

Politicians, Taxes and Debt

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The standard analysis of the efficient management of income taxes and debt assumes a benevolent government and ignores potential distortions arising from rent-seeking politicians. This paper departs from this framework by assuming that a rent-seeking politician chooses policies. If the politician chooses extractive policies, citizens throw him out of power. We analyse the efficient sustainable equilibrium. Unlike in the standard economy, temporary economic shocks generate volatile and persistent changes in taxes along the equilibrium path. This serves to optimally limit rent-seeking by the politician and to optimally generate support for the politician from the citizens. Taxes resembling those of the benevolent government are very costly since the government over-saves and resources are wasted on rents. Political distortions thus cause the complete debt market to behave as if it were incomplete. However, in contrast to an incomplete market economy, in the long run, taxes do not converge to zero, and under some conditions, they resemble taxes under a benevolent government.

1. INTRODUCTION

Economists should cease proffering policy advice as if they were employed by a benevolent despot, and they should look to the structure within which political decisions are made. (Buchanan, 1987, p. 243)

Economists have historically based their fiscal policy recommendations on research that assumes governments spend wisely and efficiently to benefit society. In practice, however, governments run inefficiently, policymakers are partly motivated by re-election prospects and personal ambition, and citizens cannot perfectly regulate policymakers.¹ This paper characterizes optimal tax and debt management in the presence of rent-seeking politicians. What results is an analysis that describes the best economic policies citizens can realistically expect from their government.²

We study the economic environment of Lucas and Stokey (1983), and instead of assuming the existence of a benevolent government, we introduce an electoral accountability model as originally developed in Barro (1973) and Ferejohn (1986). We characterize the stochastic time path of taxes and debt, and we highlight why conventional policy prescriptions which do not account for the self-interest of politicians can induce excessive rent-seeking and be socially costly.

We consider a closed economy with no capital, with shocks to the productivity of public spending, and with complete markets. The economy is managed by a rent-seeking politician

1. For a discussion of the self-interested behaviour of politicians and the implications for corruption and public goods provision, see Acemoglu (2003), Acemoglu, Johnson and Robinson (2004), Acemoglu and Verdier (2000), Banerjee (1997), Buchanan and Tullock (1962), North (1981), Persson and Tabellini (2000) and Shleifer and Vishny (1993).

2. This is the same motivation as that of Acemoglu, Golosov and Tsyvinski (2008a,b).

whose utility increases in rents, which we define as excessive public spending with no social value. While citizens discipline the politician by threatening to remove him from office for misbehaviour, the policies of the benevolent government cannot be implemented because of limited commitment: a politician cannot commit to policies once in office and citizens cannot commit to keeping the incumbent in power in the future. In order to focus our attention on pure rent-seeking distortions, we impose no limit on financial instruments, we abstract from default, we assume perfect information, and we fix the (stochastic) interest rate by assuming quasi-linear preferences.³

We consider an infinitely repeated game between citizens and politicians with double-sided lack of commitment in which reputation sustains equilibrium policies. We examine efficient sustainable equilibria. In such an equilibrium, a politician who pursues extractive policies is thrown out of power, and a politician who pursues the policies expected by citizens is rewarded with future power. Thus, the incumbent politician follows equilibrium policies as long as rents are sufficiently high, since this raises the value of cooperation, and as long as government debt is sufficiently high, since this limits what he can acquire through maximally extractive policies prior to removal from office. Moreover, citizens agree to reward a well-behaving incumbent by not replacing him as long as equilibrium taxes are sufficiently low and public spending is sufficiently high. Efficient sustainable policies thus solve the standard programme of the benevolent government subject to an incentive compatibility constraint for the politician and an incentive compatibility constraint for a representative citizen at every history.

In our environment, the optimal taxes under a benevolent government are constant, so that they exhibit no volatility and no persistence. Raising revenue distorts labour allocation, and it is optimal for the benevolent government to smooth these distortions over time by raising a constant revenue and accommodating public expenditure needs by trading contingent claims with the private sector. For example, if the government expects a temporary stochastic war at a future date, it purchases war insurance from the private sector with revenue which it raises in all periods. Therefore, the revenue raised under peace before, during (if the war does not occur), and after the stochastic war is used to finance the stochastic war.

Our first result is that efficient taxes in the presence of political economy constraints are not constant but volatile. This is because a constant revenue policy such as the one described for the benevolent government is associated with too much rent-seeking by politicians. Consider the example of a temporary stochastic war. Imagine that taxes were constant and the government received insurance payments from the private sector in the event of war. Every single unit of insurance that the politician can appropriate during the war must be matched by one unit of equilibrium rents during or after the war, since the politician could otherwise pursue extractive policies (i.e. tax heavily and appropriate the insurance payments) and be thrown out. In other words, the insurance payments finance equilibrium rents as opposed to the war. Matching every unit of insurance with one unit of rents is inefficient since citizens can be made strictly better off if the government cuts taxes under peace, and therefore cuts both insurance payments and rents during the war. Thus, taxes are not constant, and they stochastically increase in the event of war.

3. As is well known, departures from the Lucas and Stokey (1983) equilibrium occur even under a benevolent government if contingent debt is unavailable as in Barro (1979), if default is possible, if there is imperfect information about public spending shocks, or if the government cannot commit to the interest rate. Examples of papers that explore these frictions include—though by no means are limited to—Aguiar, Amador and Gopinath (2009), Angeletos (2002), Aiyagari *et al.* (2002), Bohn (1988), Buera and Nicolini (2004), Chari, Christiano and Kehoe (1994), Chari and Kehoe (1993a,b), Kydland and Prescott (1977), Krusell, Martin and Rios-Rull (2005), Shin (2007), Sleet (2004) and Sleet and Yeltekin (2006).

Our second result is that efficient taxes in the presence of political economy constraints respond persistently to shocks. Consider again the example of a temporary stochastic war. Imagine that taxes returned to their initial level following the war. Households could be made better off if taxes did not rise by as much during the war, and if instead the government issued debt during the war which it repays with a persistent increase in taxes following the war. This serves two purposes. First, it smooths revenue raising distortions into future periods after the war ends. Second, the increase in debt makes it easier for citizens to provide incentives for the politician into the future since it leaves him with less to potentially appropriate. Thus, even though future taxes are unchanged by the past realization of war under a benevolent government, they are persistently higher after the war in the presence of rent-seeking politicians.

Note that, nevertheless, there may be a limit to the extent to which the rise in taxes during the war can be made entirely persistent. More specifically, though citizens will support an incumbent who raises taxes during the war, they are less likely to support high taxes afterwards under peace since they benefit less from public goods and potentially gain by replacing the incumbent. Thus, taxes may revert downward (though not to their original level) after the war in order to generate support for the incumbent by the citizens. More generally, we show that efficient taxes accommodate the incentives of politicians by persistently increasing into the future, and they accommodate the incentives of citizens by persistently decreasing into the future. Thus, the volatile and persistent time path of taxes in our economy is qualitatively similar to that generated in the absence of contingent debt, so that the presence of political economy constraints introduces a form of endogenous market incompleteness.

Our last result is that even though the short-run behaviour of taxes in our economy resembles that under exogenously incomplete markets, the long-run behaviour of taxes is very different. Specifically, Aiyagari *et al.* (2002) show that in an economy without state-contingent debt, the benevolent government accumulates assets along the equilibrium path, until it can finance the entire stream of public spending with zero taxes. In our environment, there is no economic motive for the accumulation of assets since state contingent claims are available, and furthermore, the accumulation of assets is not politically sustainable since it ignores the incentives of politicians.⁴ To illustrate the long-run behaviour of our economy, consider the best sustainable equilibrium for the citizens. Along the equilibrium path, the government goes into further debt and the politician extracts more rents whenever he requires incentives (i.e. whenever his incentive compatibility constraint binds). If citizens do not gain enough by replacing the incumbent (i.e. their incentive compatibility constraint does not bind), then they will tolerate the tax rate increases until taxes reach a maximum. In the long run, the tax rate is constant as under a benevolent government. In contrast, if citizens stand to gain by replacing the incumbent along the equilibrium path, then taxes cannot consistently increase towards a maximum and they will be volatile and potentially persistent even in the long run.

This paper makes three contributions. First, it characterizes optimal policies in the presence of rent-seeking politicians. These are also studied in Acemoglu, Golosov and Tsyvinski (2008*a,b*). The current paper is different from their work in two important respects. First, it focuses on the dynamics of government debt, which is an essential element of macroeconomic fiscal policies and is ruled out in their model. Second, it introduces aggregate shocks which are not present in their work. The optimal fiscal policy response to aggregate shocks is studied in the work described in footnote 3, and we depart from this work by focusing on the role of pure

4. Our discussion in Section 5.3 considers the implications of not allowing for state-contingent claims within our framework and shows that debt rises along the equilibrium path.

rent-seeking distortions. Second, our paper complements the literature on the political economy of debt by highlighting how the incentives of politicians affect optimal policy *prescriptions*. This literature has emphasized how certain critical forces such as political risk, polarization and demographics cause governments to accumulate more debt than is socially optimal. The current paper shows that a similar force towards debt accumulation arises in an electoral accountability framework, and moreover that the rise in debt is actually efficient subject to political economy constraints. The closest work to ours is that of Battaglini and Coate (2008) who study the dynamics of taxes and debt in a political economy model; but they focus on the Markov perfect equilibrium in an environment with incomplete markets in which competing groups stochastically take power. The distinguishing feature of the current paper is the focus on efficient sustainable policies in an environment with complete markets and with electoral accountability. These differences enable us to obtain different predictions for the short and long run.⁵ Finally, this paper establishes a connection between our model of electoral accountability in general equilibrium under aggregate shocks and related models of consumption risk-sharing with double-sided lack of commitment (see Alvarez and Jermann, 2001; Kocherlakota, 1996). The methods used in this related literature allow us to explicitly characterize the path of policies in our framework.⁶

The paper is organized as follows. Section 2 describes a simple three-period example with a stochastic war to illustrate our main results. Section 3 describes the dynamic general equilibrium model. Section 4 defines a sustainable equilibrium. Section 5 characterizes the efficient sustainable equilibrium. Section 6 extends the model to incorporate default and exogenous political replacement. Section 7 concludes, and the Appendix contains all the proofs and additional material.

2. A SIMPLE THREE-PERIOD MODEL

We begin by describing the intuition for our main results in a very stylized three-period version of our infinite horizon model. This allows us to easily illustrate the constraints imposed by the incentives of politicians and the constraints imposed by the incentives of citizens. We show that each one of these constraints on its own generates volatility and persistence in optimal taxes.

2.1. Environment

Consider an economy with $t = 0, 1, 2$. At every date t , the government collects tax revenue $r_t \in [0, r^{\max}]$, chooses socially beneficial public spending $g_t \geq 0$, and socially wasteful rents $x_t \geq 0$. Household welfare is declining in revenues and increasing in public spending and is equal to

$$E_0 \sum_{t=0,1,2} \beta^t u(r_t, g_t, \theta_t) = E_0 \sum_{t=0,1,2} \beta^t \left(-\frac{r_t^2}{2} + \theta_t \left(g_t - \frac{1}{2} g_t^2 \right) \right). \quad (1)$$

$\beta \in (0, 1)$ is the social rate of discounting and the inverse gross interest rate. $\theta_t \geq 0$ parameterizes the exogenous social value of public spending. Let $\theta_0 = \theta_2 = 0$, and let θ_1

5. For related work on the political economy of debt, see Aghion and Bolton (1990), Alesina and Perotti (1994), Alesina and Tabellini (1990), Lizzeri (1999) and Persson and Svensson (1989). Song, Storesletten and Zilibotti (2009) also consider a dynamic model of debt, though they abstract from economic shocks and focus on the role of demographic conflict in a representative democracy.

6. See also Dixit, Grossman and Gul (2000), Kehoe and Levine (1993), Kletzer and Wright (2000) and Thomas and Worrall (1988).

be stochastically revealed at date 1 with $\Pr\{\theta_1 = 1\} = \Pr\{\theta_1 = 0\} = 0.5$. In normal times, there is no social value to public spending; however, with probability 0.5, the economy may experience a public spending shock such as a war at date 1.

Conditional on being in power, the rent-seeking politician receives flow utility x_t from rents and no direct utility from social welfare. Therefore, the welfare of the rent-seeking politician—conditional on remaining in power in all three periods—is

$$E_0 \sum_{t=0,1,2} \beta^t x_t.$$

The self-interested politician can be interpreted as an individual or a group of individuals who choose fiscal policy. Like the benevolent ruler, the self-interested politician has the unique ability to improve household welfare by financing and providing public goods, but unlike the benevolent ruler he derives no utility from this endeavour.

The government finances public spending, rents, and past debt by raising tax revenue and borrowing from the private sector. Since policies for $t \geq 1$ depend on the realization of the shock, let $j = L, H$ parameterize the state of the world with L corresponding to $\theta_1 = 0$ and H to $\theta_1 = 1$. The government's dynamic budget constraints in periods 0, 1, and 2, respectively, are:

$$g_0 + x_0 + b_0 = r_0 + 0.5\beta \sum_{j=1,2} b_1^j, \quad (2)$$

$$g_1^j + x_1^j + b_1^j = r_1^j + \beta b_2^j, \text{ and} \quad (3)$$

$$g_2^j + x_2^j + b_2^j = r_2^j + \beta b_3^j \quad (4)$$

for $j = H, L$. $b_t^j \geq 0$ represents state-contingent government debt, which means that the government can effectively purchase war insurance in order to smooth the distortions from raising revenue. We let $b_0 = b_3^j = 0$.

2.2. Benevolent government benchmark

Consider the optimal policies of the benevolent government that maximize social welfare (1) subject to (2)–(4). The benevolent government chooses $x_t^b = 0$ in every period since it does not care about the welfare of the politicians. Since public spending is only useful at date 1 during the war, it will let $g_1^{H,b} > 0$ and let public spending equal to zero otherwise. Moreover, the government equates the marginal deadweight loss of revenue generation in period 1 to the marginal benefit of public spending in period 1 so that $r_1^{H,b} = 1 - g_1^{H,b}$. Finally, the government uses state-contingent debt to smooth the distortions from raising revenue by letting revenues be constant. Since the present discounted value of the government's liabilities is $0.5\beta g_1^{H,b}$, it chooses a constant revenue to finance the liabilities, which equals $r^b = 0.5\beta g_1^{H,b} / (1 + \beta + \beta^2)$. In order to achieve this, the government lets $b_1^H = r^b(1 + \beta) - g_1^{H,b}$, $b_1^L = r^b(1 + \beta)$, and $b_2^H = b_2^L = r^b$. The path of revenue is displayed in Panel A of Figure 1. The dotted line represents the path of revenue in the event of war and the solid line depicts the path of revenue in the event of peace, and these two lines coincide in the case of the benevolent government.

There are three important features of the optimal policy under the benevolent government. First, $r_1^{H,b} = r_1^{L,b}$, so that revenues exhibit no volatility in response to the shock. Second, debt b_2^j and revenue r_2^j are independent of j , so that policies in period 2 exhibit no history

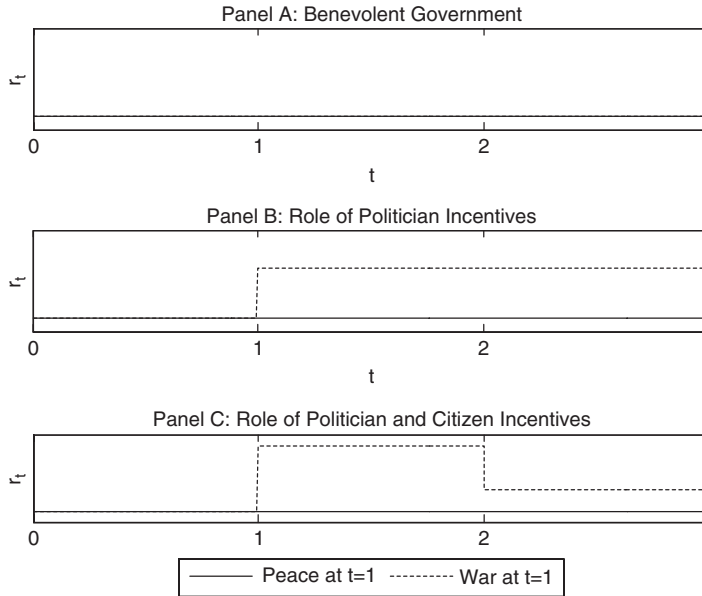


FIGURE 1
Path of revenue

dependence since they are not a function of the period 1 shock. Finally, $b_1^H < b_1^L$, so that the government purchases war insurance in period 0. Therefore, the optimal government policy would not be possible in the absence of contingent debt if the government were constrained to setting $b_1^H = b_1^L$.

2.3. *Role of politician incentives*

Now let us consider the best equilibrium from the perspective of citizens under a rent-seeking government, and let us begin by focusing on the incentives of politicians. In every period t , the incumbent politician chooses policies and markets open and clear. Though the politician is self-interested and cannot commit to policies, he knows that citizens will remove him from office at the beginning of the following period if he misbehaves.⁷ A politician who is thrown out after period t receives period t rents and a punishment equal to $-\chi^p \sum_{k=t}^2 \beta^{k-t}$ for $\chi^p > 0$ which parameterizes the strength of political institutions.

Therefore, the best policies for the citizens have to satisfy the constraint that the politician does not want to extract maximal rents and be thrown out. Note that to extract maximal rents, the politician would raise as much revenue as possible today, take out as much debt as possible today, deliver zero public goods, and repay current debt (or collect debt payments from the private sector).⁸ We let $b_1^j \leq r^{\max} (1 + \beta)$ and $b_2^j \leq r^{\max}$ so that the rents generated by taxing and borrowing maximally in a given period t are equal to $r^{\max} \sum_{k=t}^2 \beta^{k-t} - b_t$. In order for a politician to not extract maximal rents and be thrown out, he must receive a sufficiently

7. We are implicitly assuming the existence of a fourth period in which citizens would punish a misbehaving incumbent in the third period.

8. See Section 6.1 for an extension to a setting with default.

high level of rents in equilibrium. Thus, the three incentive compatibility constraints for the politician in periods 0, 1, and 2, respectively, are:

$$x_0 + \frac{1}{2} \sum_{j=L,H} \sum_{t=1,2} \beta^t x_t^j \geq (r^{\max} - \chi^p) (1 + \beta + \beta^2) - b_0, \quad (5)$$

$$\sum_{t=1,2} \beta^{t-1} x_t^j \geq (r^{\max} - \chi^p) (1 + \beta) - b_1^j, \text{ and} \quad (6)$$

$$x_2^j \geq (r^{\max} - \chi^p) - b_2^j \quad (7)$$

for $j = L, H$. Therefore, the politician is less likely to deviate from equilibrium policies if he is receiving a high level of equilibrium rents since this increases the value of cooperation or alternatively if government debt is high (or government assets are low) since this reduces the value of deviation. These two forms of incentive provision are intimately linked. Specifically, substitution of equations (2)–(4) into equations (5)–(7) yields

$$r_0 - g_0 - (r^{\max} - \chi^p) + \frac{1}{2} \sum_{j=L,H} \sum_{t=1,2} \beta^t (r_t^j - g_t^j - (r^{\max} - \chi^p)) \geq 0, \quad (8)$$

$$\sum_{t=1,2} \beta^{t-1} (r_t^j - g_t^j - (r^{\max} - \chi^p)) \geq 0, \text{ and} \quad (9)$$

$$r_2^j - g_2^j - (r^{\max} - \chi^p) \geq 0 \quad (10)$$

for $j = L, H$. Therefore, the incentive compatibility constraints on the politician come in the form of lower bounds on the size of taxes and upper bounds on the size of public spending. This is because there is a limit on the size of resources owed to the government in any given period, since if these resources are large, the politician will have a high incentive to deviate and appropriate them as rents. Since resources cannot be too high, the government has to finance more of its obligations with current and future taxes, as opposed to past taxes.

The best policies from the perspective of citizens maximize equation (1) subject to equations (2)–(4) and (8)–(10) since politicians must agree to follow equilibrium policies. Note that if χ^p is very high (i.e. political institutions are very strong), then equations (8)–(10) never bind and the best policy corresponds to that of a benevolent government.

Now imagine χ^p is sufficiently low so that the exact policy of the benevolent government does not satisfy incentive constraints (8)–(10). Specifically, let $0 < r^{\max} - \chi^p < 1$. Are policy prescriptions of revenue smoothing like those of a benevolent government still optimal for citizens? Imagine that this were the case. For a given level of public spending $g_1^{H,p}$, the lowest level of revenue that satisfies equations (2)–(4) and (8)–(10) would be optimal since citizens dislike taxes. Since public spending is only positive in period 1 under $j = H$ (since it is otherwise useless), incentive constraint (9) under $j = H$ would be the most binding constraint under a constant revenue, which means that the lowest constant incentive compatible level of revenue would equal $r^p = \frac{g_1^{H,p}}{1+\beta} + r^{\max} - \chi^p$, making equation (9) an equality. In this circumstance, resources are being diverted towards rents at some date, and from equations (2)–(4), the period 0 value of these rents is $r^p (1 + \beta + \beta^2) - \frac{1}{2} \beta g_1^{H,p} > 0$.

A constant revenue policy such as the one described is sub-optimal from the perspective of citizens since it is associated with too much rent-seeking. To see why such a policy is suboptimal, note that constraint (8) is slack under a constant revenue policy. Since revenues in period 0 are used in part to finance rents at some date, it is possible for the government

to both reduce revenues in period 0 as well as rent-seeking at some date and improve the welfare of households, and this is acceptable to politicians as long as equation (8) is satisfied. Analogous reasoning applies to constraints (9) and (10) under $j = L$, which are both slack under a constant revenue policy. In other words, the government is taxing too much during peace and the revenues are financing rents which are not necessary to ensure good behaviour by politicians. Citizens are better off if the government cuts taxes and rents.

Following this logic, the best policies for citizens can be achieved by reducing revenues until incentive constraints bind under peace, so that $r_0^p = r_1^{L,p} = r_2^{L,p} = r^{\max} - \chi^p$. Once the war takes place, then revenues permanently increase with $r_1^{H,p} = r_2^{H,p} = \frac{g_1^{H,p}}{1+\beta} + r^{\max} - \chi^p > r_0^p$, so that equation (9) binds. The path of revenue under war and under peace is displayed in Panel B of Figure 1.

Note that the three important features of the economy under a benevolent government are overturned. First, $r_1^{H,p} > r_1^{L,p}$, so that revenues cease to be constant and exhibit volatility. If they were constant, there would be too much rent-seeking. Second, $r_2^{H,p} > r_2^{L,p}$, so that policies in period 2 cease to be history independent. This occurs because $r_1^{H,p} > r_1^{L,p}$ and it is optimal to smooth revenue distortions going forward. This is also tied to the fact that $x_2^{H,p} + b_2^{H,p} > x_2^{L,p} + b_2^{L,p}$, so that debt plus rents in period 2 are also higher in the event of a war since this not only relaxes incentive constraints at date 1, but it also relaxes incentive constraints at date 2. Finally, an efficient equilibrium does not require the existence of insurance markets. From date 1 onward, the government is effectively paying for the public spending shock of size $g_1^{H,p}$ with resources which it raises *after* the shock occurs of size $r_1^{H,p} + \beta r_2^{H,p} > g_1^{H,p}$. For example, imagine that the government were constrained to using only non-contingent debt so that $b_1^H = b_1^L$. Then the government could easily set $b_1^H = b_1^L = (r^{\max} - \chi)(1 + \beta)$, $x_t^H = x_t^L = 0$ for $t = 1, 2$, and $x_0 = (r^{\max} - \chi)(1 + \beta + \beta^2)$ under the best policy. In this sense, the incentives of politicians generate an endogenous form of market incompleteness since insurance markets are no longer needed despite their availability.

2.4. Role of citizen incentives

In practice, citizens may have an incentive to replace an incumbent politician even under good behaviour so that they cannot commit to a plan to keep an incumbent in power. In such a circumstance, the incumbent would have to provide a sufficiently low level of taxation or a sufficiently high level of the public good to guarantee being able to remain in power. For simplicity, imagine that replacing an incumbent at the beginning of period t provides a benefit $\chi^c \sum_{k=t}^2 \beta^{k-t}$ to citizens, where χ^c parameterizes the lack of popularity of the incumbent. For citizens to support a well-behaving incumbent, the following incentive compatibility constraints need to be satisfied in period 0, 1, and 2, respectively:

$$u(r_0, g_0, \theta_0) - \chi^c + \frac{1}{2} \sum_{j=L,H} \sum_{t=1,2} \beta^t \left(u(r_t^j, g_t^j, \theta_t^j) - \chi^c \right) \geq 0, \tag{11}$$

$$\sum_{t=1,2} \beta^{t-1} \left(u(r_t^j, g_t^j, \theta_t^j) - \chi^c \right) \geq 0, \quad \text{and} \tag{12}$$

$$u(r_2^j, g_2^j, \theta_2^j) - \chi^c \geq 0 \tag{13}$$

for $j = L, H$. Since citizens dislike taxes and like public spending, equations (11)–(13) impose upper bounds on revenues and lower bounds on public spending.

Note that the incentive constraints on citizens in equations (11)–(13) function in an analogous fashion to the incentive constraints on politicians in equations (8)–(10). For example, imagine that instead of the current set-up, we let $\theta_0 = \theta_2 = 1$ and θ_1 be determined as before, so that there is a stochastic peace in period 1 and war otherwise. In such an environment, the benevolent government would also choose constant revenues for the reasons discussed in Section 2.2. In the presence of equations (11)–(13), however, the incentives of citizens would begin to matter for sufficiently high χ^c , and in particular, equation (12) would bind during the period of peace since it is more difficult for the government to generate support when it is providing zero public goods. Analogous arguments as those of Section 2.3 would imply that under equations (11)–(13), revenues cease to be constant since they decline in a persistent fashion conditional on the realization of peace at date 1.⁹ Therefore, constraints on citizens (11)–(13) alone can generate volatility and persistence in taxes.

Now consider how the incentives of citizens and those of politicians interact in the original set-up with a stochastic war and peace in normal times. Satisfying the incentives of politicians requires sufficiently high revenues and sufficiently low levels of public spending. In contrast, satisfying the incentives of citizens requires sufficiently low revenues and sufficiently high levels of public spending. The best policy taking both sets of incentives into account maximizes equation (1) subject to equations (2)–(4), (8)–(10), and (11)–(13). If χ^p and χ^c are low, then politicians' incentives need to be taken into account, and citizens' incentives can be ignored. Thus, the description of the equilibrium in Section 2.3 holds. Nevertheless, an increase in χ^c implies that both sets of incentives need to be taken into account and equations (11)–(13) are not satisfied under the policy described in Section 2.3. More specifically, equation (13) under $j = H$ is the most binding constraint out of equations (11)–(13) since taxes are very high and the government is not providing public goods. Therefore, equation (13) binds under $j = H$ and $r_2^{H,p} = \sqrt{-2\chi^c}$ so as to satisfy the incentives of the citizens once the war ends. Moreover, equation (9) also binds under $j = H$ so as to satisfy the incentives of the politicians once the war begins, which implies that $r_1^{H,p} = g_1^{H,p} + (r^{\max} - \chi^p)(1 + \beta) - \beta\sqrt{-2\chi^c} > r_2^{H,p}$.¹⁰ Finally, for analogous reasons as in Section 2.3, $r_0^p = r_1^{L,p} = r_2^{L,p} = r^{\max} - \chi^p$. The path of revenue under war and under peace is displayed in Panel C of Figure 1.

In sum, revenues increase during the war in order to satisfy the incentive compatibility constraints of the politician that limit the resources with which the government can enter period 1. The permanence of this increase is limited, however, by the incentives of citizens who are unwilling to tolerate high taxes once the war ends. Consequently, while the incentives of politicians limit the extent to which the government can finance the war with *past* revenues (which puts upward pressure on future revenues), the incentives of citizens limit the extent to which the government can finance the war with *future* revenues (which puts downward pressure on future revenues). Each constraint on its own generates volatility and persistence in revenues.

3. DYNAMIC GENERAL EQUILIBRIUM MODEL

In this section, we develop a dynamic general equilibrium model in the spirit of the model of Section 2. While the insights of the simpler model will be preserved, the more sophisticated model allows us to micro-found the economic environment and the strategic interaction between citizens and politicians. More importantly, the presence of dynamics allows us to analyse the

9. In other words, the path of revenue in Panel B of Figure 1 would be inverted.

10. We are implicitly assuming that $\chi^c < -1/2$ so that the solution admits an interior level of public spending.

long-run properties of the equilibrium and to better understand the relationship between our endogenously incomplete market economy and an exogenously incomplete market economy.

3.1. *Economic environment*

The economic environment is identical to that of Lucas and Stokey (1983) under quasi-linear preferences and with a single modification: the government can finance rents that are beneficial to politicians.

3.1.1. Time and uncertainty. There are discrete time periods $t = \{0, \dots, \infty\}$ and a stochastic state $s_t \in S \equiv \{1, \dots, N\}$ which follows a first-order Markov process with full support for $N \geq 2$. s_0 is given. Let $s^t = \{s_0, \dots, s_t\} \in S^t$ represent a history, and let $\pi(s^k | s^t)$ represent the probability of s^k conditional on s^t for $k \geq t$.

3.1.2. Households. There is a continuum of mass 1 of identical households that, with some abuse of notation, derive the following utility from economic activity:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t, n_t, g_t, s_t) \right), \quad \beta \in (0, 1). \quad (14)$$

c_t is consumption, n_t is labour, and g_t is government spending. Our model considers a special case of this preference: $u(c_t, n_t, g_t, s_t) = c_t - \eta n_t^\gamma / \gamma + \theta(s_t) g_t^\alpha / \alpha$, for $0 < \alpha < 1 < \gamma$ and $\theta(s_t) > 0$. $\theta(s_t)$ is high (low) when public spending is more (less) productive. This utility function allows us to abstract away from bond price manipulation by the government—which can potentially cause additional distortions even under a benevolent government—and to explicitly characterize equilibrium dynamics.¹¹

3.1.3. Politicians. There is a large number of potential and identical self-interested politicians who derive the following utility from economic activity:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t v(x_t) \right) \quad (15)$$

for x_t which represents rents. $v(x_t)$ is increasing and weakly concave, and for simplicity, let $v(x_t) = x_t$.¹² A politician who is out of power receives zero rents.

3.1.4. Markets. Household wages are normalized to 1 and are taxed at a linear rate τ_t . $b_t^h(s_{t+1}) \geq 0$ represents debt owned by the household at t , which is a promise to repay 1 unit of consumption at $t + 1$ conditional on the realization of s_{t+1} , and $q_t(s_{t+1})$ is its price at t . We ignore bonds of longer maturity structure only for notational simplicity. At every t , the

11. An extension in previous versions of this paper confirms our results in the presence of risk aversion. Details available upon request.

12. The strict concavity of $v(\cdot)$ affects only the timing of rents but not the timing of taxes or public spending in the solution to the problem.

household's allocation $\omega_t = \{c_t, n_t, \{b_t^h(s_{t+1})\}_{s_{t+1} \in S}\}$ must satisfy the household's dynamic budget constraint

$$c_t + b_{t-1}^h(s_t) = (1 - \tau_t)n_t + \sum_{s_{t+1} \in S} q_t(s_{t+1})b_t^h(s_{t+1}), \quad (16)$$

subject to $n_t \geq 0$.

$b_t^g(s_{t+1}) \geq 0$ represents debt owned by the government at t , defined analogously to the household's debt. At every t , government policies $\rho_t = \{\tau_t, \{b_t^g(s_{t+1})\}_{s_{t+1} \in S}, g_t, x_t\}$ must satisfy the government's dynamic budget constraint

$$g_t + x_t + b_{t-1}^g(s_t) = \tau_t n_t + \sum_{s_{t+1} \in S} q_t(s_{t+1})b_t^g(s_{t+1}), \quad (17)$$

subject to $g_t \geq 0$ and $x_t \geq 0$. The only difference between these budget constraints and those of the standard economy is that the rent x_t is included on the left-hand side of equation (17). We discuss the implications of allowing for default in Section 6.1.¹³

The economy is closed, and bonds are in zero net supply:

$$b_t^g(s_{t+1}) + b_t^h(s_{t+1}) = 0, \quad (18)$$

which combined with equations (16) and (17) implies the aggregate resource constraint

$$c_t + g_t + x_t = n_t. \quad (19)$$

For notational simplicity, we let $b_t^g(s_{t+1}) = b_t(s_{t+1})$ for the remainder of the discussion. Initial debt $b_{-1}(s^0)$ is exogenous. The following debt limits rule out Ponzi schemes

$$b_t(s_{t+1}) \in [\underline{b}, \bar{b}]. \quad (20)$$

Let \underline{b} to be sufficiently low and \bar{b} to be sufficiently high so that (20) does not bind. More specifically, \bar{b} is the natural debt limit so that $\bar{b} = r^{\max} / (1 - \beta)$ for r^{\max} which represents the maximal tax revenue that can ever be collected by the government (see Section 4.3).

3.2. Political environment

As in the electoral accountability models of Barro (1973) and Ferejohn (1986), citizens control politicians by potentially removing them from office. Let $P_{t+1} = \{0, 1\}$ represent the decision by citizens to remove the incumbent in period t with $P_{t+1} = 0$ representing replacement. The interaction between citizens and politicians is a game:

1. Nature chooses the state s_t .
2. Citizens make the replacement decision P_{t+1} .
3. The incumbent politician chooses policies ρ_t .
4. Markets open and clear.
5. If $P_{t+1} = 0$, the incumbent politician is replaced.

13. As is well known, allowing for default creates distortions even under a benevolent ruler (see Kydland and Prescott, 1977).

The important feature of this game is that even though citizens make their economic decisions independently, they make their political decisions regarding the replacement of the politician jointly. Since citizens are identical, there is no conflict of interest between them. These joint political decisions can be achieved by a variety of formal or informal procedures such as elections or protests. We simplify the discussion by assuming that the decision is taken by the same single representative citizen in every period.¹⁴

Politicians and citizens derive payoffs associated with P_{t+1} . If $P_{t+1} = 0$, the incumbent endures an exogenous cost $\chi^p (1 - \beta) / \beta > 0$ from t onward. χ^p captures institutional constraints on politicians, which vary across societies and are empirically important determinants of economic activity around the world.¹⁵ In addition, if $P_{t+1} = 0$, citizens receive an exogenous benefit $\chi^c (1 - \beta) \geq 0$ at t . χ^c parameterizes the social benefit of political turnover, which is related to the fact that, even though all candidate politicians are identical, citizens may derive non-pecuniary benefits from replacement. Throughout the text, we will often consider the special case in which χ^c equals zero.

Our timing of events implies that an incumbent can choose policies prior to removal from power. If instead P_{t+1} is chosen after the choice of policies ρ_t , none of our results changes with the exception that incentives for politicians and citizens are no longer provided contemporaneously after the realization of s_t , and this affects the exact characterization of policies. We prefer this timing since it facilitates the exposition of our results.¹⁶

Consider the incentives of politicians and citizens implied by this game. If citizens keep the incumbent in power, they cannot control his policies. If the incumbent limits rents and provides public goods, he cannot control citizens' future replacement decisions. In the following section, we investigate how reputational considerations can alleviate the double-sided commitment problem faced by citizens and politicians.

4. SUSTAINABLE EQUILIBRIA

As in Chari and Kehoe (1993*a,b*) we consider a sustainable equilibrium. Individual households are anonymous and non-strategic in their private market behaviour, though the representative citizen is strategic in his replacement decision. The politician in power is strategic in his choice of policies, and he must ensure that the government's dynamic budget constraint is satisfied given the anonymous market behaviour of households. Using this definition, we characterize the entire set of sustainable equilibria using the primal approach. This allows us to select the efficient sustainable equilibrium in Section 5.

4.1. Definition

Define $h_t^0 = \{s^t, P^t, \rho^{t-1}\}$ as the history of shocks, replacement decisions, and policies after the realization of s_t .¹⁷ Define $h_t^1 = \{s^t, P^{t+1}, \rho^{t-1}\}$ and $h_t^2 = \{s^t, P^{t+1}, \rho^t\}$. A representative citizen's replacement strategy Υ assigns a replacement decision for every h_t^0 . The incumbent politician's strategy σ assigns policies for every h_t^1 . A representative household's allocation

14. This is identical to the decision being made via majoritarian elections with sincere voting.

15. An alternative interpretation is that χ^p is negatively related to the non-pecuniary value of holding political power or positively related to an exogenous limit on the debt that can be issued by a politician deviating from equilibrium policies.

16. See footnote 26 for an explanation of how the alternative timing affects our results.

17. Without loss of generality, we let incumbent politicians follow identical strategies conditional on holding power so that their identity need not enter the public history.

sequence f assigns an allocation at every h_t^2 . A state price sequence ζ assigns a vector of state prices at every h_t^2 . Let $\Upsilon|_{h_t^0}$ represent the continuation strategy of the representative citizen at h_t^0 and define $\sigma|_{h_t^1}$, $f|_{h_t^2}$, and $\zeta|_{h_t^2}$ analogously.

The representative citizen's replacement strategy Υ solves the representative citizen's problem if at every h_t^0 , the continuation strategy $\Upsilon|_{h_t^1}$ maximizes household welfare given $\{\sigma, f, \zeta\}$. The incumbent politician's strategy σ solves the incumbent politician's problem if at every h_t^1 , the continuation strategy $\sigma|_{h_t^2}$ maximizes the incumbent politician's welfare given $\{\Upsilon, f, \zeta\}$ and given the government's sequential budget constraint. A representative household's allocation sequence f solves the representative household's problem if at every h_t^2 , the continuation allocation $f|_{h_t^2}$ maximizes household welfare given $\{\Upsilon, \sigma, \zeta\}$ and given the household's sequential budget constraint. Note that because households are anonymous, public decisions are not conditioned on their allocation. Finally, a state price sequence ζ clears the bond market if at every h_t^2 , the continuation sequence $\zeta|_{h_t^2}$ implies a zero net supply of bonds given $\{\Upsilon, \sigma, f\}$. Note that these conditions must hold for all histories including those that occur with probability zero. Additional formalism of our equilibrium concept is included in the Appendix.

Definition 1. A sustainable equilibrium is a 4-tuple $\{\Upsilon, \sigma, f, \zeta\}$ for which Υ solves the representative citizen's problem, σ solves the incumbent politician's problem, f solves the household's problem, and ζ clears the bond market.

4.2. Competitive equilibria

The equilibrium path of a given sustainable equilibrium is characterized by:

$$\xi = \{c(s^t), n(s^t), g(s^t), x(s^t)\}_{t=0}^\infty$$

which is feasible if $n(s^t)$, $g(s^t)$, and $x(s^t)$ are non-negative $\forall s^t$. In this section, we review the necessary and sufficient conditions established by Lucas and Stokey (1983) for a competitive equilibrium sequence, and in the next section we consider the additional conditions required for sustainability. In a competitive equilibrium, the household's equilibrium allocation $\omega = \{\omega(s^t)\}_{t=0}^\infty$ maximizes its utility as a function of the equilibrium tax rates $\tau = \{\tau(s^t)\}_{t=0}^\infty$ and state prices $q = \{q(s^{t+1}|s^t)\}_{t=0}^\infty$, for $q(s^{t+1}|s^t)$ which represents the price of a bond traded at s^t with payment conditional on s^{t+1} . The solution to the household's problem leads to the following result. All proofs are in the Appendix.

Proposition 1 (Competitive Equilibrium). ξ is a competitive equilibrium if and only if it is feasible and it satisfies

$$c(s^t) + g(s^t) + x(s^t) = n(s^t) \quad \forall s^t, \quad \text{and} \tag{21}$$

$$\sum_{t=0}^\infty \sum_{s^t \in S^t} \beta^t \pi(s^t|s^0) (r(n(s^t)) - g(s^t) - x(s^t)) = b_{-1}(s^0), \tag{22}$$

$$\text{for } r(n) = n - \eta n^\gamma.$$

Equation (21) is the resource constraint of the economy. $r(n)$ is the revenue generated by labour n derived from the household's intratemporal condition. It is independent of consumption because of risk neutrality in consumption. Equation (22) is the present value budget constraint

of the government. It states that total public spending, rents, and initial government debt are serviced by total revenues. Present values are calculated using probabilities because of risk neutrality in consumption. Together with equation (21), equation (22) implies the satisfaction of the household's present value budget constraint. Because of the completeness of financial markets, the satisfaction of equations (21) and (22) is sufficient to imply the satisfaction of the government's present value budget constraint in the future:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k | s^t) (r(n(s^k)) - g(s^k) - x(s^k)) = b(s^t | s^{t-1}) \quad \forall s^t, \quad (23)$$

for $b(s^t | s^{t-1})$ representing a bond traded at s^{t-1} with payment conditional on s^t .

4.3. Sustainable equilibria

A competitive sequence ξ and replacement rule $P = \{P(s^t)\}_{t=0}^{\infty}$ need not be sustainable.¹⁸ For instance, in any given period, the incumbent politician can maximize his rents by choosing the revenue-maximizing tax rate τ^{\max} associated with revenue r^{\max} and labour allocation n^{\max} .¹⁹ Furthermore, he can set public spending to zero, repay the current debt $b(s^t | s^{t-1})$, and borrow the maximal amount of debt $\bar{b} = r^{\max} / (1 - \beta)$. Since the government is maximally indebted in the future, public spending and rents are zero forever. From equation (23), if the politician is thrown out of power in the next period, his continuation welfare is today is

$$\underline{V}(b(s^t | s^{t-1})) = r^{\max} / (1 - \beta) - b(s^t | s^{t-1}) - \chi^p.$$

Analogously, in any given period, the representative citizen can throw out the incumbent politician, and the exiting politician will maximize his current rents using the policies described above. Since public spending and rents are zero and taxes are maximal forever independently of replacement decisions, the representative citizen will throw out all future politicians and receive χ^c forever. From equations (21) and (23), the representative citizen's continuation welfare today is

$$\underline{U}(b(s^t | s^{t-1})) = U^{\text{AUT}} + b(s^t | s^{t-1}) + \chi^c$$

for $U^{\text{AUT}} = (n^{\max} - \eta n^{\max \gamma} / \gamma - r^{\max}) / (1 - \beta)$. These facts imply a lower bound on equilibrium continuation value for the politician and the representative citizen. Specifically, let $V(\{\xi, P\} | s^t)$ and $U(\{\xi, P\} | s^t)$ be the equilibrium continuation welfare to the incumbent politician and citizens, respectively, implied by $\{\xi, P\}$ at a history s^t .

Lemma 1 (Worst Punishment). *If $\{\xi, P\}$ is a sustainable equilibrium, then $V(\{\xi, P\} | s^t) \geq \underline{V}(b(s^t | s^{t-1})) \quad \forall s^t$ and $U(\{\xi, P\} | s^t) \geq \underline{U}(b(s^t | s^{t-1})) \quad \forall s^t$.*

Lemma 1 implies that there cannot be a sustainable equilibrium in which the incumbent politician receives a lower welfare than what he would receive from maximally extractive policies and replacement and in which the representative citizen expects a lower welfare than what he could receive from throwing out the current and all future governments.

18. $P(s^t)$ refers to the choice of P_{t+1} at history s^t .

19. Formally, $n^{\max} = \arg \max_n r(n)$, $r^{\max} = r(n^{\max})$, and $\tau^{\max} = 1 - \eta n^{\max \gamma - 1}$.

These observations allow us to provide necessary and sufficient conditions for a sustainable equilibrium using the methods developed by Abreu (1988).

Proposition 2 (Sustainable Equilibrium). $\{\xi, P\}$ is a sustainable equilibrium if and only if ξ is competitive and $\{\xi, P\}$ satisfies

$$V(\{\xi, P\} |_{s^t}) \geq \underline{V}(b(s^t | s^{t-1})) \quad \forall s^t, \text{ and} \quad (24)$$

$$U(\{\xi, P\} |_{s^t}) \geq \underline{U}(b(s^t | s^{t-1})) \quad \forall s^t \quad (25)$$

for $b(s^t | s^{t-1})$ determined by equation (23).

Proposition 2 implies that the lower bounds on continuation utilities established in Lemma 1 are not only necessary for a sustainable equilibrium, but when combined with equations (21) and (22), they are sufficient. More specifically, equations (24) and (25) emerge from the following punishment strategy. Whenever the incumbent politician or the representative citizen deviates from prescribed policies and replacement rules, the politician and the representative citizen revert to an equilibrium in which the incumbent is thrown out, taxes are maximal, public spending is zero, and debt is maximal forever. Conditions (24) and (25) guarantee that every deviation by the politician or representative citizen is weakly dominated.²⁰

Note that equations (24) and (25) illustrate the endogenous debt limits generated by the presence of politicians. Government debt cannot be too low since this tightens the incentive compatibility constraint of the incumbent politician by providing him with more resources to potentially appropriate. Government debt cannot be too high, since this tightens the incentive compatibility constraint of the representative citizen by reducing the cost of throwing out the incumbent politician. Let Λ represent the set of sustainable equilibrium sequences $\{\xi, P\}$.

5. EFFICIENT SUSTAINABLE EQUILIBRIA

Having characterized the entire set of sustainable equilibria, we select the efficient one so as to compare our economy to that under a benevolent ruler. To do this, we reduce the problem into one involving a single endogenous state variable that allows for a simple recursive representation of the problem (Section 5.2). We characterize the solution to this program, and we consider short-run dynamics (Section 5.3) and long run dynamics (Section 5.4). We relate our results to some of the previous literature (Section 5.5). Finally, we consider a numerical example (Section 5.6).

5.1. Definition

Definition 2. $\{\xi, P\} \in \Lambda$ is an efficient equilibrium if $\nexists \{\xi', P'\} \in \Lambda$ s.t.

$$U(\{\xi', P'\} |_{s_0}) > (\geq) U(\{\xi, P\} |_{s_0}) \text{ and } V(\{\xi', P'\} |_{s_0}) \geq (>) V(\{\xi, P\} |_{s_0}).$$

Our definition ignores the welfare of candidate politicians and considers only the welfare of the incumbent politician in period 0. An efficient equilibrium solves

$$\max_{\xi, P} U(\{\xi, P\} |_{s_0}) \text{ s.t. } V(\{\xi, P\} |_{s_0}) \geq V_0 \text{ and } \{\xi, P\} \in \Lambda. \quad (26)$$

20. Note that while we focus on the worst punishment, our analysis holds under any punishment for which $\underline{V}(b(s^t | s^{t-1}))$ equals a constant minus $b(s^t | s^{t-1})$ and $\underline{U}(b(s^t | s^{t-1}))$ equals a constant plus $b(s^t | s^{t-1})$.

In comparison, the original problem of Lucas and Stokey (1983) sets $x(s^t) = 0 \forall s^t$ and ignores constraints (24) and (25). For this reason, we will often consider the special case of $V_0 = 0$.

Assumption 1. The solution to equation (26) requires $P(s^t) = 1 \forall s^t$.

In the Appendix, we provide a condition under which Assumption 1 is satisfied. As an example, Assumption 1 is always satisfied if χ^c equals 0, since there is little benefit to the representative citizen from throwing out a politician and since it is optimal to keep the same politician in power forever so as to minimize the amount of rents required to induce his cooperation. If Assumption 1 were not satisfied, then an incumbent politician facing replacement would choose the extractive policies described in Section 4.3. Thus, we are effectively ignoring equilibria in which such policies are chosen.²¹ In practice, politicians might be replaced for exogenous reasons such as term limits, and we explore this possibility in Section 6.2.

5.2. Recursive program

We now show that equation (26) has a recursive representation. Let

$$z(s^t|s^{t-1}) = b(s^t|s^{t-1}) + \sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k|s^t) x(s^k). \tag{27}$$

We refer to $z(s^t|s^{t-1})$ as debt net of rents. A benevolent ruler sets $z(s^t|s^{t-1}) = b(s^t|s^{t-1})$. Let $n = \{n(s^t)\}_{t=0}^{\infty}$ and define c , g , and x analogously.

Lemma 2. $\{n, g\}$ solves (26) s.t. $b_{-1}(s^0) = b$ and $V(\{\xi, P\}|_{s^0}) = V$ if and only if $\{n, g\}$ solves (26) s.t. $b_{-1}(s^0) = b + V$ and $V(\{\xi, P\}|_{s^0}) = 0$.

Lemma 2 states that the sequence of taxes and public spending which are efficient subject to initial debt b and promised utility V to the politician also are efficient, subject to initial debt $b + V$ and promised utility 0 to the politician. This result is useful since it implies that we can write the recursive program as a function of a single state variable corresponding to the value of debt plus the continuation value to the politician.

To understand Lemma 2 more formally, use equations (21) and (27) to rewrite equations (23), (24), and (25), respectively:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k|s^t) (r(n(s^k)) - g(s^k)) = z(s^t|s^{t-1}) \forall s^t, \tag{28}$$

$$z(s^t|s^{t-1}) \geq \frac{r^{\max}}{1 - \beta} - \chi^p \forall s^t, \text{ and} \tag{29}$$

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k|s^t) u(n(s^k) - g(s^k), n(s^k), g(s^k), s_k) - z(s^t|s^{t-1}) \geq U^{\text{AUT}} + \chi^c \forall s^t. \tag{30}$$

21. Equilibria that also consider the welfare of candidate politicians might feature replacement.

Rewrite the period 0 welfare of households as

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s^0) u(n(s^t) - g(s^t), n(s^t), g(s^t), s_t) - z_{-1}(s^0) + b_{-1}(s^0). \quad (31)$$

Lemma 2 is a consequence of the fact that the optimal $\{n, g\}$ that maximizes equation (31) s.t. equations (28)–(30) only depends on $z_{-1}(s^0)$. Note that given this *unique* optimal sequence $\{n, g\}$, any sequence $\{c, x\}$ that satisfies the resource constraint and equation (27) in period 0 is optimal.²² The reason for this flexibility is that the politician is invested in the assets used to pay for his rents, and households are invested in the assets used to pay for their consumption. Consequently, it is possible to relax equation (24) by increasing rents (i.e. the value of cooperation) while holding debt constant or by increasing debt (i.e. the punishment from deviation) while holding rents constant. Because of risk neutrality, both of these methods are equivalent from a welfare perspective and have the same implications for the incentive compatible sequence of labour, public spending, and debt net of rents. Analogous arguments hold with respect to the relaxation of equation (25).²³ Note that with respect to the three-period economy of Section 2, equation (29) is analogous to equations (8)–(10) and equation (30) is analogous to equations (11)–(13).

An implication of Lemma 2 is that equation (26) can be written recursively.²⁴ Given equations (28)–(31), and letting $\pi_{ks} = \Pr\{s_{t+1} = k | s_t = s\}$, we can write:

$$J(s, z) = \max_{n, g, \{z_k\}_{k \in S}} n - g - \eta \frac{n^\gamma}{\gamma} + \theta(s) \frac{g^\alpha}{\alpha} + \beta \sum_{k \in S} \pi_{ks} J(k, z_k) \quad (32)$$

s.t.

$$z = r(n) - g + \beta \sum_{k \in S} \pi_{ks} z_k \quad (33)$$

$$z_k \geq r^{\max} / (1 - \beta) - \chi^p \quad \forall k \in S \text{ and} \quad (34)$$

$$J(k, z_k) - z_k \geq U^{\text{AUT}} + \chi^c \quad \forall k \in S. \quad (35)$$

$J(s, z)$ represents the highest possible welfare to households—net of the stream of rents—that can be achieved conditional on the state s and on the value of debt net of rents being equal to z .²⁵ Equation (32) represents this program written in a recursive fashion. n and g represent labour and public spending today, respectively. z_k represents the value of debt net of rents conditional on the realization of the state k following state s . Equation (33) ensures that the value of debt net of rents is z . Equations (34) and (35) represent recursive versions of equations (29) and (30), respectively. We can characterize $J(s, z)$.

Lemma 3. $J(s, z)$ is defined over a compact interval $[\underline{z}_s, \bar{z}_s]$, is strictly concave, and is continuously differentiable in $z \in (\underline{z}_s, \bar{z}_s)$.

22. $z_{-1}(s^0)$ is chosen to minimize $V(\{\xi, P\} |_{s^0})$ subject to feasibility and $V(\{\xi, P\} |_{s^0}) \geq \max\{V_0, \underline{V}(b_{-1}(s^0))\}$.

23. This is in contrast with the work of Ray (2002) and Acemoglu, Golosov and Tsyvinski (2008a,b) who show that payments must be backloaded to provide incentives. This is not the case here because of risk neutrality and the existence of financial markets.

24. This simplification is a consequence of the quasi-linearity of preferences. In the presence of concave preferences, the relevant program would have three state variables: s , b , and V .

25. The welfare of households in period 0 is $J(s_0, z_{-1}(s^0)) - z_{-1}(s^0) + b_{-1}(s^0)$.

Remark 1. *The Lucas and Stokey (1983) solution is a solution to equations (32)–(33).*

This problem adds two distortions to the original problem of Lucas and Stokey (1983). First, the generation of rents can crowd out household consumption and increase taxes by raising $z_{-1}(s^0)$ above $b_{-1}(s^0)$. Second, the constraints on z function like endogenous debt limits. Specifically, if z is interpreted as debt, then equations (32)–(35) are isomorphic to an economy under a benevolent ruler facing *ad hoc* debt limits.

Let λ , $\beta\pi_{ks}\phi_k$, and $\beta\pi_{ks}\psi_k$ represent the Lagrange multipliers for equations (33), (34), and (35), respectively. First-order conditions and the envelope condition yield:

$$n : \eta n^{\gamma-1} = \frac{1 + \lambda}{1 + \gamma\lambda} \tag{36}$$

$$g : \theta(s) g^{\alpha-1} = 1 + \lambda \tag{37}$$

$$z_k : J_z(k, z_k) = -\frac{\lambda + \phi_k - \psi_k}{1 + \psi_k} \tag{38}$$

$$z : J_z(s, z) = -\lambda. \tag{39}$$

Equations (36) and (37) pin down output (and consequently the tax rate) and public spending, respectively, as a function of λ . By equation (39), higher z is associated with higher λ , higher taxes, and lower public spending. Equations (38) and (39) show that the slope of equation $J(\cdot)$ only changes if an incentive compatibility constraint binds. For example, under a benevolent ruler, the slope of equation $J(\cdot)$ never changes. Define

$$\tilde{g}(s^t) = \frac{g(s^t)}{\theta(s_t)^{1/(1-\alpha)}},$$

the value of public spending normalized by its productivity.

Remark 2. *The Lucas and Stokey (1983) solution is*

$$\tau(s^t) = \tau(s^{t-1}) \text{ and } \tilde{g}(s^t) = \tilde{g}(s^{t-1}) \forall s^t.$$

A benevolent government smoothes the deadweight loss from revenue generation by choosing constant taxes and it fixes the marginal product of public spending over time. We now consider how political distortions alter this prediction.

5.3. Equilibrium dynamics

In the simple three-period economy of Section 2, the incentive compatibility constraints on politicians and citizens created volatility and persistence in taxes. This is true here in the dynamic economy as well since policy dynamics are characterized by S, s rules.²⁶

26. Note that our timing of events delivers an exact S, s rule for taxes. If instead citizens make the replacement decision P_{t+1} after markets open and clear, then the worst punishment remains the same, though $\bar{\tau}(s_t)$ is replaced by $\bar{\tau}(s_t, s_{t-1})$ in equation (40) and $\underline{g}(s_t)$ is replaced by $\underline{g}(s_t, s_{t-1})$ in equation (41).

Proposition 3 (Optimal Policy Dynamics). *The unique sequences τ and g which solve equation (26) have the following property:*

$$\tau(s^t) = \begin{cases} \bar{\tau}(s_t) & \text{if } \tau(s^{t-1}) > \bar{\tau}(s_t) \\ \tau(s^{t-1}) & \text{if } \tau(s^{t-1}) \in [\underline{\tau}(s_t), \bar{\tau}(s_t)] \\ \underline{\tau}(s_t) & \text{if } \tau(s^{t-1}) < \underline{\tau}(s_t) \end{cases} \quad (40)$$

$$\tilde{g}(s^t) = \begin{cases} \bar{\tilde{g}}(s_t) & \text{if } \tilde{g}(s^{t-1}) > \bar{\tilde{g}}(s_t) \\ \tilde{g}(s^{t-1}) & \text{if } \tilde{g}(s^{t-1}) \in [\underline{\tilde{g}}(s_t), \bar{\tilde{g}}(s_t)] \\ \underline{\tilde{g}}(s_t) & \text{if } \tilde{g}(s^{t-1}) < \underline{\tilde{g}}(s_t) \end{cases}. \quad (41)$$

Equations (40) and (41) are dynamic equations which characterize the time path of the tax rate and public spending. The proposition states that perfect smoothing takes place (i.e. $\tau(s^t) = \tau(s^{t-1})$ and $\tilde{g}(s^t) = \tilde{g}(s^{t-1})$) only if $\tau(s^{t-1})$ and $\tilde{g}(s^{t-1})$ are within sustainable intervals which depend on s_t . Otherwise, $\tau(s^t) \neq \tau(s^{t-1})$ and $\tilde{g}(s^t) \neq \tilde{g}(s^{t-1})$, and $\tau(s^t)$ and $\tilde{g}(s^t)$ jump to the state-dependent upper bound or lower bound of the sustainable policy range. For example, if $\tau(s^{t-1}) < \underline{\tau}(s_t)$ and $\tilde{g}(s^{t-1}) > \bar{\tilde{g}}(s_t)$, then $\tau(s^t)$ jumps to $\underline{\tau}(s_t)$ and $\tilde{g}(s^t)$ jumps to $\bar{\tilde{g}}(s_t)$.

More specifically, the tax rate cannot be below a state-dependent lower bound $\underline{\tau}(s_t)$, and normalized public spending cannot be above a state-dependent upper bound $\bar{\tilde{g}}(s_t)$. This ensures that the incentive compatibility constraint of the politician is satisfied at s^t . The government cannot enter a given state with too many assets since the politician would divert these assets towards rents. This creates upward pressure on current and future taxes and downward pressure on current and future public spending. In other words, public spending must be financed with future, not past taxes.

Furthermore, the tax rate cannot be above a state-dependent upper bound $\bar{\tau}(s_t)$ and normalized public spending cannot be below a state-dependent lower bound $\underline{\tilde{g}}(s_t)$. This ensures that the incentive compatibility constraint of the representative citizen is satisfied at s^t . More specifically, taxes cannot be too high and public spending cannot be too low going forward, since otherwise citizens will want to replace the incumbent. In other words, public spending must be financed with past, not future, taxes.

The state-dependent bounds for taxes and public spending are not a function of initial conditions but of the exogenous parameters in the environment. For instance, a reduction in χ^p makes it more difficult to satisfy the incentive compatibility constraint of politicians and puts upward pressure on the lower bound for taxes and downward pressure on the upper bound on normalized public spending. Moreover, an increase in χ^c makes it more difficult to satisfy the incentive compatibility constraint of citizens and puts downward pressure on the upper bound for taxes and upward pressure on the lower bound for normalized public spending. While the bounds for taxes and normalized public spending are independent of initial conditions, the levels of $\tau(s^0)$ and $\tilde{g}(s^0)$ are determined by the initial condition $z_{-1}(s^0)$, which depends on $b_{-1}(s^0)$ and V_0 .

Given (40) and (41), we can describe the relationship between our environment and that of a benevolent ruler. Note that if χ^p is sufficiently high, the cost to politicians of being thrown out of power is high so that the lower bound on taxes and the upper bound on public spending never bind. Furthermore, if χ^c is sufficiently low, the benefit of replacing incumbents is low, so that the upper bound on taxes and the lower bound on public spending also never bind.

Corollary 1. *If $\chi^p = \infty$ and $\chi^c = 0$, then the unique sequences τ and g that solve equation (26) have the following property:*

$$\tau(s^t) = \tau(s^{t-1}) \text{ and } \tilde{g}(s^t) = \tilde{g}(s^{t-1}) \forall s^t.$$

Therefore, if $V_0 = 0$, so that $z_{-1}(s^0) = b_{-1}(s^0)$, then the solution will exactly coincide with that under a benevolent ruler of Lucas and Stokey (1983). In contrast, if χ^p is low, then the incentive compatibility constraint on the politician occasionally binds, and if χ^c is high, then the incentive compatibility constraint on the representative citizen occasionally binds. By the same reasoning as in the example of Section 2, it is inefficient in such circumstance to maintain a fixed tax rate. More generally, the tax rate reflects the last binding incentive compatibility constraint, since shocks to the productivity of public spending create variation in the tightness of incentive constraints, and taxes respond persistently to these shocks.

Theorem 1 (Short-Run Volatility and Persistence). *If $\chi^p < \chi^{p*}$ or $\chi^c > \chi^{c*}$, $\exists V_0, s_0$ s.t. the solution to equation (26) admits $\tau(s^t) \neq \tau(s^k)$ for $s_t = s_k$.*

This theorem establishes that there are initial conditions under which the tax rate experiences persistent changes after an incentive compatibility constraint binds. For example, imagine that $\chi^c = 0$, $V_0 = 0$, and $\chi^p < r^{\max} / (1 - \beta) - b_{-1}(s^0)$, so that the politician’s incentive compatibility constraint binds at date 0 and $\tau(s^0) = \underline{\tau}(s_0)$. If the initial state s_0 chosen is such that $\underline{\tau}(s_0) < \underline{\tau}(s_1)$ for some $s_1 \neq s_0$, then the tax rate will increase in a persistent fashion whenever the state s_1 occurs.

As in the example of Section 2, taxes are volatile since the government is limited in its ability to smooth revenue raising distortions. Moreover, taxes respond persistently to shocks and are smoothed into the future the extent allowed by future incentive compatibility constraints. This persistence not only smoothes economic distortions going forward, but it relaxes future incentive compatibility constraints through the implied change in debt net of rents. A way to see this is to note that debt net of rents is always within state-dependent bounds associated with the S, s bounds for taxes and public spending:

$$z(s^t | s^{t-1}) \in [\underline{z}(s_t), \bar{z}(s_t)]. \tag{42}$$

How are the bounds on taxes related to the state of the economy? In the three-period example of Section 2, the lower bound on taxes is higher when public spending is more productive (the politician requires incentives), and the upper bound on taxes is lower when public spending is less productive (the representative citizen requires incentives). In a dynamic economy, this is generally true under independent identically distributed (i.i.d.) shocks.²⁷

Proposition 4 (i.i.d. Tax Intervals). *If $\pi_{ks} = \pi_k \forall k, s \in S$, then $\underline{\tau}(s) < \underline{\tau}(k)$ and $\bar{\tau}(s) < \bar{\tau}(k)$ if $\theta(s) < \theta(k)$.*

This means that under i.i.d. shocks, the lower and upper bound for the sustainable range of taxes will be increasing in the productivity of public spending θ . Intuitively, when θ increases, public spending increases, and this tightens the incentive compatibility constraint

27. This result stands in contrast to Battaglini and Coate (2008) in which politicians require less pork during high public expenditure shocks.

on the politician since taxes must rise into the future to finance this spending. In contrast, when θ decreases, public spending decreases, and this tightens the incentive compatibility constraint on the representative citizen since taxes must decrease into the future in order to generate support for the government. Given the policy rule (40), this means that the equilibrium tax rate is more likely to increase (decrease) tomorrow if θ increases (decreases) tomorrow.

While the underlying driver of the persistence of taxes in our economy is the conflict between politicians and citizens, operationally, the persistence emerges because the government in our economy effectively under-insures, and this brings the economy closer to an incomplete market economy in which contingent debt is unavailable. An easy way to see this is to construct an example in which the government does not actually use contingent debt, despite its availability.²⁸ Consider the setting in which $\chi^c = 0$, and consider the solution described in Proposition 3 for $V_0 = 0$. Define

$$\underline{b}(s) = \frac{\underline{r}(s) - \max_{k \in S} \{\theta(k)^{1/(1-\alpha)}\} \bar{g}(s)}{1 - \beta}$$

for $\underline{r}(s)$ which corresponds to the level of revenue associated with $\underline{\tau}(s)$. Assume that $b_{-1}(s^0) \leq \underline{b}(s_0)$. We can construct an efficient equilibrium in which

$$b(s^t | s^{t-1}) = b(s^{t-1}) = \max \{b(s^{t-2}), \underline{b}(s_{t-1})\},$$

so that debt is no longer state-contingent and it is increasing along the equilibrium path. In such an environment, the government permanently increases taxes and debt whenever the politician requires incentives. When incentive constraints on politicians do not bind, debt is held constant and rents increase (decrease) when public spending decreases (increases). When incentive constraints on politicians bind, both debt and rents permanently increase into the future.²⁹ This example thus shows that political economy distortions generate endogenous market incompleteness since contingent claims are not used despite their availability.

5.4. Long-run dynamics

We have shown that taxes in the short run are volatile and persistent. To what extent is this true in the long run? The simple example of Section 2 shows that if we ignore the incentive compatibility constraint of the representative citizen, revenues permanently increase following a one-time shock and they do not revert back down. Analogous reasoning implies that if we ignore the incentive compatibility constraint of the politician. We use this observation to explore the long-run properties of the tax rate. By Proposition 3, the tax rate converges if the sustainable intervals for the tax rate have an overlapping region. A tax rate in this region satisfies all incentive compatibility constraints under all states.

Theorem 2 (Long Run). $\exists \hat{\chi}^c(\cdot)$ and $\hat{\chi}^p(\cdot)$ which are weakly increasing continuous functions s.t. τ which solves equation (26) converges almost surely if and only if $\chi^c \leq \hat{\chi}^c(\chi^p)$ and $\chi^p \geq \hat{\chi}^p(\chi^c)$.

Theorem 2 states that the tax rate converges if χ^c is sufficiently low (i.e. the incentive compatibility constraints on the representative citizen are sufficiently slack) and if χ^p is

28. Given the assumption of quasi-linearity, there are many sequences of rents and debt which are associated with the unique sequence of labour and public spending which solve the program.

29. This follows from the fact that in this environment, the government's dynamic budget constraint implies that $x(s^t) = \tau(s^t)n(s^t) - g(s^t) + \beta b(s^t) - b(s^{t-1})$.

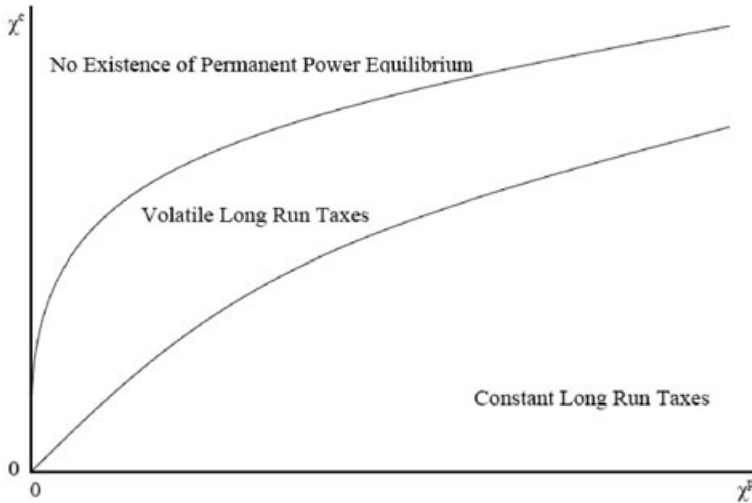


FIGURE 2
Long run taxes

sufficiently high (i.e. the incentive compatibility constraints on the politician are sufficiently slack). These conditions guarantee that the sustainable tax rate intervals have an overlapping region. Moreover, the theorem states that if χ^c decreases (χ^p increases), then the tax rate converges for a weakly larger range of χ^p (χ^c). In other words, if incentive compatibility constraints are sufficiently slack, then even though tax rates are volatile and persistent along the equilibrium path, they converge to a constant level in the long run. If these conditions are not satisfied, then a constant tax rate cannot simultaneously satisfy both the politician and the representative citizen, so that taxes are volatile even in the long run. The theorem is displayed in Figure 2, which plots χ^c against χ^p with the relevant regions of long run tax convergence and long run tax volatility. Note that in addition to these regions, we also display the range over which an equilibrium without replacement does not exist.³⁰

Intuitively, incentive provision for the politician puts upward pressure on the tax rate under some shocks and incentive provision for the representative citizen puts downward pressure on the tax rate under some shocks. When there is sufficiently little benefit to the representative citizen from throwing out the politician, the representative citizen will tolerate very high tax rates. Analogously, when there is sufficiently little benefit to the politician from additional rent-seeking, he will tolerate low tax rates. For example, imagine if χ^c is low. Along the equilibrium path, the government accumulates debt and rents to accommodate the politician's incentives, and the representative citizen receives sufficiently little benefit from throwing out the incumbent so that he accepts the gradual increase in the tax rate and decrease in public spending which accompany the government's accumulation of debt and rents. In the long run, this economy is qualitatively similar to an economy managed by a benevolent ruler but with more debt net of rents than that associated with a benevolent ruler.

The long-run behaviour of this economy stands in contrast to that of Aiyagari *et al.* (2002), in which markets are exogenously incomplete. They show that in an economy without state-contingent debt, taxes respond persistently to shocks along the equilibrium path, as they do

30. In our setting, such an equilibrium involves the tax rate converging to the maximum.

here. However, the benevolent ruler accumulates assets along the equilibrium path until he can finance the entire stream of public spending with zero taxes. In our economy, taxes do not converge to zero and they can remain volatile even in the long run.

5.5. Predicting tax rate movements

Our model predicts that tax rates should sometimes adjust persistently to shocks, and this is in line with what we observe empirically. As mentioned in the introduction, both Barro (1979) and Aiyagari *et al.* (2002) also predict persistent tax rates, and they achieve this by ruling out state-contingent debt. A natural question is how the stochastic process of tax rates in our economy compares with theirs.

To simplify the discussion, let the shock θ map one to one with the state s . According to Lucas and Stokey (1983), the tax rate covaries one to one with θ (in the quasi-linear model, the covariance is zero), which means that tax rates tomorrow are best predicted by today's shock used to forecast tomorrow's shock:

$$E(\tau_t | \tau_{t-1}, \dots, \tau_0, \theta_{t-1}, \dots, \theta_0) = E(\tau_t | \theta_{t-1}).$$

In contrast, according to Barro (1979)'s intuitions, taxes are a random walk, which means that yesterday's tax rate alone can predict today's tax rate:

$$E(\tau_t | \tau_{t-1}, \dots, \tau_0, \theta_{t-1}, \dots, \theta_0) = E(\tau_t | \tau_{t-1}).$$

Our model combines features of both of these statistical processes. Given (40), both past tax rates and past shocks are required to forecast tomorrow's tax rate:

$$E(\tau_t | \tau_{t-1}, \dots, \tau_0, \theta_{t-1}, \dots, \theta_0) = E(\tau_t | \tau_{t-1}, \theta_{t-1}).$$

This statistical process for the tax rate in our model is qualitatively similar to that of Aiyagari *et al.* (2002), even though there are no exogenous limits on the contingency of government debt in our model. The crucial distinction between our model and theirs is in the *long-run* implications for the tax rate. In their model, the tax rate converges to zero and the government holds more assets in the long run than would be implied under a benevolent ruler with complete markets. In our model, the tax rate may not converge, and if it converges, it will not generally converge to zero. Our model therefore links the existence of politicians to the endogenous incompleteness of markets, and it provides different implications than an economy with exogenous market incompleteness.

5.6. Numerical example

In this section, we illustrate the mechanics of our model using a numerical simulation. Let $(\eta, \gamma, \alpha, b_0, V_0) = (0.75, 2, 0.5, 0, 0)$, $\beta = 0.95$, and $\theta_t = \{4, 5, 6\}$. Normalize the resource constraint to $c + g + x = 10n$. The transition matrix for θ is

$$\begin{bmatrix} 0.98 & 0.02 & 0 \\ 0.01 & 0.98 & 0.01 \\ 0 & 0.02 & 0.98 \end{bmatrix},$$

so that each shock is very persistent, and a path between the highest to the lowest shock must pass through the middle shock. Let $\theta_0 = 4$. We compare three cases: $\chi^P = \infty$ and $\chi^c = 0$ (case 1), $\chi^P = 633$ and $\chi^c = 0$ (case 2), and $\chi^P = 633$ and $\chi^c = 1200$ (case 3).

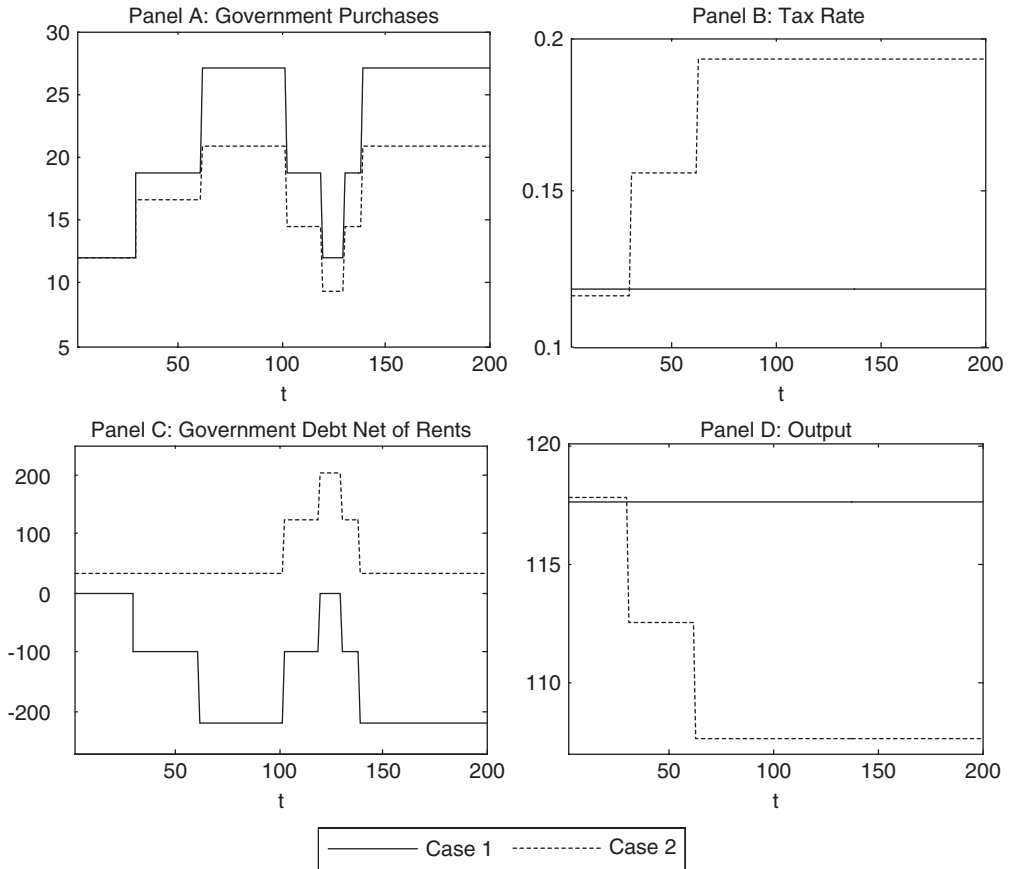


FIGURE 3
Comparing cases 1 and 2

As a reminder, case 1 corresponds to the economy under a benevolent ruler, since the incentive compatibility constraint on neither the politician nor the representative citizen binds. Case 2 ignores the incentive compatibility constraint of the representative citizen, and case 3 takes the incentive compatibility constraint of the representative citizen into account. In cases 2 and 3, the size of χ^P is chosen such that a deviation in period zero yields r^{\max} to the politician off the equilibrium path. In case 3, χ^c is chosen such that the representative citizen's constraint binds in the low state only.

Figure 3 compares cases 1 and 2 for a realized sequence of θ shocks. In case 1, rents are zero, public spending (Panel A) and government debt (Panel C) vary only with the state, and taxes (Panel B) and output (Panel D) are constant. These policies are not incentive-compatible in case 2 since they violate equation (29). In case 2, policies reflect the last binding incentive compatibility constraint on the politician, until the tax rate reaches a maximum and the economy becomes qualitatively the same as in case 1. Since $\chi^c = 0$, the representative citizen's incentive compatibility constraint never binds.

Figure 4 compares cases 1 and 3. Because χ^c is large, the representative citizen's incentive compatibility constraint binds in the lowest state in a transition path from the high state to the low state. In the low state, the government is less productive, and citizens need incentives to

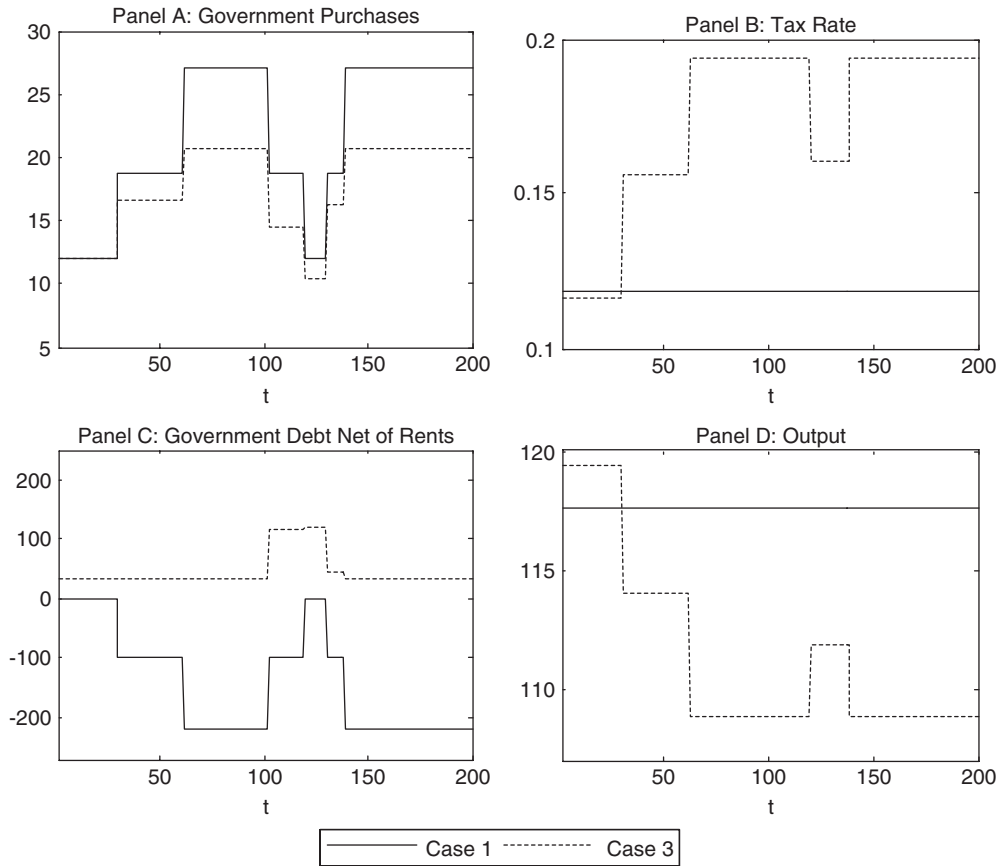


FIGURE 4
Comparing cases 1 and 3

not throw out the politician, so that the tax rate decreases. As a consequence, the tax rate in the middle state depends on whether the highest or the lowest state occurred most recently. This means that even in the long run, the tax rate and output continue to be volatile and continue to reflect the history of shocks.

We compare the period 0 welfare of households in different economies to determine the welfare cost due to the existence of rent-seeking politicians. We calculate the fraction of consumption that would be sacrificed in the economy of case 1 to make a household indifferent between living in case 1 (i.e. under a benevolent ruler with complete markets) and living in a given economy. This welfare cost is 1.94% both in case 2 and in case 3. Since rents represent around 1.6% of case 1 consumption in both cases 2 and 3, this suggests that the welfare cost is primarily due to the transfer of consumption away from households towards politicians, and it is not primarily due to the extra volatility in taxes or reduction in public spending.³¹ As a comparison, the welfare cost of imposing a balanced budget on a benevolent government is

31. More specifically, if one replaced case 1 with an economy managed by a benevolent government constrained to providing the politician with the same period 0 welfare as under case 2, then the welfare cost under case 2 becomes 0.06%.

0.07%, and the welfare cost of excluding non-contingent debt for a benevolent government is 0.04%. Therefore, the welfare cost due to political economy distortions relative to these constraints is high, though the primary source of this additional cost is a pure transfer of resources to politicians.

This does not however imply that political economy distortions are small. It only means that they are small under the optimal policy. For example, consider the solution to equation (26) subject to $\tau(s^t) = \tau(s^{t-1}) \forall s^t$ and $\tilde{g}(s^t) = \tilde{g}(s^{t-1}) \forall s^t$, which is a policy appropriate for a benevolent government under which there is no volatility in taxes. The cost of this suboptimal sustainable policy is very high and equal to 8.77%. This cost is high because there is an increase in the transfer of rents from citizens to politicians which equals 5.2% of case 1 consumption. Moreover, in addition to this pure transfer, the economy suffers because public spending declines. In other words, under the suboptimal sustainable policy, the government provides fewer public goods and wastes more resources on rents than in the best sustainable policy.³² This is because the politician cannot be trusted to pledge accumulated revenues for public use, and this simultaneously limits the size of the government while making the government more wasteful.

6. EXTENSIONS

Our analysis thus far has ignored two important frictions that could realistically interact with politicians' rent-seeking incentives: politicians can default on outstanding debt, and politicians can be replaced even after good behaviour. In this section, we explore the effect of allowing for these two frictions, and we highlight how our main results are insensitive to allowing for these possibilities. For simplicity, we let $\chi^c = 0$ so as to ignore the incentives of the representative citizen, we let $V_0 = 0$, and we let citizens to throw a politician out at the end of the period t so that the politician receives $-\chi^p$ once thrown out.³³

6.1. Default

Imagine that in addition to its policy choices, the government chooses an indicator $D_t = \{0, 1\}$ which represents a decision to default on outstanding debt. In such a setting, one can define sustainable equilibria, as we do in Section 4, and an important implication of this definition is that, in their savings decisions, households will take into account their expectations of *future* default. This is important in characterizing the optimal deviation as well as the optimal punishment for the politician. It is obvious that a politician's best deviation will involve defaulting on outstanding debt in addition to choosing maximal taxes and minimal public spending. More subtle, however, is that if the politician attempts to extract the maximal amount of debt \bar{b} , the optimal punishment strategy will involve households expecting default by future governments, and this induces a market clearing price of zero for this debt. Consequently, the politician's best deviation yields:

$$\underline{V}(b(s^t|s^{t-1})) = r^{\max} - \min\{0, b(s^t|s^{t-1})\} - \chi^p,$$

32. Another way to see this is to perform the same exercise as in footnote 31, so that the welfare cost of political economy becomes 2.77%.

33. This does not alter the incentive compatibility constraint on the incumbent in our original framework.

which implies that the politician's incentive compatibility constraint is equivalent to the following two constraints for $z(s^t|s^{t-1})$ defined in Section 5.2:

$$z(s^t|s^{t-1}) \geq r^{\max} - \chi^p \forall s^t, \text{ and} \quad (43)$$

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k|s^t) x(s^k) \geq r^{\max} - \chi^p \forall s^t. \quad (44)$$

Note that equation (43) is analogous to equation (29), though the constraints on the values of $z(s^t|s^{t-1})$ are now *weaker* so that the possibility of default is actually allowing the government to be less constrained in its savings. The additional constraint of equation (44) puts discipline on incentive-compatible sequences of x which can be chosen for a given unique optimal sequence of n and g , and such a constraint can be easily satisfied by choosing a constant level of rents. Consequently, our analysis of the case without default can be applied to this case with default, and the time path of taxes and public spending is characterized by updating rules analogous to equations (40) and (41). The reason why the possibility of default does not affect our results is that our results are driven in large part by the fact that large government asset positions are costly since they are associated with high rents. This fact is unaffected by the government's ability to renege on its debt.

6.2. Equilibrium replacement

Imagine that at the end of every period, the incumbent is exogenously replaced with probability $1 - \delta(s_t) \in (0, 1)$, and let the realization of replacement be independent of all actions and the identity of the politician. The economy we study in the text considers the special case for which $\delta(s_t) = 1 \forall s_t$.³⁴

Note that equation (24) in the presence of replacement can only become tighter at every history since the politician facing potential replacement assigns a weakly lower weight to $x(s^t)$ relative to the politician guaranteed to remain in power forever. Nonetheless, despite the tighter constraints, the efficient path of taxes and public spending in our original economy can be sustained in the presence of replacement. To see why, choose taxes and public spending as in the original economy, and let $x(s^0) > 0$ and $x(s^t) = 0$ for all s^t for $t > 0$ so that the entire equilibrium stream of rents is paid to the incumbent in period 0 and so that $b(s^t|s^{t-1}) = z(s^t|s^{t-1})$ for $t > 0$. For $t > 0$, equilibrium rents are zero and the replacement probability plays no role in the incumbent's calculus, and since equation (24) is satisfied in the original economy, the politician has no incentive to deviate. If $t = 0$, the politician facing exogenous replacement assigns the same weight to his assured initial rent $x(s^0)$ as the politician permanently in power, so that again the replacement probability plays no role. Thus, the presence of replacement does not change the characterization of our results.

This analysis changes somewhat in the presence of default as in Section 6.1. In this circumstance, equations (43) and (44) must accommodate the survival probability $\delta(s_t)$. If $r^{\max} > \chi^p$, it can be shown that in this circumstance, equation (44)—which now includes $\delta(s_t)$ —always binds so that $x(s^t) = (r^{\max} - \chi^p)(1 - \delta(s_t)\beta)$ for all s^t . Intuitively, the politician is less patient than the representative household, so it is best to front-load rents to the extent allowed by incentive compatibility constraints. This requires paying him more when

34. Debt could in principle depend on whether or not a politician was replaced. Nonetheless, it can be shown that there is no efficiency gain from letting debt be contingent on replacement since every incumbent receives the lowest continuation value.

his survival probability is low. Given this stream of rents, equation (43) can be rewritten as:

$$z(s^t | s^{t-1}) \geq (r^{\max} - \chi^p) \sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k | s^t) (1 - \delta(s_k) \beta) \forall s^t. \quad (45)$$

Consequently, our analysis of the case without exogenous replacement and without default can be applied to this case, and the time path of taxes and public spending is characterized by updating rules analogous to equations (40) and (41). Moreover, note that equation (45) is tighter (i.e. the upward pressure on taxes and debt rises) whenever survival probabilities going forward are projected to be low.

We emphasize that this comparative static refers to *exogenous* changes in the survival probability of the government such as term limits. A more detailed model of political cycles should also take into account an additional force embedded in χ^c which parameterizes the popularity of the current incumbent. One can show, for example, that exogenous fluctuations in χ^c will affect the sustainable bounds on taxes and public spending in equations (40) and (41), where a reduction in the popularity of the incumbent pushes the government towards reducing taxes and increasing public spending.

7. CONCLUSION

In this paper, we have developed a theoretical framework that studies the optimal management of taxes and debt in an environment with self-interested politicians and citizens. In doing this, we have argued that incentive compatibility for the incumbent politician and the representative citizen takes the form of endogenous debt limits on the government, and this creates distortions which generate more macroeconomic persistence and volatility than under a benevolent ruler under optimal policies. Our model predicts that taxes respond persistently to shocks even though financial markets are complete, and long-run taxes are non-zero, which is in contrast to an economy with exogenously incomplete financial markets. While we have made our arguments in a setting in which households have quasi-linear preferences, many of the insights achieved here transmit to an economy which allows for risk aversion.³⁵

Our analysis leaves some natural directions for future research. We have assumed the perfect observability of the politician's actions, although in practice rent-seeking is a private activity. Relaxing this assumption would generate even further distortions in our economy and provide more limits on financial markets. It would also potentially generate endogenous political replacement and a political business cycle. Second, our model ignores the important interaction between fiscal policy and monetary policy by focusing on the real economy. We plan to explore these extensions in future research.

APPENDIX A. DEFINITIONS

The following definitions simplify notation:

$$\begin{aligned} \Gamma(s, \lambda) &= n - \eta n^\gamma - g, \text{ and} \\ W(s, \lambda) &= \eta n^\gamma (1 - 1/\gamma) + \theta(s) g^\alpha / \alpha \end{aligned}$$

35. We have shown this result in previous versions of this paper for a particular class of preferences. Intuitively, even if politicians can manipulate the interest rate, it is still the case that taxes respond persistently to shocks since the government cannot save as much to prepare for shocks due to the incentive compatibility constraint of politicians.

for n and g which satisfy equations (36) and (37) given λ . By the implicit function theorem, $\Gamma_\lambda(s, \lambda) > 0$ and $W_\lambda(s, \lambda) < 0$ if $n \geq n^{\max}$. Also, $\Gamma(s, \lambda) > \Gamma(k, \lambda)$ and $W(s, \lambda) < W(k, \lambda)$ if $\theta(s) < \theta(k)$.

APPENDIX B. PROOFS OF SECTION 4

B.1. Proof of Proposition 1

Step 1. For necessity, equation (21) follows from equation (19). The intra-temporal and inter-temporal conditions for the household at s^t , respectively, are:

$$1 - \tau(s^t) = \eta n(s^t)^{\gamma-1} \quad (46)$$

$$q(s^{t+1}|s^t) = \beta \pi(s^{t+1}|s^t). \quad (47)$$

Equation (46) implies the function $r(n)$. For the necessity of equation (22), let $q(s^t|s^0) = q(s^t|s^{t-1}) \times \dots \times q(s^1|s^0)$. By equation (47), $q(s^t|s^0) = \beta^t \pi(s^t|s^0)$. Substitute $\beta \pi(s^{t+1}|s^t)$ in for $q(s^{t+1}|s^t)$ and $r(n(s^t))$ in for $\tau(s^t)n(s^t)$ in equation (17) at s^t , multiply both sides of equation (17) at s^t by $\beta^t \pi(s^t|s^0)$, and take the sum of all constraints (17) subject to the transversality condition implied by equation (20)

$$\lim_{t \rightarrow \infty} \beta^{t+1} \pi(s^{t+1}|s^0) b(s^{t+1}|s^t) = 0, \quad (48)$$

for $b(s^{t+1}|s^t)$ which represents a bond traded at s^t with a payment contingent on the realization of s^{t+1} . This yields equation (22). Similar arguments imply equation (23).

Step 2. For sufficiency, choose $\tau(s^t)$ which satisfies equation (46). Let $b(s^t|s^{t-1})$ satisfy equation (23). Equation (17) is satisfied. Given equation (21), equation (16) is satisfied by Walras's law. \parallel

B.2. Proof of Lemma 1 and Proposition 2

Step 1. A sustainable equilibrium must be competitive so as to satisfy equations (21) and (22).

Step 2. For the necessity of equation (24), the politician at history s^t can choose a deviation to $\tau'(s^t) = \tau^{\max}$, $b'(s^{t+1}|s^t) = \bar{b} \forall s^{t+1}$, $g'(s^t) = 0$, and $x'(s^t) = r^{\max}/(1-\beta) - b(s^t|s^{t-1})$, for $x'(s^t)$ derived from equations (17), (46), (47), and the definition of \bar{b} . Since $g'(s^k) \geq 0$ and $x'(s^k) \geq 0 \forall k > t$, then $\tau'(s^k) = \tau^{\max}$, $b'(s^{k+1}|s^k) = \bar{b}$, $g'(s^k) = 0$, and $x'(s^k) = 0 \forall k > t$. Since $\chi^p > 0$, the lowest welfare to the politician after the deviation is $\underline{V}(b(s^t|s^{t-1}))$.

Step 3. For the necessity of equation (25), the representative citizen can throw out the current incumbent. Following this decision, the current incumbent's best response sets $x'(s^t) = r^{\max}/(1-\beta) - b(s^t|s^{t-1})$ with the same policies as in step 2. No other response can dominate this response since by equations (23) and the definition of r^{\max} and \bar{b} ,

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k|s^t) x'(s^k) \leq r^{\max}/(1-\beta) - b(s^t|s^{t-1}), \quad (49)$$

and $x'(s^k) \geq 0 \forall k > t$. By step 2, all future governments set $\tau'(s^k) = \tau^{\max}$, $b'(s^{k+1}|s^k) = \bar{b}$, $g'(s^k) = 0$, and $x'(s^k) = 0 \forall k > t$. The best response of all future representative citizens is to throw out future incumbents. By equations (21) and (23), the representative citizen receives $\underline{U}(b(s^t|s^{t-1}))$ after the deviation.

Step 4. For the sufficiency of equations (24) and (25), consider the following equilibrium. Any deviation by the incumbent at s^t results in the representative citizen throwing out the incumbent at s^{t+1} as in step 3. Any deviation by the representative citizen at s^t results in the incumbent choosing extractive policies as in step 2. Given this punishment, the best deviation by the incumbent at s^t yields the right-hand side of equation (24). This is because the politician's welfare from rents under the best deviation, $x'(s^t) + \sum_{s^{t+1} \in S^{t+1}} \beta \pi(s^{t+1}|s^t) x'(s^{t+1})$, cannot exceed the right-hand side of equation (49), and the right-hand side of equation (49) can be achieved with the same policies as described in step 2. By equation (24), this deviation is weakly dominated. If the representative citizen deviates by throwing out the current incumbent at s^t , then he achieves the right-hand side of equation (25) by step 3, but this deviation is weakly dominated by equation (25). \parallel

APPENDIX C. SUFFICIENT CONDITION FOR ASSUMPTION 1

The required condition is that $V_0 \geq 0$ and that $\exists \{\lambda(s^t)\}_{t=0}^\infty$ with $\lambda(s^t) \geq 0 \forall s^t$ s.t.

$$\sum_{k=t}^\infty \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k | s^t) \Gamma(s_k, \lambda(s^k)) > r^{\max} / (1 - \beta) - \chi^p \forall s^t, \tag{50}$$

$$\sum_{k=t}^\infty \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k | s^t) W(s_k, \lambda(s^k)) > U^{\text{AUT}} + \chi^c \forall s^t, \text{ and} \tag{51}$$

$$b_{-1}(s^0) < \sum_{k=0}^\infty \sum_{s^k \in S^k} \beta^{k-t} \pi(s^k | s^t) \Gamma(s_k, \lambda(s^k)) \forall s^t. \tag{52}$$

We can show that this condition implies Assumption 1.

Step 1. This condition implies that $\exists \xi \in \Lambda$ s.t. $V(\{\xi, P\} |_{s_0}) > \underline{V}(b_{-1}(s^0))$ and $U(\{\xi, P\} |_{s_0}) > \underline{U}(b_{-1}(s^0))$ starting from any s_0 . Starting from s_0 , choose the sequences $\{n(s^t)\}_{t=0}^\infty$ and $\{g(s^t)\}_{t=0}^\infty$ to satisfy equations (36) and (37) for a sequence $\{\lambda(s^t)\}_{t=0}^\infty$ satisfying the condition. Let $P(s^t) = 1 \forall s^t$. All incentive compatibility constraints are satisfied and $V(\{\xi, P\} |_{s_0}) > \underline{V}(b_{-1}(s^0))$ and $U(\{\xi, P\} |_{s_0}) > \underline{U}(b_{-1}(s^0))$. Finally, given equation (52), an implied sequence $\{x(s^t)\}_{t=0}^\infty$ which satisfies equation (22) is non-negative so that it is feasible.

Step 2. Starting from any s_0 , $\exists \xi' \in \Lambda$ s.t. $V(\{\xi', P\} |_{s_0}) > \underline{V}(b'_{-1}(s^0))$ and $U(\{\xi', P\} |_{s_0}) > \underline{U}(b'_{-1}(s^0))$ given $b'_{-1}(s^0) < b_{-1}(s^0)$. Consider ξ' identical to ξ with the exception that $x'(s^0) = x(s^0) + b_{-1}(s^0) - b'_{-1}(s^0)$ and $c'(s^0) = c(s^0) - x'(s^0) + x(s^0)$. Then $\xi' \in \Lambda$ and $V(\{\xi', P\} |_{s_0}) > \underline{V}(b'_{-1}(s^0))$ and $U(\{\xi', P\} |_{s_0}) > \underline{U}(b'_{-1}(s^0))$.

Step 3. The solution to the program admits $P(s^0) = 1$. If instead $P(s^0) = 0$, then $V(\{\xi, P\} |_{s_0}) = \underline{V}(b_{-1}(s^0))$ and $U(\{\xi, P\} |_{s_0}) = \underline{U}(b_{-1}(s^0))$ by steps 2 and 3 of the proof of Proposition 2, and by step 1 there exists an allocation which makes everyone strictly better off.

Step 4. Imagine that the solution admits $P(s^T) = 0$ for some s^T in which the incumbent in period 0 is thrown out under an optimal sequence ξ . Let ξ' be identical to ξ with the exception that

$$x'(s^T) = \sum_{t=0}^\infty \sum_{s^t \in S^t} \mathcal{I}_0(s^t) \beta^t \pi(s^t | s^0) x(s^t) / (\beta^T \pi(s^T | s^0))$$

for $\mathcal{I}_0(s^t)$ which is an indicator which equals 1 if the incumbent in period 0 holds power at t . Also, let $x'(s^t) = 0$ if $\mathcal{I}_0(s^t) = 1$ and $s^t \neq s^T$, and let $c'(s^t) = c(s^t) - x'(s^t) + x(s^t)$. It can be verified that $\xi' \in \Lambda$ and the perturbation leaves both agents as well off.

Step 5. Given the perturbation of step 4, incentive compatibility for the politician in period 0 implies that

$$\beta^T \pi(s^T | s^0) (x'(s^T) - \chi^p) \geq \max \{V_0, r^{\max} / (1 - \beta) - b_{-1}(s^0) - \chi^p\}. \tag{53}$$

From steps 2 and 3 from the proof of Proposition 2 and from equation (23), $x'(s^T) = r^{\max} / (1 - \beta) - b'(s^T | s^{T-1})$, so that substitution into equation (53) implies that $b'(s^T | s^{T-1}) \leq b_{-1}(s^0)$. However, by steps 2 and 3, $\exists \xi'' \in \Lambda$ which is identical to ξ' but which differs from s^T onward under which $V(\{\xi'', P\} |_{s^T}) > \underline{V}(b'(s^T | s^{T-1}))$ and $U(\{\xi'', P\} |_{s^T}) > \underline{U}(b'(s^T | s^{T-1}))$. Therefore, ξ'' yields strictly higher welfare for the politician and the households relative to ξ . Therefore, $P(s^T) = 0$ is sub-optimal. By forward induction, $P(s^t) = 1 \forall s^t$.

APPENDIX D. PROOFS OF SECTION 5

D.1. Proof of Lemma 2

Step 1. Consider the original solution and perform the same substitutions as in the text.

Step 2. Consider the solution s.t. $b_{-1}(s^0) = b + V$ and $V(\{\xi, P\}|_{s^0}) = 0$ and imagine if $\{n, g\}$ does not attain the optimum but $\{n', g'\} \neq \{n, g\}$ attains it. From equation (31), this implies that

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s^0) u(n'(s^t) - g'(s^t), n'(s^t), g'(s^t), s_t) > \\ & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s^0) u(n(s^t) - g(s^t), n(s^t), g(s^t), s_t), \end{aligned}$$

which contradicts the optimality of $\{n, g\}$ in the original solution. \parallel

D.2. Proof of Lemma 3

Step 1. The solution ξ to equation (26) sets $n(s^t) \geq n^{\max} \forall s^t$. If this is not the case, then $\exists \xi' \in \Lambda$ identical to ξ with the exception that $n'(s^t) > n(s^t)$ and $r(n'(s^t)) = r(n(s^t))$ for all s^t s.t. $n(s^t) < n^{\max}$ so that $c'(s^t) = c(s^t) + (n'(s^t) - n(s^t))$, $g'(s^t) = g(s^t)$, and $x'(s^t) = x(s^t)$ for all such s^t . This perturbation strictly increases the welfare of the households and leaves the politician as well off.

Step 2. Define the sequence program implied by equations (32)–(35) for a given $z_{-1}(s^0)$ in terms of $\{r, g\}$ so that the instantaneous utility to the household becomes $\tilde{u}(r) - g + \theta \frac{g}{\alpha}$ for $\tilde{u}(r) = n - \eta \frac{n}{\gamma}$ s.t. $r(n) = r$. $\tilde{u}''(r) < 0$ by the implicit function theorem given step 1.

Step 3. Let z_s be the set of feasible values of z for our program given s . If $z', z'' \in z_s$, then $z^{\kappa} = \kappa z' + (1 - \kappa) z'' \in z_s \forall \kappa \in (0, 1)$ since $\{r^{\kappa}, g^{\kappa}\} = \kappa \{r', g'\} + (1 - \kappa) \{r'', g''\}$ is feasible and satisfies all constraints. Moreover, $\{r^{\kappa}, g^{\kappa}\}$ yields strictly greater welfare than $\kappa J(s, z') + (1 - \kappa) J(s, z'')$, establishing concavity.

Step 4. To show that z_s is closed, consider a sequence $z_s^j \in z_s$ such that $\lim_{j \rightarrow \infty} z_s^j = z'_s$. There is a corresponding stochastic sequence $\{r^{j,j}, g^{j,j}\}$ which converges to $\{r^{\infty}, g^{\infty}\}$ since social welfare net of rents and debt net of rents are continuous in $\{r^{j,j}, g^{j,j}\}$. Since every element of $\{r^{j,j}, g^{j,j}\}$ at s^t is contained in $[r^{\min}, r^{\max}] \times [0, g^{\max}]$ for some arbitrarily low r^{\min} and arbitrarily high g^{\max} , and since equations (34) and (35) are weak inequalities, then $\{r^{\infty}, g^{\infty}\}$ is incentive compatible. Since $\beta \in (0, 1)$, then by the Dominated Convergence Theorem, $\{r^{\infty}, g^{\infty}\}$ achieves z'_s and the household welfare net of rents associated with z'_s . Therefore, $z'_s \in z_s$.

Step 5. Consider a sequence $\{r, g\}$ associated with the solution for some $z \in (z_s, \bar{z}_s)$. Consider the sequence $\{r^{\epsilon}, g^{\epsilon}\}$ for which the only difference between $\{r, g\}$ and $\{r^{\epsilon}, g^{\epsilon}\}$ is that $r_0^{\epsilon} = r_0 + \epsilon$ for $\epsilon \geq 0$ arbitrarily low. Define $F(s, z, \epsilon) = \tilde{u}(r_0^{\epsilon}) - g_0 + \theta(s) \frac{g_0}{\alpha} + \beta \sum \pi_{ks} J(k, z_k)$, so that $F(s, z, 0) = J(s, z)$. Optimality implies that $F(s, z, \epsilon) \leq J(s, z + \epsilon)$ for $F(s, z, \epsilon)$ which is concave and differentiable. By Lemma 1 of Benveniste and Scheinkman (1979), $J(\cdot)$ is differentiable. \parallel

D.3. Proof of Proposition 3

Step 1. By equations (36), (39), and step 1 of the proof of Lemma 3,

$$J_z(s, z) = -(1 - \eta n^{\gamma-1}) / (\gamma \eta n^{\gamma-1} - 1) \leq 1/\gamma,$$

which implies that $J(s, z) - z$ is decreasing in z .

Step 2. Define $\underline{\lambda}(s) = -J_z(s, r^{\max} / (1 - \beta) - \chi^p)$ and define $\bar{\lambda}(s) = -J_z(s, z)$ for z which solves $J(s, z) - z = U^{\text{AUT}} + \chi^c$.

Step 3. Suppose that $\lambda(s^{t-1}) < \underline{\lambda}(s_t)$. Then by the concavity of $J(\cdot)$, for (34) to hold, we require $\lambda(s^t) \geq \underline{\lambda}(s_t) > \lambda(s^{t-1})$, so that from equation (38), $\phi(s^t) > 0$. This implies that $z(s^t | s^{t-1}) = r^{\max} / (1 - \beta) - \chi^p$ and therefore $\lambda(s^t) = \underline{\lambda}(s_t)$. Analogous arguments hold for $\lambda(s^{t-1}) > \bar{\lambda}(s_t)$.

Step 4. Suppose that $\lambda(s^{t-1}) \in [\underline{\lambda}(s_t), \bar{\lambda}(s_t)]$. If $\phi(s^t) > 0$, we have $z(s^t | s^{t-1}) = r^{\max} / (1 - \beta) - \chi^p$ and consequently $\lambda(s^t) = \underline{\lambda}(s_t)$. But from equation (38), $\phi(s^t) > 0$ implies that $\lambda(s^t) > \lambda(s^{t-1})$ which is a contradiction. Therefore, $\phi(s^t) = 0$. Analogous arguments hold if $\psi(s^t) > 0$.

Step 5. Therefore

$$\lambda(s^t) = \begin{cases} \bar{\lambda}(s_t) & \text{if } \lambda(s^{t-1}) > \bar{\lambda}(s_t) \\ \lambda(s^{t-1}) & \text{if } \lambda(s^{t-1}) \in [\underline{\lambda}(s_t), \bar{\lambda}(s_t)] \\ \underline{\lambda}(s_t) & \text{if } \lambda(s^{t-1}) < \underline{\lambda}(s_t) \end{cases}, \tag{54}$$

which implies equations (40) and (41) from equations (36), (37), and (46). \parallel

D.4. Proof of Corollary 1

Step 1. The solution to the problem which ignores equations (34) and (35) is associated with λ in equations (36) and (37) which solves

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s^0) \Gamma(s_t, \lambda) = z_{-1}(s^0)$$

for $z_{-1}(s^0) = b_{-1}(s^0) + \max\{0, V_0\}$.

Step 2. Choose $\chi^p = \infty$ $\chi^c = 0$, so that equations (34) and (35) do not bind under the implied allocation of step 1. \parallel

D.5. Proof of Theorem 1

Step 1. Choose $V_0 = 0$ and choose $s_0 = \arg \min_s \theta(s)$. Choose χ^p sufficiently low so that $r^{\max}/(1 - \beta) - b_{-1}(s^0) - \chi^p > 0$ and $z_{-1}(s^0) = r^{\max}/(1 - \beta) - \chi^p$. Since equations (34) binds in period 0, $\tau(s^0) = \underline{\tau}(s_0)$ associated with $\underline{\lambda}(s_0)$.

Step 2. Since $\Gamma(k, \lambda) < \Gamma(s, \lambda)$ if $\theta(s) < \theta(k)$ then necessarily $\underline{\lambda}(s_1) > \underline{\lambda}(s_0)$ and $\underline{\tau}(s_1) > \underline{\tau}(s_0)$ for some s_1 in order that equation (34) be satisfied for such s_1 .

Step 3. Consider a path s_0, s_1, s_0 for s_1 for which $\underline{\tau}(s_1) > \underline{\tau}(s_0)$. From equation (40), $\tau(s^2) = \min\{\underline{\tau}(s_1), \bar{\tau}(s_0)\} > \tau(s^0) = \underline{\tau}(s_0)$, where we have used Assumption 1 and the definition of $\bar{\tau}(s_0)$ in Proposition 3 to establish $\bar{\tau}(s_0) > \underline{\tau}(s_0)$.

Step 4. Choose V_0 sufficiently large that $z_{-1}(s^0) = \bar{z}(s_0)$ for $\bar{z}(s_0)$ associated with $\bar{\lambda}(s_0)$ and choose $s_0 = \arg \max_s \theta(s)$. Analogous arguments to steps 1–3 imply that for χ^c sufficiently high, there exists a path s_0, s_1, s_0 with taxes $\bar{\tau}(s_0), \bar{\tau}(s_1), \max\{\bar{\tau}(s_1), \underline{\tau}(s_0)\} < \bar{\tau}(s_0)$. \parallel

D.6. Proof of Proposition 4

Step 1. Given equation (54) and the i.i.d. assumption, $E\{\lambda_k | s_t, \lambda_t\}$ for $k > t$ is independent of s_t and is weakly increasing in λ_t .

Step 2. Let Γ_k represent the realized value of $\Gamma(s^k)$ and let W_k represent the realized value of $W(s^k)$. Then $E\{\sum_{k=t+1}^{\infty} \beta^{k-t} \Gamma_k | s_t, \lambda_t\}$ weakly increases in λ_t and $E\{\sum_{k=t+1}^{\infty} \beta^{k-t} W_k | s_t, \lambda_t\}$ weakly decreases in λ_t , and both are independent of s_t .

Step 3. If $\theta(s_t)$ is increasing in s_t , then $E\{\sum_{k=t}^{\infty} \beta^{k-t} \Gamma_k | s_t, \lambda_t\}$ strictly decreases in s_t conditional on λ_t and $E\{\sum_{k=t}^{\infty} \beta^{k-t} W_k | s_t, \lambda_t\}$ strictly increases in s_t conditional on λ_t .

Step 4. By the definitions of $\underline{\lambda}(s_t)$ and $\bar{\lambda}(s_t)$, these solve

$$E\left\{\sum_{k=t}^{\infty} \beta^{k-t} \Gamma_k | s_t, \underline{\lambda}(s_t)\right\} = r^{\max}/(1 - \beta) - \chi^p \text{ and}$$

$$E\left\{\sum_{k=t}^{\infty} \beta^{k-t} W_k | s_t, \bar{\lambda}(s_t)\right\} = U^{\text{AUT}} + \chi^c,$$

for a given s_t , which by step 3 implies that $\underline{\lambda}(s_t)$ and $\bar{\lambda}(s_t)$ are increasing in s_t which implies that $\underline{\tau}(s_t)$ and $\bar{\tau}(s_t)$ are increasing in s_t by (54). \parallel

D.7. Proof of Theorem 2

Step 1. If $\cap_{s \in S} [\underline{\tau}(s), \bar{\tau}(s)]$ is non-empty, then $\{\tau(s^t)\}_{t=0}^{\infty}$ converges almost surely. Equation (40) implies that $\tau(s^t) \in [\min_{s \in S} \underline{\tau}(s), \max_{s \in S} \bar{\tau}(s)]$ so that it is bounded. Moreover, since $\max_{s \in S} \underline{\tau}(s) \leq \min_{s \in S} \bar{\tau}(s)$, then $\tau(s^{t-1}) \leq \tau(s^t) \leq \max_{s \in S} \underline{\tau}(s)$ or $\tau(s^{t-1}) \geq \tau(s^t) \geq \min_{s \in S} \bar{\tau}(s)$. Since $\tau(s^t)$ is monotone and bounded, then $\tau(s^t)$ converges to a limit for every history s^{∞} . Therefore, it converges almost surely.

Step 2. If $\cap_{s \in S} [\underline{\tau}(s), \bar{\tau}(s)]$ is empty, then $\{\tau(s^t)\}_{t=0}^{\infty}$ converges with probability 0. Consider two state k and l for which the intervals $[\underline{\tau}(k), \bar{\tau}(k)]$ and $[\underline{\tau}(l), \bar{\tau}(l)]$ are mutually exclusive with $\bar{\tau}(k) < \underline{\tau}(l)$. Imagine if $\{\tau(s^t)\}_{t=0}^{\infty}$ converges with positive probability for a subset of histories $\varkappa \in S^{\infty}$. For a given history $\varkappa_i \in \varkappa$ in which $\{\tau(s^t)\}_{t=0}^{\infty}$ converges to $\tau(\varkappa_i)$, $\exists T(\varkappa_i)$ s.t. for every $t > T(\varkappa_i)$, $|\tau(\varkappa_i) - \tau(s^t)| < (\underline{\tau}(l) - \bar{\tau}(k))/2$. Given equation (40) this implies that either $s_t \neq k \forall t > T(\varkappa_i)$ or $s_t \neq l \forall t > T(\varkappa_i)$. Let $T^{\min} = \min_{\varkappa_i \in \varkappa} T(\varkappa_i)$. Since there is full support, $\Pr\{s^{\infty} \in \varkappa\} \leq \Pr\{s_t \neq k \forall t > T^{\min} \text{ or } s_t \neq l \forall t > T^{\min} \forall s_{T^{\min}} \in S\} = 0$, yielding a contradiction.

Step 3. If $\exists \widehat{\lambda} \geq 0$ s.t.

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi (s^k | s^t) \Gamma (s_k, \widehat{\lambda}) \geq r^{\max} / (1 - \beta) - \chi^p \text{ and} \tag{55}$$

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi (s^k | s^t) W (s_k, \widehat{\lambda}) \geq U^{\text{AUT}} + \chi^c, \tag{56}$$

$\forall s^t$, then $\cap_{s \in S} [\underline{\tau}(s), \overline{\tau}(s)]$ is non-empty. Starting from initial conditions s_t and $z (s^t | s^{t-1}) = \sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi (s^k | s^t) \Gamma (s_k, \widehat{\lambda})$, the unconstrained solution which ignores equation (34) and (35) sets $\tau (s^k) = \widehat{\tau}$ and $\widehat{g} (s^k) = \widehat{g} \forall s^k$ for $\widehat{\tau}$ and \widehat{g} associated with $\widehat{\lambda}$ through equations (36) and (37), respectively. Since this solution satisfies equations (34) and (35) $\forall s^t$, it is the constrained solution which by equation (40) implies that $\widehat{\tau} \in \{ \cap_{s \in S} [\underline{\tau}(s), \overline{\tau}(s)] \} \neq \emptyset$.

Step 4. If $\exists \widehat{\lambda} \geq 0$ which satisfies equations (55) and (56), then $\cap_{s \in S} [\underline{\tau}(s), \overline{\tau}(s)]$ is empty. If $\cap_{s \in S} [\underline{\tau}(s), \overline{\tau}(s)]$ were non-empty, then associated with every $\widehat{\tau} \in \{ \cap_{s \in S} [\underline{\tau}(s), \overline{\tau}(s)] \}$ is a value of $\widehat{\lambda}$ which satisfies (55) and (56), yielding a contradiction.

Step 5. Define $\widehat{s}^p(\widehat{\lambda})$ as the state $s_t \in S$ in which the left-hand side of equation (55) is minimized for a given $\widehat{\lambda}$, and define $\widehat{s}^c(\widehat{\lambda})$ as the state $s_t \in S$ in which the left-hand side of equation (56) is minimized for a given $\widehat{\lambda}$. Since $\Gamma (\cdot)$ and $W (\cdot)$ are monotonic in $\widehat{\lambda}$, the left-hand side of equation (55) evaluated at $\widehat{s}^p(\widehat{\lambda})$ is increasing in $\widehat{\lambda}$ and the left-hand side of equation (56) evaluated at $\widehat{s}^c(\widehat{\lambda})$ is decreasing in $\widehat{\lambda}$. For a given $\widehat{\lambda} \in \mathbb{R}^+$, there exists a value $\widehat{\chi}^p$ which is decreasing in $\widehat{\lambda}$ which sets equation (55) to an equality at $\widehat{s}^p(\widehat{\lambda})$. Moreover, there exists a value $\widehat{\chi}^c$ which is decreasing in $\widehat{\lambda}$ which sets (56) to an equality at $\widehat{s}^c(\widehat{\lambda})$. Define $\widehat{\chi}^p(\chi^c)$ as the level of $\widehat{\chi}^p$ associated with $\widehat{\lambda}$ for which $\widehat{\chi}^c = \chi^c$, and define $\widehat{\chi}^c(\chi^p)$ analogously.

Step 6. By step 5, $\widehat{\lambda}$ which satisfies equations (55) and (56) if and only if $\chi^c \leq \widehat{\chi}^c(\chi^p)$ and $\chi^p \geq \widehat{\chi}^p(\chi^c)$. The rest then follows from steps 1–4. \parallel

APPENDIX E. EXTENSION OF SECTION 4.1

In this section, we continue the formal definition of the equilibrium. We define how strategies induce histories, we define continuation strategies, and we define the problem of each agent.

Strategies induce histories as follows. Given h_t^0, Υ induces $h_t^1 = \{h_t^0, \Upsilon_t(h_t^0)\}$, and given h_t^1, σ induces $h_t^2 = \{h_t^1, \sigma_t(h_t^1)\}$ and $h_{t+1}^0 = \{h_t^1, \sigma_t(h_t^1), s_{t+1}\}$, and so on. Continuation strategies are generated as follows. Given h_t^0 and σ , a continuation of Υ is $\{\Upsilon_t(h_t^0), \Upsilon_{t+1}(h_t^0, \sigma_t(h_t^0, \Upsilon_t(h_t^0), s_{t+1}), \dots)\}$. Given h_t^1 and Υ , a continuation of σ is $\{\sigma_t(h_t^1), \sigma_{t+1}(h_t^1, s_{t+1}, \Upsilon_{t+1}(h_t^1, \sigma_t(h_t^1), s_{t+1})), \dots\}$. Given h_t^2, Υ , and σ , a continuation of f is $\{f_t(h_t^2), f_{t+1}(h_t^2, s_{t+1}, \Upsilon_{t+1}(h_t^2, s_{t+1}), \sigma_{t+1}(h_t^2, s_{t+1}, \Upsilon_{t+1}(h_t^2, s_{t+1}))), \dots\}$. Given h_t^2, Υ , and σ , a continuation of ζ is defined analogously.

Consider the private household solving its market problem in period t . Given h_t^2, Υ, σ , and ζ , a household chooses a continuation of f to maximize:

$$E \left\{ \sum_{k=t}^{\infty} \beta^{k-t} [u (c_k (h_k^2), n_k (h_k^2), g_k (h_k^1), s_k) + (1 - P_{k+1} (h_k^0)) \chi^c (1 - \beta)] | h_t^2, \Upsilon, \sigma, f, \zeta \right\}$$

s.t.

$$c_t (h_t^2) + b_{t-1}^h (s_t) (h_{t-1}^2) = (1 - \tau_t (h_t^1)) n_t (h_t^2) + \sum_{s_{t+1} \in S} q_t (s_{t+1}) (h_t^2) b_t^h (s_{t+1}) (h_t^2),$$

$$c_k (h_k^2) + b_{k-1}^h (s_k) (h_{k-1}^2) = (1 - \tau_k (h_k^1)) n_k (h_k^2) + \sum_{s_{k+1} \in S} q_k (s_{k+1}) (h_k^2) b_k^h (s_{k+1}) (h_k^2)$$

for $k > t$, and $n_t (h_t^2), n_k (h_k^2) \geq 0$. For $k > t$ all future histories are induced by Υ and σ from h_t^2 . Note that we have taken into account that the household achieves $\chi^c (1 - \beta)$ from throwing out the current politician. Since future histories do not depend on f , households are non-strategic in this allocation. Let $\bar{W}_t (h_t^2; \Upsilon, \sigma, f, \zeta)$ represent the welfare of households at h_t^2 implied by f given Υ, σ , and ζ .

Consider the politician in power at t . Such a politician takes into account whether he will be in power in the future, and we can define $\mathcal{I}_t (h_t^1, h_t^1) = 1$ and $\mathcal{I}_k (h_t^1, h_k^1) = \prod_{l=t+1}^k P_l (h_{l-1}^0)$ for $k \geq t + 1$, an indicator which equals 1 if the politician in power at t is still in power at date k . Note that if $\mathcal{I}_k (h_t^1, h_k^1) = 1$, the politician effectively receives flow

utility $v(x(h_k^1))$, and if $\mathcal{I}_k(h_t^1, h_k^1) = 0$, the politician effectively receives flow utility $-\chi^p(1-\beta)/\beta^2$.³⁶ Therefore, given h_t^1 , Υ , f , and ζ , he chooses a continuation of σ to maximize:

$$E \left\{ \sum_{k=t}^{\infty} \beta^{k-t} \left(\mathcal{I}_k(h_t^1, h_k^1) v(x(h_k^1)) - (1 - \mathcal{I}_k(h_t^1, h_k^1)) \chi^p \frac{1-\beta}{\beta^2} \right) \middle| h_t^1, \Upsilon, \sigma, f, \zeta \right\},$$

s.t.

$$g_k(h_k^1) + x_k(h_k^1) + b_{k-1}^g(s_k)(h_{k-1}^1) = \tau_k(h_k^1) n_k(h_k^2) + \sum_{s_{k+1} \in S} q_k(s_{k+1})(h_k^2) b_k^g(s_{k+1})(h_k^1),$$

$g_k(h_k^1) \geq 0$, $x_k(h_k^1) \geq 0$, and $b_k^g(s_{k+1})(h_k^1) \in [\underline{b}, \bar{b}]$ for $k \geq t$. For all $k \geq t$, future histories are induced by Υ and σ from h_t^1 .

Consider the representative citizen solving his political problem in period t . Given h_t^0 , σ , f , and ζ , he chooses a continuation of Υ to maximize

$$\bar{W}_t(h_t^0, \Upsilon_t(h_t^0), \sigma_t(h_t^0, \Upsilon_t(h_t^0)); \Upsilon, \sigma, f, \zeta)$$

for future histories which are induced by Υ and σ from h_t^0 .

Given h_t^2 , Υ , σ , and f , ζ must clear the bond market:

$$b_t^g(s_{t+1})(h_t^1) + b_t^h(s_{t+1})(h_t^2) = 0 \quad \forall s_{t+1} \in S.$$

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36. If $\mathcal{I}_{k-1}(h_t^1, h_{k-1}^1) = 1$ and $\mathcal{I}_k(h_t^1, h_k^1) = 0$, the politician pays at cost $\chi^p(1-\beta)/\beta$ from $k-1$ onward, which, given risk neutrality, is equivalent a cost of $\chi^p(1-\beta)/\beta^2$ from k onward.

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