mt} to identify the cost function parameters. These instruments are at the bank-level and must be independent of banks' liquidity cost shocks in the cross-section.

The first instrument is simply the natural disaster loss instrument taken to the bank level. For bank m at time t, we have

$$z^1_{mt} \quad = \quad \frac{1}{N_{mt}} \log\left(\sum_n damage_{nt} \cdot \frac{Q_{D,nmt}}{\sum_{n_0} Q_{D,n_0mt}}\right),$$

where N_{mt} is the number of branches and $\sum_n damage_{nt} \cdot \frac{Q_{D,nmt}}{\sum_{n_0} Q_{D,n_0mt}}$ is the sum of disaster losses accrued to branches of bank m in county n. Notice that unlike in the instrument for demand systems, we are no longer in need of a branch-level supply shock. Rather, losses from disasters predominantly comprise a bank-level demand shock for loans, and their distribution is plausibly exogenous to unobserved variation in banks' liquidity cost in the cross-section.

We also use a Bartik deposit instrument based on the average growth rates of deposits in markets where bank m has branches:

$$z_{mt}^2 = \frac{1}{N_{mt}} \left(\sum_n \frac{Q_{D,nt} - Q_{D,nt-1}}{Q_{D,nt-1}} \right),$$

where where N_{mt} is the number of branches and $\frac{Q_{D,nt} - Q_{D,nt-1}}{Q_{D,nt-1}}$ is the deposit market growth rate in county n. To remove outliers, we winsorize $\frac{Q_{D,nt} - Q_{D,nt-1}}{Q_{D,nt-1}}$ at the 1% level.

Intuitively, a bank's deposit size may very well be a result of shocks to its cost of supplying deposits. Instead, we make use of the fact that counties experience different rates of deposit growth and that banks operate branches in different counties to construct our Bartik deposit instrument Specifically, the identifying assumption is that banks' differential exposure to the deposit growth rates in the counties they have branches in is not correlated with shocks to their cost of supplying deposits, mortgages, and loans. In the baseline specification, we use a simple average to compute the bank-level exposure to county-level deposit growth, but our qualitative results are robust to using value-weighted exposures as well.

4.5 Estimation Results

Table 6 reports the parameter estimates for $(\kappa^{i,D}, \kappa^{i,M}, \kappa^{i,L}, \gamma^{i,D}, \gamma^{i,M}, \gamma^{i,L}, \gamma^{i,Q})$. Since these parameters are instrument-specific, we report the parameter values corresponding to the bank-level natural disaster shock in Panel (a), and the parameter values corresponding to the bank-level Bartik deposit shock in Panel (b).

According to Panel (a), banks incurring larger losses from natural disasters also increase their deposits, mortgages, loans, and securities. Based on the effect on costs, we infer that the increase in volumes is consistent with an increase in loan and mortgage demand following natural disasters (e.g., to meet reconstruction needs). Specifically, mortgage and loan costs both increase, while deposit costs become more negative (i.e., deposits become more valuable for the bank). From Panel (b), banks experiencing a positive Bartik deposit shock also increase their deposits, mortgages, loans, and securities. In this case, the increase in balance sheet size is aligned with a positive deposit demand shock, as expected from the Bartik instrument. Deposit costs become less negative, implying that they are less valuable to the bank. At the same time, the costs of lending to firms and issuing mortgage loans declines as deposits become more abundant.

Based on these coefficient estimates, Table 7 reports the cost function's Hessian H. All diagonal terms are positive, which means that a higher stock of deposits leads to a higher marginal cost on deposits, a higher mortgage stock leads to a higher marginal cost on mortgages, etc.[14] Regarding the off-diagonal terms, the marginal cost of mortgages, loans, and securities are decreasing in deposits, which reflects a lower cost of lending and holding securities when deposit funding is more abundant. Notice also that the marginal cost of loans and mortgages are increasing in securities holdings, which suggests that banks' holdings of reserves and other liquid assets make it not cheaper but more expensive to give out loans and mortgages.

Lastly, we consider the change to marginal costs when we distribute \$1 trillion in reserves across banks. In 2007, there are 5,445 bank-counties in our sample. If bank branches in each county receive the same amount of reserves, our cost function parameters imply there would be a $0.0125 \times 184 = 2.30$ basis point decrease in the marginal cost of deposits, a $0.0060 \times 184 = 1.10$ basis point increase in the marginal cost of mortgages, a $0.0081 \times 184 = 1.49$ basis point increase in the marginal cost of loans, and a $0.0203 \times 184 = 3.73$ basis point decrease in the marginal benefit of securities. To map these cost changes to the equilibrium impact of QE on the banking system, we present a counterfactual analysis using both our estimated cost function and demand systems.

[14]Notice that based on our cost function estimates in the Hessian H, a \$1 billion increase in deposit quantity per county is associated with a 62 bps change in the marginal cost of deposits. In comparison, the Bartik deposit shock raises the deposit cost by 63 bps and the deposit quantity per branch by \$1.4 billion. The similarity in magnitudes confirm that the Bartik shock is predominantly an exogenous shock to deposit demand.

5 Counterfactual Exercise

We use our estimated model to compute the effect of an increase in the supply of bank reserves, as was caused by the Federal Reserve's Quantitative Easing Programs. These bank reserves are safe, liquid assets that must only be held by banks, so this increased supply forces banks to hold a larger portfolio of safe assets.[15] The impact of this increased reserve supply has two main effects. First, an increase in bank reserve holdings changes banks' marginal cost of providing deposits, mortgages, and loans. This change in marginal cost is quantified by our estimated cost function (equation reference?). Second, because of these cost changes, banks change the interest rates they choose to charge on loans and mortgages and choose to pay on deposits. Given our estimated demand systems, we can compute how the equilibrium quantities of deposits, loans, and mortgages respond to these changes in the rates that banks choose. As a result, our model tells us how an increase in the supply of bank reserves passes through to changes in both the quantities of deposits, mortgages, and loans provided by the banking system as well as the rates charges on these products.

5.1 Computational Strategy

To compute our counterfactual, we need to determine each bank's holdings of reserves as well as the quantity and interest rate each bank charges for loans, deposits, and mortgages in each market. This is an over 38,000-dimensional problem. Nevertheless, dimensionality can be considerably reduced and the model is tractable to solve. We define a function that maps the set of bank-level deposit, mortgage, and loan quantities to itself whose fixed point yields the equilibrium of our model.

We posit an increase R in the interest paid on securities above the yield earned in the data. We then compute the quantity of reserves the central bank must add to the financial system to increase

[15]While there are other safe and liquid assets that are held in practice both by banks and other investors (such as Treasury securities), in our counterfactual we assume that banks simply increase their holdings of bank reserves without selling any other securities.

this interest rate increase. Let $Q^i_{D,mt}, Q^i_{M,mt}, Q^i_{L,mt}, Q^i_{S,mt}$ (where i stands for initial) be the bank level quantities of deposits, mortgages, loans, and securities actually observed in the data. First, start with a hypothesized vector of bank-level quantities $Q_{D,mt}, Q_{M,mt}, Q_{L,mt}$. Second, for each bank, compute a security quantity $Q_{S,mt}$ so that the bank's marginal cost of holding securities is consistent with the rise R in the yield on securities. Third, given the vector of bank-level quantities $Q_{D,mt}, Q_{M,mt}, Q_{L,mt}, Q_{S,mt}$ use our estimated cost function to compute a bank's marginal cost of holding deposits, mortgages, loans, and securities. Fourth, compute the optimal interest rates banks choose that are jointly consistent with all of their marginal costs. Fifth, given the rates chosen in each market, compute the bank-market-level quantities demanded by depositors/borrowers. Finally, sum up the bank-market level quantities from the previous step and compute the difference from the hypothesized bank-level quantities $Q_{D,mt}, Q_{M,mt}, Q_{L,mt}$. The market is in equilibrium when this difference is 0. Please refer to Appendix 8.4 for further details.

5.2 Estimation Results

In our benchmark counterfactual, we use data on the state of the banking system in 2007 to compute the effects of providing \$4.24 trillion of added bank reserves. This quantity was chosen so that it would increase the interest rate paid on reserves by exactly 15 basis points, which is roughly the average spread between the interest paid on excess reserves above the federal funds rate in the post-crisis period of QE. Because only banks can hold reserves while non-banks can invest at the federal funds rate, this spread is an ideal measure of the degree to which banks can earn a higher rate of return than other market participants due to the increase in reserves caused by QE. During QE, the supply of excess reserves peaked at \$2.7 trillion, which is of the same order of magnitude as our quantity increase. This quantitative similarity is not mechanical; our model is identified entirely from cross-sectional variation in how banks respond to natural disaster shocks and Bartik shocks to deposit demand. No data directly from the implementation of QE or on the excess reserves spread was used in estimation. Nevertheless, the model yields estimates of how the excess reserves spread responds to the quantity of reserves that are in the same ballpark as a casual eyeballing of data on reserve rates and reserve quantities.

One salient feature of our results is that mortgages and corporate loans are crowded out by increases in central bank reserves in QE, which suggests that empirically, the synergies between liquid and illiquid assets on bank balance sheets suggested by the theoretical literature are limited. On net, liquid securities and illiquid loans are substitutes rather than complements for commercial banks as shown by the negative coefficients for loans and mortgages in Table 8. While QE may certainly have other channels of transmission, it is important to consider the "reserves channel" we find, by which central bank reserves take up balance sheet space to reduce, rather than expand, the capacity for bank lending to the real economy. [16] The potential crowding out of lending to firms is especially important in light of QE's renewed expansion in the aftermath of the Covid-19 crisis, where reserves increased from \$1.72 trillion in February to \$3.22 trillion in May 2020.

Quantitatively, the response in corporate loans makes up 13% of the size of the reserve supply increase. The response in deposits and mortgages change considerably less than the \$4.24 trillion increase in reserve holdings, even though much of the 15 basis point increase in reserve yields are passed through to the interest rates banks choose. In table 8 we report these changes in rates and quantities. Deposit, mortgage, and loan quantities increase by \$15.4 billion, decrease by \$6.1 billion, and decrease by \$555.9 billion, respectively. The branch-level average of deposit, mortgage, and loan rates increase by 6.193 basis points, 3.857 basis points, and 5.195 basis points, respectively.

One key driver of these magnitudes is that the demand for corporate loans is more price-elastic than that for mortgages and deposits. While the rate drop in corporate loans is only 1 basis point more than that of mortgages, and their price disutility parameters are similar at -556 and -519 respectively, the outside option parameter for mortgages is only .08 while for corporate loans it is .351 in 2007. This implies that the same rate increase leads to approximately 35/8=4.375 larger of a change in

[16]For example, QE may reduce the yields on long-maturity bonds, which passed through to lower mortgage rates. This reduction in long term yields happens through general equilibrium forces in asset markets that are outside of our model.

corporate loan quantities than mortgage quantities. In other words, even though loans and mortgages are close substitutes implied by the cost function and the rates charged on them change by similar amounts in the counterfactual, the quantity of corporate loans responds considerably more because of its more elastic demand curve. In addition, although deposit rates move the most (6.193 basis points), their quantities change very modestly. This is because of the inelastic deposit demand curve, which results from both a small price disutility parameter (46), and a small outside option parameter (.29).

This counterfactual also suggests that the traditional business model of commercial banks like deposit-taking and loan-making are relatively disconnected from their activities in the reserves market. In terms of the variables we track, the increased supply of reserves is larger than the changes in any other quantity.[17] A large expansion or contraction of banks' activities in securities markets (e.g., arbitrage trade of borrowing at the Fed funds rate and lending at the IOER rate) can occur with minimal impact on the traditional functions of the banking system. This is consistent with the finding of Anderson et al. (2019) that banks' securities positions or arbitrage trades are primarily financed by borrowing from money market funds.

6 Conclusion

This paper develops and estimates a structural model of the U.S. banking system and uses the model to analyze the transmission of central bank policies, such as quantitative easing. We provide the first framework that captures two important determinants of the policy impact on the quantity and price of loans, mortgages, and deposits supplied by the banking sector to the real economy. The first one concerns the demand elasticity banks face in their respective deposit and loan markets. The second one is the synergy between the various components of bank balance sheets motivated by a large theoretical literature.[18] We find that a $4.76 trillion increase in the supply of bank reserves increases

[17] Banks may do a mix of selling securities (which are less liquid and money-like than reserves), raising wholesale funding or other debt financing, retaining payouts to equity, and issuing equity.

[18] See for example Diamond and Rajan (2000); Kashyap et al. (2002); Hanson et al. (2015) and Diamond (2019).

deposit supply by \$15.4 billion but crowds out lending by \$562 billion. Our findings suggest that the synergies between liquid and illiquid assets on bank balance sheets are limited so that an increase in the supply of reserves crowds out rather than crowds in illiquid assets such as loans and mortgages. The limited increase in deposits further reflects how a highly inelastic retail deposit demand constrains the expansion in funding for banks.

One main challenge in the evaluation of central bank policy is their endogenous nature. For example, quantitative easing by the Federal Reserve was implemented in response to the 2008 financial crisis, which directly affected banks through the demand for loans and mortgages, amongst others. To this end, the identification of our structural model only relies on cross-sectional variation exogenous to changes in the time series. The demand systems are identified using demand shocks from natural disasters to bank branches in one region, which transmit through banks' internal capital markets to become supply shocks for branches in other regions. For estimating the supply-side cost function, we use shocks from natural disasters at the bank level as well as a Bartik instrument for deposit demand.

Imperfect competition in deposit and loan markets and the synergies between banks' assets and liabilities not only affect the transmission of quantitative easing but influence banks' decision making in general. Our framework can be further applied and extended to address a number of important questions. Future work may explore the effect of dynamic considerations, especially regarding the costly issuance of bank equity. With available data, bank balance sheets may also be studied at a more granular level. For example, different types of securities may bear varying degrees of liquidity. The composition of the outside option to borrowing from banks (e.g. not borrowing versus borrowing via bond markets) is another promising avenue of future research.

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7 Appendix: Tables

Table 1: Summary Statistics (Market-Bank-Year Level)

This table reports summary statistics of bank deposits, mortgages, and loans at the market-bank-year level. Rates are reported in basis points and volumes are in millions. The instrument refers to property losses due to natural disasters as explained in Section 3.3. The sample period is from 2001 to 2017.

	Num. of Obs.	Mean	25th Pct.	50th Pct.	75th Pct.	Std. Dev.
Log Deposit Market Share	80441	−2.67	−3.45	−2.33	−1.50	1.69
Deposit Volume	80441	202.41	23.54	48.80	105.85	2691.91
Deposit Rate	50897	53.62	9.00	19.50	71.71	75.36
Deposit Instrument	80441	2.54	0.00	0.81	3.66	3.58
Log Mortgage Market Share	35316	−3.90	−5.07	−3.50	−2.35	2.08
Mortgage Volume	35316	24.53	1.25	3.86	11.81	219.03
Mortgage Rate	10603	469.23	337.50	476.67	578.13	126.01
Mortgage Instrument	35316	2.94	0.16	1.46	4.15	3.69
Log Loan Market Share	27761	−4.96	−6.46	−4.67	−3.23	2.16
Loan Volume	27761	5022.61	206.16	900.00	3703.75	13846.27
Loan Spread	27761	175.42	99.15	161.56	235.00	117.04
Loan Instrument	27761	7.09	1.95	5.02	12.42	5.60

Table 2: Demand System Estimates

This table reports the two-stage least squares results for estimating price disutility of deposit, mortgage, and loan demand systems as in Equations (10) and (11). These regressions are run at the market-bank-year level. Loan loss provision is the ratio of loan loss provision over total loans, lag deposit market share is the deposit market share in the county lagged by 1 year, lag insured deposit ratio is the ratio of insured deposits over total liabilities lagged by 1 year, and log property damage is the direct property loss from natural disasters at the county level. For the deposit, mortgage and loan rates, 0.01 means 1%. The sample period is from 2001 to 2017. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel (a): First Stage Panel Regression

	(1) Deposit Rate	(2) Mortgage Rate	(3) Loan Rate
IV	1.64***	10.93***	1.88***
	(0.22)	(2.15)	(0.29)
Loan Loss Provision	106.58***	−161.53	133.83*
	(33.46)	(118.57)	(75.87)
Lag Deposit Market Share	1.57***		
	(0.44)		
Lag Insured Deposit Ratio	44.07***		
	(10.02)		
Log Property Damage	−4.58***	−2.74***	
	(1.00)	(0.62)	
Observations	234,857	70,519	23,829
R^2	0.83	0.91	0.20
Adjusted R^2	0.78	0.85	0.16

Panel (b): 2SLS Panel Regression

	(1) Deposit Market Share	(2) Mortgage Market Share	(3) Loan Market Share
Rate (with IV)	46.45***	−556.81***	−519.04***
	(9.49)	(96.72)	(82.94)
Loan Loss Provision	−1.58***	−9.80	7.15
	(0.27)	(8.38)	(4.86)
Lag Deposit Market Share	0.91***		
	(0.01)		
Lag Insured Deposit Ratio	−0.32***		
	(0.05)		
Log Property Damage	0.11***	0.76***	
	(0.01)	(0.06)	
Observations	234,857	70,519	23,829
R^2	0.98	-0.78	-5.65
Adjusted R^2	0.97	-1.96	-5.92

Table 3: Outside Option Estimates (Deposits and Mortgages)

This table reports the two-stage least squares results for estimating the sensitivity of market-level quantities to the aggregate observed desirability parameter $\delta^{o}_{p,nt}$ for deposits and mortgages as in Equations (15) and (16). The regression is run at the market-year level. We include market-year level controls, including the average age and income of the population, the fraction of residents college degree, the log population, the annual house price growth, log property loss due to natural disaster, and lag log deposit quantity. For the deposit and mortgage rates, 0.01 means 1%. The sample period is from 2001 to 2017. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel (a): First Stage Panel Regression		
	(1)	(2)
	Deposit Rate	Mortgage Rate
IV	0.01***	−0.78***
	(0.002)	(0.20)
Age	0.003**	0.09***
	(0.001)	(0.03)
Income	0.003	0.47***
	(0.003)	(0.02)
College	−0.001	0.08***
	(0.003)	(0.03)
Population	0.001	0.40***
	(0.01)	(0.03)
House Price Growth	0.0001	0.12***
	(0.003)	(0.04)
Log Property Dmg	0.10***	0.63***
	(0.0003)	(0.01)
Lag Log Deposit	0.01***	
	(0.003)	
Observations	41,773	24,701
R^2	0.93	0.87

Panel (b): 2SLS Panel Regression		
	(1)	(2)
	Deposit Share	Mortgage Share
δ^o (with IV)	0.29**	0.08*
	(0.13)	(0.04)
Age	0.003*	−0.02***
	(0.002)	(0.01)
Income	0.002	0.36***
	(0.001)	(0.02)
College	0.01***	0.17***
	(0.002)	(0.02)
Population	0.03***	1.25***
	(0.01)	(0.01)
House Price Growth	0.01***	0.06*
	(0.002)	(0.04)
Log Property Dmg	−0.03**	−0.06**
	(0.01)	(0.03)
Lag Log Deposit	0.98***	
	(0.01)	
Observations	41,773	24,701
R^2	1.00	0.90

Table 4: Outside Option estimates (Loans)

This table reports the outside option size for loans as described in Subsection 3.2 in trillions of dollars. The Implied β_o is obtained following Subsection 3.1.2.

Year	Size of Outside Option	Implied β_o
2001	0.75	0.42
2002	0.79	0.46
2003	0.85	0.50
2004	0.75	0.37
2005	0.76	0.34
2006	0.83	0.33
2007	1.00	0.35
2008	1.61	0.66
2009	1.90	0.78
2010	1.56	0.59
2011	1.18	0.39
2012	1.30	0.46
2013	1.15	0.35
2014	1.23	0.37
2015	1.51	0.42
2016	1.58	0.43
2017	1.56	0.39
2018	1.50	0.36

Table 5: Summary Statistics (Bank-Year Level)

This table reports summary statistics for deposits, mortgages, and loans at the bank level. Markups and marginal costs are defined in Equations (18) and (7) respectively. Marginal costs and mark-up are in basis points. Bank-level volumes are normalized by the number of markets and denoted in millions. The instrument refers to property losses due to natural disasters as explained in Subsection 4.4. The sample period is from 2001 to 2017. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Num. of Obs.	Mean	25th Pct.	50th Pct.	75th Pct.	Std. Dev.
Deposit Volume per Branch	119860	198.87	35.96	64.28	117.81	2413.72
Deposit Markup	52564	−250.24	−257.72	−234.85	−222.14	52.00
Deposit Cost	52564	−336.25	−387.40	−302.97	−253.55	108.15
Mortgage Volume per Branch	119874	78.26	15.21	34.40	71.67	462.43
Mortgage Markup	11113	19.30	18.14	18.60	19.62	2.21
Mortgage Cost	11113	474.57	333.01	501.69	584.09	136.45
Loan Volume per Branch	119874	93.46	6.79	12.86	24.07	1927.00
Loan Markup	2841	59.64	40.54	49.59	63.36	65.44
Loan Cost	2841	162.10	93.33	155.59	225.33	126.47
Securities Volume per Branch	119874	60.40	6.63	12.28	23.46	1165.65
Sheldus Instrument	119874	5.58	2.40	5.10	8.83	4.02
Bartik Instrument	62281	1.05	1.02	1.04	1.08	0.06

Table 6: Cost Function Estimate

This table reports the sensitivity of bank-level costs and quantities to losses from natural disasters and a bartik deposit shock as in Equations 30 to 36. Sheldus Instrument refers to property losses due to natural disasters as explained in Subsection 4.4. Bartik Deposit Instrument refers to a bartik-style instrument of deposit growth as explained in Subsection 4.4. Rates are in basis points and quantities are in millions. the sample period is from 2001 to 2017. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel (a): Results using Natural Disaster Instrument

	Deposit Cost	Mtg Cost	Loan Cost	Deposit Vol	Mtg Vol	Loan Vol	Security Vol
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sheldus Instrument	-1.09^{***}	1.18^{***}	1.98^{***}	11.11^{***}	1.09^{***}	8.84^{***}	3.62^{***}
	(0.10)	(0.20)	(0.66)	(1.77)	(0.33)	(1.40)	(0.81)
Loan Loss Provision	-1.13	-16.52^{***}	5.06^{**}	8.10^{**}	27.00^{***}	536.38^{***}	1.13
	(1.27)	(2.96)	(2.39)	(3.81)	(4.18)	(17.48)	(1.74)
Observations	52,564	11,113	2,841	118,942	119,236	119,236	118,923
R^2	0.60	0.76	0.20	0.002	0.002	0.01	0.001

Panel (b): Results using Bartik Deposit Shock

	Deposit Cost	Mtg Cost	Loan Cost	Deposit Vol	Mtg Vol	Loan Vol	Security Vol
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Bartik Deposit Instrument	62.98^{***}	-44.41^{***}	-1.70	$1,402.46^{***}$	369.69^{***}	332.85^{***}	432.46^{***}
	(5.18)	(13.38)	(42.17)	(174.67)	(18.68)	(45.82)	(86.59)
Loan Loss Provision	-0.03	-16.29^{***}	6.10	30.97	26.16^{***}	161.98^{***}	-16.84
	(1.24)	(3.21)	(7.84)	(36.48)	(4.43)	(10.86)	(18.08)
Observations	49,095	9,074	2,273	62,104	62,209	62,209	62,098
R^2	0.47	0.74	0.22	0.002	0.01	0.005	0.001

Table 7: Cost Function Estimate

This table reports the cost function estimates including parameters K and ω, and the implied Hessian matrix H. Please refer to Section 4 for a detailed description of the estimation.

K_1	K_2	K_3	K_4	K_5	ω_M	ω_L
0.0060	0.0143	−0.0053	0.0239	−0.0304	−0.0022	0.1444

Implied Hessian $H(D, M, L, Q)$			
0.0616	−0.0365	−0.0330	−0.0125
−0.0365	0.0060	0.0060	0.0060
−0.0330	0.0060	0.0063	0.0081
−0.0125	0.0060	0.0081	0.0203

Table 8: Counterfactual Results

This table reports the results of the counterfactual analysis in Section 5, which we computes the equilibrium response to a hypothetical \$4.76 trillion increase in central bank reserves in the US banking system in 2007. Rates are in basis points and quantities are in trillions.

Average Change in Rates			
Deposits	Mortages	Loans	Securities
6.193	3.857	−5.195	15.0000

Total Change in Quantities			
Deposits	Mortages	Loans	Securities
0.0154	−0.0061	−0.5559	4.2468

8 Appendix: Estimation Strategy and Derivations

8.1 Mortgage and Loan FOC's

$$-\frac{\partial Q_{M,nmt}}{\partial Q_{S,mt}}\frac{\partial}{\partial Q_{M,nmt}}(R_t^{M,m} - R_{M,nmt} - \frac{Q_{M,nmt}}{\frac{\partial Q_{M,nmt}}{\partial R_{M,nmt}}}) = \frac{\partial^2 C(\Theta_{mt})}{\partial Q_{M,nmt}\partial Q_{S,mt}} + \sum_{Y,n'}\frac{\partial^2 C(\Theta_{mt})}{\partial Q_{M,nmt}\,\partial Q_{Y,n'mt}}\frac{\partial Q_{Y,n'mt}}{\partial Q_{S,mt}} \quad (28)$$

$$-\frac{\partial Q_{L,nmt}}{\partial Q_{S,mt}}\frac{\partial}{\partial Q_{L,nmt}}(R_t^{L,m} - R_{L,nmt} - \frac{Q_{L,nmt}}{\frac{\partial Q_{L,nmt}}{\partial R_{L,nmt}}}) = \frac{\partial^2 C(\Theta_{mt})}{\partial Q_{L,nmt}\partial Q_{S,mt}} + \sum_{Y,n'}\frac{\partial^2 C(\Theta_{mt})}{\partial Q_{L,nmt}\,\partial Q_{Y,n'mt}}\frac{\partial Q_{Y,n'mt}}{\partial Q_{S,mt}} \quad (29)$$

8.2 Market Size

This section provides an alternative expression for $\delta^o_{P,nt}$, with which we can compute it in the presence of missing data. Dividing equation 9 by equation 12 yields

$$\frac{Q_{P,nmt}}{Q_{P,nt}} = \frac{\exp(\alpha_P R_{P,nmt} + X_{P,nmt}\beta_P + \delta_{P,nmt})}{\sum_{m'}\exp(\alpha_P R_{P,nm't} + X_{P,nm't}\beta_P + \delta_{P,nm't})}$$

However, $\delta_{P,nt} = \delta^o_{P,nt} + \delta^u_{P,nt}$ was defined so that $exp(\delta_{P,nt}) = \sum_{m'}\exp(\alpha_P R_{P,nm't} + X_{P,nm't}\beta_P + \delta_{P,nm't})$. It follows that

$$\log\frac{Q_{P,nmt}}{Q_{P,nt}} \quad = \quad \alpha_P R_{P,nmt} + X_{nmt}\beta_P + \delta_{P,nmt} - \delta^u_{P,nt} - \delta^o_{P,nt}.$$

If we average this expression across all observations in a market we get

$$\frac{1}{M_{P,nt}}\sum_m \log(\frac{Q_{P,nmt}}{Q_{P,nt}}) = \frac{1}{M_{nt}}\sum_m(\alpha_P R_{P,nmt} + X_{nmt}\beta_P) - \delta^o_{P,nt}$$

since $\delta^u_{P,nt}$ is defined to equal the mean of the $\delta_{P,nmt}$ in its market, so $(\delta_{D,nmt} - \delta^u_{D,nt})$ is mean zero. This implies

$$\delta^o_{P,nt} = \frac{1}{M_{P,nt}} \sum_m (\alpha_P R_{P,nmt} + X_{nmt}\beta_P) - \frac{1}{M_{P,nt}} \sum_m \log(\frac{P_{nmt}}{P_{nt}}).$$

The two terms in this depression foor expressions are averages of quantities within a market. We average the first term over only observations that have data on interest rates and covariates. We average the second term over all observations, including those missing interest rates or covariates.

8.3 Cost Function

We regress all of the marginal costs and quantities on our two instruments z^i_{mt} (indexed by $i = 1, 2$).

$$\frac{1}{N_{mt}} \sum_n \left(\frac{\partial C}{\partial Q_{D,nmt}} - R_t \right) = \theta^D_t + \kappa^{i,D} z^i_{mt} + u^Q_{D,mt} \tag{30}$$

$$\frac{1}{N_{mt}} \sum_n \left(\frac{\partial C}{\partial Q_{M,nmt}} + R^{M,m}_t \right) = \theta^M_t + \kappa^{i,M} z^i_{mt} + u^Q_{L,mt} \tag{31}$$

$$\frac{1}{N_{mt}} \sum_n \left(\frac{\partial C}{\partial Q_{L,nmt}} + R^{L,m}_t \right) = \theta^L_t + \kappa^{i,L} z^i_{mt} + u^Q_{L,mt} \tag{32}$$

and

$$Q_{D,mt} = \alpha^D_t + \gamma^{i,D} z^i_{mt} + \varepsilon^Q_{D,mt} \tag{33}$$

$$Q_{M,mt} = \alpha^M_t + \gamma^{i,M} z^i_{mt} + \varepsilon^Q_{M,mt} \tag{34}$$

$$Q_{L,mt} = \alpha^L_t + \gamma^{i,L} z^i_{mt} + \varepsilon^Q_{L,mt} \tag{35}$$

$$Q_{S,mt} = \alpha^S_t + \gamma^{i,S} z^i_{mt} + \varepsilon^Q_{S,mt} \tag{36}$$

The coefficients from regressions 30 to 36 solve a system of equations that identifies our cost function:

$$\kappa^{i,D} = -K_1\gamma^{i,\mathcal{E}} + K_3\gamma^{i,D} + K_4\gamma^{i,\mathcal{I}} + K_5[\gamma^{i,\mathcal{E}} - \gamma^{i,D}] \tag{37}$$

$$\kappa^{i,M} = K_1\gamma^{i,\mathcal{E}} + K_2\gamma^{i,\mathcal{I}}\omega_M + K_4\gamma^{i,D}\omega_M + K_5\gamma^{i,D} \tag{38}$$

$$\kappa^{i,L} = K_1\gamma^{i,\mathcal{E}} + K_2\gamma^{i,\mathcal{I}}\omega_L + K_4\gamma^{i,D}\omega_L + K_5\gamma^{i,D} \tag{39}$$

$$0 = K_1\gamma^{i,\mathcal{E}} + K_2\gamma^{i,\mathcal{I}} + K_4\gamma^{i,D} + K_5\gamma^{i,D} \tag{40}$$

where $\gamma^{i,\mathcal{E}} = \gamma^{i,Q} + \gamma^{i,M} + \gamma^{i,L} - \gamma^{i,D}$ and $\gamma^{i,\mathcal{I}} = \gamma^{i,Q} + \omega_M\gamma^{i,M} + \omega_L\gamma^{i,L}$. The final equation has a left hand side of 0 because it represents the rate of return that banks earn on a securities investment, for which there is no cross-sectional variation across banks.

This yields a system of 8 equations which we use to identify the 7 parameters of our cost function. To see why we only are able to estimate 7 parameters, re-organize these equations to get

$$\kappa^{i,M} = (K_2\gamma^{i,\mathcal{I}} + K_4\gamma^{i,D})(\omega_M - 1) \tag{41}$$

$$\kappa^{i,L} = (K_2\gamma^{i,\mathcal{I}} + K_4\gamma^{i,D})(\omega_L - 1) \tag{42}$$

which implies [19]

$$\omega_L = 1 + \frac{\kappa^{i,L}}{\kappa^{i,M}}(\omega_M - 1) \tag{43}$$

which yields a relationship between ω_L and ω_M separately from each instrument. We average these two equations and plug in

$$\omega_L = 1 + \frac{1}{2}\left(\frac{\kappa^{1,L}}{\kappa^{1,M}} + \frac{\kappa^{2,L}}{\kappa^{2,M}}\right)(\omega_M - 1) \tag{44}$$

The remaining 6 parameters of the model are now computed by solving an exact solution to the

[19]This overidentifying restriction is not specific to the functional form of our cost function. It is a consequence of the fact that the Hessian of any cost function is a symmetric matrix.

remaining system of 6 equations

$$
\begin{aligned}
\kappa^{i,D} &= -K_1\gamma^{i,\mathcal{E}} + K_3\gamma^{i,D} + K_4\gamma^{i,\mathcal{I}} + K_5[\gamma^{i,\mathcal{E}} - \gamma^{i,D}] && (45)\\
\kappa^{i,M} &= K_1\gamma^{i,\mathcal{E}} + K_2\gamma^{i,\mathcal{I}}\omega_M + K_4\gamma^{i,D}\omega_M + K_5\gamma^{i,D} && (46)\\
0 &= K_1\gamma^{i,\mathcal{E}} + K_2\gamma^{i,\mathcal{I}} + K_4\gamma^{i,D} + K_5\gamma^{i,D}. && (47)
\end{aligned}
$$

8.4 Counterfactual

8.4.1 Demand Systems under Log-linear Approximation

Each bank m has deposits $Q_{D,nmt}$ in region n at time t. The total quantity of deposits in the region is $\boldsymbol{Q}_{D,nt} = \sum_m Q_{D,nmt}$. Let δ_{nmt} denote the desirability of its deposit:

$$
\delta_{nmt} = \alpha_D R_{D,nmt} + X_{nmt}\beta_D + \delta_{D,nmt} \tag{48}
$$

and deposits $Q_{D,nmt}$ can be expressed as

$$
Q_{D,nmt} = \boldsymbol{Q}_{D,nt}\frac{\exp(\delta_{nmt})}{\sum_{m'}\exp(\delta_{nm't})}. \tag{49}
$$

Let $\boldsymbol{Q}^i_{D,nt}$ and $\delta^{o,i}_{nt}$ denote the actual value in the data (i for initial). Next, we approximate the variation in $\boldsymbol{Q}_{D,nt}$ by

$$
\frac{\partial \log \boldsymbol{Q}_{D,nt}}{\partial \delta^o_{D,nt}} = \beta_o \tag{50}
$$

which implies that

$$
\begin{aligned}
\boldsymbol{Q}_{D,nt} &= \boldsymbol{Q}^{i}_{D,nt}\exp(\beta_o(\delta^{o}_{D,nt}-\delta^{o,i}_{nt})) && (51)\\
&= \boldsymbol{Q}^{i}_{D,nt}\exp(\beta_o(\log\sum_{m'}\exp(\delta_{nm't})-\log\sum_{m'}\exp(\delta^{i}_{nm't}))) && (52)
\end{aligned}
$$

and thus

$$
Q_{D,nmt}=\boldsymbol{Q}_{D,nt}\frac{\exp(\delta_{nmt})}{\sum_{m'}\exp(\delta_{nm't})}=\boldsymbol{Q}^{i}_{D,nt}\frac{(\sum_{m'}\exp(\delta_{nm't}))^{\beta_o-1}}{(\sum_{m'}\exp(\delta^{i}_{nm't}))^{\beta_o}}\exp(\delta_{nmt}). \tag{53}
$$

Note that the value of this expression is unchanged if we add a constant to all δ and δ^i variables in region n at time t. We also have the the difference between the δ of any two goods in the same market is the difference in their log quantities sold. It follows that we can simply use $\delta^i_{nmt}=\log(Q^i_{D,nmt})$ to compute it (since $\delta^i_{nmt}-\log(Q^i_{D,nmt})$ is the constant across all goods in each market):

$$
\delta_{nmt}=\delta^{i}_{nmt}+\alpha_D(r_{nmt}-r^{i}_{nmt}) \tag{54}
$$

Under our maintained assumption that only prices and not product qualities change in counterfactuals, we can write $\delta_{nmt}=\delta^i_{nmt}+\alpha(\Delta r_{nmt})$ where $\Delta r_{nmt}=R_{D,nmt}-R^i_{D,nmt}$ is the change in interest rates relative to the pre-counterfactual data. We can therefore write $Q_{D,nmt}$ as

$$
Q_{D,nmt}=\boldsymbol{Q}^{i}_{D,nt}\frac{(\sum_{m'}\exp(\delta^{i}_{nm't}+\alpha(\Delta r_{nm't})))^{\beta_o-1}}{(\sum_{m'}\exp(\delta^{i}_{nm't}))^{\beta_o}}\exp(\delta^{i}_{nmt}+\alpha(\Delta r_{nmt})). \tag{55}
$$

8.4.2 Marginal Cost from Optimality Condition

The optimal pricing-implied marginal cost comes from the first order condition is

$$
R_{D,nmt} = R^{D}_{t}-\frac{Q_{D,nmt}(R_{D,nmt})}{Q'_{D,nmt}(R_{D,nmt})}-\frac{\partial C(Q_{D,nmt}(R_{D,nmt}),\ldots)}{\partial Q_{D,nmt}}. \tag{56}
$$

Because

$$\log(Q_{D,nmt}) = \log(\boldsymbol{Q}^i_{D,nt}) + (\beta_o - 1)\log(\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))) \quad (57)$$

$$- \beta_o \log(\sum_{m'} \exp(\delta^i_{nm't})) + (\delta^i_{nmt} + \alpha(\Delta r_{nmt})). \quad (58)$$

we have

$$\frac{\partial \log(Q_{D,nmt})}{\partial \Delta r_{nmt}} = \alpha + \alpha(\beta_o - 1)\frac{\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} \quad (59)$$

This implies

$$\frac{\partial C}{\partial Q_{D,nmt}} = R^D_t - \left[\frac{\partial \log(Q_{D,nmt})}{\partial r_{nmt}}\right]^{-1} - R_{D,nmt} \quad (60)$$

$$= R^D_t - \left[\alpha + \alpha(\beta_o - 1)\frac{exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))}\right]^{-1} - R_{D,nmt} \quad (61)$$

and thus this demand system on its own implies a marginal cost of providing deposits coming from the optimal rate setting first order condition:

$$\frac{\partial C}{\partial Q_{D,nmt}} - \frac{\partial C^i}{\partial Q_{D,nmt}} = \left[\alpha + \frac{\alpha(\beta_o - 1)\exp(\delta^i_{nmt})}{\sum_{m'} \exp(\delta^i_{nm't})}\right]^{-1} \quad (62)$$

$$- \left[\alpha + \frac{\alpha(\beta_o - 1)\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))}\right]^{-1} - \Delta r_{nmt}$$

8.4.3 Jacobian of marginal cost from optimality condition

For numerical accuracy, the Jacobian of Eq. (62) is needed. The derivative of this marginal cost is only non-zero with respect to other rates in the same region and time. The change of bank m's marginal cost with respect to bank $m*$'s rate is give by

$$\begin{aligned}
\frac{\partial}{\partial \Delta r_{nm^*t}} \frac{\partial C}{\partial Q_{D,nmt}} &= \frac{\partial}{\partial r_{nm^*t}} \left(- \left[\alpha + \frac{\alpha(\beta_o - 1)\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} \right]^{-1} - \Delta r_{nmt} \right) \\
&= - \left[1 + \frac{(\beta_o - 1)\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} \right]^{-2} \\
&\cdot \quad (\beta_o - 1) \left(\frac{\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))\exp(\delta^i_{nm^*t} + \alpha(\Delta r_{nm^*t}))}{\left(\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))\right)^2} \right. \\
&+ \quad \left. 1_{\{m=m^*\}} \frac{\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} \right) - 1_{\{m=m^*\}} \\
&= - \left[1 + \frac{(\beta_o - 1)\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} \right]^{-2} (\beta_o - 1) \\
&\cdot \quad \frac{\exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} \left(\frac{\exp(\delta^i_{nm^*t} + \alpha(\Delta r_{nm^*t}))}{\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't}))} + 1_{\{m=m^*\}} \right) - 1_{\{m=m^*\}}.
\end{aligned} \tag{63}$$

8.4.4 Appendix: Computation of Counterfactual

Let B be the number of banks and V be the space of 3B dimensional vectors representing each bank's deposit, loan, and mortgage quantities. We want to compute how these quantities change when the central bank raises the supply of bank reserves so that increases security yields by R. We define a function $f_R : V \to V$ that equals 0 after the economy equilibrates in response to this increased reserve supply.

First, we define a function $f_1^{*,R}$ from bank level deposit, mortgage, and loan quantities to an associated security quantity consistent with the rate rise R. For each bank, this function is given by (where B_i is the number of branches of the bank) $R = \frac{1}{B_i} \begin{pmatrix} \frac{\partial^2 C}{\partial Q_D \partial Q_S} & \frac{\partial^2 C}{\partial Q_M \partial Q_S} & \frac{\partial^2 C}{\partial Q_L \partial Q_S} & \frac{\partial^2 C}{\partial Q_S \partial Q_S} \end{pmatrix} * \begin{pmatrix} Q_{D,i} - Q^o_{D,i} \\ Q_{M,i} - Q^o_{M,i} \\ Q_{L,i} - Q^o_{L,i} \\ Q_{S,i} - Q^o_{S,i} \end{pmatrix}$ This implies $S_i = S_o + \frac{B_i}{\frac{\partial^2 C}{\partial Q_S \partial Q_S}} (R - \frac{1}{B_i} \begin{pmatrix} \frac{\partial^2 C}{\partial Q_D \partial Q_S} & \frac{\partial^2 C}{\partial Q_M \partial Q_S} & \frac{\partial^2 C}{\partial Q_L \partial Q_S} \end{pmatrix} * \begin{pmatrix} Q_{D,i} - Q^o_{D,i} \\ Q_{M,i} - Q^o_{M,i} \\ Q_{L,i} - Q^o_{L,i} \end{pmatrix})$

The Jacobian of this function is $\frac{-1}{\frac{\partial^2 C}{\partial Q_S \partial Q_S}} \begin{pmatrix} \frac{\partial^2 C}{\partial Q_D \partial Q_S} & \frac{\partial^2 C}{\partial Q_M \partial Q_S} & \frac{\partial^2 C}{\partial Q_L \partial Q_S} \end{pmatrix}$ for the effect of bank i's quantities on bank i's security quantity and 0 for the effect of any other bank j on bank i's quantities. Let f_1^R be given by ($id : V \to V$, $f_1^{*,R}$)- which maps each banks 3 given quantities to themselves together with this implied security quantity.

Next, we define a map f_2 from each bank's quantities D_i, M_i, L_i, S_i to the change in its marginal costs from those before the counterfactual. This change in marginal costs is given by
$$\begin{pmatrix} MC_{D,i} - MC^o_{D,i} \\ MC_{M,i} - MC^o_{M,i} \\ MC_{L,i} - MC^o_{L,i} \end{pmatrix} = \frac{1}{B_i} \begin{pmatrix} \frac{\partial^2 C}{\partial Q_D \partial Q_D} & \frac{\partial^2 C}{\partial Q_M \partial Q_D} & \frac{\partial^2 C}{\partial Q_L \partial Q_D} & \frac{\partial^2 C}{\partial Q_S \partial Q_D} \\ \frac{\partial^2 C}{\partial Q_D \partial Q_D} & \frac{\partial^2 C}{\partial Q_M \partial Q_M} & \frac{\partial^2 C}{\partial Q_L \partial Q_M} & \frac{\partial^2 C}{\partial Q_S \partial Q_M} \\ \frac{\partial^2 C}{\partial Q_D \partial Q_L} & \frac{\partial^2 C}{\partial Q_M \partial Q_L} & \frac{\partial^2 C}{\partial Q_L \partial Q_L} & \frac{\partial^2 C}{\partial Q_S \partial Q_L} \end{pmatrix} * \begin{pmatrix} Q_{D,i} - Q^o_{D,i} \\ Q_{M,i} - Q^o_{M,i} \\ Q_{L,i} - Q^o_{L,i} \\ Q_{S,i} - Q^o_{S,i} \end{pmatrix}$$
. The Jacobian of f_2 is $\frac{1}{B_i} \begin{pmatrix} \frac{\partial^2 C}{\partial Q_D \partial Q_D} & \frac{\partial^2 C}{\partial Q_M \partial Q_D} & \frac{\partial^2 C}{\partial Q_L \partial Q_D} & \frac{\partial^2 C}{\partial Q_S \partial Q_D} \\ \frac{\partial^2 C}{\partial Q_D \partial Q_D} & \frac{\partial^2 C}{\partial Q_M \partial Q_M} & \frac{\partial^2 C}{\partial Q_L \partial Q_M} & \frac{\partial^2 C}{\partial Q_S \partial Q_M} \\ \frac{\partial^2 C}{\partial Q_D \partial Q_L} & \frac{\partial^2 C}{\partial Q_M \partial Q_L} & \frac{\partial^2 C}{\partial Q_L \partial Q_L} & \frac{\partial^2 C}{\partial Q_S \partial Q_L} \end{pmatrix}$ from a bank's own quantities to its marginal cost changes and 0 for all other terms in the Jacobian matrix.

In each market, given the marginal cost changes of each bank in the market, we now compute the change in the bank's chosen interest rates that are consistent with the marginal cost changes. That is, each bank's change in interest rates Δr_{nmt} from that observed in the data is chosen so that they all solve equation 62. This system of equations must be solved numerically, but it is tractable since it can be solved seperately market by market. In market n, equation 62 defines a function g from a vector of rate changes for each bank in the market to an expression for that bank's change in marginal cost from that implied in the data. By solving g to equal our vector of marginal cost changes, we are computing the function $f_3 = g^{-1}$. The Jacobian of $f_3 = g^{-1}$ is the inverse of the Jacobian of g, which is given by equation 63.

Having solved in each market for the change in bank-market-level interest rate changes that are consistent with our marginal cost changes, we next compute the bank-level quantities implies by plugging these new interest rate changes into our demand system. The total quantity of deposits on a bank's balance sheet is, summing equation 55 across markets.

$$Q_{D,mt} = \sum_n Q_{D,nmt} = \sum_n Q^i_{D,nt} \frac{(\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't})))^{\beta_o - 1}}{(\sum_{m'} \exp(\delta^i_{nm't}))^{\beta_o}} \exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt})) \quad (64)$$

Analogous expressions for the quantity of mortgages and loans also hold.

$$Q_{M,mt} = \sum_n Q_{M,nmt} = \sum_n Q^i_{M,nt} \frac{(\sum_{m'} \exp(\delta^{i,M}_{nm't} + \alpha^M(\Delta r_{nm't})))^{\beta^M_o - 1}}{(\sum_{m'} \exp(\delta^{i,M}_{nm't}))^{\beta^M_o}} \exp(\delta^{i,M}_{nmt} + \alpha^M(\Delta r^Q_{M,nmt})) \quad (65)$$

This defines a function f_4 from the rate changes we computed above back to a list of bank-level deposit, mortgage, and loan quantities. The Jacobian of this function is given by

$$\begin{aligned}
& \frac{\partial}{\partial \Delta r_{nm^*t}} D_{mt} && (66) \\
= \; & (\beta_o - 1)\alpha Q^i_{D,nt} \frac{(\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't})))^{\beta_o - 2}}{(\sum_{m'} \exp(\delta^i_{nm't}))^{\beta_o}} \exp(\delta^i_{nm^*t} + \alpha(\Delta r_{nm^*t})) \exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt})) \\
+ \; & 1_{\{m=m^*\}} \alpha Q^i_{D,nt} \frac{(\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't})))^{\beta_o - 1}}{(\sum_{m'} \exp(\delta^i_{nm't}))^{\beta_o}} \exp(\delta^i_{nmt} + \alpha(\Delta r_{nmt})) \\
= \; & \alpha Q^i_{D,nt} \left((\beta_o - 1) \left(\sum_{m'} \exp(\delta^i_{nm't} + \alpha(\Delta r_{nm't})) \right)^{-1} \exp(\delta^i_{nm^*t} + \alpha(\Delta r_{nm^*t})) + 1_{\{m=m^*\}} \right).
\end{aligned}$$

Thus, $f_R = f^R_1 \circ f_2 \circ f_3 \circ f_4$ maps V to V, and a fixed point of f_R yields a counterfactual equilibrium of the economy. The Jacobian of this function is (by the expression for the Jacobian of composed functions) $J(f^R_1) \times J(f_2) \times J(f_3) \times J(f_4)$, where $J(.)$ denotes the Jacobian of each individual function. We provided closed form expressions for all of these Jacobians except f_3, which was a function defined by solving a system of equations (that must be computed numerically). However, f_3 is given by the inverse of our function g that does have a closed form Jacobian, which can be used to give the Jacobian of f_3 at its computed numerical solution. We compute our counterfactual by solving the equation $f_R(v) - v = 0$ numerically, using our analytic expression for its Jacobian to speed computation.