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Volume Title: Exchange Rates and International Macroeconomics

Volume Author/Editor: Jacob A. Frenkel, ed.

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-26250-2

Volume URL: <http://www.nber.org/books/fren83-1>

Publication Date: 1983

Chapter Title: Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models

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Chapter URL: <http://www.nber.org/chapters/c11378>

Chapter pages in book: (p. 113 - 152)

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# 4 Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models

Lars Peter Hansen and Robert J. Hodrick

## 4.1 Introduction

In this paper we study the determination of forward foreign exchange rates. An exchange rate is the price of one currency in terms of another currency, and a forward rate is a contractual exchange rate established at a point in time for a transaction that will take place at the maturity date on the contract in the future. Well-organized forward markets exist for all major currencies of the world for various maturities, with the most active contract lengths being one, three, six, and twelve months.

The existence and efficiency of organized forward markets for foreign exchange were critical links in the case for flexible exchange rates. Friedman (1953) stated,

Under flexible exchange rates traders can almost always protect themselves against changes in the rate by hedging in a futures market. Such futures markets in foreign currency readily develop when exchange rates are flexible.

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The authors would like to thank several individuals and the participants in workshops where early versions of this paper were presented for useful comments and discussions: Russell S. Boyer, John Floyd, Jacob A. Frenkel, Sanford Grossman, Dale W. Henderson, Robert E. Lucas, Jr., Dan Peled, Scott F. Richard, Kenneth J. Singleton, and Robert Townsend, and participants in seminars at the University of Chicago, the University of Toronto, the Board of Governors of the Federal Reserve System, and the International Monetary Fund. Robert Leikar, Ravi Jagannathan, and Frank Russell assisted with the computations. Terry Hill of Data Resources, Inc. supplied updates of the data. Financial support from the National Science Foundation under grants SES-8007016 and DAR-8010760 is gratefully acknowledged.

He then argued that the cost of such hedging was “the price that must be paid to speculators for assuming the risk of future changes in exchange rates.”<sup>1</sup> The price of hedging can be thought of as the deviation of the forward rate from the expected future spot rate, and advocates of flexible exchange rates obviously thought that this price or risk premium would be kept small by competition. One purpose of this paper is to employ modern ideas of asset pricing to determine the nature of this risk premium in a way that leads to statistical representations with testable hypotheses. We then estimate parameters of these representations and test the hypotheses with data from the recent experience with flexible exchange rates.

This recent experience with flexible exchange rates was surprising to economists in many respects. Its most notable characteristics have been the volatility of spot exchange rates and the magnitude of forward rate forecast errors. While we now know that volatility of exchange rates can be produced in a variety of models and is characteristic of the asset market approach to exchange rate determination, we still have little solid evidence on how well the flexible exchange rate system is working or that movements in exchange rates reflect the market fundamentals of our new theories.<sup>2</sup> There is also little evidence on the nature of risk premiums in the forward market. Understanding the importance of risk in this market should facilitate the development of empirical models of spot exchange rate determinations.

Substantial evidence exists against the hypothesis of “simple market efficiency” in which either the forward rate or its logarithm is equated with the conditional expectation of the level or logarithm of the future spot exchange rate. Published studies employing a variety of techniques and data sets which provide evidence against this type of hypothesis include Geweke and Feige (1979), Frankel (1980), Hansen and Hodrick (1980), Bilson (1981), Cumby and Obstfeld (1981), Hakkio (1981), and Longworth (1981). The contribution of this literature is to show that while the deviations between forward rates and expected future spot rates may be small relative to the movement of spot rates, it is possible to devise and implement econometric procedures that are powerful enough to reject the notion that these deviations are zero. Although it has often

1. A subtle yet economically important distinction must be drawn between forward markets and futures markets. Black (1976) discusses the differences in the payoffs from forward and futures contracts while Jarrow and Oldfield (1981), Cox, Ingersoll, and Ross (1981), French (1981), and Richard and Sundaresan (1981) examine the theoretical issues in detail. The essential difference is that forward contracts have a payoff only at maturity, whereas futures contracts involve daily payoffs between the time at which the contract is written and the maturity date.

2. Surveys of the state of knowledge on exchange rate determination are provided by Mussa (1979), Dornbusch (1980), and Frenkel (1981). Each stresses that exchange rates are asset prices and consequently should be expected to be volatile.

been noted that empirical rejection of this notion of efficiency cannot be identified with market failure, developing testable hypotheses that incorporate the relevant intertemporal risk considerations has proven to be very difficult. For instance, using a traditional approach to measuring risk with a static capital asset pricing model cannot adequately characterize the intertemporal movements in risk premiums.

Our approach to characterizing risk premiums in the forward market has as its foundation the theoretical intertemporal asset pricing models. In section 4.2 we analyze the first-order conditions of an economic agent who has the opportunity to trade forward foreign exchange contracts in competitive equilibrium. We use these conditions to develop three linear econometric models of the risk premiums which are analyzed in sections 4.3, 4.4, and 4.5. We choose to focus on linear models because of their tractability and their preeminence in the empirical international economic and time series econometric literatures. Evaluating the performance of linear representations of risk premiums is an important first step in understanding the role of risk in the forward foreign exchange market. Each statistical representation of the risk premiums relies on special auxiliary assumptions to derive a testable hypothesis. Each auxiliary assumption leads to a different estimation procedure, but each procedure can be thought of as the natural extension of the estimation strategy proposed and implemented in Hansen and Hodrick (1980). This strategy is particularly useful in estimating forecasting equations in which the time interval between observations is much shorter than the forecast interval. Employing such data sets increases the effective degrees of freedom relative to procedures that equate the sampling interval to the forecast interval. In the problem at hand we have employed a data set in which observations on spot and one-month forward rates are sampled semi-weekly. The formal justification for the econometric procedures can be found in Hansen (1982), and we provide some details of our procedures in appendix A.

In section 4.6 we summarize our results and provide some concluding comments.

## **4.2 An Intertemporal Equilibrium Condition**

This section develops the relationship between forward exchange rates and expected future spot exchange rates that will prevail in a competitive market with zero transactions costs and rational use of information. As a theoretical foundation we rely on the discrete time asset pricing models of Rubenstein (1976), Lucas (1978), Breeden (1979), Brock (1980), and Richard (1981). In these models investors maximize expected utility subject to sequential budget constraints. In equilibrium, assets are priced such that the product of the price of the asset in terms of a numéraire good

and the conditional expectation of the marginal utility of the numéraire consumption good is equal to the conditional expectation of the product of the marginal utility of consumption  $k$  periods in the future and the  $k$  period payoff on the asset for each investor. Equivalently, the equilibrium condition can be represented by:

$$(1) \quad E_t(Q_{c,t+k} r_{t+k,k}) = 1,$$

where

$$Q_{c,t+k} = \frac{E_{t+k}(U_{c,t+k})}{E_t(U_{c,t})}$$

which is the marginal rate of substitution of time  $t+k$  consumption of the numéraire good for time  $t$  consumption of the same good for a particular investor,  $r_{t+k,k}$  is the  $k$ -period return on an asset purchased at  $t$ , and  $E_t(\cdot)$  is the conditional expectation based on the information set,  $\Phi_t$ , available to the investor at  $t$ .<sup>3</sup>

An analogous expression to (1) for nominal returns is:

$$(2) \quad E_t(Q_{m,t+k} R_{t+k,k}) = 1,$$

where  $Q_{m,t+k}$  is the marginal rate of substitution of money between  $t+k$  and  $t$  for a particular investor, and  $R_{t+k,k}$  is a  $k$ -period nominal return. In referring to  $Q_{m,t+k}$  as a marginal rate of substitution, we are not implying that nominal balances are the arguments of a utility function, instead we indicate indirect intertemporal valuation of the currency. In some models, such as Lucas (1982), a simple link exists between the marginal rate of substitution of money and the marginal rate of substitution of the numéraire good such that

$$Q_{m,t+k} = \frac{\Pi_{t+k}}{\Pi_t} Q_{c,t+k},$$

where  $\Pi_t$  is the purchasing power of the numéraire currency, where purchasing power means the price of the money in units of the numéraire good. Such a link would also occur if real balances are placed in the utility function, in which case results of the real asset pricing models apply directly. Although Townsend (1983) considers monetary models in which the simple link breaks down, by suitably redefining the marginal rate of substitution of money expression (2) remains intact.<sup>4</sup> If a  $k$ -period nominal risk-free asset is available at  $t$ , its return  $R_{t+k,k}^f$ , will satisfy:

3. We define the payoff on an asset as the future price plus interest payments or dividends denominated in the numéraire good. The return is the payoff divided by the current price of the asset. We take the conditional expectation of the marginal utility of consumption at  $t$  to allow for nonseparability of preference over time.

4. Townsend (1983) considers a model with a cash-in-advance constraint. In his model the nominal value of a subset of consumption goods must be less than or equal to the amount of nominal money balances chosen in the previous period. Letting  $\lambda_t$  denote the value of the Kuhn-Tucker multiplier of this constraint implies that

$$(3) \quad E_t(Q_{m,t+k}) = (R_{t+k}^f)^{-1}.$$

Now consider the pricing of a forward contract in the foreign exchange market. At time  $t$  the investor has access to a forward exchange market in which he can buy or sell various foreign currencies with delivery and payment at time  $t+k$ .<sup>5</sup> Let  $F_{t,k}^j$  be the forward exchange rate which is the domestic currency price of a unit of currency  $j$  established at  $t$  for payment at  $t+k$ . The spot exchange rate at  $t$  is  $S_t^j$  and the profit in the numéraire currency from a long position in the forward market of currency  $j$  is the difference between the uncertain future spot exchange rate and the forward rate, that is  $S_{t+k}^j - F_{t,k}^j$ .

Since no investment is required at time  $t$ , the first-order condition for the representative investor requires that the conditional expectation of the marginal utility of the nominal profit from contracting in the forward exchange market of currency  $j$  must be zero. The marginal utility of the nominal profit is the indirect valuation of the currency times the profit on the contract. Since the indirect valuation of the currency at time  $t$  is in the information set, we can divide the conditional expectation of the marginal utility of the profit by that to yield:

$$(4) \quad E_t[Q_{m,t+k} (S_{t+k}^j - F_{t,k}^j)] = 0.$$

A necessary condition for  $E_t(S_{t+k}^j - F_{t,k}^j) \neq 0$  is that the intertemporal marginal rate of substitution of money,  $Q_{m,t+k}$ , have a nonzero conditional variance. This conditional variance can be nonzero even if the intertemporal marginal rate of substitution of consumption is constant, as would be the case if the investor were risk neutral, because of the uncertainty about the purchasing powers of the currencies, a point stressed by Frenkel and Razin (1980).<sup>6</sup> In light of the difficulties in

$$Q_{m,t+k} = \left[ \frac{E_{t+k}(U_{c,t+k}) \cdot \Pi_{t+k} + \lambda_{t+k}}{E_t(U_{c,t}) \cdot \Pi_t + \lambda_t} \right].$$

Considerable controversy exists in the literature regarding how the use of fiat money in a model should be motivated. The implications for the determination of spot exchange rates of various alternative strategies, such as placing real balances in the utility function, cash-in-advance constraints, or physical and intertemporal barriers to trade, do differ. We conjecture that motivating and introducing forward markets into the various models may result in additional differences in the joint spot and forward exchange rate processes. In this paper we abstract from these differences, which must be investigated using explicit solutions to general equilibrium models, to focus on intertemporal risk aversion.

5. We abstract from any margin requirements which might affect the investor's current budget constraint. Typically, margin requirements can be met by allowing a broker to hold an investor's securities, such as Treasury bills, from which the investor continues to receive interest. If the optimizing amount of these securities which the investor would hold in the absence of considering forward contracts is greater than the margin requirement, the margin requirement is not a constraint. Margin requirements are also more common on futures contracts for which settlement occurs daily.

6. The intertemporal marginal rate of substitution of money can be uncertain even if investors are risk neutral because of uncertainty in the purchasing power of money. Letting

accurately measuring the relative purchasing powers of currencies, we do not attempt to control for uncertainty about the purchasing powers in investigating deviations of the forward rate from the expected future spot rate. In spite of this qualification, we refer to these deviations as risk premiums because of the substantial body of evidence from other financial markets indicating that investors are risk averse.

Equation (4) is our fundamental representation of the international, intertemporal equilibrium condition that must hold for all investors regardless of their country of residence. Without additional assumptions this condition has little, if any, empirical content. Ideally, one would like these additional assumptions to be made explicitly on the preferences, technology, or the stochastic behavior of any exogenous forcing processes in a general equilibrium approach, but this is not currently a feasible empirical modeling strategy for this problem.<sup>7</sup> As an alternative, the next sections develop testable statistical models which embody auxiliary assumptions to simplify interpretation of the equilibrium condition. These assumptions take the form of either specific distributional properties on the endogenous variables of the system or constant conditional covariances of these variables. Since we are not conducting an explicit general equilibrium analysis, we do not investigate whether our auxiliary assumptions can even be produced by a specification of preferences, technologies, and exogenous forcing processes. It is important to remember in interpreting the results of our statistical tests that evidence for or against a particular representation is evidence for or against the joint hypothesis of the model as specified in (4) and the auxiliary assumptions that are employed to implement it. These statistical representations allow us to characterize empirically the nature of risk premiums in the forward market.

### 4.3 The Lognormal Model

In this section we develop testable implications using our first representation of the equilibrium condition, equation (4), and a joint lognormality assumption which we chose because of the multiplicative nature of (4). Let  $Z_t^j = (S_{t,k}^j, \dots, S_{t,k}^p, F_{t,k}^1, \dots; F_{t,k}^p, Q_{m,t})$ . Assume that this vector stochastic process has the logarithmic autoregressive representation:

$\Pi_t^j$  be the purchasing power of currency  $j$  and  $\Pi_t$  that of the numéraire, the condition (4) under risk neutrality can be written as

$$E_t(S_{t+k}^j) - F_{t,k}^j = E_t(\Pi_{t+k}^j / \Pi_{t+k}) - E_t(\Pi_{t+k}^j) / E_t(\Pi_{t+k}).$$

While these terms will in general be nonzero, we do not think they are the sole source of our results.

7. Although Grossman and Shiller (1981), Hall (1981), and Hansen and Richard (1983) describe strategies for testing the real asset pricing theory by restricting preferences and using data on aggregate consumption, their procedures cannot easily be modified to study the forward foreign exchange market using the data set we employ here.

$$(5) \quad z_t = A_0 - A(L)z_{t-1} + u_t,$$

where lowercase letters represent the natural logarithms of their uppercase counterparts,  $A_0$  is a vector of constants, and  $A(L)$  is a matrix with elements that are possibly infinite-order polynomials in the lag operator, and  $u$  is a sequence of mean zero, independent, identically distributed, normal random vectors. The zeros of  $\det[I - \zeta A(\zeta)]$  are not necessarily assumed to be outside the unit circle to allow nonstationarity of the  $z$  process. Allowing for nonstationarity may be important since Meese and Singleton (1982) found evidence that the autoregressive univariate processes for the logarithms of spot exchange rates contain unit roots.

Let  $E_t^z(\cdot)$  be the conditional expectation based on the information set  $\Phi_t^z = (z_t, z_{t-1}, \dots)$ . Since  $\Phi_t^z$  is a subset of  $\Phi_t$ , (4) implies that:

$$(6) \quad E_t^z(S_{t+k}^j Q_{m,t+k}) = F_{t,k}^j E_t^z(Q_{m,t+k}).$$

Then, employing the distributional assumption (5):

$$(7) \quad E_t^z(S_{t+k}^j Q_{m,t+k}) = \exp[E_t^z(s_{t+k}^j) + E_t^z(q_{m,t+k}) \\ + 1/2V_t^z(s_{t+k}^j) + 1/2V_t^z(q_{m,t+k}) \\ + C_t^z(s_{t+k}^j; q_{m,t+k})],$$

and

$$F_{t,k}^j E_t^z(Q_{m,t+k}) = \exp[f_{t,k}^j + E_t^z(q_{m,t+k}) \\ + 1/2V_t^z(q_{m,t+k})],$$

where  $V_t^z(\cdot)$  and  $C_t^z(\cdot; \cdot)$  are the variance and covariance conditioned on the information set  $\Phi_t^z$ . Substituting (7) into (6) and taking logarithms of both sides gives:

$$(8) \quad E_t^z(s_{t+k}^j) - f_{t,k}^j = -1/2V_t^z(s_{t+k}^j) - C_t^z(s_{t+k}^j; q_{m,t+k}).$$

From representation (5), the right-hand side of (8) is a constant which we denote  $a_j$ . Therefore, this representation of the theory predicts that the logarithm of the forward exchange rate is equal to the conditional expectation of the logarithm of the future spot exchange rate plus a constant.

Hansen and Hodrick (1980) discuss alternative strategies for testing hypotheses such as the one derived above. The technique exploits the property that the forecast error,  $u_{t,k}^j = s_{t+k}^j - E_t^z(s_{t+k}^j)$ , is orthogonal to elements of the information set  $\Phi_t^z$ . Consequently, the null hypothesis developed in this section can be examined by testing the hypothesis that  $b_j = \mathbf{0}$  in the regression equation:

$$(9) \quad s_{t+k}^j - f_{t,k}^j = a_j + b_j' x_t + u_{t,k}^j,$$

where  $x_t$  is a vector of information in  $\Phi_t^z$ . Since the key requirement for consistency of the ordinary least squares (OLS) estimator of  $b_j$  is the



orthogonality of  $x_t$  and  $u_{t,k}^l$ , standard OLS computations will provide a consistent estimator.<sup>8</sup> However, asymptotic justification of the conventional computation of standard errors in the OLS regression requires serially uncorrelated errors. It can be verified that  $E(u_{t,k}^l u_{t+h,k}^l)$  is zero for  $h > k$  under the null hypothesis. Hence, unless the sampling interval equals the forecast interval, that is  $k = 1$ , the errors in (9) will be serially correlated. In Hansen and Hodrick (1980) we discuss how to compute estimates of the correct asymptotic covariance matrix relying on the asymptotic distribution theory developed in Hansen (1982).

While choice of the auxiliary variables,  $x_t$ , is arbitrary and dictated in most cases by availability of data, Geweke (1980) has demonstrated the desirability of including the forward premium,  $f_{t,k}^i - s_t^i$ , in the set of regressors. In our previous work with three-month forward rates, lagged multicountry forward rate forecast errors,  $s_{t-k}^i - f_{t-k,k}^i$ , were found to contain significant explanatory power. Hence, in table 4.1 we present the estimated OLS regressions using the modifications described above of the forward rate forecast error for a currency on a lagged value of its own forecast error, the lagged forecast errors of four other currencies, and its own and four other forward premiums as in:

$$(10) \quad s_{t+9}^i - f_{t,9}^i = a_i + \sum_{i=1}^5 b_{ij}(s_t^i - f_{t-9,9}^i) + \sum_{i=1}^5 c_{ij}(f_{t,9}^i - s_t^i) + u_{t,9}^i.$$

The exchange rates are U.S. dollars per unit of foreign currency. The data set is a semiweekly sample in which Tuesday forward rates predict Thursday spot rates thirty days in the future and Friday forward rates predict Monday spot rates.<sup>9</sup> Under the null hypothesis, the disturbance terms in (10) are consequently an eighth-order moving average process.<sup>10</sup> Table 4.1 presents evidence for a sample of 512 observations which corresponds to 5 February 1976 to 29 December 1980.

Out sample excludes the transitional early years of the floating exchange rate era. Our reasoning for the exclusion centers on the possibilities that agents may have been expecting a return to a fixed rate regime after the breakdown of the Bretton Woods system in February 1973, and that they may not have known or fully understood the intervention

8. One possible set of requirements to ensure the consistency of the OLS estimator in (9) is: (i)  $s_{t+k}^i - f_{t,k}^i$  and  $x_t$  are stationary and ergodic; (ii)  $E[1, x_t]' [1, x_t]$  is nonsingular; (iii)  $E(x_t u_{t,k}^l) = 0$ ; and (iv)  $E(u_{t,k}^l) = 0$ .

9. See appendix B for a description of the data.

10. In Hansen and Hodrick (1980) we prove that the procedure of sampling the data to generate a serially uncorrelated error term, thereby justifying the conventional OLS computation of standard errors, is dominated asymptotically by the procedure employing all the data. We also noted that serial correlation correction generalized least-squares procedures would lead to inconsistent estimators in this example.

Table 4.1

$$s_{t+s}^i - f_{t,s}^i = a_i + \sum_{j=1}^s b_{ij}(s_t^i - f_{t-s,s}^i) + \sum_{j=1}^s c_{ij}(f_{t,s}^i - s_t^i) + u_{t,s}^i$$

Exchange Rates: US\$/Foreign Currency; Sample: 5 February 1976 to 29 December 1980; Number of Observations: 512

Currency	$\hat{a}_i$		$\hat{b}_{i2}$		$\hat{b}_{i3}$		$\hat{b}_{i4}$		$\hat{b}_{i5}$		$\hat{c}_{i1}$		$\hat{c}_{i2}$		$\hat{c}_{i3}$		$\hat{c}_{i4}$		$\hat{c}_{i5}$		$\chi^2(10)$	
	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence	(Std. Err.)	Confidence
French franc	0.132 (1.311)	-0.173 (0.194)	0.003 (0.104)	0.223 (0.133)	-0.069 (0.116)	-0.275 (0.207)	-1.995 (1.232)	2.738 (1.845)	0.304 (2.780)	0.407 (1.403)	-2.988 (4.169)	12.871 (0.769)	0.132	5.403								
Japanese yen	2.245 (1.628)	-0.112 (0.243)	0.037 (0.130)	0.392 (0.165)	0.167 (0.145)	-0.615 (0.258)	-0.640 (1.530)	-2.561 (2.292)	-5.906 (3.453)	0.403 (1.742)	6.994 (5.184)	24.578 (0.994)	0.261	8.403								
Swiss franc	2.376 (1.677)	-0.054 (0.249)	-0.034 (0.134)	0.525 (0.170)	-0.009 (0.148)	-0.919 (0.265)	-2.359 (1.584)	2.345 (2.352)	-4.502 (3.544)	1.802 (1.785)	0.142 (5.325)	33.386 (0.999)	0.280	9.681								
U.K. pound	0.537 (1.469)	0.008 (0.217)	-0.065 (0.117)	0.249 (0.149)	0.186 (0.130)	-0.423 (0.231)	0.793 (1.380)	0.920 (2.067)	-2.842 (3.116)	-1.775 (1.575)	2.976 (4.666)	8.745 (0.444)	0.095	6.829								
Deutsche mark	1.065 (1.354)	-0.296 (0.201)	-0.081 (0.108)	0.408 (0.137)	0.021 (0.120)	-0.451 (0.214)	-1.309 (1.275)	3.487 (1.904)	-0.730 (2.869)	0.008 (1.447)	-4.585 (4.306)	22.177 (0.986)	0.199	5.905								

NOTE: Confidence is one minus the marginal level of significance. Values of the confidence term which are close to one indicate evidence against the null hypothesis that one or a set of coefficients equals zero. All forward exchange rate forecast errors and forward premiums are multiplied by 100, making them interpretable as percent at a monthly rate.

strategies of the central banks in the beginning of the era. The exact choice of a starting date was dictated by the following reasoning.

After the movement to flexible exchange rates in March 1973 there was considerable uncertainty regarding the future of the international monetary system.<sup>11</sup> The Committee of Twenty, created by the International Monetary Fund (IMF) in July 1972 to study restructure of the international monetary system, published an Outline of Reform in June 1974 suggesting that the restructure be based on stable but adjustable par values with limited floating. This solution was not consistent with the preferences of all countries, particularly the United States, and negotiations continued through 1975, climaxing with the agreement at Rambouillet in November of that year. At Rambouillet the governments of France, the Federal Republic of Germany, Italy, Japan, the United Kingdom, and the United States produced an agreement which led directly to the amendments to the Articles of Agreement of the IMF which formally ratified the flexible exchange rate system. After the agreement was ratified by the Interim Committee of the IMF in January 1976 in Jamaica, the countries of the world were free to adopt the exchange rate regime of their choice.

The Rambouillet ratification of a system of flexible exchange rates possibly could have been an important piece of information to economic agents in terms of forecasting the behavior of future exchange rates. After the agreement, market participants may have been more secure in their forecasts of governmental actions that influence the determination of exchange rates. Thus the forecasting properties of forward exchange rates as well as the monetary and exchange market intervention policy rules of the countries may have been different before and after the agreement.

In table 4.1 we examine the joint hypothesis that the  $b_{ji}$ 's and the  $c_{ji}$ 's are zero for each currency. For the Japanese yen, the Swiss franc, and the Deutsche mark the hypothesis can be rejected at all levels of significance greater than .02. The evidence for the French franc and the U.K. pound does not indicate significant evidence against the null hypothesis.

Having soundly rejected the null hypothesis, it is appropriate to reiterate the joint nature of its derivation. Equation (4) can be true, but the assumptions of joint lognormality or the time invariant representation (5) which implied constancy of conditional covariances may be false, which would cause a rejection of the combined hypotheses.

To determine whether the rejection of the forward foreign exchange market efficiency hypothesis characterized by (4) and (5) was due strictly to the movement of the U.S. dollar relative to all other currencies, the OLS regressions for the sample of 512 observations were examined with the Swiss franc as the numéraire currency. These results are presented in

11. See Murphy (1979) for a discussion of the recent history.

table 4.2. While these regressions are merely linear combinations of the regressions in table 4.1, they do indicate that failure of the hypothesis is not purely a U.S. dollar phenomenon. The null hypothesis that the coefficients other than the constant are zero can be rejected for the Swiss franc/French franc exchange rate, for the Swiss franc/U.K. pound exchange rate, and for the Swiss franc/Deutsche mark exchange rate at all levels of significance greater than .001. The similar rejection for the Swiss franc/U.S. dollar exchange rate reproduces the result of table 4.1.

The results presented above indicate that there is considerable evidence against the hypothesis that the log of the one-month forward exchange rate is equal to the conditional expectation of the log of the future spot rate plus a constant. This is consistent with the investigation of three-month forward rates conducted in Hansen and Hodrick (1980). In that study we focused on the empirical properties of forward rates as predictors of future spot rates, recognizing that risk premiums could exist in theory but we did not make any attempt to model them. The analysis demonstrates that we can reinterpret our previous tests as precise tests of the international equilibrium condition for a risk averse investor and the lognormality distributional assumption.

#### 4.4 The Nominal Risk-Free Return Model

In this section we derive a second representation of the intertemporal equilibrium condition that can potentially reconcile the empirical results of the previous section and the equilibrium theory. This reconciliation requires variation over time in expected profits in the forward market, and the expected profits reflect the compensation that risk averse investors require for holding the contracts.

To develop the testable hypothesis of this section, we first divide (4) by  $S_t^j$  which is in the information set  $\Phi_t$ . We do this strictly for its potentially desirable effect on the statistical properties of the forward rate forecast errors. Using the definition of conditional covariance, which is signified  $C_t(\cdot; \cdot)$ , relation (4) can be rewritten as follows:

$$(11) \quad E_t \left( \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right) = -C_t \left[ \left( \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right); Q_{m,t+k} \right] \\ / E_t(Q_{m,t+k}).$$

Substituting the expression for the risk-free nominal return given by (3) into (11), we obtain our second representation of an investor's first-order condition:

$$(12) \quad E_t \left( \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right) = -C_t \left[ \left( \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right); Q_{m,t+k} \right] \\ R_{t+k,k}^f.$$

Table 4.2

$$s_{t+9} - f_{t,9} = a_9 + \sum_{i=1}^8 b_{9i}(s_t^i - f_{t-9,9}) + \sum_{i=1}^8 c_{9i}(f_{t,9} - s_t^i) + u_{t,9}$$

Exchange Rates: Swiss Franc/Foreign Currency; Sample: 5 February 1976 to 29 December 1980; Number of Observations: 512

Currency	$\hat{a}_9$ (Std. Err.) Conf- dence	$\hat{b}_{91}$ (Std. Err.) Conf- dence	$\hat{b}_{92}$ (Std. Err.) Conf- dence	$\hat{b}_{93}$ (Std. Err.) Conf- dence	$\hat{b}_{94}$ (Std. Err.) Conf- dence	$\hat{b}_{95}$ (Std. Err.) Conf- dence	$\hat{c}_{91}$ (Std. Err.) Conf- dence	$\hat{c}_{92}$ (Std. Err.) Conf- dence	$\hat{c}_{93}$ (Std. Err.) Conf- dence	$\hat{c}_{94}$ (Std. Err.) Conf- dence	$\hat{c}_{95}$ (Std. Err.) Conf- dence	$\chi^2(10)$ All $\hat{b}_{9i} =$ $\hat{c}_{9i} = 0$ Conf- dence	Resid. R <sup>2</sup> Var.
French franc	-2.244 (1.163) 0.946	-0.118 (0.173) 0.506	0.037 (0.093) 0.311	-0.202 (0.112) 0.928	-0.060 (0.103) 0.438	0.644 (0.184) 0.999	0.365 (1.101) 0.259	0.393 (1.629) 0.191	-1.038 (1.365) 0.553	-1.395 (1.233) 0.742	-3.131 (3.694) 0.603	29.402 0.999	0.258 4.873
Japanese yen	-1.131 (1.678) 0.062	-0.058 (0.249) 0.184	0.071 (0.134) 0.406	-0.362 (0.161) 0.975	0.177 (0.149) 0.765	0.305 (0.265) 0.750	1.719 (1.582) 0.723	-4.907 (2.357) 0.963	-0.861 (1.976) 0.337	-1.399 (1.791) 0.565	6.852 (5.331) 0.801	12.606 0.753	0.135 9.393
U.S. dollar	-2.376 (1.677) 0.844	0.054 (0.249) 0.173	0.034 (0.134) 0.202	-0.492 (0.161) 0.998	0.009 (0.148) 0.049	0.919 (0.265) 0.999	2.359 (1.584) 0.863	-2.345 (2.352) 0.681	-2.571 (1.972) 0.796	-1.802 (1.785) 0.687	-0.142 (5.325) 0.021	33.386 0.999	0.280 9.681
U.K. pound	-1.839 (1.531) 0.770	0.063 (0.228) 0.216	-0.031 (0.122) 0.201	-0.448 (0.148) 0.997	0.196 (0.136) 0.851	0.496 (0.243) 0.959	3.152 (1.453) 0.970	-1.425 (2.142) 0.494	-2.643 (1.793) 0.859	-3.577 (1.618) 0.973	2.834 (4.862) 0.440	32.716 0.999	0.277 8.707
Deutsche mark	-1.311 (0.933) 0.840	-0.242 (0.140) 0.915	-0.046 (0.751) 0.463	-0.094 (0.092) 0.696	0.030 (0.083) 0.287	0.469 (0.149) 0.998	1.050 (0.888) 0.763	1.141 (1.304) 0.618	0.577 (1.091) 0.391	-1.793 (0.982) 0.932	-4.728 (2.969) 0.889	30.856 0.999	0.258 3.383

NOTE: See table 4.1.

This condition indicates that the expected profit in terms of the numéraire currency on a forward contract in currency  $j$  is proportional to the risk-free return in the numéraire currency where the factor of proportionality depends on the conditional covariance of the profit with the marginal rate of substitution on the numéraire currency. In general the factor of proportionality will depend on information in  $\Phi_t$ . Equation (12) indicates that there are two potential sources of time varying risk premiums or expected profits in the forward market: movements in the conditional covariance and movements in the nominal risk-free return. Our approach in this section is to assume that the conditional covariance is a constant and to investigate whether time variation in the nominal return on one-month U.S. Treasury bills is sufficient to capture the time variation in the risk premiums on one-month forward contracts for purchases of foreign currency with U.S. dollars. We assume that the U.S. Treasury bill return is a nominal risk-free return.

The null hypothesis of this section of the paper is:

$$(13) \quad E_t \left( \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right) = b_j R_{t+k,k}^f,$$

where

$$b_j = -C_t \left[ \left( \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right); Q_{m,t+k} \right]$$

which is assumed to be constant.<sup>12</sup> Realizations of the left-hand side of (13) have a larger variation than  $\ln(S_{t+k}^j) - \ln(F_{t,k}^j)$ , but the correlation between the two representations of the forward exchange rate forecast error is 0.999 for all five U.S. dollar-denominated exchange rates in this study. Hence, if (13) is to be a successful reconciliation of the results in section 4.3 with the equilibrium model, we should expect to find estimates of  $b_j$  that are quite significantly different from zero and explanatory power similar to that found in the regressions reported in the previous section.<sup>13</sup>

Since  $R_{t+k,k}^f$  is also an element of  $\Phi_t$ , the specification (13) can be examined with the appropriately modified OLS technique discussed in section 4.3.<sup>14</sup> Table 4.3 presents the analysis of the regressions,

12. Roll and Solnik (1977) refer to  $(S_{t+k}^j - F_{t,k}^j)/S_t^j$  as the "extraordinary exchange return" and Geweke and Feige (1979) call it the "realized rate of exchange gain in forward market  $j$ ."

13. The calculated  $R^2$ 's in this section are not strictly comparable to those of section 4.3 since the regressions in section 4.3 included a larger set of explanatory variables, did not include the nominal interest rate among the variables, and employed a different specification of the left-hand side variable.

14. The modified OLS procedure used in computing estimates in section 4.3 used the maintained assumption of conditional homoscedastic error processes which was justified by the assumed representation (11). In this section we allow for conditional heteroscedasticity in conducting our statistical inference. See appendix A for further details.

$$\frac{S_{t+9}^i - F_{t+9}^i}{S_t^i} = a_j + \sum_{i=1}^5 b_{ji} \left( \frac{S_t^i - F_{t-9}^i}{S_{t-9}^i} \right) + c_j R_{t+9,9}^i + u_{t,9}^i$$

Table 4.3

Exchange Rates: US\$/Foreign Currency; Sample: 5 February 1976 to 29 December 1980; Number of Observations: 512

Currency	$\hat{a}_j$ (Std. Err.) Confidence	$\hat{b}_{j1}$ (Std. Err.) Confidence	$\hat{b}_{j2}$ (Std. Err.) Confidence	$\hat{b}_{j3}$ (Std. Err.) Confidence	$\hat{b}_{j4}$ (Std. Err.) Confidence	$\hat{b}_{j5}$ (Std. Err.) Confidence	$\hat{c}_j$ (Std. Err.) Confidence	$\chi^2(6)$ $\hat{a}_j = \hat{b}_{j5} = 0$ Confidence	$R^2$	Resid. Var.
French franc	1.335 (1.179) 0.742	-0.150 (0.146) 0.696	-0.030 (0.125) 0.189	0.243 (0.146) 0.904	-0.094 (0.079) 0.768	-0.214 (0.216) 0.677	-0.122 (0.151) 0.579	7.026 0.682	0.083	5.620
Japanese yen	2.508 (1.467) 0.912	-0.257 (0.299) 0.609	0.149 (0.155) 0.662	0.444 (0.204) 0.970	0.147 (0.178) 0.592	-0.678 (0.313) 0.969	-0.245 (0.166) 0.861	30.328 0.999	0.196	9.288
Swiss franc	3.418 (1.473) 0.979	-0.058 (0.291) 0.157	-0.015 (0.168) 0.069	0.597 (0.154) 0.999	-0.028 (0.142) 0.158	-0.976 (0.311) 0.998	-0.364 (0.193) 0.941	21.171 0.998	0.237	10.173
U.K. pound	0.262 (1.141) 0.182	0.010 (0.196) 0.039	-0.047 (0.094) 0.383	0.217 (0.114) 0.942	0.181 (0.124) 0.853	-0.394 (0.171) 0.978	0.036 (0.132) 0.214	11.304 0.921	0.073	6.949
Deutsche mark	2.548 (1.231) 0.962	-0.249 (0.172) 0.852	-0.109 (0.129) 0.601	0.397 (0.158) 0.988	0.026 (0.098) 0.215	-0.411 (0.182) 0.975	-0.262 (0.163) 0.891	14.038 0.971	0.180	6.000

NOTE: See table 4.1.

$$(14) \quad \frac{S_{t+9}^j - F_{t,9}^j}{S_t^j} = a_j + \sum_{i=1}^5 b_{ji} \left( \frac{S_t^i - F_{t-9,9}^i}{S_{t-9}^i} \right) + c_j R_{t+9,9}^f + u_{t,9}^j,$$

where a constant and five lagged forward rate forecast errors have been added to (13) and  $u_{t,9}^j$  is the forecast error from the conditional expectation in (13). If (13) captures the time variation in expected profits in the forward markets, the  $a_j$  and the five  $b_{ji}$ 's should be zero in each regression. Also, the  $c_j$  should be significantly different from zero, especially for the regression using the Japanese yen, the Swiss franc, and the Deutsche mark. The data are sampled semiweekly as before.

The results of the regressions reported in table 4.3 do not support the null hypothesis of this section. The chi-square statistic which tests the hypothesis that  $a_j$  and the five  $b_{ji}$ 's are zero indicates strong rejection of the hypothesis since the marginal levels of significance are .001 for the yen, .002 for the Swiss franc, and .03 for the Deutsche mark. Also, the coefficients on the nominal return do not have particularly low marginal levels of significance for tests of the hypothesis that the coefficients are zero.

These results provide substantial evidence against the constant coefficient, nominal risk-free return model. A reasonable explanation is that the assumption of a constant conditional covariance is too strong. The next section investigates a statistical model of the intertemporal risk return relation in the forward exchange market that links the analysis to the empirical literature in finance.

#### 4.5 A Latent Variable Risk Return Model

In the empirical finance literature it has been commonplace to characterize the risk return trade-off facing investors with a single beta model. In such a model the riskiness of any asset is measured by the covariation of the excess return on the asset with the excess return from some benchmark portfolio. More precisely,

$$(15) \quad E_t^\gamma (R_{t+k,k} - R_{t+k,k}^z) = \beta_t^* E_t^\gamma (R_{t+k,k}^b - R_{t+k,k}^z),$$

where  $E_t^\gamma(\cdot)$  is the expectation operator conditioned on an information set  $\Phi_t^\gamma$  which is a subset of economic agents' information set, where

$$\beta_t^* = \frac{C_t^\gamma (R_{t+k,k}, R_{t+k,k}^b)}{V_t^\gamma (R_{t+k,k}^b)},$$

and where  $R_{t+k,k}^z$  is the return on an asset that is conditionally uncorrelated with the return on the benchmark asset  $R_{t+k,k}^b$ . If a nominal risk-free return is in the information set, it can be chosen as  $R_{t+k,k}^z$ . As has been demonstrated by Roll (1977) and extended to conditional



environments by Hansen and Richard (1983), the content of the restriction embodied in (15) is that the benchmark return  $R_{t+k,k}^b$  lies on the conditional mean variance frontier and is not the conditional minimum variance portfolio. A benchmark return is on the conditional mean variance frontier if any other return that has the same conditional mean as the benchmark also has a conditional variance that is at least as large as that of the benchmark. Since  $\beta_t^*$  is the ratio of a conditional covariance to a conditional variance, it will, in general, depend on elements in the conditioning information set  $\Phi_t^y$ .

The static capital asset pricing model [CAPM] is given empirical content through the assertion that the return on the aggregate wealth portfolio measured by the econometrician is mean variance efficient. Typically, it is assumed that observations on a vector of returns, including the return on the aggregate wealth portfolio, are normally distributed with probability distributions that are independent and identical over time. Since we have found evidence for time variation in the risk premiums in the forward foreign exchange market, it is important that we relax the requirement that returns be temporarily independent. Also, intertemporal asset pricing models do not have the implication that the return on the aggregate wealth portfolio be mean variance efficient. For this reason, we shall not require that observations on a benchmark return for a single beta model be available a priori. Instead we treat  $R_{t+k,k}^b$  as a latent variable in a time series version of what Zellner (1970) and Goldberger (1972) refer to as a multiple indicator, multiple cause (MIMIC) model. This model is similar in spirit to what Sargent and Sims (1977) refer to as an index model.<sup>15</sup> Our approach assumes that all of the time variation in risk premiums in the foreign exchange market and in a suitable benchmark portfolio can be captured by movements in conditional means. As with most empirical formulations of the static CAPM, we assume that conditional betas are constant. Thus, we maintain some of the ingredients of the static CAPM without its restrictive assumptions of zero temporal covariances and observability of the return on a benchmark portfolio.

To study empirically representation (15), we postulate the following statistical model: Let

$$y_{t+k} = (y_{t+k}^1, y_{t+k}^2, \dots, y_{t+k}^p),$$

where

$$y_{t+k}^j = \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j},$$

and let  $R_{t+k,k}^f$  denote a vector of the nominal risk-free returns in the  $p$  currencies. We suppose that the information set  $\Phi_t^y$  contains  $(y_t,$

15. The latent variable procedures which we propose use a different set of orthogonality restrictions than the ones employed by Sargent and Sims (1977). The procedure is also not a strict application of the MIMIC model.

$y_{t-1}, \dots$ ), and  $(R_{t+k,k}^f, R_{t+k-1,k}^f, \dots)$ . An investor can translate foreign-currency-denominated, risk-free returns into domestic currency returns in two ways. He can sell the proceeds in the forward market, or he can wait until the time of the payoff and sell the proceeds in the spot market. Each of these strategies generates a numéraire currency return that satisfies (15). Now consider the difference between the two returns generated by buying foreign currency, investing in the risk-free, foreign-currency-denominated asset, and either leaving the proceeds uncovered or covering them in the forward market. This difference in returns, when combined with (15) and after some manipulation, satisfies

$$(16) \quad E_t^y(y_{t+k}) = \beta_t^* E_t^y(R_{t+k,k}^b - R_{t+k,k}^f),$$

where  $\beta_t^* = (\beta_t^{*1}, \beta_t^{*2}, \dots, \beta_t^{*p})'$ , and  $\beta_t^{*j} = C_t^y(R_{t+k,k}^b, y_{t+k}^j) / V_t^y(R_{t+k,k}^b)$ . In our statistical model we shall treat  $\beta_t^*$  as a vector of constants and  $E_t^y(R_{t+k,k}^b - R_{t+k,k}^f)$  as a latent variable.

As noted above, a necessary condition for a return to be a legitimate benchmark for a single beta representation is that it resides on the mean variance frontier. We now interpret this requirement within the framework of intertemporal asset pricing models. Suppose that  $R_{t+k,k}^c$  is the minimum second-moment return conditioned on the information set of economic agents. That is, suppose that  $E_t(R_{t+k,k}^c)^2 \leq E_t(R_{t+k,k})^2$  for all returns  $R_{t+k,k}$ . Further, suppose that the probability of the event  $\{E_t^y(R_{t+k,k}^c) = R_{t+k,k}^f\}$  is zero. Then Hansen and Richard (1983) establish that any return  $R_{t+k,k}^b$  on the mean variance frontier conditional on  $\Phi_t^y$  satisfies:

$$(17) \quad R_{t+k,k}^b = \omega_t R_{t+k,k}^c + (1 - \omega_t) R_{t+k,k}^f,$$

where  $\omega_t$  is in  $\Phi_t^y$ . Hence, we can characterize the conditional mean variance frontier by characterizing the minimum conditional second-moment return. In cases in which an investor can trade a portfolio with a nominal return:

$$(18) \quad R_{t+k,k}^m = \frac{Q_{m,t+k}}{E_t(Q_{m,t+k})^2},$$

where  $Q_{m,t+k}$  is the intertemporal marginal rate of substitution of money for an investor, then  $R_{t+k,k}^m = R_{t+k,k}^c$ . This is established in Hansen and Richard as a straightforward implication of relation (2). In a world with heterogeneous investors but homogeneous information sets, as long as equilibrium allocations are consistent with the existence of complete markets,  $R_{t+k,k}^m$ , as defined in (18), is not investor specific since investors will have the same intertemporal marginal rates of substitution. If there exist certain assets that are not freely traded, then it may be implausible to assume the existence of a portfolio with a return  $R_{t+k,k}^m$ . In this case we can either define  $R_{t+k,k}^c = R_{t+k,k}^m$ , where  $R_{t+k,k}^m$  is defined for one investor, or we can assume that  $R_{t+k,k}^c$  is the minimum conditional second-

moment return conditional on agents' homogeneous information sets. Under the former interpretation,  $R_{t+k,k}^c$  is not a return but a transformation of an investor's intertemporal marginal rate of substitution. In either case the single beta representation (16) is valid for  $R_{t+k,k}^b$  given in (17), as long as probability of the event ( $\omega_t = 0$ ) is zero.

Equation (17) suggests that  $R_{t+k,k}^b$  is not uniquely defined. Recall, however, that in our statistical model we require  $\beta_t^*$  in (16) to be constant. This restricts the class of admissible  $\omega_t$ 's in (17) used in defining candidates for benchmark returns. Obviously, one of our maintained assumptions is that this set of admissible  $\omega_t$ 's is not empty.<sup>16</sup> Let  $\omega_t^*$  be such an admissible random weight that leaves  $\beta_t^*$  constant, and suppose  $h$  is a constant different from zero.

Then it can be shown that

$$(19) \quad R_{t+k,k}^b = h\omega_t^* R_{t+k,k}^c + (1 - h\omega_t^*) R_{t+k,k}^f$$

will satisfy (16) for  $\beta_t^*$  constant. Hence the parameter  $h$  is not identified in our model, and given that there is one legitimate benchmark return there are infinitely many such benchmarks. Since  $\omega_t^*$  in (19) can depend on information in  $\Phi_t^\gamma$ , there is no restriction on the sign of

$$E_t^\gamma(R_{t+k,k}^b - R_{t+k,k}^f) = \omega_t^* E_t^\gamma(R_{t+k,k}^c - R_{t+k,k}^f),$$

even though it can be shown that  $E_t^\gamma(R_{t+k,k}^c - R_{t+k,k}^f) \leq 0$ . It should be apparent that testing the restrictions implied by our risk return statistical model cannot be construed as a test of an explicit intertemporal equilibrium asset pricing model of forward foreign exchange rates. Instead, the tests should be interpreted as tests of the validity of a parsimonious characterization of risk in the foreign exchange market.

Equation (16) implies that the following is true:

$$(20) \quad y_{t+k} = \beta^* x_t + u_{t,k},$$

where  $x_t = E_t^\gamma(R_{t+k,k}^b - R_{t+k,k}^f)$ , and where  $u_{t,k}$  is the vector of forecast errors,  $(u_{t,k}^1, \dots, u_{t,k}^p)$ . These forecast errors satisfy the following conditions:

$$(21) \quad E(u_{t,k} u'_{t-j,k}) = \begin{cases} \Omega_j & j = 0, \dots, k-1 \\ 0 & j \geq k \end{cases}$$

$$(22) \quad E(u_{t,k} h_t) = 0 \text{ for all } h_t \text{ in } \Phi_t^\gamma.$$

16. It may well be that the set of admissible  $\omega_t$ 's is empty in explicit equilibrium analyses in which returns are represented as functions of underlying forcing variables. One can argue that the overidentifying restrictions in our model emerge completely from the requirement that this set not be empty. Consequently, our approach to modeling in this section as well as in the previous sections is best construed as interpreting alternative statistical representations of risk premiums using intertemporal asset pricing theory. Our statistical tests are not, however, tests of explicit equilibrium models.

Condition (21) indicates that, in general, the forecast errors will be contemporaneously correlated, and if  $k > 1$ , that is, if the forecast interval is greater than the sampling interval, the forecast errors will be serially correlated. Condition (22) merely reiterates that the conditional forecast errors are orthogonal to all information contained in the information set  $\Phi_t^y$  which includes  $x_t$ .

Since  $x_t$  is unobservable to the econometrician, we substitute into (20) the best linear prediction of  $x_t$  based on an observable subset of the information in  $\Phi_t^y$ . We choose this parsimonious subset based on the fact that in our previous study and in section 4.2 past forward rate forecast errors of currencies were useful in predicting  $y_{t+k}$ , but we also need to keep the information set small for computational purposes. Consequently, let

$$(23) \quad x_t = \alpha_0^* + \alpha_1^{*'} y_t + \epsilon_t,$$

where  $\epsilon_t$  is the prediction error which has mean zero and is orthogonal to  $y_t$ . Substituting (23) into (20) gives:

$$(24) \quad y_{t+k} = \beta^* \alpha_0^* + \beta^* \alpha_1^{*'} y_t + v_{t,k},$$

which is a constrained vector regression of  $y_{t+k}$  on a constant and  $y_t$  where  $v_{t,k} = u_{t,k} + \beta^* \epsilon_t$ , which implies that  $v_{t,k}$  is orthogonal to  $y_t$  also. The specification of the model does not imply that  $v_{t,k}$  is orthogonal to  $y_{t-i}$  for  $i \geq 1$ .

Our goal is to estimate  $\beta^*$  and  $\alpha^{*'} = (\alpha_0^*, \alpha_1^{*'})$ . Estimation of the  $k$ -step-ahead forecasting equation for  $y_{t+k}$ , given  $y_t$  subject to the non-linear cross-equation restrictions embedded in (24), allows us to recover consistent estimators of  $\beta^*$  and  $\alpha^*$  once one of the elements of  $\beta^*$  is normalized to one which is necessary because of the lack of identification of  $h$  discussed above. For the discussion of estimation, take this normalized  $\beta$  to be  $\beta^1$ . Once again, the  $\beta^*$  parameters provide us with information about the relative importance of risk across currencies, and the knowledge of  $\alpha_1^{*'}$  indicates the nature of time variation in the risk premiums.

Several strategies are available for estimating  $\delta^* = (\beta^{*2}, \dots, \beta^{*p}, \alpha^{*'})$  in (24). We now discuss the relative merits of alternative possibilities and in the process describe our estimation procedures. One strategy is to impose the additional requirement:

$$(25) \quad E(v_{t,k} / y_t, y_{t-1}, \dots) = 0,$$

and to estimate the parameters via maximum likelihood, that is, employ a Gaussian density function but not necessarily assume that  $y_t$  is Gaussian. It is customary to employ some time or frequency domain approximation to the likelihood function to ease the computational burden. Even with these approximations, maximum likelihood estimation can be difficult for values of  $k$  and  $p$  that substantially exceed one. The reason for this is that

all the parameters of the vector moving average error process must be estimated simultaneously with the structural parameters of interest. Furthermore, if (25) is false, the technique is misspecified.

A second strategy is to estimate the parameters of (24) using generalized method of moments (GMM) estimators. The procedure which we employ is a generalization of nonlinear three-stage least squares and is included in the class of GMM estimators studied by Hansen (1982) who derived their large sample properties. This procedure allows us to impose the cross-equation restrictions implied by our latent variable model either with or without imposition of the auxiliary assumption (25). As in maximum likelihood, it requires a numerical search algorithm to compute estimates, but the search is undertaken over a much smaller parameter space. On the other hand, in circumstances in which (25) holds, maximum likelihood is asymptotically more efficient. We now develop the GMM estimator for  $\delta^*$ .

A GMM estimator can be thought of as arising from the minimization of a criterion function that exploits the orthogonality conditions of the model. To see this, we construct a family of criterion functions that employ the same set of orthogonality conditions. Let  $\mathbf{z}'_t = (y_t, 1)$  and define the matrix of reduced form parameters:

$$\Theta(\delta) = \begin{bmatrix} \alpha'_1 & \alpha_0 \\ \beta^2 \alpha'_1 & \beta^2 \alpha_0 \\ \beta^p \alpha'_1 & \beta^p \alpha_0 \end{bmatrix},$$

where  $\delta$  is the  $m = 2p$  dimensional parameter vector. Construct the vector function,  $f(y_{t+k}, \mathbf{z}_t, \delta)$ , of the data and the parameters that summarize the  $r = (p+1)p$  orthogonality conditions of the model,

$$(26) \quad f(y_{t+k}, \mathbf{z}_t, \delta) = [y_{t+k} - \Theta(\delta)\mathbf{z}_t] \otimes \mathbf{z}_t,$$

where  $\otimes$  is the Kronecker product. Since  $\mathbf{z}_t$  is in  $\Phi_t$ , we are assured that the expectation of  $f$  evaluated at the true parameters  $\delta^*$  is zero:

$$(27) \quad E[f(y_{t+k}, \mathbf{z}_t, \delta^*)] = E(\mathbf{v}_{t,k} \otimes \mathbf{z}_t) = \mathbf{0}.$$

The  $r$  orthogonality restrictions in (27) are used to estimate the parameters  $\delta^*$ . Let

$$(28) \quad g_T(\delta) = \frac{1}{T} \sum_{t=1}^T f(y_{t+k}, \mathbf{z}_t, \delta),$$

where  $T$  is the sample size of the data set. Let  $a_T$  be an  $r$ -dimensional symmetric matrix which is allowed to be dependent on the sample data.

We can estimate  $\delta^*$  by choosing  $\delta = \delta_T$  where  $\delta_T$  minimizes the criterion function:

$$(29) \quad g_T(\delta)' a_T g_T(\delta).$$

Under regularity conditions specified in Hansen (1982), it is demonstrated that if a random sequence of matrices  $\{a_T: T \geq 1\}$  converges almost surely to a constant  $r$ -dimensional nonsingular symmetric matrix  $a^*$ , the estimator proposed above is strongly consistent. Furthermore,  $\sqrt{T}(\delta_T - \delta^*)$  converges in distribution to a normally distributed random vector with mean zero and covariance matrix  $\Sigma(a^*)$ . Details on how to compute and estimate  $\Sigma(a^*)$  are provided in appendix A. We followed Hansen (1982) in choosing  $a^*$  optimally to produce the smallest asymptotic covariance matrix among the class of estimators that exploit the orthogonality conditions defined by (27).

To motivate a test of the model, recognize that  $\delta_T$  is the parameter vector that sets a linear combination of the sample orthogonality conditions  $g_T(\delta)$  to zero via the first-order conditions for the minimization of (29). More precisely, the first-order conditions require that  $\delta_T$  sets  $m$  linear combinations of the  $r$  orthogonality restrictions equal to zero. Thus there are  $r - m$  linearly independent combinations of  $g_T(\delta)$  that are not necessarily equated to zero but which should be close to zero if the restrictions are true. Under an alternative hypothesis, the elements in the reduced form parameter matrix  $\Theta$  are unrestricted and can be estimated using equation-by-equation ordinary least squares. These estimates are provided in table 4.4. Relaxing the restrictions in this manner is equivalent to setting the  $r$  sample orthogonality conditions equal to zero. Therefore a test of the restrictions can be conducted by examining the minimized value of the criterion function when the restrictions are imposed relative to zero, which is its unrestricted value.<sup>17</sup>

Hansen demonstrates that  $T g_T(\delta_T)' a_T g_T(\delta_T)$  is asymptotically chi-square distributed with  $r - m$  degrees of freedom, where  $\delta_T$  is a minimizer of (29) and  $a_T$  is an estimator of an optimal choice of  $a^*$ . We use this as a test of our model restrictions.

An additional set of tests that we performed amounts to examination of whether subsets of the reduced form parameters in the matrix  $\Theta^* = \Theta(\delta^*)$  equal zero. First we examined the unrestricted reduced forms and tested whether the coefficients other than the constants were zero. Consistent with our earlier results, we found substantial evidence against the hypothesis of no time variation in the risk premiums. These same tests were repeated with the restrictions imposed.

The results for the latent variable model are presented in table 4.5. The

17. Gallant and Jorgensen (1979) propose tests similar to this for the case in which disturbances are serially uncorrelated.

$$\frac{S_{t+9} - F_{t,9}}{S_t} = a_i + \sum_{j=1}^8 b_{ij} \left( \frac{S_j - F_{j-9,9}}{S_{j-9}} \right) + u_{i,9}$$

Table 4.4

Exchange Rates: US\$/Foreign Currency; Sample: 5 February 1976 to 29 December 1980; Number of Observations: 512

Currency	$\hat{a}_i$ (Std. Err.) Confi- dence	$\hat{b}_{i1}$ (Std. Err.) Confi- dence	$\hat{b}_{i2}$ (Std. Err.) Confi- dence	$\hat{b}_{i3}$ (Std. Err.) Confi- dence	$\hat{b}_{i4}$ (Std. Err.) Confi- dence	$\hat{b}_{i5}$ (Std. Err.) Confi- dence	$\chi^2(5)$ All $\hat{b}_{ij}$ 's $j \geq 1 = 0$ Confi- dence	$R^2$	Resid. Var.
French franc	0.297 (0.320) 0.646	-0.165 (0.142) 0.754	-0.003 (0.116) 0.018	0.252 (0.151) 0.904	-0.122 (0.088) 0.835	-0.173 (0.225) 0.557	6.738 0.759	0.068	5.698
Japanese yen	0.428 (0.420) 0.691	-0.287 (0.162) 0.932	0.204 (0.148) 0.832	0.463 (0.162) 0.996	0.090 (0.126) 0.528	-0.596 (0.272) 0.972	20.303 0.999	0.165	9.626
Swiss franc	0.328 (0.414) 0.572	-0.102 (0.292) 0.272	0.067 (0.161) 0.323	0.626 (0.161) 0.999	-0.113 (0.155) 0.533	-0.853 (0.325) 0.991	16.100 0.993	0.178	10.939
U.K. pound	0.568 (0.371) 0.874	0.014 (0.199) 0.056	-0.055 (0.097) 0.430	0.214 (0.112) 0.944	0.190 (0.132) 0.849	-0.406 (0.182) 0.974	8.390 0.864	0.072	6.943
Deutsche mark	0.328 (0.341) 0.664	-0.281 (0.167) 0.907	-0.051 (0.123) 0.320	0.417 (0.162) 0.990	-0.034 (0.108) 0.245	-0.323 (0.194) 0.904	9.462 0.908	0.124	6.394

NOTE: See table 4.1.

**Table 4.5** The Latent Variable Model

Exchange Rates: US\$/Foreign Currency; Sample: 5 February 1976 to 29 December 1980; Number of Observations: 512

Currency	$\hat{\beta}_i$ (Std. Err.)	Reduced Form Coefficients										$\chi^2(5)$ All $\hat{\theta}_j$ 's $j \neq 1 = 0$ Conf- dence	$R^2$
		$\hat{\theta}_{10}$ (Std. Err.) Conf- dence	$\hat{\theta}_{11}$ (Std. Err.) Conf- dence	$\hat{\theta}_{12}$ (Std. Err.) Conf- dence	$\hat{\theta}_{13}$ (Std. Err.) Conf- dence	$\hat{\theta}_{14}$ (Std. Err.) Conf- dence	$\hat{\theta}_{15}$ (Std. Err.) Conf- dence						
French franc	0.348 (0.178)	0.084 (0.108) 0.565	-0.033 (0.062) 0.406	0.045 (0.040) 0.733	0.157 (0.099) 0.889	0.036 (0.041) 0.617	-0.249 (0.157) 0.888	2.745 0.261	0.023				
Japanese yen	1.0	0.242 (0.281) 0.611	-0.095 (0.172) 0.421	0.128 (0.092) 0.836	0.452 (0.148) 0.998	0.103 (0.104) 0.677	-0.715 (0.232) 0.998	12.943 0.976	0.152				
Swiss franc	1.164 (0.316)	0.282 (0.325) 0.614	-0.111 (0.200) 0.421	0.149 (0.106) 0.843	0.526 (0.157) 0.999	0.120 (0.121) 0.681	-0.832 (0.245) 0.999	17.205 0.996	0.146				
U.K. pound	0.421 (0.242)	0.102 (0.129) 0.569	-0.040 (0.075) 0.406	0.054 (0.048) 0.742	0.190 (0.116) 0.899	0.043 (0.049) 0.622	-0.301 (0.183) 0.901	2.941 0.291	0.009				
Deutsche mark	0.659 (0.229)	0.160 (0.188) 0.603	-0.063 (0.114) 0.419	0.084 (0.063) 0.818	0.298 (0.115) 0.991	0.068 (0.070) 0.667	-0.471 (0.182) 0.990	8.398 0.864	0.066				

NOTE: Test of the constrained model  $\chi^2(20) = 18.834$ . See table 4.1.



value of the test statistic of the model restrictions is 18.834, which is below the mean of a chi-square variable with 20 degrees of freedom. Therefore, there is little evidence in the sample against the restrictions imposed by our model. Using the estimates of the betas as measures of relative risk, we find that the Swiss franc and Japanese yen were the most risky contracts. The French franc and the U.K. pound were least risky, and the Deutsche mark was intermediate between the two sets of currencies. Tests of the reduced form parameters for time variation in the risk premiums are quite consistent with the evidence of table 4.4. The restrictions of our model still capture most of the significant time variation in the risk premiums found in the unrestricted estimates.

#### 4.6 Summary and Conclusions

In this paper we investigated risk premiums in the forward foreign exchange market using linear time series models. We relied on first-order conditions of a rational investor to interpret alternative statistical restrictions about the divergence of forward exchange rates from expected future spot rates. Three basic conclusions emerged from our analysis. First, risk premiums are not adequately characterized by constants as was implied by a time invariant lognormal model. Second, time variation in the risk premiums are not accurately summarized by movements in the nominal interest rate of the currency of denomination of the forward contract. Third, using a single beta latent variable model to measure risk, we found risk premiums to be important in at least two of the five currencies studied. It may be that longer time series or more powerful econometric procedures will alter some of our conclusions. Also from a practical standpoint, in all of our statistical inference we are forced to rely on asymptotic distribution theory in computing significance and confidence levels, and knowledge of the correct small sample distributions of the test statistics we have employed might overturn some of our conclusions. Nonetheless, we believe that we have been successful in providing additional insights into the nature of the forward foreign exchange market.

While we used first-order conditions of the intertemporal maximization problems of investors to motivate and interpret alternative restrictions on time series representations, our statistical tests cannot be construed as direct tests of an equilibrium model. An explicit equilibrium analysis would require that we write down specifications of stochastic forcing variables that generate the restrictions on the endogenous time series that are imposed and tested here. An obvious criticism of our approach is that by placing our auxiliary assumptions on endogenous variables, we are in danger of analyzing empirically specifications that may not be either internally consistent nor consistent with plausible

specifications of the stochastic forcing variables. Even though we take this criticism seriously, we view our analysis as a useful starting point in studying equilibrium models of the forward foreign exchange market. Recent theoretical work of Lucas (1982) has begun to integrate monetary theory and modern general equilibrium financial theory. We interpret our results as demonstrating the potential importance of this integration, although additional work needs to be done in obtaining general equilibrium models with implications that are susceptible to formal statistical inference. Such models would allow us to investigate the ultimate sources of risk premiums and would provide a vehicle for interpretation of movements in spot exchange rates. While a direct and explicit equilibrium econometric study will be of great interest, such an exercise may be overly ambitious at this time, given problems in calculating dynamic, stochastic equilibria.

Our research to date has focused on time series representations that do not require measurements of intertemporal marginal rates of substitution of money. Incorporating such measurements could lead to valuable extensions of this paper. Unfortunately, obtaining such measurements is particularly difficult because much of the existing consumption and price data are aggregated over time and across commodities. Although Hansen and Singleton (1982) have described distribution-free procedures for testing intertemporal asset pricing models, their procedures require point-in-time measurements of consumption and purchasing power. Alternatively, distributional assumptions on the point-in-time data coupled with a priori specification of the possibly nonlinear averaging filters might lead to testable implications of the intertemporal asset pricing models using the available data.

## Appendix A

In this appendix we describe how the asymptotic covariance matrices for the parameter estimators of the various statistical models were estimated.

### *Lognormal Model*

The parameters were estimated equation by equation using ordinary least squares. Consider the  $k$ -step-ahead regression equation

$$y_{t+k} = \mathbf{z}_t' \boldsymbol{\beta}_0 + u_{t,k}.$$

Under the assumptions of the lognormal model, the conditional covariances

$$E(u_{t,k} u_{t-j,k} / \mathbf{z}_t, u_{t-k,k}, \mathbf{z}_{t-1}, u_{t-k-1,k}, \dots)$$

are constant. The asymptotic covariance matrices were estimated using the formula in Hansen and Hodrick (1980), pp. 833–35.

### *Nominal Risk-Free Return*

Again the parameters were estimated equation by equation using ordinary least squares. We relaxed the assumption that the conditional covariances were constant. Let  $u_{i,k}^T$  denote the estimated least-squares residual of  $t$  using a sample of size  $T$ . The asymptotic covariance matrices were estimated using the formula.

$$\Sigma_T^{-1} S_T \Sigma_T^{-1},$$

where

$$\Sigma_T = \frac{1}{T} \sum_{t=1}^T \mathbf{z}_t \mathbf{z}_t';$$

$$S_T = \sum_{j=-k+1}^{k-1} R_T(j);$$

$$R_T(j) = \frac{1}{T} \sum_{t=j+1}^T u_{i,k}^T u_{i-k}^T \mathbf{z}_t \mathbf{z}_{t-j}', \quad 0 \leq j < k;$$

and

$$R_T(j) = R_T(-j)', \quad -k < j < 0.$$

### *Latent Variable Model*

In the previous two models, we assumed that the least-squares regression equations were optimal  $k$ -step-ahead forecasting equations. In conducting inference in the latent variable model, we no longer made that assumption. We allowed the disturbances to be arbitrarily serially correlated. For this reason we relied on spectral estimators of the asymptotic covariance matrix.

Because of the restrictions across the projection equations, an equation system estimation procedure was needed to estimate the parameters. This meant that a relatively large number of orthogonality conditions, thirty, were used simultaneously in estimation. Although it appears desirable to allow for conditional heteroscedasticity, it is difficult, if not impossible, to employ this many orthogonality conditions without adding some more restrictive assumptions. For this reason we imposed a joint normality assumption on the observable variables which rules out conditional heteroscedasticity. It allowed us to estimate the optimal weighting matrix and the asymptotic covariance matrix by estimating the cross-spectral density matrix of  $(\mathbf{v}', \mathbf{z}')$  and then applying the formulas under case (v) in Hansen (1982), p. 1045.

Since the  $\mathbf{v}$ 's were not observable, they were replaced by the least-squares residuals from unrestricted least-squares regressions. The sample

means were subtracted from the  $z$ 's and all series were prewhitened using a second-order univariate filter estimated by ordinary least squares. The cross-spectral density matrix of the prewhitened series was estimated by using a tent averaging filter applied to the periodogram ordinates with a bandwidth of twenty-five harmonic frequencies. The cross-spectral density matrix was then recolored and the formulas in Hansen (1982) were applied. The appropriate modifications of these formulas were used that allowed the  $z$ 's to have nonzero means.

The asymptotic covariance matrices for the unrestricted least-squares estimators were estimated using the same procedure as was used in the lognormal model.

## Appendix B Data Sources

The data set was obtained from Data Resources, Inc. The data were daily observations of spot exchange rates and one-month forward rates and were supplied to Data Resources, Inc. by Bank of America. The forward rates are from the interbank forward market. Bid and ask rates were averaged to form the quotations used in the analysis. A semiweekly sample was constructed in which Tuesday forward rates were matched with Thursday spot rates thirty days in the future and Friday forward rates were matched with Monday spot rates.

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## Comment Craig S. Hakkio

### Introduction

Several studies of the foreign exchange market, typically using weekly data, have concluded that the exchange rate has deviated from simple market efficiency. Simple efficiency implies that the forecast error,  $(S_{t+k}^j - F_{t,k}^j)/S_t^j$ , should be zero ex ante. Several models generate the prediction that this forecast error may be nonzero because of risk aversion or stochastic price levels. Empirically, we find that the forecast error is approximately zero, but that variables dated  $t$  or earlier can explain the forecast error, that is, the foreign exchange market appears inefficient. The paper by Hansen and Hodrick is an attempt to document and explain these observations. In light of the time series nature of the rejections, they develop several dynamic arbitrage conditions, derived from the intertemporal capital asset pricing model. The different arbitrage conditions rely on various distributional assumptions, such as lognormality or the constancy of a conditional variance or covariance. This paper is an important study for aiding our understanding of the risk premium. The importance stems from three aspects of the paper: (1) the arbitrage conditions are derived from an optimizing equilibrium model; (2) the econometric and theoretical assumptions underlying each hypothesis are clearly stated so we know exactly what is being accepted or rejected; and (3) the econometric and empirical work is done very carefully.

### The Theoretical Model

The model employed by Hansen and Hodrick is based on an intertemporal asset pricing model. There are two reservations I have about the use of this model to explain the risk premium in the foreign exchange market. First, Hansen and Hodrick begin with an intertemporal arbitrage condition, derived from a real (nonmonetary) model (see, for example, Brock 1979). They then convert this real return condition to a nominal return condition. As the authors realize, such a step is nontrivial and depends on

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The author would like to thank Robert Chirinko and Lauren J. Feinstone for helpful comments.

the way money is introduced into the model. Consequently, their results are conditional on money being introduced in an appropriate manner. Second, the model is based on a representative individual. To go from an individual to an economy-wide aggregate requires very stringent assumptions on preferences, technologies, or information sets. Such assumptions may not be reasonable to make in an international context. If one assumes that agents are different between countries (and, as Paul Krugman pointed out, consumption bundles in different countries are different), then it is not clear that this arbitrage condition will hold. Simple aggregation may be invalid in this model.

Before discussing the particular models, let me comment on the data set. The data are sampled twice a week: "Tuesday forward rates predict Thursday spot rates thirty days in the future and Friday forward rates predict Monday spot rates." (Section 4.3 and appendix B). Unfortunately, Friday forward rates should predict *Tuesday* spot rates. In a detailed study such as this, using data sampled twice a week, such a misalignment may be critical.

#### Discussion of the Lognormal and the Nominal Risk-Free Return Models

The lognormal model assumes that the relevant variables are jointly lognormal, and therefore the risk premium is constant. The methodology and results are similar to Hansen and Hodrick (1980). Although no system-wide test is provided, it is likely that such a test would fail. My question is: What was the *nature* of the rejection and what caused the rejection? To answer this, we can only look at the individual results that are given in table 4.1. The hypothesis is rejected for Japan, Switzerland, and Germany, and accepted for France and the United Kingdom. (I am embarrassed to put it this way, because I believe we should look at the foreign exchange *market*.) In addition, the rejection seems to be the result of lagged Swiss and German forecast errors. According to some European economists, the Swiss franc was closely tied to the Deutsche mark and the Deutsche mark was the dominant currency. If we accept this, what happened to Germany during this period? Can we identify some structural change that occurred, such as higher energy prices or government policy changes?<sup>1</sup>

In addition, there is a question of the degrees of freedom. Although 512 observations were used, there are only 59 nonoverlapping observations. As the authors realize, the small sample properties of their procedures are unknown.

1. In addition, there is one bit of evidence that the U.S. dollar was a problem: for the Swiss franc/U.S. dollar exchange rate and the Japanese yen/U.S. dollar exchange rate the null hypothesis was rejected, yet for the Japanese yen/Swiss franc exchange rate the null hypothesis was accepted.



A final issue concerns the stability of the coefficients across different time periods. Of course, instability of the parameter estimates would imply a rejection of the lognormal distribution assumption, but it is important for determining whether the *observed* time variation is an indication of a profit opportunity. Unfortunately, such tests, while feasible, are not reported.

The nominal risk-free return model attempts to explain the time variation in the risk premium by movements in the U.S. Treasury-bill rate. The tests reported in table 4.3 reject this model. The explanatory power of the Treasury-bill rate is insignificant. Again, no system-wide test is provided, although one would probably reject the system joint hypothesis. However, looking again at the single-country results, the null hypothesis was rejected for Germany, Switzerland, and Japan, because of German and Swiss lagged forecast errors, as was the case in table 4.1.

#### Discussion of the Latent Variable Model

I found the latent variable model to be both the most interesting and puzzling aspect of the paper. As was discussed, this model assumes that the risk premium for *each* currency is proportional to a common unobserved variable. To make the model operational, one must obtain a predictor of the unobserved variable that explains the movement in the risk premium. The authors use their previous results which showed that the time variation of the risk premium could be explained by lagged forecast errors; their best linear predictor for time  $t$  is the vector of lagged forecast errors. The additional structure imposed by the assumption of a single unobserved variable is a set of cross-equation restrictions.

Using a quite general econometric procedure developed by Hansen (1982), they are able to estimate the constrained model and test the restrictions implied by a single latent variable. They are not able to reject the hypothesis of a single unobserved variable that explains the time variation of the risk premium. This, I believe, is their main finding. However, they find that the Japanese yen and the Swiss franc are the most risky and that the U.K. pound and French franc are the least risky, with Germany in the middle. This seems to be the *opposite* of what I would have thought, yet it seems consistent with the results of tables 4.1–4.3.

The latent variable model attempts to explain the time variation in the risk premium using an unobserved variable. The choice of the variables used to explain the latent variable is arbitrary and appears to be dictated by what was known from table 4.1: lagged forecast errors “explain” the risk premium. What is the economic context of such an assumption? One should not be surprised at their results, given their findings in table 4.1, even after taking into account the cross-equation restrictions. The previous results showed that for each currency, lagged German and Swiss forecast errors were significant variables in explaining the risk premium.

An interesting further test would be to project the latent variable against just the German, or the German and Swiss, forecast error. My guess is that such a model could not be rejected. Such a test would narrow the range of explanations for why the hypothesis of simple market efficiency is rejected.

### General Discussion

As is always the case in such tests of efficiency, a joint hypothesis is being tested. In the case at hand, we take the general equilibrium to be "correct," which, as stated earlier, may be incorrect. To obtain testable implications, various assumptions must be made. Since the joint hypotheses for the lognormal and nominal risk-free return models are rejected, we must ascertain which aspect of the *joint* hypotheses is suspect.

The combination of tables 4.1 and 4.5 act to focus our attention on Germany. If we accept the auxiliary hypothesis of a constant conditional covariance, then we must explain why lagged German and Swiss forecast errors "explain" the risk premium. If we reject the auxiliary hypothesis of a constant conditional covariance, then we must explain why the conditional covariance changed over the sample period of February 1976 to December 1980. The conditional covariance between variables  $x$  and  $y$  is simply the covariance between  $x$  and  $y$ , conditional on the information set at time  $t$ . Therefore, since the information set varies, the conditional covariance may also vary. But why should it? If during the sample period countries were still adapting to the flexible exchange rate system and were undergoing structural changes, that is, adjusting to the higher energy prices and supply shocks, then as people learn the new processes, the conditional covariance may change. Suppose one tested for coefficient stability in either the lognormal model or the nominal risk-free return model and found the coefficients were unstable—this would lead one to believe that the rejection of the joint hypothesis may be the result of the conditional covariance not being constant. This would seem to be a plausible explanation of the rejection.<sup>2</sup> The same argument could be applied to any other of the three models.

### Conclusion

What additional information or knowledge do we now possess concerning the behavior of the risk premium? The results here and elsewhere seem to indicate that the forecast error is on average very close to zero, but it *fluctuates* around zero, and these fluctuations in forecast errors can be "explained" by lagged forecast errors. The latent variable model indicates that the lagged forecast errors influence each risk premium in a

2. In fact, Hansen and Hodrick do recognize this possibility: "A reasonable explanation is that the assumption of a constant conditional covariance is too strong" (conclusion of section 4.4).

particular way through a single unobserved latent variable. But, what does this tell us about a foreign exchange risk premium?

In addition, what is the implication of these results for our understanding of exchange rate behavior? Should we conclude that the foreign exchange market is inefficient? Should we reject the use of the forward premium as a measure of the expected rate of depreciation? I think the answer to both questions is no!

For some questions of exchange rate behavior this finding will be important. For other questions, such as the use of the forward premium as a measure of expected depreciation, this finding is less significant. True, one must be careful: there does appear to be information *not* incorporated in the forward rate. However, it is not clear that this information is exploitable. Put another way, are the deviations significant for all purposes?

To summarize, this paper by Hansen and Hodrick is an important addition to our stock of knowledge concerning the time series behavior of the risk premium. It is a serious, systematic examination of the foreign exchange risk premium and serves to focus our attention on certain empirical regularities: the deviation from simple market efficiency can be "explained" by the use of a single latent variable, which is itself "explained" by lagged forecast errors. Unfortunately, the authors do not provide an economic interpretation of this finding; what can we conclude about exchange rate behavior? The implication of this paper for further study would seem to be to explain why the latent variable should be related to lagged forecast errors. (1) What economic information is contained in the lagged forecast errors that is related to the current risk premium? (2) Is the time variation of the information set and structure sufficient to generate a time variation in the conditional covariance which could also explain these results?

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## Comment Kenneth J. Singleton

### Introduction

While there is now substantial empirical evidence against the "simple market efficiency hypothesis," much less attention has been given to modeling the risk premiums that many believe underlie recent rejections of this hypothesis. Hansen and Hodrick have provided an important contribution to our understanding of the recent behavior of floating exchange rates by investigating relations among exchange rates and other economic variables deduced from an equilibrium, intertemporal asset pricing relation. The particular models considered are chosen so that the exchange rate relations that are estimated are linear in the levels or logarithms of exchange rates. The derivation of these relations, together with their careful econometric analyses, provide a number of insights into the limitations of several linear models to explain the temporal behavior of exchange rates in a world of risk averse agents and nontrivial production technologies. In my comments on this paper I will elaborate on some of the theoretical properties of the three models investigated by Hansen and Hodrick and then discuss ways of testing nonlinear, intertemporal models of exchange rate determination that do not impose the restrictive assumptions underlying their linear relations. To avoid confusion, I will denote the  $j$ th equation in Hansen and Hodrick's paper by (Hj).

### The Linear Representations of Exchange Rates

The fundamental equilibrium relation from which all of the empirical models are derived is their equation (H4):

$$(1) \quad E_t[Q_{m,t+k}(S_{t+k}^j - F_{t,k}^j)] = 0.$$

In (1),  $Q_{m,t+k}$  is the marginal rate of substitution of money between dates  $t$  and  $t+k$  for an individual investor,  $S_{t+k}^j$  is the spot exchange rate for currency  $j$  at date  $t+k$ ,  $F_{t,k}^j$  is the forward exchange rate for currency  $j$  established at date  $t$  for payment at date  $t+k$ , and  $E_t[\cdot]$  is the mathematical expectation conditioned on an investor's information set at date  $t$ ,  $I_t$ . Equation (1) is derived from the necessary conditions of an individual investor's intertemporal utility maximization problem, without assuming that  $Q_{m,t+k}$ ,  $F_{t,k}^j$ ,  $S_{t+k}^j$ , or the elements of the information set are drawn from any particular distribution. Therefore, in general (1) does not imply that  $(S_{t+k}^j - F_{t,k}^j)$  or the corresponding expression in logarithms,  $(s_{t+k}^j - f_{t,k}^j)$ , is linearly related to variables observed by the econometrician. To deduce such relations, Hansen and Hodrick impose additional

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Robert Hodrick provided very helpful comments on a preliminary draft of this comment.

structure on the joint distributions of the variables in (1). I will comment on their special cases of (1) in the order of their introduction in their paper.

### *The Lognormal Model*

The first special case of the equilibrium condition (1) is obtained by assuming that the vector  $Z_t$  of  $p$  spot and forward rates and  $Q_{m,t}$ ,  $Z_t' = (S_t^1, \dots, S_t^p, F_{t,k}^1, \dots, F_{t,k}^p, Q_{m,t})$ , is a stationary, lognormal stochastic process. Under this assumption, Hansen and Hodrick show that

$$(2) \quad E_t^z(s_{t+k}^j) - f_{t,k}^j = a_j, \quad j = 1, \dots, p,$$

where  $a_j$  is a constant, lowercase letters denote logarithms of their uppercase counterparts, and  $E_t^z(\cdot)$  is the expectation conditioned on  $I_t^z = (z_{t-s}, s \geq 0)$ . The fact that  $Q_{m,t}$  enters (2) only through the information set  $I_t^z$  is a consequence of the fact that no investment is required at date  $t$  to contract in the foreign exchange market. The counterpart to (2) for a lognormally distributed return on a common stock or bond that satisfies their Euler equation (H2) involves the expected value of the logarithm of the marginal rate of substitution,  $E_t^z(\ln Q_{m,t+k})$  (see Hansen and Singleton 1982b).

Equation (2) states that the logarithm of the forward rate is equal to the expected future value of the logarithm of the spot rate plus a constant. As Hansen and Hodrick note, this condition has been tested in several recent studies and, in their own previous work (1980), has been rejected. What is new to this part of their paper is the reinterpretation of these tests as tests of a special case of an intertemporal asset pricing model. In light of the prevalence of tests for "simple market efficiency" in the literature, this reinterpretation would seem to warrant further discussion.

Previous tests of the null hypothesis that the coefficients in the linear least-squares projection of  $(s_{t+k}^j - f_{t,k}^j)$  onto the past histories of economic variables are zero have been viewed by some as tests for the absence, or at least constancy, of risk premiums. The theoretical discussion in Hansen and Hodrick suggests (to me) that there is an important sense in which this interpretation of the evidence may be misleading. The assumptions of perfect financial markets, rational expectations, risk neutral agents, and no uncertainty about future purchasing power do not imply equation (2). Rather, they imply that  $E_t(s_{t+k}^j - f_{t,k}^j) = 0$  (Frenkel and Razin 1980), which is equivalent to the null hypothesis of no risk premiums when risk premiums are defined to be the expected gain from contracting in the forward market,  $E_t(s_{t+k}^j) - f_{t,k}^j$ .

Furthermore, the restrictions embodied in (2) do not imply that risk premiums, as just defined, are zero or even constant. This can be seen by inspection of equation (H2):

$$(3) \quad E_t \left| \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right| = -C_t \left| \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right|, Q_{m,t+k} \left| R_{t+k,k}^f \right|$$

Since  $S_t^j \in I_t$ , it follows immediately from (3) that  $E_t(S_{t+k}^j - F_{t,k}^j)$  will in general vary over time as a function of the elements of  $I_t$ , even though  $E_t^z(s_{t+k}^j - f_{t,k}^j)$  is a constant. That this is the case should be reassuring, since in the derivation of (2) Hansen and Hodrick made no assumptions about the risk aversion of investors. The marginal rate of substitution dropped out of their expressions as a consequence of assuming lognormality.

### *The Nominal Risk-Free Return and Latent Variable Models*

The second and third approaches to testing the equilibrium pricing relations are based on equation (3). Equation (3) is deduced directly from (1). Therefore, in the absence of additional assumptions, tests of the model based on (3) amount to testing a subset of the restrictions embodied in (1). Heuristically, (3) implies that the risk premium  $E_t(S_{t+k}^j) - F_{t,k}^j$  is due to consumption risk. To see this, note that the risk-free rate  $R_{t+k,k}^f$  is always positive. Now suppose that the gain  $(S_{t+k}^j - F_{t,k}^j)$  covaries positively with the marginal utility of money. Then, the expected gain from contracting in the forward market, or equivalently the risk premium  $E_t(S_{t+k}^j) - F_{t,k}^j$ , must be negative. This is because the gain  $(S_{t+k}^j - F_{t,k}^j)$  will be relatively large when  $Q_{m,t+k}$  is large and money is considered relatively valuable. The equilibrium foreign exchange prices will reflect this insurance role of exchange markets, and with a zero investment requirement in forward markets the expected gain must be negative. Over time, of course, the conditional covariance in (3) is generally changing and may be positive or negative at each point in time. Therefore, in practice, (3) allows for the expected gain, and hence the risk premium, to change signs over time.

Because the conditional covariance in (3) will in general vary over time as a nonlinear function of the variables in the investor's information set at date  $t$ , direct tests of the pricing model for foreign exchange based on (3) are difficult. To circumvent this difficulty, Hansen and Hodrick assume that certain conditional covariances are constant in the second and third tests of their model, while allowing conditionally heteroscedastic errors. They are careful to point out in their note 16 that, because these statistical representations of risk premiums are not explicitly deduced from an underlying model, an explicit equilibrium model is not being tested. For this reason, more motivation for the particular characterizations of risk considered, and discussion of sufficient conditions that might lead to their restrictions, would have been helpful.

One possible set of sufficient conditions that leads to their equation

(H13), with conditionally homoscedastic errors, is that the joint process  $Z_t^j = [(S_t^j - F_{t-k,k})/S_{t-k}^j, Q_{m,t}, R_{t+k,k}^j]$  is normal. An unsatisfactory feature of this assumption is that it requires that  $R_{t,k}^j$  and  $Q_{m,t}$ , which are always positive, be normally distributed. Nevertheless, since it leads to the second representation, I briefly consider it here. Taking expectations in (1) conditional on the information set  $I_t^Z = \{Z_{t-s} : s \geq 0\}$ , using the law of iterated expectations, and then mimicking the steps that lead to (3), it can be shown that

$$(4) \quad E_t^Z \left\{ \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j} \right\} = -C_t^Z \left\{ \frac{S_{t+k}^j - F_{t,k}^j}{S_t^j}, Q_{m,t+k} \right\} R_{t+k,k}^j.$$

Now under the joint normality assumption, the conditional covariance term in (4) is a constant  $b_j$  and relation (4) is equivalent to Hansen and Hodrick's equation (H13), except that it applies for the smaller information set  $I_t^Z$ .

Interestingly, a similar distributional assumption implies a version of their latent variable model, which is the third representation of exchange rates that is tested. According to this representation of the risk premium,  $y_{t+k}^j = (S_{t+k}^j - F_{t+k}^j)/S_t^j$  is given by

$$(5) \quad E_t^y y_{t+k}^j = \beta_{jt}^* E_t^y (R_{t+k,k}^b - R_{t+k,k}^j),$$

where  $R_{t+k,k}^b$  is a special "benchmark" return, the superscript  $y$  indexes the information set, and

$$(6) \quad \beta_{jt}^* = \frac{\text{Cov}_t^y(R_{t+k,k}^b, y_{t+k}^j)}{\text{Var}_t^y(R_{t+k,k}^b)}.$$

If the assumption that  $(y_{t+k}^j, \dots, y_{t+k}^p, R_{t+k,k}^b, R_{t+k,k}^j)$  is normal is now imposed on the model, then  $\beta_{jt}^*$  in (6) is constant when expectations are conditioned on current and past values of the  $(S_t^j - F_{t-k,k}^j)/S_{t-k}^j$ ,  $R_{t,k}^b$ , and  $R_{t+k,k}^j$ . Thus, a normality assumption is also sufficient to deduce the latent variables representation. Again, I emphasize that these observations are intended only to illustrate possible sets of sufficient conditions that lead to the second and third representations. Before taking the normality assumption seriously, one may wish to verify that such a distributional assumption can be supported by a plausible equilibrium model.

Implicitly, the linear relations (4) and (5) with constant conditional covariances may embody very different sets of underlying assumptions about the economy. However, since Hansen and Hodrick do not exploit the particular forms of the constant covariances in (4) and (5) when deducing their restrictions, it is easy to see that (4) is a special case of the latent variable model (5). More precisely, their latent variable equation is

$$(7) \quad y_{t+k} = \beta^* x_t + u_{t,k},$$

where  $\beta^*$  is a vector of unrestricted constants and  $x_t$  is a scalar latent variable. The error  $u_{t,k}$  is orthogonal to their information set  $\Phi_t^y$ , which includes at least  $(y_{t-s}, s \geq 0)$  and current and past values of the vector of nominal risk-free rates  $R_{t+k,k}^f$ . The latent variable  $x_t$  is given by  $E_t^y(R_{t+k,k}^b - R_{t+k,k}^f)$ , where  $R_{t+k,k}^f$  is the risk-free return in the domestic currency. To make (7) operational, they form the linear least-squares projection of  $x_t$  onto elements of  $\Phi_t^y$ . For their empirical analysis, they restrict attention to current and past values of  $y_t$ . Notice, however, that  $R_{t+k,k}^f \in \Phi_t^y$  and, thus, they could just as well have included  $R_{t+k,k}^f$  in their projection equation (H23), which would then become:

$$(8) \quad y_{t+k} = \beta^* \alpha_0^* + \beta^* \alpha_1^{*'} y_t + \beta^* \alpha_2^* R_{t+k,k}^* + v_{t,k}.$$

Had they done this, then the nominal risk-free relation (H14) in their paper would be a special case of their latent variable representation. Specifically, a test of the null hypothesis  $\alpha_0^* = 0$  and  $\alpha_1^* = \mathbf{0}$  in (8) is equivalent to a test of the nominal risk-free representation.

These observations highlight the difficulty of interpreting their results in the absence of more information about the underlying assumptions that lead to the linear exchange rate representations. The nominal risk-free and latent variable representations are presented as if they represent very different theoretical models of exchange rate determination and, indeed, the underlying models clearly may be very different. However, since so little structure is imposed on the empirical representations of the theoretical models, one representation can be interpreted as a special case of the other.

### *Approaches to Testing the Nonlinear Model of Exchange Rates*

In this final section I will briefly describe some alternative approaches to testing the restrictions on exchange rate behavior implied by the equilibrium condition (1). These approaches involve adaptations of the test procedures discussed in Hansen and Hodrick (1980) and in Hansen and Singleton (1982a), and the interested reader is referred to these papers for details. An important feature of the tests based directly on (1) is that it is not necessary to impose distributional assumptions and, in particular, the conditional covariances in (4) and (5) need not be constant. One disadvantage of these alternative approaches is that they require a measure of aggregate consumption. Hansen and Hodrick were able to test their models using biweekly data and without having to measure consumption. Moreover, they implicitly allowed for considerable heterogeneity across investors. To use monthly observations on aggregate consumption, it must be assumed that investors have monthly decision intervals and that they are sufficiently alike for tests to be based on aggregate consumption.

If the parameters characterizing the preference function of a repre-



sentative investor are specified a priori, then  $Q_{m,t+k}(S_{t+k}^j - F_{t,k}^j)$  is observed by the econometrician at date  $t+k$ , and can be used as the dependent variable in a least-squares projection. An implication of equation (1) is that the coefficients in the projection of  $Q_{m,t+k}(S_{t+k}^j - F_{t,k}^j)$  onto a vector of variables in agents' information sets at date  $t$  are zero. This hypothesis can be tested using procedures that account for serial correlation in the projection error when  $k > 1$  and conditional heteroscedasticity, just as Hansen and Hodrick accounted for these econometric difficulties in their analyses.

The exchange rate relation (1) can also be tested without having to specify the values of parameters characterizing the preference function a priori. Suppose  $Q_{m,t+k}$  depends on a  $k \times 1$  vector of parameters  $\gamma_0$ ,  $Q_{m,t+k}(\gamma_0)$ , that are unknown to the econometrician. Also, let  $z_t$  be an  $r \times 1$  vector of variables in  $I_t$  that are observed by the econometrician, with  $r > k$ . Then (1) implies that

$$(9) \quad E[Q_{m,t+k}(\gamma_0)(S_{t+k}^j - F_{t,k}^j)z_t] = 0,$$

where  $E[\cdot]$  is the unconditional expectation. Equation (9) represents  $r$  nonlinear equations in  $k$  unknowns. Hansen and Singleton (1982a) describe how the sample counterparts of these orthogonality conditions can be used to obtain consistent and asymptotically normal estimates of  $\gamma_0$  under fairly weak regularity conditions. Their procedures turn out to set  $k$  linear combinations of the sample orthogonality conditions equal to zero in estimation. Hence, there are  $r - k$  independent linear combinations that were not used in estimation, but should be close to zero if the restrictions (9) are valid. These  $r - k$  conditions can form the basis of a test of the model (1), using a chi-square statistic with  $r - k$  degrees of freedom.

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