OUTSIDE AND INSIDE LIQUIDITY*

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First Draft: May 7th 2009
This draft: April 9th 2010

Abstract

We propose an origination-and-contingent-distribution model of banking, in which liquidity demand by short-term investors can be met with cash reserves (inside liquidity) or sales of assets (outside liquidity) to long-term investors. Outside liquidity is a more efficient source, but asymmetric information about asset quality can introduce a friction in the form of excessively early asset trading in anticipation of a liquidity shock, excessively high cash reserves, and too little origination of assets by banks. The model captures elements of the current financial crisis and yields policy prescriptions on public liquidity provision to overcome liquidity crises.

I. INTRODUCTION

The goal of this paper is to propose a tractable model of origination and contingent distribution of assets by financial intermediaries, and the liquidity demand arising from the maturity mismatch between asset payoffs and desired redemptions. When financial intermediaries invest in long-term assets they may face redemptions before these assets mature. Early redemptions can be met either with an intermediary's own reserves – what we refer to as *inside liquidity* – or with the proceeds from asset sales to other investors with a longer horizon–what we refer to as *outside liquidity*. The purpose of our analysis is to determine the relative importance of inside and outside liquidity in a competitive equilibrium of the financial sector.

We consider two different groups of agents that differ in their investment horizons. One class of agents is short-run investors (SRs) who prefer early asset payoffs, and the second class is long-run investors (LRs) who are indifferent to the timing of payoffs. One may think of the long-run investors as wealthy individuals, endowments, hedge funds, pension funds or sovereign wealth funds, and of the short-run investors as financial intermediaries, banks or mutual funds, catering to investors with shorter horizons. Within this model the key questions are, what determines the mix of inside and outside liquidity in equilibrium? And how is the mix of inside and outside liquidity linked to the origination of assets by financial intermediaries?

In our model SRs invest in risky projects and a set of LR investors, those with sufficient knowledge to value and oversee the risky projects, may stand ready to buy them at a relatively good price. An important potential source of inefficiency in reality and in our model is asymmetric information between SRs and LRs about project quality. LRs cannot always tell whether the SR asset sale is motivated by a sudden liquidity need or whether the SR investor is trying to pass on a *lemon*. This problem is familiar to market participants and has been widely studied in the literature in different contexts.

A novel aspect our model is the focus on the *timing* of liquidity trades. Over time, SRs learn (asymmetrically) more about the value of the assets they originated. Therefore, when at the onset of a liquidity shock they choose to hold on to their assets in the hope of riding out a temporary liquidity need, SRs run the risk of having to go to the market in a much worse position later. Yet, it makes sense for SRs not to rush to sell their projects, as these may mature and pay off soon enough so that SRs ultimately may not face a liquidity shortage. This timing decision by SRs as to *when* to sell their assets creates the main tension in the model.

We capture the unfolding of a liquidity crisis by establishing the existence of two types of rational expectations equilibria: an *immediate-trading equilibrium*, where SRs are expected

to trade at the onset of the liquidity shock, and a *delayed-trading equilibrium*, where they are instead expected to try to ride out the crisis and only trade as a last resort.

We show that under complete and symmetric information about asset values the unique equilibrium involves delayed trading. Under asymmetric information, however, an immediate-trading equilibrium always exists and under some parameter values, both an immediate and delayed-trading equilibrium may coexist. In the delayed trading equilibrium the anticipation of future asymmetric information induces an acceleration of trade.[1]

When two different rational expectations equilibria can coexist one naturally wonders how they compare in terms of efficiency. The answer to this question is crucially related to the amount of risky projects originated by the SRs. In a nutshell, under the expectation of immediate liquidity-trading, LRs expect to obtain the assets originated by SRs at close to fair value. In this case the returns of holding outside liquidity are low and the LRs hold little cash. On the other side of the trade, SRs will then expect to be able to sell a relatively small fraction of assets at close to fair value, and therefore respond by relying more heavily on inside liquidity and originating fewer projects. In an immediate-trading equilibrium there is less *cash-in-the-market pricing* (to borrow a term from Allen and Gale, 1998) and a lower supply of outside liquidity. The anticipated reduced supply of outside liquidity causes SRs to originate fewer projects and, thus, bootstraps the relatively high equilibrium price for the assets.

In contrast, under the expectation of delayed liquidity trading, SRs rely more on outside liquidity. Here the bootstrap works in the other direction, as LRs decide to hold more cash in anticipation of a larger future supply of the assets held by SRs. These assets will be traded at lower prices in the delayed-trading equilibrium, even taking into account the lemons problem. The reason is that in this equilibrium SRs originate more projects and therefore end up trading more assets following a liquidity shock. They originate more projects in this delayed trading equilibrium because the expected return for SRs to investing in a project is higher in the delayed-trading equilibrium, due to the lower overall probability of liquidating assets before they mature.

Our model predicts the typical pattern of liquidity crises, where asset prices progressively deteriorate throughout the crisis.[2] Because of this deterioration in asset prices one would expect that welfare is also worse in the delayed-trading equilibrium. However, the delayed-trading

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[1]An analogy with Akerlof's famed market for second hand cars is helpful to understand these results. When sellers of second hand cars can time their sales they tend to sell their cars sooner, when they are less likely to have become aware of flaws in their car, so as to reduce the lemons discount at which they can sell their car.

[2]SRs' decision to delay trading has all the hallmarks of *gambling for resurrection*. But it is in fact unrelated to the idea of excess risk taking as SRs will choose to delay whether or not they are levered.

equilibrium is in fact Pareto superior. What is the economic logic behind this result? The fundamental gains from trade in our model are between SRs who undervalue long term assets and LRs. The more SRs can be induced to originate projects the higher are the gains from trade and therefore the higher is welfare. In other words, the welfare efficient form of liquidity provision is outside liquidity. Since the delayed-trading equilibrium relies more on outside liquidity it is more efficient. As the lemons' problem worsens, however, the cost of outside liquidity for SRs rises. There may then come a point when the cost is so high that SRs are better off postponing the redemption of their investments altogether rather than realize a very low fire-sale price for their valuable projects. At that point the delayed-trading equilibrium collapses, as only lemons are traded for early redemption.

Our analysis sheds light on the recent transformation of the financial system towards more origination and greater reliance on distribution of assets as evidenced in Adrian and Shin (2009). This shift can be understood in our model in terms of a move from an immediate-trading equilibrium, with little reliance on outside liquidity, to a delayed-trading equilibrium. The consequences of this shift is more origination and distribution but also a greater fragility of the financial system, to the extent that assets are distributed at larger discounts under delayed trading. Our analysis highlights that greater fragility does not necessarily imply greater inefficiency. On the contrary, the move to more distribution and reliance on outside liquidity is a welfare improving move even if it means that liquidity crises may be more severe when they occur. That being said, an important concern with origination and distribution that is omitted from our model is the greater moral hazard in origination that arises with greater distribution.

In this paper we do not take an optimal mechanism design approach. We attempt instead to specify a model of trading opportunities that mimics the main characteristics of actual markets. The advantage of this approach is that it facilitates interpretation and considerably simplifies aspects of the model that are not central to the questions we focus on. Nevertheless, we do consider one long-term contracting alternative to markets, in which SRs write a long-term contract for liquidity with LRs. Such a contract takes the form of an investment fund set up by LRs, in which the initial endowments of one SR and one LR are pooled, and where the fund promises state-contingent payments to its investors. Under complete information such a fund arrangement always dominates any equilibrium allocation achieved through future spot trading of assets for cash.

However, when the investor who manages the fund also has private information about the realized returns on the fund's investments then, as we show, the long-term contract cannot

always achieve a more efficient outcome than the delayed-trading equilibrium. Indeed, the fund manager's private information then constraints the fund to make only incentive compatible state-contingent transfers to the SR investor, thus raising the cost of providing liquidity. We show in particular that the fund allocation is dominated by the delayed-trading equilibrium in parameter regions for which there is a high level of origination and distribution of risky assets.

Given that neither financial markets nor long-term contracts for liquidity can achieve a fully efficient outcome, the question naturally arises whether some form of public intervention may provide an efficiency improvement. There are two market inefficiencies that public policy might mitigate. An ex-post inefficiency, which arises when the delayed-trading equilibrium fails to exist, and an ex-ante inefficiency in the form of an excess reliance on inside liquidity. It is worth noting that a common prescription against banking liquidity crises–to require that banks hold cash reserves or excess equity capital–would be counterproductive in our model. Such a requirement would only force SRs to rely more on inefficient inside liquidity and would undermine the supply of outside liquidity.

We discuss policy interventions and use this model to interpret the current crisis in Section VII and, in greater depth, in Bolton, Santos and Scheinkman (2009). We point out that the best form of public liquidity intervention relies on a complementarity between public and outside liquidity. Public liquidity in the form of a price support (or guarantee) for SR assets can restore existence of the delayed-trading equilibrium and thereby induce LRs to hold more outside liquidity. Such a policy would induce long-term investors to hold more cash in the knowledge that SRs rely less on inside liquidity, and thus help increase the availability of outside liquidity. Thus, far from being a substitute for privately provided liquidity, a commitment to providing a price support in secondary asset markets in liquidity crises can be a complement and give rise to positive spillover effects in the provision of outside liquidity.

II. Related Literature

Our paper is related to the literatures on banking and liquidity crises, and the limits of arbitrage. Our analysis differs from other contributions in these literatures mainly in two respects: first, our focus on ex-ante efficiency and equilibrium portfolio composition, and second, the endogenous timing of liquidity trading. Still, our analysis shares several important themes and ideas with previous papers.

Diamond and Dybvig (1983) and Bryant (1980) provide the first models of investor liquidity demand, maturity transformation, and inside liquidity. In their model a bank run may occur if there is insufficient inside liquidity to meet depositor withdrawals. In contrast to

our model, investors are identical ex-ante, and are risk-averse with respect to future liquidity shocks. The role of financial intermediaries is to provide insurance against idiosyncratic investors' liquidity shocks.

Bhattacharya and Gale (1986) provide the first model of both inside and outside liquidity by extending the Diamond and Dybvig framework to allow for multiple banks, which may face different liquidity shocks. In their framework, an individual bank may meet depositor withdrawals with either inside liquidity or outside liquidity by selling claims to long-term assets to other banks who may have excess cash reserves. An important insight of their analysis is that individual banks may free-ride on other banks' liquidity supply and choose to hold too little liquidity in equilibrium.

More recently, Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) (see also Aghion, Bolton and Dewatripont, 2000) have analyzed models of liquidity provided through the interbank market, which can give rise to contagious liquidity crises. The main mechanism they highlight is the default on an interbank loan which depresses secondary-market prices and pushes other banks into a liquidity crisis. Subsequently, Acharya (2009) and Acharya and Yorulmazer (2008) have, in turn, introduced optimal bailout policies in a model with multiple banks and cash-in-the-market pricing of loans in the interbank market.

While Diamond and Dybvig considered idiosyncratic liquidity shocks and the risk of panic runs that may arise as a result of banks' attempts to insure depositors against these shocks, Allen and Gale (1998) consider aggregate business-cycle shocks and point to the need for equilibrium banking crises to achieve optimal risk-sharing between depositors. In their model aggregate shocks may trigger the need for asset sales, but their analysis does not allow for the provision of both inside and outside liquidity.

Another strand of the banking literature, following Holmstrom and Tirole (1998 and 2008) considers liquidity demand on the corporate borrowers' side rather than on depositors' side, and asks how efficiently this liquidity demand can be met through bank lines of credit. This literature emphasizes the need for public liquidity to supplement private liquidity in case of aggregate demand shocks.

Most closely related to our model is the framework considered in Fecht (2004), which itself builds on the related models of Diamond (1997) and Allen and Gale (2000). The models of Diamond (1997) and Fecht (2004) seek to address an important weakness of the Diamond and Dybvig theory, which cannot account for the observed coexistence of financial intermediaries and securities markets. Liquidity trading in secondary markets undermines liquidity provision by banks and obviates the need for any financial intermediation in the Diamond and

Dybvig setting, as Jacklin (1987) has shown. In Diamond (1997) banks coexist with securities markets because households face costs in switching out of the banking sector and into securities markets. Fecht (2004) extends Diamond (1997) by introducing segmentation between financial intermediaries' investments in firms and claims issued directly by firms to investors though securities markets. Also, in his model banks have local (informational) monopoly power on the asset side, and subsequently can trade their assets in securities markets for cash–a form of outside liquidity. Finally, Fecht (2004) also allows for a contagion mechanism similar to Allen and Gale (2000) and Diamond and Rajan (2005),[3] whereby a liquidity shock at one bank propagates itself through the financial system by depressing asset prices in securities markets.

Two other closely related models are Gorton and Huang (2004) and Parlour and Plantin (2007). As us, Gorton and Huang consider liquidity supply in a general equilibrium model and argue that publicly provided liquidity can be welfare enhancing if the private supply of liquidity involves a high opportunity cost. However, in contrast to our analysis they do not look at the optimal composition of inside and outside liquidity, nor do they consider the dynamics of liquidity trading. Parlour and Plantin (2007) consider a model where banks may securitize loans, and thus obtain access to outside liquidity. As in our setting, the efficiency of outside liquidity is affected by adverse selection. But in the equilibrium they characterize liquidity may be excessive for some banks–as it undermines their loan origination standards–and too low for other banks, who may be perceived as holding excessively risky assets.

Our model is also related to the literature on liquidity and the dynamics of arbitrage by capital or margin-constrained speculators as in Dow and Gorton (1993) and Shleifer and Vishny (1997). The typical model in this literature (e.g. Kyle and Xiong, 2001 and Xiong, 2001) also allows for outside liquidity and generates episodes of fire-sale pricing – even destabilizing price dynamics – following negative shocks that tighten speculators' margin constraints. However, models in this literature do not address the issue of deteriorating adverse selection and the timing of liquidity trading, nor do they explore the question of the optimal mix between inside and outside liquidity. The most closely related articles to the present paper, besides Kyle and Xiong (2001) and Xiong (2001), are Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009) and Kondor (2007). In particular, Brunnermeier and Pedersen (2009) also focus on the spillover effects of inside and outside liquidity, or what they refer to as funding and market liquidity.

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[3]Another feature in Diamond and Rajan (2005) in common with our setup is the idea that financial intermediaries possess superior information about their assets, which is another source of illiquidity.

III. The Model

III.A Agents

There are two sets of agents, short and long-run investors, each with unit mass. *Short run investors* (SRs) have preferences over consumption in period $t = 1, 2, 3$, $C_t \geq 0$, represented by the following utility function:

$$(1) \qquad u(C_1, C_2, C_3) = C_1 + C_2 + \delta C_3,$$

where $\delta \in (0, 1)$. These investors have one unit of endowment at date 0 and no endowments at subsequent dates. *Long run investors* (LRs) have a utility function over $C_t \geq 0$,

$$\hat{u}(C_1, C_2, C_3) = \sum_{t=1}^{3} C_t.$$

LRs have $\kappa > 0$ units of endowment per-capita at $t = 0$, and no endowments at subsequent dates. The limit on the aggregate endowment by the LRs reflects our hypothesis that only investors with sufficient knowledge of the risky projects would stand ready to buy them, although we do not model here the determinants of κ.

III.B Assets and Information

The two sets of investors have access to different investment opportunity sets. LRs can hold *cash*, with a unit gross per-period rate of return, and invest in a decreasing-returns-to-scale *long-maturity* asset that returns $\varphi(x)$ at date 3 for an initial investment of $x = (\kappa - M)$ at date 0, where $M \geq 0$ denotes the LRs' cash holding. We refer to M as *outside liquidity*. As LRs are risk neutral, the assumption that the long run project is riskless is without loss of generality.

SR investors can hold cash and invest in a *risky* asset that they originate, a scalable constant returns-to-scale project with unit returns $\widetilde{\rho}_t$ at dates $t = 1, 2, 3$, where $\widetilde{\rho}_t \in \{0, \rho\}$ and $\rho > 1$. The return on risky assets is the only source of uncertainty in the model and is shown in Figure I. An aggregate *maturity shock* affects all risky assets: all risky assets may either mature at date 1 or at some later date. If risky assets mature at date 1 they all yield the certain return ρ. If they mature at a later date, the realized return of an individual risky asset and whether it matures at date 2 or 3 is determined by an idiosyncratic shock.

Formally, an SR chooses a size $\nu \leq 1$ for the risky project at date 0. The project then either pays $\rho\nu$ at date 1 (in state $\omega_{1\rho}$) with probability λ, or it pays at a subsequent

date with probability $(1 - \lambda)$. In that case, the asset yields either a return $\widetilde{\rho}_2 \in \{0, \rho\}$ at date 2, or a late return $\widetilde{\rho}_3 \in \{0, \rho\}$ at date 3 per unit invested. After date 1, shocks are idiosyncratic (i.e. independently and identically distributed across SRs). They are represented by two independent random variables: $(1 - \theta)$, the probability that the asset matures at date 3 (the idiosyncratic state ω_{2L}); and η, the probability that the asset returns $\widetilde{\rho}_t = \rho$ when it matures at either dates $t = 2, 3$ (in idiosyncratic states $\omega_{2\rho}$ and $\omega_{3\rho}$, respectively). Thus, $\widetilde{\rho}_t = 0$ with probability $(1 - \eta)$ at $t = 2, 3$ (in idiosyncratic states ω_{20} and ω_{30}). The realization of idiosyncratic shocks is private information to the SR originating the risky asset. We denote by m the amount of cash held by SRs and by $\nu = 1 - m$ the amount invested in the risky asset; m is thus our measure of *inside liquidity.*

Under our assumptions about asset returns and observability of idiosyncratic states, SRs and LRs have symmetric information at date 1 but asymmetric information at dates 2 and 3 about expected and realized returns of risky assets. In other words, while there is no adverse selection at date 1, there will be at dates 2 and 3. This change in information asymmetry is meant to capture in a simple way the idea that in liquidity crises the extent of asymmetric information grows over time.

The notion that adverse selection problems worsen during a liquidity crisis is intuitive, as originators learn more about the quality of their assets over time. It is also broadly consistent with how the financial crisis of 2007 and 2008 has played out. To be sure, the risk profile and asset quality of many financial intermediaries became difficult to ascertain as the residential real estate and mortgage markets' implosion unfolded in 2007 and 2008 (see Gorton 2007 and 2008). Marking assets to market became more difficult. Determining the extent of unsold inventory of assets was also difficult, and the value of any insurance or swap agreements was undermined by growing counter-party risk. The freezing up of the interbank loan market was one clear symptom of the difficulty of assessing the direct and indirect exposure of financial institutions to these toxic assets.

III.C Assumptions

We impose assumptions on payoffs to focus the analysis on the economically interesting situations. First, for the long run asset we assume that:

ASSUMPTION 1.

$$\varphi'(\kappa) > 1 \quad \text{with} \quad \varphi''(x) < 0 \quad \text{and} \quad \lim_{x \longrightarrow 0} \varphi'(x) = +\infty.$$

The assumption $\varphi''(\cdot) < 0$ captures the idea that the long assets represent scarce investment opportunities. The assumption $\lim_{x \to 0} \varphi'(x) = +\infty$ ensures that LRs always want to invest some fraction of their endowment in the long asset. The key assumption here though is that $\varphi'(\kappa) > 1$. This implies that LRs incur a strictly positive opportunity cost of carrying cash. They will only hold cash in equilibrium if they expect be able to acquire assets at dates 1 or 2 with expected returns at least as high as $\varphi'(\kappa)$. Given our assumption of risk neutrality this can only occur if asset purchases occur at *cash-in-the-market* prices. That is, assets must trade in equilibrium at prices that are below their expected payoff, for otherwise LRs would have no incentive to hold cash.

Second, for the risky asset we assume that:

ASSUMPTION 2.

$$\rho \left[\lambda + (1 - \lambda)\eta\right] > 1 \qquad \text{and} \qquad \lambda\rho + (1 - \lambda)\left[\theta + (1 - \theta)\delta\right]\eta\rho < 1.$$

These assumptions imply that SRs would not invest in the risky asset in autarchy, even though investment in the risky asset may be more attractive than holding cash when the asset can be resold for it's expected payoff. Assumption 2 captures the economically interesting situation where liquidity of secondary markets at dates 1 and 2 affects asset allocation decisions at date 0.[4]

Third, we assume that there are gains from trading risky assets for cash at least at date 1 following an aggregate liquidity shock (the realization of state ω_{1L}). This is the case when $\varphi'(\kappa)$ is not so high to make it unattractive for LRs to carry cash to purchase risky assets at date 1:

ASSUMPTION 3.
$$\frac{\varphi'(\kappa) - \lambda}{(1 - \lambda)\eta\rho} < \frac{1 - \lambda}{1 - \lambda\rho}.$$

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[4]If we assume instead that
$$\lambda\rho + (1 - \lambda)\left[\theta + (1 - \theta)\delta\right]\eta\rho \geq 1,$$
then SRs would always choose to put all their funds in a risky asset irrespective of the liquidity of the secondary market at date 1.

IV. OPTIMIZATION

Given that all SRs are ex-ante identical, we restrict attention to equilibria that treat all SRs symmetrically. Similarly, we assume that all LRs get the same (expected) profit in equilibrium. We also restrict attention to *pooling equilibria*, in which observable actions cannot be used to distinguish among SRs with worthless risky assets (in state ω_{20}), and SRs with valuable assets maturing at date 3 (in state ω_{2L}).

We denote by P_1 the price of one unit of risky asset traded at date 1 in state ω_{1L}, and by P_2 the price of one unit of risky asset traded at date 2. Similarly, we denote by Q_1 and Q_2 the amount of risky assets demanded by an LR investor at dates 1 and 2, respectively.[5] Finally, we denote by q_1 the amount of risky asset supplied by an SR at date 1 (in state ω_{1L}) and by q_2 the amount supplied at date 2. Given that SRs learn at date 2 the realized returns of the risky asset they have originated, SRs can condition their trading policy on the realization of their idiosyncratic state ω_2. An SR in state ω_{20} would always sell his risky asset at any price, as he knows that the asset is worthless. An SR in state $\omega_{2\rho}$ has no reason to sell a valuable risky asset that has already matured. He may as well hold on to the asset and consume its output. An SR in state ω_{2L} will only sell a positive quantity of the risky asset $q_2 > 0$ if the price P_2 is greater than or equal to the discounted expected value of the asset $\delta\eta\rho$. We assume that SRs always sell their entire risky investment whenever they are indifferent between selling or holding on to their risky asset.[6] For expositional ease, we do allow LRs to buy a fraction of a risky project, but in section IX.D below we show how to treat the constraint that LRs also acquire an integer number of indivisible projects.

IV.A The SR Optimization Problem

At date 0, SRs must determine how much of their unit endowment to hold in cash and how much to invest in a risky asset. At date 1, they must decide how much of the risky asset to trade at price P_1, and at date 2 how much to trade of what they still own at price P_2.

Their objective function is as follows:

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[5]More formally, we could have written $P_1(\omega_{1L})$ and $P_2(\omega_{1L})$ to denote the prices of the risky asset at dates 1 and 2 and similarly $Q_1(\omega_{1L})$ and $Q_2(\omega_{1L})$ to denote the quantities acquired by LRs at different dates. Given that all trading occurs in the "lower branch" of the tree we adopt the simpler notation as there is no possible ambiguity.

[6]One interpretation of this assumption is that once a scale is chosen, a risky project is indivisible. This indivisibility is consistent with our assumption that each risky project has at most one SR owner, who is the only agent that observes the state of the risky project in period 2.

$$
\begin{aligned}
\pi\left[m, q_{1}, q_{2}\right] \;=\;& m+\lambda\left(1-m\right) \rho \\
&+\;\left(1-\lambda\right) q_{1} P_{1} \\
&+\;\left(1-\lambda\right) \theta \eta\left[\left(1-m\right)-q_{1}\right] \rho \\
&+\;\left(1-\lambda\right) \theta\left(1-\eta\right)\left[1-m-q_{1}\right] P_{2} \\
&+\;\left(1-\lambda\right)\left(1-\theta\right) q_{2} P_{2} \\
&+\;\delta\left(1-\lambda\right)\left(1-\theta\right) \eta\left[\left(1-m\right)-q_{1}-q_{2}\right] \rho .
\end{aligned}
$$

(2)

Recall that an SR liquidates his remaining position in the risky asset in state ω_{20}. Also, in states where the asset yields ρ, SRs hold on to the risky asset and consume ρ.

The SR's investment program $\mathcal{P}_{SR}$ is then given by:

Program $\mathcal{P}_{SR}$

$$
\max_{m, q_{1}, q_{2}} \pi\left[m, q_{1}, q_{2}\right]
$$

subject to

$$
m \in [0, 1]
$$

and

$$
q_{1}+q_{2} \leq 1-m \qquad \text{and} \qquad q_{1}, q_{2} \in \{0, 1-m\}.
$$

The constraints simply state that SRs cannot invest more in the risky asset than their endowment and that they cannot sell more than what they hold. The last condition ensures that when an SR sells his risky asset, he sells everything he owns.

IV.B The LR Optimization Problem

At date 0 LRs determine how much of their endowment to hold in cash, M, and how much in the long term asset, $\kappa - M$. LRs must also decide at dates 1 and 2 how much of the risky assets to purchase at prices P_1 and P_2. Given that holding cash involves a strictly positive opportunity cost LRs will not carry cash that they will never use. That is, in the states of nature in which trade is profitable LRs will completely exhaust their cash reserves to purchase risky assets. With this observation in mind we can write the payoff of an LR investor that purchases Q_1 at date 1 and Q_2 at date 2, as follows:

$$
\Pi\left[M, Q_{1}, Q_{2}\right] \;=\; M+\varphi\left(\kappa - M\right)
$$

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$$\text{(3)} \qquad\qquad\qquad + \quad (1-\lambda)\left[\eta\rho - P_1\right]Q_1$$

$$+ \quad (1-\lambda)E\left[\tilde{\rho}_3 - P_2 | \mathcal{F}\right]Q_2.$$

The first line in (3) is simply what the LR investor gets by holding an amount of cash M until date 3 without ever trading in secondary markets at dates 1 and 2. The second line is the net return from acquiring a position Q_1 in risky assets at unit price P_1 at date 1. Indeed, the expected payoff of a risky asset in state ω_{1L} is $\eta\rho$. The last line is the net return from trading at date 2. This net return depends on the expected realized payoff of the risky asset at date 3, or in other words on the expected quality of assets purchased at date 2. As we postulate rational expectations, the LR investor's information set, $\mathcal{F}$, includes the particular equilibrium that is being played. In computing conditional expectations, LRs assume that the mix of assets offered at date 2 corresponds to the one observed in equilibrium. We also impose a standard and weak refinement on LR out-of-equilibrium beliefs, that if they purchase a risky asset at date 2, in an equilibrium that prescribes no trade at that date, at a price for which SRs in state ω_{2L} strictly prefer to hold the asset until date 3, then LRs assume that the asset is worthless.

The LR investor's program is thus:

Program $\mathcal{P}_{LR}$

$$\max_{M,Q_1,Q_2} \Pi\left[M, Q_1, Q_2\right]$$

subject to

$$\text{(4)} \qquad\qquad\qquad 0 \leq M \leq \kappa$$

and

$$\text{(5)} \qquad\qquad Q_1 P_1 + Q_2 P_2 \leq M \qquad \text{and} \qquad Q_1 \geq 0,\ Q_2 \geq 0.$$

The first constraint (4) is simply the LR's wealth constraint: LRs' cannot carry more cash than their initial capital κ and they cannot borrow. The second constraint (5) says that LRs cannot purchase more risky projects than their money, M, can buy and that LRs cannot short risky projects.

V. EQUILIBRIUM

We establish the existence of two stable rational expectations equilibria: an *immediate-trading equilibrium*, in which all trade takes place at date 1, and a *delayed-trading equilibrium*, in which all trade takes place at date 2.

V.A Definition of Equilibrium

A rational expectations competitive equilibrium is a vector of portfolio policies $[m^*, M^*]$, supply and demand choices $[q_1^*, q_2^*, Q_1^*, Q_2^*]$ and prices $[P_1^*, P_2^*]$ such that (i) at these prices $[m^*, q_1^*, q_2^*]$ solves $\mathcal{P}_{SR}$ and $[M^*, Q_1^*, Q_2^*]$ solves $\mathcal{P}_{LR}$ and (ii) markets clear in all states of nature.

V.B Equilibrium Under Full Information

We begin by showing that when all agents are fully informed about the realization of idiosyncratic shocks at date 2, then the unique equilibrium is the delayed-trading equilibrium. Thus, suppose for now that both SRs and LRs can observe whether a risky project is in state ω_{2L} or ω_{20}. Then the following result holds.

PROPOSITION 1. (Unique full information equilibrium) Assume that both SRs and LRs observe whether a risky asset is in state ω_{2L} or ω_{20}, that Assumptions 1-3 hold, and that δ is small enough.[7] Then the unique equilibrium is the delayed-trading equilibrium.

We provide a formal proof of when the delayed-trading equilibrium exists in the appendix. For our purposes now it is sufficient to show that an immediate-trading equilibrium cannot exist under full information. Note first that the expected payoff of acquiring assets in state ω_{2L} for LRs is $\eta\rho$, the same expected payoff as at date 1. It follows that LRs prefer to purchase risky assets at date 1 instead of date 2 whenever the price at the earlier date is lower than at the later date:

$$(6) \qquad\qquad P_{2i}^* \geq P_{1i}^*.$$

Similarly, SRs sell their risky asset at date 1 whenever the price they can obtain at date 1 is higher than the expected utility of holding the asset until date 2, which is the payoff in state $\omega_{2\rho}$ times $\theta\eta$ plus the price at which SRs sell the risky asset in state ω_{2L}, P_{2i}^*, times $(1-\theta)$:

$$(7) \qquad\qquad P_{1i}^* \geq \theta\eta\rho + (1-\theta)P_{2i}^*.$$

The conditions (6) and (7) must hold in any putative immediate-trading equilibrium.

But note that these two conditions together imply that $P_{1i}^* \geq \eta\rho$, and thus that $P_{2i}^* \geq \eta\rho$. Hence, for an SR, investing in a risky-project and selling it at either period 1 or 2 dominates holding cash and thus $m_i^* = 0$. However, given that the expected gross payoff of the asset at $t = 1$ is $\eta\rho$, the expected return of carrying cash for LRs cannot be greater than one, so

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[7]In the proof of the proposition an exact, strictly positive, bound is given.

that $M_i^* = 0$ because by Assumption 1, $\varphi'(\kappa) > 1$. Hence SRs that have projects that will mature in date 3, cannot find any buyers. In sum, there cannot exist an immediate-trading equilibrium when LRs are fully informed about the value of risky assets at date 2. We show next that when instead there is asymmetric information about the true value of risky assets at date 2, an immediate-trading equilibrium always exists.

V.C Equilibrium Under Asymmetric Information

We now consider the more plausible situation where only the originating SR can observe whether its risky asset is in state ω_{2L} or ω_{20}. LRs at date 2 can only tell that if an asset is put up for sale it can be in either state ω_{2L} or ω_{20}.

In what follows and for the remainder of the article we restrict our analysis to this situation of asymmetric information. In the presence of asymmetric information the following fundamental result obtains.

PROPOSITION 2. (The immediate-trading equilibrium) Suppose that LRs only observe the information set $\{\omega_{2L}, \omega_{20}\}$ at date 2, while SRs can observe the true state ω_{2L} or ω_{20}. Suppose also that Assumptions 1-3 hold. Then there always exists an immediate-trading equilibrium, such that

$$M_i^* > 0 \qquad q_1^* = Q_1^* = 1 - m_i^* \qquad \text{and} \qquad q_2^* = Q_2^* = 0.$$

In this equilibrium cash-in-the-market pricing obtains and

$$(8) \qquad\qquad P_{1i}^* = \frac{M_i^*}{1 - m_i^*} \geq \frac{1 - \lambda\rho}{1 - \lambda}.$$

Moreover the cash positions m_i^* and M_i^* are unique.

To gain some intuition on the construction of the immediate-trading equilibrium notice first that the first order conditions for m and M are respectively:

$$(9) \qquad\qquad P_{1i}^* \geq \frac{1 - \lambda\rho}{1 - \lambda} \qquad \text{and} \qquad \lambda + (1 - \lambda)\frac{\eta\rho}{P_{1i}^*} = \varphi'(\kappa - M_i^*),$$

when $m_i^* < 1$ and $M_i^* > 0$.[8] These expressions follow immediately from the maximization problem $\mathcal{P}_{SR}$ when we set $q_1^* = 1 - m_i^*$, and from problem $\mathcal{P}_{LR}$. Note, in particular, that the

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[8]The proof of Proposition 2 establishes that Assumption 3 rules out the possibility of a "no trade" immediate-trading equilibrium in which $M_i^* = 0$ and $m_i^* = 1$.

LR portfolio must be such that the expected return of holding cash is the same as the return obtained by investing an additional dollar in the long run asset.

Next, to determine the equilibrium price, let P_{1i} be the unique solution to the equation:

$$(10) \qquad \lambda + (1 - \lambda)\frac{\eta\rho}{P_{1i}} = \varphi'(\kappa - P_{1i}),$$

which, given our assumptions, always exists. Assume first that the solution to (10) is such that

$$P_{1i} > \frac{1 - \lambda\rho}{1 - \lambda}.$$

In that case, we can set $P_{1i}^* = P_{1i}$, $m_i^* = 0$, so that SRs are fully invested in the risky asset, and also $M_i^* = P_{1i}^*$, which by construction satisfies the LR's first order condition. Moreover, by Assumption 1 it must also be the case that $M_i^* < \kappa$.

The key step in the construction of the immediate-trading equilibrium then, is that the price at date 2, P_{2i}^*, has to be such that both SRs and LRs have incentives to trade at date 1 and not at date 2. That is, it has to be the case that

$$(11) \qquad P_{1i}^* \geq \theta\eta\rho + (1 - \theta\eta)P_{2i}^* \qquad \text{and} \qquad \frac{\eta\rho}{P_{1i}^*} \geq \frac{\mathbf{E}\left[\widetilde{\rho}_3|\mathcal{F}\right]}{P_{2i}^*}.$$

The first expression in (11) states that SRs prefer to sell their risky assets at date 1 for a price P_{1i}^* rather than carrying it to date 2: if SRs carry the asset to date 2, then with probability $\theta\eta$ the risky asset pays off ρ, and with probability $(1 - \theta\eta)$ the asset is either in state ω_{2L} or ω_{20}, when SRs choose to sell the asset at price P_{2i}^*. Hence, if the price P_{2i}^* is low enough then SRs prefer to sell the asset at date 1.

The second condition in (11) states that LR expected returns from acquiring a risky asset at date 1 (in state ω_{1L}) is higher than at date 2. To guarantee this outcome it is sufficient to set $P_{2i}^* < \delta\eta\rho$ for in this case SRs in state ω_{2L} would prefer to carry the asset to date 3 rather than selling it for that price. This means that only "lemons" (risky assets in state ω_{20}) get traded at date 2. LRs, anticipating this outcome, set their expectations accordingly to $\mathbf{E}\left[\widetilde{\rho}_3|\mathcal{F}\right] = 0$, and therefore at any price $0 \leq P_{2i}^* < \delta\eta\rho$ LRs (weakly) prefer to acquire assets at date 1. Hence $P_{2i}^* = 0$ clears markets in period 2 and supports the immediate trading equilibrium.

Assume next that the solution to (10) is such that

$$(12) \qquad P_{1i} \leq \frac{1 - \lambda\rho}{1 - \lambda},$$

and set P_{1i}^* equal to the right hand side of (12). At this price, SRs are indifferent on how much cash $m \in [0, 1]$ to carry. Then the solution to the LRs first order condition (9) is such that:

$$M_i^* < P_{1i}^* = \frac{1 - \lambda \rho}{1 - \lambda}.$$

It is then sufficient to set $m_i^* \in [0, 1)$ such that:

$$\frac{M_i^*}{1 - m_i^*} = \frac{1 - \lambda \rho}{1 - \lambda},$$

which is always possible.[9] Finally, we may choose again $P_{2i}^* = 0$.

Why does an immediate-trading equilibrium emerge under asymmetric information when it does not exist under full information? The reason is simply that under full information SRs get to trade the risky asset at date 2 at a sufficiently attractive price to make it worthwhile for them to delay trading until that date. By trading at date 1, SRs give up a valuable option not to trade the risky asset at all. This option is available if they delay trading to date 2 and has value in the event that the asset matures at date 2 with a payoff ρ. Under asymmetric information the price at which risky assets are traded at date 2 may be so low (due to lemons problems) that SRs prefer to forego the option not to trade and to lock in a more attractive price for the risky asset at date 1. Thus, the expectation of future asymmetric information can bring about an *acceleration of trade* which we show in the next section is inefficient.

Under full information the price of the risky asset at date 2 must be bounded below by the price at date 1. The reason is that the expected gross value of a risky asset to LRs is always $\eta \rho$ whether it is traded at date 1 (in state ω_{1L}) or at date 2 (in state ω_{2L}). But the opportunity cost of trading the risky asset for SRs is higher at date 1 than at date 2, as SRs forego the option not to trade when they trade at date 1 and SRs can expect to sell their asset in state ω_{2L} at an even higher price than at date 1. To compensate SRs for these foregone options the price at date 1 has to be at least $P_{1i} \geq \eta \rho$, but at this price LRs don't want to carry cash to acquire risky assets at date 1. In sum, in the presence of asymmetric information the price at date 2 may be lowered sufficiently to make trade at date 1 attractive for *both* SRs and LRs.

While an immediate trading equilibrium always exists under asymmetric information, the next proposition establishes that a delayed-trading equilibrium exists only if the underpricing

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[9]Notice that Assumption 2 implies that $1 - \lambda \rho > 0$.

of risky assets in state ω_{2L} due to asymmetric information is not too large.[10]

PROPOSITION 3. (Delayed-trading equilibrium) Suppose that LRs only observe the information set $\{\omega_{2L}, \omega_{20}\}$ at date 2, while SRs can observe the true state ω_{2L} or ω_{20}. Assume also that Assumptions 1-3 hold and that δ is small enough[11] then there always exists a delayed-trading equilibrium, where $m_d^* \in [0, 1)$, $M_d^* \in (0, \kappa)$, and

$$q_1^* = Q_1^* = 0 \qquad \text{and} \qquad q_2^* = Q_2^* = (1 - \theta\eta)(1 - m_d^*).$$

In this equilibrium cash-in-the-market pricing obtains and

(13)
$$P_{2d}^* = \frac{M_d^*}{(1 - \theta\eta)(1 - m_d^*)} \geq \frac{1 - \rho[\lambda + (1 - \lambda)\theta\eta]}{(1 - \lambda)(1 - \theta\eta)}.$$

Moreover the cash positions m_d^* and M_d^* are unique.

The construction of the delayed-trading equilibrium is broadly similar to the immediate-trading equilibrium, with a few differences that we emphasize next. First, as stated in the proposition, δ must be small enough. Specifically, it must be such that $\delta\eta\rho < P_{2d}^*$. Otherwise SRs in state ω_{2L} prefer to carry the risky asset to date 3 rather than selling it at date 2. This would destroy the delayed-trading equilibrium, as only lemons would then be traded at date 2. Second, a key difference with the immediate-trading equilibrium is that the aggregate supply of risky assets by SRs is reduced by an amount $\theta\eta$ under delayed trading. This is the proportion of risky assets that pay ρ at date 2. As a result, cash-in-the-market pricing under delayed trading is given by:

$$P_{2d}^* = \frac{M_d^*}{(1 - \theta\eta)(1 - m_d^*)}.$$

The supply of risky assets at date 2 is given by $(1 - \theta\eta)(1 - m_d^*)$, so that delaying asset sales introduces both an adverse selection effect which depresses prices, and a lower supply of the risky assets which increases prices.

As under the immediate-trading equilibrium, to support a delayed-trading equilibrium requires that both SRs and LRs have incentives to trade at date 2 rather than at date 1, which

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[10]Note that we are assuming that $q_1, q_2 \in \{0, 1 - m\}$. If instead we let $0 \leq q_1, q_2 \leq 1 - m$ there would also be a third equilibrium, which involves positive asset trading at both dates 1 and 2. We do not focus on this equilibrium as it is unstable.

[11]The proof of the proposition clarifies the upper bound on δ that guarantees existence, see expression (33) in the Appendix and the discussion therein.

entails that

(14) $$P_{1d}^* \leq \theta\eta\rho + (1 - \theta\eta) P_{2d}^* \qquad \text{and} \qquad \frac{\eta\rho}{P_{1d}^*} \leq \frac{\mathbf{E}\left[\tilde{\rho}_3|\mathcal{F}\right]}{P_{2d}^*},$$

where now the expected payoff of the risky asset, conditional on a trade at date 2 is given by

(15) $$\mathbf{E}\left[\tilde{\rho}_3|\mathcal{F}\right] = \frac{(1 - \theta)\,\eta\rho}{(1 - \theta\eta)}.$$

If (14) is to be met, the price P_{1d}^* in state ω_{1L} has to be in the interval

$$\left[\frac{1 - \theta\eta}{1 - \theta}P_{2d}^*, \theta\eta\rho + (1 - \theta\eta)P_{2d}^*\right].$$

The key step of the proof of Proposition 2 is to show that this interval is non empty.

V.D Outside and Inside Liquidity in the Immediate and Delayed Trading Equilibria

How does the composition of inside and outside liquidity vary across equilibria? To build some intuition on this question it is helpful to consider the following numerical example.

EXAMPLE 1. Our parameter values are:

$$\lambda = .85 \qquad \eta = .4 \qquad \rho = 1.13 \qquad \kappa = .2 \qquad \delta = .1920 \qquad \varphi(x) = x^\gamma \quad \text{with} \quad \gamma = .4$$

We also set $\theta = 0.35$. In our subsequent numerical examples we leave all parameter values unchanged except for θ. The parameter θ plays a critical role in our analysis, as it affects both the expected maturity of the risky assets and the informational rent of SRs at date 2. To ensure that Assumption 2 holds we always restrict the values of θ to the interval $0 \leq \theta \leq \bar{\theta}$, where $\bar{\theta}$ is the solution to

(16) $$1 = \rho\left[\lambda + (1 - \lambda)\eta\rho\left(\bar{\theta} + (1 - \bar{\theta})\delta\right)\right].$$

Under our chosen parameter values we have $\bar{\theta} = .4834$. It is immediate to check that Assumptions 1 to 3 also hold for these parameter values. In particular, we have

$$\varphi'(\kappa) \approx 1.05 \qquad \text{and} \qquad \rho\left[\lambda + (1 - \lambda)\eta\right] \approx 1.03.$$

In this example both the immediate and delayed-trading equilibrium exist for $\theta \in [0, \bar{\theta} = .4834)$. Moreover in the delayed-trading equilibrium we have $m_d^* > 0$ when $\theta \in [0, \hat{\theta} =$

.4196). For $\theta \in [\widehat{\theta} = .4196, \overline{\theta} = .4834]$ the delayed-trading equilibrium is such that $m_d^* = 0$. Finally, for $\theta \in (.4628, .4834]$ the delayed-trading equilibrium does not exist. As we explain below, for this range of θ, the discount factor δ is not sufficiently small to induce SRs in state ω_{2L} to trade their asset at date 2; instead these SRs hold on to their risky asset until maturity at date 3. □

Figure II represents the immediate and delayed-trading equilibria in a diagram where the x axis measures M, the amount of cash carried by LRs, and the y axis m, the amount of cash carried by SRs. The dashed lines are the *isoprofit curves* of LRs and the straight (continuous) lines are the *isoprofit lines* of SR.[12] To see the direction in which payoffs increase as one moves from one isoprofit curve to another, it is sufficient to observe that LRs prefer that SRs carry more risky projects for a given level of outside liquidity, M. In other words, that m is lower. Along the other axis, LRs also prefer to carry less outside liquidity (lower M) for a given supply of risky projects by SRs. The converse is true for SRs. In the figure we display the isoprofit lines for both the immediate and delayed-trading equilibrium (this is why the isoprofit lines appear to cross in the plot; the lines that cross correspond to different dates). Equilibria are located at the tangency points between the SR and LR isoprofit curves.

Consider first the immediate-trading equilibrium, located at the point marked $(M_i^*, m_i^*) = (.0169, .9358)$. Note that the SR isoprofit curve is for the SR reservation utility, $\overline{\pi} = 1$. In other words, the gains from trade in the immediate-trading equilibrium go entirely to LRs. Next, note that the delayed-trading equilibrium at $(M_d^*, m_d^*) = (.0540, .4860)$ has more outside and less inside liquidity relative to the immediate-trading equilibrium.

One way of understanding these equilibrium portfolio choices is to note that in state ω_{1L} the risky asset is of higher ex-ante value to LRs $(\eta\rho)$ than to SRs $(\theta\eta\rho + (1-\theta)\delta\eta\rho)$. In the immediate-trading equilibrium, SRs must be compensated with a relatively high price to be willing to originate risky assets.[13] But this higher price can only come at the expense of lower returns to holding cash for LRs, who are therefore induced to hold less cash. This, in turn, makes it less attractive for SRs to invest in the risky asset, and so on. The outcome is that in the immediate-trading equilibrium most of the liquidity is *inside liquidity* held by SRs,

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[12] To generate these isoprofit lines note that we can construct an indirect expected profit function for SRs and LRs as a function of outside and inside liquidity, $\pi[M, m]$ and $\Pi[M, m]$ respectively. The lines plotted in Figures 2 and 3 simply give the combinations of m and M such that $\pi[m, M] = \overline{\pi}$ and $\Pi[m, M] = \overline{\overline{\Pi}}$. Assumption 3 then simply says that the slope of the isoprofit lines at $M = 0$ at date 1 are such that there are gains from trade: the LR isoprofit curve is "flatter" than the SR isoprofit line.

[13] This observation is reflected in the slope of the isoprofit lines in Figure II: The SRs' isoprofit line in the immediate-trading equilibrium is *flatter* suggesting that SRs require a higher price per unit of risky asset sold at that date.

whereas the delayed-trading equilibrium features relatively more *outside liquidity* than inside liquidity.

The next proposition formalizes this discussion and characterizes the mix of inside and outside liquidity across the two equilibria. For the sake of exposition it is convenient to impose one additional assumption:[14]

ASSUMPTION 4.
$$\frac{1 - \lambda\rho}{1 - \lambda} > \kappa.$$

PROPOSITION 4. (Inside and outside liquidity across equilibria.) Assume that Assumptions 1-4 hold and that δ is small enough that a delayed-trading equilibrium exists for all $\theta \in (0, \overline{\theta}]$ then there exists a cutoff $\theta' \in (0, \overline{\theta}]$ such that $m_i^* > m_d^*$ and $M_i^* < M_d^*$ for all $\theta \in (0, \theta']$.

Thus, for the range $\theta \in [0, \theta']$ there is more outside and less inside liquidity in the delayed-trading equilibrium than the immediate-trading equilibrium. In our example $\theta' = \overline{\theta}$ so that Proposition 4 holds for the entire range of admissible θs.[15]

Finally, note that under Assumption 4 we do not necessarily have $m_d^* > 0$. Figure III shows the immediate and delayed-trading equilibrium when θ takes the higher value $\theta = .45$. The delayed-trading equilibrium is then $(M_d^*, m_d^*) = (.0716, 0)$, while the immediate-trading equilibrium is the same as in example 1, as this equilibrium is independent of θ. Note also that, unlike in Figure II, gains from trade do not entirely accrue to LRs in this example. In Figure III the isoprofit line marked IP_{SR} corresponds to the profit level $\overline{\pi} = 1$ for SRs, which is the same as under autarky. The isoprofit line through the delayed-trading equilibrium, however, lies strictly to the right of IP_{SR}, which means that SRs now obtain strictly positive profits in the delayed-trading equilibrium. The reason is that at the corner when $m_d^* = 0$ SRs are at "full capacity" in originating risky assets. They may then earn *scarcity rents*, as LRs compete for the limited supply of risky assets originated by SRs.

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[14]As we show in the Result in the appendix, under Assumption 4 the immediate-trading equilibrium is such that $m_i^* \in (0, 1)$, ruling out the corner outcome where $m_i^* = 0$ and thus the situation where $m = 0$ under both the immediate and the delayed-trading equilibrium. Assumption 4 holds in all our numerical examples

[15]Although we have been unable to prove it formally, we have not found an example of an economy that meets assumptions 1-4 for which $\theta' < \overline{\theta}$.

VI. Welfare

To begin with, note that all equilibria are *interim efficient*. That is, conditional on trade occurring at either dates there is no reallocation of the risky asset that would make both sides better off. Figure II shows that it is not possible to improve the ex-post efficiency of either equilibrium, as in each case the equilibrium allocation is located at the tangency point of the isoprofit curves. In our model inefficiencies arise through distortions in the *ex-ante* portfolio decisions of SRs and LRs and through the particular timing of liquidity trades they give rise to. When agents anticipate trade in state ω_{1L}, SRs lower their investment in the risky asset and carry more *inside liquidity* m_i. In contrast LRs, carry less liquidity M_i as they anticipate fewer units of the risky asset to be supplied in state ω_{1L}.

When the immediate and delayed-trading equilibrium coexist, an interesting question to consider is whether the two equilibria can be Pareto ranked. We are able to establish that indeed the delayed-trading equilibrium *Pareto dominates* the immediate-trading equilibrium. But the delayed-trading equilibrium may not exist. When the delayed-trading equilibrium does not exist we show, however, that a more efficient outcome can be attained under a LR monopoly.

VI.A Pareto Ranking of the Immediate and Delayed-trading Equilibria

The clear Pareto ranking of the two equilibria is somewhat surprising, as delayed trade is hampered by the information asymmetry at date 2, and takes place at lower equilibrium prices. While lower prices clearly benefit LRs it is not obvious a priori that they also benefit SRs. The next proposition establishes that this is the case. The economic reason behind this clear Pareto ranking is that SRs are induced to originate more risky assets when they expect to trade at date 2. This higher supply of risky assets benefits SRs sufficiently to compensate for the lower price at which risky assets are sold.

PROPOSITION 5. (Pareto ranking of equilibria.) Assume that Assumptions 1-4 hold and that δ is small enough so that a delayed-trading equilibrium exists for all $\theta \in [0, \overline{\theta}]$, then there exists a $\theta' \in (0, \overline{\theta}]$ such that $\pi_i^* \leq \pi_d^*$ and $\Pi_i^* < \Pi_d^*$ for all $\theta \in (0, \theta')$.

In our numerical example $\theta' = \overline{\theta}$ so that the delayed-trading equilibrium Pareto dominates the immediate-trading equilibrium for all $\theta \in (0, \overline{\theta}]$. This is illustrated in Figure VI, where the expected profits of both SRs and LRs are plotted for a particular range of θs.[16] The

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[16]The value $\theta = .35$ is chosen simply to show the figures in a convenient scale.

top panel shows the SRs' expected profits. Notice that for all $\theta \le \widehat{\theta} = .4196$ SRs only obtain their reservation profits, when they were to be fully invested in cash. The SRs' risky asset is a constant returns to scale technology and, a shown in Proposition 4, in this range of θ SRs are not fully invested in the risky asset. The lower panel shows the LRs' expected profit. The flat line corresponds to the LR's expected profit in the immediate-trading equilibrium, which is everywhere strictly below the expected profit in the delayed-trading equilibrium.

Somewhat surprisingly, in the range of $\theta \in (0, \widehat{\theta})$ LRs' expected profits are increasing in θ. As θ increases, the adverse selection problem at date 2 worsens, yet LRs' obtain higher ex-ante expected profits. This is due to the fact that when θ increases, the expected maturity of risky assets is also shorter, so that risky assets become more attractive investments for SRs. Therefore SRs originate and distribute more risky assets to LRs at date 2, which can only make them better off.

For $\theta > \widehat{\theta}$ SRs are fully invested in the risky asset and acquire equilibrium rents. In this range $\pi_d^* > 1$ and increasing with θ, whereas LRs' expected profits are decreasing in θ. Note however that $\Pi_d^* > \Pi_i^*$ throughout the relevant range for θ.

In our setup a higher total surplus can be achieved when the aggregate amount of cash held by investors is lower and when investment in risky and long run projects is increased. But under Assumption 2, SRs only want to hold cash in autarchy and do not want to originate risky projects. They are only willing to invest in risky projects if enough outside liquidity is provided by LRs at either dates 1 or 2. SRs are endowed with an investment opportunity they don't want to exploit, unless they can *distribute* the investment to LRs in exchange for cash in some contingencies. The SR investment technology is a constant returns to scale technology. Therefore, from a social point of view efficiency requires minimization of inside liquidity. Thus the key trade-off is between the efficiency gain from lowering inside liquidity and the efficiency loss from raising outside liquidity.

In the delayed-trading equilibrium, inside liquidity is lower and the amount of risky projects originated is larger than in the immediate-trading equilibrium. But, there is also more outside liquidity. The higher amount of risky projects originated is an efficiency gain, while the larger amount of outside liquidity is an efficiency loss. However, the efficiency gain more than offsets the efficiency loss. The reason is that the amount of outside liquidity that LRs hold in the delayed-trading equilibrium is not that much larger than the amount of cash they hold in the immediate-trading equilibrium. LRs don't need to hold much more cash as they expect to acquire only risky assets in states ω_{2L} and ω_{20}. In other words, they expect that SRs retain the risky asset in state $\omega_{2\rho}$ in the delayed trading equilibrium. In contrast, in

the immediate trading equilibrium the price of the risky asset must be relatively high, and the expected returns to LRs relatively low, to compensate SRs for the foregone option that the asset may pay off at date 2. This lowers the amount of outside liquidity that LRs are willing to hold to trade at date 1, and this in turn decreases the incentives of SRs to invest in risky assets.

VI.B Existence of the Delayed Trading Equilibrium

Adverse selection at date 2 plays a fundamental role in our framework and introduces two sources of inefficiency. The first is the main contribution of this paper: the anticipation of adverse selection problems at a future date may lead to an inefficient acceleration of liquidity trades. This acceleration of trade is inefficient from an ex-ante perspective for it induces SRs to rely less on distribution as a source of liquidity and more on inside cash reserves. The second source of inefficiency is more standard and is related to *the lemons problem* in Akerlof (1970): when the adverse selection discount is too large good risks (SRs in state ω_{2L}) withdraw their supply leaving only *lemons* in the market. This then leads to a market breakdown.[17] SRs in state ω_{2L} prefer to hold the risky asset to date 3 whenever the candidate delayed-trading equilibrium price P_{2d}^C as defined in (13) is small enough that $P_{2d}^C < \delta\eta\rho$. In our example this occurs for the range of economies for which $\theta \in (.4628, .4834)$.

To illustrate the welfare costs associated with this breakdown in the secondary market at date 2, Figure VII plots the expected profits for SRs and LRs as a function of θ in the delayed-trading equilibrium. There are three regions in the plot. The first two correspond to the cases already discussed. In region A, $\theta \in (0, .4196)$ and the delayed-trading equilibrium is such that $m_d^* > 0$. In region B, where $\theta \in [.4196, .4628)$, the delayed-trading equilibrium is such that $m_d^* = 0$. In region C a delayed-trading equilibrium does not exist, so that the unique equilibrium outcome is the immediate-trading equilibrium. The dashed line in both panels of Figure VII shows the additional expected profits that SRs and LRs would obtain if SRs could commit ex-ante to sell their risky assets at the candidate price P_{2d}^C in state ω_{2L}. In this case, LRs–anticipating that the pool of assets supplied at date 2 also includes high quality assets–would be willing to hold more outside liquidity than in the immediate-trading equilibrium, which is a Pareto improvement as we have shown.

VI.C Monopolistic Supply of Liquidity and Efficiency

Another way of ensuring trade at date 2 in state ω_{2L} is to have a monopoly LR set prices

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[17]We have so far assumed that δ is small enough that good risks prefer to trade at date 2 at the (candidate) price P_{2d}^* rather than hold on the risky asset to maturity (date 3).

instead of an auctioneer in a competitive market. A monopoly LR would internalize the effect of an excessively low price on the quality of assets exchanged by SRs and may choose to keep its price P_2^M above $\delta\eta\rho$ to support the market at date 2. The obvious question then is whether a monopoly LR may be more efficient ex-ante than a competitive market.

When $\theta < \widehat{\theta}$, where $\widehat{\theta}$ is the lowest value of θ such that $m_d^* = 0$, SRs carry a strictly positive amount of inside liquidity $m_d^* > 0$ and make zero profits. All the surplus then goes to LRs, whether they behave competitively or not. It follows then that in this range the competitive and monopoly solutions are identical. In contrast, when $\theta \geq \widehat{\theta}$, the level of inside liquidity in the competitive equilibrium is $m_d^* = 0$, LRs compete for a fixed supply of the risky assets, and SRs obtain some of the surplus from trade. In this situation, a monopoly LR would be able to generate higher returns by restricting its supply of outside liquidity and thereby raising the price P_{2d}. This can be seen in Figure VIII, where the top panel plots the profits of a monopoly LR along with the profits under perfect competition, and the bottom panel plots the respective prices in states $(\omega_{20}, \omega_{2L})$.

Notice first that in region **A** ($\theta < \widehat{\theta}$) prices and profits under a monopoly are identical to those under perfect competition. In region **B** (where $m_d^* = 0$) the monopoly LR restricts the supply of outside liquidity so as to fully capture all the gains from trade. Therefore, the price of the risky asset at date 2 under a monopoly LR is below the competitive price.

When θ exceeds the threshold where the competitive equilibrium ceases to exists, the monopoly LR sets the price for the risky asset equal to $\delta\eta\rho$ to guarantee a profitable trade at date 2. In this parameter region, region **C** in Figure VIII, a monopoly LR improves ex-ante efficiency, by avoiding the break down of the delayed exchange market. As shown in the top panel of Figure VIII, the monopoly's profits in this region are above those that obtain in the immediate-trading equilibrium, which is the only one that exists with competitive LRs.[18]

VII. APPLICATIONS AND MOTIVATION

Although our model is highly stylized and abstracts from many institutional aspects of financial markets, it does shed light on the unfolding of the current crisis. Our model builds on the interconnections between the reversal in real estate price growth and the liquidity shock to financial intermediaries over this period. The central source of uncertainty in our model comes from SRs' origination of risky projects. This uncertainty takes the form of both payoff

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[18]It is worth emphasizing than in this region SR profits are such that $\pi > 1$. The reason is that the monopolist has to "leave some rents" to the SRs precisely to elicit trade of quality assets in state ω_{2L}.

uncertainty and maturity risk. When risky assets mature late this results in a liquidity shock for SRs.

The analogy with the financial crisis here is that prior to the crisis banks have originated a growing proportion of loans – subprime mortgages, leveraged buyouts, or commercial real estate loans – which were structured to be refinanced within a relatively short horizon, on the expectation that real estate and asset prices would continue to appreciate and thus enable the borrowers to refinance the initial loan with a new loan collateralized by a more valuable asset.[19] When real estate prices unexpectedly started to decline, loan refinancing was no longer possible, resulting in both a maturity and liquidity shock for banks. This is what our aggregate liquidity shock at date 1 represents.

Banks (or SRs in our model) at that point had the choice of quickly selling the loans they had originated, but at fire-sale prices, or hold on to their assets in the hope that the decline in real estate prices would not affect much their own portfolio. This is what SRs choice to trade or wait until date 2 represents in our model. At the same time intermediaries became aware that the initial valuation of assets by rating agencies was seriously flawed and that it would pay to invest on learning the quality of the specific securities they held. Some of these assets, such as CDOs and CDO^2, were so complex that they required substantial resources to determine their value. Strikingly, the financial stability office of the Bank of England has estimated that the documents underlying a typical CDO^2 amounted to over 1,1 billion pages (Haldane, 2009). Inevitably, in the discovery process of underlying asset values, originators such as Merrill Lynch and Citi, holding large quantities of a particular CDO^2, were expected to develop an informational advantage, as they would benefit from scale economies in appraising these assets.[20] This informational asymmetry in turn undermined liquidity trading:

> In a market that is supposed to roll over billions of dollars of debt each day, a
> sudden need to evaluate counterparty collateral can be devastating. These markets
> operate on trust, that is, faith that the counter-party is creditworthy, with no time
> for detailed evaluations. [Bengt Holmstrom (2008) pp: 3-4][21]

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[19]Originating financial institutions also kept super senior tranches of asset backed debt on their balance sheet. These tranches, as well as the special investment vehicles backed by commercial paper facilities were asset risks that banks remained exposed to until the securities were sold to third parties.

[20]The complexity of CDO^2 is a necessary but not sufficient condition for asymmetric information problems to arise, and in fact, common equity is a more complicated security than a CDO^2. We are arguing that buyers worried that originators of certain derivatives were insiders, just as buyers of stocks worry that sellers may have inside knowledge.

[21]Echoing Bengt Holmstrom, Martin Hellwig (2008) has drawn attention to the same mechanism: "As the crisis unfolded, participants in the various relevant markets behaved as one would expect them to behave when

Thus, the dilemma for bank originators over the summer and fall of 2007 in particular was whether to immediately respond to the liquidity shock by raising new funds through asset sales at fire-sale prices, or to take a chance that the liquidity shock might be shortlived at the risk of having to raise liquidity at a later date under much worse conditions, such as those prevailing after the collapse of Lehman Brothers. Banks were aware that the longer they waited in trading assets the more they would be perceived to be trading based on superior information about asset quality. In september of 2007 the 10 largest US banks attempted to resolve this dilemma by setting up a *superconduit* called the *Master-Liquidity Enhancement Conduit* which would pool a large fraction of their non-refinanced assets and which would use these assets as collateral to raise new funds.[22] The plan eventually collapsed as the participating banks could not find a way of avoiding the *lemons'* problem in distributing the worst assets to the superconduit.

As we have shown, the delayed-trading equilibrium in our model Pareto dominates the immediate-trading equilibrium, even though secondary market prices for risky assets are higher under early trading. The reason is that while some SRs are forced to sell at even lower prices in the delayed-trading equilibrium, others are able to hold on to their assets as they learn that their liquidity needs are only temporary. The delayed-trading equilibrium, thus, economizes on aggregate liquidity. The important implication of this observation is that lower secondary market prices do not imply that the liquidity crisis is more severe. On the contrary.

To our knowledge, our model is the first in which origination and the timing of the resolution of the liquidity crisis are explicitly linked. This is to us a main feature of the present crisis: It is precisely because the economy was in a delayed-trading equilibrium that banks were originating a large amount of risky assets (mortgages) to be sold, if necessary, at severely distressed prices at $t = 2$. If instead banks were expecting to sell at date 1, at better prices, this would come at the expense of the expected returns of outside providers of liquidity who then would bring little cash to the market which in turn would elicit low investment in risky assets. The reason for the low expected returns is that when selling at date 1, our SRs sell also the good outcome at $t = 2$, that is, they sell the contingency when risky assets pay off, in state $\omega_{2\rho}$. This "expensive" contingency is costly for SRs to let go. In sum, efficient origination can

there is significant apprehensiveness about the quality of the assets, the quality of counterparties, and the evolution of the financial system in the near future. They withdrew funding and insisted on large discounts on any assets of unknown quality ... this behaviour can be seen as an instance of Akerlofs lemons problem: In a crisis situation, in which there is asymmetric information about the quality of assets that are being traded, any potential investor must fear that the seller is trying to unload his rotten apples while keeping the good ones."

[22]See "Rescued Readied By Banks Is Bet To Spur Market"By Carrick Mollenkamp, Deborah Solomon and Robin Sidel, *Wall Street Journal* October 15, 2007

only come at the expense of truly distressed selling. Of course, many factors that our model ignores also contributed to the current crisis, in particular moral hazard at origination.

Our model also underscores the importance of correctly timing government intervention and public liquidity provision. If a delayed-trading equilibrium prevails, then public injections of liquidity at any date are counterproductive: at date 2 they will crowd out liquidity provision by LRs, and at dates 1 or 3 they may undermine the equilibrium by shifting assets trades to inefficient states of nature. In contrast, if an immediate-trading equilibrium prevails, then public intervention in the form of a price support at date 2, helps shift trade to an efficient state of nature and *crowds in* liquidity supplied by LRs at date 2. This form of intervention is welfare improving, as it raises the quality of the average asset for sale at date 2 and thus increases private liquidity provision by LRs. Just as with the delayed-trading equilibrium, however, interventions at date 1 or 3 are counterproductive. At date 1 public liquidity would only crowd out private outside liquidity and at date 3 it would undermine outside liquidity altogether.

Our model also highlights that by supporting secondary market trading and the reliance on outside liquidity by banks, monetary authorities can encourage banks to do new lending. In other words, they can induce banks to originate more assets. Our analysis thus helps put into context the new forms of intervention by the federal reserve during the crisis, ranging from the commercial paper funding facility (CPFF), the money-market investor funding facility (MMIFF), to the public-private investment program for bank *legacy* assets (PPIP). All these interventions are aimed at restoring the outside liquidity channel for banks and make new origination of loans more attractive.

Finally, one natural interpretation of the parameter δ in the model is that it equals $\frac{1}{1+r}$ where r is the interest rate faced by SR's at date 2. Lowering r, that is increasing δ, makes it more likely that SRs with good projects will choose to hold on to their assets rather than trade them for outside liquidity at date 2, undermining the delayed trading equilibrium.

In sum, as we emphasize in Bolton, Santos and Scheinkman (2009), our analysis highlights that when governments intervene as lenders of last resort–as opposed to market makers of last resort–they risk crowding out rather than crowding in the private provision of liquidity.

VIII. Comparative Statics

We examine next in greater detail how changes in θ affect delayed-trading equilibrium cash holdings, supply of risky assets and returns, where expected returns on acquiring a risky

asset at date 2 are defined as

$$R_{2d}^* \equiv \frac{(1-\theta)\,\eta\rho}{(1-\theta\eta)\,P_{2d}^*}.$$

Several important effects are at work as θ changes, some of which we have already mentioned. First, SRs' incentives to hold onto their assets until date 2 are affected. As θ rises the risky asset is more likely to mature at date 2 and thus becomes more attractive to SRs. Other things equal, SRs are then both more likely to invest in the risky asset and to carry the asset from date 1 to date 2. Second, as θ rises SRs are more likely to trade lemons at date 2 and therefore equilibrium prices P_{2d}^* are lower. These lower prices in turn reduce SR incentives to invest in the risky asset and to carry it to date 2. An additional complication is that as θ increases the supply of risky assets at date 2,

(17) $$s_{2d}^* \equiv (1 - m_d^*(\theta))\,(1 - \theta\eta)$$

diminishes as more risky assets mature early and are then not traded. The next proposition establishes how these countervailing effects net out and how M_d^*, m_d^*, s_{2d}^*, P_{2d}^* and R_{2d}^* vary with θ. Throughout we assume that $\theta \leq \overline{\theta}$, as defined in (16).

PROPOSITION 6. (Comparative statics.) Assume that Assumptions 1-4 hold and that δ is small enough so that a delayed-trading equilibrium exists for all $\theta \in [0, \overline{\theta}]$. Then there exists a unique $\widehat{\theta} \in [0, \overline{\theta}]$ such that:

1. The SRs cash position m_d^*: (a) is a (weakly) decreasing function of θ, (b) $m_d^* > 0$ for all $\theta \in [0, \widehat{\theta})$ and $m_d^* = 0$ for all $\theta \in [\widehat{\theta}, \overline{\theta}]$, and (c) s_{2d}^* is a strictly increasing function of θ for $\theta \in [0, \widehat{\theta})$ and a strictly decreasing function of θ for $\theta \in [\widehat{\theta}, \overline{\theta}]$.

2. The LRs cash position: M_d^* is a strictly increasing function of θ for $\theta \in [0, \widehat{\theta})$ and a strictly decreasing function of θ for $\theta \in (\widehat{\theta}, \overline{\theta}]$.

3. Expected returns at date 2: R_d^* is an increasing function of θ for $\theta \in [0, \widehat{\theta})$ and a decreasing function of θ for $\theta \in (\widehat{\theta}, \overline{\theta}]$.

We illustrate the comparative statics results in Proposition 6 in Figures IV and V. Consider first Figure IV. As expected, the amount of cash carried by SRs is a decreasing function of θ, and $m_d^* = 0$ for $\theta \geq \widehat{\theta} = .4196$. It is less obvious how cash carried by LRs varies with θ. Consider first the case where $\theta \leq \widehat{\theta}$. The amount of cash carried by LRs is then an increasing function of θ. This is surprising: the more severe the lemons problem at date 2 the more cash is carried by LRs. What is the logic behind this result?

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Although an increase in θ worsens the lemons problem and would push LRs to reduce their supply of liquidity other things equal, there is the countervailing effect of the increase in θ on the higher origination and greater supply of risky assets by SRs at date 2. As Proposition 6-1-(c) establishes, s_d^* is an increasing function of θ in the range where $\theta \leq \widehat{\theta}$.[23] This higher supply of risky assets makes LRs want to increase their holdings of outside liquidity. The latter effect dominates and thus results in an increasing M_d^* as a function of θ in the range $\theta \leq \widehat{\theta}$. Instead, when $\theta > \widehat{\theta}$ the supply effect is reversed and s_d^* is a decreasing function of θ. Both the supply side and the adverse selection effect then reduce the benefits for LRs of carrying cash. This is why M_d^* is a decreasing function of θ over the range where $\theta > \widehat{\theta}$.

Figure V illustrates that the price P_{2d}^* is a decreasing function of θ. Note that the decline is more pronounced when $\theta < \widehat{\theta}$ due to the increased supply of risky assets by SRs when θ increases. When $\theta \geq \widehat{\theta}$ SRs hit a corner solution, $m_d^* = 0$, and there can be no further investment in the risky asset. At that point the price P_{2d}^* keeps falling as θ increases, but at a lower rate since now only adverse selection is present.

The pattern of returns is revealing about LRs incentives to carry outside liquidity in the delayed-trading equilibrium. For $\theta < \widehat{\theta}$, R_{2d}^* is an increasing function of θ. The expected payoff of the risky asset at date 2 is given by (15), which is a decreasing function of θ. But the price P_{2d}^* is falling faster, so that returns R_{2d}^* are increasing in θ. This is why LRs want to carry more cash when θ increases. Instead when $\theta > \widehat{\theta}$, the expected payoff is still decreasing in θ but the price P_{2d}^* is falling more slowly so that R_{2d}^* is a decreasing function of θ in this range.

In sum, for $\theta \in [0, \widehat{\theta}]$ the more severe the lemons problem, as measured by θ, the higher the amount of outside liquidity brought to the market by LRs and the lower the amount of inside liquidity carried by SRs. This counterintuitive result is due to the fall in prices, P_{2d}^*, which makes the risky asset more attractive to LRs at date 2. The larger the liquidity discount at date 2, the more attractive it is for LRs to carry cash and trade opportunistically.

IX. ROBUSTNESS

IX.A Trading of Risky Assets at Date 0

We have so far only allowed for the distribution of risky assets originated by SRs at dates 1 (in state ω_{1L}) and 2 (in states ω_{20} and ω_{2L}). A natural question is whether distribution could

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[23]There are two effects on s_{2d}^* when $\theta \leq \widehat{\theta}$. When θ increases, SRs carry more risky projects; that is m_d^* decreases as the risky project is more likely to pay off at date 2. On the other hand, the higher θ, the lower the fraction of risky projects carried by SRs that is supplied at date 2. Note that the second term in (17), $1 - \theta\eta$, is a decreasing function of θ. Proposition 4-I shows that the first effect dominates the second over this range.

also take place instantaneously at date 0 and whether this might not be welfare improving. We show next that, in fact, if a market is open, no trading will occur at date 0.

To see that instantaneous trading cannot be supported in a competitive equilibrium, suppose to the contrary that there is a profile of equilibrium prices $\left[\widehat{P}_0, \widehat{P}_1, \widehat{P}_2 \right]$ that supports instantaneous-trading. In an instantaneous-trading equilibrium it must the case that SRs and LRs weakly prefer to trade at date 0 rather than at date 1, that is,

$$(18) \qquad \widehat{P}_0 \geq \lambda \rho + (1 - \lambda) \widehat{P}_1 \qquad \text{and} \qquad \frac{[\lambda + (1 - \lambda) \eta] \rho}{\widehat{P}_0} \geq \frac{\eta \rho}{\widehat{P}_1}.$$

When the first inequality in (18) holds SRs weakly prefer to sell their risky asset at date 0 rather than date 1; under the second condition the expected return of acquiring the risky assets for LRs are not lower at date 0 than at date 1. Trivial manipulations of these inequalities then imply that

$$\widehat{P}_0 \geq [\lambda + (1 - \lambda) \eta] \rho.$$

As $\varphi'(\kappa) > 1$, LRs then strictly prefer to invest their capital in the long-run asset to purchasing any risky assets at price $\widehat{P}_0$ at date 0. It follows from this argument that neither the immediate-trading equilibrium, nor the delayed-trading equilibrium is unraveled by the introduction of possible trading of risky assets at date 0.

In summary, LRs prefer to hold cash to acquire assets opportunistically at depressed prices at dates 1 and 2. The gains from trade between SRs and LRs occur in states of nature ω_{1L} and ω_{2L} when SRs suffer a negative maturity shock. Not surprisingly, therefore, this is when LRs want to be in the market for risky assets. In other words, our model represents a particular form of modern banking: origination and *contingent* distribution of assets in the presence of liquidity shocks.

IX.B General Investment Opportunity Sets for Both LRs and SRs

If instantaneous distribution of risky assets cannot be supported as an equilibrium, the next question is whether LRs would want to invest in risky assets directly at date 0 if they could? So far we have ruled out this possibility by assuming that asset markets are segmented: only SRs can invest in a risky asset, and only LRs can invest in the long-maturity asset. Interestingly, this separation in investment opportunity sets is less restrictive than it seems.

Consider first LRs. Even if LRs can invest in risky assets at date 0, they may still choose not to hold these assets if the return on risky assets is low relative to the return on holding cash, as is the case for a large subset of our parameter values in our model. If, however, the

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supply of risky assets by SRs is so low that SRs earn a scarcity rent from investing in risky assets, then LRs may also invest a positive amount of their endowment in risky assets at date 0. In this case SRs are fully invested in risky assets and hold no cash. Even in this case, LRs will continue to hold cash sufficient to equalize the return on the marginal dollar held in cash with the expected return on risky assets at date 0. The prospect of purchasing risky assets from SRs at distressed prices at dates 1 or 2 provides a sufficiently high expected return on cash to LRs to induce them to hold positive amounts of cash.

Consider next SRs. If they are allowed to invest in the long-maturity asset, they may still choose not to invest in these assets if the discounted return on the long-maturity asset from their point of view is sufficiently low. If they buy and hold long-run assets, a sufficient condition for SRs to prefer not to fully invest in the long run asset is $\delta \varphi'(1) < 1$.

Similarly, even if SRs buy long-run assets to sell them to LRs at date 1 or 2, as a substitute for holding cash, they may still choose to only hold cash and originate risky assets if the shadow cost of cash for LRs $\varphi'(\kappa - M)$ is very large. Indeed, in this case SRs have to sell their long-run assets at such discounts at dates 1 or 2 that holding only cash and risky assets is preferred to holding long run assets that they sell at dates 1 or 2.

If however, the shadow cost of cash for LRs is not too high then SRs may choose to buy long-run assets to sell them to LRs at date 1 or 2, as a substitute for holding cash. In this case our analysis with respect to SRs demand for liquidity with respect to the risky assets they originate would still go through virtually unchanged. In this case, cash is a dominated asset for SRs but not for LRs, as the latter continue to benefit from buying risky assets in secondary markets at distressed prices. Also, the Pareto dominance of the delayed-trading equilibrium would still obtain. The only difference is that liquidity for SRs is held in the form of a tradable long-run asset instead of cash.

The basic point is that what makes an investor an SR or LR is almost by definition the investor's preferences for short versus long-maturity assets. These preferences in turn drive portfolio choices whether or not we assume that asset markets are segmented.

IX.C Arbitrage Contagion: The Price of the Long Run Asset

In the main analysis of the model we have only considered secondary markets for risky assets at dates 1 and 2. We now briefly discuss the implications of also opening secondary markets for the long-run asset at those dates. We show that cash-in-the-market pricing in one market then translates into cash-in-the-market pricing in other markets with potentially large "balance sheet" effects for LRs. Consider first the immediate trading equilibrium. In this

equilibrium the price at dates 1 and 2 for claims to date 3 output from the long run asset are

$$S_{1i}^* = \frac{P_{1i}^*}{\eta\rho} < 1 \qquad \text{and} \qquad S_{2i}^* = 1,$$

respectively. As LRs are risk neutral, the expected returns of all the assets they may hold have to be equated otherwise there would be an arbitrage gain. If for instance $S_{1i}^* = 1$ then LRs could sell claims to date 3 output from the long run asset to obtain the cash and then acquire the risky assets at date 1, which offer higher expected returns. Similarly, in the delayed trading equilibrium secondary market prices for date 3 consumption are

$$S_{1d}^* = \frac{P_{1d}^*}{\eta\rho} \qquad \text{and} \qquad S_{2d}^* = \frac{P_{2d}^*(1 - \theta\eta)}{(1 - \theta)\eta\rho}.$$

Similarly, claims to date 3 output from the long-run asset also trade at depressed prices at date 1, even if fire sales of risky assets only take place at date 2.

In sum, a unit of output from the long-run project at date 3 has to trade at a discount at dates 1 and 2 because of arbitrage. Thus, in our setup cash-in-the-market pricing is necessarily transmitted in the form of *arbitrage contagion* across different secondary asset markets, even if no trading of the long-run asset actually occurs in equilibrium. In other words, liquidity events affect prices of assets other than the ones where distressed sales are taking place. Liquidity crises thus cannot be contained across markets and time when these markets are linked via arbitrageurs.

IX.D Trading of Indivisible Risky Projects

In this subsection we explore the consequences of restricting LRs to buying an integer number of indivisible projects. This restriction parallels the constraint we imposed on SRs and is similarly motivated by the fact that assets may in practice be physically indivisible, and more importantly, that information about each risky project is itself indivisible.

Consider for example the delayed-trading equilibrium. If only indivisible assets can be traded then only a fraction $(1 - \theta\eta)$ of LRs will be acquiring risky assets (recall that there is a unit mass of SRs and LRs). These risky assets will be purchased with their own cash reserves M_d^*, and with the cash reserves of the other $\theta\eta$ fraction of LRs, obtained by exchanging a share of long-run assets held by the fraction $(1 - \theta\eta)$ of LRs. If there is such a feasible exchange of cash for long-run assets then there is no difficulty in supporting a delayed-trading equilibrium with indivisible risky projects.

Thus, we need to verify that the value of long-run assets $(1 - \theta\eta)S_{2d}^*\varphi(\kappa - M_d^*)$ held by

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LRs acquiring risky assets is greater than or equal to the value of cash held by LRs who do not acquire risky assets: $\theta\eta M_d^*$. In other words, we need to verify that the following inequality holds:

$$(1 - \theta\eta)\frac{P_{2d}^*(1 - \theta\eta)}{(1 - \theta)\eta\rho}\varphi(\kappa - M_d^*) \geq \theta\eta M_d^* \equiv \theta\eta P_{2d}^*(1 - m_d^*)$$

or:

(19)
$$\frac{(1 - \theta\eta)^2}{(1 - \theta)\eta\rho}\varphi(\kappa - M_d^*) \geq \theta\eta(1 - m_d^*).$$

Note that inequality (19) holds for theta sufficiently small. In addition it can be verified that inequality (19) holds for all our numerical examples for the delayed-trading equilibrium.

Alternatively, we can also interpret the decreasing returns to scale of the long run asset as due to a pecuniary externality that depends on the average amount invested by all LRs. That is, the output produced at date 3 with x units invested at date 0 equals $x\phi(\bar{x})$, where $\bar{x}$ is the average LR investment and ϕ is a concave function. Under this interpretation, every LR is indifferent between holding cash or investing in the long run project in equilibrium. Besides capturing an important aggregate economic effect, this formulation also makes it easier to accommodate the discreteness of long-run projects.

X. LONG-TERM CONTRACTS FOR LIQUIDITY

X.A Long-term Contracts

As Section VI.C highlights, a commitment by LRs to purchase risky assets at date 2 at a pre-determined price can improve ex-ante welfare in situations where a delayed-trading equilibrium fails to exist due to severe lemons problems. A natural question therefore is what form of long term contract between an SR and LR at date 0 can improve on the allocations obtained in the immediate and delayed-trading equilibria?

Allowing for bilateral contracts between an SR and LR expands the set of allocations that can be attained as transfers can be made contingent on the realization of $\omega_{2\rho}$, ω_{20}, and ω_{2L}. It therefore seems to follow that ex-ante contracting will always give rise to more efficient outcomes than under the immediate and delayed-trading equilibria. A key and surprising observation of this section, however, is that optimal incentive-compatible, ex-ante contracts do not generally give rise to strict efficiency improvements over the equilibrium allocations in the delayed-trading equilibrium.

We consider long-term bilateral contracts between one SR and one LR such that SR transfers to LR both his risky investment opportunity and unit of endowment at date 0 in

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exchange for the commitment by LR to offer SR a state-contingent consumption stream $C_t(\omega)$, where $t = 1, 2, 3$ and $\omega \in \{\omega_{1\rho}, \omega_{1L}, \omega_{2\rho}, \omega_{20}, \omega_{2L}, \omega_{30}, \omega_{3\rho}\}$. In other words, the contract sets up a fund with total assets $(1 + \kappa)$ managed by LR and invested in a portfolio of assets that may comprise the long-run asset, a risky asset, and cash.[24] As LR is managing the fund, LR can observe the realized idiosyncratic states of nature for the risky asset, but not SR. The fund manager LR therefore faces incentive compatibility constraints, which limit the efficiency of the long-term contract.[25]

Finally we assume throughout that $\delta\varphi(\kappa) < 1$, so that LR would not simply invest the whole endowment $(1 + \kappa)$ in the long asset and repay the SR at date 3.

X.B Feasibility, Participation and Incentive Compatibility Constraints

We begin with date 3 incentive compatibility constraints, in situations where LR has previously announced the state of nature ω_{2L}. Incentive compatibility at date 3 then requires that

(20) $$C_3(\omega_{3\rho}) = C_3(\omega_{30}),$$

for otherwise LR simply announces the state which involves the lower payment to SR.

Given this constraint, date 2 incentive compatibility in turn requires that:

(21) $$C_2(\omega_{2L}) + C_3(\omega_{30}) = C_2(\omega_{20}) + C_3(\omega_{20}) = C_2(\omega_{2\rho}) + C_3(\omega_{2\rho}).$$

Otherwise, again, LR would simply announce the state at date 2 which involves the lowest total payout.

Turning next to feasibility constraints, we can without any loss in generality impose the restriction that $C_1(\omega_{1\rho}) = C_1(\omega_{1L}) = 0$ given that SR is indifferent between consumption at

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[24]Note that we do not allow for more general multilateral contracts such that, for example, a giant financial intermediary contracting with all LRs and SRs simultaneously. In the absence of any organizational frictions in managing such a large institution, this arrangement is bound to achieve a better outcome, as it can pool all the idiosyncratic risks and thereby virtually eliminate asymmetric information between the parties.

It is clearly unrealistic, however, to suppose that such an institution can be run without a hitch, and that it can magically overcome all existing informational constraints. In other words, such an institution in practice would be constrained by the same informational problems present in competitive bilateral exchange, but this time inside the organization. Explicitly modeling these informational frictions and solving for the optimal informationally efficient multilateral organization is beyond the scope of this paper.

[25]If SR can also observe the realization of idiosyncratic shocks then the asymmetric information problem in the delayed-trading equilibrium would not be present, so that the long-term contract at date 0 clearly yields a superior outcome. The more consistent and interesting situation, however, is when the observation of idiosyncratic shocks is private information to the manager of the risky asset.

date 1 and 2. Under this restriction the feasibility constraints in state $\omega_{1\rho}$ are as follows:

$$C_{2\rho}\left(\omega_{1\rho}\right) \leq \alpha_x \rho + M_x \quad \text{and} \quad C_2\left(\omega_{1\rho}\right) + C_3\left(\omega_{1\rho}\right) \leq \alpha_x \rho + M_x + \varphi\left(y_x\right),$$

where $\alpha_x \leq 1$ is the amount that LR invests in the risky project, M_x the cash position and $y_x = \kappa + 1 - \alpha_x - M_x$ the amount invested in the long project. Note that since α_x, M_x and y_x are all observable to SR and LR and verifiable, the long term contract between the two parties will specify a particular portfolio allocation. The other feasibility constraints follow along similar lines and in the interest of space we write them out explicitly only in the Appendix.

Finally, participation constraints at date 0 must also be met. Without loss of generality we give LR all the bargaining power. He can make a take-it-or-leave-it offer to SR, who in turn accepts the contract if and only if she gets at least the same payoff as under the delayed trading equilibrium. The long-term contract then dominates the allocation under the delayed-trading equilibrium if and only if the surplus under the long term contract to LR, Π_x^*, exceeds LR's expected payoff under the delayed-trading equilibrium, Π_d^*.[26]

X.C Long-term Contracts Versus Market Liquidity

When SR expects the delayed-trading equilibrium, then the long-term contract cannot always replicate the allocation under delayed trading. The reason is that under delayed trading, SR is constrained by different incentive constraints at date 2 than those faced by LR under the long-term contract. Under delayed-trading SR must trade the risky asset at the same price in both states ω_{20} and ω_{2L}, and in state $\omega_{2\rho}$ there is no trade between SR and LR. Under the long-term contract, however, LR promises transfers $C_t(\omega)$ to SR which must satisfy the incentive compatibility constraints (20) and (21). It is immediate from these constraints that LR cannot replicate the delayed-trading equilibrium allocation under a long-term contract.

Given that the delayed-trading equilibrium allocation is not in the feasible set for the long-term contract it is not obvious a priori which allocation is superior. To be able to answer this question we must first characterize the optimal long-term contract. Solving the long-term contracting problem is a somewhat tedious constrained optimization problem, as it involves two investment variables (α, M) and seven state-contingent transfers to SR. This problem can be simplified to some extent, as the next proposition establishes, since the combination of

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[26]When SR expects the immediate-trading equilibrium, then any pair of LR and SR are weakly better off writing a long-term contract at date 0. At worst the contract simply replicates the allocation under immediate trading. But, the contract can also implement other allocations that are not feasible under the immediate-trading equilibrium. Therefore, the optimal long-term contract weakly (and sometimes strictly) dominates the equilibrium allocation under immediate trading.

all the incentive and feasibility constraints reduce the long-term contracting problem to the determination of optimal values for only: i) the amount $\alpha \in [0,1]$ invested in the risky SR project, ii) the amount M of cash held by the fund, and iii) payments to SR in states $\omega_{1\rho}$, $\omega_{2\rho}$ and ω_{30}.

PROPOSITION 7. (Characterization of the long term contract)

1. Without loss of generality, any feasible, incentive-compatible long-term contract between LR and SR takes the form:

	$\omega_{1\rho}$	$\omega_{2\rho}$	ω_{20}	ω_{2L}, ω_{30}	$\omega_{2L}, \omega_{3\rho}$
$C_2(\omega)$	$M + \alpha\rho$	$C_2(\omega_{2\rho})$	M	M	M
$C_3(\omega)$	$C_3(\omega_{1\rho})$	$C_3(\omega_{2\rho})$	$C_3(\omega_{30})$	$C_3(\omega_{30})$	$C_3(\omega_{3\rho})$

2. Suppose that δ is close to zero and that

$$(22) \qquad \eta(1-\lambda)\rho + \varphi(0) \leq \varphi(\kappa),$$

then the optimal long-term contract is such that $C_3(\omega_{1\rho}) = C_3(\omega_{2\rho}) = 0$.

As SR discounts date 3 consumption by δ it seems inefficient to offer any date 3 consumption to SR. Still, we cannot rule out that $C_3(\omega) > 0$ for either $\omega \in \{\omega_{1\rho}, \omega_{2\rho}, \omega_{30}, \omega_{3\rho}\}$ since a date 3 transfer in one state may be required for LR to satisfy all incentive constraints he faces. To be able to credibly disclose that the realized state is ω_{20}, for example, LR may have to promise a high transfer $C_3(\omega_{30})$ at date 3. Nevertheless, intuition suggests that if δ is very small, λ is sufficiently large, and the opportunity cost of holding cash for LR is bounded, then the optimal contract ought to specify $C_3(\omega_{1\rho}) = C_3(\omega_{2\rho}) = 0$. This is what Proposition 7.2 establishes.

With this characterization we are able to numerically solve for the optimal contract and to compare LR payoffs under the contract and under the delayed-trading equilibrium. The numerical solution is such that the long-term contract is dominated by the delayed-trading equilibrium for high values of θ, but not for low values of θ. The economic logic behind this result is that when θ is high the risky asset is likely to mature at dates 1 or 2. The added value of additional liquidity to SR offered by LR through a long-term contract is then not that high. In addition, when θ is high LR also faces high costs of meeting incentive constraints under the long-term contract. To be able to credibly claim that the risky asset did not yield a return ρ at either dates 1 or 2, LR must commit to wasteful date 3 payments $C_3(\omega_{3\rho}) = C_3(\omega_{30}) > 0$,

which SR does not value much. The deadweight cost of these distortions then exceeds the benefit of extra liquidity insurance when θ is high.

EXAMPLE 2. In our example we keep θ as a free parameter and fix the other parameters to the following values:

$$\lambda = .7 \qquad \eta = .4 \qquad \rho = 1.25 \qquad k = .12 \qquad \delta = .1 \quad \text{and} \quad \varphi(x) = x^\gamma \quad \text{with} \quad \gamma = .19.$$

Note that all our assumptions are then met as long as $\theta \le .8148$. Accordingly our plots below are restricted to the interval $\theta \in [0, .8148]$. The payoffs of SR and LR under the long-term contract are given by respectively:

$$
\begin{aligned}
\pi_x^* \;=\;\; & \lambda[M + \alpha\rho + \delta C_3(\omega_{1\rho})] \\
+ \;\; & (1-\lambda)[\theta\eta(C_2(\omega_{2\rho}) + \delta C_3(\omega_{2\rho})) + (1-\theta\eta)(M + \delta C_3(\omega_{30}))]
\end{aligned}
$$

and

$$
\begin{aligned}
\Pi_x^* \;=\;\; & M + \varphi[\kappa + (1-\alpha) - M] + \lambda[\alpha\rho - (C_2(\omega_{1\rho}) + C_3(\omega_{1\rho}))] \\
+ \;\; & (1-\lambda)[\eta\alpha\rho - (M + C_3(\omega_{30}))].
\end{aligned}
$$

We set $\pi_x^* = \pi_d^*$, the SR payoff in the delayed-trading equilibrium. The numerical solution for the chosen parameter values is such that $C_3(\omega_{1\rho}) = C_3(\omega_{2\rho}) = C_3(\omega_{30}) = 0$, and therefore that $C_2(\omega_{2\rho}) = M$.

Note that for these parameter values a delayed-trading equilibrium always exists. In the top panel of Figure IX we plot the expected utility of SR in the delayed-trading equilibrium and in the bottom panel we plot the expected utility of LR in the delayed-trading equilibrium, Π_d^*, together with LR's expected payoff under the long-term contract, Π_x^*. For $\theta > \tilde{\theta}$ this payoff is less than what LR gets in the delayed-trading equilibrium. The bottom panel of Figure X shows that when θ increases, the amount of cash carried by LR to fulfill his commitments under the long-term contract increases, making the contract less efficient, in sharp contrast with the total amount of cash $m_d^* + M_d^*$ carried by both LR and SR in the delayed-trading equilibrium, shown in the top panel of the same figure. This increase in cash under the long-term contract is due to the incentive constraints LR faces, which restrict the difference in payments in states $\omega_{2\rho}$ and ω_{20}. As

payments at date 3 are highly inefficient, the contract specifies higher payments at date 2, which requires carrying more cash. □

XI. Conclusions

This paper is concerned with two questions. First, what determines the mix of inside and outside liquidity in equilibrium? Second, does the market provide an efficient mix of inside and outside liquidity? In addition we asked whether the provision of market liquidity can be Pareto improved upon by long term contracts between those with potential liquidity needs and those who are likely to supply it.

Our model departs from the existing literature by considering the endogenous timing of asset sales and the deterioration of adverse selection problems over time. Financial intermediaries face the choice of raising liquidity early before adverse selection problems set in, or in the midst of a crisis at more depressed prices. The benefit of delaying asset sales and attempting to ride through the crisis is that the intermediary may be able to entirely avoid any sale of assets at distressed prices should the effect of the crisis on its portfolio be mild. We show that when the adverse selection problem is not too severe there are multiple equilibria, an immediate-trading and a delayed-trading equilibrium. In the first equilibrium, intermediaries liquidate their positions in exchange for cash early in the liquidity crisis. In the second equilibrium, liquidation takes place late in the liquidity event and in the presence of adverse selection problems.

We show that, surprisingly, the latter equilibrium Pareto-dominates the former because it saves on cash reserves, which are costly to carry.[27] However, the delayed-trading equilibrium does not exist when the adverse selection problem is severe enough. The reason is that in this case prices are so depressed as to make it profitable for the agents holding good assets to carry them to maturity even when it is very costly to do so. We show that if they were able to do so, intermediaries would be better off committing ex-ante to liquidating their assets at these depressed prices in the distressed states. We also show that a monopoly supplier of liquidity may be able to improve welfare.

We argued in Bolton, Santos and Scheinkman (2009) that the role of the public sector as a provider of liquidity has to be understood in the context of the competitive provision of liquidity by the private sector. In particular the public provision of liquidity can act as a complement for private liquidity in situations where lemon's problems are so severe that

---

[27]This Pareto dominance must be qualified by the fact that we ignore the greater moral hazard problems at origination that may arise in the delayed-trading equilibrium.

the market would break down without any public price support. For the intervention to be effective, the public liquidity provider needs to know whether the crisis is at date 1 or 2. An important remaining task is then to analyze the benefits of public policy in our model under the assumption that the public agency may be ignorant about the true state of nature in which it is intervening.

Another central theme in our analysis is the particular timing of the liquidity crisis that we propose. Liquidity crises tend in our view to be triggered by real shocks. In our framework the onset of the liquidity event starts with a real deterioration of the quality of the risky asset held by financial intermediaries. The assumption that adverse selection problems worsen during the liquidity crisis is a feature of our analysis that, as we have argued, seems plausible in the context of the current crisis. Our model captures the fact that intermediaries were holding securities which had a degree of complexity that made for a costly assessment of the actual risk that they were exposed to (see Gorton (2008) for an elaboration of this point). Once problems in the mortgage market were widely reported in early 2007 banks turned to an assessment of the actual risks buried in their books. As emphasized by Holmstrom (2008) the opacity of these securities was also initially the source of their liquidity. Once the crisis started though, banks and intermediaries started the costly process of risk discovery in their books, which immediately led to an adverse selection problem. Financial institutions here faced a choice of whether to liquidate early or ride out the crisis in the hope that the asset may ultimately pay off. This trade-off is unrelated to the incentives that may force institutions to liquidate at particular times, due to accounting and credit quality restrictions in the assets they can hold, that have featured more prominently in the literature. Understanding the effect that these restrictions have on the portfolio decisions of the different intermediaries remains an important question to explore in future research.

Finally, in our model LRs are those agents with sufficient knowledge to be able to value and absorb the risky assets for sale by financial intermediaries. Only their capital and liquid reserves matter for equilibrium pricing to the extent that they are the only participants with the knowledge to perform an adequate valuation. Other, less knowledgeable, capital will only step in at steeper discounts. Our current research attempts to understand how different knowledge-capital gets "earmarked" to specific markets. What arises is a theory of market segmentation and contagion that may shed new light on the behavior of financial markets in crisis situations.

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Appendix

Proof of Proposition 1. First define P_2 to be the solution to[28]

$$\lambda + (1 - \lambda)\frac{\eta\rho}{P_2} = \varphi'\left(\kappa - (1-\theta)P_2\right),$$

which always exists, is unique, and immediately implies that $P_2 < \eta\rho$ by Assumption 1. Define next

$$\underline{P}^{fi} = \frac{1 - \rho\left[\lambda + (1-\lambda)\theta\eta\right]}{(1-\theta)\left(1-\lambda\right)},$$

where fi stands for full information. We have two cases.

- Case 1: $P_2 < \underline{P}^{fi}$; then set $P_2^* = \underline{P}^{fi}$ and set M^* to be the unique solution to

$$(23) \qquad\qquad \lambda + (1 - \lambda)\frac{\eta\rho}{P_2^*} = \varphi'\left(\kappa - M^*\right),$$

  the LR's first order condition, which now takes into account the fact that the acquired assets have expected payoff $\eta\rho$ as there is no asymmetric information (compare this with the conditional expected payoff under asymmetric information, expression (15)). Clearly $M^* < (1-\theta)P_2^*$. Then, set $m^* \in (0,1)$ to solve

$$P_2^* = \frac{M^*}{(1-\theta)(1-m^*)},$$

  which, obviously exists, is unique, and satisfies the SR's first order condition. SRs and LRs postpone trading to date 2 as long as

$$(24) \qquad\qquad P_1^* \in [P_2^*, \theta\eta\rho + (1-\theta)P_2^*],$$

  which is non-empty by Assumption 2. Finally we show that $M^* > 0$. Notice that the LR's first order condition (23) can be written as

$$\psi\left(\theta\right) = \varphi'\left(\kappa - M^*\right) \qquad \text{where} \qquad \psi\left(\theta\right) = \lambda + (1-\lambda)^2\frac{(1-\theta)\,\eta\rho}{1 - \rho\left[\lambda + (1-\lambda)\,\theta\eta\right]}$$

  and notice that again, Assumption 3 can be written as $\psi\left(0\right) > \varphi'\left(\kappa\right)$. Differentiating, rearranging and by Assumption 2 we obtain that $\psi_\theta > 0$, which proves that $M^* > 0$.

---

[28]Throughout we drop the subscript d to emphasize that now the only equilibrium is a delayed one.

- Case 2: $P_2 \geq \underline{P}^{fi}$; then set $P_2^* = P_2$, $M^* = (1-\theta)P_2^*$ and $m^* = 0$, which by construction satisfy the LR's and SR's first order condition, respectively. Notice that given that $P_2^* \leq \eta\rho$, it immediately follows that the interval in (24) is non-empty. Finally, to support the equilibrium at date 2 it has to be the case that $\delta \leq \bar{\bar{\delta}}$ where $\bar{\bar{\delta}} = P_2^*/\eta\rho$, which concludes the proof.   QED

Proof of Proposition 2. We proceed by constructing an immediate-trading equilibrium with prices P_{1i}^* and P_{2i}^*. We show that under those prices SRs prefer to sell the risky asset at date 1, rather than selling at date 2 or alternatively carrying the asset to date 2, taking the chance that the asset may payoff in $\omega_{2\rho}$, or to date 3 if in ω_{2L}, or swapping the risky asset for units of the long asset (trading at S_{1i}^*).

The first order condition of the LR is

(25) $$\lambda + (1-\lambda)\frac{\eta\rho}{P_{1i}} \leq \varphi'(\kappa - M).$$

First we establish that it is not possible to support an equilibrium with $M_i^* = 0$ and $m_i^* = 1$. Indeed if $m_i^* = 1$ it has to be the case that the price in state ω_{1L} is such that

$$P_{1i}^* \leq \frac{1-\lambda\rho}{1-\lambda}$$

but by Assumption 3 this implies

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_{1i}^*} > \varphi'(\kappa),$$

and thus $M_i^* > 0$ a contradiction.

Having ruled the no trade immediate-trading equilibrium we proceed next as follows. Start by solving the following equation in P_{1i}

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_{1i}} = \varphi'(\kappa - P_{1i}),$$

and define

$$\underline{P} = \frac{1-\lambda\rho}{1-\lambda},$$

a positive number by Assumption 2.

- Case 1: Assume first that $P_{1i} \geq \underline{P}$, then set $P_{1i}^* = M_i^* = P_{1i}$ and $m_i^* = 0$, which meets the first order condition of the SRs as can be checked by inspection of expression (2).

- Case 2: Assume next that $P_{1i} < \underline{P}$, then set $P_{1i}^* = \underline{P}$ and M_i^* to be the solution to

$$\lambda + (1 - \lambda)\frac{\eta\rho}{\underline{P}} \leq \varphi'\left(\kappa - M_i^*\right),$$

which by Assumption 3 is such that $M_i^* > 0$ and clearly it has to be such that $M_i^* < \underline{P}$. Because, given these prices, the SRs are indifferent on the level of cash carried set m_i^* so that

$$P_{1i}^* = \frac{1 - \lambda\rho}{1 - \lambda} = \frac{M_i^*}{1 - m_i^*}$$

As for prices at date 2 they have to be such that both the SRs and the LRs prefer to trade at ω_{1L}. For this set $P_{2i}^* = 0$. Given this price the LR investors expect only lemons (assets with zero payoff) in the market at $t = 2$ and thus the demand is equal to zero $Q_2^* = 0$. As for the SRs notice that if they wait to liquidate at $t = 2$ they obtain

$$\theta\eta\rho < \frac{1 - \lambda\rho}{1 - \lambda} \leq P_{1i}^*,$$

where the first inequality follows from Assumption 2. Thus SRs set $q_2^* = 0$ and $q_1^* = 1 - m_i^* = Q_1^*$.

Notice as well that under these prices SRs prefer to liquidate rather than carry the asset to date 2 or 3. Indeed, given that we have established that SRs do not want to sell at $t = 2$, if instead they were to carry the asset to dates $t = 2$ (where the asset pays with probability $\theta\eta$) or take its chances at date $t = 3$ (in which case the asset is worth $\delta\eta\rho$ in ω_{2L}) it must be because:

$$(26) \qquad\qquad P_{1i}^* < \theta\eta\rho + (1 - \theta)\delta\eta\rho$$

Recall that

$$(27) \qquad\qquad P_{1i}^* \geq \frac{1 - \lambda\rho}{1 - \lambda}.$$

Then substitution yields

$$(28) \qquad\qquad \frac{1 - \lambda\rho}{1 - \lambda} < \theta\eta\rho + (1 - \theta)\delta\eta\rho$$

which, once rearranged, yields

$$1 < \lambda\rho + (1 - \lambda)\left[\theta + (1 - \theta)\delta\right]\eta\rho, \tag{29}$$

a contradiction with Assumption 2. Finally, it is obvious that the SRs do not want to trade into the long asset. Indeed, assume they do. In this case the number of units of the long asset that they can acquire is $\eta\rho$, which are only worth $\delta\eta\rho$ to them which is clearly below P_{1i}^{*}, by Assumption 2.   QED

Proof of Proposition 3. We first construct a *candidate* delayed-trading equilibrium and then establish the conditions on δ under which the candidate delayed-trading equilibrium is indeed an equilibrium.

First notice that since $\varphi'(\kappa) > 1$ in any delayed-trading equilibrium there must be cash-in-the-market pricing thus

$$M_d^* = P_{2d}^*\left(1 - \theta\eta\right)\left(1 - m_d^*\right)$$

Define P_{2d} to be the solution to

$$\lambda + (1 - \lambda)\frac{(1 - \theta)\,\eta\rho}{(1 - \theta\eta)\,P_{2d}} = \varphi'\left(\kappa - (1 - \theta\eta)\,P_{2d}\right)$$

This equation always a unique solution which in addition satisfies

$$P_{2d} \in \left(0, \frac{\kappa}{1 - \theta\eta}\right).$$

There are two cases to consider:

- Case 1: P_{2d} is such that

$$P_{2d} < \frac{1 - \rho\left[\lambda + (1 - \lambda)\theta\eta\right]}{(1 - \lambda)(1 - \theta\eta)} = \underline{P}. \tag{30}$$

    In this case set

$$P_{2d}^* = \underline{P},$$

and set M_d^* to be the solution of

$$(31) \qquad \lambda + (1-\lambda) \frac{(1-\theta)\,\eta\rho}{(1-\theta\eta)\,P_{2d}^*} = \varphi'\left(\kappa - M_d^*\right),$$

which from the strict concavity of $\varphi(\cdot)$ is

$$M_d^* < (1-\theta\eta)P_{2d}^*.$$

By Assumption 3 $M_d^* > 0$. Indeed, define

$$\psi\left(\theta\right) = \lambda + (1-\lambda)^2 \left(\frac{(1-\theta)\eta\rho}{1 - \rho\left[\lambda + (1-\lambda)\theta\eta\right]} \right),$$

which is the left hand side of the LR's first order condition as shown in (31). Notice that Assumption 3 can be simply written as $\psi(0) > \varphi'(\kappa)$. Straightforward algebra shows that

$$\psi_\theta \propto \rho\left[\lambda + (1-\lambda)\eta\right] - 1 > 0,$$

by Assumption 2.

Then choose m_d^* such that

$$P_{2d}^* = \frac{M_d^*}{(1-\theta\eta)\left(1 - m_d^*\right)}$$

Notice that because $P_{2d}^* = \underline{P}$ the SRs are indifferent in the level of cash held. Both types of traders would prefer to wait to trade at date 2 provided that P_{1d}^* is in the interval

$$\left[\frac{(1-\theta\eta)\,P_{2d}^*}{1-\theta}, \theta\eta\rho + (1-\theta\eta)\,P_{2d}^* \right],$$

which is non empty if and only if

$$(32) \qquad P_{2d}^* \le \frac{(1-\theta)\,\eta\rho}{1-\theta\eta} = \overline{P}.$$

Clearly, given Assumption 1, specifically the fact that $\varphi'(\kappa) > 1$, and equation (31), equation (32) is trivially met. Clearly, given Assumption 1, specifically the fact that $\varphi'(\kappa) > 1$, and equation (31), equation (32) is trivially met.

Notice that P_{2d}^* is independent of δ and for $\delta \le \bar{\delta}$, where

$$(33) \qquad \bar{\delta} = \frac{P_{2d}^*}{\eta\rho},$$

the SR (weakly) prefers to trade at date 2 for a price P_{2d}^* than carrying the asset to date 3.

- Case 2: $P_{2d} \ge \underline{P}$, then choose

$$P_{2d}^* = P_{2d} \qquad M_d^* = P_{2d}^* \left(1 - \theta\eta\right) > 0 \qquad \text{and} \qquad m_d^* = 0.$$

Except for establishing inequality (32), the remainder of the proof follows as in the previous case. To establish that P_{2d}^* meets (32) it is enough to substitute $\underline{P}$ in (32) and appeal to Assumption 2. \hfill QED

Before proceeding it is useful to establish the following

Result. Let Assumptions 1-4 hold. Then the immediate-trading equilibrium is such that $m_i^* \in (0,1)$.

Proof. By the SR's first order condition if the price at date 1 is given by

$$P_{1i}^* = \frac{1 - \lambda\rho}{1 - \lambda}$$

then the SR investor is indifferent about the cash position carried. Let M_i^* be the solution to

$$\lambda + (1 - \lambda)^2 \frac{\eta\rho}{1 - \lambda\rho} = \varphi'\left(\kappa - M_d^*\right),$$

which by Assumption 3 exists and is unique. By Assumption 4,

$$\frac{1 - \lambda\rho}{1 - \lambda} > \kappa > M_i^*.$$

Then set $m_i^* \in (0,1)$ so that

$$\frac{1 - \lambda\rho}{1 - \lambda} = \frac{M_i^*}{1 - m_i^*}.$$

The construction now of the immediate-trading equilibrium follows as in the proof of Proposition 1. \hfill QED

We prove Proposition 6 first. The proofs of Propositions 4 and 5 following trivially after that.

Proof of Proposition 6. First notice that by the result above, the immediate-trading equilibrium is such that $m_i^* > 0$ (and, obviously, $M_i^* > 0$). Thus because the delayed-trading equilibrium specializes to the immediate-trading equilibrium when $\theta = 0$, it follows that there exists a neighborhood $(0, \tilde{\theta})$ such that $m_d^* > 0$. Then from the LR's and SR's first order conditions, combined with cash in the market pricing, M_d^* and m_d^* are fully determined by

$$(34) \qquad \psi^{(M)} = \lambda + (1 - \lambda) R_d^* (\theta) - \varphi' (\kappa - M_d^*) = 0$$

$$(35) \qquad \psi^{(m)} = (1 - m_d^*) (1 - \rho (\lambda + (1-) \theta \eta)) - (1 - \lambda) M_d^* = 0$$

Expression (34) is the LR's first order condition. Expression (35) is the SR's first order condition combined with the cash-in-the-market pricing equation. These two equations determine M_d^* and m_d^*. In the above expression

$$R_d^* = \frac{(1 - \theta) \eta \rho}{(1 - \theta \eta) P_{2d}^*},$$

where P_{2d}^* is given by $\underline{P}$ (see expression (30). Then basic algebra shows that

$$R_{d,\theta}^* = \frac{\partial R_d^*}{\partial \theta} \propto \rho [\lambda + (1 - \lambda) \eta] - 1 > 0,$$

by Assumption 2.

$$(36) \qquad \partial_x \psi = \begin{pmatrix} \psi_M^{(M)} & \psi_m^{(M)} \\ \psi_M^{(m)} & \psi_m^{(m)} \end{pmatrix} \qquad \text{and} \qquad \partial_\theta \psi = \begin{pmatrix} \psi_\theta^{(M)} \\ \psi_\theta^{(m)} \end{pmatrix},$$

where

$$\psi_M^{(M)} = \varphi'' (\kappa - M_d^*) < 0$$
$$\psi_m^{(M)} = 0$$
$$\psi_m^{(m)} = - [1 - \rho (\lambda + (1 - \lambda) \theta \eta)] < 0$$
$$\psi_M^{(m)} = -(1 - \lambda)$$
$$\psi_\theta^{(M)} = (1 - \lambda) R_{d,\theta}^* > 0$$

$$\psi_\theta^{(m)} \;=\; -(1 - m_d^*)(1 - \lambda)\eta\rho < 0,$$

First,
$$|\partial_x \psi| = - \left[1 - \rho \left(\lambda + (1 - \lambda)\theta\eta \right) \right] \varphi''\left(\kappa - M_d^* \right) > 0$$

Second, by an application of the implicit function theorem

$$M_{d,\theta}^* = \frac{\partial M_d^*}{\partial \theta} = -[\; 1 \quad 0 \;] \left(\partial_x \psi \right)^{-1} \partial_\theta \psi \qquad \text{and} \qquad m_{d,\theta}^* = \frac{\partial m_d^*}{\partial \theta} = -[\; 0 \quad 1 \;] \left(\partial_x \psi \right)^{-1} \partial_\theta \psi.$$

After some algebra:

$$
\begin{aligned}
m_{d,\theta}^* \;&=\; -|\partial_x \psi|^{-1} \left[-\psi_M^{(m)} (1 - \lambda)\, R_{d,\theta}^* - \psi_M^{(M)} (1 - m)(1 - \lambda)\eta\rho \right] \\
(37) \qquad &=\; -|\partial_x \psi|^{-1} \left[(1 - \lambda)^2\, R_{d,\theta}^* - \varphi''\left(\kappa - M_d^* \right)(1 - m_d^*)(1 - \lambda)\eta\rho \right] \\
&<\; 0
\end{aligned}
$$

and

$$
\begin{aligned}
M_{d,\theta}^* \;&=\; -|\partial_x \psi| \left[\psi_m^{(m)} \psi_\theta^{(M)} - \psi_m^{(M)} \psi_\theta^{(m)} \right] \\
&=\; -|\partial_x \psi| \psi_m^{(m)} \psi_\theta^{(M)} \\
&>\; 0
\end{aligned}
$$

Because m_d^* is strictly decreasing in θ if $m_d^* = 0$ for some $\widehat{\theta}$, then $m_d^* = 0$ for all $\theta \geq \widehat{\theta}$. For $\theta \geq \widehat{\theta}$ the LR's first order condition is given by

$$\lambda + (1 - \lambda) \frac{(1 - \theta)\,\eta\rho}{M_d^*} = \varphi'\left(\kappa - M_d^* \right),$$

where we have made use of the fact that cash-in-the-market pricing obtains and $m_d^* = 0$. Then a basic application of the implicit function theorem shows that $M_{d,\theta}^* < 0$ for $\theta > \widehat{\theta}$. As for the behavior of expected returns when $\theta > \widehat{\theta}$, notice that the LR's first order condition is written as

$$\lambda + (1 - \lambda) R_d^* = \varphi'\left(\kappa - M_d^* \right),$$

and thus given that $M_{d,\theta}^* < 0$ for $\theta > \widehat{\theta}$, it follows that $R_{d,\theta}^* < 0$ for that range.

We turn now to the properties of the aggregate supply of the risky asset at date 2 in the delayed-trading equilibrium s_d^*. Using (37),

$$
\begin{aligned}
(38) \quad s_{d,\theta}^* &= |\partial_x \psi|^{-1} (1-\lambda)^2 R_{d,\theta}^* (1-\theta\eta) \\
&- |\partial_x \psi|^{-1} \varphi'' (\kappa - m_d^*) (1 - m_d^*) (1-\lambda) \eta\rho (1-\theta\eta) - \eta (1 - m_d^*)
\end{aligned}
$$

Tedious algebra shows that (38) is equal to

$$
(1 - m_d^*) \eta \left[\frac{\rho - 1}{1 - \rho\left(\lambda + (1-\lambda)\theta\eta\right)} \right],
$$

which is positive by Assumption 2. This completes the proof of Proposition 3.   QED

Proof of Proposition 4. That $m_i^* > m_d^*$ follows immediately from the fact that $m_i^* = m_d^* (\theta = 0)$ and Proposition 3. Clearly for $\theta \leq \widehat{\theta}$ where $\widehat{\theta}$ was defined in the proof of Proposition 3, $M_i^* < M_d^*$. For $\theta > \widehat{\theta}$ M_d^* is a decreasing function of θ and thus, by continuity there exists a (unique) θ', possibly higher than $\bar{\theta}$, for which $M_d^* (\theta') = M_i^*$; for any $\theta < \theta'$, $M_i^* < M_d^*$.   QED

Proof of Proposition 5. Under Assumption 4, $m_i^* > 0$ and thus $\pi_i^* = 1 \leq \pi_d^*$. As for the expected profits of the LR investors, firs notice that

$$
\frac{\partial \Pi_d^*}{\partial \theta} = \Pi_{d,\theta}^* = (1-\lambda) R_{d,\theta}^* M_d^*.
$$

Given that $\Pi_i^* = \Pi_d^* (\theta = 0)$ and the characterization of expected returns in Proposition 3 the result follows immediately.   QED

Proof of Proposition 7.

1. Given a choice M of cash carried by LR and α invested in the SR risky project $0 \leq \alpha \leq 1$, the feasibility constraints on transfers to SR are given by:

$$
\begin{aligned}
C_1 (\omega_{1\rho}) &\leq \alpha\rho + M, \\
C_1 (\omega_{1\rho}) + C_3 (\omega_{1\rho}) &\leq \alpha\rho + M + \varphi\left[\kappa + (1-\alpha) - M\right], \\
C_1 (\omega_{1L}) + C_2 (\omega_{20}) &\leq M, \\
C_1 (\omega_{1L}) + C_2 (\omega_{2L}) &\leq M, \\
C_1 (\omega_{1L}) + C_2 (\omega_{2\rho}) &\leq \alpha\rho + M,
\end{aligned}
$$

$$C_1\left(\omega_{1L}\right) + C_2\left(\omega_{2\rho}\right) + C_3\left(\omega_{2\rho}\right) \;\leq\; \alpha\rho + M + \varphi\left[\kappa + (1 - \alpha) - M\right],$$

$$C_1\left(\omega_{1L}\right) + C_2\left(\omega_{20}\right) + C_3\left(\omega_{20}\right) \;\leq\; M + \varphi\left[\kappa + (1 - \alpha) - M\right],$$

$$C_1\left(\omega_{1L}\right) + C_2\left(\omega_{2L}\right) + C_3\left(\omega_{30}\right) \;\leq\; M + \varphi\left[\kappa + (1 - \alpha) - M\right].$$

Consider next the following observations concerning equilibrium contracts:

(a) State $\omega_{1\rho}$ is observable and since there is no discounting between periods 1 and 2 we may assume without any loss of generality that $C_1(\omega_{1\rho}) = C_1(\omega_{1L}) = 0$.

(b) If $C_3(\omega_{1\rho}) > 0$ then $C_2(\omega_{1\rho}) = \alpha\rho + M$. For if $C_2(\omega_{1\rho}) < \alpha\rho + M$, both agents can be made better off by increasing $C_2(\omega_{1\rho})$ and decreasing $C_3(\omega_{1\rho})$.

(c) Incentive compatibility requires that $C_3(w_{30}) = C_3(w_{3\rho})$. Hence any feasible and incentive compatible payment in histories that follow from w_{2L} is also feasible in histories that follow w_{20}. Incentive compatibility also requires that

$$C_2\left(\omega_{2L}\right) + C_3\left(\omega_{30}\right) = C_2\left(\omega_{20}\right) + C_3\left(\omega_{20}\right).$$

Therefore any payment prescribed for the histories starting at w_{2L} must also be prescribed for histories starting at w_{20}:

$$C_2(\omega_{2L}) = C_2(\omega_{20}),$$

and

$$C_3(\omega_{30}) = C_3(\omega_{20}).$$

(d) If $C_3(\omega_{30}) > 0$ then $C_2(\omega_{2L}) = M$. For if $C_2(\omega_{2L}) < M$ SR can be made better off, while keeping LR indifferent, by increasing the payment at date 2 and decreasing by the same amount the payments in states ω_{30} and $\omega_{3\rho}$ at date 3. The same reason, together with observation 3, implies that if $C_3(\omega_{20}) > 0$ then $C_2(\omega_{20}) = M$. We can also use the same reasoning to show that if $C_3(\omega_{2\rho}) > 0$ then $C_2(\omega_{2\rho}) = M + \alpha\rho$.

(e) Since $(\lambda + (1 - \lambda))\rho > 1$ and $\varphi'(\kappa) > 1$, if cash is carried by the LR it must be distributed in some state (at either dates 1 or 2). Hence either $C_2\left(\omega_{1\rho}\right) = M + \alpha\rho$, or $C_2(\omega_{2\rho}) = M + \alpha\rho$ or $C_2(\omega_{20}) = C_2(\omega_{2L}) = M$. Note furthermore from observation 4 and incentive compatibility that we must have $C_2(\omega_{2\rho}) > 0$

and $C_2(\omega_{20}) > 0$ unless SR consumption is zero in all histories starting at ω_{1L}. However, in the latter case, because of discounting and $\delta\phi'(k) < 1$ the ex-ante contract is dominated by autarky. Hence we may assume that $C_2(\omega_{2\rho}) > 0$ and $C_2(\omega_{20}) > 0$. In an analogous fashion we can establish that $C_2(\omega_{1\rho}) > 0$.

(f) Suppose, that $C_2(\omega_{1\rho}) \le M + \alpha\rho - \mu$ for some $\mu > 0$, and let $\gamma > 0$ be small enough that $\gamma < \frac{\mu}{2}$ and $\gamma\frac{\lambda}{1-\lambda} < \min\{C_2(\omega_{2\rho}); C_2(\omega_{20})\}$. Consider the payment

$$\hat{C}_2(\omega_{1\rho}) = C_2(\omega_{1\rho}) + \gamma$$

and lower date 2 payments for all realizations following ω_{1L} by $\gamma\frac{\lambda}{1-\lambda}$. This new contract, leaves SR indifferent and economizes in cash. This cash can be invested in the LR project, which has a marginal product above one, and yield extra utility for LR at date 3. Hence the initial contract cannot be optimal.

(g) Suppose that $C_2(\omega_{2L}) < M$, then from observation 4, $C_3(\omega_{30}) = 0$. Hence $C_2(\omega_{20}) < M$ and $C_2(\omega_{2\rho}) < M + \alpha\rho$. Using the same logic as in observation 6 we may then show that this contract is not optimal.

(h) Incentive compatibility requires that

$$C_2(\omega_{2\rho}) + C_3(\omega_{2\rho}) = M + C_3(\omega_{30}).$$

Since $C_2(\omega_{2\rho}) = M$ satisfies the LR budget constraint, it follows that

$$C_3(\omega_{2\rho}) \le C_3(\omega_{30}).$$

2. Under assumption (22) LR's opportunity cost of holding cash, $\varphi'(\kappa + (1-\alpha) - M)$, is bounded. To see this, note first from lemma 7 that LR must pay SR at least M following the realization of state ω_{1L}. LR's date 0 expected payoff therefore cannot exceed:

$$\eta(1-\lambda)\alpha\rho + \varphi(\kappa + (1-\alpha) - M).$$

Since participation by LR requires that

$$\eta(1-\lambda)\alpha\rho + \varphi(\kappa + (1-\alpha) - M) \ge \varphi(\kappa),$$

we must have $\kappa + (1 - \alpha) - M > 0$, by assumption (22). It follows that

$$\varphi'(\kappa + (1 - \alpha) - M) < B, \quad \text{for some } B > 0.$$

Now, suppose by contradiction that $C_3(\omega_{1\rho}) \geq \epsilon > 0$. Then lowering $C_3(\omega_{1\rho})$ by ϵ and increasing M by $\delta\lambda\epsilon$ keeps SR indifferent, but makes LR strictly better off if $B\delta < 1$. Similarly, if $\min\{C_3(\omega_{2\rho}); C_3(\omega_{30})\} = C_3(\omega_{2\rho}) \geq \epsilon$, a decrease of $C_3(\omega_{30})$ and $C_3(\omega_{2\rho})$ by ϵ and an increase of M by $(1 - \lambda)\delta\epsilon$, again keeps SR indifferent but makes LR better off (provided that $B\delta < 1$). QED

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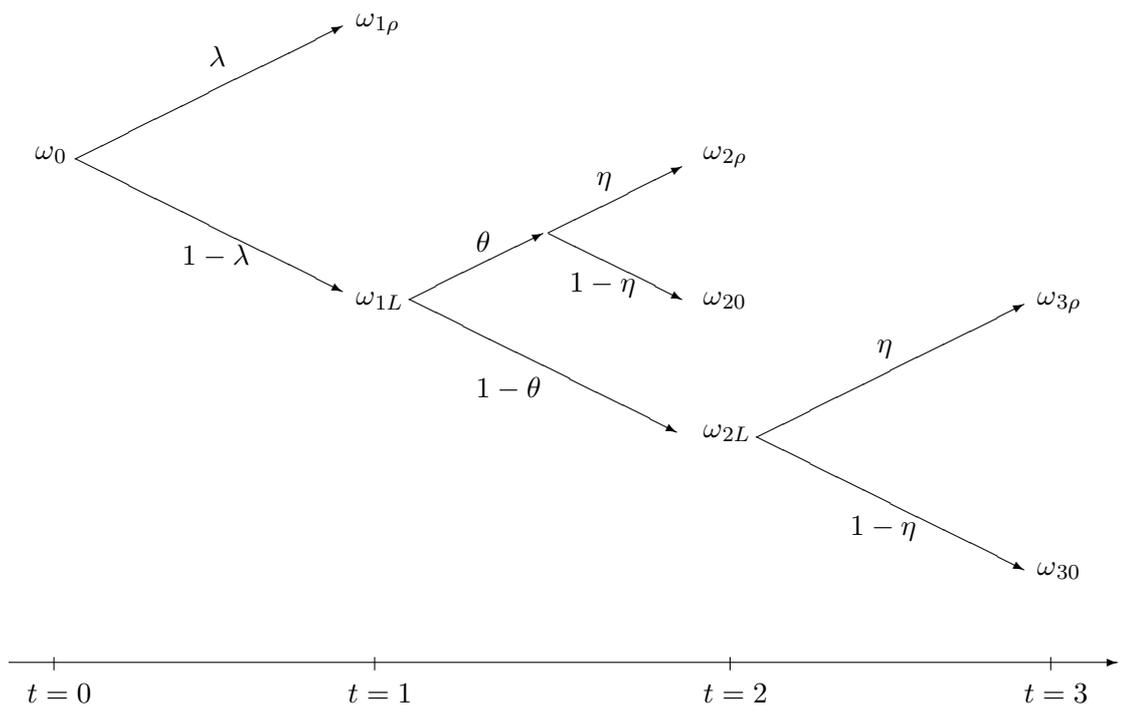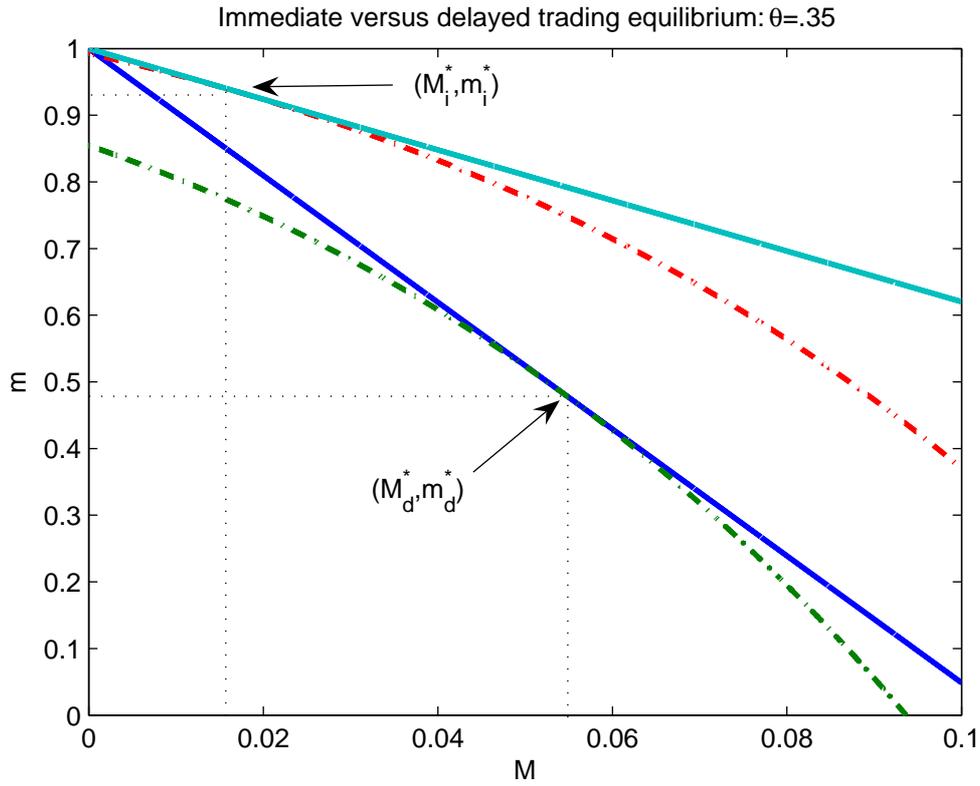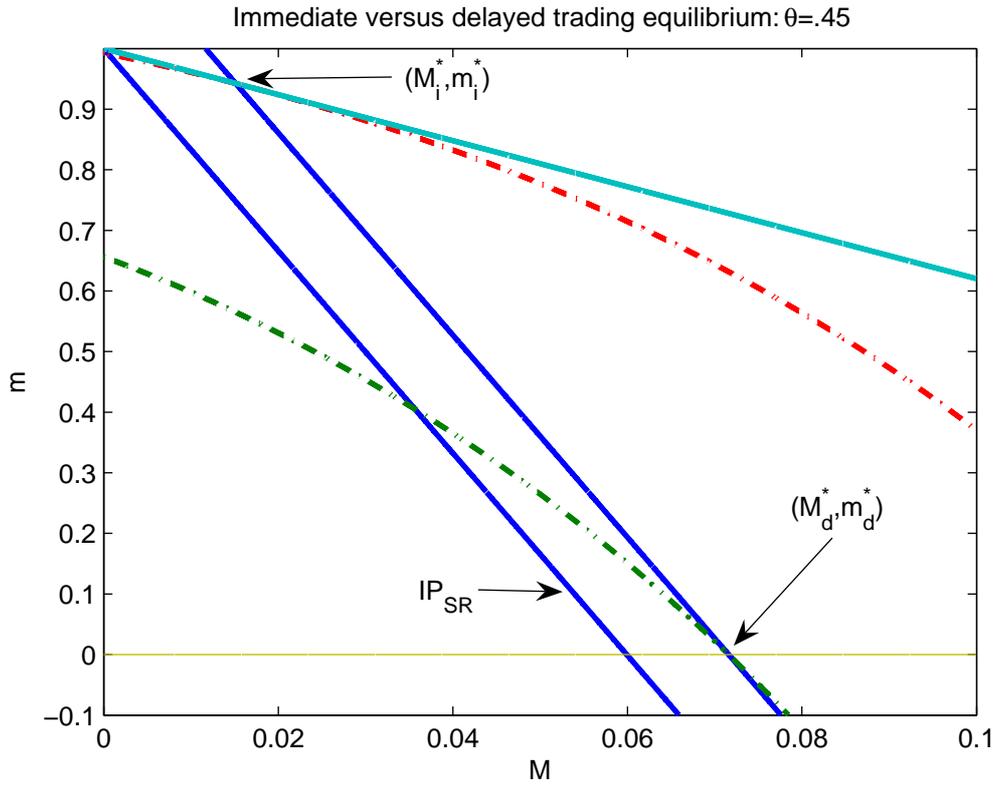
FIGURE I

The risky asset

FIGURE II

Immediate and delayed-trading equilibria in Example 1 for the case $\theta = .35$. The graph represents cash holdings, with the cash holdings of the LRs, M, in the x-axis and the cash holdings of the SRs, m in the y-axis. The dashed curves represent isoprofit lines for the LR and the straight continuous lines represent the SR's isoprofit lines, for both when the exchange occurs in state ω_{1L} and in date 2. The isoprofit lines for the SR correspond to its reservation profits $\pi_i^* = \pi_d^* = 1$. The immediate and delayed-trading equilibrium cash holdings are marked (M_i^*, m_i^*) and (M_d^*, m_d^*), respectively.

FIGURE III

Immediate and delayed-trading equilibria in Example 1 when $\theta = .45$. The graph represents cash holdings, with the cash holdings of the LRs, M, in the x-axis and the cash holdings of the SRs, m in the y-axis. The dashed curves represent isoprofit lines for the LR and the straight continuous lines represent the SR's isoprofit lines, for both when the exchange occurs in state ω_{1L} and in date 2. As opposed to the case in Figure II now the delayed-treading equilibrium, marked (M_d^*, m_d^*), has the SRs commanding strictly positive profits, $\pi_d^* > 1$. The line marked IP_{SR} denotes the SR's reservation isoprofit line in states $(\omega_{20}, \omega_{2L})$.

Cash position of the SRs in the delayed trading equilibrium as a function of θ

Cash position of the LRs in the delayed trading equilibrium as a function of θ
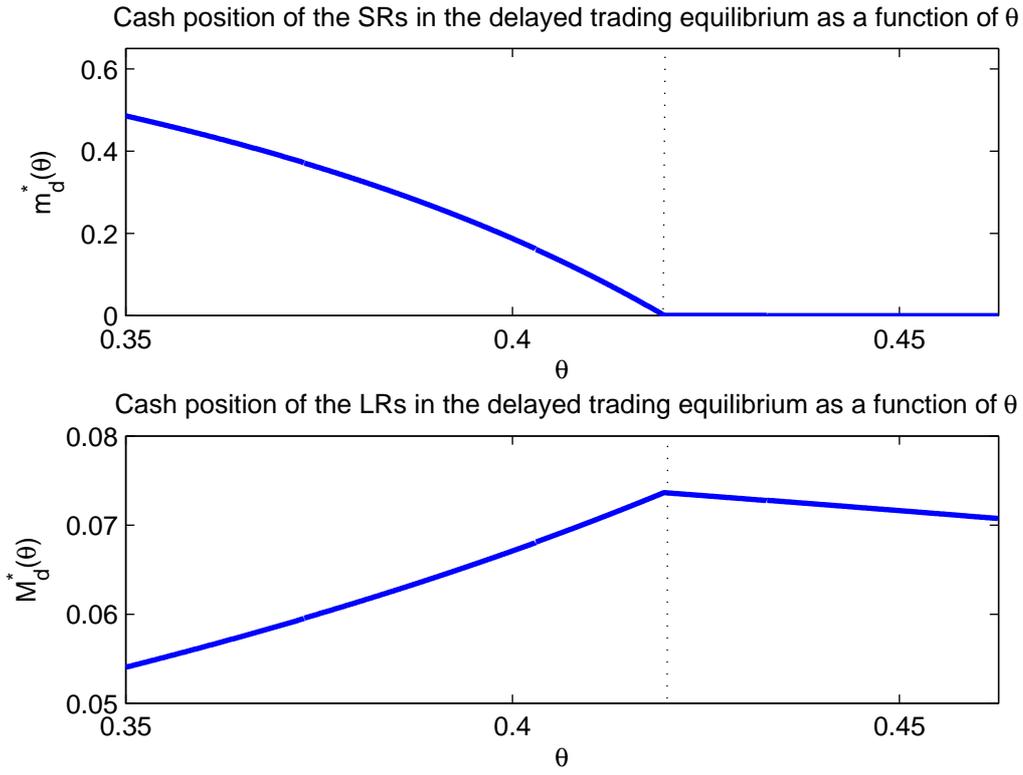
FIGURE IV

Cash holdings as a function of θ for Example 1. Panel A represents the SR's cash holdings in the delayed-trading equilibrium, m_d^* as a function of θ and Panel B does the same for the LR, M_d^*. The dashed vertical line, which sits at $\widehat{\theta} = .4196$ delimits the set of θs for which $m_d^* > 0$ and the one for which $m_d^* = 0$.

Expected return of the risky asset at t=2 in the delayed trading equilibrium, $R^*_{2,d}$

Price of the risky asset at t=2 in the delayed trading equilibrium, $P^*_{2,d}$
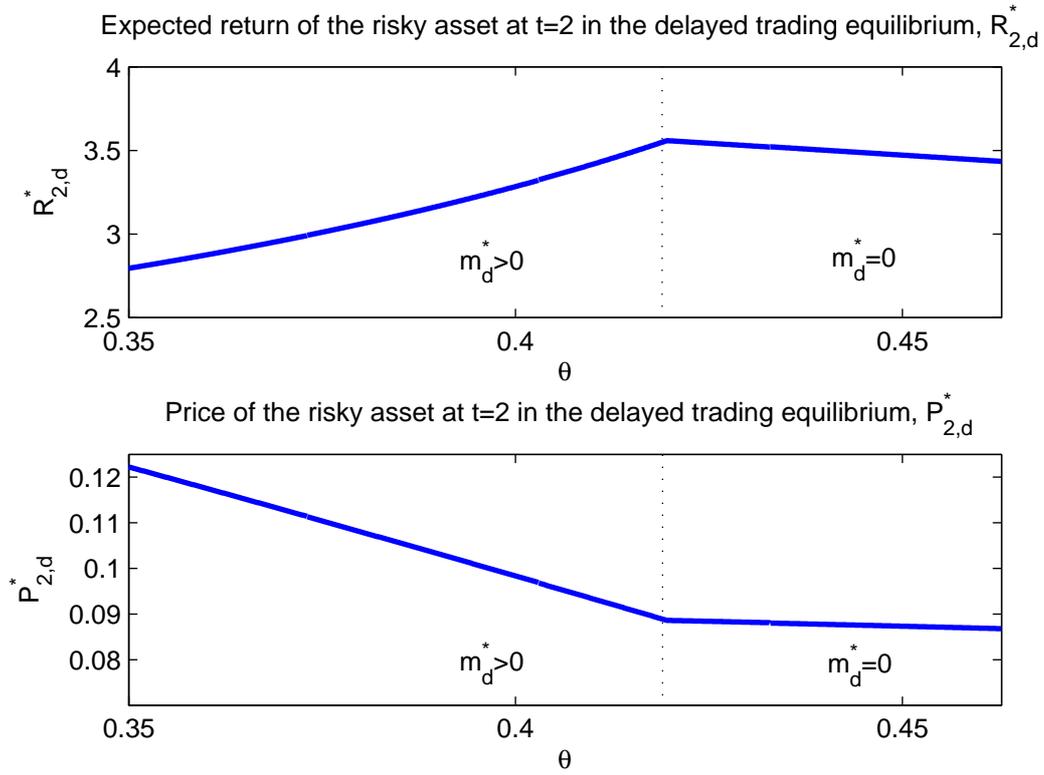
FIGURE V

The top panel shows the expected return of the risky asset, R^*_{2d}, as a function of θ at date 2 in the delayed-trading equilibrium. The bottom panel shows the price of the risky asset at $t = 2$, P^*_{2d}, as a function of θ at date 2 in the delayed-trading equilibrium. The dashed vertical line corresponds to $\widehat{\theta} = .4196$. Both panels correspond to the case considered in Example 1.
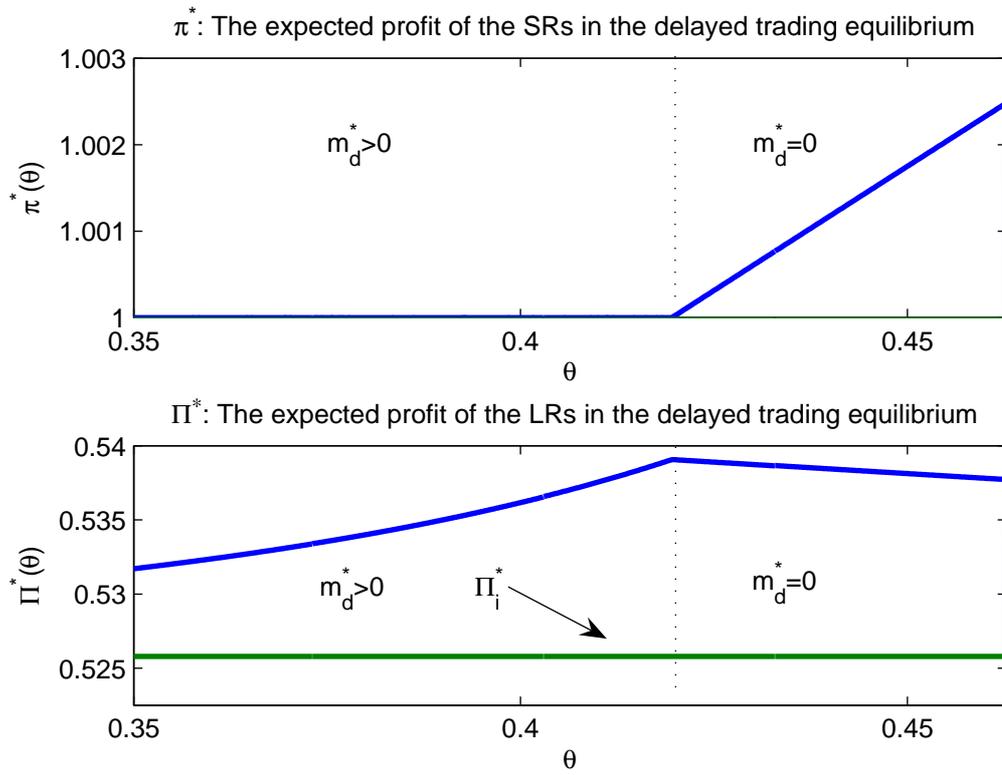
FIGURE VI

Expected profits for the SR, π^*, (top panel) and the LR (bottom panel), Π^*, as a function of θ in the delayed-trading equilibrium for the case considered in Example 1. The dashed vertical line corresponds to $\widehat{\theta} = .4196$.
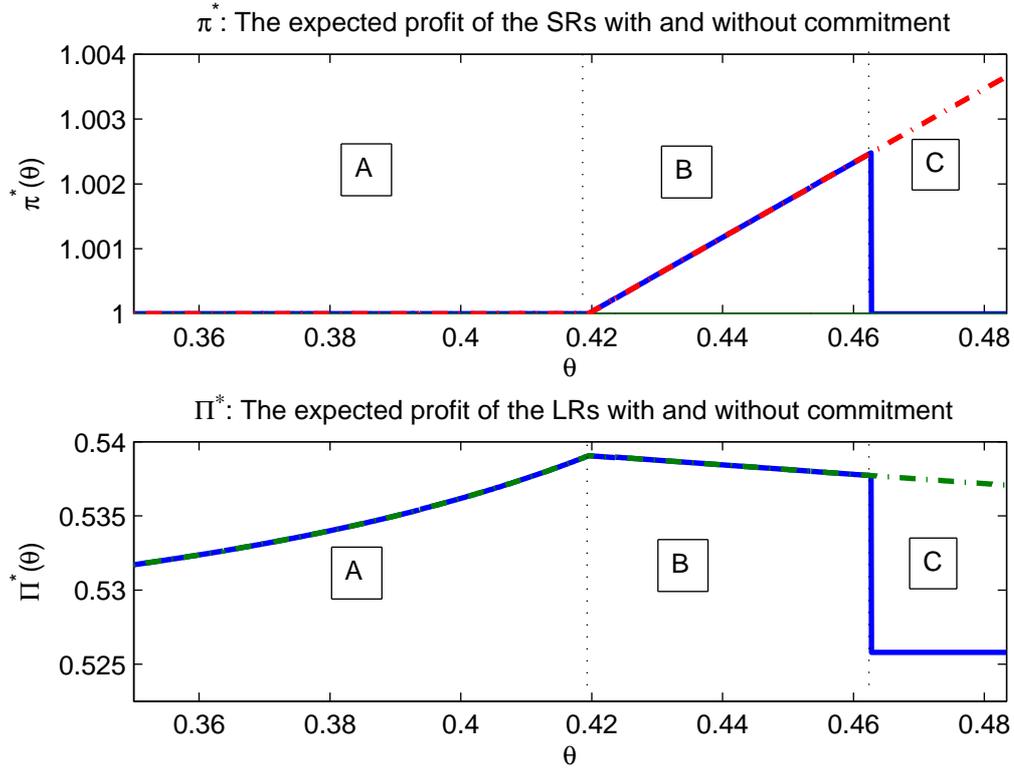
FIGURE VII

Expected profits for the SR, π^*, (top panel) and the LR (bottom panel), Π^*, as a function of θ for the case considered in Example 1. The first dashed vertical line corresponds to $\widehat{\theta} = .4196$. The continuous line plots the expected profits when the Pareto superior equilibrium is chosen. In regions A and B, the delayed-trading equilibrium exists and it is the Pareto superior equilibrium. In region C, which corresponds to $\theta \in (.4628, .4834]$, the delayed-trading equilibrium no longer exists as $P_d^* < \delta\eta\rho$ and the sole equilibrium is the immediate-trading equilibrium. The dashed line corresponds to the expected profits when the SRs can commit to liquidate assets in state ω_{2L}.
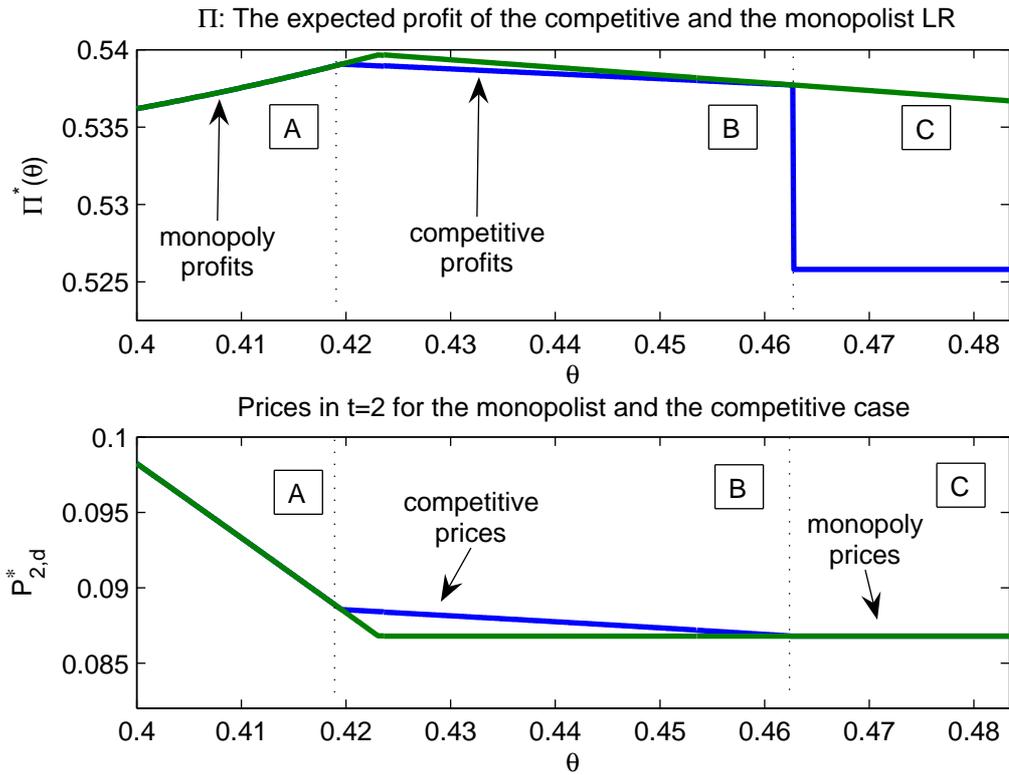
FIGURE VIII

Top panel: Expected profits of the monopolist (the thick line) and the competitive LR (the thin line) as a function of θ. Bottom panel: Prices at date 2, P_{2d}^*, in the monopolist (the thick line) and the competitive (the thin line) LR case. Both panels correspond to the case considered in Example 1.
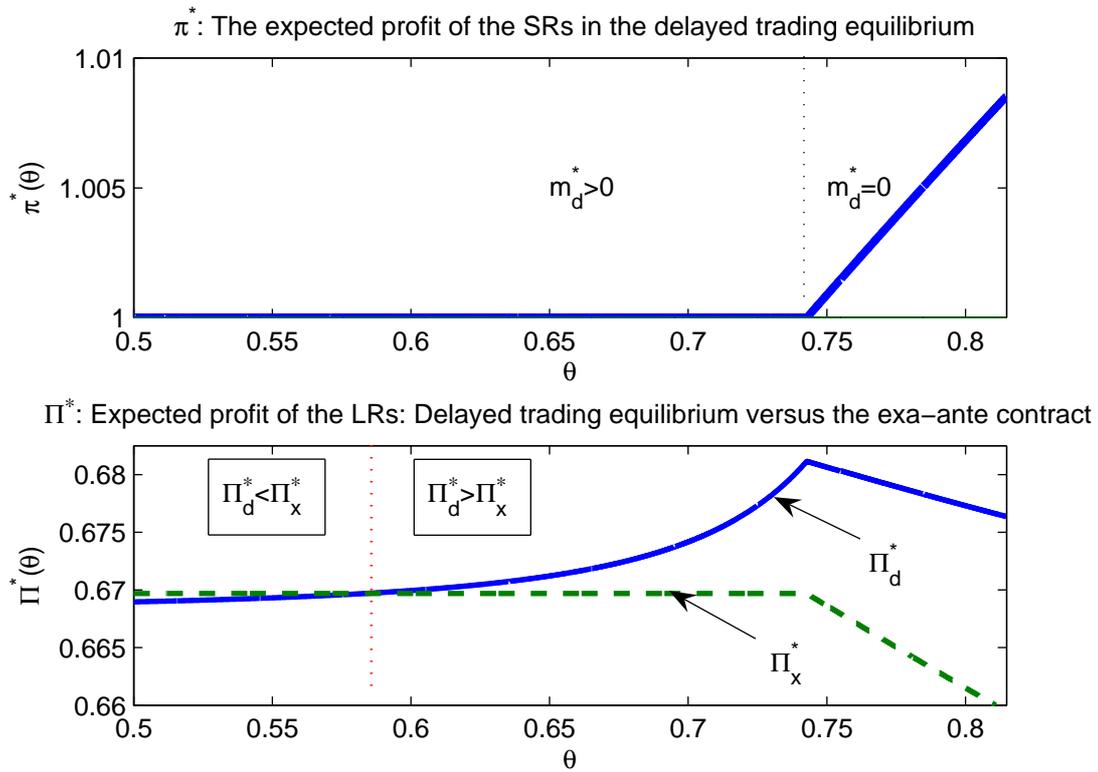
FIGURE IX

Top panel: Expected profits for the SR in the ex-ante contract when the outside value is the expected profit associated with the delayed-trading equilibrium. Bottom panel: Expected profits of the LR in the ex-ante contract, Π_x^*, when the outside value of the SR is the expected profit associated with the delayed-trading equilibrium (π_d^*). Also included are the expected profit of the LR in the delayed-trading equilibrium, Π_d^*, and in the immediate-trading equilibrium, Π_i^*. Both panels correspond to the case considered in Example 2.
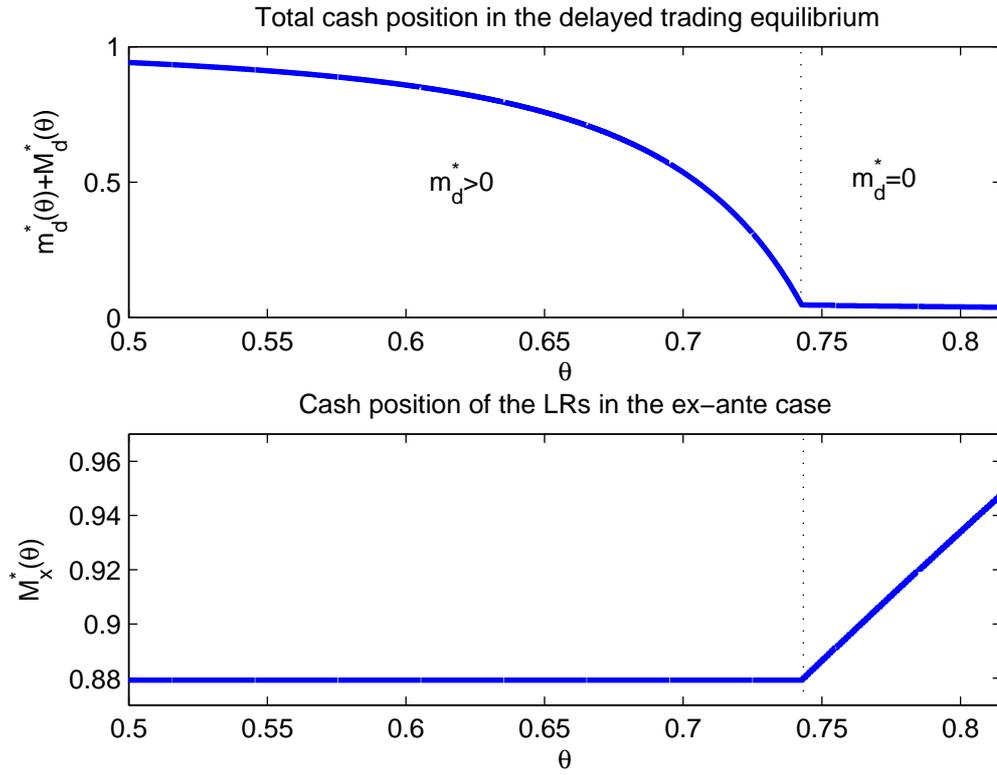
Top panel: Total cash position, $m_d^* + M_d^*$ in the delayed-trading equilibrium as a function of θ. Bottom panel: Cash position of the LR in the ex-ante contract case as a function of θ when the outside opportunity of the SR is the expected profit in the delayed-trading equilibrium. Both panels correspond to the case considered in Example 2.