# Inflation and the Stock Market: Understanding the "Fed Model"\*

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This Draft: April 2009

## JEL Classifications G12, G15, E44

#### Keyphrases

Money illusion, Equity premium, Countercyclical risk aversion, Fed model, Inflation, Economic Uncertainty Dividend yield, Stock-Bond Correlation, Bond Yield

#### Abstract:

The so-called Fed model postulates that the dividend or earnings yield on stocks should equal the yield on nominal Treasury bonds, or at least that the two should be highly correlated. In US data there is indeed a strikingly high time series correlation between the yield on nominal bonds and the dividend yield on equities. This positive correlation is often attributed to the fact that both bond and equity yields comove strongly and positively with expected inflation. While inflation comoves with nominal bond yields for well-known reasons, the positive correlation between expected inflation and equity yields has long puzzled economists. We show that the effect is consistent with modern asset pricing theory incorporating uncertainty about real growth prospects and also habit-based risk aversion. In the US, high expected inflation has tended to coincide with periods of heightened uncertainty about real economic growth and unusually high risk aversion, both of which rationally raise equity yields. Our findings suggest that countries with a high incidence of stagflation should have relatively high correlations between bond yields and equity yields and we confirm that this is true in a panel of international data.

<sup>\*</sup>This work does not necessarily reflect the views of the Federal Reserve System or its staff. In particular, our use of the term "Fed Model" reflects only conventional parlance among finance practicioners for this common valuation model.

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We thank the participants of seminars at Columbia University, the Federal Reserve Board, Tilburg University, the Fourth Annual Asset Pricing Retreat at the University of Amsterdam, and the Federal Reserve System's Day Ahead Conference in San Francisco for helpful comments. All errors are the sole responsibility of the authors.

## 1 Introduction

The so-called Fed model postulates that the dividend or earnings yield on stocks should equal the yield on nominal Treasury bonds, or at least that the two should be highly correlated.<sup>1</sup> Both investment professionals (see for instance Asness (2003)) and academics (see for instance Thomas and Zhang (2008)) have long been struck by the strength of the empirical regularity. Figure 1 shows a graph of the yield on a 10-year nominal bond and the equity yield (using dividends) for the US aggregate stock market. The correlation between the two yields is 0.78! While some investment professionals are using the Fed model as a model of equity valuation (see the references in Estrada (2005)), both practitioners and academics have concluded that the model is inconsistent with a rational valuation of the stock market (see for instance, Asness (2003), Feinman (2005), Campbell and Vuolteenaho (2004), Cohen, Polk and Vuolteenaho (2005), Ritter and Warr (2002) and Sharpe (2002)).

The difficulty in squaring the model with rational valuation can be illustrated using a simple decomposition of the dividend yield and the nominal bond yield. Using the Gordon model, we can write the equity cash yield, EY, on the aggregate stock market as consisting of three components:

$$EY = -EDIV + RRF + ERP \tag{1}$$

where EDIV is the expected growth rate of real equity dividends, RRF is the real risk free rate of interest and ERP is the equity risk premium. Similarly, the yield on a nominal bond is:

$$BY = EINF + RRF + IRP \tag{2}$$

where EINF is expected inflation, RRF is again the real interest rate, and IRP is the inflation risk premium. The high correlation between dividend yields and nominal bond yields is difficult to reconcile with rational models because expected inflation is a dominant source of variation in nominal yields and the extant literature seems to have concluded that it is impossible for expected inflation to have a large (rational) effect on any of the *real* components that drive the equity cash yield. In fact, the aforementioned authors all resort to the simple behavioral model proposed by Modigliani and Cohn in 1979 to explain the empirical regularity: inflation (or money) illusion. Inflation illusion suggests that when expected inflation increases, bond yields duly increase, but because equity investors incorrectly discount real cash flows using nominal rates, the increase in nominal

 $<sup>^{1}</sup>$ The Fed Model may have gained its moniker from Prudential Securities strategist Ed Yardeni in 1997 who noted that in the Federal Reserve Humphrey Hawkins Report for July 1997, a chart plotted the time series for the earnings-price ratio of the S&P 500 against the 10-year constant-maturity nominal treasury yield.

yields leads to equity underpricing (the equity yield rises with bond yields to levels that are too high) and vice versa. Alternatively, one can view equity investors as correctly discounting nominal cash flows and using nominal discount rates, but failing to increase expected nominal cash payouts in response to increases in expected inflation.

The importance of this conclusion extends beyond the narrow confines of testing the Fed model. If behavioral biases induced by inflation cause misvaluation in the equity market, then the potential exists for informed practitioners to devise trading strategies to take advantage of the mispricing. For policy makers, if money illusion causes undue variation in equity prices during periods of inflation uncertainty, this suggests another motive for inflation stabilization policies, as Campbell and Vuolteenaho (2004) point out.

In this article, we carefully re-examine the evidence by constructing dynamic versions of Equations (1) and (2) in a vector autoregressive (VAR) framework, building on Campbell and Shiller's (1988) seminal work. The benchmark VAR includes earnings growth and survey expectations of earnings to help predict cash flow growth and uses empirical proxies for real rates and expected inflation. However, we construct the risk premium components of yields as residuals since they are not directly measurable. We find that expected inflation is indeed the primary bond yield component responsible for the high stock-bond yield correlation. This is a remarkable stylized fact that any macro-economic model of the stock market must seek to explain. In the context of a rational model, expected inflation must be positively correlated with the dividend yield through some combination of positive correlation with the real rate and the equity risk premium, or a negative correlation with expected cash flow growth. We find that only a relatively small portion of the overall comovement between expected inflation and the dividend yield can be ascribed to the correlation between expected inflation and real rates or expected cash flow growth.<sup>2</sup> The bulk of the positive covariance between the dividend yield and expected inflation comes from positive comovement between expected inflation and the equity risk premium. Importantly, because we measure the equity premium as a residual, these initial results do not identify whether money illusion-induced misvaluation or rational equity risk premiums are responsible for the high correlation with expected inflation.

However, our subsequent analysis strongly supports the latter explanation. We demonstrate that the high correlation between expected inflation and the dividend yield is almost entirely due to the positive correlation between expected inflation and two plausible proxies for rational time-varying risk premiums: a measure of economic uncertainty (the uncertainty among professional forecasters regarding real GDP growth) and a

 $<sup>^{2}</sup>$ This confirms Modigliani and Cohn's careful work that the effect is not due to expected real cash flow growth rates being adversely affected by expected inflation.

consumption-based measure of risk aversion. These measures of rationally time-varying risk premiums feature prominently in recent asset pricing articles showing that they help to explain a number of salient asset return features. Bansal and Yaron (2004, BY henceforth) have stressed the importance of economic uncertainty and Campbell and Cochrane (1999, CC henceforth) have built a model of external habit, leading to a measure of time-varying risk aversion that can be constructed from current and past consumption data and is countercyclical. Bekaert, Engstrom and Xing (2009) combine both measures in one model.<sup>3</sup> Consequently, a rational channel explains why the Fed model "works:" high expected inflation coincides with periods of high risk aversion and/or economic uncertainty. Therefore our explanation is very different from the prevailing explanations based on money illusion. Our work is related to but distinct from another "old" hypothesis regarding the relationship between inflation and the stock market: Fama's (1981) proxy hypothesis. Fama argues that the strong negative relationship between stock returns and inflation is due to stock returns anticipating future economic activity and inflation acting as a proxy for expected real activity; hence, the hypothesis also relies on stagflation being an important part of US data. Using our VAR's implicit cash flow expectations to capture real activity, we show that the proxy hypothesis is part of the explanation, but that our risk-based story dominates. We also provide an out-of-sample test of our interpretation of the US data. Specifically, our results suggest that the correlation between equity and bond yields ought to be higher in countries with a higher average incidence of stagflation. We confirm that this is the case. We also make sure that our US results are robust, investigating a wide variety of alternative VAR specifications. The concluding section summarizes our results and discusses how they hold up during the 2008-2009 episode.

## 2 Empirical Methodology

In this section, the first sub-section presents a dynamic version of the Gordon model alluded to in the introduction. In the second sub-section, we describe how we decompose the different yields using a VAR methodology. The third sub-section describes how our framework generates estimates of equity-bond yield correlations and their components. The fourth sub-section shows how we identify a rational component in the equity yield to test our main hypothesis. In the fifth sub-section, we focus on alternative hypotheses involving cash flow expectations that we can test using our framework.

 $<sup>^{3}</sup>$ Note that all these articles feature tightly parameterized models that are not designed to fit the comovements between equity and bond yields and their components.

#### 2.1 Yield Decompositions

Our goal is to construct dynamic versions of Equations (1) and (2). Beginning with the latter task, we simply assume the nominal yield decomposition relationship holds at each point in time using continuously compounded rates, which we denote with lower case letters. In particular, we model  $by_t$ , the continuously compounded bond yield, as,

$$by_t = einf_t + rrf_t + irp_t. \tag{3}$$

where  $rrf_t$  is a real risk free rate assumed to have maturity equal to that of the nominal bond,  $einf_t$  is the average (annualized) expected inflation over the life of the bond, and  $irp_t$  is the inflation risk premium associated with the bond. In principle, all three components are unobserved. We achieve identification by finding observable proxies for the real rate and expected inflation, and use equation (3) to infer the inflation risk premium.<sup>4</sup> We describe all empirical variable definitions and data sources in the next section.

To decompose the equity yield into its components, we use the Campbell-Shiller (1988, CS henceforth) decomposition. CS arrive at the following formula for the logarithmic equity yield,  $ey_t$ :

$$ey_t = -\frac{k}{1-\rho} + E_t \left[ \sum_{j=0}^{\infty} \rho^j \left( r_{t+j+1} - \Delta d_{t+j+1} \right) \right].$$
(4)

where k and  $\rho$  are linearization constants,  $r_t$  is the one-period real return to holding equity, and  $\Delta d_t$  is logarithmic one-period real dividend growth. Without loss of generality, we can split the expected rate of return on equity into risk-free and risk premium components,

$$E_t \left[ r_{t+1} \right] = rrf_t + erp_t \tag{5}$$

where  $erp_t$  is the continuously compounded one-period equity risk premium. Given the implicit definition of  $rrf_t$  in Equation (3), the equity premium is defined relative to a long-term real risk free rate. Substituting,

$$ey_t = -\frac{k}{1-\rho} - E_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+j+1} + E_t \sum_{j=0}^{\infty} \rho^j rrf_{t+j} + E_t \sum_{j=0}^{\infty} \rho^j erp_{t+j}$$
(6)

which is the dynamic version of Equation (1). Here too, we treat the risk premium component as the residual, with the two other components constructed empirically using our assumed data generating process, described

 $<sup>^{4}</sup>$ In a robustness exercise, we also conduct our main analysis using a different identification scheme for real rates that assumes we can measure the inflation risk premium more directly as a function of inflation uncertainty. See Section 5 for details.

next.

#### 2.2 Empirical Model: VAR

To model the joint dynamics of stock and nominal bond yields and their components, we stack the following variables into a vector,  $Y_t$ ,

$$Y_t = [einf_t, rrf_t, \Delta d_t, erp_t, irp_t, x'_t]', \qquad (7)$$

with  $x_t$  denoting a vector of time-t observable information variables that will be useful in interpreting the results:

$$x_t = [ra_t, vr_t, \Delta ern_t, gern_t^{su}]'.$$
(8)

Hence, there are a total of nine variables in the VAR. The first two elements of the information vector,  $x_t$ , are designed to capture rational components of the equity risk premium,  $erp_t$ . First,  $ra_t$ , is a measure of rational risk aversion based on the specification of external habit persistence in CC. Second,  $vr_t$  is a measure of uncertainty about real economic growth. BY use uncertainty in the context of a data generating process for dividend and consumption growth and demonstrate that a modest amount of time-varying uncertainty about real growth can, under some assumptions about investor preferences, generate nontrivial variation in the equity risk premium. The other two variables in  $x_t$  represent contemporaneous realized real earnings growth,  $\Delta ern_t$ , and a subjective measure of expected earnings growth,  $gern_t^{su}$ . These variables help predict future dividends and help us test some alternative hypotheses. Further details are provided in section 2.5.

We proceed by assuming a simple data generating process for  $Y_t$ , and using the fully observable vector,

$$W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t']', \qquad (9)$$

to identify the dynamics of  $Y_t$ . Specifically, we assume a first-order VAR for  $Y_t$ ,

$$Y_t = AY_{t-1} + u_t \tag{10}$$

where we are suppressing drift terms since we are only interested in variance decompositions. The matrix A is square and is comprised of parameters governing the conditional mean of  $Y_t$ , and  $u_t$  is a vector of i.i.d shocks with covariance matrix  $\Omega$ . Once the  $Y_t$  dynamics are specified to take this form, a simple linear translation between  $Y_t$  and the observable vector,  $W_t$  is available. In particular, Equations (3) and (4) imply that  $W_t$  is a linear combination of concurrent values of  $Y_t$  as well as expectations of future values of  $Y_t$ :

$$W_t = M_1 Y_t + M_2 E_t \sum_{j=0}^{\infty} \rho^j Y_{t+j+1}$$
(11)

where we continue to suppress constant terms, and the matrices  $M_1$  (9 × 9) and  $M_2$  (9 × 9) are comprised of known constants. Under the VAR(1) structure for  $Y_t$ , this has the implication that  $Y_t$  and  $W_t$  are related by a linear transformation, which we denote as

$$Y_t = \Theta W_t \tag{12}$$

and we must solve for  $\Theta$ . Consequently,  $W_t$  also follows a linear VAR:

$$W_t = A^w W_{t-1} + u_t^w$$

where  $u_t^w$  has covariance matrix  $\Omega^w$ . Under the mapping in Equation (12), we can express A and  $\Omega$  as:

$$A = \Theta A^{w} \Theta^{-1}$$
  

$$\Omega = \Theta \Omega^{W} \Theta'.$$
(13)

To solve for  $\Theta$ , we combine Equations (11) and (12) to obtain,

$$W_t = M_1 \Theta W_t + M_2 E_t \sum_{j=0}^{\infty} \rho^j \Theta W_{t+j+1}$$
(14)

Defining for notational convenience  $\Phi_1 = A^w \left(I - \rho A^w\right)^{-1}$  and solving the expectations terms yields

$$W_t = M_1 \Theta W_t + M_2 \Theta \Phi_1 W_t$$

Equating  $W_t$  coefficients on both sides of the equations yields a solution for  $\Theta$ :

$$vec\left(\Theta\right) = \left(I' \otimes M_1 + \Phi_1' \otimes M_2\right)^{-1} vec\left(I\right).$$

$$\tag{15}$$

Using Equations (13) and (15), we can completely specify the dynamics of  $Y_t$  in terms of parameters estimated from the data. That is,  $\left\{\widehat{A}, \widehat{\Omega}\right\} = F\left\{\widehat{A^w}, \widehat{\Omega^w}\right\}$ .

## 2.3 Decomposing Yields under the VAR

As stated above, the nominal bond yield is affine in components of  $Y_t$ , as the right hand side terms of Equation (3) are direct elements of  $Y_t$ . We can also now more explicitly describe our decomposition of the equity yield into three components,

$$ey_t = const + ey_t^{\Delta d} + ey_t^{rrf} + ey_t^{erp} \tag{16}$$

where  $ey_t^{\Delta d} = -E_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+j+1}$  represents the total effect of cash flow expectations,  $ey_t^{rrf} = E_t \sum_{j=0}^{\infty} \rho^j rrf_{t+j}$ , represents the total effect of real interest rates, and  $ey_t^{erp} = E_t \sum_{j=0}^{\infty} \rho^j erp_{t+j}$  represents the total effect of equity risk premiums. We use objective conditional expectations under the VAR to calculate each of these quantities, and because of the simple VAR structure, the three equity yield components are affine in  $Y_t$ . For example, ignoring constant terms, and defining  $e'_{\Delta d}$  such that  $\Delta d_t = e'_{\Delta d} Y_t$ ,

$$ey_{t}^{\Delta d} = -e_{\Delta d}' E_{t} \sum_{j=0}^{\infty} \rho^{j} Y_{t+j+1} = -e_{\Delta d}' \rho A \left(I - \rho A\right)^{-1} Y_{t}$$

which is indeed a linear function of  $Y_t$ . For our baseline specification then,  $M_1$  is an identity matrix and  $M_2$  is the zero matrix except for the rows pertaining to  $ey_t$  and  $by_t$ :

$$M_{1}^{ey} = e'_{rrf} + e'_{erp}, \quad M_{2}^{ey} = -e'_{\Delta d} + \rho e'_{rrf} + \rho e'_{erp}$$
$$M_{1}^{by} = e'_{einf} + e'_{rrf} + e'_{irp}, \quad M_{2}^{by} = 0$$
(17)

where  $M_1^{ey}$  denotes the relevant row of  $M_1$  for the equity yield, and similarly for the other superscripts.

To determine the source of the high covariance between stock and bond yields, we decompose it into its nine components:

$$COV(ey_t, by_t) = COV(ey_t^{\Delta d}, einf_t) + COV(ey_t^{\Delta d}, rrf_t) + COV(ey_t^{\Delta d}, irp_t) + COV(ey_t^{rrf}, einf_t) + COV(ey_t^{rrf}, rrf_t) + COV(ey_t^{rrf}, irp_t) + COV(ey_t^{erp}, einf_t) + COV(ey_t^{erp}, rrf_t) + COV(ey_t^{erp}, irp_t)$$
(18)

Each of these covariances is readily calculated using VAR arithmetic. For instance,

$$COV\left(ey_t^{\Delta d}, einf_t\right) = -e'_{\Delta d}\rho A \left(I - \rho A\right)^{-1} COV\left(Y_t\right) e'_{einf}$$
<sup>(19)</sup>

where  $vec[COV(Y_t)] = (I - A \otimes A)^{-1} vec(\Omega)$ . Note that every element of  $COV(ey_t, by_t)$  is ultimately a function of the parameters of the observable VAR,  $\{\widehat{A^w}, \widehat{\Omega^w}\}$ .

## 2.4 Orthogonalizing the Equity Risk Premium

The equity risk premium component of equity yields in our decompositions above,  $ey_t^{erp}$ , is essentially a residual, the difference between the observed equity yield and the summed presented values, calculated under the VAR, of future cash flows and real risk free rates. A disadvantage of this approach is that model misspecification could contaminate the equity risk premium estimates. To try to isolate the component of the equity risk premium that is consistent with rational pricing, we draw on recent theoretical advances in the empirical asset pricing literature. CC and BY suggest that  $erp_t$  is approximately linear in risk aversion,  $ra_t$ , or real uncertainty,  $vr_t$ respectively.

Let's start with describing our fundamental measure of risk aversion; more details can be found in a selfcontained data appendix. In CC's external habit model, (logarithmic) risk aversion is a negative affine function of the log "consumption surplus ratio," which in turn is aggregate consumption minus the "habit stock" divided by consumption. As aggregate consumption moves closer to the habit stock (as would happen in recessions), aggregate risk aversion increases. CC model the surplus ratio as a heteroskedastic autoregressive process, with its shocks perfectly correlated with consumption shocks. We use data on nondurables and services consumption growth and CC's parameters and model to create an empirical proxy for risk aversion. The resulting measure is clearly counter-cyclical.

In BY, it is the heteroskedasticity in consumption growth itself that leads to time-variation in risk premiums. BY introduce two latent variables, a time-varying mean for consumption growth, and time-varying volatility for consumption (and dividend) shocks. The volatility process follows an AR(1) process. In the robustness section, we report results from a system in which we literally use BY 's model, parameters and U.S. consumption data to filter out an economic uncertainty process. However, there are more direct measures of economic uncertainty available using the Survey of Professional Forecasters that do not rely on consumption data or a specific ARIMA model. As we detail in the data appendix, for our benchmark specification, we combine information from a survey about the probability of a recession the next quarter and from the dispersion across respondents about next year's real GDP growth.

In a recent article by Bekaert, Engstrom and Xing (2009), both economic uncertainty and risk aversion drive equity risk premiums. However, in their model, risk aversion is imperfectly correlated with fundamentals. For our exercise here, it is important to keep the rational part of the equity premium tied to fundamentals. Therefore, we parse  $ey_t^{erp}$  into two components: one spanned-by and one orthogonal-to the vector  $[ra_t, vr_t]$ . Figure 2 plots the two series. Because this vector is a subset of the information variable vector in the VAR,  $x_t$ , we can easily decompose  $ey_t^{erp}$  into these two pieces without any further estimation. Conceptually, the process is analogous to running a regression of  $ey_t^{erp}$  on  $ra_t$  and  $vr_t$  and interpreting the regression residual as the orthogonal component, which we denote  $ey_t^{erp-re}$ . For example, we calculate

$$ey_t^{erp-sp} = \beta^{erp'} [1, ra_t, vr_t]$$

$$ey_t^{erp-re} = ey_t - ey_t^{erp-sp}$$
(20)

where the coefficients,  $\beta^{erp}$  are given under OLS as,  $E\left(\left[1, ra_t, vr_t\right]\left[1, ra_t, vr_t\right]'\right)^{-1} E\left(ey^{erp}\left[1, ra_t, vr_t\right]'\right)$  and the two unconditional expectations that comprise the coefficients are readily calculated from the VAR. With this additional decomposition, there are now six potential components to the covariance between the equity premium component of stock yields and bond yields,

$$COV(ey_t^{erp}, by_t) = COV(ey_t^{erp-sp}, einf_t) + COV(ey_t^{erp-sp}, rrf_t) + COV(ey_t^{erp-sp}, irp_t) + COV(ey_t^{erp-re}, einf_t) + COV(ey_t^{erp-re}, rrf_t) + COV(ey_t^{erp-re}, irp_t)$$

If money illusion were present in the data, we would expect to find a positive covariance between the residual equity yield and expected inflation,  $COV\left(ey_t^{erp-re}, einf_t\right)$  as all the other covariances with expected inflation are constructed in a manner consistent with rational pricing.

### 2.5 Cash Flow Expectations

Our model for cash flow expectations is much richer than the models featured in CC and BY. All the variables in the VAR can affect expected future dividends, including realized and expected earnings growth. We do this for several reasons. First, in our decomposition we measure cash flow expectations directly and must make sure we have predictive power for future dividends. Both realized and expected earnings growth are helpful in this respect. In an Appendix table, we report regressions of one quarter and one year dividend growth on these variables, finding significant coefficients for at least one of the variables in each regression and at least 10 percent significant joint predictability in both. Having a reasonable model for cash flow expectations is also helpful in distinguishing Fama's proxy hypothesis from our interpretation of the data. If Fama is correct, inflation may be negatively correlated with real future activity when stagflations dominate the data and the correlation between equity yields and inflation really reflects a link between equities and future real activity. In our decomposition, the proxy hypothesis effect can be measured using the covariance between expected inflation,  $einf_t$  and,  $ey_t^{\Delta d}$ .

Second, we can use our framework and the difference between "subjective" and "objective" cash flow forecasts to cast some direct doubt on "money illusion" as an alternative interpretation of the data. We compute the equity premium residual assuming that agents use "correct" cash flow forecasts. However, some descriptions of money illusion suggest that the effect comes through incorrect subjective cash flow predictions by market participants which are correlated with inflation expectations. Of course, in our VAR system, subjective errors in cash flow forecasts would end up in the "residual," the equity premium, and if not related to  $ra_t$  and  $vr_t$ , they will still be attributed to the residual component of the equity premium,  $ey_t^{erp-re}$ . To shed light on whether a subjective bias in cash flow expectations is related to the variation in equity yields and expected inflation, we use our VAR to estimate the bias and then check for comovement of the bias with inflation and equity yields. Specifically, we calculate the subjective bias in profit expectations as the difference between the subjective measure of real profit expectations and an objective growth estimate under the VAR,  $gern_t^{ab}$ , at the same horizon (four quarters). The latter is readily calculated using VAR mathematics because we include realized real earnings growth,  $\Delta ern_t$ , as an element of the information vector in the VAR,  $x_t$ . Because the subjective earnings expectations measure predicts annual earnings, and we use quarterly data, we compute (ignoring constant terms):

$$gern_t^{ob} = e'_{\Delta ern} \left( A + A^2 + A^3 + A^4 \right) Y_t.$$
(21)

We define the subjective bias as

$$bias_t = gern_t^{su} - gern_t^{ob} \tag{22}$$

which is clearly affine in  $Y_t$  given that  $gern_t^{su}$  is also in the information vector,  $x_t$ . If this bias is not significantly related to either equity yields or expected inflation, it is hard for money illusion to play a major role in explaining equity-bond yield correlations.

## 3 VAR Results

In this section, we first briefly discuss the data and the estimation methodology. We then move to the main results regarding the equity-bond yield correlations.

### **3.1** Data and Empirical Methods

We estimate the VAR using quarterly data, extending from the 4th quarter, 1968 through the end of 2007. The data are described in detail in Appendix 7.1. Here we give a short overview. The bond yield is the yield to maturity on a nominal 10 year US Treasury bond<sup>5</sup>. As a proxy for the real rate, we use the estimate for the 5 year zero coupon real rate provided in Ang, Bekaert and Wei (2008). As is well known, real term structures are relatively flat at longer maturities so that this maturity is a reasonable proxy for a coupon bond with duration significantly lower than 10 years. There is a voluminous literature on inflation forecasting, but recent work by Ang, Bekaert and Wei (2007) strongly suggests that professional surveys provide the best out-of-sample forecasts of inflation. Therefore, we use a proxy for inflation expectations from the Survey of Professional Forecasters (SPF). The availability of the SPF data determines the starting point for our sample. Section 5 considers several robustness checks to the measurement of real rates and inflation expectations.

The equity data we use are standard and represent information on the S&P500 Index. In our base results we use dividends not accounting for repurchases, but we discuss results with an adjusted measure in Section 5. Consequently, real earnings, dividend growth and the equity yield all refer to the S&P500 Index. Subjective expectations regarding earnings growth are also extracted from the SPF.

Finally, the empirical proxies for "fundamental risk aversion" and for economic uncertainty we described earlier also use standard data sources. We use CC's risk aversion specification together with nondurables and services consumption data from the NIPA tables. We started the process in 1947, so that the effect of initial conditions has died out by the time our sample starts.

We estimate the VAR on  $W_t$  using OLS. Table 1 reports a few specification tests on the VAR residuals (Appendix Table A2 reports some summary statistics of the 9 endogenous variables.). In Panel A, we report the standard Schwarz (BIC) and Akaike (AIC) criteria. The BIC criterion clearly selects a first-order VAR whereas the AIC criterion selects a second-order VAR. In the second panel, we report Cumby-Huizinga (1987) tests on the residuals of a first and second-order VAR for each variable separately. We use 4 autocorrelations.

<sup>&</sup>lt;sup>5</sup>While the coupon bonds on which these yields are based have a roughly stable maturity, their duration naturally varies over time. We can roughly gauge the degree of this variation under some simplifying assumptions: If (1), the bonds pay semi-annual coupons, and (2) trade at par, then the bonds' duration is function of yield alone. These calculations yield a Macaulay duration series for the bonds that has a mean of around 7.5 years and a standard deviation of about 0.8 years.

While the selection criteria in Panel A suggest that a VAR(1) adequately describes the dynamics of the data, the Cumby-Huizinga tests in Panel B suggest some serial correlation remains with a first-order VAR and that a second-order VAR may be more appropriate. Nevertheless, given the length of our sample, we use a first-order VAR as the benchmark specification and consider a second order VAR only as a robustness check.

Our data sample is comprised of 157 quarterly observations of a nine-variable vector. In addition to the 9 unconditional means, the first-order VAR feedback matrix,  $A^w$ , has 81 elements and the innovation covariance matrix,  $\Omega^w$ , has 45 distinct elements. The "saturation ratio," or the ratio of the number of the total number of data points to the number of estimated parameters, is thus  $(157 \cdot 9)/(9 + 81 + 45) = 10.5$ . This is satisfactory but suggests many VAR coefficients may not be statistically significant. To make sure our results are not due to over-fitting, the robustness section considers VARs with insignificant coefficients zeroed out and smaller VARs.

In the results discussion, we immediately focus on the comovements statistics derived from the VAR. Because all of these statistics are functions of the VAR parameters, it is possible to derive standard errors for them using the parameter standard errors and the delta method. However, there are many reasons to suspect asymptotic theory may not work well in this context: some of the variables are very persistent, the saturation ratio is not exceedingly large and the residuals are likely fat-tailed. Therefore, we use standard errors derived from a standard bootstrap procedure, which is further described in Appendix 7.2. The bootstrap procedure yields 90 percent confidence intervals for all our state variables.

### **3.2** Main Results

Table 2 contains the main results. In Panel A, the top line simply reports the variance of the bond and equity yields, their covariance and their correlation. The heart of the puzzle is that the correlation between  $ey_t$  and  $by_t$  is 78 percent. Under the VAR point estimates, a (bootstrapped) 90 percent confidence interval for this correlation ranges from 34 to 90 percent. This is puzzling because, as shown under the variance decompositions for the two yields, 55 percent of the variance of the bond yield is driven by expected inflation, whereas 80 percent of the variation of the equity yield is driven by the equity risk premium.<sup>6</sup>

Let's first comment on the realism of the variance decompositions. That discount rate variation is the dominant source of equity yield variation is by now well accepted (see Cochrane 1992). Nevertheless, different theoretical models imply starkly different predictions. The CC model has no predictable cash flow variation, so that the dividend yield variation is entirely driven by discount rate variation. The persistent time-varying mean

 $<sup>^{6}</sup>$ Note that when we use the concept of "equity premium" here, we refer to the summation of current and (expected) future equity premiums, as defined in Equation (17).

for consumption (and dividend) growth in BY naturally implies that cash flows constitute a more important fraction of equity yields variation, with the BY article claiming a roughly 50-50 split. Models that fit the data more closely such as Bekaert, Engstrom and Xing (2009) imply that discount rate variation dominates. Our confidence interval encompasses the estimates in the literature. For bonds, it is generally accepted that expected inflation is a dominant source of bond yield variation, although concrete estimates are actually hard to find. Ang, Bekaert and Wei (2008) report that 71 percent is accounted for by expected inflation, and indirect estimates by Mishkin (1990) and many others also suggest expected inflation is the dominant source of bond yield variation, especially at longer horizons. Again, our estimates are consistent with the extant literature.

With the equity premium the main driver of equity yields and expected inflation the main driver of bond vields, for the yields to comove so strongly, expected inflation, a nominal concept, must correlate highly with the equity premium, a real concept. This fact is confirmed in the covariance decomposition on the right side of Panel A. More than half of the comovement comes from the positive correlation between expected inflation and the equity premium. The other two relatively large contributors are the covariance between the real rate and the equity premium, which is positive and contributes 17 percent to the  $ey_t - by_t$  covariance, and the covariance between expected inflation and the cash flow component of the equity yield, which contributes 12 percent. The latter effect implies that expected inflation is on average positively correlated with periods of low cash flow expectations, as the cash flow component of the equity yield is negatively related to cash flow projections. This in itself already suggests that above-average inflation in the US has occurred often at times of depressed earning (and dividend) expectations. This effect is of course closely related to the "proxy hypothesis" of Fama (1981) and Kaul (1987), and shows that while it definitely plays a role, its explanatory power is rather limited. Finally, expected inflation and the real rate are positively correlated, which contributes 7 percent to the comovement between the bond and equity yield. While this number is small, it is relatively precisely estimated. This result is inconsistent with the well-known Mundell-Tobin effect that suggests a negative relation. However, our measures here are long-term (proxying for a 5 to 10 year horizon) and Ang, Bekaert and Wei (2008) also find a positive correlation between expected inflation and long-term real rates.

Looking at the last row of the covariance decomposition matrix, we note that 79 percent of the comovement between equity yields and bond yields comes through the equity premium, a residual in the equity yield decomposition. While it is tempting to conclude that irrational forces are at work, the next panel proves otherwise. In Panel B, we decompose the equity yield into a part spanned by risk aversion and uncertainty and an unspanned part. Note that the spanned part represents more than 66 percent of the total variation in the equity premium (53/(53+27)); in the spanned part the contributions of risk aversion and uncertainty are not statistically different from one another, with risk aversion accounting for 42 percent and uncertainty for the remainder of the variation. More importantly, 80 percent of what the equity premium explains of the total  $ey_t - by_t$  covariance comes from the spanned, rational part<sup>7</sup>. If we focus on  $COV(ey_t^{erp}, einf_t)$ , the expected inflation component, about 86 percent (51/59) can be ascribed to the rational component,  $COV(ey_t^{erp-sp}, einf_t)$  with the rest, potentially, coming from money illusion.

In panel C, we explore the comovements among equity yields, expected inflation, and the subjective earnings bias. On the left side, we see that the subjective earnings bias is barely correlated with either the equity yield or expected inflation. This suggests that subjective bias in cash flow expectations (1) is not an important driver of the equity yield and (2) does not comove strongly with expected inflation. Both of these effects are in sharp contrast with the assumptions of money illusion. Still, equity yields *are* highly correlated with expected inflation. In fact, we show the correlation to be 85 percent. On the right hand side of Panel C, we decompose this comovement because the Fed model puzzle essentially is due to the high correlation between expected inflation and equity premiums. The Panel shows that about 10 percent of their comovement comes from the positive correlation between expected inflation, 16 percent of the comovement can be ascribed to the negative correlation between expected inflation and cash flow expectations, but 66 percent can be ascribed to the fact that risk aversion and uncertainty are high in times of high expected inflation. The unexplained residual is a paltry 10 percent, which severely limits the potential role of money illusion.

Given previous results in the literature, our findings are perhaps surprising. For example, Campbell and Vuolteenaho (2004, CV henceforth) perform a closely related VAR-based analysis and interpret their findings as clearly suggestive of money illusion. How can their results be so different from ours? We believe there are four main reasons. First, CV treat cash flows as residuals. All unexplained variation is hence assigned to cash flow variation. In contrast, we attempt to measure cash flows directly and leave the equity premium as the residual component. We prefer the latter method because, although they are highly seasonal cash flows are clearly measurable. Second, CV measure the equity risk premium with a variable due to Cohen, Polk and Vuolteenaho (2005) that may be subject to considerable measurement error and is not, to date, widely used in the literature. Third, CV work directly in terms of excess returns, and therefore ignore one potentially important rational source of common variation in the two yield variables: real rates. Our results in Table 2 indicate that they therefore "miss" about 20 percent of the comovement between equity and bond yields. Finally, subsequent research has found that CV's results are not robust to the post-war subsample on which we focus (Wei and Joutz, 2007).

Finally, the positive correlation between the "equity premium piece" of the equity yield and expected inflation

<sup>&</sup>lt;sup>7</sup>Calculated as the sum of the first line in Panel B divided by the sum of the last line in Panel A (64/81).

may also appear, at first glance, inconsistent with an older literature showing that expected equity returns and (expected) inflation are negatively correlated, see Fama and Schwert (1977) and Fama (1981). However, our results are entirely consistent with the literature. What we call the "equity premium" for short is the sum of the current equity premium and all future premiums necessary to discount future cash flows (see the definition of  $ey_t^{erp}$  after Equation (17)). In Figure 3, we plot the different components of this sum. At lag 0, the correlation between expected inflation and the current equity premium is indeed negative, and this is the finding stressed in the extant literature. However, the correlation between expected inflation and expected future equity premiums quickly turns positive and obviously the sum of all these components correlates positively with expected inflation. It is also interesting to note that the negative short-term correlation is driven by the part of the equity premium not spanned by  $ra_t$  and  $vr_t$ , both of which correlate positively with expected inflation for our U.S. sample (see Table A2 in the Appendix).

## 4 International Results

We first motivate why it can be useful to examine international data and comment on our data sources. Then, we demonstrate how the cross-sectional variation in the correlation between bond and equity yields actually confirms our main hypothesis: high correlations stem from the incidence of periods in which high inflation and recessions (which drive up risk premiums) coincide.

### 4.1 Motivation

Our work analyzes one US based data set, with one history of inflation, bond yields and equity yields. Using this data set alone, it is hard to definitively exclude the money illusion story in favor of our story. International data offer an interesting out-of-sample test of our hypothesis. Essentially, we argue that the US experienced high correlations between equity yields and bond yields because higher inflation happened to occur during recessions, so that in recessions equity and bond premiums are both relatively high. In other words, the Fed model "works" in countries with a high incidence of stagflation.

Estrada (2009) shows that there is indeed substantial cross-sectional variation in the strength of the correlation between bond and equity yields across countries. He focuses on statistical problems in interpreting the correlations in a panel of international data. We now explore the possibility that 'stagflation incidence' accounts for part of the cross-sectional variations in stock-bond yield correlations using data similar to the Estrada sample. Specifically, we collect four variables for 20 countries over the period from December 1987 to June 2005. First, we use the dividend yield,  $ey_{i,t}$ , provided by Thomson for each country's equity index. The measure is not perfectly available, but 97 percent of all possible country-months are populated. We also use a long term risk free local currency nominal bond yield,  $by_{i,t}$ , from Thomson. Third, we measure the inflation rate for each country-month as reported by the local governments,  $infl_{i,t}$ . Where available, we use the continuously compounded change in the CPI index. If no such series is available for a particular country, we use the GDP deflator. If this variable is available only quarterly, we divide the quarterly inflation rate by three and use repeated values for months in that quarter. Finally, we measure real activity using the recession indicator  $recess_{i,t}$  published by the Economic Cycle Research Institute, which provides monthly indicator series for the incidence of recession. Where recession indicators are not available (8 countries and in 2005 for all countries), we define recessions as two consecutive quarters of negative real GDP growth.

## 4.2 Cross-Country Analysis

We start with a heuristic analysis of the cross-sectional association between "Fed model effect intensity" and "stagflation intensity." To capture the intensity of the Fed model effect, we compute the time series correlation between the dividend yield and the nominal long bond yield for each country. To measure the intensity of stagflation for a country, we similarly compute the time series correlation of the recession indicator with inflation for each country. Figure 3 plots each country along these two dimensions. Although there are only 20 country observations, a positive relationship seems evident. In fact, the cross-sectional correlation between Fed model intensity and stagflation intensity on this plot is 0.50, and significant at the 5 percent level (not accounting for the sampling uncertainty in the time series correlations). Moreover, a cross sectional OLS regression of Fed model intensity on stagflation intensity produces a positive slope coefficient of 1.35 which is also significant at the 5 percent level (again, not accounting for the sampling uncertainty in the time series correlations). The significance of the slope coefficient is robust to the (sequential) exclusion of Japan and Austria, potential outliers. We interpret these results as supportive of a positive relationship. The relationship exists even though the U.S. itself has not exhibited stagflation in the post-1987 sample while retaining a high  $by_t - ey_t$  correlation.

We add more statistical formality to this analysis by estimating two sets of cross-sectional regressions with the cross-section of countries' stock-bond yield correlations as the dependent variable. The results for both sets of regressions are reported in Table 3. The first regression set (numbers on the left of the table) focuses on the incidence of stagflation, defined as the percent of observations where a recession occurs simultaneously with high inflation. Our cut off value for high inflation is 10 percent, but we also conducted the analysis using an inflation level of 5 percent as the cut-off with largely similar results. Regression (3) shows that stagflation by itself has a huge effect on the equity –bond yield correlation: a country with 1 percent higher stagflation incidence than the average has a 21 percentage point higher equity-bond yield correlation. Of course, the stagflation effect could be due to its separate components, recession or simply inflation. Regressions (1) and (2) show that the percent of high inflation months by itself does increase the equity yield-bond yield correlation whereas a high frequency of recessions actually reduces it, but the latter effect is not significant. Regression (4) includes all three dependent variables in one regression. This regression provides a nice test of our stagflation story versus just money illusion. If money illusion drives the correlation, the coefficient on inflation should be significant, but there is little reason for stagflation to have a particular effect on the bond-equity yield correlation. However, we find that inflation has an insignificant effect on the correlation. The recession effect is still negative but not significant, and the stagflation effect is large and significantly different from zero. While the associated t-statistic is large, the regression suffers from three econometric problems. First, the sample is small (20 observations). Second, the regressors and regressands involve pre-estimated statistics. Third, the different observations arise from correlated time series. Therefore, we conduct a Monte Carlo analysis, described in detail in the Appendix 7.3, and generate a small sample distribution for the t-statistics in the regressions. This Monte Carlo analysis uses the asymptotic variance-covariance matrix for estimating the independent and dependent variables in the regression to draw new regression variables and it imposes the null hypothesis of no cross-sectional dependence. Significant t-statistics according to the small sample distribution are indicated with asterisks. The stagflation coefficient remains significant when using the small sample distribution for the t-statistics.

The second set of regressions, replace "high inflation incidence" by average inflation, and "stagflation" by the interaction of inflation and the recession indicator. The univariate regression, Regression (5), reveals that countries with high average inflation do have significantly higher equity yield-bond yield correlations, but when this variable is added to a regression that includes the inflation-recession interaction, Regression (7), the direct effect of inflation disappears. The inflation-recession interaction comes in very significantly and the significance survives at the 5 percent level under the small sample distribution. The direct effect of the frequency of recessions continues to be negative but insignificant.

## 5 Robustness Checks

The first three sub-sections describe a set of robustness exercises against which we have tested our main results in Table 2. The final subsection focuses on the robustness of the international results.

## 5.1 VAR Specification

The results in Table 2 are essentially unchanged under four alternative VAR specifications. Results for all our robustness exercises are reported in Table 4. We only focus on the critical statistics from Table 2: the percent contribution of the covariance between expected inflation and the equity premium to the total yield covariation, and the percent contribution of the covariance between expected inflation and the non-spanned, residual part of the equity premium,  $erp_t^{re}$ . For ease of comparison, the first line repeats the results from the main VAR reported in Table 2. First, given the VAR specification tests reported earlier, we repeat the analysis using a VAR(2) data generating process. The results in Table 2 are essentially unchanged. Our second and third experiments focus on the fact that with a VAR of large dimension relative to the sample size, insignificant coefficients could affect the statistics of interest. Our bootstrapping procedure for calculating standard errors should address this issue to a large extent, but we also conduct two exercises to directly verify the robustness of the point estimates. First, we calculate the results presented in Table 2 after zeroing-out any element of A which has an OLS t-statistic less than one. Second, we repeat the calculations using a smaller VAR excluding the information variables, that is dropping  $x_t$ . This procedure of course precludes us from decomposing the equity risk premium and calculating the subjective earnings bias. Under both experiments, the results of Panel A of Table 2 are essentially unchanged. Finally, we also use an alternative economic uncertainty proxy that is directly derived from BY's article (see data appendix). The contribution of the covariance between expected inflation and the equity yield decreases and the relative contribution of the covariance between expected inflation and the residual equity premium increases somewhat. However, this is mostly due to the limited ability of the BY-based uncertainty measure to help span the equity premium component of the equity yield.

#### 5.2 Bond Yield Decomposition

We conduct three exercises to check the robustness of results to alternative bond yield decompositions, with our results remaining materially unaffected in each case. First, we add an additional information variable to the VAR, a measure of inflation uncertainty based on SPF data (using a procedure similar to that which we used for real uncertainty). Second, we substitute a longer-term measure of survey-based inflation expectations (our standard measure looks ahead only four quarters) as our measure of expected inflation. The longer-term measure is not available early in the sample, so we must first filter its early values (see data appendix for a description of this procedure). Third, we use a completely different measure of the real rate, by assuming we can measure the inflation risk premium directly – as proportional to inflation uncertainty.

inflation expectations and a constant times inflation uncertainty from nominal rates. We use the residual as an alternative real rate measure. We choose the constant of proportionality to match the unconditional mean of the real rate to that of our standard measure from Ang, Bekaert, and Wei (2008).

#### 5.3 Cash Flow Measurement

We use two alternative measures of the cash flow from equity. First, we use earnings instead of dividends, both for constructing cash flow growth and calculating the equity yield. That is, we now investigate the earnings yield. We are motivated to do this, in part, because practitioners overwhelmingly focus on earnings as the unit of fundamental analysis for equity valuation. However, to do formal analysis using earnings in the CS framework, we make the not-entirely satisfactory assumption of a constant payout ratio. The results for earnings-based equity yields are largely consistent with our main results. (1) The stock-bond yield covariance is very high, (2) the majority of the comovement comes through the covariance of the equity yield with expected inflation, and (3) very little of the covariance involves the  $ey_t^{erp-re}$  component of the equity yield. One difference from our main results is that the contribution of  $COV\left(ey_t^{\Delta d}, einf_t\right)$  to the total  $ey_t - by_t$  covariance is substantially larger when using earnings rather than dividends, accounting for 41 percent of the covariance versus just 12 percent under our baseline VAR as reported in Table 2. Hence, rather than the covariance between expected inflation and the equity risk premium being the main driver for the stock-bond yield covariance, it is now comovement between expected inflation and expected cash flow growth. This is consistent with Fama's (1981) proxy hypothesis. Nevertheless, even if this is the correct interpretation of the data, stagflation remains a critical ingredient: Inflation happens to occur at times of depressed earnings expectations. Note that we use objective. not subjective, earnings forecasts, so that this cannot be caused by money illusion.

Second, we add repurchases to dividends in calculating cash flow, because repurchases have been an important channel by which companies have returned cash to shareholders in the past few decades, and this can have important asset pricing implications (see Boudoukh et al, 2007). The correlation of the resulting equity yield measure with the bond yield remains positive but not statistically significant. This owes to the fact that repurchases have, on a quarterly basis, been extremely volatile, especially over the past few years. The point estimates of our main results are broadly similar to those presented in Table 1, but the estimates of all the  $ey_t - by_t$  covariance components are very imprecisely estimated and none are individually statistically different from zero. While this is a disappointing result, it is likely similarly due to the excessive volatility of repurchases.

#### 5.4 International Results

For robustness to our use of dividends as the relevant equity cash flow in the international data, we also conduct the analysis using one year-ahead analyst-expected earnings in calculating the equity yield. This change does not affect the results of Table 2 very much. Finally, because the dependent variable in the cross-sectional regressions are correlations and thus limited to the interval [0, 1], we conducted the OLS regressions using a transformation of the correlation,  $\ln(1 + corr.) / \ln(2 - corr.)$ , which effectively spreads the range of the dependent variable to  $(-\infty, +\infty)$ . The OLS t-statistics using this transformation are very similar to those reported in Table 2.

## 6 Conclusion

In this article, we re-examine potential explanations for the surprisingly high correlation between the "real" equity yields and nominal bond yields in US post-war data. We show that the prevailing explanation, money illusion, actually has rather limited explanatory power. We ascribe a large part of this covariation to the rather high incidence of stagflations in the US data. We postulate that in recessions economic uncertainty and risk aversion may increase leading to higher equity risk premiums, which, in turn, increase yields on stocks. If expected inflation happens to also be high in recessions, bond yields will increase through their expected inflation and, potentially, their inflation risk premium components, and positive correlations emerge between equity and bond yields and inflation. We establish this result using a VAR methodology that uses measures of inflation expectations and two proxies for rational variation in risk premiums, one based on economic uncertainty, one based on the habit model formulated by Campbell and Cochrane (1999). Our confidence in these findings is bolstered by a cross-country analysis that demonstrates that "stagflation incidence" accounts for a significant fraction of the cross-sectional variation in equity - bond yield correlations.

Our findings have potentially important policy implications. If money illusion afflicts pricing in the stock market, inflation stabilization also helps prevent distortions and mis-pricing in the stock market. If money illusion does not affect the stock market, the Federal Reserve's inflation policy has no bearing on the equity market beyond its implications for real economic growth.

To conclude, we point out that the 2008-2009 crisis period is consistent with our interpretation of the data. This period witnessed extremely low correlations between equity yields and bond yields. Given our main hypothesis, this is to be expected, as this period experienced a recession (and hence high equity premiums) but coupled with subdued inflation pressures and thus low expected inflation.

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## 7 Appendices

## 7.1 US Data

The empirical work uses quarterly data over 1968Q4-2007Q4. This section describes our data construction and notation.

#### 7.1.1 Stock and Bond Data

The equity data we use are based on the S&P 500 index. We measure dividends, earnings and repurchases on a quarterly, per-share, seasonally adjusted basis, and price on a quarter-end, per-share basis. The earnings are "as reported" prior to 1985, and "operating" thereafter. Repurchase data are available quarterly from Standard and Poors beginning only in 2001Q2. Prior to that, we estimate repurchases by using estimates (from Boudoukh, et al 2007) of the annual ratio of repurchases to dividends for the Compustat universe, applying this ratio to quarterly dividend series for S&P 500 firms.

We take the quarter-end yield on a constant maturity nominal 10-year Treasury coupon bond from the St. Louis Fed FRED webpage, and estimates of the real risk-free long-term rate provided by Ang, Bekaert and Wei (2007). The rate yield data end in 2004. To extend the series, we filter the missing values using the Kalman filter, assuming a stable VAR describes the comovements of real yields, nominal yields, expected inflation, and inflation uncertainty.

#### 7.1.2 Real Growth Uncertainty

We use two imperfect SPF measures of uncertainty about future real growth to generate a real uncertainty index. First, respondents are asked to report their subjective assessment of the probability of negative real GDP growth over the next quarter. Assuming a binomial distribution for real GDP growth (+1.0 percent growth in expansion, -0.5 percent growth in contractions), we calculate the implied standard deviation of real growth for each respondent and then take the cross-sectional average in each quarter. This measure is denoted  $sd_t$ . The second measure we use is the dispersion in respondents' expectation for real GDP growth over the next four quarters. The dispersion measure we use is the difference between the 90th percentile and the 10th percentile of all responses, and is denoted  $dp_t$ . To aggregate these two measures, we assume that "true" uncertainty,  $vr_t^*$ , follows an AR(1) process, and both empirical measures are noisy indicators of  $vr_t^*$ .

$$\begin{array}{lll} vr_t^* &=& b\,vr_{t-1}^* + \varepsilon_t^{vr*} \\ \left[\begin{array}{c} sd_t \\ dp_t \end{array}\right] &=& \left[\begin{array}{c} f^{v\pi} \\ f^{d\pi} \end{array}\right] vr_t^* + \left[\begin{array}{c} \sigma^{sd}\varepsilon_t^{sd} \\ \sigma^{dp}\varepsilon_t^{dp} \end{array}\right] \end{array}$$

where all variables are demeaned and  $[\varepsilon^{vr*}, \varepsilon^{ds}, \varepsilon^{dp}]$  are distributed i.i.d. N(0, I). Conditional (not smoothed) filtered estimates for  $vr_t^*$  are easily estimable by standard Kalman filter methods. We make no attempt to correct for the filtering error.

#### 7.1.3 Bansal Yaron (2004) Volatility Measure

We also create an alternative  $vr_t$  process that is directly-based on the model and parameters in BY. Under BY, the consumption growth process is:

$$\Delta c_{t+1} = \mu_c + \phi x_t + \sigma_t u_{t+1}$$

$$x_t = \rho_x x_{t-1} + \varphi_e \sigma_t e_{t+1}$$

$$\sigma_t^2 = \sigma^2 + \nu_1 \left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1}$$

We treat the system above as a stochastic mean and volatility model that fits into the general state-space form: The top equation is the measurement equation, and the latter two equations are the state equations. We use this model to filter estimates of  $x_t$  and  $\sigma_t^2$  using a nonlinear Kalman filter. We use the BY calibrations of all the system parameters (stated at quarterly rates), and use quarterly NIPA consumption data. The filtered values  $\sigma_t^2$  are treated as the BY volatility series.

#### 7.1.4 Inflation Data

We measure expected inflation using the Survey of Professional Forecasters (SPF). Specifically, in our main results, we use the median survey response for the four-quarter ahead percent change in the GDP price deflator. As a robustness check in Table 4, we use the forecast of the 10-year annualized average rate of CPI inflation which is only available since 1980 (to complete the sample, we filter the early sample values using the Kalman filter, assuming four-quarter inflation expectations, long-term inflation expectations, long term nominal rates, and long term real rates evolve according to a stable VAR). We use actual inflation to deflate the equity cash flows. For this we use the GDP deflator (for consistency with the SPF forecast) published by the BEA. We also measure inflation uncertainty using SPF responses in a manner exactly analogous to that used for the construction of the real uncertainty measure. The first of two indicators we have is the mean variance of one year-ahead inflation as measured over histograms filled out by SPF respondents. The second measure we use is the dispersion in respondents' expectation for real inflation growth over the next four quarters.

#### 7.1.5 Subjective Profit Growth Expectations

We measure subjective profit growth expectations using the Survey of Professional Forecasters (SPF). Specifically, we use the median survey response for the four-quarter ahead percent change in the NIPA measure of nominal corporate profits. To calculate a real profit growth measure, we subtract, at the respondent level, the four-quarter rate of expected GDP deflator inflation.

#### 7.1.6 Habit-Based Risk Aversion

We construct a habit-based model of local relative risk aversion following Campbell and Cochrane (1999, CC hereafter). CC use a model of external habit to motivate stochastic risk aversion, the log of which we denote as  $ra_t$ . Risk aversion is a function of the log 'surplus consumption' ratio,  $s_t$ ,

$$ra_t = \ln\left(\gamma\right) - s_t \tag{23}$$

where  $\gamma$  is the instantaneous utility curvature parameter, and the surplus consumption ratio is:

$$s_t = \ln\left(\left(C_t - H_t\right)/C_t\right) \tag{24}$$

where  $C_t$  is real nondurable consumption and  $H_t$  is the 'habit stock' which is roughly speaking a moving average of past consumption levels. Rather that modelling  $H_t$  directly, CC model  $s_t$  as an autoregressive, heteroskedastic process which is perfectly (conditionally) correlated with consumption growth innovations,  $\varepsilon_t^c$ 

$$s_{t} = (1 - \phi)\overline{s} + \phi s_{t-1} + \lambda_{t-1}\varepsilon_{t}^{c}$$

$$\lambda_{t} = \begin{cases} \frac{1}{\overline{s}}\sqrt{1 - 2(s_{t} - \overline{s})} - 1 & s_{t} \leq s_{\max} \\ 0 & s_{t} > s_{\max} \end{cases}$$

$$(25)$$

where the parameters,  $\gamma, \phi, \overline{s}, \overline{S}$  and  $s_{\max}$  are calibrated by CC to fit several salient features in the data. We use the parameter values in CC to create our empirical proxy for  $ra_t$ . The innovation term,  $\varepsilon_t^c$ , is the shock to consumption growth, and following CC we use demeaned values for real nondurables and services consumption log growth from the NIPA tables. The sensitivity of  $s_t$  to  $\varepsilon_t^c$  is governed by the  $\lambda_t$  process, which is always non-negative. Consequently, risk aversion tends to behave counter-cyclically. Because the starting point of  $s_t$ is not specified, we start the process at its unconditional mean,  $\overline{s}$ , at the beginning of the consumption growth sample, 1947Q2. Given that our analysis only starts in 1968Q4, the level of  $s_t$  is not sensitive to that choice.

### 7.2 Bootstrapping Procedure for Vector Autoregressions

The procedure we employ is as follows. Recall that the VAR we estimate on observed data is

$$W_t = \mu^w + A^w W_{t-1} + \Sigma^w \varepsilon_t \tag{26}$$

- 1. Calculate, by OLS, point estimates for the VAR parameters,  $\widehat{\mu_0^w}$ ,  $\widehat{A_0^w}$ , and  $\widehat{\Sigma_0^w}$  using the raw data. Also extract values for the residuals,  $\{\widehat{\varepsilon}_t\}_0$
- 2. Calculate all the reported statistics as  $\widehat{\Psi_0}$
- 3. For 10,000 iterations indexed by i
  - (a) randomly select the vector  $\{\hat{\varepsilon}_t\}$  (with replacement) across time to generate  $\{\hat{\varepsilon}_t\}_i$
  - (b) Generate a simulated sequence for  $\{W_t\}_i$  under the assumed VAR data generating process and the shuffled innovations,  $\{\hat{\varepsilon}_t\}_i$ , beginning the  $\{W_t\}_i$  sequence at the first data observation,  $W_1$
  - (c) Calculate, by OLS, point estimates for the VAR parameters,  $\widehat{\mu_i^w}, \widehat{A_i^w}$ , and  $\widehat{\Sigma_i^w}$  using the drawn data,  $\{W_t\}_i$ .
  - (d) Calculate all the reported statistics as  $\widehat{\Psi_i}$
- 4. Report a confidence interval for  $\widehat{\Psi_0}$  as the spread between the 95th and 5th percentile across  $\widehat{\Psi_i}$  draws.

## 7.3 Monte Carlo Procedure for Country Cross-Sectional Regressions

The panel data set is comprised of monthly observations of  $ey_{i,t}$ ,  $by_{i,t}$ ,  $\pi_{i,t}$ , and  $recession_{i,t}$  (as defined in the text) from December 1987 through June 2005 for 20 countries. The regressions we report in Table 3 are of the

form,

$$corr_i \left( ey_t, by_t \right) = a + b \,\overline{infl_i} + c \, recess_i^{percent} + d \, \left( \overline{infl_i} \cdot recess_i^{percent} \right) + u_i \tag{27}$$

where  $corr_i (ey_t, by_t)$  is the time-series correlation between ey and by for country i,  $infl_i$  denotes the fullsample country-specific mean of inflation and  $recess_i^{percent}$  denotes the percentage of observations during which the country was in recession. OLS statistics may be poorly behaved in this regression given (1) the small sample of 20 countries, (2) sampling error in the generated regressors and regressand, and (3) the presence of limited dependent variables (correlations are confined to the unit interval). To account for this, we report OLS coefficients and t-ratios in Table 3, but then use the following Monte Carlo procedure to assess the significance of the results.

First, we use the panel data to calculate estimates (and an estimate of their covariance matrix) for the vector,

$$\left\{corr_i\left(ey, by\right), \overline{infl_i}, recess_i^{percent}, \overline{infl_i} \cdot recess_i^{percent}\right\}_{i=1}^{20}.$$
(28)

That is, we jointly estimate 80 statistics: four for each of 20 countries. We use standard GMM techniques allowing for generalized heteroskedasticity and autocorrelation and assume that these estimates are well-behaved<sup>1</sup>. From these estimates and covariance matrix, we generate 10,000 draws from the associated normal distribution. For each draw, we run the OLS regression in Equation (27) and examine the properties of the OLS t-ratios. However, our aim is to simulate the data under the null hypothesis that none of the explanatory variables are related to  $corr_i (ey_t, by_t)$  in the cross-section. Note that the null hypothesis will not necessarily hold in the draws (for instance, if Country X has a high  $corr_i (ey_t, by_t)$  and high  $infl_i$ , in the data sample, this information will be preserved, in expectation, for every draw). To impose the null, we randomize the matching of  $corr_i (ey_t, by_t)$  with  $[infl_i, recess_i^{percent}, infl_i \cdot recess_i^{percent}]$  cross-sectionally for each draw. For instance, Country X's  $corr_i (ey_t, by_t)$  draw is randomly reassigned to Country Y's draw of the triple,  $[infl_i, recess_i^{percent}, infl_i \cdot recess_i^{percent}]$ . In this way, relationships among the explanatory variables are preserved, but the null hypothesis holds in expectation for every draw.

For each simulated regression, we collect t-ratios for each regression coefficient. We then count the number of times the simulated t-ratios exceed the sample OLS t-ratios. If the portion of simulated t-ratios exceeding the sample t-ratio is greater than 10 percent, we conclude that the estimate is insignificant. If the portion of simulated t-ratios which exceed the sample t-ratio is greater than 5 percent, but less than 10 percent, we conclude

<sup>&</sup>lt;sup>1</sup>This may be justified by noting that the data used for the estimates are comprised of about 240 monthly observations of 4 series  $(ey, by, \pi, recess)$  over 20 countries, or about 19,000 data points, whereas the 80 estimates and covariance matrix require 80 + 80\*81/2 or about 3000 parameters. The saturation ratio is therefore about 6.

that the estimate is significant at the 10 percent level, etc.

Table 1: VAR Specification Tests

Panel	Panel A: VAR lag length								
	VAR(1)	VAR(2)	VAR(3)	VAR(4)					
BIC	-73.5	-72.3	-70.7	-70.0					
AIC	-75.2	-75.7	-75.7	-75.6					
Panel	B: Cumby-	Huizinga te	ests (p-value	es)					
	VAR	2(1)	VAR(2)						
$einf_t$ ,	0.4	4	0.38						
$rrf_t$	0.0	1	0.26						
$\Delta d_t$	0.0	4	0.04						
$ra_t$	0.0	0	0.08						
$vr_t$	0.0	0	0.57						
$\Delta ern_t$	0.3	7	0.21						
$gern_t^s$	0.0	4	0.01						
$ey_t$	0.7	9	0.79						
$by_t$	0.7	1	0.44						

Results in this table are based on the observable VAR,  $W_t = \mu^w + A^w W_{t-1} + \Sigma^w \varepsilon_t$ , where  $W_t = [einf_{tt}, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$ and  $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]$ .' Panel A presents information criteria for optimal VAR lag length. The row labeled BIC contains standard Schwartz test results and the row labeled AIC reports results for the Akaike test. In Panel B, p-values for Cumby-Huizinga (1987) tests for residual autocorrelation are presented. Each VAR equation is tested separately. We test for autocorrelation at up to four lags.

Table 2: U.S. VAR Results

	$VAR(by_t)^* \\ 0.45 \\ (0.20, 0.60)$		$VAR(ey_t) \\ 0.63 \\ (0.35, 0.80)$		$\begin{array}{c} COV\left( by_{t},ey_{t}\right) ^{*}\\ 0.22\\ (0.03,0.43)\end{array}$	$CORR(by_t, ey_t) \\ 0.78 \\ (0.37, 0.90)$	
			Frac	tional C	ontributions		
	$VAR\left( by_{t} ight)$		$VAR\left( ey_{t} ight)$		$einf_t$	$COV(by_t, ey_t) \\ rrf_t$	$irp_t$
$einf_t$	0.55	$ey_t^{\Delta d}$	0.14	$ey_t^{\Delta d}$	0.12	0.02	0.00
$rrf_t$	(0.28, 0.71) 0.22	$ey_t^{rrf}$	(-0.10, 0.40) 0.07	$ey_t^{rrf}$	(-0.09, 0.42) 0.07	(-0.06, 0.08) 0.02	(-0.13, 0.13) 0.00
'' Jt	(0.18, 0.27)	$e y_t$	(0.02, 0.11)	$eg_t$	(0.03, 0.11)	(0.01, 0.03)	(-0.02, 0.03)
$irp_t$	0.22	$ey^{erp}$	0.80	$ey^{erp}$	0.59	0.17	0.03
	(0.09, 0.48)		(0.52, 1.07)		(0.21, 1.16)	(0.05, 0.26)	(-0.40, 0.26)

Panel A: Decomposing Yield (Co-)Variation

Panel B∙	Decomposing	$eu^{erp}$	into $eu^{erp-sp}_{t}$	and $eu_{i}^{erp-re}$
I and D.	Docomposing	$C_{H}$	$mo cq_4$	and $cq_4$

Fractional Contributions									
	$VAR\left( ey_{t}\right)$			$COV(by_t, ey_t)$					
			$einf_t$	$rrf_t$	$irp_t$				
$ey^{erp-sp}$	0.53	$ey_t^{erp-sp}$	0.51	0.13	0.00				
	(0.13, 0.76)		(0.15, 0.95)	(0.06, 0.18)	(-0.34, 0.13)				
$ey^{erp-re}$	0.27	$ey^{erp-re}$	0.08	0.05	0.03				
	(0.12, 0.73)		(-0.01, 0.36)	(-0.02, 0.16)	(-0.19, 0.22)				

Panel C: Equity Yields, Expected Inflation and Subjective Earnings Expectations Biases

Correl	Correlations		Fractional Contributions to $einf_t$		
$einf_t - bias_t$	-0.04 (-0.35, 0.27)	$ey_t^{\Delta d}$	$ey_t^{rrf}$	$ey^{erp-sp}$	$ey^{erp-re}$
$einf_t - ey_t$	0.85	0.16	0.09	0.66	0.10
	(0.48, 0.93)	(-0.11, 0.53)	(0.04, 0.15)	(0.25, 0.89)	(-0.01, 0.38)
$bias_t - ey_t$	0.02				
	(-0.29, 0.34)				

Results in this table are based on the latent VAR,  $Y_t = \mu + AY_{t-1} + \Sigma \varepsilon_t$ , where  $Y_t = [einf_{tt}, rrf_t, \Delta d_t, erp_t, irp_t, x_t]'$  and  $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]'$ ,  $\varepsilon \sim (0, I)$  and  $irp_t$  and  $erp_t$  are unobserved. The  $Y_t$  system parameters are derived from VAR estimates on the observable vector  $W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$  using the data and methodology described in Section 2 and the Appendix. The procedure for decomposing  $ey_t$  and  $by_t$  into their component pieces (e.g.  $ey_t^{\Delta d}$  for  $ey_t$ , and  $rrf_t$  for  $by_t$ ) is described in Section 2 as is the procedure for decomposing  $ey_t^{erp}$  into parts spanned-by and orthogonal-to proxies of rational equity risk premiums. Bootstrapped 90 percent confidence intervals are reported in parentheses. \* denotes that the reported statistic has been multiplied by 100 for readability.

Table 3: Cross-Country Results

Specification	$hinfl_i^{percent}$	$recess_i^{percent}$	$stag_i^{percent}$	$\overline{infl}_i$	$\overline{infl \ rec}$	$R^2$
<u></u>	<i>J</i> -1	11	<i>3</i> 1		···j·_·	
(1)	3.95					0.07
	$(1.10)^*$					
(2)		-0.40				0.01
		(0.43)				
(3)			21.37			0.23
			$(2.24)^{**}$			
(4)	-0.68	-1.59	30.52			0.37
	(0.19)	(1.70)	$(2.55)^{***}$			
(5)				3.06		0.32
(-)				$(2.74)^{**}$		
(6)					8.78	0.41
		1.07		0 50	$(3.38)^{***}$	0 70
(7)		1.25		-0.50	7.93	0.52
		(0.62)		(0.37)	$(1.85)^{**}$	

This table presents results for cross-sectional regressions of the general form

$$corr_i (ey_t, by_t) = a + b hinfl_i^{percent} + c \, recess_i^{percent} + d \, stag_i^{percent} + u_i$$
<sup>(29)</sup>

and

$$corr_i(ey_t, by_t) = a + b \overline{infl_i} + c \, recess_i^{percent} + d \, \overline{infl\_rec} + u_i$$

where  $by_t$  is the locally nominally risk free long bond yield for country i at time t and  $ey_t$  is the dividend yield. The variable  $corr_i(ey_t, by_t)$  is the time-series correlation between  $ey_t$  and  $by_t$  for country i. The variable  $hinfl_i^{percent}$  denotes the percentage of observations during which the country exhibited high inflation, defined as 10 percent or more (annualized) inflation per month. The variable  $recess_i^{percent}$  denotes the percentage of observations during which the country exhibited recess<sub>i,t</sub>). The variable  $stag_i^{percent}$  denotes the percentage of observations during which the country exhibited stagflation, defined as the coincidence of high inflation and recession. The variable  $infl_i$  denotes the full-sample country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the full-sample country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the full-sample country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the full-sample country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the full-sample country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the full-sample country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the country-specific mean of inflation,  $infl_{i,t}$ . The variable  $infl_i$  denotes the country-specific mean of the interaction,  $infl_{i,t} \cdot recess_{i,t}$ . Data are monthly from 1987-2005 for 20 countries. OLS coefficients and t-ratios (in parentheses) are reported. The superscripts \*, \*\* and \*\*\* denote significance at the 10, 5, and 1 percent level. Significance is determined using corrections for the small sample and pre-estimation effects of the regressors and regressand utilizing a Monte-Carlo method detailed in the appendix.

Specification	$COV\left(einf_{t},ey_{t}^{erp}\right)$	$COV\left(einf_t, ey_t^{erp-re}\right)$
Main VAR	0.59	0.08
	(0.21, 1.16)	(-0.01, 0.36)
VAR(2)	0.58	0.06
	(0.24, 1.08)	(-0.04, 0.25)
Small VAR	0.47	-NA-
	(0.19, 1.05)	-NA-
Zeroed-out	0.56	0.08
	(0.22, 1.14)	(-0.01, 0.15)
alternative $vr_t^*$ measure	0.30	0.13
-	(-0.03, 0.69)	(0.00, 0.75)
w/inflation uncertainty	0.57	0.07
	(0.18, 1.11)	(-0.01, 0.30)
long-term inflation exp.	0.47	0.08
	(0.19, 0.88)	(0.00, 0.34)
alternative real rate	0.58	0.08
	(0.15, 1.08)	(-0.03, 0.28)
$\cosh flow = earnings$	0.42	0.10
	(-0.20, 1.21)	(-0.20, 1.21)
$\cosh flow = \operatorname{div} + \operatorname{repo}$	0.36	0.35
	(-3.79, 4.78)	(-1.29, 2.37)

 Table 4: U.S. VAR Robustness Exercises

Percent Contribution to  $ey_t - by_t$  Covariance under Alternative Specifications

This table reports two key statistics (and their confidence intervals) reported for our main specification in Table 1 under a variety of alternative VAR specifications. The "Main VAR" row simply reproduces the statistics of interest from Table 1: the percent contribution to total  $ey_t - by_t$  covariance of COV ( $einf_t, ey_t^{erp}$ ) and COV ( $einf_t, ey_t^{erp-re}$ ). The "VAR(2)" specification expands the Main VAR to include two lags of all the dependent variables. The "Small VAR" specification drops the  $x_t$  vector from the VAR list (without  $x_t$ , the COV ( $einf_t, ey_t^{erp-re}$ ) contribution cannot be calculated). The "Zeroed-out" specification employs a two-step estimation procedure for our main VAR: first estimate the VAR by OLS, noting all elements of  $A^W$  with OLS t-statistics less than 1. In the second step, re-estimate the VAR imposing that the low t-statistic coefficients are zero. The "alternate  $vr_t^*$ " line replaces the measure of real uncertainty in the observable VAR to a measure filtered from actual consumption growth using the consumption growth model of Bansal and Yaron (2004) and a nonlinear Kalman filter. The "w/inflation uncertainty" specifications "specification replaces our usual four-quarter expected inflation measure with a longer-term survey-based inflation expectations measure (see data appendix). The "cash flow = earnings" specification replaces the dividend yield and dividend growth in the Main VAR with earnings growth and the earnings-price ratio. The "cash flow = div + repo" specification adds repurchases to dividends before calculating dividend growth and the dividend yield.

Figure 1: Equity and Bond Yield Time Series for the U.S.



This figure plots time series for the equity yield,  $ey_t$  (blue, left scale), and the bond yield,  $by_t$  (green, right scale). We measure the equity yield,  $ey_t$  as the dividend yield for the S&P500, and the nominal bond yield,  $by_y$ , as that of the 10-year constant-maturity Treasury. For illustration, both yields have been plotted in levels (that is, the  $ey_t$  series has been exponentiated), and in units of percentage points at annual rates.

Figure 2: Risk Aversion and Real Uncertainty



This figure plots time series for risk aversion,  $ra_t$  (blue, left scale), and real uncertainty,  $vr_t$  (green, right scale). Data construction is described in the appendix.

Figure 3: Term Structure of Correlation between Expected Excess Equity Returns and Expected Inflation



This chart plots  $corr\left(E_t\left[erp_{t+j}\right], einf_t\right)$  as a function of j = 0, ...80. Results in this table are based on the latent VAR,  $Y_t = \mu + AY_{t-1} + \Sigma \varepsilon_t$ , where  $Y_t = [einf_t, rrf_t, \Delta d_t, erp_t, irp_t, x'_t]'$  and  $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]'$ ,  $\varepsilon \sim (0, I)$  and  $irp_t$  and  $erp_t$  are unobserved. The  $Y_t$  system parameters are derived from VAR estimates on the observable vector  $W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x'_t]'$  using the data and methodology described in Section 2 and the Appendix.



Figure 4: Multi-Country Relationship between Stagflation and the Fed Model

This figure plots countries in the panel data set along two dimensions: (1) the country specific time-series correlation between the dividend yield and the long term (locally risk free) nominal bond yield, and (2) the time series correlation between inflation and a recession indicator. The sample is monthly from December 1987 through June 2005. The slope of the regression line is 1.35 with an OLS standard error of 0.59. A regression (line not shown) estimated excluding the Japan (Austria) observation has a slope of 1.04 with an OLS standard error of 0.54 (1.10 with a standard error of 0.66).

	coef	t - NW	t - H92
1-quarter	$\operatorname{growth}(I$	$R^2 = 0.03,$	pval = 0.02)
const	0.0082	0.79	0.88
$\Delta ern$	0.1604	1.09	1.07
$gern_t^s$	0.3013	2.24	2.21
4-quarter	growth (1	$R^2 = 0.11,$	pval = 0.07)
const	0.0103	0.98	1.23
$\Delta ern$	0.1492	1.75	2.59
$gern_t^s$	0.2020	1.49	1.79

Table A1: Dividend Growth Predictability

Results in this table are from regressions investigating the predictability of dividend growth,  $\Delta^i d_t$ , with respect to lagged realized earnings,  $\Delta ern_t$ , and survey forecasts of earnings,  $gern_t^s$ . We simultaneously investigate predictability at horizons  $i = \{1, 4\}$ . Specifically, we estimate the system,

$$\begin{bmatrix} \Delta^1 d_{t+4} \\ \Delta^4 d_{t+4} \end{bmatrix} = a_d + b_d \begin{bmatrix} \Delta ern_t \\ gern_t^s \end{bmatrix} + u_{t+4}$$
(30)

where  $\Delta^i$  denotes the  $i^{th}$  difference operator and all differences are calculated at an annual rate. The matrices  $a_d$  and  $b_d$  contain the parameters to be estimated. We use OLS to calculate point estimates and present two alternative sets of t-statistics. The first, reported in the column labeled t - NW reports Newey West (1987) based t-statistics using 4 Newey West lags. The second, labeled t - H92, reports Hodrick (1992) t-statistics. The  $R^2$  statistics are calculated in the usual manner, while the equation-by-equation *pvals* statistics report the equation-by-equation joint significance tests for the instruments and are based on the Hodrick (1992) estimate of the parameter covariance matrix.

	$einf_t$	$rrf_t$	$\Delta d_t$	$ra_t$	$vr_t$	$\Delta ern_t$	$gern_t^{su}$	$erp_t$	$irp_t$
Std.	0.0043	0.0016	0.0311	0.1807	0.0863	0.0816	0.0137	0.0280	0.0028
Dev.	(0.0043)	(0.0010)	(0.0076)	(0.1212)	(0.0763)	(0.0236)	(0.0054)	(0.0165)	(0.0014)
Auto	0.98	0.95	-0.33	0.95	0.99	0.05	0.80	0.69	0.82
Corr.	(0.04)	(0.09)	(0.25)	(0.09)	(0.03)	(0.27)	(0.16)	(0.42)	(0.21)
				Correlation	ns				
	$einf_t$	$rrf_t$	$\Delta d_t$	$ra_t$	$vr_t$	$\Delta ern_t$	$gern_t^{su}$	$erp_t$	$irp_t$
c	0.40								
$rrf_t$	0.49								
	(0.65)								
$\Delta d_t$	-0.07	-0.07							
	(0.22)	(0.23)							
$ra_t$	0.34	0.42	-0.11						
	(0.61)	(0.68)	(0.22)						
$vr_t$	0.87	0.38	-0.11	0.09					
	(0.30)	(0.78)	(0.20)	(0.77)					
$\Delta ern_t$	-0.11	-0.05	0.12	-0.14	-0.05				
	(0.26)	(0.26)	(0.23)	(0.25)	(0.26)				
$gern_t^{su}$	-0.20	0.20	-0.12	0.29	-0.15	0.18			
	(0.53)	(0.58)	(0.20)	(0.49)	(0.47)	(0.26)			
$erp_t$	-0.06	0.08	-0.14	0.47	-0.16	-0.45	-0.17		
1 0	(0.42)	(0.55)	(0.65)	(0.48)	(0.42)	(0.59)	(0.64)		
$irp_t$	-0.16	0.58	-0.04	0.15	-0.15	0.08	0.11	0.13	
Γu	(0.64)	(0.43)	(0.21)	(0.70)	(0.75)	(0.28)	(0.56)	(0.65)	

Table A2: Unconditional Moments of Endogenous Variables

Results in this table are based on the latent VAR,  $Y_t = \mu + AY_{t-1} + \Sigma \varepsilon_t$ , where  $Y_t = [einf_{tt}, rrf_t, \Delta d_t, erp_t, irp_t, x_t]'$  and  $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]'$ ,  $\varepsilon \sim (0, I)$  and  $irp_t$  and  $erp_t$  are unobserved. The  $Y_t$  system parameters are derived from VAR estimates on the observable vector  $W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$  using the data and methodology described in the Appendix. The width of bootstrapped 90 percent confidence intervals are reported in parentheses.

# References

 Hodrick, R., 1992, Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement, Review of Financial Studies, 5(3), 357-386.