

A Conjoint Approach to Multi-Part Pricing

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ABSTRACT

Multi-part pricing is commonly used by providers of such services as car rentals, prescription drug plans, HMOs and wireless telephony. The general structure of these pricing schemes is a fixed access fee, which sometimes entitles users to a certain level of product use; a variable fee for additional use; and still another fee for add-on features that are priced individually and/or as bundles. The authors propose a method using conjoint analysis for multi-part pricing. The method reflects the two-way dependence between prices and consumption, and incorporates the uncertainty consumers have about their use of a service. The proposed method estimates both choice probabilities and usage levels for each individual as functions of the product features and the different price components. These estimates are then used to evaluate the expected revenues and profits of alternative plans and pricing schemes. The method is illustrated using data from a conjoint study concerning cellular phone services. Its results are compared with those obtained from using several competing models. The proposed procedure is used to identify the optimal set of features in a base plan, and the pricing of optional features, for a provider of cellular phone services.

Keywords: conjoint analysis; multi-part pricing; optimal product design.

The use of conjoint analysis to study the effect of price changes on market demand dates back to at least the 1970's (Johnson 1972, Jones 1975). Since then, marketing researchers have developed a variety of pricing models using conjoint analysis. Mahajan, Green and Goldberg (1982) describe a method for estimating self- and cross-elasticities using conjoint analysis. Kohli and Mahajan (1991) introduce an approach for optimal pricing using conjoint data. Jedidi and Zhang (2002) further develop this method to allow for the effect of new product introduction on category-level demand. Jedidi, Jagpal and Manchanda (2003) describe a method for estimating consumer reservation prices for product bundles.

Each of these methods assumes that a product is sold at a single price, and that a consumer cannot upgrade or add features to a product by paying an additional fee. Another assumption common to these methods is that consumer usage rates do not depend on price. These assumptions are approximately, if not perfectly, satisfied for some products, for example such durable goods as washing machines and refrigerators. However, there are also categories of products and services where one or more of these assumptions is not appropriate. For example, some services charge not one price but two prices, and charge additional fees for add-on features. Examples are car rentals, some HMO plans, prescription drug plans, Xerox copying services, memberships to health clubs, museums and zoos, and telephone services (Danaher 2002; Narayanan, Chintagunta and Miravete 2007). Some of these services charge an additional variable fee. For example, institutional users pay a per page charge for copies on a Xerox machine; and members of an HMO pay a deductible for each visit to a doctor or each purchase of a prescription drug. Other services, like car rentals, cellular phone services, museum and health club memberships, charge a fixed fee and allow "free" use up to a certain level, beyond which consumers have to pay a usage-based unit rate. This induces a two-way dependence of price and consumption --- the price charged by a provider influences consumption while the price a consumer pays depends on his or her usage level. Some services allow customers to purchase optional features, such as rollover minutes for cell phone services and

extra life insurance for car rentals and air travel. Other services, like HMO and prescription drug plans, do not allow service enhancements but offer alternative plans with bundles of add-on features. Still other services, such as cable television, allow unlimited use for a monthly fee but allow consumers additional subscriptions to such options as digital channels, high-definition broadcasts, pay-per-view, broadband internet access and IP telephony.

The purpose of this paper is to describe a conjoint model for the multi-part pricing of products and services. Our demand analysis approach has its roots in labor economics (Burtless and Hausman 1978, Hall 1973, Hausman 1985, Moffitt 1990). Labor economists are concerned with the prediction of changes in the labor supply when a new, typically multi-part, tax structure is imposed on people. The two-way dependence between prices and consumption in the present instance parallels the simultaneity between tax rate and hours of work. We represent the mutual dependence of prices and consumption in our proposed conjoint model using an approach developed by Burtless and Hausman (1978) and Hausman (1985). We extend this approach in three ways. First, we incorporate the effect of consumption on consumer choice. Second, we allow for the possibility that consumers are unsure about how much of a service they will use, an important aspect in models of multi-part pricing (Lambrecht and Skiera 2006, Lambrecht, Seim, and Skiera 2007, Lemon, White and Winer 2002, Narayanan, Chintagunta, and Miravete 2007, Nunes 2000). Third, we estimate both choice and usage from choice-set experiments: consumers are presented a series of choice sets, and are asked to choose at most one multi-attribute alternative from each set. As is common in conjoint analysis, we do not require consumers to estimate their consumption of a chosen alternative. Instead, we infer the latter within the model, using information on the choices that are made, and not made, by consumers. The present paper is, to the best of our knowledge, the first to suggest how the simultaneity of prices and consumption, and the uncertainty consumers have about their usage levels, can be incorporated in conjoint analysis, in choice simulations, and in the product-design decisions for which conjoint analysis is often used.

In its most general form, the proposed model permits (i) a fixed fee that is charged by a firm for a duration of time (e.g., a daily rate for car rental, a monthly charge for cellphone or cable TV use, an annual HMO membership fee); (ii) a base level of use to which a person paying the fixed fee is entitled; (iii) a variable fee for use beyond the base level; and (iv) an option for consumers to add service features at an additional price. How much of the service a consumer uses depends on the utility obtained from additional use. We allow the usage rate to differ from one person to another. An important feature of the proposed model is that it does not require users to consume all of the "free" units to which they are entitled upon paying a fixed charge. This is consistent with empirical evidence. For example, there are no usage limits to cable service, but most people do not use it all the time. Similarly many cell phone users do not exhaust all their free minutes (Iyengar, Ansari and Gupta 2007). We accommodate this kind of behavior by assuming a quadratic utility function. The parameter values of the linear and quadratic terms determine if a consumer has constant or diminishing marginal utility, and if there is an (unobserved) level of use beyond which the utility decreases for a consumer.

The usage rate for a consumer depends in our model on both the number of free units available upon payment of the fixed fee and the per unit price for use of the service beyond the free units. We allow individual differences in usage rates and in their preferences for alternative services.

Consumer choice among alternative services, and the level to which a service is used by a consumer, are modeled in a probabilistic framework. We aggregate across consumers to (i) obtain estimates of the market penetration and usage of a service; (ii) identify an optimal plan that maximizes profit for a service provider; and (iii) find the optimal prices for such service options as internet access and rollover minutes that are offered by providers of cellular phone services. We compare the proposed method to six other competing models.

The rest of the paper is organized as follows. We begin by describing the proposed model. Next, we report an application of the model to the pricing of cellular phone services, and compare the results with those obtained using other null models. Thereafter, the implications of our model are examined for the design of optimal service plans and the pricing of service features. We then report the results of a small Monte Carlo simulation assessing the performance of our estimation procedure. The paper concludes with our contributions and key results.

MODEL

We consider a three-part pricing scheme which comprises a base (access) fee, a free usage allowance, and a per-unit (variable) charge for the use of a service in excess of the allowance. For example, the Basic Plus cellphone plan from T-Mobile charges a monthly access fee of \$29.99, offers 300 monthly free minutes, and charges 40 cents per minute for any excess usage. A two-part tariff plan is a special case when there is no usage allowance. A plan with zero variable fee but unlimited usage allowance is called a flat-fee plan; and a plan with zero access fee and zero usage allowance is called a pay-per-use plan.

Consider a choice set with J service plans.¹ Let $x_j = (x_{j1}, \dots, x_{jm})'$ denote a vector of m *non-price* attributes (e.g., service features or service providers) associated with service plan j . Let I denote the number of consumers and v_{ij} denote the attribute-based utility consumer i associates with service j .

We assume the following functional form for v_{ij} :

$$(1) \quad v_{ij} = \gamma_{ij0} + \sum_{k=1}^m \gamma_{ik} x_{jk}, \text{ for all } i = 1, \dots, I, j = 1, \dots, J.$$

We emphasize that v_{ij} does not depend on the price of service plan j . The γ_{ij0} term is a constant specific to service plan j . It represents the value of a service plan that is not explained by the vector

x_j of features. The γ_{ik} are regression (part-worth) coefficients that capture the effect of the non-price attributes on utility.

We assume that consumer i cannot choose more than one service plan. Let f_j denote the base fee (fixed cost). We assume that there is an individual-specific composite (outside) good with unit price p_i^w . We also assume that consumer i has a budget w_i . The budget need not be observable, but one has to postulate its existence to develop an economic model. A consumer can spend the entire budget on the composite good, or spend some of it on the composite good and the rest to buy one of the J service plans.

Let A_j denote the number of free units that are offered with plan j . Let p_j denote the per-unit price for the consumption of each unit of the service exceeding the quantity A_j . Let z_{ij} denote the number of units of the composite good; and let n_{ij} denote the number of units of the service consumer i expects to use if he or she selects plan j . Consistent with the literature on multi-part pricing (Lambrecht and Skiera 2006, Lambrecht, Seim, and Skiera 2007, Narayanan, Chintagunta, and Miravete 2007, Nunes 2000), we assume that the consumer is uncertain about the value of n_{ij} . Later in this section, we describe a model for this uncertainty.

Let $u_i(n_{ij}, z_{ij})$ represent the utility consumer i obtains from consuming n_{ij} units of service j and z_{ij} units of the composite good. We assume that the consumer maximizes his or her utility, subject to a budget constraint. This constraint takes one of the following forms, depending on whether the quantity consumed is above or below the free units:

- (2) Constraint I: $p_i^w z_{ij} + f_j \leq w_i$ if $0 \leq n_{ij} \leq A_j$
(3) Constraint II: $p_i^w z_{ij} + f_j + p_j(n_{ij} - A_j) \leq w_i$ if $n_{ij} > A_j$.

Note that, on the one hand, the marginal cost of using an extra unit of the service depends on the level of consumption --- it is p_j if consumption exceeds A_j , and it is zero otherwise. On the other hand, the consumption itself depends on the pricing scheme. It is this simultaneity between price and consumption that we want to capture in our model.²

A utility maximizing consumer will exhaust the budget. That is, equations (2) and (3) will be satisfied as equalities. Without loss of generality, we normalize the unit price of the composite good to $p_i^w = 1$. We assume that the utility function for consumer i has the following form:

$$(4) \quad u_i(n_{ij}, z_{ij}) = v_{ij} + \beta_{i1}n_{ij} + \beta_{i2}n_{ij}^2 + \beta_{i3}z_{ij},$$

where

$$(5) \quad z_{ij} = \begin{cases} w_i - f_j & \text{if } 0 \leq n_{ij} \leq A_j, \\ w_i - f_j - p_j(n_{ij} - A_j) & \text{if } n_{ij} > A_j. \end{cases}$$

Observe that the utility function reduces to $u_i(0, w_i) = \beta_{i3}w_i$ when the consumer chooses no alternative from a choice set.

The quadratic utility specification of equation (4) is often used in both the marketing and the economics literatures (see, for example, Erdem and Keane 1996, Jensen 2006, Miravete and Rölller 2004).³ We require $\beta_{i2} < 0$ to represent the preferences of consumers who have diminishing marginal utility in the consumption of the service; and we require $\beta_{i3} > 0$ to denote a positive income effect.⁴ Observe that if β_{i2} is sufficiently small, the utility increases up to a level of consumption, beyond which it decreases. This is useful for representing the preferences of those consumers for whom consumption beyond a particular level (e.g., of cell phone use or of television viewership) is a nuisance.

We now consider the uncertainty in n_{ij} , the number of units of the service consumer i expects to use if he or she selects plan j . We assume that n_{ij} is a random variable with mean n_{ij}^* , the quantity that maximizes the consumer utility for plan j (equation 4) subject to the budget constraint (equation 5). That is,

$$(6) \quad n_{ij} = n_{ij}^* + \delta_{ij},$$

where δ_{ij} is an error term. We assume that δ_{ij} has a normal distribution with a mean 0 and a consumer-specific variance, θ_i^2 . Observe that this approach to modeling usage has the benefit of allowing both the mean consumption and the usage uncertainty to affect plan choice.⁵

We obtain the value of n_{ij}^* using the method described in Hausman (1985). Let n_{ij}^{*a} denote the value of n_{ij}^* when $n_{ij} \leq A_j$, and let n_{ij}^{*b} denote its value when $n_{ij} > A_j$. At most one of these two candidate optima will be feasible (i.e., will lie in the proper consumption interval). The values of n_{ij}^{*a} and n_{ij}^{*b} are obtained from the first order conditions with respect to n_{ij} that maximize (4) subject to (5):

$$(7) \quad n_{ij}^{*a} = \frac{-\beta_{i1}}{2\beta_{i2}},$$

$$(8) \quad n_{ij}^{*b} = \frac{-\beta_{i1} + p_j\beta_{i3}}{2\beta_{i2}}.$$

Observe that n_{ij}^{*a} can take any real value, but it is a feasible solution to the consumer's decision problem only if it lies between 0 and A_j . Similarly, n_{ij}^{*b} can take any real value but it is feasible only if it is greater than A_j . If $n_{ij}^{*a} > A_j$ and $n_{ij}^{*b} < A_j$, there is no interior solution and the consumer will choose A_j . It follows that the optimal quantity for consumer i under plan j is given by:

$$(9) \quad n_{ij}^* = \begin{cases} n_{ij}^{*a} & \text{if } 0 \leq n_{ij}^{*a} \leq A_j, \\ n_{ij}^{*b} & \text{if } n_{ij}^{*b} > A_j, \\ A_j & \text{otherwise.} \end{cases}$$

This solution is unique because the budget set is convex and the utility function is quasi-concave (see Hausman 1985, p. 1257).

Both n_{ij} and z_{ij} are random variables, the latter because its value depends on n_{ij} . The uncertainty in consumption implies that (4) is a stochastic utility function. In such a situation, consumers will use expected utility, which incorporates their usage beliefs, for making a choice decision. The expected utility of plan j for consumer i is given by:

$$(10) \quad E[u_i(n_{ij}, z_{ij})] = v_{ij} + \beta_{i1}E[n_{ij}] + \beta_{i2}E[n_{ij}^2] + \beta_{i3}E[z_{ij}].$$

A discrete choice model predicts that the consumer will choose a service plan if and only if he or she obtains greater expected utility from having the service plan than from not having it, and if that plan has the maximum expected utility in the choice set. This choice depends on both the features of a plan and on a consumer's expected consumption. Nunes (2000, p. 398) provides empirical evidence supporting such a simultaneous choice process. For example, cell-phone plans advertise the number of free minutes presumably because consumers take expected consumption into account when choosing plans. Note that consumer's choice provides information on expected consumption. As Wilson (1993) suggests, heavy users tend to prefer service plans with high access fees and low per-unit prices whereas light users will tend to prefer those plans with low access fees and higher per-unit prices.

A notable benefit of the proposed model is its ability to infer consumption at different prices from choice data. This is important in situations where the objective of the firm is either profit or share maximization. If the objective is the latter, it is important that different consumers be weighed differentially based on their expected consumption when inferring aggregate market shares. Self-stated consumption data from consumers can be used in this regard. This method, however, treats consumption as independent of prices and can lead to meaningless results as we will show in the empirical section. An additional benefit from the model is quantifying the amount of usage uncertainty and its impact on choice. A final benefit is the proper characterization of the impact of changes in the prices and free units. Equations (4) and (5) imply that the per-unit price (p_j) and the free units (A_j) have an impact on utility only if $n_{ij} > A_j$. Thus, our model predicts that if all consumers use no more than A_j units, then an increase in the per-unit rate will have no effect on the choice probability of a service plan. This contrasts with the predictions of choice models that do not capture these nonlinear effects in the budget constraint.

Model Estimation

We estimate the proposed conjoint model using a hierarchical Bayesian, multinomial probit approach. Consider a sample of I consumers, each choosing at most one plan from a set of J service plans. Let t indicate a choice occasion. If consumer i contributes T_i such observations, then the total number of observations in the data is given by $T = \sum_{i=1}^I T_i$. Let $y_{ijt} = 1$ if the choice of plan j is recorded for choice occasion t ; otherwise, $y_{ijt} = 0$. Let $j = 0$ denote the index for the no-choice alternative. Thus, $y_{i0t} = 1$ if the consumer chooses none of the service plans. The random utility of plan j on the t^{th} choice occasion is given by:

$$(11) \quad U_{it}(n_{ijt}, z_{ijt}) = u_{it}(n_{ijt}, z_{ijt}) + \varepsilon_{ijt},$$

where $u_{it}(n_{ijt}, z_{ijt})$ is specified by equation (4) and ε_{ijt} is a random error term. The utility of the no-choice option is given by $U_{it}(0, w_i) = u_{it}(0, w_i) + \varepsilon_{i0t}$. We assume that $\varepsilon_{it} = (\varepsilon_{i0t}, \varepsilon_{i1t}, \dots, \varepsilon_{iJt})$ is normally distributed with null mean vector and covariance matrix Σ . Note that there are two error components in equation (11). The first component is the consumer-level uncertainty in usage (equation 6) and is embedded in $u_{it}(n_{ijt}, z_{ijt})$. The second component is the choice error, ε_{ijt} , which is known to the consumer but unknown to the researcher. This interpretation of the error structure is consistent with the structural modeling tradition (see Erdem and Keane 1996, p. 6).

The choice of a service plan depends on its overall expected utility, which in the presence of choice error is given by

$$(12) \quad \tilde{U}_{ijt} = E[U_{it}(n_{ijt}, z_{ijt})] = E[u_{it}(n_{ijt}, z_{ijt})] + \varepsilon_{ijt},$$

where $E[u_{it}(n_{ijt}, z_{ijt})]$ is given by equation (10). Note that ε_{ijt} appears in the overall expected utility because it is unobservable to the researcher. As $u_{it}(0, w_i)$ does not depend on consumption, the overall expected utility of no-choice alternative is $\tilde{U}_{i0t} = U_{it}(0, w_i)$.

Following the random utility framework, consumer i will choose service plan j if and only if he or she obtains (i) greater expected utility from having the service plan than from not having it; and (2) the highest expected utility from plan j across the J available plans. Equivalently, consumer i will choose service plan j if

$$(13) \quad \tilde{U}_{ijt} = \max_{1 \leq l \leq J} \tilde{U}_{ilt} \geq \tilde{U}_{i0t};$$

and will not choose any plan if

$$(14) \quad \tilde{U}_{ijt} < \tilde{U}_{i0t}, j = 1, \dots, J.$$

Let $\beta_i = (\gamma_{i10}, \dots, \gamma_{iJ0}, \gamma_{i1}, \dots, \gamma_{im}, \beta_{i1}, \beta_{i1}, \beta_{i3})$ denote the vector of regression parameters, and let θ_i denote the parameter for the uncertainty in the quantity used by consumer i . Let P_{ijt} be the probability that consumer i chooses service plan j on choice occasion t , given β_i , θ_i , and Σ . Let P_{i0t} be the no-choice probability (see Jedidi, Jagpal and Manchanda (2003) for the derivation of these probabilities when there is no uncertainty). Then the conditional likelihood, $L_i | (\beta_i, \theta_i, \Sigma)$, of observing the choices consumer i makes across the T_i choice occasions is given by

$$(15) \quad L_i | (\beta_i, \theta_i, \Sigma) = \prod_{t=1}^{T_i} \prod_{j=0}^J P_{ijt}^{y_{ijt}}.$$

To capture consumer heterogeneity, we assume that the individual-level regression parameters, β_i , are distributed multivariate normal with mean vector $\bar{\beta}$ and covariance matrix Ω . We further assume that $\log(\theta_i)$ is normally distributed with mean μ_θ and variance τ_θ^2 . The unconditional likelihood, L , for a random sample of I consumers is then given by

$$(16) \quad L = \prod_{i=1}^I \int L_i | (\beta_i, \theta_i, \Sigma) f(\beta_i | \bar{\beta}, \Omega) g(\log(\theta_i) | \mu_\theta, \tau_\theta^2) d\beta d\theta,$$

where $f(\beta_i | \bar{\beta}, \Omega)$ is the multivariate normal $N(\bar{\beta}, \Omega)$ density function and $g(\log(\theta_i) | \mu_\theta, \tau_\theta^2)$ is the univariate normal $N(\mu_\theta, \tau_\theta^2)$ density function.

The likelihood function in equation (16) is complicated as it involves multidimensional integrals, making classical inference using maximum likelihood methods difficult. We circumvent this

complexity by using MCMC methods, which avoid the need for numerical integration. We adopt a Bayesian framework for inference about the parameters. The MCMC methods yield random draws from the joint posterior distribution and inference is based on the distribution of the drawn samples. We use a combination of data augmentation (Albert and Chib 1993), the Gibbs sampler (Geman and Geman 1984) and the Metropolis-Hastings algorithm (Chib and Greenberg 1995). Finally, we use proper but noninformative priors.

Our model estimation approach follows the standard Bayesian estimation of the multinomial probit model except for three differences. First, as consumption is not observed, we calculate its value using the draws of β_i and θ_i from the MCMC sampler. We use the value of β_i to calculate optimal consumption n_{ijt}^* using equation (9). We use the value of θ_i to generate the quantity uncertainty, δ_{ij} . These two quantities are input to compute the consumption value n_{ij} using equation (6). Second, as the choice decision is based on expected utility, we generate a large sample of n_{ij} (as described above). For each sample value of n_{ij} , we calculate the utility using equations (4) and (5). The average of these utilities produces an estimate of the expected utility. Third, to ensure the quasi-concavity of the utility function, we enforce the two Slutsky restrictions on the individual-level parameters: $\beta_{i2} < 0$ and $\beta_{i3} > 0$. We enforce these restrictions by reparametrizing $\beta_{i2} = -\exp(b_{i2})$ and $\beta_{i3} = \exp(b_{i3})$ where b_{i2} and b_{i3} are unconstrained individual-level parameters. With these two restrictions, the normality assumption holds for parameters b_{i2} and b_{i3} but no longer holds for β_{i2} and β_{i3} .

For the Bayesian estimation, we use the following set of noninformative priors for all the population-level parameters. Suppose $\bar{\beta}$ is a $p \times 1$ vector and Ω^{-1} is a $p \times p$ matrix. Then the prior for $\bar{\beta}$ is a multivariate normal with mean $\eta_{\beta} = 0$ and covariance $C_{\beta} = \text{diag}(100)$. The prior for Ω^{-1} is a Wishart distribution, $W((\rho \mathbf{R}^{-1}), \rho)$ where $\rho = p+1$ and \mathbf{R} is a $p \times p$ identity matrix. For μ_{θ} , we

set a univariate normal prior with mean $\eta_\theta = 0$ and variance $C_\theta = 100$. The prior for τ_θ^2 is an inverse gamma $IG(a,b)$ with $a = 3.0$ and $b = 2.0$. Finally, we assume that the utilities of the plans are independent given β_i and θ_i i.e., Σ is a block diagonal matrix. Let σ_j^2 be the variance of choice error ε_{ijt} (see Equation 12). We set an inverse gamma prior $IG(3.0, 2.0)$ on this variance.⁶

In a later section, we report the results of a small simulation assessing the robustness of our MCMC estimation procedure. These results indicate that our MCMC algorithm does well in recovering the true parameters.

AN APPLICATION

We illustrate the proposed model using data from a conjoint study concerning cellular phone plan choices. The participants in the study were seventy two undergraduate marketing students at two large northeastern universities.⁷ We use the data from this study to estimate the proposed model and compare its results to those obtained using six null models. Some of these null models use information concerning the current plans used by the participants, the attributes of these plans, and self-reported usage (in minutes) of their respective plans. On average, our subjects report using 540 minutes of cell phone services per month. This is very close to the national average of 600 minutes per month in 2005 (Cellular Telecommunications and Internet Association, 2006).

To design our conjoint experiment, we conducted a pilot study using a convenience sample of thirty three undergraduate students, each of whom was a subscriber to a cellular phone service plan. We determined the attributes to include in our conjoint design by asking these subjects to state the three most important attributes when choosing among service plans. We also asked them to indicate the three most popular service providers in this category. Access fee, per-minute rate, monthly free-minutes, the service provider's name, roll-over and internet access, were the most frequently

mentioned attributes; Verizon, Cingular, AT&T, and T-Mobile were the most popular service providers. To establish an empirically viable range for the pricing components of a cellular phone service, we asked each subject to state the maximum access fee and per-minute rate that he or she would be willing to pay for a cell phone service. From the results, we identified \$15 to \$90 as a feasible range for access fee and 15 cents to 60 cents as a feasible range for per-minute rate. The market rates at the time of the study fell within these ranges.

Study design

Following the results of the above pilot study, we selected six attributes for creating the conjoint profiles: (1) access fee, (2) per-minute rate, (3) plan minutes, (4) service provider, (5) internet access and (6) rollover of unused minutes. After defining these six attributes in the questionnaire, each participant was then presented a sequence of eighteen individualized choice sets in show card format. Each choice set had three wireless plans, described using the six attributes above.⁸ The task of a participant was to choose at most one alternative (i.e., no choice is permitted) from each choice set he or she was shown. We controlled for the order and position effects by counterbalancing the position of the service providers and randomizing the order of profiles across subjects. Figure 1 presents an example of a choice set used in our study.

Figure 1 here

To ensure that no choice set had a dominating alternative, we used a utility-balance type approach for designing the choice sets (see Huber and Zwerina, 1996). We first generated three orthogonal plans with 18 profiles each from the full factorial design (Addelman 1962). We ordered the eighteen profiles from each orthogonal plan from least to most preferred using the average attribute importance weights from the pilot survey. We then selected the three alternatives with equal ranks

to form a choice set.

Specification of attribute levels. We retained Verizon, Cingular, and T-Mobile as service providers, but dropped AT&T because it had already merged with Cingular at the time of the study. We included internet access and rollover as additional binary (yes/no) features. Access fee refers to the monthly charge to a customer for using the wireless service. Per-minute rate is the marginal cost to the consumer for each minute of use in excess of the free minutes. We divided the ranges of each variable into “low,” “medium” and “high” categories for both the access fee (\$15-\$40, \$40-\$65, and \$65-\$90) and the per-minute rate (\$.15-\$.30, \$.30-\$.45, and \$.45-\$.60). Service plans with higher access fee typically have more free minutes (plan minutes). To reflect this realism in the design of our stimuli, we computed the cost of a free minute (i.e., the ratio of the access fee and the number of free minutes) for a large number of plans that were available on the market at the time of the study. The empirical range of this cost varies from 4 cents to 15 cents; the corresponding number of free minutes per month vary between 100 minutes to 2000 minutes. We created three levels for this cost range: “low” (\$.04-\$.06), “medium” (\$.06-\$.09) and “high” (\$.09-\$.15). We randomly selected a value from the appropriate range for each attribute level appearing in a hypothetical service plan. To determine the monthly free minutes, we divided the randomly generated access fee value of the plan by its generated cost per free minute. Thus, the actual prices and the number of free minutes vary continuously across choice sets and respondents. Note that we do not present the pricing and free minute attributes using their low, medium, and high levels, but rather using their exact values, which we draw randomly from their respective intervals in the conjoint design.

Model Specifications

We use the above data to estimate the proposed model and six null models.⁹ The latter were selected to examine if the proposed method provides any improvements in predictions and/or offers new

insights.

Let f_j denote the monthly access fee; p_j the per-minute rate for usage beyond the free minutes; and A_j the number of free minutes per month (in hundreds of minutes) for plan j . Let $CING_j$, $TMOB_j$, VER_j , $ROLL_j$, and INT_j denote 0/1 dummy variables representing the absence or presence of Cingular, T-Mobile, Verizon, Roll-Over and Internet access, respectively, in plan j . We select Verizon as the base level for brand name.

We specify the following utility function for the proposed model (for simplicity we omit the subscript t denoting choice occasion):

$$(17) \quad u_{ij} = \beta_{i0} + \beta_{i1}n_{ij} + \beta_{i2}n_{ij}^2 + \beta_{i3}z_{ij} + \beta_{i4}CING_j + \beta_{i5}TMOB_j + \beta_{i6}ROLL_j + \beta_{i7}INT_j, \quad j = 1, 2, 3.$$

Each of the parameters in the above equation is specified at the individual-level¹⁰; n_{ij} is the quantity as defined in equation (6); and

$$(18) \quad z_{ij} = \begin{cases} -f_j & \text{if } 0 \leq n_{ij} \leq A_j, \\ -f_j - p_j(n_{ij} - A_j) & \text{if } n_{ij} > A_j. \end{cases}$$

Note that, in contrast with the budget constraint (5), the empirical budget constraint (18) does not contain a w_i term. This is because in a choice model setting, the $\beta_{i3} w_i$ term enters the utility of each alternative and hence cancels out.

We estimate two special cases of our proposed model. First, to assess the impact of uncertainty, we estimate a model in which we assume that consumers have no consumption uncertainty, or

equivalently, $n_{ij} = n_{ij}^*$. We call this model the “Proposed Model—No Uncertainty.” Second, as suggested by empirical evidence (e.g., Lambrecht, Seim and Skiera 2007), our model allows the possibility that consumers do not use all their free minutes (underage). As a benchmark, we estimate the special case of our model that requires consumers to exhaust all free minutes (i.e., no underage but overage is possible). We call this model the “Proposed Model—No Underage.”

We also compare our model with three alternative choice-based conjoint specifications. The first model is the following standard, main-effects conjoint model:

$$(19) \quad U_{ij} = \alpha_{i0} + \alpha_{i1}f_j + \alpha_{i2}p_j + \alpha_{i3}A_j + \alpha_{i4}CING_j + \alpha_{i5}TMOB_j + \alpha_{i6}ROLL_j + \alpha_{i7}INT_j + e_{ij} \quad j = 1, 2, 3.$$

We assume that the person-specific vector $\alpha_i = (\alpha_{i0}, \dots, \alpha_{i7})'$ of parameter estimates in this conjoint model follows a multivariate normal distribution with mean vector $\bar{\alpha}$ and covariance matrix Ω_α ; and we assume that $e_i = (e_{i0}, e_{i1}, e_{i2}, e_{i3})'$ is a vector of error terms, normally distributed with zero mean and covariance matrix Ψ . We call this model the “Standard Conjoint Model.”

The next two benchmark models extend the above main-effects conjoint model in different ways. One of these, labeled “Interaction Effects Model,” additionally includes interaction effects between pairs of the access-fee, per-minute-rate and free-minutes attributes. The other, called the “Nonlinear Effects Model,” reflects nonlinear price effects in a manner analogous to Goett, Hudson and Train (2000, p. 9). This model adds to the conjoint model in (19) logarithmic terms in the access fee, per-minute rate and free minutes.¹¹

Finally, we test a “Monthly Cost Model,” in which the utility is a function of the monthly cost of the plan and its features ($CING_j$, $TMOB_j$, VER_j , $ROLL_j$, and INT_j). To calculate the cost of a plan, we use the self-stated consumption for each consumer along with the plan free minutes, access fee

and per-minute rate.

Model performance results

We used Markov Chain Monte Carlo (MCMC) methods for estimating the above models. For each model, we ran sampling chains for 150,000 iterations. In each case, convergence was assessed by monitoring the time-series of the draws. We report results based on 100,000 draws retained after discarding the initial 50,000 draws as burn-in iterations.

Goodness of fit. We use Bayes Factor (BF) to compare the models. This measure accounts for model fit and automatically penalizes model complexity (Kass and Raftery 1995). In our context, BF is the ratio of the observed marginal densities of a particular null model and our model. We use the MCMC draws to obtain an estimate of the log-marginal likelihood (LML) for each of the models. Table 1 reports the log-marginal likelihood for all the models and log-BF relative to our proposed model. The results provide evidence of the empirical superiority of our proposed models with and without uncertainty (see Kass and Raftery 1995, p. 777).¹²

Table 1 here

The standard conjoint and the monthly cost models perform very poorly. The former fails to capture important nonlinear and interaction effects; and the latter does not correct for the effect of prices on consumption (which is fixed at the self-stated monthly usage for each consumer). The interaction- and the nonlinear-effects models perform better than these two models. However, both models, despite having a larger number of parameters, have poorer fits than that of the proposed models, with or without uncertainty. A comparison of the log-BF for the proposed model with uncertainty relative to models without uncertainty or underage suggests that it is more important to capture

satiation effects than it is to model the uncertainty in consumption.

Predictive validity. For each subject, we randomly select 16 of the 18 choice sets for model estimation and use the remaining 2 for out-of-sample prediction. The last column of Table 1 reports the mean hit rate across subjects and holdout choice sets for each model. All models have hit rates that are significantly higher than the 25% chance criterion. Consistent with the earlier LML results, the standard conjoint and the monthly cost models have relatively poor predictive validity. All other models (except the No Underage Model) have holdout hit rates that are statistically indistinguishable.

The results in Table 1 collectively suggest that it is important to reflect the effect of plan characteristics as well as various aspects of consumption such as usage quantity, uncertainty and satiation in a choice model. Although satisfactory in terms of holdout hit rates, the nonlinear- and interaction-effects models are ad-hoc in their specification. Later in the paper, we discuss the implications of this on demand estimation and optimal pricing recommendation.

Reliability of estimates. As already noted, an important benefit of our model is its ability to infer expected consumption for each subject. We assess the reliability of these estimates by correlating consumers' self-stated, monthly consumptions (minutes of cell phone use) with our model estimates. The latter are computed given each subject's self-stated per-minute rate p_j and free minutes A_j for their current wireless service plan. For our sample, this correlation is .70, which provides good evidence for the reliability of our consumption estimates.

As a further check for the reliability of our consumption estimates, we perform a regression relating the self-stated monthly consumption and the model predicted consumption (\hat{n}_{ij}). We obtain the following regression equation (standard error in parentheses).

$$\text{Self – Stated Consumption} = \underset{(94.89)}{35.80} + \underset{(.20)}{1.14} \times \hat{n}_{ij}.$$

The results suggest that respondents slightly overstate their monthly consumption. However, we fail to reject a test of a null intercept and a slope of 1.0.

Estimation results

We now discuss the parameter estimation results from our proposed models. As is common in Bayesian analysis, we summarize the posterior distributions of the parameters by reporting their posterior means and 95% posterior confidence intervals. Table 2 reports the results.

Table 2 here

In all three models, most of the unconstrained parameter estimates have the expected sign and are “significant.” The main effect of quantity ($\hat{\beta}_1$) is positive and its significance validates the importance of accounting for consumption in a choice model. For the brand effects, in general consumers are indifferent between Verizon and T-Mobile, which they marginally prefer to Cingular. This weak brand effect is consistent with a Harris Interactive study which reports a yearly 14% switching rate among service providers and a 32% customer satisfaction rate (Harris Interactive 2005). The presence of Rollover and Internet Access adds significantly to the utility of a wireless service. Both of the constrained parameters ($\hat{\beta}_2$ and $\hat{\beta}_3$) are significant and the constraints are binding. Thus, there is a significant diminishing utility from consuming additional minutes and a positive income effect.

A comparison of the models with and without uncertainty in usage quantities suggests that a failure

to account for uncertainty is likely to result in an underestimation of the magnitude of quantity effects. This result is consistent with the findings in the psychometric literature that ignoring measurement error can lead to biased regression parameter estimates (Jedidi, Jagpal, and DeSarbo 1997). Similarly, the comparison between the no uncertainty model and the no underage model suggests that not allowing consumers to leave free minutes on the table produces a downward bias in the magnitude of the quantity effects. This happens because the latter model forces consumers to exhaust all their free minutes, thus artificially increasing consumption (n_{ij}).

Recall that the parameter θ_i^2 captures consumption uncertainty. We have assumed that $\log(\theta_i)$ is distributed across consumers according to a normal distribution with a mean μ_θ and a variance τ_θ^2 .

We obtain the following estimates for these parameters (where monthly consumption is measured in 100s of minutes): $\hat{\mu}_\theta = .06$ and $\hat{\tau}_\theta^2 = .22$. Using the MCMC draws for $\log(\theta_i)$, we calculate the individual-specific estimates of θ_i^2 . Across consumers, we find a mean monthly uncertainty $\bar{\theta}^2 = 167$ minutes and a standard deviation $\text{std}(\theta^2) = 133$ minutes. Using market level data for a wireless service company, Iyengar et al. (2007, p. 25) find an average monthly uncertainty of 181 minutes, which is quite close to our estimate.

The results for heterogeneity (not reported in this paper due to space constraints but are available from the authors upon request) suggest that consumers in this sample appear to be more heterogeneous in the squared quantity and income effects than in the effects of quantity, wireless service provider, and service features.

Table 3 here

Table 3 reports the estimation results from the conjoint null models.¹³ For the standard conjoint model, all the parameter estimates have the expected sign. Access Fee and Free Minutes are

significant at the $p < .05$ level; Per Minute Rate is only significant at the $p < .10$ level. The lack of significance in the latter case may be due to the ad-hoc way the per-minute price enters the utility function. Recall that, for our proposed model, the per-minute price has a no effect (negative effect) on utility if consumption is lower (higher) than the plan free minutes. In contrast, for the standard conjoint model, as the per-minute rate is specified as a covariate and always affects the utility function irrespective of consumption, there is an aggregation of effects over these two consumption regions and this may have led to this insignificance. Preference for wireless services is higher for plans with higher number of free minutes, lower access fee and per-minute rate. The results also suggest that consumers are indifferent between the brands, and that Rollover adds significantly to the utility of a wireless service. Internet Access, however, has an insignificant effect. Observe that the results from the standard conjoint model do not provide information about expected consumption of services by consumers.

Within the interaction-effects model, only the interaction between access fee and free minutes is significant. The inclusion of this interaction term renders the main effect of access fee insignificant and increases by almost six fold the impact of the main effect of free minutes. Usage allowance in this model is still the most important driver of choice, but its importance diminishes with higher access fee. The results of the nonlinear effects model show that consumer responses to changes in access fee and free minutes are nonlinear. However, at the population-level, the nonlinearity in response to changes in per minute rate is not significant.

Demand estimation

We compare the predictions of the models for the demand of a wireless plan offered by T-Mobile, with the attribute levels for Verizon and Cingular set at the values associated with their most popular wireless plans. At the time of the study, the most popular plan offered by Cingular provided

400 free minutes, charged \$40 for access fee and 40 cents per minute for excess calling time, and allowed for rollover. Verizon's most popular plan offered identical attribute levels as that of Cingular but did not permit rollover. T-Mobile offered a plan similar to that of Verizon but with 500 free minutes per month. We examine how the demand for T-Mobile is affected by variations, one factor at a time, in the access fee, the per-minute rate, and the number of free minutes offered to consumers. For example, holding access fee at \$40 and per-minute rate at 40 cents, we seek to predict how variations in the number of free minutes affect consumer choice and the sales revenues of T-Mobile. Consumer choice is simply the mean predicted choice probability for a given T-Mobile wireless plan. Similarly, expected sales revenues is the dollar amount that a consumer is expected to pay given his or her consumption level, the access fee and the per-minute rate of the T-Mobile wireless plan times the choice probability of the plan.

Impact on Choice Probability. Figure 2 shows how the mean choice probability of a T-Mobile wireless plan varies as a function of per-minute rate, free minutes, and access fee for the proposed model and each of the conjoint null models. For all models, this choice probability decreases with increasing per-minute rate and access fee. Except for the nonlinear-effects model, this probability increases with increasing free plan minutes.

There are differences in the shapes and the magnitudes of the response curves. We first examine the impact of a change in per-minute rate (marginal price). The proposed model and the nonlinear-effects model suggest a nonlinearity in consumers' response to changes in per-minute rate. The other two null models indicate a perfectly decreasing linear trend. In contrast to these latter models, changes in marginal price do not have any effect beyond the 25 cents (30 cents) per-minute for the proposed (nonlinear-effects) model. Why is this happening? The answer lies in how each model specifies the effect of per-minute prices.

Figure 2 here

Recall that the proposed model accounts for the effect of per-minute price through the budget constraints which are used to find consumption. Thus, depending on where consumption lies on the budget set, two possible outcomes can result from a marginal price change. It is possible that consumption for an individual lies beyond the allowable free minutes (i.e., $n_{ij} > A_j$), in which case a reduced per-minute price would lead to an increase in consumption and utility and hence increased choice probability. It is also quite possible that consumption lies on the flat part of the budget constraint (i.e., $n_{ij} < A_j$), in which case a change in per-minute price could leave the consumption and choice probability of a consumer unaffected. In fact, we have examined the occurrence of such scenario in this simulation and found that expected consumption is always less than A_j for all consumers when the per-minute rate exceeds 25 cents. The nonlinear-effects model appears to capture some of these aspects; the interaction-effects model fails to do so. In contrast, situations of this kind do not arise under the standard conjoint model, as the per-minute price enters linearly in the utility function (i.e., as a covariate) and therefore increases in per-minute price would always result in decreased choice probability.

We next examine the impact of a change in number of free minutes. Except for the standard conjoint model, all models suggest a nonlinearity in consumers' response to changes in plan free minutes. The proposed (interaction-effects) model shows an S-shaped response curve where increases in free minutes have no impact if A_j exceeds 900 (1300) minutes per month. Surprisingly, the nonlinear-effects model suggests an ideal-point type utility function whereby the choice probability increases up to around 1000 minutes and then decreases afterwards.

These results stem from the different ways in which the models account for free minutes: they are part of the budget constraint in the proposed model, but appear only as covariates in the other

models. In our model, an increase in the number of free minutes has no effect on the level of consumption and utility for consumers for whom $n_j \leq A_j$. For the other consumers, this change increases the utility, and thus the choice probability, for a plan. However, it is important to note that as A_j increases, the proportion of unaffected consumers gets larger, possibly reaching 100% after a certain threshold (900 minutes in our case). Increasing the number of free minutes beyond this level makes little difference on either the consumption or the choice probabilities of consumers. Here, unlike the results for the effect of marginal price, the interaction-effects model appears to capture some of these aspects whereas the nonlinear-effects model fails to do so. As expected, the standard conjoint model does not capture these nonlinear effects as the free minutes enters linearly in the utility function.

Finally, the models also differ in the magnitude of the predicted choice probability (at 1500 monthly free minutes, the predicted choice probabilities are 37%, 56%, 96% and 74% for the proposed, nonlinear-effects, interaction-effects and the standard conjoint models, respectively).

Changes in access fee result in response functions that are (reverse) S-shaped for all the models. This similarity in shape is not surprising because, in all models, access fee affects consumer utility irrespective of the level of consumption.¹⁴

Impact on Expected Revenues. The dollar revenue from a particular plan is obtained by an accounting formulae which adds the access fee of the plan to the revenues from consumption in excess of the free minutes, if applicable. For the proposed model (null models), this excess is the difference between the expected (self-stated) consumption and the plan's allowable free minutes. To obtain the expected revenue, we simply multiply the dollar revenue by the expected choice probability of the plan. Figure 3 shows how the mean expected revenue varies as we vary the per-minute price, the number of free minutes, and access fee. For the proposed model, there is always a

revenue maximizing value for these variables (while keeping two fixed) -- about 6 cents for the per-minute price, 900 minutes for the free plan minutes and \$30 for the monthly access fee.

The results from the null models are quite peculiar. They are mainly due to the fact that intended (self-stated) consumption is treated as exogenous and is not adversely affected by increases in per-minute rate or by decreases in free minutes. First, expected revenues are always increasing with increasing per-minute price. For the null models, increasing per-minute rate has two effects: (1) it reduces the choice probability (see Figure 2) and in contrast to the proposed model (2) it always increases the dollar revenues from a chosen plan if consumption exceeds the free minutes. The net effect of these two forces resulted in an upward sloping revenue curve for the null models. Second, in the standard conjoint (nonlinear-effects) model, the impact of free minutes on expected revenues is U-shaped (inverted U-shaped).

Finally, expected revenues are declining with increasing access fee. This result is due to the tradeoff between probability of choice, which decreases with increasing access fee, and revenue, which increases with increasing access fee. The net result of this tradeoff appears to be a decrease in expected revenue.

Figure 3 here

Summary. The empirical analysis highlights two important benefits of the proposed model. First, it parsimoniously captures the nonlinear effects of the various components of a multi-part pricing scheme in a theoretically meaningful way. While the Nonlinear-Effects Model captures the nonlinearity in the effect of marginal price, and the Interaction-Effects model captures the nonlinearity in free minutes, neither of these models capture both effects simultaneously. Second, our model infers consumer expected consumption while allowing for usage uncertainty. None of the

null models is capable of such inference. Most importantly, these benefits are achieved with no loss in model fit and predictive validity.¹⁵

Optimal service plan

For the purpose of illustration, suppose T-Mobile were a new entrant in a market in which Cingular and Verizon already offered cellular phone services. What plan should T-Mobile offer its customers, and how should it price the plan? Should it include internet access and/or rollover minutes as part of its basic plan? If it offers these services as add-ons, what prices should it charge for each? To examine these questions, we consider the data from those of our respondents who do not have T-Mobile as their current service. This subset of consumers comprises 89% of our sample. We set T-Mobile's monthly variable cost for offering services to its customers at \$30 (T-Mobile Quarterly Report 2005). We then perform the following grid search to identify the optimal service plan for T-Mobile.

We evaluate all possible combinations of the design factors at discrete points: (1) access fee in increments of \$3, ranging from \$25 - \$90 per month; (2) per-minute rate in increments of 2 cents, ranging from 1 cent per minute to 50 cents per minute; (3) free minutes in increments of 85 minutes, ranging from 100 minutes to 2250 minutes; (4) internet access (present or absent); and (5) rollover minutes (present or absent). Thus, there are $20 \times 20 \times 25 \times 2 \times 2 = 40,000$ grid points. We then identify the plan with the highest total expected contribution margin for T-Mobile. We do so by using each person's current service plan as his/her status-quo option. For the proposed model, we use the estimated utility functions to compute the choice probabilities and usage quantity for each person and target plan. Similarly, for the conjoint null models, we use their respective estimated utility functions to infer the choice probability for T-Mobile versus the status quo; and then use the self-stated usage rates to compute the expected margin for each consumer. We average the expected

profit across consumers. Table 4 describes the service plans with the highest expected contribution for T-Mobile, one obtained using the proposed method and the others using the conjoint null models.

Table 4 here

All plans identify internet access and rollover minutes as features of an optimal plan. But they differ substantially on the other three features -- access fee, per-minute rate and free minutes.

In 2005, T-Mobile reported an average profit of \$27 per customer per month (T-Mobile Quarterly Report 2005). This is closer to the \$13.40 expected profit per customer per month for the optimal plan obtained using the proposed method than it is to the \$128, \$48 and \$57 per customer per month obtained by the standard conjoint, interaction- and nonlinear-effects models, respectively. These latter numbers appear to be relatively high because they ignore the effect of the per-minute rate and the free minutes on the usage rate. And it is this reason that the latter models select a substantially higher per-minute rate. If we use the proposed model to assess the profitability of the optimal plans identified by the conjoint null models, we find that it should make an expected profit of -58 cents, 55 cents and -\$3.12 per customer per month for the standard conjoint, interaction- and nonlinear-effects models, respectively. We also find that the latter null models, respectively, project a profit of \$8.81, \$6.71 and \$7.38 per customer per month for the optimal plan identified by the proposed model. This suggests that the optimal plans identified by the null models will fare poorly if we include the effect of per-minute rate on consumption and choice.

Next, we examine the sensitivity of the optimal solution identified by the proposed method to changes in the price per minute. The reason for doing this is that the 4 cents per-minute rate identified by the optimal solution appears quite far from the 40 cents per-minute that is typical in

the cellular phone industry. One possible reason for this deviation is that the profit function might be flat over a range of the per-minute rate charged to consumers. To examine this issue, we vary the per-minute rate in 10 cent increments, from 10 cents per minute to 50 cents per minute. We also perform an analysis using a 1 cent per-minute rate. We then use the grid search described above to identify the profit-maximizing plan, given a fixed, per-minute rate.

The results suggest that the average profit per customer is relatively flat over the range of the variable rate. When the per-minute rate is 1 cent (50 cents) per-minute, we obtain an expected profit per customer per month of \$13.25 (\$13.00). In addition, at a rate of 40 cents per minute, this expected profit is less than the optimal value of \$13.40 by only 40 cents. The effect on profits of this per-minute rate increase is compensated by offering 806 (= 1175-369) extra free minutes. The corresponding optimal T-Mobile plan has its highest profit when the access fee is \$66, which is \$7 higher than the fixed fee for the proposed optimal plan in Table 4. A comparison of the optimal plans obtained using the proposed model and the conjoint null models, constraining the per-minute rate to 40 cents per minute, appears in Table 5. These plans, like those shown in Table 4, are quite different.

Table 5 here

Feature pricing

A common practice is to unbundle features and offer one or more of these as add-on features to a base plan. In the present example, a single plan featuring both internet access and rollover minutes allows customers no choice of add on features. It might sometimes be beneficial for customers, and possibly the firm, to have a base plan that excludes one or both of these features, and offers them, separately, together or both, as add-ons for additional fees. The optimization problem for T-Mobile in this instance requires the consideration of mixed bundling and the additional pricing of optional

features for users who can buy those features they find most useful. We use the following method for determining the base plan and the prices of internet access and rollover minutes as optional features.

For each consumer in our sample, we introduce four T-Mobile plans in his or her consideration set. Each consumer then has five plans in his/her choice set - a status quo option and the four T-Mobile plans. In the following discussion, we refer to a plan with no rollover and no internet connection as a “basic” plan. The other three plans include either the rollover feature or internet or both. We constrain the free minutes to be the same for all the four T-Mobile plans and set the per-minute rate at \$0.40 per minute. We calculate the access fees for the four plans, and their *common* free minutes, to maximize the sum of the expected profits across the four T-Mobile plans. Table 6 describes these plans.

Table 6 here

These results suggest that T-Mobile should include internet access in its base plan. It should charge a monthly fee of just over \$62 and give 1250 free minutes. It should offer rollover minutes as an optional feature and price it at $\$66.79 - \$62.14 = \$4.65$. From the results in Table 6 we see that the probability of choice for T-Mobile across the four plans is .41 ($= .01 + .03 + .14 + .23$) and the overall expected profits per customer is just over \$14.37 per month ($= \$3.31 + \$9.00 + \$4.54 + \8.62). Both values are higher than the corresponding values from a single optimal plan with a \$.40 per-minute rate (see Table 6). For instance, the expected profit of \$14.37 is \$1.37 higher than the \$13.00 expected profit per customer from the single plan.

Simulated testing

We performed a small-scale simulation experiment to assess how well the estimation procedure

recovers the true simulated parameters. For generating choice data, we used the same conjoint design as was used in our empirical application. For simplicity, however, we ignored brand name, internet access, and roll-over and only kept access fee, free minutes, and marginal price as plan features. We set $\beta = (\bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3)' = (2.5, -2.5, 1.5)'$, $\Omega = \text{diag}(.2, .2, .2)$, $\mu_\theta = 0$, $\tau_\theta^2 = .2$, and $\Sigma = \text{diag}(1.0, .5, .5, .5)$ to generate the choice data. Note that the variance of the utility of the no-choice option is fixed to 1.0.

We used the same priors that we presented in the model estimation section. We used a total of 20 Monte Carlo replications to study the variations of parameter estimates across the generated samples. For each of the 20 data sets we estimated the model parameters based on 100,000 draws retained after discarding 50,000 draws as burn-in iterations. We checked convergence by visual inspection of the trace plots of the various parameters. For each of the 20 replications convergence was achieved before the burn-in period.

Table 7 reports the true parameters and their respective average estimates across the 20 Monte Carlo samples. The table also includes 95% coverage for each parameter. This coverage is the proportion of the Monte Carlo samples in which the 95% posterior interval spanning the 2.5th to the 97.5th percentile of the MCMC draws covers the true parameter. Table 7 shows an excellent recovery of the true population-level parameters. The average mean squared error (MSE) is .01, which is quite low. In addition, the coverage properties are also excellent. On average, 98% of the posterior intervals contain the true parameter. Finally, the recovery of the individual-level parameters (not reported in Table 7) is also excellent. (The average MSE is .02.) In sum, the results of this simulation suggest that our estimation procedure performs very well in recovering the true parameters.

Conclusions

Multi-part pricing is used by providers of such services as wireless telephony, xerography, car rentals, HMO plans, and prescription drug plans. These pricing schemes require consumers to pay a per-unit fee for usage beyond some “free” (possibly zero) number of units. We propose a method using conjoint analysis for assessing the impact of such pricing schemes on consumer choice and usage. An important aspect of our model is that it takes into account the two-way dependence between consumption and price. That is, it accounts for the fact that the price charged by the provider influences consumption while the price a consumer pays depends on his or her usage level. We incorporate this simultaneity by proposing a model in which consumers allocate budgets taking into account the structure of nonlinear pricing schemes. The model allows inference of consumption and usage uncertainty. It is estimated using choice data.

We describe an application comparing the proposed model, and two of its special cases, to four null models. We find that the proposed model has better fit and predictive validity, and that it produces reliable, individual-level consumption estimates. A standard conjoint model that uses service attributes to predict choices does especially poorly, suggesting that it might not be appropriate for modeling nonlinear pricing schemes. Extensions of the standard conjoint model that include nonlinear and interactions effects do improve model fit and predictive validity. However, these models are not parsimonious, and they do not permit estimation of usage quantity. In addition, they do not fully capture the nonlinear effects induced by multi-part pricing schemes.

A comparison of the two special cases of our model suggests that consumption satiation is more important than consumption uncertainty when modeling nonlinear pricing effects. Ignoring consumption uncertainty results in an underestimate of the quantity used by a consumer.

We use the proposed and conjoint null models to assess the impact of plan features on demand for a

wireless service provider. In contrast to the null models, the demand curves produced by the proposed model properly capture the impact of changes in per-minute rate, free minutes, and access fee, and always identify finite values for which these features maximize revenue. We use the parameter estimates to characterize an optimal wireless plan that maximizes profit for a service provider. The optimal plan obtained using the proposed model is more consistent with industry practice than the optimal plans identified using the null models. We illustrate how the proposed model can be used to price optional features such as rollover and internet access.

A useful area of future research is to examine computationally efficient methods for optimal selection of product features and prices. The present approach of explicit enumeration is reasonable if, as in our application, there are a small number of attributes. For larger problems, more efficient procedures are necessary. A related area of research concerns the development of methods for optimal design of product lines and feature bundles. Models that consider the effect of competitive actions and reactions (e.g., Choi, DeSarbo and Harker 1990, Choi and DeSarbo 1994) on multi-part pricing can also be useful to examine in future research.

Finally, a potential area of related research concerns the use of alternative preference functions than those used in the paper. There has been substantial recent interest in the mathematical representation of non-compensatory preference models and choice processes (e.g. Kohli and Jedidi 2007, Yee, Dahan, Orlin and Hauser 2007). It is likely that consumers use some form of non-compensatory processes for screening and evaluating service plans, especially those which have many features and part prices.

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FOOTNOTES

1. In practice, multi-part tariff pricing mechanisms are used for pricing both products and services. For simplicity, we shall use service or service plan to indicate products as well.
2. Our model can be generalized to situations where the pricing scheme has a multi-part (more than two), increasing-block structure; i.e., a pricing scheme where the per-unit price increases with increasing consumption (see Reiss and White 2005).
3. The individual-specific term $\beta_{i3}w_i$, which captures the effect of income on utility, cancels out in a choice model. This is because only differences in utilities are relevant.
4. These restrictions imply risk aversion and arise from the Slutsky negativity constraints, which ensure quasiconcavity of the utility function (Hurwicz and Ozawa 1971).
5. We thank an anonymous reviewer for suggesting the inclusion of usage uncertainty in our

- model.
6. The details of the Bayesian estimation procedure are available from the authors upon request.
 7. The use of students as sample subjects limits the generalization of our results to other populations.
 8. The participants were told that all the three plans offer free nights and weekend minutes. For all other plan features that they may be thinking of (e.g., free in-network calls, assortment of handsets), the respondents were told to assume that all the plans are equivalent.
 9. We thank the review team for suggesting several of these null models.
 10. The intercept β_{i0} is estimable because the data collection allows a no-choice option (Haaijer, Kamakura and Wedel 2000). With this specification, the utility of the no-choice option is set to zero.
 11. We have also estimated a nonlinear model with a quadratic specification. However, due to multicollinearity, we encountered convergence problems in estimation. Another approach to capture nonlinearity is to treat the attributes as discrete variables with three levels each as in traditional conjoint. Although this is feasible, it is difficult to implement in the context of our study. First, the usage allowance A_j is not independent of access fee and per-minute rate. Second, our model requires continuous attributes. Therefore, by using continuous attributes for some models and discrete ones for others, the comparison of results across models gets muddled due to the loss of information.
 12. Kass and Raftery suggest that a value of log BF greater than 5.0 provides very strong evidence for the superiority of the proposed model.
 13. We do not report the results for the monthly cost model because of its poor model fit and holdout prediction. The details can be obtained from the authors.
 14. It is important to note here that respondents exhibit higher degree of price (access fee and per minute rate) sensitivity in hypothetical conjoints tasks than they do in real life (Verlegh,

Schifferstein, and Wittink 2002).

15. The figures for the expected quantity for the four models is available from the authors upon request.

TABLES

TABLE 1: MODEL PERFORMANCE COMPARISON¹

Model	LML	Log - BF	Hit rate
Proposed-Uncertainty	932.65	--	68.2
Proposed- No Uncertainty	944.71	12.06	69.4
Proposed - No Underge	977.79	45.14	65.9
Standard Conjoint	1186.19	253.54	58.3
Interaction Effects	977.82	45.17	67.3
Nonlinear Effects	952.03	19.38	68.9
Monthly Cost	1280.1	347.45	52.1

1. LML denotes Log Marginal Likelihood and Log-BF denotes Log Bayes Factor.

TABLE 2: PARAMETER ESTIMATES FOR PROPOSED MODELS: POSTERIOR MEANS
AND 95 % CONFIDENCE INTERVALS

Variable	Variable Label	Parameter Label	Uncertainty Model	No Uncertainty Model	No Underage Model
Quantity	n_{ij}	β_1	2.59 (2.44, 2.73)	1.73 (1.60, 1.84)	.87 (.71, 1.04)
Quantity ²	n^2_{ij}	β_2	-.29 (-.32, -.25)	-0.12 (-0.13, -0.10)	-.03 (-.05, -.02)
Income effect	z_{ij}	β_3	.07 (.05, .08)	.07 (.05, .08)	.06 (.05, .08)
Cingular	CING	β_4	-.19 (-.30, -.06)	-.29 (-.47, -.14)	-.28 (-.50, -.11)
T-Mobile	TMOB	β_5	-.12 (-.21, -.02)	-.15 (-.33, .02)	-.13 (-.30, .01)
Verizon	VER		0	0	0
Roll-Over	ROLL	β_6	.38 (.26, .49)	.37 (.20, .55)	.30 (.16, .44)
Internet	INT	β_7	.27 (.09, .46)	.23 (.02, .44)	.21 (.04, .38)
Intercept		β_0	-.78 (-1.16, -.39)	-1.38 (-1.96, -.82)	-.27 (-.25, .86)
Uncertainty	θ	μ_θ	.06 (-.07, .19)	–	–

Parameters in bold face are “significant” at the 95% level.

TABLE 3: PARAMETER ESTIMATES FOR CONJOINT NULL MODELS: POSTERIOR MEANS AND 95 % CONFIDENCE INTERVALS

Variable	Variable Label	Standard Conjoint	Interaction Model	Nonlinear Model
Access Fee	f_j	-6.40 (-7.90, -5.10)	-.40 (-2.80, 1.70)	-15.50 (-19.90, -11.80)
Per Minute Rate	p_j	-.59 (-1.24, .09)	-1.12 (-3.23, 0.62)	.01 (-.03, .04)
Free Minutes	A_j	16.90 (12.90, 21.90)	92.90 (72.80, 117.01)	8.54 (6.81, 11.11)
Cingular	CING	-.08 (-.32, .14)	-.34 (-.59, -.10)	-.33 (-.60, -.09)
T-Mobile	TMOB	.09 (-.13, .35)	-.21 (-.46, .03)	-.14 (-.38, .09)
Verizon	VER	0	0	0
Roll-Over	ROLL	.30 (.11, .49)	.42 (.20, .66)	.42 (.19, .67)
Internet	INT	.16 (-.06, .38)	.24 (-.03, .52)	.27 (-.01, .57)
Intercept		2.18 (1.65, 2.72)	.77 (-0.43, 1.80)	-7.55 (-8.51, -6.66)
Access X Minutes	$f_j \times A_j$	–	-135.02 (-170.01, -109.50)	–
Access X Rate	$f_j \times p_j$	–	-.50 (-5.40, 3.90)	–
Rate X Minutes	$p_j \times A_j$	–	15.10 (-6.21, 41.10)	–
Log Access	$\ln(f_j)$	–	–	2.90 (1.73, 4.27)
Log Minute Rate	$\ln(p_j)$	–	–	-.40 (-1.68, .94)
Log Free Minutes	$\ln(A_j)$	–	–	-3.67 (-4.99, -2.75)

Parameters in bold face are “significant” at the 95% level.

TABLE 4 : OPTIMAL T-MOBILE PLANS OBTAINED USING THE PROPOSED MODEL AND THE CONJOINT NULL MODELS

	Proposed Model	Standard Conjoint	Interaction Effects	Nonlinear Effects
Access Fee	\$59	\$25	\$90	\$25
Per Minute Rate	\$.04	\$.50	\$.50	\$.50
Free Minutes	369	100	100	369
Roll Over	Yes	Yes	Yes	Yes
Internet	Yes	Yes	Yes	Yes
Choice Probability	.38	.62	.17	.65
Expected Profit Per Customer	\$13.40	\$128	\$48	\$57

TABLE 5 : OPTIMAL T-MOBILE PLANS OBTAINED USING THE PROPOSED MODEL AND THE CONJOINT NULL MODELS WHEN THE PER-MINUTE RATE IS \$0.40

	Proposed Model	Standard Conjoint	Interaction Effects	Nonlinear Effects
Access Fee	\$66	\$25	\$90	\$25
Free Minutes	1175	100	100	369
Roll Over	Yes	Yes	Yes	Yes
Internet	Yes	Yes	Yes	Yes
Choice Probability	.36	.61	.18	.65
Expected Profit Per Customer	\$13.00	\$104	\$44	\$46

TABLE 6: FEATURE PRICING FOR THE PROPOSED PLANS

Plans	Access Fee	Prob of Choice	Mean Expected Profit (\$)
Basic	\$62.14	.01	\$.31
Basic + Internet	\$62.14	.03	\$.95
Basic + Roll	\$66.79	.14	\$4.54
Basic + Roll + Internet	\$66.79	.23	\$8.62

Per minute rate is set at 40 cents per minute.

Optimal Free Minutes is 1250 minutes.

TABLE 7: PARAMETER RECOVERY FOR SIMULATED DATA USING THE PROPOSED

MODEL

Variable	Parameter Label	True Value	Average Estimate	95% Coverage
Quantity	β_1	2.50	2.59	.95
Quantity ²	β_2	-2.50	-2.49	1.00
Income Effect	β_3	1.50	1.35	.95
Heterogeneity (Quantity)	ω_{11}	.20	.15	.95
Heterogeneity (Quantity ²)	ω_{22}	.20	.19	1.00
Heterogeneity (Income)	ω_{33}	.20	.18	1.00
Utility Variance	σ_{11}	.50	.56	.95
Utility Variance	σ_{22}	.50	.58	1.00
Utility Variance	σ_{33}	.50	.56	1.00
Uncertainty (Mean)	μ_0	.00	.03	1.00
Uncertainty (Variance)	τ_0^2	.20	.23	1.00

FIGURE 1

AN EXAMPLE OF A CHOICE SET OF THREE WIRELESS PLANS

Plan Features	Plan 1	Plan 2	Plan 3
Service Provider	Verizon	Cingular	T-Mobile
Access Fee	\$31.99	\$30.99	\$49.99
Plan Minutes	230 min	430 min	360 min
Per Minutes Rate	\$0.38/min	\$0.47/min	\$0.40/min
Internet Access	No	No	Yes
Roll Over	No	No	No

I choose: None Plan 1 Plan 2 Plan 3

FIGURE 2

COMPARISON OF FOUR MODELS ON PREDICTED CHOICE PROBABILITY AS A
FUNCTION OF MARGINAL PRICE, FREE MINUTES AND ACCESS FEE

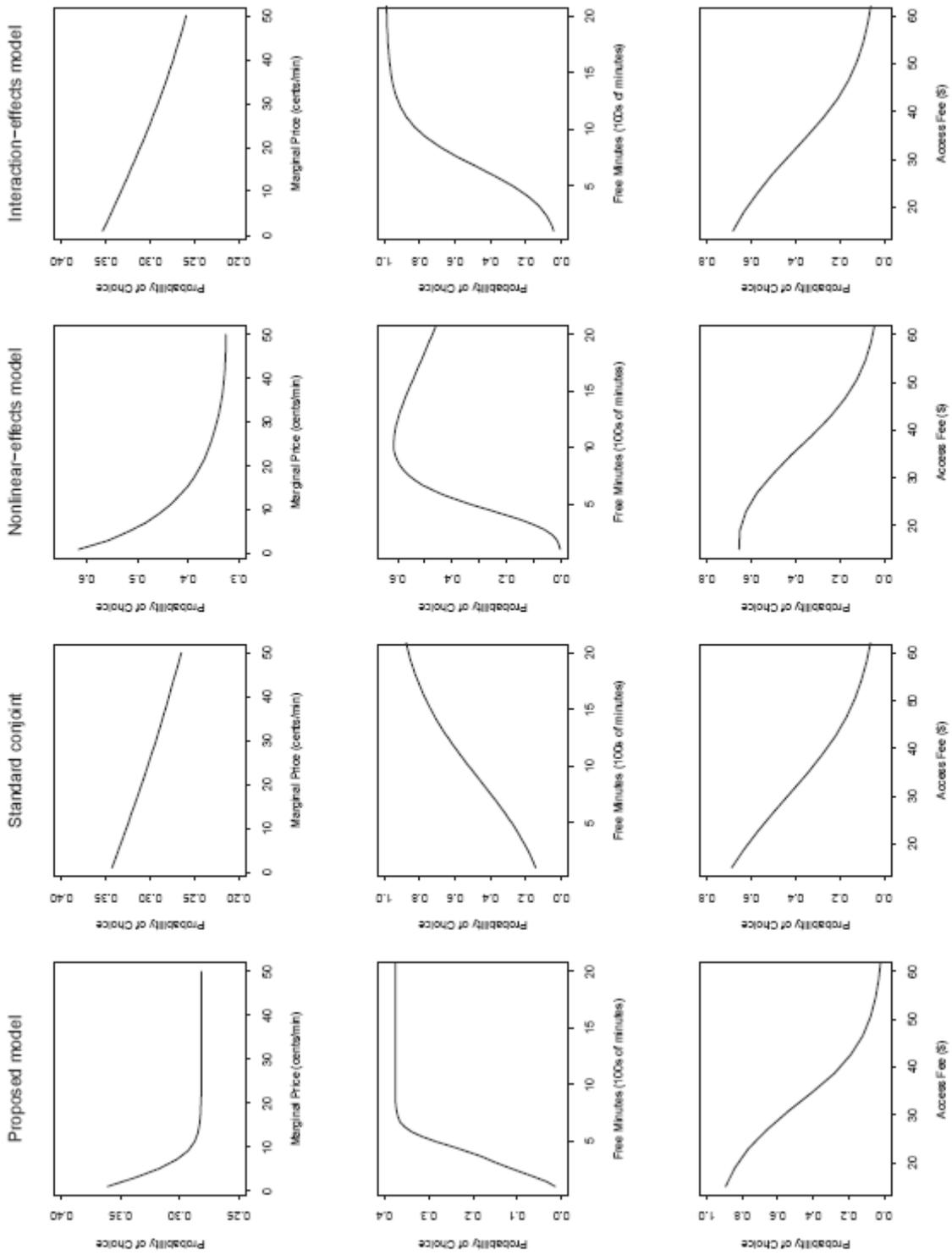


FIGURE 3

COMPARISON OF FOUR MODELS ON EXPECTED REVENUE AS A FUNCTION OF MARGINAL PRICE, FREE MINUTES AND ACCESS FEE

