SAMPLING INFORMATION GOODS: HOW MUCH SHOULD BE FREE?*

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Abstract

This paper examines optimal sampling and pricing of information goods for firms that generate revenues from both sales and advertising. We develop a model where the demand for paid information goods is influenced by the sample portion that the firm offers for free. Taking into account the consumers' initial valuation and experienced quality of the free version, we characterize the firm's optimal sampling and pricing decisions. We find that the effect of advertising revenues on optimal sample portion and price depends on expected and experienced quality. Moreover, a firm should never offer a free sample when the expected quality exceeds the experienced quality. Our model assumptions and results are generally consistent with an empirical analysis of firm data on the market for news. We compare the models' implications with managerial decisions and show that managers have a tendency to offer too small samples when consumers use them extensively in updating their expectations about product quality.

Keywords: Information Goods, Sampling, Pricing, Advertising, Quality Expectations, Quality Experience, Quality Updating

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1 Introduction

One of the major impacts of the Internet has been the vast increase in the amount of information available for free. The provision of free information, or more specifically how much information to give away for free as a viable business model, has led to a debate among observers as well as managers. For example, over the past few years publications such as the *Wall Street Journal* have gone from having primarily paid versions to providing a substantial amount for free and recently back to more of a "pay for information" model. A firm producing information goods has two principle sources of revenues: advertising and sales. In some cases, the firm generates revenues solely from selling information goods, while in other cases firms offer their content for free and receive advertising revenues. Quite often, however, a free content sample and a paid (complete) version is offered. For instance, the *Wall Street Journal* offers news articles for free combined with advertising on its website *wsj.com*. Background information, dossiers and archived articles, by contrast, are in most cases sold for a fee. The purpose of this paper is to explore of exactly how much content should be offered for free and at what price the paid version should be sold.

Existing literature addresses properties of information goods that make pricing decision complex. Part of the complexity results from the fact that they are experience goods (Nelson 1970; Darby and Karni 1973) and consumers learn about the quality by using the goods themselves (Shapiro 1983). The characteristics of experience goods also lead to the information paradox (Akerlof 1970): While consumers have to experience an information good in order to value it, after they have experienced it, they have less or no incentive to purchase the product. Taking this specific characteristic of information goods into account, the literature has suggested many forms of differential pricing. Quality discrimination or versioning, offering a product line with vertically differentiated quality levels, has been analyzed and discussed by several authors (Varian 2000a; Bhargava and Choudhary 2001, 2008) and is recognized as a critical business strategy for information goods (Shapiro and Varian 1998; Wei et al. 2007). Among others, Varian (2000a) analyzes the strategy of vertical differentiation and shows how a supplier can maximize profits by offering a low- and a high-quality version of the same good at different prices.

While pricing strategies are well understood, the literature on information goods on sampling is sparse. A notable exception is Boom (2009), who shows via a quality signalling model that the firm should offer a free sample if the number of informed consumers is small. Although the literature has suggested several forms of "sampling" such as previews and browsing (Varian 2000b), the question of

Table 1: The key dimensions of information goods.

	Single Use	*Multiple Use*
One Time / Single Edition	Event Programming, Mystery Movies	Reference Books, Music
Renewed	Newspapers, TV Drama Series	“How To Do It” Magazines

how much information to offer as a free sample has been ignored.

In contrast, sampling is extensively discussed in the marketing literature on consumer and durable goods. Free samples are considered one of the most effective ways to introduce consumers to try a new product (Lawrence and Kamins 1988; Marks and Kamins 1988; Jain et al. 1995) and reduce their uncertainty about its quality (McGuiness et al. 1992). Research has examined which sample size (Heiman et al. 2001; Bawa and Shoemaker 2004) and trial rates (Meyer 1982; McGuiness et al. 1992) optimize demand for a product. Jain et al. (1995) show that a high sampling level is only appropriate if a durable product has a high coefficient of imitation or the firm has a high discount rate or a large gross margin. On the other hand, heavy sampling makes little sense if a durable product has a high coefficient of innovation. Heiman et al. (2001) show that although sampling effort should decline over a product’s life cycle, it may continue for mature products. For both consumer and durable goods, product sampling is expensive due to variable costs. The cost characteristics of information goods (especially digital information goods), high fixed costs and low variable costs, are quite different.

This paper contributes to these two strands of literature and addresses two related marketing questions: First, what portion of a information good should be offered as a free sample? Second, how much should a firm charge for the paid (complete) version? To set the stage, we classify information goods along two key dimensions. These goods can be either “perishable” or “durable”, depending on whether their value declines once it is consumed or whether their value holds up over time. Information goods can also be “one time” events (e.g., a particular book or movie) or renewed (e.g., newspapers, magazines, or TV series). These four combinations are illustrated in Table 1. The focus of this paper is on single use, single edition information goods.

We model the firm’s choice of the price and sample portion of an information good. An interesting feature of the model is that the firm generates revenues from two sources: the price charged for the paid version and advertising revenues gained from the free version. We start by modeling consumers’ utility and derive the demand for the paid product and for the free sample. Consumers either sample a portion of the product for free or buy the paid version. Consumers who sample the free version update their quality expectation. If the updated expectation are high enough, they might purchase the product whereas absent the sample they would not have bought. We characterize optimal price and sample portion, and also examine two extensions of the model which allow for a more general updating rule and for the firm to receive advertising revenues from the paid version as well.

Several key insights emerge from the model. For example, when advertising revenues per consumer increase, the effect on the sample portion depends on consumers' experienced quality: If consumers' experienced quality noticeably exceeds expected quality, the firm should offer a larger sample. In contrast, if consumers' experienced quality is close to expected quality, the firm should offer a smaller sample portion. Further, when consumers have higher quality expectations, the firm should offer a larger sample portion when expected quality and experienced quality are similar and a smaller sample portion if experienced quality exceed expected quality. Importantly, a firm should never offer a free sample when the expected quality exceeds the experienced quality. We provide further evidence that supports some of our assumptions and results. Using a data set of news websites, we find that a higher demand for the free sample goes along with a higher demand for the paid version. Importantly, when we compare the model with managers' decisions, we find that decision makers have a tendency to provide too small sample portions when consumers highly weight experienced quality.

The paper proceeds as follows. Section 2 introduces the model and details the consumers' quality updating upon sample experience. Section 3 derives consumers' demand for the free and the paid version, which consists of initial and sampling-induced demand. Section 4 studies firm behavior, characterizes the optimal sampling and pricing policy, and derives comparatives statics properties. The section concludes with a discussion of optimal strategies. Section 5 extends the model by allowing for more general updating of consumers' quality experience and including advertising revenues from the paid version. Section 6 provides relevant empirical evidence from news websites and a managerial study and Section 7 discusses conclusions and directions for future research.

2 Model

We develop an analytical model for a single profit-maximizing firm producing a single information good. To produce the first copy of the information good the firm incurs fixed costs $F \geq 0$. The marginal cost of creating a copy of the information good or of providing access to it is negligible and assumed being zero (Varian 2000b). Furthermore, we assume that the firm can modify and sample the information good at zero cost.

The firm offers simultaneously a full version of an information good at price p and a free sample of the content α. The sample may consist, for example, of an abstract (summary) or a portion of the information good (teaser). We normalize the content of the information good to unity and define $\alpha \in (0,1)$ as the portion of the content contained in the free sample. In its simplest form, α would be the percent of the total information. However, in the case of a "summary", a small sample can provide more content than its proportion or, in the case of a "teaser", less than that proportion.

We consider a market with a mass of N consumers and normalize N to unity without loss of generality. Consumers have an initial valuation (capturing quality expectations) v for the information good that varies across consumers. Following Jain and Kannan (2001), we assume that v is uniformly distributed on the interval $[0, \bar{v}]$.[1] If a firm offers only a paid version, consumers will purchase the information good if their basic valuation exceeds the price ($v > p$). However, when a free sample is offered, this allows consumers to evaluate product quality before they decide whether to purchase the paid version. Thus consumers have two options, either to purchase the product at price p or to take the free sample portion α. Consumers' utility U^I from these two options is given by

$$U^I = \begin{cases} v - p & \text{if the consumer buys the paid version at price } p \\ \alpha v & \text{if the consumer takes the free sample portion } \alpha. \end{cases}$$

Thus, a consumer will purchase the paid version if $v - p > \alpha v$ and take the free version otherwise. This condition can be rewritten as $v(1-\alpha) > p$, which means that the value offered beyond the free sample has to exceed the price for the full version. Importantly, we assume here that consumers are able to observe the size of sample offered for free *before* they eventually learn about sample quality.

Consumers who chose to take the free sample update their initial valuations. We follow Kopalle and Lehmann (2006) and assume the updating rule is given as

$$v' = \alpha q + (1-\alpha)v, \tag{1}$$

[1] In the spirit of Mussa and Rosen (1978), we can write $v = \theta\bar{v}$, where $\theta \sim U(0,1)$ is the willingness to pay for quality and $\bar{v}$ is a real-valued index describing intrinsic product quality.

where v' is the consumer's updated valuation given experienced quality q, which we assume to be uniformly distributed on the interval $[0, \bar{q}]$. We assume that v and q are independent random variables, i.e. that prior expectation and experienced quality are uncorrelated.[2] This specification features two intuitive properties: First, consumers' puts more weight on experienced quality q and less on initial valuation v the larger the sample portion is. Second, consumers' updated quality expectation v' explicitly depends on q with the extent of the impact determined by the sample portion α.

After updating, consumers have two options: either purchase the product given sample experience at price p or stay with the free sample. Consumers' utility U^S from these two options is given by

$$U^S = \begin{cases} v' - p & \text{if the consumer buys the paid version at price } p \\ \alpha v' & \text{if the consumer stays with the free sample } \alpha. \end{cases}$$

The variation in initial valuation and quality experience are exogenous to the firm. We assume that the firm has private information about the distributions of $\bar{v}$ and $\bar{q}$. For instance, the firm can learn about such information employing standard market research techniques such as surveys. We assume further that the firm generates revenues from two sources. First, it receives advertising revenues a_f per consumer who take (e.g. download) the free sample.[3] Second, it receives revenues from selling the paid version. We assume that a_f is exogenously given and constant. This price-taking behavior is justified in markets in which the CPM (cost per thousand customers) is the outcome of competitive interaction.[4] In such an environment, the firm makes two decisions: the price p to charge for the paid version and the portion α of the good to provide as a free sample.

3 Consumer Demand

The section derives the demand for the paid version and the demand for the free sample as a function of the price p and the sample portion α. Demand for the paid version stems from two groups of buyers. The first group consists of those consumers who buy the product without taking the free sample, and

[2]Alternatively, one could argue either that those with high expectations tend to confirm this (positive correlation) or that those with the highest expectations are the most critical and hence tend to be disappointed (negative correlation). Similarly those with low expectations may be accurate in those expectations or pleasantly surprised by actual quality.

[3]In Section 5, we extend our model and consider also advertisement revenues from the paid version

[4]Godes et al. (2009) model advertising revenues as an outcome of two-sided competition. While endogenizing the price of advertising yields interesting insights when studying optimal bundling of content and advertisements, we assume that the provider of the information good has no market power on the advertising market.

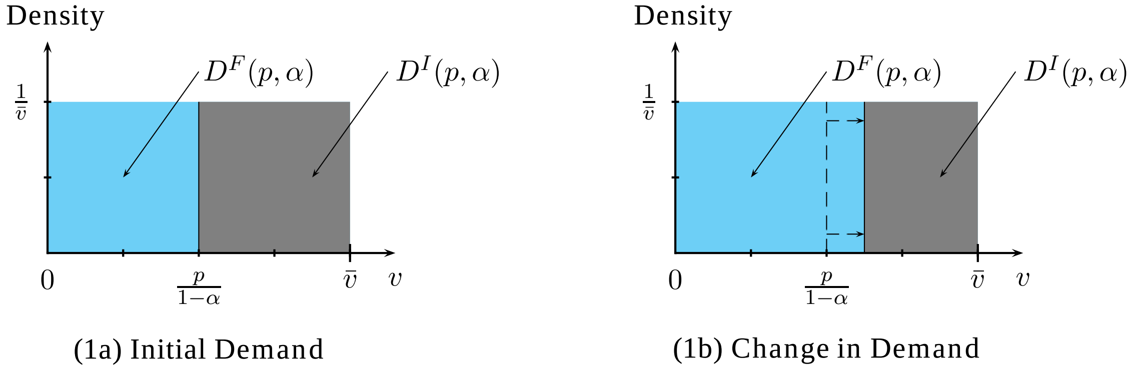


Figure 1: Initial demand and the demand for the free sample.

the second group consists of those who buy the product after first experiencing the sample. We refer to these components of demand for the paid version, respectively, as "initial demand" and "sampling induced demand".

3.1 Initial Demand

Given an initial valuation v, a consumer buys the paid version at price p if and only if she enjoys a surplus $v-p$ that exceeds the value obtained by choosing the free sample αv. Initial demand is thus given by

$$D^I(p,\alpha) \equiv \Pr\left\{v > \frac{p}{1-\alpha}\right\} = 1 - \frac{p}{(1-\alpha)\bar{v}}.$$

We assume that initial demand is strictly positive. Put differently, this assumption requires that the consumers with the highest valuations in the market buy the product without taking the free sample, which happens if $\bar{v} > p/(1-\alpha)$. Note that the firm loses consumers that might initially buy when either the price or the sampling portion are increased, as these consumers become more inclined to take the free sample.

The demand for the free samples then becomes the number of consumers minus those who choose to buy the product initially:

$$D^F(p,\alpha) = 1 - D^I(p,\alpha) = \frac{p}{(1-\alpha)\bar{v}}.$$

Offering a free sample increases the demand for the free version per se, thereby reducing the demand for the paid version. Figure 1 illustrates the respective demands (panel a) and shows that the demand for the free sample is increasing in both sample size α and price p (panel b). Further note that initial demand is increasing in $\bar{v}$ and therefore that the demand for the free sample is decreasing in $\bar{v}$ by construction. It is important to note that the firm's choice of p and α separates consumers into two groups: a group of consumers that initially buy and a group of consumers that take the free sample. The demand of the latter group determines advertising revenue. To obtained the demand and hence revenues for the paid version, we have to take into account those consumers who buy after experiencing the sample. The next section addresses this and derives sampling-induced demand.

3.2 Sampling-Induced Demand

This section derives the sampling-induced demand that stems from those consumers who buy the paid version after experiencing the quality of the sample portion α. The probability that a consumer buys at price p crucially depends on $\bar{q}$. Depending on the value of $\bar{q}$, two cases may arise. The first case is where all consumers may buy. The second case is where low-valuation consumers never buy.

3.2.1 Consumers' Updating of Quality Expectations

Importantly, only consumers who take the free sample can learn about product quality and update their initial valuation. Given sample experience, a consumer buys if and only if his or her surplus $v' - p$ from buying given revised expectations exceeds the value $\alpha v'$ derived from staying with the free sample. Using the updating rule given in Eq. (1), the probability of buying after experiencing product quality can be expressed as

$$\Pr\left\{\alpha q + (1-\alpha)v \geq \frac{p}{1-\alpha}\right\}.$$

We follow Buehler and Halbheer (2009) to compute this probability and note that the joint density of the bivariate uniform distribution of (v, q) on the rectangular type space

$$T \equiv [0, \frac{p}{1-\alpha}] \times [0, \bar{q}]$$

is given by

$$f(v,q) = \begin{cases} \frac{1}{\bar{v}\bar{q}} & \textit{if } (v,q) \in T \\ 0 & \textit{otherwise,} \end{cases}$$

where the definition of T follows by noting that only consumers with valuation $v < p/(1-\alpha)$ take the free sample. With this in mind, we can express the probability of buying given sample experience in more compact notation as

$$\Pr\left\{\alpha q + (1-\alpha)v \geq \frac{p}{1-\alpha}\right\} = \frac{1}{\bar{v}\bar{q}} \int_0^{\frac{p}{1-\alpha}} \int_0^{\bar{q}} \mathbf{1}_{\{\alpha q+(1-\alpha)v \geq \frac{p}{1-\alpha}\}} dq dv,$$

where $\mathbf{1}_A$ is the indicator function taking value 1 on the set A (which is here the set of consumers that buy given sample experience) and 0 otherwise. We show in the next section that this probability crucially depends upon the experienced quality $\bar{q}$.[5]

3.2.2 Sampling as a Persuasion Device

We assume that sampling persuades some of the consumers with the highest post sample experience and hence valuations to buy. Given sample experience, consumers buy if the following condition holds.

Condition 1. *Consumers with the highest valuations buy the product only if they experience a sufficiently high sample quality, that is, if* $\bar{q} > p/(1-\alpha) > 0$.

To intuitively grasp the meaning of this condition, it is useful define the set of consumers that buy the product given sample experience as

$$\left\{(v,q) \in T \mid \alpha q + (1-\alpha)v \geq \frac{p}{1-\alpha}\right\}.$$

The boundary of this set defines the set of indifferent consumers and hence an "indifference curve", which can be written as

$$q = f(v) = \frac{p}{\alpha(1-\alpha)} - \frac{(1-\alpha)v}{\alpha},$$

implying

$$f\left(\frac{p}{1-\alpha}\right) = \frac{p}{1-\alpha}.$$

This condition ensures that the consumers with the highest valuation (among those who take the free sample) buy the product if they experience a sufficiently high sample quality. A graphical illustration of this condition is given in Figure 2. Also, the figure visualizes that all consumers may buy if $\bar{q}$ is

[5] Buonocore et al. (2009) provide a closed-form solution for the convolution integral. However, the derived distribution function also depends on specific parameter values.

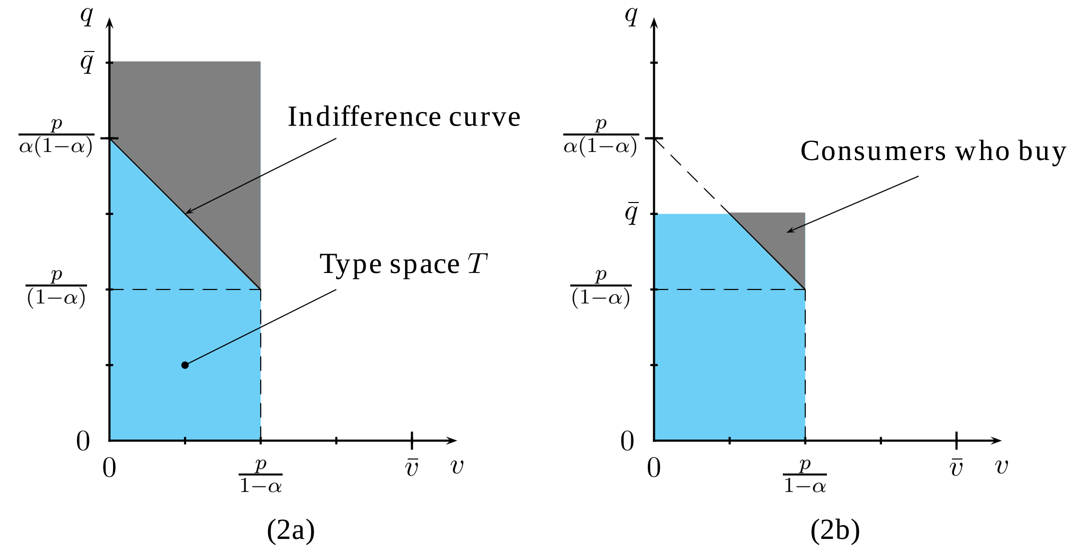


Figure 2: Consumers who buy after sampling.

sufficiently large (dark shaded area in Panel 2a) and that low-valuation consumers never buy when $\bar{q}$ is relatively small (Panel 2b).

Case I: All Consumers May Buy

Even the consumers with the lowest valuations may buy the product given sample experience if the following condition holds.

Condition 2a. *Consumer with the lowest valuation in the market can be persuaded to buy the product if they experience a high enough sample quality, that is, if $\bar{q} > p/(\alpha(1-\alpha))$.*

This condition is best understood by inspection of the indifference curve drawn in Figure 2a. The condition requires that the indifference curve hits the vertical axis below $\bar{q}$, which is equivalent to assuming

$$f(0) = \frac{p}{\alpha(1-\alpha)} < \bar{q}.$$

The next lemma gives the demand induced by letting consumers evaluate product quality.

Lemma 1. *When all consumers may buy, the firm faces sampling-induced demand*

$$D_1^S(p,\alpha) = D^F(p,\alpha) - \frac{(1+\alpha)\,p^2}{2\alpha\,(1-\alpha)^2\,\bar{v}\bar{q}}.$$

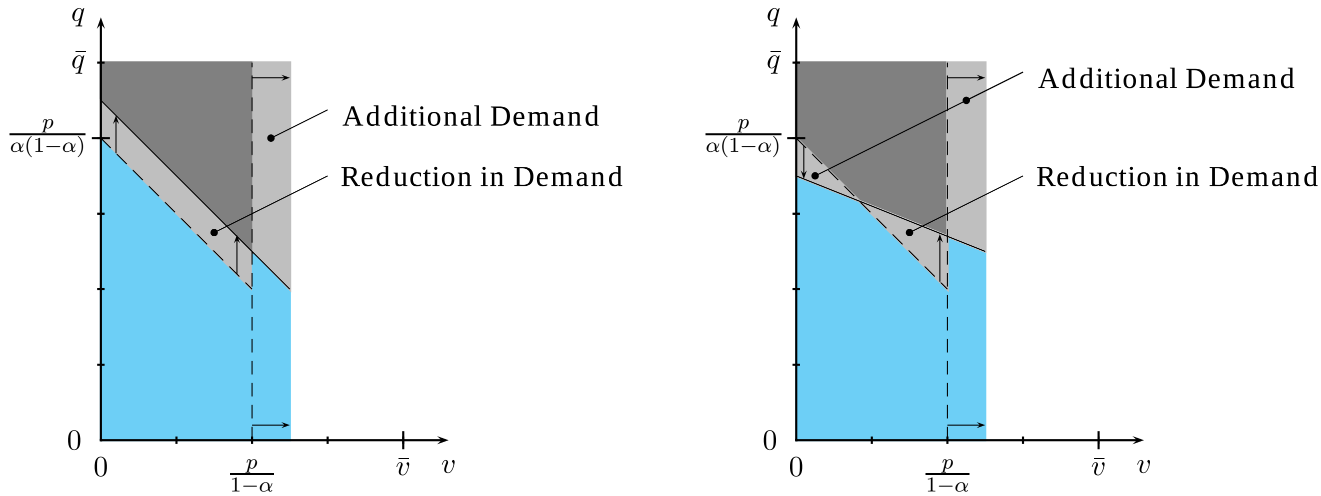


Figure 3: The change in sampling induced demand due to an increase in price p (panel 3a) and an increase in the sample portion α (panel 3b).

Proof. See Appendix A.[6] □

Lemma 1 shows that the sampling-induced demand is equal to the demand for the free sample less the number of consumers that do not buy given sample experience. To understand the effects of a price change in sampling-induced demand, observe that increasing the price induces consumers to switch from the paid to the free version. This in turn increases sampling-induced demand. At the same time, the price increase induces some consumers who bought at the lower price to refrain from buying, which leads to a reduction in sampling-induced demand. Which of these two countervailing effects dominates depends on the price level before the price change: At low initial prices the demand-enhancing effect dominates, whereas at high initial prices the demand-reducing effect does (see Figure 3a). Intuitively, the demand-enhancing effect dominates at low prices because most consumers initially buy, so an increase in price does not lead to a large loss of consumers that buy upon sample experience. At high prices, however, the demand-reducing effect dominates because most consumers buy given sample experience, so increasing the price leads to a large reduction in sampling-induced demand.[7] Increasing the sample portion also has a countervailing effect on sampling-induced demand, where the sign depends on the size of the sample portion before the price change (see Figure 3b). In sum, increasing the sample portion has a demand-enhancing effect when the sample size is initially

[6] All proofs can be found in Appendix A.

[7] To see this geometrically, observe that the area of the parallelogram is smaller than the area of the trapezoid at low prices, and vice versa at high prices (see again Figure 3a).

small and a demand-reducing effect when the sample portion is initially large.

We now derive the firm's (total) demand for the paid version. This demand function is easily obtained by adding initial and sampling-induced demand, and the comparative statics properties immediately follow.

Lemma 2. *When all consumers may buy, the firm's total demand for the paid version is*

$$\begin{aligned} D_1^P(p,\alpha) &\equiv D^I(p,\alpha)+D_1^S(p,\alpha) \\ &= 1-\frac{(1+\alpha)\,p^2}{2\alpha\,(1-\alpha)^2\,\bar{v}\bar{q}}. \end{aligned}$$

Total demand is (i) decreasing in price p, implying that potential demand-enhancing effects via increases in sample-induced demand are always dominated by the reduction in initial demand; (ii) increasing (decreasing) in the sample portion α when the sample portion is initially small (large); (iii) increasing in valuation $\bar{v}$; and (iv) increasing in quality experience $\bar{q}$.

Case II: Low-Valuation Consumers Never Buy

The consumers with the lowest valuations never buy the product given sample experience if the following condition holds.

Condition 2b. *Consumers with the lowest valuation in the market cannot be persuaded to buy the product even if they experience the highest possible sample quality, that is, if $\bar{q} < p/(\alpha(1-\alpha))$.*

This is visualized by inspection of the indifference curve drawn in Figure 2b. In contrast to the previous case, we now require that the indifference curve hits the vertical axis above $\bar{q}$, implying consumers with $v = 0$ do not buy.

The next lemma describes the demand generated by letting consumers evaluate product quality via a sample.

Lemma 3. *When low-valuation consumers never buy, sampling induced demand is given by*

$$D_2^S(p,\alpha)=\frac{\alpha((1-\alpha)\bar{q}-p)^2}{2(1-\alpha)^3\bar{v}\bar{q}}.$$

Lemma 3 shows that sampling-induced demand is decreasing in price. Since the firm offers a large sample portion, only consumers with high valuations will buy. Given the large sample portion, however, the demand-reducing effect of a price increase dominates the demand-enhancing effect that results from an increase in the number of consumers that take the free sample.[8] Again, sampling-induced demand can either be increasing or decreasing due to a change in the sample portion, depending on how large the sample portion initially is. Increasing the sample portion has a demand-enhancing effect when the sample size is initially small and a demand-reducing effect when it is initially large.

The next lemma describes the firm's total demand for the paid version and its comparative statics properties.

Lemma 4. *When low-valuation consumers never buy, the firm's (total) demand for the paid version is*

$$\begin{aligned} D_2^P(p,\alpha) &\equiv D^I(p,\alpha) + D^S(p,\alpha) \\ &= 1 - \frac{p}{(1-\alpha)\bar{v}} + \frac{\alpha((1-\alpha)\bar{q}-p)^2}{2(1-\alpha)^3\bar{v}\bar{q}}. \end{aligned}$$

Total demand is (i) decreasing in price p; (ii) increasing (decreasing) in the sample portion α when the sample portion is initially small (large); (iii) increasing in valuation $\bar{v}$; and (iv) increasing in quality experience $\bar{q}$.

To sum up, it is useful to note that the demand functions for the paid versions have the same comparative statics properties in both cases. We now shift focus to firm decisions.

4 Firm Decisions

This section examines optimal firm decisions. Depending on consumers' experienced quality, the firm potentially derives sampling-induced demand from either all consumers or high-valuation consumers only. Thus, the firm essentially has two strategies available: either to de facto target all consumers or to target high-valuation consumers only.

We proceed as follows: First, we characterize the firm's optimal pricing and sampling decision. Second, we analyze the "Targeting All Consumers"-strategy (Case I) and the "Targeting High-Valuation Consumers"-strategy (Case II). Finally, we discuss the conditions under which the firm should adopt either one of the two strategies.

[8] To see this geometrically, inspection of Figure 3a is useful: Since the area of the parallelogram is larger than the area of the trapezoid at high prices in Case I, this is a fortiori so in Case II (where prices are even higher for a given sample size).

4.1 Optimal Pricing and Sampling

The firm's decision variables price p and sample portion α are modeled as the outcome of a two-stage decision problem. We suppose that the decision about the sample portion precedes the pricing decision.[9] We therefore first determine the price for a given sample portion by maximizing the *product market profit*

$$\pi_i(p_i, \alpha_i) = pD_i^P(p_i, \alpha_i) + a_f D^F(p_i, \alpha_i), \quad i = 1, 2. \tag{2}$$

The product market profit is the sum of revenues from selling the paid version and advertising revenues, respectively.[10] Subscript i refers to the two cases and we let (p_i, α_i) denote the corresponding decisions. The solution to the first-order condition with respect to p

$$D_i^P(p_i, \alpha_i) + p_i \frac{\partial D_i^P(p_i, \alpha_i)}{\partial p_i} + a_f \frac{\partial D^F(p_i, \alpha_i)}{\partial p_i} = 0$$

yields the best-response function $p_i(\alpha_i)$, which indicates how the price p_i varies with sample size. Substituting $p_i(\alpha_i)$ into the product market profit function yields $\pi_i(\alpha_i)$, the product market profit in terms of α_i. The firm derives the optimal sample portion by maximizing its *total profit*

$$\Pi_i(\alpha) = \pi_i(\alpha) - F$$

where F denotes the fixed costs to produce the information good.[11] Making the dependence on model parameters explicit, we let profit-maximizing choice $\alpha_i^* = \alpha_i\left(a_f, \bar{v}, \bar{q}\right)$ denote the optimal sample portion, with the corresponding optimal price being $p_i^*\left(a_f, \bar{v}, \bar{q}\right) = p_i\left(\alpha_i(a_f, \bar{v}, \bar{q}), a_f, \bar{v}, \bar{q}\right)$.

To understand how changes in parameter values affect the optimal price, we decompose the price change into a *sampling-mediated effect* and a *direct effect*. To make this more explicit, let the generic variable x stand for one of the model parameters a_f, $\bar{v}$, or $\bar{q}$. Then, the change in price due to an exogenous variation of x is given by

$$\frac{dp_i^*}{dx} = \frac{\partial p_i\left(\alpha_i^*, x\right)}{\partial \alpha_i} \frac{d\alpha_i}{dx} + \frac{\partial p_i\left(\alpha_i^*, x\right)}{\partial x},$$

where the first term on the right-hand side is the sampling-mediated effect and the second term is the

[9] However, the solution to the problem would be the same when the decisions are made simultaneously as the decision has no external effects. We merely use this assumption for convenience.

[10] Note that the product market profit is equal to revenue under the assumption of zero unit production costs.

[11] We assume that the fixed cost do not exceed the product market profit. Hence, they do not change the analysis and can therefore be omitted.

direct effect. Importantly, the sign of the sampling-mediated effect depends on the slope of the best-response function evaluated at the optimal sample portion. We now proceed to characterize optimal sampling and pricing for our two targeting strategies.

4.2 Case I: Targeting All Consumers

When all consumers may buy, the firm's product market profit is

$$\pi_1(p_1, \alpha_1) = p_1 \left(1 - \frac{(1+\alpha_1)\, p_1^2}{2\alpha_1 (1-\alpha_1)^2\, \bar{v}\bar{q}}\right) + a_f \left(\frac{p_1}{(1-\alpha_1)\,\bar{v}}\right).$$

The best-response function for price is given by

$$p_1(\alpha_1) = \sqrt{\frac{2\alpha_1 (1-\alpha_1)\left(\bar{v}(1-\alpha_1) + a_f\right)\bar{q}}{3(1+\alpha_1)}}, \tag{3}$$

and the reduced-form profit function is

$$\Pi_1(\alpha_1) = \sqrt{\frac{8\alpha_1 \left(\bar{v}(1-\alpha_1) + a_f\right)^3 \bar{q}}{27(1-\alpha_1)(1+\alpha_1)\bar{v}^2}} - F. \tag{4}$$

The next result characterizes the optimal sample portion and comparative statics properties thereof.

Proposition 1. *When targeting all consumers, the optimal sample portion is*

$$\alpha_1^*(a_f, \bar{v}) = \sqrt{\frac{24\bar{v}^2 + (\bar{v}+a_f)^2}{9\bar{v}^2}} \cos\left(\frac{\pi+\varphi}{3}\right) - \frac{(\bar{v}+a_f)}{6\bar{v}}, \tag{5}$$

where

$$\varphi = \arccos\left(\frac{(\bar{v}+a_f)\left(90\bar{v}^2 + (\bar{v}+a_f)^2\right)}{\left(24\bar{v}^2 + (\bar{v}+a_f)^2\right)^{\frac{3}{2}}}\right).$$

The optimal sample portion is (i) increasing in a_f, (ii) decreasing in $\bar{v}$, and (iii) independent of $\bar{q}$.

The comparative statics results have intuitive explanations. First, when a_f increases, the firm has an incentive to offer a larger sample portion to increase the demand for the free version. While the increase in the sample portion may reduce the revenues from the paid version, the increase in advertising revenues overcompensates the potential loss. Second, an increase in $\bar{v}$ leads to higher initial demand. To further increase revenues from the paid version, the firm has an incentive to offer a smaller sample portion. Third, and less intuitive, increases of $\bar{q}$ affect neither initial demand nor the demand for the free sample. Therefore, the firm has no incentive to adjust the size of the sample portion when the sampling-induced demand increases.

To study how changes in parameter values affect the optimal price, we use the slope of the best-response function evaluated at the optimal sample portion to determine the sign of the sampling-mediated effect.

Claim 1. *Offering a larger sample portion goes along with a lower price for the paid version of the product, since the slope of the best-response function is negative when evaluated at the optimal sample portion α_1^*, that is, $dp_1(\alpha_1)/d\alpha_1|_{\alpha_1=\alpha_1^*} < 0$.*

Inspection of the response function shows that the sign of all direct effects is positive. Adding sampling-mediated and direct effects, we can compute the total effect on price.

Proposition 2. *When targeting all consumers, the optimal price $p_1^*(a_f, \bar{v}, \bar{q})$ is (i) increasing in a_f if $0 < a_f/\bar{v} < 0.35$ and decreasing in a_f if $0.35 < a_f/\bar{v} < 0.45$, (ii) increasing $\bar{v}$, and (iii) increasing in $\bar{q}$.*

The ambiguity of the first comparative statics result stems from countervailing sampling-mediated and direct effects. The direct effect is positive since the firm has an incentive to induce the consumers to take the free sample. The sampling-mediated effect is negative as a higher a_f leads to a larger sample portion, which in turn decreases the price. Which of the two effects dominates depends on the initial level of advertising revenues: If a_f is low, the firm should shift demand to the free version whereas the firm should not do so if a_f is high. If the ratio of $a_f/\bar{v}$ exceeds 0.45, the optimal sample portion will be a boundary solution, that is either equal to 0 (no sample) or equal to 1 (no paid version). However, it is unlikely that corner solutions occur as we can predict an optimal sample portion for cases for advertising revenues as high as 45% of the highest valuation $\bar{v}$. Second, as a larger $\bar{v}$ calls for a smaller sample portion, the sampling-mediated effect is positive, which further reinforces the positive direct effect. Therefore, the firm should increase its price to skim consumers' higher valuations and accept lower sales from the paid version. Third, as changes in $\bar{q}$ do not affect the optimal sample portion, there is only a (positive) direct effect of $\bar{q}$ on price which leads the firm to increase its price to benefit from higher advertising revenues from the free sample.

4.3 Case II: Targeting High-Valuation Consumers

When low-valuation consumers never buy, the firm's product market profit is

$$\pi_2(p_2, \alpha_2) = p_2 \left(1 - \frac{p_2}{(1-\alpha_2)\bar{v}} - \frac{\alpha_2(p_2 - (1-\alpha_2)\bar{q})^2}{2(1-\alpha_2)^3\bar{v}\bar{q}}\right) + a_f \left(\frac{p_2}{(1-\alpha_2)\,\bar{v}}\right),$$

and the best-response function is given by

$$p_2(\alpha_2) = \frac{1-\alpha_2}{3\alpha_2}\left(2\bar{q} + \sqrt{\left((4-3\alpha_2^2)\bar{q} - 6\alpha_2((1-\alpha_2)\bar{v} + a_f)\right)\bar{q}}\right). \tag{6}$$

Proposition 3. *When targeting high-valuation consumers, the optimal sample portion is characterized by*

$$\alpha_2^*(a_f, \bar{v}, \bar{q}) = \frac{2\sqrt{\bar{q}(2\bar{v}-\bar{q})} - (a_f + \bar{v})}{(2\bar{v}-\bar{q})}. \tag{7}$$

If $\bar{q} < 2\bar{v}$, the optimal sample portion is (i) decreasing in a_f, (ii) increasing in $\bar{v}$, and (iii) increasing in $\bar{q}$.

We now provide an intuitive explanation for these comparative statics results.[12] First, an increase in a_f leads to higher advertising revenues. However, the firm has an incentive to shift demand from the free to the paid version, since it offers a large sample portion when targeting high-valuation consumers. Second, an increase in $\bar{v}$ leads to a reduction in the demand for the free sample since more consumers choose to buy initially. To compensate for the corresponding loss in revenue, the firm has an incentive to offer a larger sample portion. Third, a larger experienced quality $\bar{q}$ leads to a greater demand for the paid version, which gives the firm an incentive to increase α and thereby the demand for the free sample.

Similar to the targeting-all-consumers strategy, there is an inverse relationship between the price of the paid version and the sample portion, as follows.

Claim 2. *When targeting high-valuation consumers, offering a larger sample portion goes along with a lower price for the paid version of the product, since the slope of the best-response function is negative when evaluated at the optimal sample portion α^*, that is, $dp_2(\alpha_2)/d\alpha_2|_{\alpha_2=\alpha_2^*} < 0$.*

The next proposition characterizes the optimal price and its comparative statics properties.

[12] We show in Section 4.4 that the restriction $\bar{q} < 2\bar{v}$ is always fulfilled under the conditions of Case II.

Proposition 4. *When targeting high-valuation consumers and $\bar{q} < 2\bar{v}$, the optimal price $p_2^*(a_f, \bar{v}, \bar{q})$ is (i) increasing in a_f, (ii) decreasing in $\bar{v}$, and (iii) increasing in $\bar{q}$.*

First, the comparative statics effect of a_f on price results from the positive sampling-mediated effect being dominated by the negative direct effect. To understand this at first glance counter-intuitive effect, note that the demand of the free version is large and the sampling-induced demand is small when targeting high-valuation consumers. As the firm already earn possible advertising revenues from the free sample, the firm has an incentive to increase its price to increase the revenues from initial and sample induced demand. Further, an increase in $\bar{v}$ leads to higher initial demand. Intuitively, the firm's incentive to lower the price stems from the fact that a reduction in price further increases initial demand (both the sampling-mediated and the direct effect are negative). Third, the direct effect of a higher $\bar{q}$ on price is positive, i.e. higher experienced quality results in a higher price. Although the sampling-mediated effect is negative, the firm has an incentive to increase its price to further increase its sampling-induced demand.

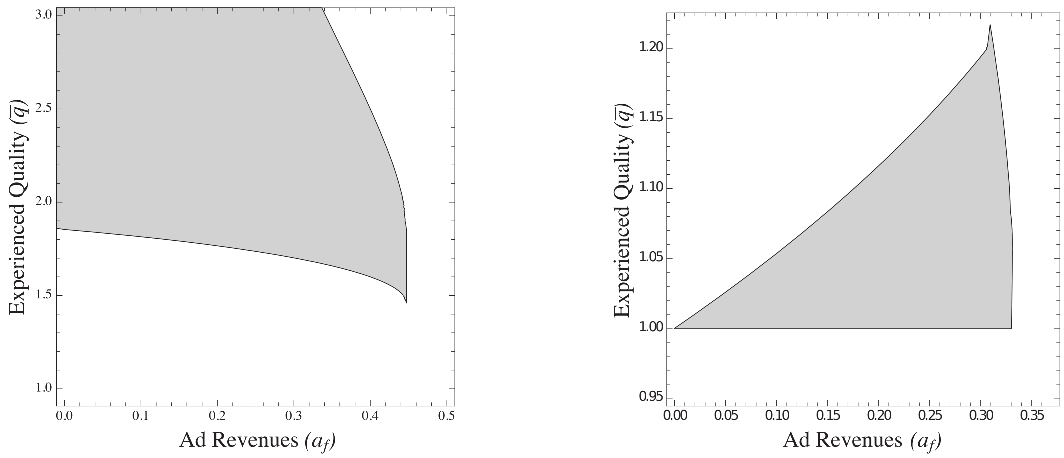
4.4 Optimal Strategy

We have shown how quality experience $\bar{q}$ determines whether the firm should focus on demand from all consumers or high-valuation consumers only for a given price p and sample portion α. In this section we show that the ratio of experienced to expected quality determines the firm's targeting strategy and that the firm should never offer a free sample when experienced quality is lower than expected quality.

The following lemma characterizes how optimal firm strategy is determined by the proportion of the two quality levels $\bar{v}$ and $\bar{q}$.

Lemma 5. *The optimal sample portion $\alpha_i^*(a_f, \bar{v}, \bar{q})$ is homogeneous of degree 0 in ($a_f, \bar{v}, \bar{q}$) and the optimal price $p_i^*(a_f, \bar{v}, \bar{q}) = p_i(\alpha_i^*(a_f, \bar{v}, \bar{q}); a_f, \bar{v}, \bar{q})$ is homogeneous of degree 1 in $(a_f, \bar{v}, \bar{q})$.*

The first statement allows us to scale parameters by any positive number—and thus specifically by a factor of $1/\bar{v}$—without changing the optimal sample portion, so that $\alpha_i^*(a_f, \bar{v}, \bar{q}) = \alpha_i^*(a_f/\bar{v}, 1, \bar{q}/\bar{v})$. We therefore reduce the dimension of the parameter space and study the set of admissible parameter constellations a_f and $\bar{q}$ measured in terms of $\bar{v}$ graphically. The second statement simply means that a given proportional change in parameter values leads to the same proportional change in the optimal price.



(4a) Case I

(4b) Case II

Figure 4: Parameter Combinations and Optimal Strategies.

With this in mind, we determine the parameter combinations that lead to targeting all consumers (Case I) vs. targeting high valuation consumers (Case II) as strategy, illustrated in Figure 4, which utilized the normalization $\bar{v} \equiv 1$. The two shaded areas define, respectively, the set of admissible parameter constellations for the targeting-all-consumers and the targeting-high-valuations-consumers strategies. The borders of these areas are defined by the conditions

$$\bar{q} \geq \frac{p_1^*}{1-\alpha_1^*}, \quad \bar{q} \geq \frac{p_1^*}{\alpha_1^*(1-\alpha_1^*)}, \quad \text{and} \quad \bar{v} \geq \frac{p_1^*}{1-\alpha_1^*} \tag{Case I}$$

and

$$\bar{q} \geq \frac{p_2^*}{1-\alpha_2^*}, \quad \bar{q} \leq \frac{p_2^*}{\alpha_2^*(1-\alpha_2^*)}, \quad \text{and} \quad \bar{v} \geq \frac{p_2^*}{1-\alpha_2^*}, \tag{Case II}$$

respectively, which are unaffected by proportional changes in parameter values.[13] Inspection of Figure 4 reveals that the ratio of $\bar{q}/\bar{v}$ (here reduced to just $\bar{q}$) determines optimal firm strategy, independent of the level of advertising revenues per consumer. The firm should target all consumers if experienced quality noticeably exceeds expected quality.[14] In contrast, the firm should target only high-valuation consumers if consumers' experienced quality is close to expected quality. The figure reveals further that the firm should never offer a free sample when expected quality exceeds experienced quality. To put this result into perspective, recall that the firm has private information about the distribution of

[13]The first restriction is derived from Condition 1, the second from Conditions 2a and 2b, and the third from our assumption that initial demand is positive, which is $\bar{v} > p/(1-\alpha)$.

[14]Put differently, this condition requires that average experienced quality exceeds average expected quality under our assumptions on the distribution of qualities.

expected and experienced quality and hence about $\bar{v}$ and $\bar{q}$. Given this knowledge, the firm can easily choose the appropriate sampling strategy.

More specifically, inspection of the figure shows that targeting-high-valuation-consumers strategy is appropriate if the consumers have "rational" expectations in the sense that the distributions of expected and experienced qualities coincide ($\bar{v} = \bar{q}$). However, if $\bar{q} > \bar{v}$, the optimal sampling strategy depends on the specific parameter constellations, as follows. The targeting-all-consumers strategy is an optimal strategy if and only if experienced quality exceeds experienced quality by a factor of at least 1.5. In contrast, the targeting-high-valuation-consumers strategy is appropriate if $\bar{q}$ is close to $\bar{v}$ and does not exceed it by a factor of more than 1.2. The next section presents a more general model which covers quality ratios between 1.2 and 1.5.

5 Model Extensions

We broaden our proposed model by relaxing and generalizing some of the assumptions regarding consumers updating behavior and firms' revenue streams. Specifically, we allow both updating of expectations to be more general and for a firm to receive advertising revenues from the paid version as well as the free sample. First, consumers who take a free sample of an information good may not update their initial valuation as a linear multiple of the sample portion as proposed in (1). To allow consumers to put a different weight on experienced quality based on the sample portion, we generalize the updating rule as follows:

$$v' = \alpha^b \, q + (1 - \alpha^b) \, v, \tag{8}$$

where $b > 0$ is the weight consumers place on experienced quality in updating their prior valuation v. When $b < 1$, consumers put more weight on the sample portion and thus on experienced quality, whereas if $b > 1$, consumers place less weight on experienced quality (as they might if the expected quality to be highly variable). Note that $b = 1$ represents a market where the weight on the experience with the free sample is equal to the portion of the value of the information good offered for free which was the initial assumption in (1).

Second, firms receive in many cases advertising revenues not only from the free sample but also from the paid version of the information good. To consider this second source of advertising revenues we extend firm's profit function (2) in the following way:

$$\pi_i(p_i, \alpha_i) = (p_i + a_p) D_i^P(p_i, \alpha_i) + a_f D^F(p_i, \alpha_i), \tag{9}$$

where $a_p > 0$ are the advertising revenues per consumer purchasing the paid version of the information good. Note this is consistent with the pricing of media advertising in which the costs per thousand readers are fixed and exogenously given.

These extended assumptions regarding consumers' updating of initial valuation and firms' profit function lead to the following product market profit functions:

$$\pi(p, \alpha) = (p + a_p)\left(1 - \frac{p^2\left(1 + a^b\right)}{2a^b(1-a)^2\bar{q}\bar{v}}\right) + a_f\left(\frac{p}{(1-a)\bar{v}}\right) \tag{Case I}$$

and

$$\pi(p, \alpha) = (p + a_p)\left(1 - \frac{p}{(1-a)\bar{v}} + \frac{a^b((1-a)\bar{q} - p)^2}{2\left(1 - a^b\right)(1-a)^2\bar{q}\bar{v}}\right) + a_f\left(\frac{p}{(1-a)\bar{v}}\right). \tag{Case II}$$

Given the complexity of this more general model, the optimization problem does not yield closed-form solutions. The model is therefore solved using numerical techniques.[15]

Sensitivity to the model parameters was estimated by a multivariate regression on the results of the numerical simulation with the optimal price p^* and free sample portion α^* as dependent variables (see Table B-1 in Appendix B). Overall the results show that changes in the values of the parameters $\bar{v}$, $\bar{q}$, and a_f have the same directional impact on the optimal price p^* and the free sample portion α^* as the comparative statics results for Case I and Case II. In addition, the sensitivity of α^* to a change in the advertising revenues from the paid version a_p changes α^* in the expected direction in both Case I and Case II. When a_p increases, a firm should decrease the free sample portion in both cases. However, the optimal price p^* in Case I is decreasing in a_p while it is increasing in Case II.

The sensitivity of the optimal price and free sample portion to changes in the *weight consumers place on the quality experience* b was not expected a priori and is less intuitive. The more consumer weight experienced quality (the smaller b), the smaller is the optimal free sample portion α^* in Case I and the larger in Case II. With respect to the optimal price p^*, the sign of the effects are exactly also the opposite. The larger consumers' weight on experienced quality, the larger is p^* in Case I and the smaller in Case II. When a company is confident that the quality of its new product exceeds (likely)

[15]See Appendix B for details.

prior expectations of its quality by a wide amount, it should have a smaller free sample since that is all that is needed to "get the word out" and convert triers into buyers. Similarly, the firm should charge a higher price to "reflect" the true value of the product. By contrast, when expectations are in line with (but slightly lower than) actual quality, the firm should decrease its price and give a free sample.

Importantly, the model extensions expands the range of admissible parameter constellations which lead to an optimal free sample portion α^* and a related optimal price p^*, and therefore help fill the gap of admissible parameter constellations in the model presented in the previous section.

6 Empirical Investigations

In order to illustrate and confirm our model assumptions as well as the results, we present the results of two empirical investigations. First, using data on German news websites, we examine the assumption about the relation of price and sample portion. Second, we compare the results of our model with managers decisions to offer free sample portions and show certain variations both consistencies of an important difference.

6.1 Actual Firm Behavior

An interesting contrast between strategies of offering free and paid versions of information goods is evident on web sites which offer news or business content. On news websites like *wsj.com (Wall Street Journal)*, most of the information is free. Only the archives and some specially prepared articles are sold. In contrast, on business websites like *forrester.com*, the vast majority of content is sold.

In order to see how consistent the results of our model are with observable market behavior, we examine the demand for the paid versions and free samples of 45 leading nationwide news websites in Germany. The demand for the paid version is derived from a dataset of *Click&Buy*, the leading European micropayment provider. This dataset contains purchase transactions of paid information goods on news websites. To measure the demand for free samples of the information goods, data from the German association of newspaper proprietors was used. This organization collects data about the number of visits and page impressions (the number retrieved, complete website screens) from websites in Germany. Using search engine software, we measured the free sample portion offered as the number of free articles divided by the sum of free and paid articles. Only firms that had been offering paid information goods for more than 12 months between January 2004 and December 2005 were examined. The sample of 45 news websites includes 177,859 customers with 1,123,181 purchases of information goods, 20,427,309,477 page impressions and 3,184,183,422 visits. Less than 7% of all customers purchased information goods from more than one supplier.

We correlated the portion of articles available for free and price with the demand for the free and paid version of the 45 websites. As assumed by the model, the larger the free sample portion, the higher the demand for it. By contrast, a larger free sample portion only slightly and insignificantly correlates negatively with the demand for the paid version, suggesting that much of the additional demand stimulated by a larger free version converts to demand for the paid version. Unsurprisingly, higher prices are associated with lower sales of the paid version (the correlation coefficient is $r = -0.18$). Further, as $p/(1-\alpha)$ increases, the incentive to buy the paid version in addition to the free version decreases ($r = -0.14$), and the ratio of users of the free to the paid version usage becomes larger ($r = 0.56$).

We further analyzed the relationship between demand for the free version and demand for the paid version applying a cross-sectional time-series regression. Specifically, we regressed *Growth in Purchase Transactions per Website Visitor* on *Growth in Free Page Impressions per Website Visitor*. The results show a strong and significant positive relation (p-value < 0.05). The coefficient of the growth of page impressions per visit (as a measure of the growth of demand for the free sample) is approximately 1.16, i.e. growth in the number of page impressions per visit for the free version leads to slightly higher growth in the number of purchase transactions per visit.

Similar results are found in a cross-section time-series linear regression between the growth of page impressions per website visitor and the growth of paying customers per website visitor (as dependent variable). Growth in demand for the free version leads to growth in the number of paying customers with a coefficient larger than 1.5. Since page views increase when more is given for free, this suggests giving more for free indeed produces additional sales of the paid version. This finding confirms the assumption that giving samples of information goods is a viable strategy.

6.2 Managerial Intuition

In order to see if managers' instincts were consistent with model predictions, we gathered data via a survey from a panel of managers ($N = 44$). Each participant was asked what portion of a movie (or a

Table 2: Percent Managers' Would Give for Free in Different Scenarios

Scenario	*X1* Ad Revenue Free Sample (a_f)	*X2* Total Revenues Paid Version ($p + a_p$)	*X3* Weight on Quality Experience (b)	*Y* Average Free Sample Portion	*Model* Optimal Free Sample Portion
1	1	6	Little	0.20	0.32
2	1	20	High	0.23	0.42
3	1	50	Equal	0.23	0.29
4	2	6	Equal	0.22	0.37
5	2	20	Litte	0.19	0.29
6	2	50	High	0.21	0.39
7	5	6	High	0.26	0.63
8	5	20	Equal	0.32	0.37
9	5	50	Little	0.20	0.25
Average				0.23	0.37

Notes: The result of the manager study are based on *N=44* participants.

book) they would offer as a free sample (i.e. as a "trailer" or "excerpt") in different situation when they were targeting all consumers (Case I). The nine situations were drawn orthogonally from a $3 \times 3 \times 3$ design which had three levels of advertising revenues from the free sample ($1, $2, $5), three levels of revenues from the paid version ($6, $20, $50) and three levels of how influential consumer experience with the sample would be vs. prior expectation on their decision to purchase the paid version (very little, about equal, and high). These correspond to the parameters a_f, p, and b in the model.

Across the nine scenarios, the respondents gave away 23% of the goods for free on average vs. the 37% which the numerical analysis of the model suggests was optimal, showing a remarkably close correspondence (see Table 2). Overall the managers choices of how much to give for free were significantly positively correlated with the optimal ones ($r = 0.43$). On average the portion managers would give away for free is 0.14 smaller as the model suggests. The difference between managers' decisions and model suggestion is above average in Scenario 2 (difference: 0.19), Scenario 6 (difference: 0.18) and Scenario 7 (difference: 0.37), all cases where the weight on quality experience was high. Thus managers have a tendency to underuse sampling when it is most effective.

We regressed the percent given away for free vs. the per person revenues from the free and paid versions as well as the impact of the free information using .1, .5, and .9 as the implied weights on actual quality (b). We also estimated all two-way interactions among the variables. Because they did not significantly improve predictions, we dropped the interactions. The three variables had the expected and significant effect. As advertising revenues of the free version rose, the average percent given for free went from 24% to 28% to 42%. Similarly as revenue from the paid version increased, the amount given for free decreased from 40% to 29% to 24%. Further, as the weight consumers placed on the sample increased, the amount managers would give away for free increased from 24% to 34% to 36%. However, managers instincts differ most from model prescription when consumers place a high weight on quality experience. Apparently managers do not recognize the impact a high weight on quality experience has on purchasing and hence on the optimal sample size on the free sample.

7 Conclusions

This paper examined the optimal sampling and pricing of information goods. While in the past the decision of how much to sample has been largely based on intuition and rules-of-thumb, we present a formal approach for determining how much should be offered for free. We develop a model that examines a firm's optimal strategy for the price of the complete information good and the fraction of that good to offer as a free sample. The model incorporates advertising revenues and allows for consumer heterogeneity in terms of both expectations of quality and the experienced sample quality while capturing consumers' quality updating. Specifically, consumers' initial valuation rely on their expectations while their updated valuations depends on both expected and experienced quality. The larger the sample portion, the more consumers will be able to predict and value actual quality and hence reduce the uncertainty which in turn may lead to additional sales of the paid version. On the other hand, as the sample portion becomes larger, consumers incentives to buy the paid version become smaller.

The key result is that the firm follows one of two strategies, the choice of which depends on advertising revenues and most critically on the level of consumers' experienced quality relative to their expected quality. We show that the firm should targeting all consumers if experienced quality noticeably exceeds expected quality while it should stick to the targeting-high-valuation-consumers strategy if consumers' experienced quality is close to expected quality. Further, the analysis suggests that the firm should never offer a free sample when expected quality exceeds experienced quality. We also provided evidence that supports some of our assumptions and results. Using a data set from German news websites, we found that a higher demand for the free sample goes along with a higher demand for the paid version, consistent with the assumption that some consumers who initially take the sample become convinced it is worth buying the complete good. While managers' intuition is generally consistent with model prescriptions, decision makers have a tendency to provide a too small sample portion when consumers' highly weight experienced quality.

We made several assumptions in developing the model. Therefore, we presented an extension where we relax two of these assumptions, allowing for advertising revenues from the free as well as from the paid version and a more general updating rule. Our results are robust for these changes. Clearly, further research could examine the implications of relaxing other assumptions. For example, the model could be generalized to a dynamic, multi-period model that includes multiple updatings of consumers' product quality valuation. In addition, the impact of word-of-mouth could be explicitly included in the model. To the extent that consumers convey their reactions to others who "accept" their opinions, then quality expectations will be updated. In essence this would require adding a third term to consumers updating behavior reflecting average experienced quality and the number of people that had experienced the good. Another fruitful avenue for future research would be to consider the impact of competition both between providers of information goods as well as advertisers and alternative outlets for the adverting. We hope this paper encourage work in these and related directions.

References

Akerlof, G. A. 1970. The market for lemons. *Quarterly Journal of Economics* 84(3), 488–500.

Bawa, K., R. Shoemaker. 2004. The effect of free sample promotions on incremental brand sales. *Marketing Science* 23(3), 345–363.

Bhargava, H. K., V. Choudhary. 2001. Information goods and vertical differentiation. *Journal of Management Information Systems* 18(2), 89–106.

Bhargava, H. K., V. Choudhary. 2008. When is versioning optimal for information goods? *Management Science* 54(5), 1029–1035.

Boom, A. 2009. "Download for Free" – When do providers of digital goods offer free samples? Free University of Berlin, SFB / TR 15 Working Paper No. 70.

Buehler, S., D. Halbheer. 2009. Selling to consumers with multidimensional tastes. University of Zurich, mimeo.

Buonocore, A. , E. Pirozzi, L. Caputo. 2009. A note on the sum of uniform random variables. *Statistics and Probability Letters* 79(19), 2092–2097.

Bronstein, I. N., K. A. Semendjajew, G. Musiol, H. Mühlig. 2008. *Taschenbuch der Mathematik*. Wissenschaftlicher Verlag Harri Deutsch GmbH, Frankfurt am Main (7. Auflage).

Darby, M. R., E. Karni. 1973. Free competition and the optimal amount of fraud. *Journal of Law and Economics* 16(1), 67–88.

Godes, D., E. Ofek, M. Sarvary. 2009. Content vs. advertising: The impact of competition on media firm strategy. *Marketing Science* 28(1), 20–35.

Heiman, A., B. McWilliams, Z. Shen, D. Zilberman. 2001. Learning and forgetting: Modeling optimal product sampling over time. *Management Science* 47(4), 532–546.

Jain, S., P. K. Kannan. 2001. Pricing of information products on online servers: Issues, models and analysis. *Management Science* 48(9), 1123–1142.

Jain, D. C., V. Mahajan, E. Muller. 1995. An approach for determining optimal product sampling for the diffusion of a new product. *Journal of Product Innovation Management* 12(2), 124–135.

Kopalle, P. K., D. R. Lehmann. 2006. Setting quality expectations when entering a market: What should the promise be? *Marketing Science* 25(1), 8–24.

Lawrence J. M., M. A. Kamins. 1988. The use of product sampling and advertising: Effects of sequence of exposure and degree of advertising claim exaggeration on consumers' belief strength, belief confidence, and attitudes. *Journal of Marketing Research* 25(3), 266–281.

Marks, L. J., M. A. Kamins. 1988. The use of product sampling of sequence of exposure and degree of advertising claim exaggeration on consumers' belief strength, belief confidence, and attitudes. *Journal of Marketing Research* 25(3), 266–281.

McGuinness, D., P. Gendall, S. Mathew. 1992. The effect of product sampling on product trial, purchases and conversion. *International Journal of Advertising* 11(1), 83–92.

Meyer, R. J. 1982. A descriptive model of consumer information on search behavior. *Marketing Science* 1(1), 93–121.

Mussa, M., S. Rosen. 1978. Monopoly and product quality. *Journal of Economic Theory* 18(2), 301–317.

Nelson, P. J. 1970. Information and consumer behavior. *Journal of Political Economy* 78(2), 311–329.

Shapiro, C. 1983. Optimal pricing of experience goods. *Bell Journal of Economics* 14(2), 497–507.

Shapiro, C., H. R. Varian. 1998. Versioning: The smart way to sell information. *Harvard Business Review* 76(6), 106–114.

Varian, H. R. 2000a. Versioning Information Goods. In Varian, H. R. and B. Kahin, eds., *Internet Publishing and Beyond: The Economics of Digital Information and Intellectual Property*, MIT Press, Cambridge MA, 190–202.

Varian, H. R. 2000b. Markets for information goods. In Okina, K. and T. Inoue, eds., *Monetary Policy in a World of Knowledge-Based Growth, Quality Change, and Uncertain Measurement*, Palgrave Macmillan, 85–100.

Wei, X., C. Weiss, B. R. Nault. 2007. Experience information goods: Versioning and upgrading. *Proceedings of the 2007 INFORMS Conference on Information Systems and Technology*, Seattle, WA.

Appendix A: Proofs

This appendix gives the proofs. To simplify the notation, we suppress the subscript i indexing cases.

A.1 Proof of Lemma 1

The claim is established using a graphical argument based on Figure 2 (Panel 2a). Inspection of the figure shows that the number of consumers that buy for given p and α can be expressed as

$$\left(\bar{q} - \frac{p}{\alpha(1-\alpha)}\right)\frac{p}{1-\alpha} + \frac{1}{2}\left(\frac{p}{\alpha(1-\alpha)} - \frac{p}{1-\alpha}\right)\frac{p}{1-\alpha} = \left(\bar{q} - \frac{(1+\alpha)p}{2\alpha(1-\alpha)}\right)\frac{p}{1-\alpha}.$$

Recalling that the density of (v, q) is equal to $(\bar{v}\bar{q})^{-1}$ on T, the (density-weighted) number of consumers generate demand

$$D_1^S(p,\alpha) \equiv \frac{1}{\bar{v}\bar{q}}\left(\bar{q} - \frac{(1+\alpha)p}{2\alpha(1-\alpha)}\right)\frac{p}{1-\alpha} = D^F(p,\alpha) - \frac{(1+\alpha)\,p^2}{2\alpha\,(1-\alpha)^2\,\bar{v}\bar{q}}.$$

This completes the proof.

A.2 Proof of Lemma 2

Since $D^I(p,\alpha) = 1 - D^F(p,\alpha)$, the firm's demand for the paid version follows immediately from Lemma 1. (i) Observe that

$$\frac{\partial D^P(p,\alpha)}{\partial p} = -\frac{(1+\alpha)\,p}{\alpha\,(1-\alpha)^2\,\bar{v}\bar{q}} < 0$$

for all admissible parameter values. (ii) Note that

$$\frac{\partial D^P(p,\alpha)}{\partial \alpha} = -\frac{\left(2\alpha^2 + 3\alpha - 1\right)p^2}{2\alpha^2\,(1-\alpha)^3\,\bar{v}\bar{q}}.$$

Thus, offering a larger sample portion increases total demand if the condition $\alpha(2\alpha + 3) < 1$ is satisfied.[16] (iii) Increasing the valuation $\bar{v}$ upper increases total demand, as inspection of

$$\frac{\partial D^P(p,\alpha)}{\partial \bar{v}} = \frac{(1+\alpha)\,p^2}{2\alpha\,(1-\alpha)^2\,\bar{v}^2\bar{q}} > 0$$

shows. (iv) Increasing the upper bound to the quality experience stimulates demand, as

$$\frac{\partial D^P(p,\alpha)}{\partial \bar{q}} = \frac{(1+\alpha)\,p^2}{2\alpha\,(1-\alpha)^2\,\bar{v}\bar{q}^2} > 0.$$

This completes the proof.

A.3 Proof of Lemma 3

The claim is established using a graphical argument based on Figure 2 (Panel 2b). It easily shown that the indifference curve passes through the point $(v', \bar{q})$, where

$$v' = \frac{p}{(1-\alpha)^2} - \frac{\alpha}{1-\alpha}\bar{q}.$$

Inspection of the figure shows that the number of consumers that buy for given p and α can be expressed as

$$\frac{1}{2}\left(\frac{p}{1-\alpha} - \left(\frac{p}{(1-\alpha)^2} - \frac{\alpha}{1-\alpha}\bar{q}\right)\right)\left(\bar{q} - \frac{p}{1-\alpha}\right) = \frac{\alpha}{2\,(1-\alpha)}\left(\bar{q} - \frac{p}{1-\alpha}\right)^2.$$

Recalling that the density of (v,q) is equal to $(\bar{v}\bar{q})^{-1}$ on T, the (density-weighted) number of consumers generate demand

$$D_2^S(p,\alpha) \equiv \frac{1}{\bar{v}\bar{q}}\left(\frac{\alpha}{2\,(1-\alpha)}\left(\bar{q} - \frac{p}{1-\alpha}\right)^2\right) = \frac{\alpha\left((1-\alpha)\,\bar{q} - p\right)^2}{2\,(1-\alpha)^3\,\bar{v}\bar{q}}.$$

This completes the proof.

[16] This condition holds if α is "sufficiently small", that is, if α is below the threshold level $\underline{\alpha} = \frac{\sqrt{17}-3}{4} \approx 0.28$. If, in contrast, $\alpha > \underline{\alpha}$ sampling has a demand-reducing effect.

A.4 Proof of Lemma 4

The total demand for the paid version follows by adding initial demand $D^I(p,\alpha)$ and the sampling-induced demand $D^S(p,\alpha)$ as given in Lemma 3. (i) To establish the claim, observe that

$$\frac{\partial D^P(p,\alpha)}{\partial p} = -\frac{(1-\alpha)\bar{q} - \alpha p}{(1-\alpha)^3 \bar{v}\bar{q}}.$$

Making use of Condition 1 and the fact that $\alpha \in (0,1)$, the sequence of inequalities $(1-\alpha)\bar{q} > p > \alpha p$ implies the result. (ii) The claim follows by inspection of

$$\frac{\partial D^P(\alpha,p)}{\partial \alpha} = \frac{(1-\alpha)^2\bar{q}^2 + p^2(1+2\alpha) - 4p(1-\alpha)\bar{q}}{2(1-\alpha)^4 \bar{v}\bar{q}}.$$

Demand is increasing in α if $\bar{q} > 4p/(1-\alpha)$ and decreasing in α if the inequality is reversed (this is a sufficient but overly strong condition). Total demand is increasing in α if the level of α is "sufficiently small". If, in contrast, α is "large enough", sampling has a demand-reducing effect. (iii) To see this claim, observe that

$$\frac{\partial D^P(p,\alpha)}{\partial \bar{v}} = \frac{2(1-\alpha)p\bar{q} - \alpha(p^2 + (1-\alpha)^2\bar{q}^2)}{2(1-\alpha)^3 \bar{v}^2\bar{q}}.$$

By Condition 1, $(1-\alpha)^2\bar{q}^2 > p^2$, which establishes the claim. (iv) Similarly,

$$\frac{\partial D^P(p,\alpha)}{\partial \bar{q}} = \frac{\alpha\left((1-\alpha)^2\bar{q}^2 - p^2\right)}{2(1-\alpha)^3 \bar{v}\bar{q}^2} > 0$$

by Condition 1. This completes the proof.

A.5 Proof of Proposition 1

The optimal sample portion is a solution to the problem

$$\max_{\alpha} \quad \Pi(\alpha) \equiv \sqrt{\frac{8\alpha\left(\bar{v} + a_f - \bar{v}\alpha\right)^3 \bar{q}}{27\left(1-\alpha\right)\left(1+\alpha\right)\bar{v}^2}} - F$$

and satisfies the first-order condition

$$\frac{d}{d\alpha}\left(\sqrt{\frac{8\alpha\left(\bar{v} + a_f - \bar{v}\alpha\right)^3 \bar{q}}{27\left(1-\alpha\right)\left(1+\alpha\right)\bar{v}^2}}\right) = 0,$$

which can be rewritten as

$$2\alpha^3\bar{v} + (\bar{v} + a_f)\,\alpha^2 - 4\alpha\bar{v} + (\bar{v} + a_f) = 0. \tag{A.1}$$

Solving the cubical equation.[17] Dividing (A.1) by $2\bar{v}$, we obtain

$$\alpha^3 + \frac{(\bar{v} + a_f)}{2\bar{v}}\alpha^2 - 2\alpha + \frac{(\bar{v} + a_f)}{2\bar{v}} = 0.$$

Letting

$$y = \alpha + \frac{(\bar{v} + a_f)}{6v},$$

we get

$$y^3 + 3z_1 y + 2z_2, \tag{A.2}$$

where

$$\begin{aligned} 2z_2 &= \frac{2\,(\bar{v} + a_f)^3}{27(2\bar{v})^3} - \frac{(\bar{v} + a_f)\,(-4\bar{v})}{3(2\bar{v})^2} + \frac{(\bar{v} + a_f)}{2\bar{v}} \\ &= \frac{(\bar{v} + a_f)^3}{108\bar{v}^3} + \frac{5\,(\bar{v} + a_f)}{6\bar{v}} \end{aligned}$$

and

$$\begin{aligned} 3z_1 &= \frac{3(2\bar{v})(-4\bar{v}) - (\bar{v} + a_f)^2}{3(2\bar{v})^2} \\ &= -2 - \frac{(\bar{v} + a_f)^2}{12\bar{v}^2}. \end{aligned}$$

Restricting parameters: The discriminant is defined as $D \equiv z_1^3 + z_2^2$. Substituting, we have

$$\begin{aligned} D &= \left(\frac{(\bar{v} + a_f)^3}{216\bar{v}^3} + \frac{5\,(\bar{v} + a_f)}{12\bar{v}}\right)^2 - \left(\frac{2}{3} + \frac{(\bar{v} + a_f)^2}{36\bar{v}^2}\right)^3 \\ &= \frac{1}{432\bar{v}^4}\left(-68\bar{v}^4 + 122\bar{v}^3 a_f + 65\bar{v}^2 a_f^2 + 4\bar{v}a_f^3 + a_f^4\right). \end{aligned}$$

In order to have three real solutions, we need $D < 0$, that is,

$$-68\bar{v}^4 + 122\bar{v}^3 a_f + 65\bar{v}^2 a_f^2 + 4\bar{v}a_f^3 + a_f^4 < 0.$$

[17] The solution procedure follows Bronstein (2008, pp. 40–42).

Solving this equation for $\bar{v}$, we find that this condition holds if and only if

$$\frac{a_f}{v} < \sqrt{\frac{11\sqrt{33}-59}{2}} - 1. \tag{A.3}$$

Solving the cubical equation (cont'd). We know that

$$z_1 = -\left(\frac{2}{3} + \frac{(\bar{v}+a_f)^2}{36\bar{v}^2}\right) < 0$$

and that $z_2 > 0$. We therefore let

$$r = \sqrt{\left|-\left(\frac{2}{3} + \frac{(\bar{v}+a_f)^2}{36v^2}\right)\right|} = \sqrt{\frac{2}{3} + \frac{(\bar{v}+a_f)^2}{36v^2}}$$

and define

$$\cos(\varphi) = \frac{z_2}{r^3} = \frac{\frac{(\bar{v}+a_f)^3}{216\bar{v}^3} + \frac{5(\bar{v}+a_f)}{12\bar{v}}}{\left(\sqrt{\frac{2}{3} + \frac{(\bar{v}+a_f)^2}{36\bar{v}^2}}\right)^3}.$$

Simplifying, we have

$$\cos(\varphi) = \frac{(\bar{v}+a_f)\left(91\bar{v}^2 + 2\bar{v}a_f + a_f^2\right)}{\left(25\bar{v}^2 + 2\bar{v}a_f + a_f^2\right)^{\frac{3}{2}}},$$

from which we obtain

$$\varphi = \arccos\left(\frac{(\bar{v}+a_f)\left(91\bar{v}^2 + 2\bar{v}a_f + a_f^2\right)}{\left(25\bar{v}^2 + 2\bar{v}a_f + a_f^2\right)^{\frac{3}{2}}}\right).$$

Equation (A.2) has three solutions:

$$\begin{aligned} y_1 &= -2r\cos\left(\frac{\varphi}{3}\right) \\ y_2 &= 2r\cos\left(\frac{\pi}{3} - \frac{\varphi}{3}\right) \\ y_3 &= 2r\cos\left(\frac{\pi}{3} + \frac{\varphi}{3}\right). \end{aligned}$$

so that the solutions to the first-order condition given in (A.1) are

$$\begin{aligned}
\alpha_1 &= -2r\cos\left(\frac{\varphi}{3}\right) - \frac{(\bar{v}+a_f)}{6\bar{v}} \\
\alpha_2 &= 2r\cos\left(\frac{\pi}{3}-\frac{\varphi}{3}\right) - \frac{(\bar{v}+a_f)}{6\bar{v}} \\
\alpha_3 &= 2r\cos\left(\frac{\pi}{3}+\frac{\varphi}{3}\right) - \frac{(\bar{v}+a_f)}{6\bar{v}}.
\end{aligned}$$

Hence, the maximizer of problem given in (A.5) is

$$\alpha^*(a_f,\bar{v}) = 2r\cos\left(\frac{\pi}{3}+\frac{\varphi}{3}\right) - \frac{(\bar{v}+a_f)}{6\bar{v}},$$

as α_1 is negative and $\pi(\alpha)$ takes its minimal value at α_2.

Restricting parameters: The domain of the $\arccos(z)$ is the set $[-1,1]$. In our case $z>0$, so we only need to ensure that $z<1$, that is

$$\frac{(\bar{v}+a_f)\left(91\bar{v}^2+2\bar{v}a_f+a_f^2\right)}{\left(25\bar{v}^2+2\bar{v}a_f+a_f^2\right)^{\frac{3}{2}}} < 1.$$

Since

$$\begin{aligned}
\frac{(\bar{v}+a_f)\left(91\bar{v}^2+2\bar{v}a_f+a_f^2\right)}{\left(25\bar{v}^2+2\bar{v}a_f+a_f^2\right)^{\frac{3}{2}}} &> \frac{(\bar{v}+a_f)\left(91\bar{v}^2+2\bar{v}a_f+a_f^2\right)}{\left(91\bar{v}^2+2\bar{v}a_f+a_f^2\right)^{\frac{3}{2}}} \\
&= \frac{\bar{v}+a_f}{\sqrt{91\bar{v}^2+2\bar{v}a_f+a_f^2}},
\end{aligned}$$

a sufficient condition is that

$$\frac{\bar{v}+a_f}{\sqrt{91\bar{v}^2+2\bar{v}a_f+a_f^2}} < 1 \Leftrightarrow (\bar{v}+a_f)^2 < 91\bar{v}^2+2\bar{v}a_f+a_f^2 \Leftrightarrow \bar{v}>0,$$

which is satisfied by assumption. (i) Numerical simulations show that $d\alpha^*/da_f > 0$ and that (ii) $d\alpha^*/d\bar{v} < 0$ for all conceivable parameter constellations $\bar{v}$ and a_f satisfying (A.3).[18] This completes the proof.

[18]The numerical simulations are available from the authors upon request.

A.6 Proof of Claim 1

Note

$$
\begin{aligned}
& sign\left(\frac{\partial}{\partial\alpha}\left(\sqrt{\frac{2\alpha\left(1-\alpha\right)\left(\bar{v}+a_f-\bar{v}\alpha\right)\bar{q}}{3\left(1+\alpha\right)}}\right)\right) \\
= \quad & sign\left(\left(1-\alpha\right)\left(\bar{v}+a_f-\bar{v}\alpha\right)-\alpha\left(1+\alpha\right)\left(2v+a_f-2v\alpha\right)\right).
\end{aligned}
$$

The expression on the right-hand side is a cubic equation in α, which can be rewritten as

$$2\bar{v}\alpha^3+\left(\bar{v}-a_f\right)\alpha^2-\left(4\bar{v}+2a_f\right)\alpha+\left(\bar{v}+a_f\right). \tag{A.4}$$

Let $\hat{\alpha}$ denote the solution to this equation.

Solving the cubical equation.[19] Dividing the preceeding equation by $2\bar{v}$, we have

$$\alpha^3+\frac{\left(\bar{v}-a_f\right)}{2\bar{v}}\alpha^2-\frac{\left(4\bar{v}+2a_f\right)}{2\bar{v}}\alpha+\frac{\left(\bar{v}+a_f\right)}{2\bar{v}}=0.$$

Letting

$$y=\alpha+\frac{\left(\bar{v}-a_f\right)}{6\bar{v}},$$

we get

$$y^3+3z_1y+2z_2=0, \tag{A.5}$$

where

$$
\begin{aligned}
2z_2 \quad &= \quad \frac{2\left(\bar{v}-a_f\right)^3}{27(2\bar{v})^3}+\frac{\left(\bar{v}-a_f\right)\left(4\bar{v}+2a_f\right)}{3\left(2\bar{v}\right)^2}+\frac{\left(\bar{v}+a_f\right)}{2\bar{v}} \\
&= \quad \frac{\left(\bar{v}-a_f\right)^3}{108\bar{v}^3}+\frac{5\bar{v}^2+2\bar{v}a_f-a_f^2}{6\bar{v}^2}
\end{aligned}
$$

and

$$
\begin{aligned}
3z_1 \quad &= \quad \frac{3(2\bar{v})(-1)\left(4\bar{v}+2a_f\right)-(\bar{v}-a_f)^2}{3(2\bar{v})^2} \\
&= \quad -\frac{\left(5\bar{v}+a_f\right)^2}{12\bar{v}^2}.
\end{aligned}
$$

[19] The solution procedure follows Bronstein (2008, pp. 40–42).

Real-valued solutions: The discriminant is defined as $D \equiv z_1^3 + z_2^2$. Substituting, we have

$$\begin{aligned} D &= \left(\frac{(\bar{v} - a_f)^3}{216\bar{v}^3} + \frac{5\bar{v}^2 + 2\bar{v}a_f - a_f^2}{12\bar{v}^2} \right)^2 + \left(-\frac{(5\bar{v} + a_f)^2}{36\bar{v}^2} \right)^3 \\ &= -\frac{1}{216\bar{v}^4} \left(34\bar{v}^4 + 59\bar{v}^3 a_f + 51\bar{v}^2 a_f^2 + 17\bar{v}a_f^3 + a_f^4 \right). \end{aligned}$$

Since $D < 0$ for all parameter values, the three solutions of (A.5) are real-valued.

Solving the cubical equation (cont'd). We know that $z_1 < 0$ and $z_2 > 0$, where the latter inequality holds for all parameter values a_f and $\bar{v}$ satisfying (A.3). Now let

$$\hat{r} = \sqrt{\left| -\frac{(5v + a_f)^2}{36v^2} \right|} = \frac{5v + a_f}{6v}$$

and define

$$\cos(\hat{\varphi}) = \frac{z_2}{\hat{r}^3},$$

from which we obtain after rearranging

$$\cos(\hat{\varphi}) = \frac{91v^3 + 33v^2 a_f - 15va_f^2 - a_f^3}{(5v + a_f)^3},$$

which in turn implies

$$\hat{\varphi} = \arccos\left(\frac{91v^3 + 33v^2 a_f - 15va_f^2 - a_f^3}{(5v + a_f)^3} \right).$$

The solutions to equation (A.5) are:

$$\begin{aligned} y_1 &= -2\hat{r}\cos\left(\frac{\hat{\varphi}}{3}\right) \\ y_2 &= 2\hat{r}\cos\left(\frac{\pi}{3} - \frac{\hat{\varphi}}{3}\right) \\ y_3 &= 2\hat{r}\cos\left(\frac{\pi}{3} + \frac{\hat{\varphi}}{3}\right) \end{aligned}$$

Hence, the solutions to (A.4) are

$$\begin{aligned}\alpha_1 &= -2\hat{r}\cos\left(\frac{\hat{\varphi}}{3}\right) - \frac{(\bar{v}-a_f)}{6\bar{v}}\\ \alpha_2 &= 2\hat{r}\cos\left(\frac{\pi}{3}-\frac{\hat{\varphi}}{3}\right) - \frac{(\bar{v}-a_f)}{6\bar{v}}\\ \alpha_3 &= 2\hat{r}\cos\left(\frac{\pi}{3}+\frac{\hat{\varphi}}{3}\right) - \frac{(\bar{v}-a_f)}{6\bar{v}},\end{aligned}$$

and the relevant solution for our problem is

$$\hat{\alpha}(a_f,\bar{v}) \equiv \alpha_3 = 2\hat{r}\cos\left(\frac{\pi}{3}+\frac{\hat{\varphi}}{3}\right) - \frac{(\bar{v}-a_f)}{6\bar{v}}.$$

Slope of best-response function in equilibrium. The comparison of $\alpha^*(a_f,\bar{v})$ and $\hat{\alpha}(a_f,\bar{v})$ allows us to determine the sign of the best-response function at the equilibrium value α^*. Now observe that

$$\alpha^*(a_f,\bar{v}) - \hat{\alpha}(a_f,\bar{v}) = 2r\cos\left(\frac{\pi}{3}+\frac{\varphi}{3}\right) - 2\hat{r}\cos\left(\frac{\pi}{3}+\frac{\hat{\varphi}}{3}\right) - \frac{a_f}{3v}.$$

Numerical simulations show that $\alpha^*(a_f,\bar{v}) - \hat{\alpha}(a_f,\bar{v}) > 0$ for all conceivable parameter constalations satisfying condition (A.3).[20] This completes the proof.

A.7 Proof of Proposition 2

(i) From Claim 2 we know that $\partial p/\partial\alpha < 0$ at α^*. From Proposition 1, we know $d\alpha^*/da_f > 0$. Inspection of the best-response function given in (3) shows that $\partial p/\partial a_f > 0$. Thus, the sign of dp^*/da_f is generally ambiguous, but condition (A.3) implies that the set of admissible parameter values is $0 < a_f/\bar{v} < 0.45$. Numerical simulations show that $dp^*/da_f > 0$ if $0 < a_f/\bar{v} < 0.35$, and that $dp^*/da_f < 0$ if $0.35 < a_f/\bar{v} < 0.45$.[21] (ii) From Claim 2, we know that $\partial p/\partial\alpha < 0$. Furthermore, we know from Proposition 1 that $d\alpha^*/d\bar{v} < 0$. Inspection of the best-response function as given in (3) shows that $\partial p/\partial\bar{v} > 0$. Therefore, $dp^*/d\bar{v} > 0$. (iii) Inspection of the best-response function as given in (3) shows that $\partial p/\partial\bar{q} > 0$.

A.8 Proof of Proposition 3

(i) Note that

$$\frac{\partial\alpha^*(a_f,\bar{v},\bar{q})}{\partial a_f} = -\frac{1}{2\bar{v}-\bar{q}} < 0$$

[20] The numerical simulations are available from the authors upon request.
[21] The numerical simulations are available from the authors upon request.

if $2\bar{v}-\bar{q}>0$. (ii) To establish the second claim, consider

$$\frac{\partial\alpha^*(a_f,\bar{v},\bar{q})}{\partial\bar{v}}=\frac{2a_f+\bar{q}-2\sqrt{\bar{q}(2\bar{v}-\bar{q})}}{(2\bar{v}-\bar{q})^2}.$$

The sign of this expression depends on the sign of nominator, which is positive if and only if

$$2a_f+\bar{q}>2\sqrt{\bar{q}(2\bar{v}-\bar{q})}\Longleftrightarrow\bar{q}<2\bar{v}-\frac{(2a_f+\bar{v})^2}{4\bar{q}},$$

which is automatically satisfied if $2\bar{v}-\bar{q}>0$. (iii) The third claim follows from inspection of

$$\frac{\partial\alpha^*(a_f,\bar{v},\bar{q})}{\partial a_f}=-\frac{a_f\bar{q}+\left(\bar{q}-2\sqrt{\bar{q}(2\bar{v}-\bar{q})}\right)\bar{v}}{\bar{q}(2\bar{v}-\bar{q})^2}.$$

The sign of this expression depends again on the sign of nominator, which is positive if and only if

$$a_f\bar{q}+(\bar{q}-2\sqrt{\bar{q}(2\bar{v}-\bar{q})})\bar{v}<0\Longleftrightarrow\bar{q}<2\bar{v}\frac{\sqrt{\bar{q}(2\bar{v}-\bar{q})}}{a_f+\bar{v}}.$$

which is automatically satisfied if $2\bar{v}-\bar{q}>0$. This completes the proof.

A.9 Proof of Claim 2

The proof relies on numerical simulations, which are available from the authors upon request.

A.10 Proof of Proposition 4

(i) From Claim 2, we know that $\partial p/\partial\alpha<0$ at α^*. From Proposition 3 that $d\alpha^*/da_f<0$. Inspection of the best-response function as given in (6) shows that $\partial p/\partial a_f<0$. Therefore, the sign of dp^*/da_f is generally ambiguous, but numerical simulations show that the direct effect dominates the sampling-mediated effect, thus implying $dp^*/da_f<0$.[22] (ii) From Claim 2, we know that $\partial p/\partial\alpha<0$. Furthermore, we know from Proposition 3 that $d\alpha^*/d\bar{v}>0$. Inspection of the best-response function as given in (6) shows that $\partial p/\partial\bar{v}<0$. Therefore, $dp^*/d\bar{v}<0$. (iii) From Claim 2, we know that $\partial p/\partial\alpha<0$. Furthermore, we know from Proposition 3 that $d\alpha^*/d\bar{q}>0$. Numerical simulations show that $\partial p/\partial\bar{q}>0$ as well as the total effect $dp^*/d\bar{q}$ is negative.[23]

[22] The numerical simulations are available from the authors upon request.

[23] The numerical simulations are available from the authors upon request.

A.11 Proof of Lemma 5

A function $f(x_1, x_2, ..., x_n)$ of several variables is called a homogeneous function of degree m if

$$f(\lambda x_1, \lambda x_2, ..., \lambda x_n) = \lambda^m f(x_1, x_2, ..., x_n).$$

holds for arbitrary $\lambda > 0$. *Proof of Case I:* To establish the claim for α^* given in given in (5), observe that

$$\alpha^*(\lambda a_f, \lambda \bar{v}) = \sqrt{\frac{8}{3} + \frac{(\lambda \bar{v} + \lambda a_f)^2}{9(\lambda \bar{v})^2}} \cos\left(\frac{\pi}{3} + \frac{\varphi}{3}\right) - \frac{(\lambda \bar{v} + \lambda a_f)}{6\lambda \bar{v}} = \lambda^0 \alpha^*(a_f, \bar{v}),$$

where

$$\varphi = \arccos\left(\frac{(\lambda \bar{v} + \lambda a_f)\left(91(\lambda \bar{v})^2 + 2\lambda \bar{v} \lambda a_f + (\lambda a_f)^2\right)}{(25(\lambda \bar{v})^2 + 2\lambda \bar{v} \lambda a_f + (\lambda a_f)^2)^{\frac{3}{2}}}\right),$$

which shows that the optimal sample portion α^* is homogeneous of degree 0 in $(a_f, \bar{v}, \bar{q})$. To establish the claim for p^* given in (3), observe that

$$p^*(\alpha^*; \lambda a_f, \lambda \bar{v}, \lambda \bar{q}) = \sqrt{\frac{2\alpha\,(1-\alpha)\,(\lambda \bar{v} + \lambda a_f - \lambda \bar{v} \alpha)\,\lambda \bar{q}}{3\,(1+\alpha)}} = \lambda^1 p^*(\alpha^*, a_f, \bar{v}, \bar{q}),$$

which shows that p^* is homogeneous of degree 1 in $(a_f, \bar{v}, \bar{q})$. The proof of Case II is similar and therefore omitted. This completes the proof.

Appendix B: Numerical Study of the Extended Model

To understand how sensitive the optimal price p^* and optimal sample portion α^* are to changes in the values of the parameters of the extended model, especially those related to customers characteristics, we set the parameters to different values. That allows us to see how a change in the parameter impacts the optimal price p^* and free sample portion α^*. The parameters are varied as follows:

(i) range of consumers' prior valuation ($\bar{v}$) from 10 to 90 in steps of 20;

(ii) range of consumers' experienced quality exceeding the range of consumers' prior valuation ($\bar{q} > \bar{v}$) from 0 to 120 in steps of 20;

(iii) weight consumers put on experienced quality in updating their valuation (b) from 0.5 to 2 in steps of 0.1;

(iv) advertising revenues per user (e.g. reader) a_f and a_p between 0 and 40 in steps of 1.

Table B-1: Sensitivity Analysis: Determinants of Optimal Sampling and Pricing

	Optimal Sample Portion α^*		*Optimal Price p^**	
Independent Variables	*Stand. Coefficient*	*t-value*	*Stand. Coefficient*	*t-value*
Case I: Targeting All Consumers				
Range of Consumers' Prior Valuation ($\bar{v}$)	-0.337	-247.58	0.444	429.30
Range of Experienced Quality ($\bar{q}$)	0.063	52.66	0.616	678.83
Weight on Quality Experience (b)	0.925	1265.03	-0.224	-403.29
Ad Revenue of Free Sample (a_f)	0.675	640.15	0.211	263.26
Ad Revenue of Paid Version (a_p)	0.087	98.20	-0.131	-195.17
	R^2 = 0.98		R^2 = 0.98	
Case II: Targeting High-Valuation Consumers				
Range of Consumers' Prior Valuation ($\bar{v}$)	1.051	49.05	-0.025	-6.74
Range of Experienced Quality ($\bar{q}$)	1.511	70.90	0.107	28.53
Weight on Quality Experience (b)	0.969	78.66	0.647	298.82
Ad Revenue of Free Sample (a_f)	0.625	46.14	-0.272	-114.35
Ad Revenue of Paid Version (a_p)	0.518	41.02	0.245	110.46
	R^2 = 0.96		R^2 = 0.94	

Notes: The result are derived from a numerical study of the extended model based on *N=209'773* parameter value combinations in Case I and *N=57'381* parameter value combinations in Case II .

We choose the parameter values to capture a variety of situations. By allowing the range of prior expectation to vary between 10 and 90, we capture cases where the variance of the prior is small for all consumers as well as cases where the variance of the prior is large. We allow the range (and therefore also the variance) of experienced quality to exceed prior valuation by between 0 (prior valuation equals experienced quality) and 120. Regarding the weight b, $b = 1$ implies that the weight placed on the experienced quality of the free sample is equal to the proportional value of the total good that is given for free. By contrast a value of b less than 1, e.g. $b = 0.5$, implies consumers weight the experience from the sample portion of the value given for free more than their prior valuation while if b is greater than 1, they weight it less.

Across all parameter values there are 423,360 "value combinations". In our numerical analysis we focus on numerical combinations which fulfill the conditions of a profit maximum and are consistent with Conditions 1 and 2a (Case I) and Conditions 1 and 2b (Case II). This reduces our set of different parameter value combinations to 209,773 in Case I and 57,381 in Case II, respectively.

As an alternative to comparative statics and to determine how "sensitive" our model is to changes in the values of the model parameters, we run multivariate regressions with the optimal price p^* and free sample portion α^* as dependent variables. Based on this, we analyzed the impact of each parameter by observing its impact on the optimal price p^* and the free sample portion α^* over the range of values it took on in the simulation (see Table B-1). The standardized coefficients in Table B-1 show the sensitivity of the optimal price, p^*, and free sample portion, α^*, to changes in the values of the parameters $\bar{v}$, $\bar{q}$, $\bar{q}$, a_f and a_p for Case I as well as in Case II.

Appendix C: Instructions to Participants

This study will require about few minutes of your time and concerns managerial decisions about how to market a movie (or a book, respectively). Specifically we will ask you what fraction of the total value of a movie (book) you would give away as a free sample to generate (future) sales. Please note that there are no right or wrong answers. Individual responses will be kept absolutely confidential and used only for statistical analysis. Please do not discuss this exercise with others – the quality of the research we do depends on your confidentiality. We will provide you with a summary of the results once the analysis is complete.

Consumers: Consumers can download either a free sample or the complete paid movie (book) immediately on their computer. If consumers download the free sample, they will update their expectation regarding quality (value) of the full movie (book). How much consumers' update of their quality expectations depends on the amount included in the free sample as well as its quality. If consumers like the product after experiencing the sample, they may then also decide to buy the paid version to get the remaining part of it.

Firm: The firm receives advertising revenues from the free sample as well as from the paid version of the movie (book) based on the number of people who accept the free sample and buy the movie (book) respectively. In addition, the firm receives revenues from each movie (book) which is sold. As a manager you have to decide what portion of the movie (book) to give as a free sample.

Scenarios:

- Ad revenues per person receiving of the free sample is \$1 (\$2, \$5)
- Total (Sale price plus Advertising revenue) revenue from selling one paid version: \$6 (\$20, \$50)
- Consumers decision depends very little (about equal, and high) on the actual quality of the sample

Question: What portion of the total value of this movie (book) would you offer as a free sample? ....%