Package Sizes for the Bottom of the Pyramid

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Abstract

In parts of the world, single-serve packages account for significant sales of such everyday products as bread, coffee, ketchup, cream and shampoo. These markets are characterized by two conditions: low transactions costs for buyers and sellers, and the presence of poor consumers who have unsteady income streams and can afford to only buy what they immediately need. We examine if the latter condition is necessary for a firm to make single-serve packages. We develop a model for a firm selling a product with a finite usable life to consumers with heterogeneous usage rates and reservation quantities, the latter being defined as the minimum quantity per dollar a consumer must obtain to purchase a product. We show that, even if there are no constraints on the income streams of buyers, a seller can charge higher unit prices and also sell a larger quantity of a product by making single-serve packages. The higher the unit cost of the product, the more salient the effect of single-serve packages on unit price. Sales volume simultaneously increases, provided the unit cost is not too low. A higher unit price therefore need not be a poverty penalty imposed on poor buyers, but a consequence of buyers being able to better match their purchases to their desired consumption. Less waste is not only socially desirable, it also leads consumers to pay higher unit prices and buy more of a product. These results are obtained if the cost of a package increases at a constant or increasing rate with its size. Sufficiently large economies of size can offset these advantages of smaller package for the seller and can make it more profitable to offer larger than single-serve packages. A single-serve package permits the seller to discriminate among buyers with different usage requirements, each consumer self-selecting to purchase quantities that match, as closely as possible, his or her desired level of consumption.

Keywords: Bottom of the pyramid; package size; pricing; product design.

1. Introduction

> *A rapidly evolving approach to encouraging consumption and choice at the BOP (Bottom of the Pyramid) is to make unit packages that are small and, therefore, affordable. The logic is obvious. The rich use cash to inventory convenience. They can afford, for example, to buy a large bottle of shampoo to avoid multiple trips to the store. The poor have unpredictable income streams. Many subsist on daily wages and have to use cash conservatively. They tend to make purchases only when they have cash and buy only what they need for that day. Single-serve packaging— be it shampoo, ketchup, tea and coffee, or aspirin — is well suited to this population. A single-serve revolution is sweeping through the BOP markets.*
>
> — C.K. Prahalad (2005)

As the above quote from Prahalad's book *Fortune at the Bottom of the Pyramid* suggests, package sizes can affect whether or not people buy a product and how much they consume. The single-serve revolution to which Prahalad refers is in some ways a return, albeit in more convenient and branded form, to the days when products were sold without packaging — you went to the grocer who measured out how much oil or flour you wanted; you could, in effect, make your own packages. In parts of the world, single-serve packages account for a significant portion of the volume sold for such everyday products as shampoo, salt, ketchup, cookies, fruit-drink concentrate, coffee, spices, toothpaste, cosmetics, bread, cooking oil, cream — the list is long and getting longer.

BOP markets are characterized by two conditions that differ substantially from those prevailing in developed markets. First, as Prahalad notes, some — but not all — buyers purchase only what they need for the day. Second, the transactions costs are lower for buyers and sellers than they are in more developed countries. One reason is that buyers can often purchase from sellers located in close proximity to their homes. India, for example, has over 12 million retail outlets — one for approximately every 90 persons — that currently account for annual sales of over \$200 billion per year.[1] Many retail stores are open for long hours and offer such services as telephone ordering, home delivery, and credit to buyers. Some firms, such as Hindustan Levers, have deployed distribution networks that sell directly to consumers in hard-to-reach markets in India; Avon is using 800,000 "Avon Ladies" as distributors to reach the most remote regions in Brazil. The ordering costs for buyers are typically low in these markets, and many consumers make frequent, sometimes daily, purchases, even if they can afford to buy larger quantities. For retailers, order-processing costs are also low, because labor is cheap and deliveries to customers are often made by unskilled workers who walk or ride a bicycle to make home deliveries. This is in contrast with developed countries that have fewer, larger stores located at substantial distances from consumers, and where high labor cost makes home delivery infeasible for retailers.[2] For the latter markets, inventorying convenience is important for consumers. Gerstner and Hess (1987) reflect these costs in a model in which they show that differences in package sizes and unit costs can be used to discriminate between high and low

[1]see http //www.tata.com/tata_strategic/articles/20060925_retail.htm

[2]Interestingly, single-serve packages, sold in packs of multiple units, are become available in developed markets like the USA, but for very different reasons than those considered here for BOP markets. These reasons include variety seeking (e.g., for products like fruit yoghurts) and limiting consumption (e.g., the 100-calorie packs of snacks developed in response to concerns about obesity in the population).

value segments that differ in their consumption rates: segmentation induces the seller to offer multiple packages, and the larger packages can even have higher unit prices.

If consumers can only afford to buy single-serve packages in BOP markets, then of course there is no alternative for a firm — it has to make single-serve packages. But is this condition necessary? Is it possible that, even in the absence of this constraint, single-serve packages make sense in BOP markets, where ordering and order-processing costs are not the central factors affecting consumer choices and firm decisions — there are, after all, many consumers in these markets who can afford to buy larger package sizes? More generally, what factors affect package-size decisions in markets with low transactions costs for buyers and sellers? How does the unit price depend on package sizes? And what is the social cost of making packages that are larger than single-serve sizes? The answers to these questions are important not just for small, local manufacturers and retailers. Multinational companies like Unilever, Proctor & Gamble, and Nestlé, sell products worth billions of dollars in developing countries, where buying and selling occurs in very different conditions than in developed countries. We refer the reader to Prahalad (2005) and Rangan, Quelch, Herrero and Barton (2007) for further discussion of the differences between BOP and non-BOP markets.

We examine the above questions in a model that considers four factors affecting package-size decisions by a firm: (1) the usable life of a product; (2) the relationship between package size and package cost, which we allow to be linear, concave or convex; (3) the consumption rate, which we allow to differ across consumers; and (4) the reservation quantity, which refers to the minimum usable quantity per dollar required by a user to buy a package. The reservation quantity is a measure of willingness-to-pay (which in turn can be affected by income constraints), except that it considers the amount the consumer expects to consume over the usable life of the product, not the quantity he or she purchases. We allow customers to differ in their reservation quantities.

The key results from our analysis can be summarized as follows. A seller can charge higher unit prices and also sell a larger quantity of a product by making single-serve package sizes. The lower the unit cost of the product, the more salient the effect of single-serve packages on unit price. Sales volume simultaneously increases, provided the unit cost is not too low. The higher unit price in our model is not a poverty penalty imposed on poor buyers, but a consequence of buyers being able to better match purchases to their desired consumption. Less waste is not only socially desirable, it also leads consumers to pay higher unit prices and possibly buy more of a product. These results are obtained if the cost of a package increases at a constant or increasing rate with its size, and do not depend on the cash-flow constraints on BOP consumers. Sufficiently large economies of size can offset these advantages of smaller package for the seller and can make it more profitable to offer larger than single-serve packages. The reason these results are interesting is that it is not *a priori* obvious if a seller should charge high or low unit prices, or make small or large packages — it is, for example, possible that the firm might make higher profit by making larger packages for only those buyers with high consumption rates, or low price sensitivity, or both. We note that the higher profits and unit prices are in the present instance not due to any form of price discrimination among buyers, which typically require usage levels to be related to consumer price sensitivity or the availability of different competing alternatives (Wilson 1992). Our model makes no comparable assumption: the seller makes only a single product and sells it at a common price to all buyers. Still, a single-serve package size allows quantity discrimination among buyers, each of whom can match purchases to consumption by selecting the appropriate number of single-serve packages. This self selection helps a seller to discriminate among users with varying levels of product use, and enables it to extract higher profits, reduce waste, and charge higher unit prices.

The problem we examine reflects BOP markets in substantial ways, but is by no means a complete representation. It is restricted to the consideration of a single seller offering one package size to all consumers; it ignores the availability of the same product from different, proximate retailers, a factor that has an effect on both customer behavior and the pricing and package size decisions made by a firm; and it does not consider costs of storage and handling for the supply chain, or the terms of trade that determine retailer behavior. Still, the simple problem we consider is non-trivial, and captures the effects of some important factors affecting product-size decisions that, to our knowledge, have not been previously examined, separately or together, in the literature. Indeed, there is little analytical work of which we are aware that deals with package size decisions, besides the aforementioned paper by Gerstner and Hess (1987).

Organization of the paper. Section 2 describes the consumer model. Section 3 introduces heterogeneity in usage rates and reservation quantities across consumers, and obtains the demand function. Section 4 characterizes the optimal package size and price for the firm when package cost increases linearly with package size. Section 5 analyzes some key characteristics of the demand and profit, the quantity wasted by consumers and the demand not fulfilled by the firm when it offers its optimal product size; it also examines the sensitivity of the optimal solution to the parameters of the problem. Section 6 examines nonlinear cost functions, and obtain a sufficient condition under which it is optimal for the firm to produce larger than single-serve packages. Section 7 discusses the implications and concludes the paper.

2. Consumer model

The size of a single serving is in itself somewhat arbitrary, because different consumers can use different quantities on a single usage occasion. For this reason, and for analytical simplicity, we develop our analysis in terms of the units in which a product is commonly sold, such as grams or milliliters. Our objective is to determine how many units s of the product the seller should sell in a single package. We interpret a solution to imply "single-serve packaging" if the optimal solution for the firm is to make the package size s as small as possible.

Consider a buyer who uses a product at an average rate of θ units per unit time, say per day. We assume for simplicity that the desired consumption rate for a user is constant over time and independent of the package size. The actual number of units purchased is endogenous in our model, and so the quantity consumed by a buyer depends in our model on the unit price and package size. If a consumer buys one package of size s, then he can use the product for at most s/θ days, provided the product is still of usable quality for this duration of time; otherwise, he will only obtain partial use of the product. How long a product will remain usable after purchase depends on several factors. Examples of such factors are the time a package has remained on a store shelf, the inherent rate of product deterioration, and the storage conditions both before and after the consumer purchases the product. We summarize the effect of these factors in our model by assuming that the time until the product becomes unusable has an expected value of T days after purchase. For brevity, we refer to T as the *expiration time,* with the implicit understanding it refers to an expected value, and that it captures more than the expected value of the time between the purchase date and the expiry date stamped on a product. We assume that consumers have common expectations about the expiration time, a simplifying assumption because in reality different consumers can have different expectations about the time until a product becomes unusable.

The rate at which a product deteriorates determines the usable life of a product and the various costs of making, storing and transporting the packaged product. For many products, the rate of deterioration is determined by its intrinsic (often chemical) properties, by the storage conditions, and by the way in which it is handled at different points in the supply chain. Most foods and drugs deteriorate due to microbial growth, and because changes in chemical composition affect their quality, taste or efficacy. Products also deteriorate due to handling during transportation and in storage. Better packaging materials and methods can improve the usable life of a product, decrease the costs of inventory handling and product spoilage, and reduce product returns by consumers. The effect, in turn, is to allow for larger packages to be produced and sold by firms.

The present paper describes a model in which package size decisions are affected by the following four factors: (1) the expected life of the product; (2) the variable cost associated with packages of different sizes; (3) the rates at which consumers use the product; and (4) the reservation quantities of users. We allow differences in the consumption rates and reservation quantities by assuming heterogeneity distributions for each of these variables. For parametric analysis, we assume these variables to be uniformly distributed across customers. When considering cost, we first examine the case in which the variable cost for the firm increases linearly with package size. Later, we consider nonlinear costs. We do not consider transactions costs because as noted above, these factors are less important in BOP markets than they are in developed countries. We allow package and unit prices to affect purchase quantities and consumption, albeit for reasons that differ from those suggested in the literature. Folkes, Martin and Gupta (1993), Ailawadi and Neslin (1998) and Chandon and Wansink (2002) find that stockpiling increases consumption, the last of these reporting that stockpiling can increase product salience and trigger consumption incidence among high-convenience products.[3] In the present model, higher consumption occurs when a consumer finds it worth buying and consuming an additional unit of a product that he or she may not be able to fully use before it goes bad. The maximum possible increase in product use is thus equal to one package of the product.

Let $q(T)$ denote the quantity of the product a consumer uses in time T. A consumer who buys $i \geq 1$ packages expects to use at least $i-1$ packages, and some or all of the i-th package. That is,

$$q(T) = (i-1)s + f, \quad \text{where} \quad 0 < f \leq s.$$

Observe that $f = 0$ is not possible because it implies that the consumer needs to buy only $i-1$ packages; and $f = s$ implies that the consumer might possibly have forgone consumption of part of an $(i+1)$st package. Thus, if $0 < f < s$,

[3]On the other hand Ailawadi, Lehmann and Neslin (2001) report that deals and coupons, which affect purchase incidence and quantity, surprisingly have little impact on category usage by consumers. Bell, Iyer and Padmanabhan (2002) develop and test a model of price competition between firms in response to the stockpiling and subsequent consumption dynamics of consumers.

then the usage rate for a consumer is given by

$$\theta = \frac{q(T)}{T} = \frac{(i-1)s + f}{T}.$$

However, if $f = s$, then the usage rate for the consumer is at least $q(T)/T = is/T$; and it is smaller than $(i+1)s/T$, because the consumer might have forgone consumption of some part of an additional $(i+1)$st package of the product he or she did not buy. Thus,

$$(i-1)s < \theta T < (i+1)s.$$

Let p denote the price for a package of size s. Then the consumer obtains full use of each of $i-1$ packages, and consumes a quantity s/p for each dollar spent on buying these packages. However, if the consumer only uses a fraction of the i-th package, then the effective quantity per dollar he or she obtains is $r = f/p$, where $0 < f \leq s$. That is, $0 \leq r \leq s/p$, where the upper bound on r corresponds to a situation in which a consumer expects to use the entire product $q(T) = s$. The larger the value of r, the more attractive a product is for a consumer. The seller can increase the value of r by lowering the price p. Decreasing the package size s can have a direct effect on the value of r; it can also have an indirect effect if a firm reduces the price p for a smaller package. The quantity $1/r = p/q(T)$ is the price per unit of the part of the i-th package used by a consumer. Although it is in some ways conceptually more appealing to develop the model in terms of $1/r$, the effective quantity per dollar makes analysis simpler. For this reason, we will use it in the following discussion. For brevity, we refer to r as the *effective usage* in the following discussion, with the implicit understanding that it refers to the last package consumed by a user.

Let R denote the minimum value of the effective usage a consumer demands before buying the i-th package, and implicitly, each of the other $i-1$ packages, for which he or she obtains at least as much effective usage $(= s/p)$ than the i-th package. That is, the consumer buys the i packages only if

$$r = f/p > R.$$

We call R the *reservation quantity* for the consumer. The following Lemma gives the condition under which a consumer buys i packages.

Lemma 1. *Suppose a product has size s and is sold at price p. A consumer with a reservation quantity R, who expects the product to expire after T units of time, will buy i packages of the product only if: (i) the package size is $s \geq pR$; and (ii) the consumer's usage rate is*

$$\theta \geq \frac{pR + (i-1)s}{T}.$$

Proofs of Lemma 1 and all subsequent lemmas and theorems appear in the Appendix.

We use the lower bounds in Lemma 1 to determine how consumers with different usage rates and reservation quantities decide how many packages to buy. The conditions on s and θ in Lemma 1 can be combined to obtain the following lower bound on the usage rate for the consumer to buy i packages of the product:

$$\theta > \frac{(i-1)s + pR}{T} \geq \frac{ipR}{T}.$$

However, the bound $\theta > ipR/T$ is quite weak because $pR < f \leq s$ and a consumer who highly values the product (i.e., for whom R is large) might only consumer a small quantity f of the i-th package he or she buys.

3. Usage rates, reservation quantities and the demand for packages

The preceding section considers the purchase decision for a single consumer: how many, if any, packages to buy, given the usage rate θ and the reservation quantity R. We now consider how consumers differ in their usage rates and reservation quantities. We obtain the demand for a product as a function of the package size s and the price p. Later, we examine how the choice of an optimal package size, and its price, is affected by the consumer heterogeneity in usage rates and reservation quantities.

Let A_1 denote the minimum usage rate, and A_2 the maximum usage rate, across consumers, where $A_2 > A_1 \geq 0$. Let $f(\theta)$, $\theta \in [A_1, A_2]$, denote the density function, and $F(\theta)$ the cumulative density function (distribution function), characterizing the heterogeneity in consumer usage rates. Let n denote the maximum possible number of packages purchased, for use over time T, by a consumer with the largest possible usage rate, A_2. Then $n = \lceil A_2T/s \rceil$. For example, if $s > A_2T/2$, then $n < \lceil 2 \rceil = 2$, or equivalently $n = 1$. The following lemma generalizes these considerations.

Lemma 2. *If the size s of a package is $A_2T/(n+1) < s \leq A_2T/n$, then no consumer purchases more than n packages of the product.*

Let $D_i(s,p)$ denote the expected number of customers who buy at least i packages of a product of size s at price p, for all $i = 1, \ldots, n$. By definition,

$$D_1(s,p) > D_2(s,p) > \cdots > D_n(s,p),$$

and the number of consumers who buy exactly i packages is given by

$$d_i(s,p) = D_i(s,p) - D_{i+1}(s,p), \quad \text{for all} \quad i = 1, \ldots, n-1, \text{ and}$$

$$d_i(s,p) = D_i(s,p), \quad \text{for} \quad i = n.$$

The following lemma gives an expression for $D(s,p)$, the total demand for packages.

Lemma 3.

$$D(s,p) = \sum_{i=1}^{n} D_i(s,p).$$

Let $D_i(s,p|R)$ and $d_i(s,p|R)$ denote the number of consumers with reservation quantity R who buy at least i packages, and exactly i packages, respectively, when each package has size s and price p. From Lemma 1, a consumer with a given value of the reservation quantity R purchases at least i packages if his/her usage rate θ satisfies the condition

$$\theta \geq \frac{pR + (i-1)s}{T}.$$

It follows that $D_i(s,p|R)$, the number of consumers who buy at least i packages, each of size s, is

$$D_i(s,p|R) = \int_{[pR+s(i-1)]/T}^{A_2} f(\theta)d\theta.$$

The number of consumers who buy exactly i packages is

$$d_i(s,p|R) = D_i(s,p|R) - D_{i+1}(s,p|R) = \int_{[pR+s(i-1)]/T}^{[pR+si]/T} f(\theta)d\theta, \text{ for all } i = 1,\ldots,n.$$

Thus, following Lemma 3, the total demand, given R, is

$$D(s,p|R) = \sum_{i=1}^{n} D_i(s,p|R) = \sum_{i=1}^{n} \int_{[pR+s(i-1)]/T}^{A_2} f(\theta)d\theta.$$

Observe that the total demand, as well as the demand for any number i of packages, (i) decreases with the price of the package, and with the package size, (ii) increases as the time T that it remains usable increases, and (iii) is larger among consumers with a lower reservation quantity R.

We now consider the heterogeneity in the value of R, the minimum quantity per dollar a consumer seeks before buying his/her last unit of the product. Let B_1 denote the minimum value of the reservation quantity R, and B_2 the maximum value of the reservation quantity across consumers, $B_2 > B_1 > 0$. Let $g(R)$, $R \in [B_1, B_2]$, denote the density function characterizing heterogeneity in consumer reservation quantities. Let $G(R)$ denote the cumulative density function for R. We assume for simplicity that the distributions $f(\theta)$ and $g(R)$ are independent.

Theorem 1. *The demand for exactly i packages of size s, when each package is sold at price p, is given by*

$$d_i(s,p) = \int_{B_1}^{\min\{B_2,\ s/p\}} \left\{ F\left(\frac{pR+si}{T}\right) - F\left(\frac{pR+s(i-1)}{T}\right)\right\} g(R)dR, \text{ for all } i = 1,\ldots,n-1;$$

and

$$d_i(s,p) = D_i(s,p) = \int_{B_1}^{\min\{B_2,\ s/p\}} \left[\int_{[pR+s(n-1)]/T}^{A_2} f(\theta)d\theta\right] g(R)dR, \quad \text{if} \quad i = n.$$

The total demand for packages is

$$D(s,p) = \sum_{i=1}^{n} \left[\int_{B_1}^{\min\{B_2,\ s/p\}} \left\{ 1 - F\left(\frac{pR + s(i-1)}{T} \right) \right\} g(R) dR \right].$$

Observe that if $s/p \leq B_2$, then only those consumers for whom $R \in [B_1, s/p]$ will buy the product. If, on the other hand, $B_2 \leq s/p$, then all consumers, over the entire range $R \in [B_1, B_2]$, will buy the product. Thus, $s/p = B_2$ characterizes the maximum demand for the product: if $s/p < B_2$, the demand is smaller, but if $s/p > B_2$, it cannot be any larger, because by definition there are no consumers who have a higher valuation than B_2. We therefore consider only the case where $s/p < B_2$.

Let $Q(s,p)$ denote the total quantity (i.e., units) purchased by buyers. Then

$$Q(s,p) = s \cdot D(s,p),$$

where $D(s,p)$ is given by Theorem 1.

4. Optimal package size and price

Let $c(s)$ denote the variable cost of producing a product of size s. This cost includes the cost of producing the product, the cost of its packaging, and any unit inventory holding, breakage/damage and insurance costs.

Lemma 4. *The maximum package size offered by a seller is bounded by the condition*

$$s \leq \min\left\{ pB_2, \frac{A_2 T}{n} \right\}.$$

The above Lemma imposes a constraint on the optimal package size and price, which is obtained by solving the following profit-maximization problem, denoted P:

$$\max_{s,p} \pi = (p-c(s))D(s,p) = (p-c(s))\sum_{i=1}^{n}\left[\int_{B_1}^{s/p}\left\{1-F\left(\frac{pR+s(i-1)}{T}\right)\right\}g(R)dR\right].$$

$$\text{subject to : } s \leq \min\left\{pB_2, \frac{A_2T}{n}\right\}.$$

Lemma 5. *If $\alpha > 1/B_1$, then the seller should not make the product.*

Lemma 5 says that the firm will not make the product if the unit cost is higher than the price per unit that the least price sensitive consumer is willing to pay. To further characterize the solution to the seller's problem, we require the specification of cost function $c(s)$ and the heterogeneity distributions $f(\theta)$ and $g(R)$. We turn to this analysis next. For simplicity, we assume that $f(\theta)$ and $g(R)$ are uniform distributions over $[A_1, A_2]$ and $[B_1, B_2]$, respectively. That is,

$$f(\theta) = \frac{1}{A_2 - A_1,} \quad F(\theta) = \frac{\theta - A_1}{A_2 - A_1}, \quad \text{for all } \theta \in [A_1, A_2];$$

and

$$g(R) = \frac{1}{B_2 - B_1,} \quad G(R) = \frac{R - B_1}{B_2 - B_1}, \quad \text{for all } R \in [B_1, B_2].$$

We assume in this section that the variable cost increases linearly with package size: $c(s) = \alpha s, \alpha > 0$. We show that in this case it is optimal for the firm to make single-serve packages. In section 6, we consider nonlinear costs, observe that it remains optimal for the firm to make single-serve packages if $c(s)$ is a convex function of package size, s, and examine conditions under which larger package sizes are optimal for a concave cost function.

We begin by substituting $\theta = \big(pR + s(i-1)\big)/T$ and

$$F(\theta) = F\left(\frac{pR + s(i-1)}{T}\right) = \frac{pR + s(i-1) - A_1 T}{(A_2 - A_1)T}$$

into the expressions for $D(s,p)$ and $d(s,p)$ in Theorem 1. This substitution gives the following expression for the number of buyers who purchase exactly i packages:

$$d_i(s,p) = \int_{B_1}^{s/p} \left\{ \frac{pR + si - A_1 T}{(A_2 - A_1)T} - \frac{pR + s(i-1) - A_1 T}{(A_2 - A_1)T} \right\} \frac{1}{B_2 - B_1} dR.$$

The total demand for packages is thus given by

$$D(s,p) = \sum_{i=1}^{n} \left[\int_{B_1}^{s/p} \left\{ 1 - \frac{pR + s(i-1) - A_1 T}{(A_2 - A_1)T} \right\} \frac{1}{B_2 - B_1} dR \right].$$

If $B_1 \leq s/p \leq B_2$, then only those consumers for whom $R \in [B_1, s/p]$ consider the product worth buying. Thus, the number of buyers $d_i(s,p)$ who purchase exactly i packages each is given by

$$d_i(s,p) = \int_{B_1}^{s/p} \frac{s}{(A_2 - A_1)T} \frac{1}{B_2 - B_1} dR = \left(\frac{\frac{s}{p} - B_1}{B_2 - B_1} \right) \left(\frac{s}{(A_2 - A_1)T} \right), \; i = 1, \ldots, n,$$

and the total demand $D(s,p)$ for packages of size s, each sold at a price p, is

$$\begin{aligned} D(s,p) \; &= \textstyle\sum_{i=1}^{n} \left[\int_{B_1}^{s/p} \left\{ 1 - \frac{pR + s(i-1) - A_1 T}{(A_2 - A_1)T} \right\} \frac{1}{B_2 - B_1} dR \right] \\ &= \left(\frac{\frac{s}{p} - B_1}{B_2 - B_1} \right) \left(\frac{1}{1 - \frac{A_1}{A_2}} - \frac{pB_1 + sn}{2(A_2 - A_1)T} \right) n. \end{aligned}$$

The total quantity purchased by the consumers is $Q = sD(s,p)$. Thus, given s and p, the demand D for packages and Q for quantity, increases with T, the usable life of a product; A_1, the minimum usage rate across consumers; and A_1/A_2, the ratio of the lowest to the highest usage rates. On the other hand, both D and Q decrease with increases in $A_2 - A_1$, the difference between the highest and lowest usage rates; B_1, the lowest reservation quantity across consumers; and $B_2 - B_1$, the difference between the largest and smallest reservation quantity across consumers.

We now use the above expression for demand to prove the following theorem concerning the optimal package size and price for the seller.

Theorem 2. *If $\alpha \leq 1/B_1$, the seller should, for any given value of $n \geq 1$, make packages of size*

$$s^* = \frac{1-k}{n} A_2 T,$$

and for each package should charge a price of

$$p^* = \frac{k}{B_1} A_2 T,$$

where

$$k = \frac{1}{\sqrt{(n+1)(\frac{n}{B_1 \alpha} + 1)}}.$$

Note that $0 < k < 1/(n+1)$, where the upper bound of $1/(n+1)$ corresponds to $\alpha = 1/B_1$. As $B_1\alpha$ becomes smaller, k becomes larger, and the seller both charges a higher price and makes a smaller package, implying sharply increasing unit prices for products with lower unit cost and/or lower reservation quantity for the least price sensitive consumer. As k approaches its upper bound, s^* approaches the value $A_2T/(n+1)$; and as k approaches zero, s^* approaches A_2T/n. Thus, the value of k determines the optimal package size in the relevant range $s \in (A_2T/(n+1), A_2T/n)$. The optimal price for a package increases with the value of k, and lies between $0 < p^* < A_2T/B_1(n+1)$. Both the optimal package size and the optimal price increase with the usable life T of the product, and with the maximum usage rate A_2 across consumers; and both decrease with increasing values of n and the minimum reservation quantity B_1. As T is finite for any product, the ratio T/n decreases with n for all products; i.e., as n increases, the package size become smaller for all products, although products with longer shelf life should have larger package sizes. Observe also that as the unit cost α increases, the optimal package size s^* becomes smaller and the optimal price p^* becomes larger. Thus, the unit price p^*/s^* increases with unit cost. Note that the optimal package size and the price are not affected by the minimum usage rate A_1 or the maximum reservation quantity B_2, or by the range $A_2 - A_1$ or $B_2 - B_1$ for the heterogeneity in usage rates reservation quantities.

Theorem 2 implies

$$s^* \geq \frac{A_2T}{n+1}, \quad p^* \leq \frac{A_2T}{B_1(n+1)}.$$

Recall from the formulation of the seller's problem that $s \leq A_2T/n$ is an upper bound on s. Thus,

$$\frac{A_2T}{n+1} \leq s^* \leq \frac{A_2T}{n}.$$

The gap between the upper- and lower bounds on s^* becomes smaller as the value of n increases. For large values of n, $n \approx n+1$, and so $s^* \approx A_2T/n$. That is, the optimal package size is chosen so that ns^* equals exactly the quantity consumed over time T by a customer with the highest possible consumption rate, A_2. Thus, as n becomes larger, the optimal package size becomes independent of the unit cost α, the minimum usage rate A_1, and the reservation quantities B_1 and B_2 (or equivalently, the dollars per consumed unit users are willing to pay). As $s \leq pB_2$, we have

$$\frac{s^*}{B_2} \leq p^* \leq \frac{A_2T}{B_1(n+1)}.$$

As $s^* \geq A_2T/(n+1)$, we have

$$\frac{z}{B_2} \leq p^* \leq \frac{z}{B_1},$$

where $z = A_2T/(n+1)$ is a decreasing function of n. Thus, as the heterogeneity in reservation quantity decreases, the above range for the optimal price becomes smaller. If $B_2 = B_1 = B$, the optimal price converges to $p^* = A_2T/B(n+1)$, and is independent of the unit cost, α, and the minimum usage rate, A_1.

We now examine the relation between the value of n and the optimal profit for the firm. Substituting the expressions for s^*, p^* and D^* into the profit equation for the seller gives

$$\pi^* = \pi(s^*, p^*) = \frac{A_2^2T\left(\sqrt{n+B_1\alpha} - \sqrt{B_1\alpha(n+1)}\right)^2}{2(A_2-A_1)(B_2-B_1)n}.$$

which can be written as

$$\pi^* = \frac{A_2^2T}{2(A_2-A_1)(B_2-B_1)}\left(1+\left(1+\frac{2}{n}\right)\alpha B_1 - 2\sqrt{\alpha B_1\left(1+\frac{B_1\alpha}{n}\right)\left(1+\frac{1}{n}\right)}\right).$$

The optimal profit increases with n, with the usable life T of the product, and with the maximum usage rate A_2 across consumers; it decreases with the unit cost α, with the minimum reservation quantity B_1, and with the ranges $A_2 - A_1$ and

$B_2 - B_1$ over which the heterogeneity in usage rates and reservation quantities is described. As the value of n increases, the right hand side of the above expression for π^* approaches from below the value

$$\pi^*_{\max} = \lim_{n\to\infty} \pi^*_n = \frac{A_2^2(1-\sqrt{B_1\alpha})^2 T}{2(A_2-A_1)(B_2-B_1)}.$$

Let π^*_1 denote the profit π^* when $n = 1$. Figure 1 plots the "profit ratio" $r = \pi^*/\pi^*_1$ as a function of n and $B_1\alpha$. Observe that the smaller the value of $B_1\alpha$, the smaller the value of $r_{\max} = \pi^*_{\max}/\pi^*_1$, the ratio of the profits when $n = \infty$ and $n = 1$. Put another way, a seller has less reason to make a smaller package size if the unit cost α is small, and if even the least price-sensitive buyers are still quite price sensitive (i.e., have a lower value of B_1). We also observe that, for each value of $B_1\alpha$, the value of r initially increases rapidly with n, but then levels off quickly to its asymptotic value. For example, when $\alpha B_1 = 0.01$, the profit ratio r is within 1% of the maximum for $n = 10$; and when $\alpha B_1 = 0.99$, the profit ratio is about 2% of the maximum for $n = 30$. Thus, in practice, $n = 30$ is large enough for a seller to obtain about the maximum possible profit; and in most cases, $n = 10$ is likely to be large enough. The optimal package size s^* for a suitably large value of n will correspond to a single-serve package in most instances (recall that n is defined as the number of units of a package purchased over for consumption over time T by a consumer with the highest possible usage rate, A_2.

To summarize, if package cost increases linearly with package size, then the firm's profits increase with n and the firm should make single-serve packages. This conclusion also follows if the package cost is a convex function of package size; i.e., when the production costs increases in an increasing rate with product size. However, this is not the case when the cost function is concave. We examine this case later in section 6.

5. Characteristics of the optimal solution

Unit price. Theorem 2 implies the following expression for the cost per unit of the product:

$$\frac{p^*}{s^*} = \frac{1}{B_1}\left(\frac{A_2 T}{s^*} - n\right) = \frac{1}{B_1\left(\sqrt{(n+1)(\frac{n}{B_1\alpha}+1)} - 1\right)} \leq \frac{1}{B_1},$$

where the last inequality follows from the condition $B_1\alpha \leq 1$. The above expression implies that the smaller the optimal package size s^*, and the larger the value of n, the higher the price per unit that should be charged by a profit-maximizing firm. Figure 2 shows the relationship between p^*/s^* and s^*. Observe that the rate at which the unit price decreases with s^* becomes smaller as s^* becomes larger. This conclusion is consistent with the empirical observation that in BOP markets, where consumers pay higher unit prices for single serve packages. However, it is not a poverty premium that is charged to exploit poor consumers because they can only afford to buy single-serve packages — we obtain this result in the present model without imposing any constraints on income streams. This of course does not mean that such a premium is not charged; it only means that the prevalence of higher unit prices for smaller packages is not by itself an indication that firms are exploiting the poor. We note that the lower unit prices in the present instance cannot in be viewed as mirroring quantity discounts for larger package sizes (Clements 2006), because there is only one package size, and thus no reason for the seller to offer a discount in our model. Instead, the reason is that additional unit sales from reducing the unit price is large enough to compensate for the reduction in unit margin for all units sold as well as the loss in sales due fewer consumers buying a larger package size.

Number of packages and units. The demand for the number of packages associated with the optimal package size s^* and price p^* is given by the expression

$$D^* = D(s^*, p^*) = \frac{A_2 B_1}{2(A_2 - A_1)(B_2 - B_1)} \left(\sqrt{(n+1)\Big(\frac{n}{B_1\alpha} + 1\Big)} - (n+1) \right).$$

As $B_1\alpha < 1$, the term under the square root is always greater than $n+1$. And as $B_1\alpha$ increases, so does $n/B_1\alpha$ and D^*. Note that D^* also increases with n, because the package size s^* becomes smaller as n becomes larger. The total quantity $Q^* = s_n^* D^*$ purchased by consumers is

$$Q^* = \frac{A_2^2 B_1 T}{2(A_2 - A_1)(B_2 - B_1)} \left(1 - \frac{1}{\sqrt{(n+1)(\frac{n}{B_1\alpha} + 1)}} \right) \left(\sqrt{\Big(1 + \frac{1}{n}\Big)\Big(\frac{1}{B_1\alpha} + \frac{1}{n}\Big)} - \Big(1 + \frac{1}{n}\Big) \right).$$

The value of Q^* increases with n, and approaches the limiting value

$$Q^*_{\max} = \lim_{n\to\infty} Q^* = \frac{A_2^2 B_1 T}{2(A_2 - A_1)(B_2 - B_1)} \left(\frac{1}{\sqrt{B_1\alpha}} - 1 \right).$$

Note that $Q^*_{\max} > 0$ because $B_1\alpha < 1$, and that a larger value of $B_1\alpha$ implies a smaller value of $Q^*_{\max}$. A change in the value of $B_1\alpha$ has a disproportionately large effect on the value of $Q^*_{\max}$. For example, if the value of $B_1\alpha$ decreases from x to x^2, then $Q^*_{\max}$ increases by a factor of

$$\frac{\frac{1}{\sqrt{x^2}} - 1}{\frac{1}{\sqrt{x}} - 1} = \frac{1}{\sqrt{x}} + 1.$$

Thus if a seller can decrease the unit cost so that the value of $B_1\alpha$ drops from 0.5 to 0.25, then the quantity purchased by consumers for arbitrarily large n increases by a factor of 3.

Let Q_1^* denote the quantity purchased when $n = 1$. Figure 3 shows how the ratio Q^*/Q_1^* changes with $n \geq 1$ for different values of $0 < B_1\alpha < 1$. We see that if $B_1\alpha < 0.1$, the value of Q^*/Q_1^* decreases with n; and if $B_1\alpha > 0.1$ the value of Q^*/Q_1^* increases with n. That is, if the unit cost is less than a tenth of the reservation quantity B_1 of the least price sensitive customer, then the total quantity Q^* purchased by consumers decreases as n increases; otherwise, it increases with n. The reason is that as B_1 approaches zero, s/p also approaches zero, but at a faster rate than B_1. This reduces the demand to zero. Thus, if the unit cost α and the minimum reservation quantity B_1 is sufficiently low, consumers will purchase a smaller quantity Q^* and, as discussed above, pay a higher unit price, for smaller package sizes. Otherwise, consumers will both purchase a larger amount in the aggregate, and pay a higher unit price for smaller package sizes. This is an important reason for firms to offer small package sizes in BOP markets.

Product waste and unfulfilled demand. From the consumer's perspective, the product size problem shares certain aspects of inventory problems. For example, some users may not be able to buy as much as they can consume, and other may buy in excess and waste a part of the product because it is no longer usable. This is a type of inefficiency introduced by quantity bundling. Its effect is that consumers can possibly pay more, and sellers earn less profit, than they might in the absence of the inefficiency. We use the term "consumer waste" to refer to the total quantity bought but not used by consumers; and we use the term "unfulfilled demand" to refer to the lost sales to consumers who buy less than they can consume. Figure 4 graphically illustrates the difference between consumer waste and unfulfilled demand. Consider a consumer with consumption rate θ and reservation quantity R. If $\theta T \leq is$, then the consumer will buy i packages only if $p^*R + (i-1)s^* \leq \theta T \leq is$, or equivalently, if their usage rate is bounded by

$$\frac{p^*R + (i-1)s^*}{T} \leq \theta \leq \frac{is}{T}.$$

Such a consumer will waste a quantity $w = is^* - \theta T$. As only those customers for whom $s^*/p^* > R$ buy i units of the product, the total quantity wasted across consumers is given by the expression

$$W = \sum_{i=1}^{n} \int_{B_1}^{s^*/p^*} \int_{\frac{p^*R+(i-1)s^*}{T}}^{\frac{is^*}{T}} \big(is^* - \theta T\big) f(\theta) d\theta g(R) dR.$$

For uniform distributions over $\theta \in [A_1, A_2]$ and $R \in [B_1, B_2]$, the above expression simplifies to

$$W = \frac{(s^* - B_1 p^*)^3 n}{6(A_2 - A_1)(B_2 - B_1)p^* T}.$$

Substituting for p^* and s^* from Theorem 2 gives the following expression for consumer waste:

$$W = \frac{A_2^2 T \left(\sqrt{B_1\alpha(n+1)(n+B_1\alpha)} - B_1\alpha(n+1)\right)^3}{6(A_2 - A_1)(B_2 - B_1)B_1\alpha^2 n^2 (n+1)(n+B_1\alpha)}.$$

The value of W goes to zero as n becomes arbitrarily large, because each customer can then purchase exactly the amount he or she can consume in time T.

Next, if $\theta T > is^*$, then the unfulfilled demand for the customer is $c = \theta T - is^*$, where

$$\frac{is^*}{T} \leq \theta \leq \frac{p^*R + is^*}{T}.$$

Integrating over values of R and the summing over the units bought by customers gives the total unfulfilled demand across customers:

$$C = \sum_{i=1}^{n} \int_{B_1}^{s^*/p^*} \int_{\frac{is^*}{T}}^{\frac{p^*R+is^*}{T}} \left(\theta T - is^*\right) f(\theta) d\theta g(R) dR.$$

For uniform distributions $\theta \sim U[A_1, A_2]$ and $R \sim U[B_1, B_2]$, the above expression simplifies to

$$C = \left[\frac{(s^*)^3 - (B_1 p^*)^3}{6(A_2 - A_1)(B_2 - B_1)p^* T}\right] n.$$

Substituting the values of s^* and p^* from Theorem 2 into the above expression for C gives

$$C = \frac{A_2^2 T \left[\left(\sqrt{B_1\alpha(n+1)(n+B_1\alpha)} - B_1\alpha\right)^3 - (B_1\alpha n)^3\right]}{6n^2(A_2 - A_1)(B_2 - B_1)B_1\alpha^2(n+1)(n+B_1\alpha)}.$$

The value of C goes to zero as n becomes arbitrarily large. Thus, single-serve packages reduce consumer waste — each customer can purchase exactly the amount he or she can consume. Also,

$$\frac{W}{C} = \frac{(s^* - B_1 p^*)^3}{(s^*)^3 - (B_1 p^*)^3} < 1.$$

Thus, the amount wasted by customers is always less than the unfulfilled demand across customers. This is one reason the firm reduces package sizes: it stands to gain more in additional sales (from non-buyers, and from those consuming less than they can) than it stands to lose because some consumers buy more than they can use. The other reason for the firm selling smaller package sizes is that it can also charge higher unit prices, because consumers can more exactly match their purchases with their needs, and thus are willing to pay more per package when it has a smaller size.

6. Nonlinear cost functions

The above analysis considers costs that increase linearly with package size. The minimum package size result we obtain under this assumption is readily seen to extend for convex cost functions, where package costs increase at an increasing rate with package size. The reason is straightforward — if it is optimal to minimize package sizes for linear costs, then it must also be optimal to do so for convex costs, because reducing the package size leads to additional cost economies (and thus increasingly higher margins), without having any effect on the demand for the product.

The situation differs for concave cost functions, which allow economies of size, so that larger package sizes have a lower unit cost; i.e., $c(s)/s$ is smaller for a package with a larger size, s. For example, the volume of a package is a cubic function of its linear dimensions, whereas the surface area of a package is a quadratic function of its linear dimensions. Thus, the cost of packaging materials increases at a decreasing rate with the volume of a package. Larger packages may also require lesser handling and transportation costs, although it is difficult to generalize about this in all situations.

It is possible to obtain in specific instances explicit solutions for cost function reflecting cost economies in package sizes. We have, for example, obtained the results for a piecewise linear cost function that allows unit cost to become lower beyond a threshold package size. However, the solutions can become far too complicated to provide any meaningful insight.[4] We therefore consider the general conditions under which a concave cost function can create an incentive for a seller to make larger package sizes than under a linear cost function. In particular, we

[4]A Mathematica notebook with the solution for the piecewise linear cost function is available from the authors; it is far too complex to be reported in the paper.

obtain sufficient conditions under which the profit obtained by selling packages of size $s^* \in [A_2T/(n+1), A_2T/n]$ exceeds the profit from selling an arbitrarily small product. The actual value of s^* of course depends on the actual cost function, and can be numerically calculated for a given problem and cost specification.

Consider a specific, finite value of $n \geq 1$. Figure 5 plots the package size versus the package cost. The optimal package size for a linear cost function $c = \alpha s$, is indicated in the figure by the point labeled s^* on the horizontal axis, where $A_2T/(n+1) \leq s^* \leq A_2T/n$. Let $\pi^* \equiv \pi^*(s^*, p^*)$ denote the associated profit. Consider a concave cost function, $\tilde{c} \equiv \tilde{c}(s)$, which lies everywhere below the cost function $c = \alpha s$; i.e., $0 < d\tilde{c}(s)/ds < \alpha$, $d^2\tilde{c}(s)/ds^2 < 0$, for all $s > 0$. We assume that

$$\lim_{s \to 0} \frac{d\tilde{c}(s)}{ds} = \alpha,$$

so that $c = \alpha s$ corresponds to a linear cost function that minimally dominates $\tilde{c}$. Let s_A and p_A denote the optimal package size and price for the cost function $\tilde{c}(s)$, and let $\pi_A \equiv \pi_A(s_A, p_A)$ denote the associated profit. Consider the linear cost function $\hat{c} = \hat{\alpha} s$, which goes through the origin and intersects the concave cost function $\tilde{c}(s)$ at the point $s = A_2T/(n+1)$. Let

$$\begin{aligned}\alpha_{\max} = \frac{1}{B_1 n}\Bigg[& 2 + 2B_1\alpha - 4\sqrt{B_1\alpha} + n(2 + B_1\alpha - 2\sqrt{B_1\alpha}) \\ & - 2\sqrt{(n+1)\Big\{(1 - 4\sqrt{B_1\alpha}) + B_1\alpha(6 + B_1\alpha - 4\sqrt{B_1\alpha}) + n(1 + B_1\alpha - 2\sqrt{B_1\alpha})\Big\}}\Bigg]\end{aligned}$$

The following theorem gives a sufficient condition under which a seller's profit increases as n decreases and s^* increases.

Theorem 3. *Let $n = n^*$ denote a value of n for which a linear cost function $\hat{c} = \hat{\alpha}s$ satisfies $\hat{\alpha} < \alpha_{max}$. Then the seller earns a higher profit by making a package of size $s \in (A_2T/(n+1), A_2T/n)$ than by making a single-serve package.*

Each value of n specifies a range $s \in (A_2T/(n+1), A_2T/n)$ for the package size. As n decreases, the package size increases. Theorem 3 gives a sufficient condition for which the seller earns a higher profit by making bigger packages than single-serve packages. Figure 6 shows how the largest possible value of $\hat{\alpha}$ for which the above condition is satisfied changes with n and α. Observe that as n^* increases, $\hat{\alpha}$ approaches α, which implies that a small reduction in costs $(\alpha_{\max}/\alpha)$ is needed to satisfy the condition for larger than single-serve package sizes. Moreover, when the cost is high (e.g. 0.9) the firm needs to incur in a small cost reduction (less than 5%) in order to satisfy that condition. In contrast, when the cost is small (e.g. 0.1) a higher proportionate cost reduction is needed to satisfy the condition of Theorem 3.

Recall that a consumer purchases a package if $s/p > R$, or equivalently if $p < s/R$. Consider in addition the constraint $p \leq p_{\max}$, reflecting the cash flow constraint on a buyer in a BOP market who cannot afford to pay more than a price $p_{\max}$. Then the condition for such a buyer to purchase a package is $p < \min\{p_{\max},\ s/R\}$. If the constraint $p \leq p_{\max}$ is binding, then $s/R \leq p_{\max}$, and so $s \leq Rp_{\max}$ for a consumer with reservation quantity R to buy the product. This creates an incentive for the firm to make smaller package sizes when there are many buyers for whom R and/or $p_{\max}$ is small, even if there are cost economies for the seller from making larger package sizes.

7. Conclusion

A product can be characterized by such attributes as its price, quality, brand name and package size (Guadagni and Little 1983). Among these, package size has perhaps received the least attention in the analytical marketing literature. Prahalad (2005) observes that single-serve packages are creating a revolution in BOP markets because these enable the poor to buy products that they could not otherwise afford. We examine in this paper how in these markets package sizes can affect unit prices, consumption and waste by users, and sales and profits earned by a seller. BOP markets are characterized by unsteady income streams for some buyers and small transactions costs for buyers and sellers. Our analysis suggests that the latter factor alone can be sufficient for a firm to offer single-serve packages, unless there are significant economies of size for the seller. Consistent with Prahalad's observation, single-serve packages can increase the number of consumers who buy a product, increase total consumption, decrease product waste, and lead to higher profits for a seller, in part because of higher unit prices. However, these higher unit prices do not have to be a poverty penalty to buyers who do not have the income stream to buy and store larger packages. Some consumers purchase less, others stop buying, when the unit price increases. Other consumers enter the market, and still others buy more because they waste less. The latter increase is sufficiently large, even at higher unit prices, for the seller to have an incentive to make single-serve packages. Customer awareness of unit prices plays no role in the present analysis.

Nonlinear pricing is one method by which a seller can discriminate among buyers, charge them different prices, and earn higher profit (see, e.g., Wilson 1992). There is no such opportunity in our model, because the seller only makes a single product and sells it at a common price. Unlike quantity discounts, which enable price discrimination if high-usage consumers have higher price elastiticities, our model makes no comparable assumption; instead, the reservation quantity (R) is independent of the consumption rate. Still, a single-serve package allows a form of discrimination among buyers, each of whom can match purchases by selecting the appropriate number of packages for consumption. This self selection helps a seller to discriminate among users with varying levels of product use, and enables it to extract higher profits.

The parametric results in this paper are obtained assuming that the usage rates, and the reservation quantities, have uniform distributions across consumers. It is possible that there are disproportionately more consumers in BOP markets who have lower usage rates and higher reservation quantities (i.e., who seek to consume more units per dollar if they buy a product). We expect that these conditions are likely to make the rationale for smaller package sizes even more compelling. The key requirement for the results is that transactions costs be small, a condition that is more likely to be met in BOP markets where retail stores and located close to consumers and open for long hours, phone ordering is common, and labor is cheap, allowing free delivery to homes.

Consumer cash-flow constraints can become a significant incentive for package-size reductions in BOP markets, even when the seller's marginal cost decreases with package size. An alternative is for the seller to reduce package prices, but this is feasible only up to a point and can eventually lead to serious erosion in profit. Still, if only a small fraction of consumers have cash flow constraints, and if these constraints are not too severe, a seller might prefer a small price cut to stimulate demand. But if a large fraction of a market faces these constraints, then the seller will tend to favor smaller package sizes to draw more buyers into the market, even if there are cost economies from making larger package sizes.

The present analysis considers two important features of BOP markets: low transactions costs and cash-flow constraints on consumer purchases. Future research might benefit from the consideration of competition among manufacturers and retailers, and the availability of multiple package sizes to buyers. Competition is likely to create both downward pressure on prices and greater differentiation in package sizes, with firms choosing to target different segments of the market. Retail competition can lead to lower prices for all package sizes, although it is not clear if these will be disproportionately large for smaller package sizes, thereby making larger package sizes more profitable for a seller. As one might expect, multiple package sizes are likely to counter unit price increases for smaller packages (Granger and Billson 1972). Examining these issues, in both theoretical and empirical contexts, can be useful for future research.

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Appendix: Proofs of Lemmas and Theorems

Lemma 1 *Suppose a product has size s and is sold at price p. A consumer with a reservation quantity R, who expects the product to expire after time T , will buy i packages of the product only if: (i) the package size is $s \geq pR$; and (ii) the* *consumer's usage rate is*

$$\theta \geq \frac{pR + (i-1)s}{T}.$$

Proof. As the consumer uses $f \leq s$ units of the i-th package, we have

$$s/p \geq f/p > R,$$

which implies $s > pR$. Next, a consumer who buys i packages must consumer $i-1$ packages, each of size s, and $f \leq s$ units from the i-th package. Thus, the usage rate for the consumer is

$$\theta = \frac{f + (i-1)s}{T}.$$

Now $r = f/p > R$ implies $f > pR$, and so

$$\theta = \frac{f + (i-1)s}{T} > \frac{pR + (i-1)s}{T}.$$

□

Lemma 2 *If the size s of a package is $A_2T/(n+1) < s \leq A_2T/n$, then no consumer purchases more than n packages of the product.*

Proof. As $\theta \in [A_1, A_2]$, the maximum quantity a consumer can use in time T before the product is no longer usable is A_2T. Thus, the maximum number of packages of size s that a consumer can use is $n = \lceil A_2T/s \rceil$. Now

$$\frac{A_2T}{n+1} < s \leq \frac{A_2T}{n},$$

and so the maximum number i of packages a consumer can use is

$$\lceil n \rceil \leq i < \lceil n+1 \rceil,$$

which is equivalent to $i = n$. $\square$

Lemma 3

$$D(s,p) = \sum_{i=1}^{n} D_i(s,p).$$

Proof. The total number of packages purchased by consumers is thus given by

$$D(s,p) = 1 \cdot d_1(s,p) + 2 \cdot d_2(s,p) + \cdots + n \cdot d_n(s,p),$$

which can be written as

$$D(s,p) = 1 \cdot [D_1(s,p) - D_2(s,p)] + 2 \cdot [D_2(s,p) - D_3(s,p)] + \ldots$$
$$+ (n-1) \cdot [D_{n-1}(s,p) - D_n(s,p)] + n \cdot D_n(s,p)$$

or equivalently

$$D(s,p) = D_1(s,p) + \cdots + D_n(s,p).$$

$\square$

Theorem 1 *The demand for exactly i packages of size s, when each package is sold at price p, is given by*

$$d_i(s,p) = \int_{B_1}^{\min\{B_2,\ s/p\}} \left\{ F\left(\frac{pR+si}{T}\right) - F\left(\frac{pR+s(i-1)}{T}\right) \right\} g(R)dR, \text{ for all } i = 1, \ldots, n-1;$$

$$\text{and} \quad d_i(s,p) = D_i(s,p) = \int_{B_1}^{\min\{B_2,\ s/p\}} \left[\int_{[pR+s(n-1)]/T}^{A_2} f(\theta)d\theta \right] g(R)dR, \quad i = n.$$

The total demand for packages is

$$D(s,p) = \sum_{i=1}^{n} \left[\int_{B_1}^{\min\{B_2,\ s/p\}} \left\{ 1 - F\left(\frac{pR + s(i-1)}{T} \right) \right\} g(R)dR \right].$$

Proof. Suppose $s/p < B_2$. Then there are consumers for whom $r \in \{s/p, B_2\}$. None of these consumers will buy the product, because they demand at least r units per dollar, whereas the maximum they can obtain is s/p units per dollar. It follows that only those consumers buy the product for whom $B_1 \leq R \leq s/p$. Thus,

$$D_i(s,p) = \int_{B_1}^{s/p} \left[\int_{[pR+s(i-1)]/T}^{A_2} f(\theta)d\theta \right] g(R)dR, \quad i = 1, \ldots, n-1.$$

Substituting

$$\int_{[pR+s(i-1)]/T}^{A_2} f(\theta)d\theta = 1 - F\left(\frac{pR + s(i-1)}{T} \right)$$

gives

$$D_i(s,p) = \int_{B_1}^{s/p} \left[1 - F\left(\frac{pR + s(i-1)}{T} \right) \right] g(R)dR.$$

It follows that, for all $i = 1, \ldots, n-1$,

$$d_i(s,p) = D_i(s,p) - D_{i+1}(s,p) = \int_{B_1}^{s/p} \left\{ F\left(\frac{pR + si}{T} \right) - F\left(\frac{pR + s(i-1)}{T} \right) \right\} g(R)dR,$$

and

$$d_i(s,p) = D_i(s,p) = \int_{B_1}^{s/p} \left[\int_{[pR+s(n-1)]/T}^{A_2} f(\theta)d\theta \right] g(R)dR, \quad \text{for} \quad i = n.$$

Finally, Lemma 3 implies

$$D(s,p) = \sum_{i=1}^{n} D_i(s,p) = \sum_{i=1}^{n} \left[\int_{B_1}^{s/p} \left\{ 1 - F\left(\frac{pR + s(i-1)}{T} \right) \right\} g(R) dR \right].$$

□

Lemma 4 *The maximum package size offered by a seller is bounded by the condition*

$$s \le \min\left\{ pB_2, \frac{A_2 T}{n} \right\}.$$

Proof. As $s/p = B_2$ characterizes the condition for the maximum demand for the product, and as a smaller package package size has a lower total cost $c(s)$, a seller will maximize profit by restricting the package size to at most $s = pB_2$. Also, a consumer with usage rate θ can consume no more than θT units of the product before it expires. Thus, if a consumer with the highest possible usage rate, $\theta = A_2$, can consumer up to n packages of the product, then the maximum package size the seller needs to offer is $s = A_2T/n$. Otherwise, $dc(s)/ds > 0$ implies that the seller can make a larger profit by reducing the package size to $s = A_2T/n$ without changing the price of the product. □

Lemma 5 *If $\alpha > 1/B_1$, then the seller should not make the product.*

Proof. Recall $R \in [B_1, B_2]$, where $R = B_1$ corresponds to the least price sensitive customer in the market, who is willing to pay a unit price $1/B_1$ to buy the product. Thus, if the unit selling price exceeds $1/B_1$, no customer will buy the product. But if $\alpha > 1/B_1$, the seller will make a loss at a unit price below $1/B_1$. It follows that the seller will not make the product if $\alpha \geq 1/B_1$. $\square$

Theorem 2 *If $\alpha \leq 1/B_1$, the seller should, for any given value of $n \geq 1$, sell packages of size*

$$s^* = \frac{A_2 T}{n}\left(1 - \frac{1}{\sqrt{(n+1)(\frac{n}{B_1\alpha}+1)}}\right) \quad \text{units,}$$

and for each package should charge a price of

$$p^* = \frac{A_2 T}{B_1}\frac{1}{\sqrt{(n+1)(\frac{n}{B_1\alpha}+1)}}.$$

Proof. The seller's problem is to maximizes the profit over package size s and price p:

$$\begin{aligned} \text{Maximize} \quad \pi(p,s) &= (p-\alpha s)\sum_{i=1}^{n}\left[\int_{B_1}^{s/p}\left\{\frac{A_2}{A_2-A_1} - \frac{pR + s(i-1)}{(A_2-A_1)T}\right\}\frac{1}{B_2-B_1}dR\right] \\ &= (p-\alpha s)\left(\frac{\frac{s}{p}-B_1}{B_2-B_1}\right)\left(\frac{1}{1-\frac{A_1}{A_2}} - \frac{pB_1+sn}{2(A_2-A_1)T}\right)n \end{aligned}$$

$$\text{Subject to :} \qquad s \leq \frac{A_2 T}{n}, \quad s \leq pB_2.$$

To find the globally optimal solution to this problem, we identify and compare all the feasible solutions that satisfy the first order conditions obtained from the

Lagrangian:

$$L = \frac{n(B_1p - s)(B_1p + ns - 2A_2T)(p - s\alpha)}{2(A_2 - A_1)(B_2 - B_1)pT} + \lambda_1\left(pB_2 - s\right) + \lambda_2\left(\frac{A_2T}{n} - s\right),$$

where $\lambda_1 \geq 0, \lambda_2 \geq 0$ are Lagrange multipliers. The first-order conditions are:

$$\frac{\partial L}{\partial p} = \frac{\partial L}{\partial s} = \lambda_1\frac{\partial L}{\partial \lambda_1} = \lambda_2\frac{\partial L}{\partial \lambda_2} = 0,$$

where

$$\frac{\partial L}{\partial p} = \frac{B_1np^2(2B_1p + (n-1)s - 2A_2T) - ns\left(B_1^2p^2 + s(ns - 2A_2T)\right)\alpha}{2(A_2 - A_1)(B_2 - B_1)p^2T} + \lambda_1B_2,$$

and

$$\frac{\partial L}{\partial s} = \frac{n\left(-B_1^2\alpha p^2 + B_1((n-1)p + 2(-ns + s + A_2T)\alpha)p + 2A_2T(p - 2s\alpha) + ns(3s\alpha - 2p)\right)}{2(A_2 - A_1)(B_2 - B_1)pT}$$
$$- (\lambda_1 + \lambda_2).$$

We examine each of the following four cases below: (1) $\lambda_1 > 0, \lambda_2 > 0$, (2) $\lambda_1 = 0, \lambda_2 > 0$, (3) $\lambda_1 > 0, \lambda_2 = 0$, and (4) $\lambda_1 = \lambda_2 = 0$.

(1) $\lambda_1 > 0, \lambda_2 > 0$: The shadow prices are

$$\lambda_1 = \frac{A_2(-n\alpha B_2^3 + B_1(n+1)B_2 + B_1^2(B_2\alpha - 2)}{2(A_2 - A_1)(B_2 - B_1)B_2^2}, \quad \lambda_2 = \frac{A_2B_1(B_2\alpha - 1)}{(A_2 - A_1)B_2^2}.$$

From complementary slackness, we obtain

$$s = \frac{A_2T}{n},\ p = \frac{A_2T}{nB_2}.$$

The corresponding profit is

$$\pi_1 = A_2^2 T \frac{(nB_2 - B_1)(1 - B_2\alpha)}{2nB_2^2(A_2 - A_1)}.$$

Now $\lambda_2 \geq 0$ iff $B_2\alpha \geq 1$, in which case $\pi_1 < 0$, as is evident from the above expression. It follows that there is no solution with non-negative profit.

(2) $\lambda_1 = 0, \lambda_2 > 0$: We obtain $s = A_2T/n$ from complementary slackness. Substituting for s and solving for p gives $p = f(D^{1/2})$, where $f(\cdot)$ is an increasing function of D, and

$$D = -nB1\alpha(n^3 + 3n^2(1 + B_1\alpha) + (1 + B_1\alpha)^3 + 3n(1 + B_1\alpha(B_1\alpha - 7))).$$

Now $B_1\alpha > 0$ implies

$$-(n^3 + 3n^2(1 + B_1\alpha) + (1 + B_1\alpha)^3 + 3n(1 + B_1\alpha(B_1\alpha - 7))) < 0.$$

Thus, $D < 0$, p is not a real number, and there is no feasible solution.

(3) $\lambda_1 > 0$, $\lambda_2 = 0$: As $\lambda_1 > 0$, we obtain $s = pB_2$ from the complementary slackness condition. Substituting for s in terms of p and solving the equations from the first-order condition gives

$$p = \frac{A_2T}{B_1 + nB_2} \quad s = \frac{A_2B_2T}{B_1 + nB_2} = \frac{A_2T}{\frac{B_1}{B_2} + n}.$$

As $B_2 > B_1$, we have $A_2T/(n+1) < s < A_2T/n$, as desired. The total demand

$$D = \frac{A_2 n}{2(A_2 - A_1)} = \frac{n}{2(1 - \frac{A_1}{A_2})}$$

is positive because $A_1 < A_2$. Also,

$$\lambda_1 = \frac{A_2 n(B_1^2\alpha - B_2^2 n\alpha + B_1(1+n-2B_2\alpha))}{2(A_2-A_1)(B_2-B_1)(B_1+B_2 n)}.$$

The denominator of the above expression is always positive, and so $\lambda_1 > 0$ requires

$$B_1^2\alpha - B_2^2 n\alpha + B_1(1+n-2B_2\alpha) > 0,$$

which is equivalent to the condition

$$\alpha < \frac{(n+1)B_1}{nB_2^2 + 2B_1B_2 - B_1^2} = \beta,$$

where $B_2 > B_1$ implies the right hand side of the above expression is always positive. The total profit is

$$\pi_3 = \frac{A_2^2 nT(1-B_2\alpha)}{2(A_2-A_1)(B_1+B_2 n)},$$

which is non-negative if $B_2\alpha \leq 1$.

(4) $\lambda_1 = \lambda_2 = 0$: We obtain the following four solutions:

(a) $p_a = \dfrac{2A_2T}{B_1(n+1)}, \quad s_a = \dfrac{2A_2T}{n+1}, \quad \pi_a = 0;$

(b) $p_b = \dfrac{2\alpha A_2T}{n+B_1\alpha}, \quad s_b = \dfrac{2A_2T}{n+B_1\alpha}, \quad \pi_b = 0;$

(c) $p_c = -\dfrac{A_2T\alpha}{\sqrt{(n+1)B_1\alpha(n+B_1\alpha)}}, \quad s_c = \dfrac{A_2T}{n}\left(1 + \dfrac{B_1\alpha}{\sqrt{(n+1)B_1\alpha(n+B_1\alpha)}}\right), \quad \pi_c < 0;$

(d) $p_d = \dfrac{A_2T\alpha}{\sqrt{(n+1)B_1\alpha(n+B_1\alpha)}}, \quad s_d = \dfrac{A_2T}{n}\left(1 - \dfrac{B_1\alpha}{\sqrt{(n+1)B_1\alpha(n+B_1\alpha)}}\right),$

$$\pi_d = \frac{A_2^2T\left(\sqrt{n+B_1\alpha} - \sqrt{B_1\alpha(n+1)}\right)^2}{2(A_2-A_1)(B_2-B_1)n}.$$

As only $\pi_d > 0$, we consider the solution in case (d) alone. Let $\pi_4 \equiv \pi_d$. We observe that the solution to case (d) satisfies $A_2T/(n+1) < s < A_2T/n$, because $B_1\alpha > 0$ implies $s < A_2T/n$, and $B_1\alpha < 1$ implies $s > A_2T/(n+1)$. The corresponding expression for the total demand is

$$D = \frac{A_2\sqrt{(n+1)B_1\alpha(n+B_1\alpha)} - (n+1)B_1\alpha}{2\alpha(A_2-A_1)(B_2-B_1)}.$$

We note that $D > 0$ because the denominator of the above expression is always positive, and as $B_1\alpha < 1$, the numerator is also positive.

To summarize, only cases (3) and (4) have one feasible solution each. The solution for case (4) places no constraint on α whereas the solution for case (3) is obtained under the condition $\alpha < \beta$. However, even under the latter condition, the profit for the firm is higher in case (4) that it is in case (3). To see this, we write the difference in the profits π_4 and π_3 as

$$\Delta\pi = \pi_4 - \pi_3 = \frac{A_2^2TM}{2(A_2-A_1)n},$$

where

$$M = \frac{(B_2\alpha-1)n^2}{B_1+B_2n} + \frac{\left(\sqrt{B_1(n+1)\alpha} - \sqrt{n+B_1\alpha}\right)^2}{B_2-B_1}.$$

We can verify $M > 0$ if $\alpha = 0$, and $dM/d\alpha > 0$, and so $\Delta\pi > 0$. $\square$

Theorem 3 *Let $n = n^*$ denote a value of n for which a linear cost function $\hat{c} = \hat{\alpha}s$ satisfies $\hat{\alpha} < \alpha_{max}$. Then the seller earns a higher profit by making a package of size $s \in (A_2T/(n+1), A_2T/n)$ than by making a single-serve package.*

Proof. The condition

$$\lim_{s\to 0}\frac{d\tilde{c}(s)}{ds} = \alpha,$$

implies

$$\lim_{n\to\infty} \pi_A = \lim_{n\to\infty} \pi^* = \pi^*_{\max}.$$

That is, as s approaches zero, π_A, the seller's optimal profit under the concave cost function $\tilde{c}(s)$, approaches $\pi^*_{\max}$, the maximum possible profit under the linear cost function $c = \alpha s$; see Figure 5. Let $\pi_B = \pi_B(s^*, p^*)$ denote the profit at (s^*, p^*) if we use the cost $\tilde{c}(s^*)$ instead of the cost $c = \alpha s^*$ when computing the profit at (s^*, p^*). Then

$$\pi^* < \pi_B \le \pi_A,$$

where (i) $\pi^* < \pi_B$ because $\tilde{c}(s^*) < \alpha s^*$, and (ii) $\pi_B \le \pi_A$ because (s_A, p_A) is the optimal solution when the cost is given by the concave function $\tilde{c}(s)$.

Next, consider the linear cost function $\hat{c} = \hat{\alpha} s$, which goes through the origin and intersects the concave cost function $\tilde{c}(s)$ at the point $s = A_2T/(n+1)$. Then

$$c < \hat{c} < \tilde{c}(s), \quad \text{for all } s \ge \frac{A_2T}{n+1},$$

because $\hat{c} = \hat{\alpha} s$ (i) lies everywhere below $c = \alpha s$, and (ii) lies above $\tilde{c}(s)$ for $s > A_2T/(n+1)$. Let $\hat{\pi} = \hat{\pi}(s^*, p^*)$ denote the profit if we use the cost $\hat{c} = \hat{\alpha} s^*$ instead of the cost $c = \alpha s$ to compute the profit at (s^*, p^*). Then

$$\pi^* < \hat{\pi} < \pi_B,$$

where (i) $\pi^* < \hat{\pi}$ because $\hat{\alpha} s^* < \alpha s^*$, and (ii) $\hat{\pi} < \pi_B$ because $\tilde{c}(s^*) < \hat{\alpha} s^*$.

The inequalities $\pi^* < \pi_B \le \pi_A$, and $\pi^* < \hat{\pi} < \pi_B$, together imply

$$\pi^* < \hat{\pi} < \pi_B \le \pi_A.$$

Subtracting $\pi^*_{\max}$ from each term of the above inequality gives

$$\pi^* - \pi^*_{\max} < \hat{\pi} - \pi^*_{\max} < \pi_B - \pi^*_{\max} \leq \pi_A - \pi^*_{\max}.$$

We showed in the previous section that $\pi^* - \pi^*_{\max} < 0$. However, if $\hat{\pi} - \pi^*_{\max} > 0$, then the above inequality implies $\pi_A - \pi^*_{\max} > 0$. Thus, a sufficient condition under which the maximum profit for a concave cost function $\tilde{c}(s)$ is obtained for a finite value of $n = n^*$ is given by the condition $\hat{\pi} - \pi^*_{\max} > 0$, which, upon substituting

$$\hat{\pi}^* = \frac{A_2^2 T \left(\sqrt{n^* + B_1\hat{\alpha}} - \sqrt{B_1\hat{\alpha}(n^*+1)}\right)^2}{2(A_2 - A_1)(B_2 - B_1)n^*},$$

and

$$\pi^*_{\max} = \lim_{n\to\infty} \pi^*_n = \frac{A_2^2(1 - \sqrt{B_1\alpha})^2 T}{2(A_2 - A_1)(B_2 - B_1)},$$

yields the desired condition $\hat{\alpha} < \alpha_{\max}$, where

$$\alpha_{\max} = \frac{1}{B_1 n^*}\Bigg[2 + 2B_1\alpha - 4\sqrt{B_1\alpha} + n^*(2 + B_1\alpha - 2\sqrt{B_1\alpha}) \\ - 2\sqrt{(n^*+1)\Big\{(1 - 4\sqrt{B_1\alpha}) + B_1\alpha(6 + B_1\alpha - 4\sqrt{B_1\alpha}) + n^*(1 + B_1\alpha - 2\sqrt{B_1\alpha})\Big\}}\Bigg]$$

FIGURE 1. $r = \pi^*/\pi_1^*$ as a function of n and $B_1\alpha$

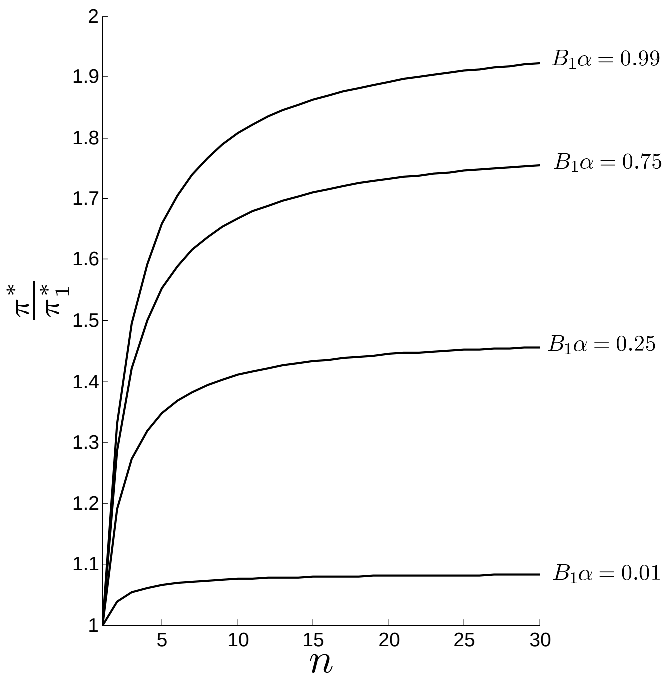


FIGURE 2. Optimal unit price versus package size

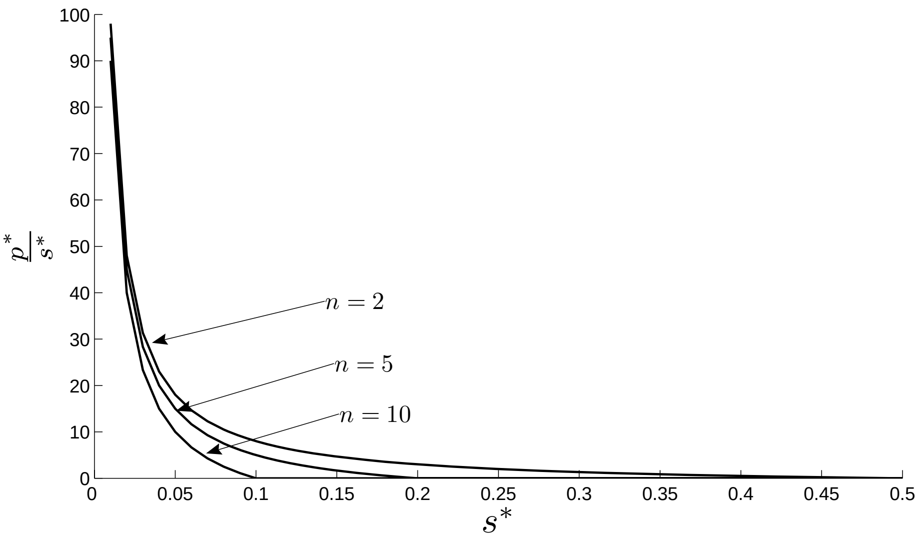


FIGURE 3. Q^*/Q_1^* as a function of n and $B_1\alpha$

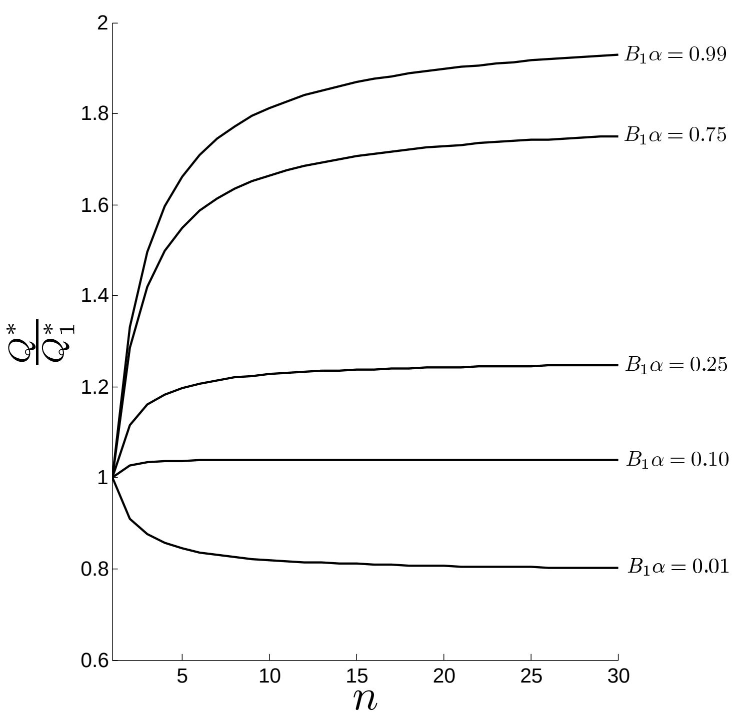


Figure 4. Consumers waste and unfulfilled demand

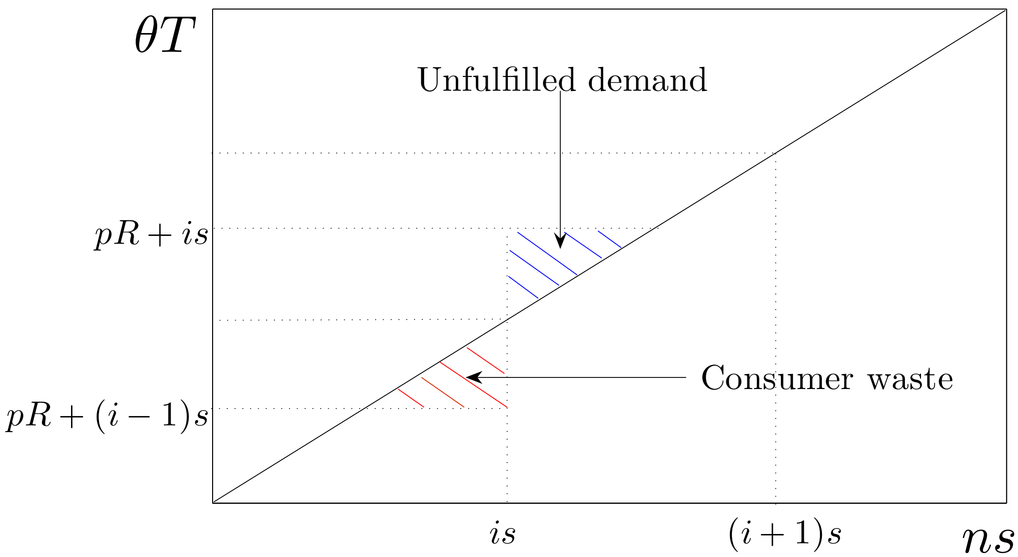
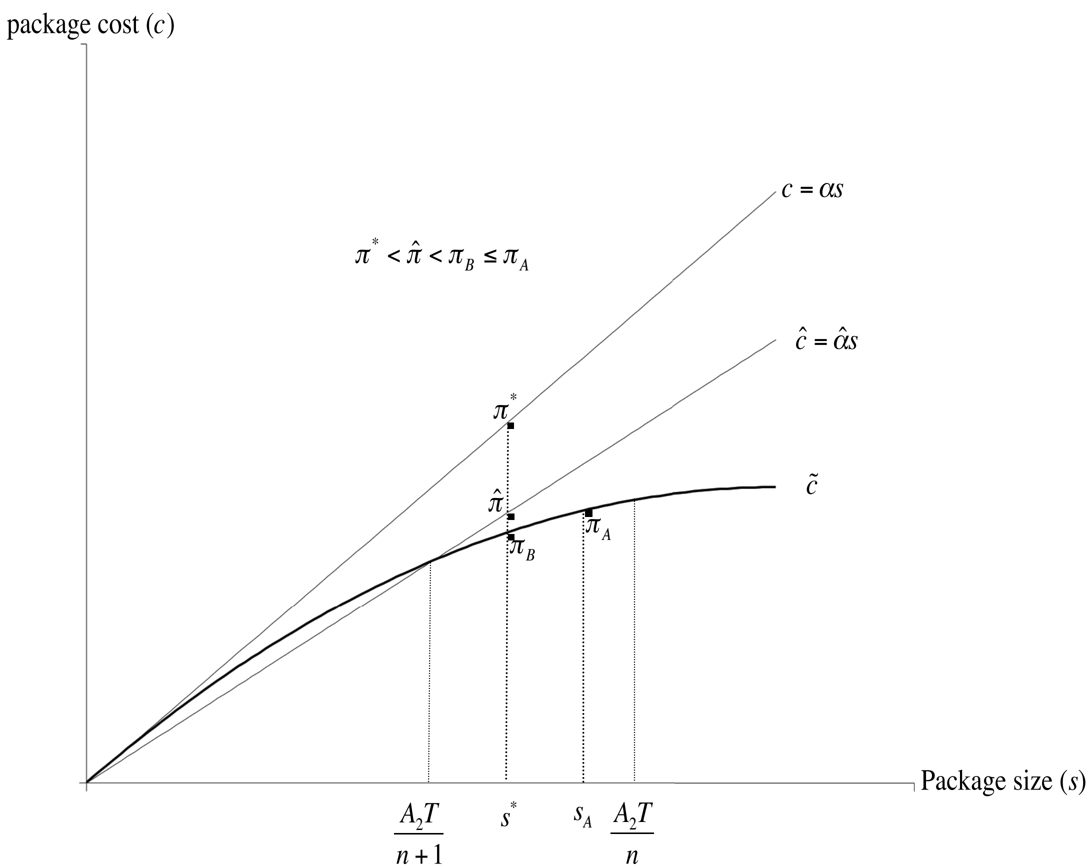


FIGURE 5. Package size and package cost relationships for linear and concave cost function



Notes: The figure illustrates the relation $\pi^* < \hat{\pi} < \pi_B \leq \pi_A$, where:

s_A denotes the optimal package size when the package cost is specified by the concave function $\tilde{c} = \tilde{c}(s)$. The associated profit is $\pi_A \equiv \pi_A(s_A, p$

s^* denotes the optimal package size when the package cost is $c = \alpha s$. The associated profit is $\pi^* \equiv \pi^*(s^*, p^*)$.

$\hat{\pi} \equiv \hat{\pi}(s^*, p^*)$ is the profit associated with (s^*, p^*) if the cost is $c = \hat{\alpha} s^*$, where $\hat{\alpha}$ satisfies $\hat{c} = \hat{\alpha} s = \tilde{c}$ for $s = A_2 T/(n+1)$.

$\pi_B \equiv \pi_B(s^*, p^*)$ is the profit associated with (s^*, p^*) if the cost is $\tilde{c} = \tilde{c}(s^*)$.

FIGURE 6. $\alpha_{\max}/\alpha$ as a function of n^*