PRICING AND REPLENISHMENT STRATEGIES IN A DISTRIBUTION SYSTEM WITH COMPETING RETAILERS

FERNANDO BERNSTEIN

The Fuqua School of Business, Duke University, Durham, North Carolina 27708, fernando@duke.edu

AWI FEDERGRUEN

Graduate School of Business, Columbia University, New York, New York 10027, af7@columbia.edu

We consider a two-echelon distribution system in which a supplier distributes a product to N competing retailers. The demand rate of each retailer depends on all of the retailers' prices, or alternatively, the price each retailer can charge for its product depends on the sales volumes targeted by all of the retailers. The supplier replenishes his inventory through orders (purchases, production runs) from an outside source with ample supply. From there, the goods are transferred to the retailers. Carrying costs are incurred for all inventories, while all supplier orders and transfers to the retailers incur fixed and variable costs. We first characterize the solution to the centralized system in which all retailer prices, sales quantities and the complete chain-wide replenishment strategy are determined by a single decision maker, e.g., the supplier. We then proceed with the decentralized system. Here, the supplier chooses a wholesale pricing scheme; the retailers respond to this scheme by each choosing all of his policy variables. We distinguish systematically between the case of Bertrand and Cournot competition. In the former, each retailer independently chooses his retail price as well as a replenishment strategy; in the latter, each of the retailers selects a sales target, again in combination with a replenishment strategy. Finally, the supplier responds to the retailers' choices by implementing his own cost-minimizing replenishment strategy. We construct a perfect coordination mechanism. In the case of Cournot competition, the mechanism applies a discount from a basic wholesale price, based on the *sum* of three discount components, which are a function of (1) annual sales volume, (2) order quantity, and (3) order frequency, respectively.

Received December 1999; revision received December 2001; accepted June 2002.

Subject classifications: Games/group decisions, noncooperative: price and quantity competition among nonidentical retailers. Inventory/production, multi-item/echelon/stage: two-echelon supply chain with procurement setup costs. Inventory/production, policies, marketing/pricing: coordinating mechanisms via discounting schemes.

Area of review: Manufacturing, Service, and Supply-Chain Operations.

1. INTRODUCTION

In their attempt to improve or optimize aggregate performance, many supply chains increasingly investigate and compare their performance under *centralized* and *decentralized* decision making. In a decentralized system, each chain member optimizes his own profit function. The challenge therefore consists of structuring the costs and rewards of all of the chain members so as to align their objectives with aggregate supply-chain-wide profits. Such a cost and reward structure is referred to as a *coordination mechanism*. If the decentralized cost and reward structure results in chainwide profits that are equal to those achieved under a centralized system, the coordination mechanism is called *perfect*.

In this paper, we address these questions for the following prototype two-echelon distribution system with competing retailers. A supplier distributes a single product or closely substitutable products to multiple retailers, which in turn sell these to the consumer. Each retailer's sales occur at a constant rate which depends on the prices charged by him as well as those charged by all other retailers, according to a given retailer specific demand function. Alternatively, the price each retailer can charge for his product depends on the sales volumes targeted by all of the retailers. The supplier replenishes her inventory through orders (purchases, production runs) from an outside source with ample supply. From there, the goods are transferred to the retailers. Carrying costs are incurred for all inventories, while all supplier orders and transfers to the retailers incur fixed and variable costs, all with facility-specific cost parameters. We consider one additional cost component: the supplier may incur a specific annual cost for managing each retailer's needs and transactions. We model the "management costs" associated with a retailer account by a concave function of the retailer's annual sales volume, reflecting economies of scale. This cost component has been considered by Chen et al. (2001); see there for a discussion of how such account management costs arise in different industries. All demand functions and cost parameters are stationary and common knowledge among all channel members.

We first characterize the solution to the centralized system in which all retailer prices, sales quantities, and the complete chainwide replenishment strategy are determined by a single decision maker, e.g., the supplier. The *exact* optimal (centralized) strategy is unknown and, in any case, of such complex structure as to preclude its implementability, even if it could be computed in a reasonable amount of time. This holds even for the far simpler case where all retailer prices and sales rates are exogenously given. We are, however, able to derive efficiently computable *lower* and *upper bounds* which are shown to be *tight*, whenever the retailers' gross profit margins [= (retail price −

wholesale price)/wholesale price] are not excessively low (say, at least 20%), and the annual holding cost rate is not excessively large (say, less than 30%). The lower bound represents the profit of a strategy with stationary retailer prices under the optimal power-of-two replenishment policy to service the corresponding sales rates at all retailers. (Under a power-of-two policy, all facilities replenish when their inventory is down to zero and each uses constant replenishment intervals, specified as a facility-specific power-of-two multiple of a given base period.) Similarly, the upper bound represents the profit of a strategy with stationary retail prices employing a lower bound for the minimum setup and holding costs incurred to service the corresponding sales at the retailers.

We proceed with the decentralized system. Here, the supplier chooses a wholesale pricing scheme; the retailers respond to this scheme by each choosing all of his policy variables. We distinguish systematically between the case of Bertrand and Cournot competition. In the former, each retailer independently chooses his retail price as well as a replenishment strategy; in the latter, each of the retailers selects a sales target, again in combination with a replenishment strategy. Finally, the supplier responds to the retailers' choices by implementing her own cost-minimizing replenishment strategy. We focus initially on *linear* wholesale pricing schemes where each retailer pays a *constant* wholesale price for each unit purchased. We show, under both Bertrand and Cournot competition that, while a Nash equilibrium may fail to exist under completely general parameter combinations, an equilibrium (in pure strategies) is, in fact, guaranteed under a condition which relates the retailer's price elasticity of demand to the ratio of his annual sales and his combined inventory and setup costs. (This condition is, again, shown to be comfortably satisfied in virtually all product categories.) A related and slightly stronger condition guarantees that the equilibrium is *unique*. We proceed with a comparison between the equilibrium under Bertrand and Cournot competition.

We next derive a *perfect* coordination mechanism. In the case of Cournot competition, the mechanism applies a discount from a basic wholesale price, based on the *sum* of three discount components, which are a function of (1) annual sales volume, (2) order quantity, and (3) order frequency, respectively. Under this discount scheme, the optimal centralized solution arises as a Nash equilibrium in the resulting retailer game. We derive conditions under which *all* Nash equilibria in the retailer game achieve optimal supply-chain-wide profits, thus giving rise to a *strong* form of perfect coordination. In the absence of retailer competition, discounts based on the annual sales volume arise only in the presence of account management costs, as demonstrated in Chen et al. (2001). On the other hand, in the presence of retailer competition, such discounts are required even if no account management costs prevail. The coordination mechanism thus provides an economic rationale, within the context of a model with complete information and symmetric bargaining power for all retailers, for wholesale prices to be discounted on the basis of annual sales volumes, one of the most prevalent forms of price discount schemes (see e.g., Brown and Medoff 1990, Stein and El-Ansary 1992, and Munson and Rosenblatt 1998). We show that for each retailer this coordinating wholesale pricing scheme is given by the per unit "indirect" costs incurred for this retailer, augmented by a markup, the magnitude of which increases with the so-called "competitive impact", a measure for the degree of competition a retailer presents to the remainder of the market. (This measure was first introduced in Bernstein et al. 2002.) If the retailers compete in price space (i.e., face Bertrand competition), perfect coordination can be achieved with a similar, albeit more complex, discount scheme.

We assess the value of (perfect) coordination within a decentralized system by analyzing settings in which a simple *linear* wholesale pricing scheme is offered to the retailers and no other measures are taken to coordinate the channel members' decisions. We analyze the performance of the system, assuming either that the supplier has the market power to specify the linear wholesale pricing scheme, or that the constant wholesale price is chosen so as to optimize the supply-chain-wide profits. In the first case, the chain members are engaged in a Stackelberg game with the supplier as the leader and the retailers following by playing the noncooperative retailer game described above. The Stackelberg solution often results in major losses in the supply-chain-wide profits.

The marketing literature on channel coordination focuses on pricing decisions. Jeuland and Shugan (1983) consider a simple channel with one supplier and one retailer. Their model does not consider any inventory replenishment decisions or resulting setup and inventory carrying costs. The authors found that a simple quantity discount results in a perfect coordination mechanism. Because theirs is a static model, the "quantity" does not refer to the size of a replenishment order but to the annual sales volume. As an alternative, Moorthy (1987) showed that in this single-retailer setting, perfect coordination can be achieved with a simple two-part tariff, i.e., by charging the retailer the marginal cost plus a fixed franchise fee. McGuire and Staelin (1983) consider the special case of *two identical* retailers, competing in price space under *linear* procurement costs. These authors assume that the two retailers are supplied by two different manufacturers which are either vertically integrated with their retailer or not. See Moorthy (1987) for further observations on this model.

Ingene and Parry (1995) generalize Jeuland and Shugan (1983), by allowing for *two* nonidentical retailers. The authors show that perfect coordination *cannot* be achieved by any constant wholesale price which is identical for both retailers. Instead, they derive a perfect coordination scheme by discounting the wholesale price as a *linear* function of the retailers' purchase volumes. While attractive, the proposed scheme fails when the number of retailers is larger than two or when the procurement costs are nonlinear, as is clearly the case in our operational model with inventory and setup costs. Raju and Zhang (1999) analyze another variant of our model with one dominant retailer capable of singlehandedly setting the retail price which is adopted by all other retailers in the market. Under a linear cost structure, the authors show that with a linear wholesale pricing scheme, perfect coordination requires that double marginalization be avoided. A nonlinear pricing scheme is offered as an alternative. Tyagi (1999) addresses the case of an arbitrary number of competing retailers, albeit that they are assumed to be identical and that, once again, procurement costs are restricted to be linear.

The marketing literature has thus restricted itself to the simplest of cost structures, i.e., to the case of *linear* costs. As summarized above and demonstrated below, more complex, yet basic, operational cost structures such as those arising under inventory carrying and fixed distribution costs introduce additional and essential complexities to the challenge of designing appropriate coordination mechanisms. This point has been brought out in an emerging stream of operations management papers. The latter, on the other hand, restrict themselves to settings where the demand processes are exogenously given, i.e., where the revenues cannot be controlled, or, in a few cases, to models in which the retailers fail to compete with each other in terms of their retail prices and/or sales targets. We refer to Chen et al. (2001) for a review of the literature on models with exogenously given, deterministic demand processes. (This stream of papers appears to have originated with Crowther 1964, examining quantity discounts from both the buyer's and the seller's perspective, and Lal and Staelin 1984 who deal with a single retailer or multiple but identical retailers.) Lee and Whang (1996), Chen (1999), and Cachon and Zipkin (1999) have developed perfect coordination schemes for a stochastic version of our model with a *single* retailer facing an exogenously given demand process and in the absence of fixed costs for deliveries from the supplier to the retailers. (This work builds on earlier coordination results by Clark and Scarf 1960 and Federgruen and Zipkin 1984.)

Weng (1995) is one of the first attempts to treat the retailers' demand rates as endogenous variables to be determined by a careful balancing of *revenue* as well as cost considerations. This model considers the special case of a single retailer or multiple, but identical and noncompeting retailers. The author asserts that an order quantity discount plus a periodic franchise fee suffice to achieve perfect coordination. This assertion, however, has not been substantiated, as Boyaci and Gallego (1997) point out. Chen et al. (2001) address the *centralized* and the *decentralized* versions of our supply-chain model, in the absence of the retailers competing in price or quantity space, i.e., when each retailer's sales volume is a function of his own price only. See Munson and Rosenblatt (1998), Boyaci and Gallego (1997), Cachon (1999), Lariviere (1999), and Tsay et al. (1999) for additional reviews of the operations management literature related to channel coordination with noncompeting retailers.

The existence and design of (perfect) coordination mechanisms in vertical supply chains is, in addition, a central topic in the industrial organization economics literature; see, e.g., Tirole (1988) and Katz (1989). See Bernstein et al. (2002) for a recent review of this part of the literature. The latter paper addresses a variant of the model of this paper in which, contrary to our setting, the operational costs can be decomposed into a part which is determined *only* by the supplier and another part which results exclusively from the retailers' replenishment strategy.

The remainder of this paper is organized as follows: §2 introduces the model and notation. Section 3 addresses the centralized system. The analysis of the decentralized system is given in §4. Section 5 develops the perfect coordination mechanism. Section 6 reports on a numerical study, comparing the performance of the supply chain under centralization and various forms of decentralization. We conclude the paper with a conclusions section.

2. MODEL AND NOTATION

We consider a distribution system with a supplier distributing a single product or closely substitutable products to N retailers. The retailers sell their product to the final consumer. The supplier replenishes his inventory from a source with ample supply. All demands and all retailer orders must be satisfied without incurring any stockouts. We assume that all orders are received instantaneously upon placement. Positive but deterministic leadtimes can be handled by a simple shift in time of all desired replenishment epochs. Thus, let

$p_i =$ retail price charged by retailer i, and

$q_i =$ consumer demand for retailer i's product.

The two sets of variables may be related to each other via the (direct) *demand functions*

$q_i = d_i(p_1, \ldots, p_N),\ i = 1, \ldots, N,$

or the *inverse demand functions*

$p_i = f_i(q_1, \ldots, q_N),\ i = 1, \ldots, N.$

We assume that all demand functions are downward sloping, a property almost invariably satisfied, with the exception of rare luxury, or Veblen goods:

$$\frac{\partial d_i}{\partial p_i} < 0, \quad i = 1, \ldots, N. \tag{1}$$

For all $i = 1, \ldots, N$, we use ϵ_{ii} to denote the absolute price elasticity of retailer i's demand, measured from the *direct* demand functions, and $\hat{\epsilon}_{ii}$ measured from the inverse demand functions:

$$\epsilon_{ii} = -\frac{\partial d_i(p)}{\partial p_i}\frac{p_i}{d_i(p)}, \quad \hat{\epsilon}_{ii} = -\frac{1}{\partial f_i(q)/\partial q_i}\frac{f_i(q)}{q_i}.$$

(Note that in the absence of retailer competition, $\epsilon_{ii} = \hat{\epsilon}_{ii}$.)

We assume that the demand volumes vary within a cube Q in the positive orthant of $\Re^N$, i.e., there exist numbers $0 \leqslant q_i^{\min} < q_i^{\max}$ such that $q_i^{\min} \leqslant q_i \leqslant q_i^{\max}$ for all $i = 1, \ldots, N$. Similarly, the set of feasible prices for each retailer i is a closed interval $[p_i^{\min}, p_i^{\max}]$, where

$p_i^{\min} = f_i(q^{\max})$ and $p_i^{\max} = f_i(q^{\min})$. The direct demand functions will be used when the retailers compete in terms of their announced retail prices, i.e., under Bertrand competition. Likewise, competition is best modeled via the inverse demand functions, when the retailers compete in terms of their sales targets, i.e., under Cournot competition.

To simplify *some* of the results, we shall confine ourselves to the case where all demand functions are *linear*. In particular,

$$d_i(p) = a_i - b_i p_i + \sum_{j \neq i} \beta_{ij} p_j \quad \text{with } a_i > 0, b_i > 0, \quad i = 1, \ldots, N. \quad (2)$$

Because the retailer products are substitutes, we have, by a common definition going back to Samuelson (1947), that

$$\beta_{ij} \geqslant 0 \quad \text{for all } i \neq j. \quad (3)$$

We assume in addition that the matrix B, with $B_{ii} = -b_i$ and $B_{ij} = \beta_{ij}$ for all $i \neq j$, is nonsingular, so that the inverse demand functions exist and are linear as well, i.e.,

$$f_i(q) = \hat{a}_i - \hat{b}_i q_i - \sum_{j \neq i} \hat{\beta}_{ij} q_j, \quad i = 1, \ldots, N. \quad (4)$$

Moreover, we would like the inverse demand functions to be downward sloping and products to be substitutes in terms of the inverse demand functions as well (see Vives 2000):

$$\hat{b}_i > 0; \quad \hat{\beta}_{ij} \geqslant 0 \quad \text{for all } i \neq j, i = 1, \ldots, N. \quad (5)$$

Unfortunately, (5) is not *necessarily* implied by the corresponding properties (1) and (3) for the direct demand functions. The implication holds, however, when

(D) $\quad b_i > \sum_{j \neq i} \beta_{ij} \quad$ for all $i = 1, \ldots, N$

(see Bernstein et al. 2002, Proposition 1). This "dominant diagonal" condition is highly intuitive and is satisfied in most industries. It states that each retailer observes a decrease in his sales volume if all retailers simultaneously increase their prices by the same amount. Another equally intuitive sufficient condition to ensure that (1) and (3) imply (5) is $b_i > \sum_{j \neq i} \beta_{ji}$ for all $i = 1, \ldots, N$: It states that a price increase by any one of the retailers results in a decrease of total sales in the market. (See Bernstein et al. 2002, Proposition 1, for the most general, necessary and sufficient condition.) We henceforth assume (D), throughout.

We now turn to a description of the cost structure. All deliveries to and from the supplier incur fixed and variable costs. In a decentralized setting, it is useful to decompose the fixed cost associated with a delivery to a retailer into a component incurred by this retailer and one incurred by the supplier (e.g., an order-processing cost). Inventory carrying costs are incurred for each location's inventory and they are proportional with the prevailing inventory level. In addition, the supplier may incur a specific annual cost for managing each retailer's account. All cost parameters are stationary. For $i = 1, \ldots, N$, define

K_0 = fixed cost incurred for each delivery to the supplier,

K_i = fixed cost incurred for each delivery to retailer i, $i = 1, \ldots, N$,

K_i^s = the component of K_i incurred by the *supplier* in a decentralized setting,

K_i^r = the component of K_i incurred by *retailer i* in a decentralized setting,

$K_i = K_i^s + K_i^r$,

h_0 = annual holding cost per unit of inventory at the supplier,

$\bar{h}_i$ = annual holding cost per unit of inventory at retailer i,

$h_i = \bar{h}_i - h_0$, incremental or echelon holding cost at retailer i,

c_0 = cost per unit delivered to the supplier,

c_i = transportation cost per unit shipped from the supplier to retailer i,

$\psi(d_i)$ = annual cost incurred for managing retailer i's account, with $\Psi(\cdot)$ nondecreasing, concave, and $\psi(0) = 0$.

We assume that $h_i \geqslant 0$ for all i, which means that the cost of carrying a unit at retailer i is at least as large as the cost of carrying it in the supplier's warehouse. We also assume, without loss of generality, that in a decentralized setting the transportation cost c_i is borne by the retailer. The above-specified costs do not include any transfer payments between the supplier and the retailers.

3. THE CENTRALIZED SOLUTION

In this section, we analyze the system, assuming that a central planner makes all decisions regarding retailer prices, sales volumes, and replenishment strategies so as to maximize supply-chain-wide profits.

The vector of retailer prices p uniquely determines the vector of sales volumes q and vice versa via the direct and inverse demand functions. Thus, in a *centralized* setting, aggregate chainwide profits may be expressed as a function of p or as a function of q, and it is immaterial which of the two functions is optimized. This is in sharp contrast to the decentralized system described in §4 in which retailer competition in price space may result in equilibria quite distinct from those achieved under competition in quantity space.

The revenue component and the variable transportation and account management costs can all easily be expressed in terms of the vector q: The revenue term is given by $\sum_{i=1}^N f_i(q) q_i$, the variable order/transportation costs by $\sum_{i=1}^N (c_0 + c_i) q_i$, and the account management costs are $\sum_{i=1}^N \psi(q_i)$.

This leaves us with the specification of the inventory and fixed delivery costs, the only components which depend on the supply-chain-wide replenishment strategy. Even with a given vector of demand rates q, it is exceedingly difficult to identify a replenishment strategy which minimizes these costs, let alone to express the optimal cost value as a simple analytical function of a limited set of decision variables. Until Roundy's (1985) seminal paper, this very problem remained poorly understood. Roundy showed, however, that while a fully optimal replenishment strategy is intractable, a near-optimal solution exists within the class of so-called power-of-two policies. Under a power-of-two policy, each facility gets replenished when its inventory level is down to zero, replenishments come after constant intervals and these intervals are chosen as power-of-two multiples of a given base period T_b; i.e., they are part of the discrete set $\{2^m T_b\colon m=-\infty,\dots,-1,0,1,\dots\}$. A power-of-two policy is thus fully characterized by the vector of replenishment intervals $T=(T_0,T_1,\dots,T_N)$, with T_0 the interval used to replenish the supplier and T_i to make deliveries to retailer i, $i=1,\dots,N$. Roundy (1985) showed that under a power-of-two policy, with interval vector T, the systemwide cost is given by the following relatively simple analytical expression:

$$\frac{K_0}{T_0}+\sum_{i=1}^{N}\left\{\frac{K_i}{T_i}+\frac{1}{2}h_0 q_i\max\{T_0,T_i\}+\frac{1}{2}h_i q_i T_i\right\}. \tag{6}$$

Thus,

$$\overline{C}(q)=\min\left\{\frac{K_0}{T_0}+\sum_{i=1}^{N}\left\{\frac{K_i}{T_i}+\frac{1}{2}h_0 q_i\max\{T_0,T_i\}+\frac{1}{2}h_i q_i T_i\right\}\;\middle|\; T_i=2^{m_i}T_b, m_i\in Z, i=1,\dots,N\right\} \tag{7}$$

represents the cost of the *best* power-of-two policy. Roundy (1985) showed, in addition, that the unconstrained minimization of (6) over *all* vectors T results in a lower bound for the cost of a *fully optimal* strategy:

$$\underline{C}(q)=\min\left\{\frac{K_0}{T_0}+\sum_{i=1}^{N}\left\{\frac{K_i}{T_i}+\frac{1}{2}h_0 q_i\max\{T_0,T_i\}+\frac{1}{2}h_i q_i T_i\right\}\;\middle|\; T>0\right\}. \tag{8}$$

Moreover, $\overline{C}(q)\leqslant 1.06\underline{C}(q)$.

We conclude that, while it is impossible to compute the supply-chain-wide profits under a *fully* optimal (replenishment) strategy, let alone to express the optimal value Π_{SC}^{opt} as an analytical function, Π_{SC}^{opt} can be approximated very closely from below and above. Let

$$\overline{\Pi}_{SC}(q)=\sum_{i=1}^{N}f_i(q)q_i-\sum_{i=1}^{N}(c_0+c_i)q_i-\sum_{i=1}^{N}\psi(q_i)-\underline{C}(q), \tag{9}$$

$$\underline{\Pi}_{SC}(q)=\sum_{i=1}^{N}f_i(q)q_i-\sum_{i=1}^{N}(c_0+c_i)q_i-\sum_{i=1}^{N}\psi(q_i)-\overline{C}(q). \tag{10}$$

Then,

$$\max_{q\in Q}\underline{\Pi}_{SC}(q)\stackrel{\text{def}}{=}\underline{\Pi}_{SC}\leqslant\Pi_{SC}^{\text{opt}}\leqslant\overline{\Pi}_{SC}\stackrel{\text{def}}{=}\max_{q\in Q}\overline{\Pi}_{SC}(q). \tag{11}$$

There exists a vector $q^l[q^u]$ which achieves the maximum to the left [right] of (11). This follows from the compactness of Q and the continuity of $\underline{\Pi}_{SC}(\cdot)$ and $\overline{\Pi}_{SC}(\cdot)$, a property which is immediate from the following characterization of $\overline{C}$ and $\underline{C}$:

Lemma 1. (a) *$\overline{C}$ and $\underline{C}$ are jointly concave.*

(b) *$\overline{C}[\underline{C}]$ is differentiable, almost everywhere on Q, i.e., whenever problem* (7) [(8)] *has a unique minimum $T^l(q)[T^u(q)]$, and for all $i=1,\dots,N$*

$$\frac{\partial\overline{C}}{\partial q_i}=\frac{1}{2}h_0\max\{T_0^l(q),T_i^l(q)\}+\frac{1}{2}h_i T_i^l(q),$$

$$\frac{\partial\underline{C}}{\partial q_i}=\frac{1}{2}h_0\max\{T_0^u(q),T_i^u(q)\}+\frac{1}{2}h_i T_i^u(q).$$

Proof. See the Appendix for the proof. □

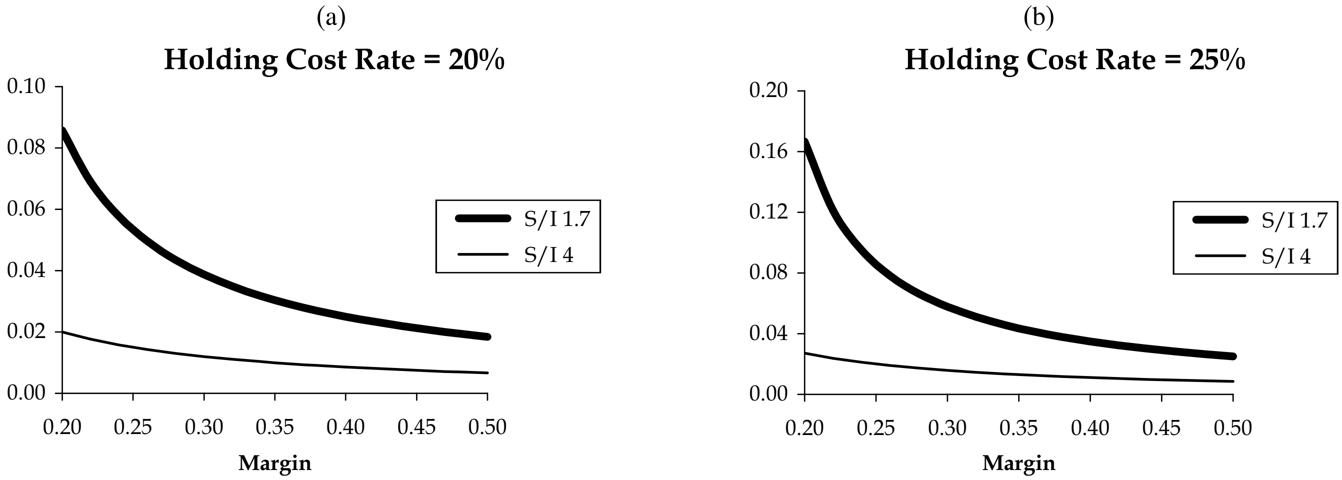
We now show that the bounds $\underline{\Pi}_{SC}$ and $\overline{\Pi}_{SC}$ tend to be very close. Write $\Pi_{SC}^{\text{opt}}=\text{grosspr}^{\text{opt}}-\text{cost}^{\text{opt}}$. (Here, the *grosspr* term is defined as the gross profits, i.e., sales minus variable costs minus account management costs, and the *cost* term refers to setup and inventory holding costs only.) Using the optimality gap results in Roundy (1985), it is easily verified that

$$\frac{\underline{\Pi}_{SC}}{\Pi_{SC}^{\text{opt}}}\geqslant 1-\frac{0.06}{(\text{grosspr}^{\text{opt}}/\text{cost}^{\text{opt}})-1}, \tag{12}$$

$$\frac{\overline{\Pi}_{SC}}{\Pi_{SC}^{\text{opt}}}\leqslant 1+\frac{0.06}{(\text{grosspr}^{\text{opt}}/\text{cost}^{\text{opt}})-1}. \tag{13}$$

While (12) and (13) do not result in an *absolute worst-case* gap for the two bounds, the gaps are very small for most product lines with reasonable *gross* profit margins. For example, we have computed a lower bound for the annual sales-to-inventory ratio for a centralized supply chain in 10 consumer goods categories, assuming the supplier's and the retailers' individual sales-to-inventory ratios are above the product category's lower quartiles reported for the wholesale and retail sectors in Dun and Bradstreet (2000–2001), respectively. This lower bound varies between 1.7 and 4.0.[1] This implies a lower bound for the average ratio $\text{grosspr}^{\text{opt}}/\text{cost}^{\text{opt}}$ between 1.85 and 4.24, assuming an average gross profit margin of 32%[2] and an annual inventory carrying cost rate of no more than 30%. Thus, the right-hand side of (12) [(13)] varies between 0.93 [1.02] and 0.98 [1.07]. Alternatively, the U.S. Census Bureau reports on retailer gross margins for 30 retail sectors; see Table 7 in U.S. Census Bureau (2002) which reports these values for the years 1993–2000. The average across all retail sectors is close to 30% and only in 17 out of the 240 cases is the gross margin lower than 20%. These rare exceptions represent high-volume sectors (e.g., the "Warehouse Clubs and Superstores" sector) where

Figure 1. Effects of gross profits on bounds (12) and (13).



sales-to-inventory ratios are *high*, offsetting the impact of a relatively low gross profit margin.

Figure 1 exhibits how the optimality gap $0.06/[(\text{grosspr}^{\text{opt}}/\text{cost}^{\text{opt}})-1]$ varies as a function of the gross retail profit margins, based on the U.S. Census data taken to vary between 20% and 50%. Figure 1(a) assumes an inventory cost rate (per sales dollar) of 20% and Figure 1(b) of 25%. Each figure shows two curves, one for a sales-to-inventory ratio of 1.7 and one for a ratio of 4. The optimality gap becomes significant only when an exceptionally low profit margin arises in conjunction with a unusually low sales-to-inventory ratio, while, in practice, low margins tend to arise under high sales-to-inventory ratios.

To compute $\overline{\Pi}_{SC}$ and $\underline{\Pi}_{SC}$ and the corresponding pair of optimizing vectors (q^u, T^u) and (q^l, T^l), first note that the common term in $\overline{\Pi}_{SC}(q)$ and $\underline{\Pi}_{SC}(q)$ can be evaluated straightforwardly for any $q \in Q$; $\underline{C}(q)$ and $\overline{C}(q)$ represent Roundy's (1985) proposed lower and upper bound for the one-warehouse multiple-retailer model with fixed demand rates. These can be evaluated in $O(N \log N)$ time for any $q \in Q$, using Roundy's algorithm (a later refinement by Queyranne (1987) shows that $\underline{C}(\cdot)$ and $\overline{C}(\cdot)$ can, in fact, be evaluated in $O(N)$ time). Moreover, Lemma 1 shows that $\underline{C}(\cdot)$ and $\overline{C}(\cdot)$ are differentiable almost everywhere, with a gradient which is easily computed in the process of calculating $\underline{C}(\cdot)$ and $\overline{C}(\cdot)$. These observations allow for the efficient usage of a gradient-based, standard unconstrained optimization algorithm to compute $\overline{\Pi}_{SC}$ and $\underline{\Pi}_{SC}$ (see, e.g., Dennis and Schnabel 1989).

4. THE DECENTRALIZED SYSTEM

In this section, we consider a decentralized system in which each retailer is responsible for his own price and sales volume decisions as well as his own replenishment strategy, while the supplier selects a wholesale pricing scheme as well as her replenishment policy in response to the retailer orders. We start with an analysis of the system under a simple *linear* wholesale pricing scheme, i.e., where retailer i is charged a constant per-unit wholesale price w_i.

4.1. The Retailer Game Under Linear Wholesale Pricing Schemes

First, assume that the retailers are engaged in price or Bertrand competition. Under a linear wholesale pricing scheme, it is clearly optimal for each retailer to replenish his inventory when it drops to zero, and at constant intervals of length T_i, say. Thus, assuming that all retailers simultaneously choose their prices and replenishment strategies, this gives rise to the following profit function for retailer i:

$$\pi_i(p_i, T_i \mid p_{-i}, w_i) = (p_i - c_i - w_i)\left(a_i - b_i p_i + \sum_{j\neq i} \beta_{ij} p_j\right) - \frac{K_i^r}{T_i} - \frac{1}{2} d_i(p)\bar{h}_i T_i, \tag{14}$$

where $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N)$. For the sake of notational simplicity, we initially assume that $\bar{h}_i$ is independent of the wholesale price w_i. In many settings, holding costs include capital costs, in which case $\bar{h}_i$ should be modeled as an *increasing* function of w_i: $\bar{h}_i(w_i)$. As shown at the end of this subsection, all of the results for the decentralized system continue to apply in this general setting.

Note from (14) that while retailer i's price p_i has an impact on the profits achieved by all retailers, his replenishment strategy affects only his *own* profits. This observation permits us to view the noncooperative retailer game as one in which each retailer competes with a *single* instrument or decision variable, i.e., his retailer price, and with simplified profit functions obtained by replacing each variable T_i with his optimal EOQ value

$$\pi_i(p_i \mid p_{-i}, w_i) = (p_i - c_i - w_i)\left(a_i - b_i p_i + \sum_{j\neq i} \beta_{ij} p_j\right) - \sqrt{2 d_i(p)\bar{h}_i K_i^r}. \tag{15}$$

In general, these profit functions fail to exhibit any known structural properties to ensure that a Nash equilibrium exists in the retailer game, let alone that this equilibrium be unique. We will show, however, that a unique equilibrium can be guaranteed in most, if not all, realistic markets assuming sales-to-inventory ratios are not excessively low and demand elasticities are not excessively large (in absolute value). More specifically, we introduce the following condition. Let

$$\begin{aligned}\mathrm{INV}_i &= \sqrt{2d_i(p)\overline{h}_i K_i^r}\\ &= \text{optimal total inventory and setup cost}\\ &\quad\ \text{for retailer } i \text{ under the price vector } p,\\ \mathrm{REV}_i &= p_i d_i(p)\\ &= \text{total gross revenue for retailer } i\\ &\quad\ \text{under the price vector } p,\end{aligned}$$

$$\text{(C1)}\quad \epsilon_{ii} \leqslant \frac{8\times\mathrm{REV}_i}{\mathrm{INV}_i}.$$

Even if the annual inventory carrying cost is as large as 40% of the dollar value of the inventory (a comfortable upper bound, in practice), the ratio $\mathrm{REV}_i/\mathrm{INV}_i$ is at least 2.5 times the (annual) sales-to-inventory ratio. For the sample of 10 consumer product lines mentioned in §3, the average lower quartile of the retailers' sales-to-inventory ratios varies between 2.8 and 6.7. Thus, for retailers in any of these sectors, with a sales-to-inventory value above the lower quartiles, the right-hand side of (C1) is bounded from below by 56, and in some product lines by 134. Compare these values with estimated elasticities of demand, which vary between 1.4 and 2.8. (These estimates are obtained from Tellis 1988.)

Condition (C1) is equivalent to the inequality $d_i(p)^{3/2} \geqslant \frac{1}{8}b_i\sqrt{2\bar{h}_i K_i^r}$, or $d_i(p) \geqslant \alpha_i \stackrel{\text{def}}{=} \frac{1}{4}(b_i\sqrt{2\bar{h}_i K_i^r})^{2/3}$. Thus, in vector notation, (C1) holds on the compact polyhedron $P = \{p \geqslant 0\colon a + Bp \geqslant \alpha\}$. This polyhedron is a Leontief-type polyhedron, and has a largest element $\bar{p}$, i.e., for all $p \in P$, $p \leqslant B^{-1}(\alpha - a) \stackrel{\text{def}}{=} \bar{p}$. The last inequality follows because $(B^{-1})_{ii} = -\hat{b}_i \leqslant 0$ and $(B^{-1})_{ij} = -\hat{\beta}_{ij} \leqslant 0$, by (5). Because, as demonstrated above, (C1) holds in equilibrium in almost all practical settings, we have that $\bar{p} > 0$ in all such cases. We henceforth assume that the cube $X_{i=1}^N[p_i^{\min}, p_i^{\max}]$ is contained in P.

THEOREM 1. *Assume* (C1) *applies. Then, the retailer game under Bertrand competition has a Nash equilibrium.*

PROOF. In view of Friedman (1977), it suffices to show that each of the profit functions $\pi_i(p_i \mid p_{-i}, w_i)$ is concave in p_i. It is easily verified that

$$\begin{aligned}&\frac{\partial^2\pi_i(p_i \mid p_{-i}, w_i)}{\partial p_i^2} = -2b_i + \frac{b_i^2}{4}\frac{\sqrt{2K_i^r\bar{h}_i d_i(p)}}{d_i^2(p)} \leqslant 0\\ &\iff \frac{\mathrm{INV}_i}{d_i^2(p)} \leqslant \frac{8}{b_i}\\ &\iff \epsilon_{ii} = \frac{p_i b_i}{d_i(p)} \leqslant \frac{8p_i d_i(p)}{\mathrm{INV}_i} = \frac{8\ \mathrm{REV}_i}{\mathrm{INV}_i}. \quad \square\end{aligned}$$

Several questions remain regarding the retailer game. First, we would like to know whether the Nash equilibrium is unique, thus guaranteeing a fully predictable (equilibrium) behavior by the retailers. In the presence of multiple equilibria, it is, in general, difficult to predict which of the equilibria will be adopted (see, e.g., Harsanyi and Selten 1988). Second, we expect an increase in the wholesale prices to result in an increase of all equilibrium retailer prices. Finally, we would like to know whether an effective procedure exists to compute the Nash equilibrium (or equilibria). All these questions can be answered in the affirmative, if the retailer game can be shown to be *supermodular* (see Topkis 1998 and Milgrom and Roberts 1990 for a precise definition and detailed discussion). In our context, the set of feasible decisions for each retailer i is a simple closed interval $[p_i^{\min}, p_i^{\max}]$ and, hence, a compact set. This implies that the retailer game is supermodular if, for each retailer i, the profit difference

$$\begin{aligned}&\pi_i(p_i^1 \mid p_{-i}, w_i) - \pi_i(p_i^2 \mid p_{-i}, w_i)\\ &\quad \text{is increasing in all } p_j, j \neq i, \quad \text{for any } p_i^1 > p_i^2. \qquad (16)\end{aligned}$$

This property is satisfied under the following slight strengthening of condition (C1):

$$\text{(C2)}\quad \epsilon_{ii} \leqslant \frac{4\times\mathrm{REV}_i}{\mathrm{INV}_i}.$$

As discussed above, (C2), like (C1), is satisfied in virtually all realistic markets and the set of prices on which (C2) applies is again a compact polyhedron P'. We now assume that $X_{i=1}^N[p_i^{\min}, p_i^{\max}] \subset P'$.

It has been well known since Topkis (1998) that if the game is supermodular, the following simple *tatônnement* scheme converges to a Nash equilibrium. In the kth iteration of this scheme, each retailer i determines a price $p_i^{(k)}$ which maximizes his own profit function $\pi_i(\cdot \mid p_{-i}^{(k-1)}, w_i)$ assuming all other retailers maintain their prices according to the current vector $p^{(k-1)}$ obtained in the $(k-1)$st iteration. This gives rise to a new vector $p^{(k)}$.

THEOREM 2. *Assume condition* (C2) *applies.*

(a) *The retailer game is supermodular and has a unique Nash equilibrium* p^*.

(b) *The tatônnement algorithm converges to* p^* *for every starting point.*

(c) p^* *is increasing in* w.

PROOF. Part (a): It suffices to show that (16) applies. Because the profit functions $\pi_i(\cdot)$ are twice differentiable, (16) is equivalent to $\partial^2\pi_i/(\partial p_j \partial p_i) \geqslant 0$ for all $i \neq j$.

Note that

$$\begin{aligned}&\frac{\partial^2\pi_i}{\partial p_j \partial p_i} = \beta_{ij} - \frac{b_i\beta_{ij}\sqrt{2K_i^r\bar{h}_i d)i(p)}}{4d_i^2(p)} \geqslant 0\\ &\quad\Leftrightarrow\quad \frac{\mathrm{INV}_i}{d_i^2(p)} \leqslant \frac{4}{b_i} \quad\Leftrightarrow\quad \epsilon_{ii} = \frac{p_i b_i}{d_i(p)}\\ &\quad\leqslant \frac{\sqrt{2K_i^r\bar{h}_i d_i(p)}}{p_i d_i(p)} = \frac{4\,\mathrm{REV}_i}{\mathrm{INV}_i}.\end{aligned}$$

Milgrom and Roberts (1990), generalizing an earlier condition by Friedman (1977), showed that a *unique* Nash equilibrium arises in a supermodular game if for all $i = 1, \dots, N$,

$$2b_i - \frac{b_i^2 \,\mathrm{INV}_i}{4d_i^2(p)} = -\frac{\partial^2 \pi_i}{(\partial p_i)^2} > \sum_{j \neq i} \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}$$
$$= \sum_{j\neq i} \beta_{ij} - \sum_{j \neq i} \frac{b_i \beta_{ij}\,\mathrm{INV}_i}{4d_i^2(p)}$$

which holds if and only if

$$b_i + \left(b_i - \sum_{j\neq i} \beta_{ij}\right)\left(1 - \frac{b_i\,\mathrm{INV}_i}{4d_i^2(p)}\right) > 0.$$

Observe now that $b_i > 0$, $b_i - \sum_{j\neq i} \beta_{ij} > 0$ by (D), while

$$\frac{b_i\,\mathrm{INV}_i}{4d_i^2(p)} \leqslant 1,$$

because by (C2),

$$\epsilon_{ii} = \frac{b_i p_i}{d_i(p)} \leqslant 4\frac{\mathrm{REV}_i}{\mathrm{INV}_i} = \frac{4p_i d_i(p)}{\mathrm{INV}_i}.$$

Because the retailer game is supermodular, part (b) is immediate and part (c) follows from $\partial^2 \pi_i/(\partial p_i \partial w_i) = b_i \geqslant 0$ for all $i = 1, \dots, N$. □

It is noteworthy that, while the equilibrium retailer prices respond monotonically to the wholesale price(s), the same *cannot* be guaranteed for the sales volumes and profits, except in the special case where retailer competition is absent, i.e., where each retailer's demand function depends on his own price only. This can be demonstrated with a simple example (see Bernstein et al. 2002).

We next consider the case where the retailers are engaged in quantity or Cournot competition. As before, it is possible to view the retailer game as one in which each retailer i competes with his *sales volume* q_i as the *single* instrument or decision variable. The retailers' profits, expressed as a function of the vector q, are easily obtained from (15), making all appropriate substitutions:

$$\pi_i^C(q_i \mid q_{-i}, w_i) = \left(\hat{a}_i - \hat{b}_i q_i - \sum_{j\neq i} \hat{\beta}_{ij} q_j - c_i - w_i\right) q_i - \sqrt{2q_i \overline{h}_i K_i^r}. \tag{17}$$

Once again, in general these profit functions fail to exhibit any known structural properties to ensure that a Nash equilibrium exists in the retailer game. As in the case of Bertrand competition, an equilibrium can, however, be guaranteed under (C1), and a *unique* equilibrium under (C2), i.e., in most markets with realistic sales-to-inventory ratios and demand elasticities.

THEOREM 3. (a) *Assume* (C1) *applies. Then the retailer game under Cournot competition has a Nash equilibrium.*

(b) *The Nash equilibrium q^* is unique under* (C2) *and* (D'): $\hat{b}_i > \sum_{j\neq i} \hat{\beta}_{ij}$ (*the counterpart of* (D)).

PROOF. In view of Friedman (1977), it suffices again to show that each of the profit functions $\pi_i^C(q_i \mid q_{-i}, \mathbf{w})$ is concave in q_i. It is easily verified that

$$\frac{\partial^2 \pi_i^C(q_i \mid q_{-i}, w_i)}{\partial q_i^2} = -2\hat{b}_i + \frac{1}{4}\frac{\mathrm{INV}_i}{q_i^2}$$

so that

$$\frac{\partial^2 \pi_i^C(q_i \mid q_{-i}, w_i)}{\partial q_i^2} < 0$$

if and only if

$$\frac{1}{4}\frac{\mathrm{INV}_i}{q_i^2} < 2\hat{b}_i$$

which holds if and only if

$$\hat{\epsilon}_{ii} < 8\frac{\mathrm{REV}_i}{\mathrm{INV}_i}.$$

By Proposition 6.1 in Vives (2000),

$$\hat{\epsilon}_{ii} < \epsilon_{ii} < 8\frac{\mathrm{REV}_i}{\mathrm{INV}_i},$$

where the second inequality follows from (C1).

To prove part (b), again following Friedman (1977), it is sufficient to show that

$$2\hat{b}_i - \frac{1}{4}\frac{\mathrm{INV}_i}{q_i^2} = -\frac{\partial^2 \pi_i^C(q_i \mid q_{-i}, w_i)}{\partial q_i^2}$$
$$> \sum_{j\neq i}\left|\frac{\partial^2 \pi_i^C(q_i \mid q_{-i}, w_i)}{\partial q_i \partial q_j}\right| = \sum_{j\neq i} \hat{\beta}_{ij},$$

or equivalently, that

$$\hat{b}_i - \sum_{j\neq i} \hat{\beta}_{ij} + \hat{b}_i - \frac{1}{4}\frac{\mathrm{INV}_i}{q_i^2} > 0$$

which follows from

$$\hat{b}_i > \sum_{j\neq i} \hat{\beta}_{ij} \quad \text{and} \quad \hat{\epsilon}_{ii} < \epsilon_{ii} < 4\frac{\mathrm{REV}_i}{\mathrm{INV}_i},$$

where the last inequality follows from (C2). □

It should be noted that the Cournot game fails to be supermodular even under (C2). As a consequence, and in contrast with Theorem 2(b) and (c), we cannot guarantee that the simple tatônnement scheme converges to q^* or that the equilibrium sales volumes are monotone in the wholesale price. Alternative methods need to be invoked to compute q^* as the unique solution of the system of equations $\partial \pi_i^C/\partial q_i = 0$, $i = 1, \dots, N$.

Finally, in the general model where $\overline{h}_i$ is an increasing function of w_i, it is easily verified that all of the above results continue to apply. This is immediate for all of Theorems 1–3 and Proposition 1, except for Theorem 2(c). Assuming $\overline{h}_i(\cdot)$ is differentiable with derivative $\overline{h}_i'$, the proof of Theorem 2(c) generalizes, now with

$$\frac{\partial^2 \pi_i}{\partial p_i \partial w_i} = b_i + b_i \bar{h}_i' \frac{1}{2}\sqrt{\frac{K_i^r}{2\bar{h}_i d_i}} > 0.$$

4.2. A Comparison Between Price and Quantity Competition

We complete this section with a brief comparison between the Bertrand and Cournot equilibria. Let $p^C = f(q^*)$ denote the price vector under the Cournot equilibrium and $q^B = d(p^*)$ the demand volume vector under the Bertrand equilibrium.

Proposition 1. *Assume condition* (C2) *applies. Then,* $p^C \geqslant p^*$.

Proof. By the proof of Theorem 2, the retailer game is supermodular under Bertrand competition and the profit functions $\pi_i(p_1, \ldots, p_N)$ are concave in p_i for all $i = 1, \ldots, N$. In view of the proof of Proposition 6.2 in Vives (2000), it thus suffices to show that $\partial \pi_i(p^C)/\partial p_i \leqslant 0$. Note that

$$\frac{\partial \pi_i}{\partial p_i} = d_i(p) + (p_i - c_i - w_i)(-b_i) + \frac{2b_i \bar{h}_i K_i^r}{2\,\mathrm{INV}_i(p)}$$

$$= d_i(p) - b_i(p_i - c_i - w_i) + \frac{b_i\,\mathrm{INV}_i(p)}{2d_i(p)}.$$

Substituting $q_i = d_i(p)$, we obtain

$$\frac{\partial \pi_i(p^C)}{\partial p_i} = q_i^* - b_i(f_i(q^*) - c_i - w_i) + \frac{b_i\,\mathrm{INV}_i(q^*)}{2q_i^*}. \qquad (18)$$

Because q^* is the Nash equilibrium in the Cournot game, it satisfies the first-order condition

$$0 = \frac{\partial \pi_i^C(q_i^* \mid q_{-i}^*, w)}{\partial q_i}$$

$$= -\hat{b}_i q_i^* + (f_i(q^*) - c_i - w_i) - \frac{\bar{h}_i K_i^r}{\mathrm{INV}_i(q^*)}$$

$$= -\hat{b}_i q_i^* + (f_i(q^*) - c_i - w_i) - \frac{\mathrm{INV}_i(q^*)}{2q_i^*}. \qquad (19)$$

Substituting (19) into (18), we conclude that $\partial \pi_i(p^C)/\partial p_i = q_i^* - b_i \hat{b}_i q_i^* = (1 - b_i \hat{b}_i) q_i^* \leqslant 0$, because $b_i \hat{b}_i > 1$ as $1 = (BB^{-1})_{ii} = b_i \hat{b}_i - \sum_{j \neq i} \beta_{ij} \hat{\beta}_{ji}$. $\square$

Thus, if the retailers compete in quantity space, each adopts a retail price that is larger than its equilibrium retail price under price competition. One might conjecture that, similarly $q^* \leqslant q^B$. A numerical example in §6 shows, however, that this relationship may fail to hold. Larger sales volumes, under price competition, can only be guaranteed in special cases, e.g., when the retailers are identical so that the equilibria are symmetric. (In this case, $p_i^* = p^*$ and $p_i^C = p^C$ for all $i = 1, \ldots, N$ so that for all $i = 1, \ldots, N$, $q_i^B = a_i - (b_i - \sum_{j \neq i} \beta_{ij}) p^* \geqslant a_i - (b_i - \sum_{j \neq i} \beta_{ij}) p^C = q_i^*$.) On the other hand, if $q^* \leqslant q^B$ is satisfied, it follows that each retailer realizes lower profits under price competition as compared to quantity competition; for all $i = 1, \ldots, N$: $\pi_i(p^*) = \pi_i^C(q^B) = \pi_i^C(q_i^B, q_{-i}^B) \leqslant \pi_i^C(q_i^B, q_{-i}^*) \leqslant \pi_i^C(q_i^*, q_{-i}^*) = \pi_i^C(q^*)$, where the first inequality follows if $q^* \leqslant q^B$ given that $\partial \pi_i^C/\partial q_j < 0$ for all $j \neq i$, while the second inequality follows from the fact that q^* is a Nash equilibrium in the Cournot game.

5. COORDINATION WITH THE SUPPLIER

We now investigate the performance of the *complete* supply chain under a linear wholesale pricing scheme, characterized by a vector of wholesale prices w. Even if this wholesale price vector is chosen to *optimize* supply-chain-wide profits, the resulting aggregate profit value is likely to be disappointing in the absence of any additional procedures to coordinate the supplier's replenishment activities with those of the retailers. Recall from §4.1 that it is optimal for each retailer to replenish his stock with constant replenishment intervals and, without any upfront restrictions, these intervals will be set according to the EOQ formula. In general, the resulting order stream for the supplier is highly nonstationary, fails to follow any simple, periodically repeating pattern and represents a difficult managerial problem for the supplier. No satisfactory solution is known for the corresponding inventory problem. Moreover, the supplier's costs are, in general, much higher than if orders arrived according to a simple pattern, e.g., if all retailers were required to choose their replenishment intervals from the discrete set of power-of-two values $\{2^m T_b\colon m = -\infty, \ldots, -1, 0, 1, \ldots, \infty\}$.

Recall from §3 that even in a *centralized* setting the proposed (heuristic) strategy is based on all channel members restricting their consecutive replenishment intervals from the set of power-of-two values. With this restriction, we showed that the best achievable strategy comes very close to an upper bound for the optimal systemwide profits; see (12) and (13) and the subsequent discussion there.

We therefore proceed to consider coordination mechanisms, based on an upfront "contract" (see, e.g., Tirole 1988), specifying that all channel members agree to choose each interval between consecutive replenishments from the above discrete set of power-of-two values. While this restriction involves major benefits for the supplier, it comes at *minimal* additional expense to the retailers. (See, e.g., Brown 1959, Roundy 1985, and the discussion below.) As a consequence, the power-of-two value restriction should be easily agreed upon by the channel members. In the worst case, the supplier may offer an annual rebate to the retailers equal to the *modest* increase in their inventory and setup costs resulting from the interval restriction. (Such rebates are most easily computed and clearly do not affect the supply-chain-wide profits.) With this restriction, we first revisit the retailer game that arises under an arbitrary linear wholesale pricing scheme, specified by a wholesale vector w. We consider this restricted game first under Bertrand competition. (As in the unrestricted game, retailers choose prices and replenishment strategies simultaneously.) Note that the "new" profit function $\hat{\pi}_i(\cdot \mid p_{-i}, w)$ for retailer i is obtained from (14) by replacing T_i by the power-of-two value $2^{m_i} T_b$ which is closest, in the relative sense, to the EOQ value. Clearly, $\hat{\pi}_i \leqslant \pi_i$, $i = 1, \ldots, N$.

In the restricted retailer game, no conditions appear to prevail to ensure that the game has any of the known structures guaranteeing existence of an equilibrium: for example, the rounding procedure introduces violations of both concavity and supermodularity at various discrete points on the parameter spectrum. Under a linear wholesale pricing scheme, we are therefore only able to guarantee a so-called δ-equilibrium, a concept introduced by Radner (1980) (see also Fudenberg and Tirole 1991, Def. 4.3).

Definition 1. In a Bertrand [Cournot] retailer game, the vector p^{eq} [q^{eq}] is a δ-Nash equilibrium for some $\delta > 0$, if no retailer $i = 1, \ldots, N$ can improve his profit value by more than a δ-fraction through a (unilateral) change of the retailer price p_i^{eq} [quantity q_i^{eq}].

(In Fudenberg and Tirole 1998, δ refers to the *absolute* amount by which a player's profit can be improved by a unilateral deviation from the equilibrium.)

Proposition 2. *Fix a wholesale price vector w. Assume condition* (C1) *applies, so that a Nash equilibrium p^* $[q^*]$ exists for the continuous game under Bertrand* [*Cournot*] *competition. Then, p^* $[q^*]$ is a δ-Nash equilibrium in the corresponding restricted retailer game, with*

$$\delta = \frac{0.06}{\min_{i=1,\ldots,N}\{(\mathrm{grosspr}_i^*/\mathrm{cost}_i^*)\} - 1.06},$$

where $\mathrm{grosspr}_i^*$ *denotes retailer i's annual gross revenues minus variable purchase and transportation costs, and* cost_i^* *his annual setup and holding costs in the continuous game under p^* $[q^*]$.*

Proof. We give the proof for the case of Bertrand competition. Fix $i = 1, \ldots, N$. It suffices to show that for all p_i-values $\hat{\pi}_i(p_i \mid p_{-i}^*, w_i) \leqslant (1+\delta)\hat{\pi}_i(p_i^* \mid p_{-i}^*, w_i)$.

Let $\mathrm{cost}_i' = K_i^r/t_{m_i} + \frac{1}{2}d_i(p^*)\bar{h}_i t_{m_i}$, the optimal annual setup and inventory holding costs for retailer i, under the power-of-two interval restriction, assuming the retailers adopt the price vector p^*. It is well known that $\mathrm{cost}_i' \leqslant 1.06\mathrm{cost}_i^*$. Note that

$$\begin{aligned}
\hat{\pi}_i(p_i \mid p_{-i}^*, w_i) &\leqslant \pi_i(p_i \mid p_{-i}^*, w_i) \leqslant \pi_i(p_i^* \mid p_i^*, w_i)\\
&= \mathrm{grosspr}_i^* - \mathrm{cost}_i^*\\
&= (\mathrm{grosspr}_i^* - \mathrm{cost}_i')\left(1 + \frac{\mathrm{cost}_i' - \mathrm{cost}_i^*}{\mathrm{grosspr}_i^* - \mathrm{cost}_i'}\right)\\
&\leqslant \hat{\pi}_i(p_i^* \mid p_{-i}^*, w_i)\left(1 + \frac{0.06\mathrm{cost}_i^*}{\mathrm{grosspr}_i^* - \mathrm{cost}_i'}\right)\\
&\leqslant \hat{\pi}_i(p_i^* \mid p_{-i}^*, w_i)(1+\delta).
\end{aligned}$$

(The second inequality follows from p^* being a Nash equilibrium in the continuous game.) $\square$

Example 1. Consider a market with $N = 2$ identical retailers, with common demand function $d_i(p) = 640 - 17p_i + 4p_{3-i}$, $i = 1, 2$. Let $w_i + c_i = 16$, $K_i = 800$, $\bar{h}_i = 16$, $i = 1, 2$. Assume $T_b = 1$, $p^{\min} = 30$, and $p^{\max} = 40$, so that $80 \leqslant d_i(p) \leqslant 290$ because $d_i(p)$ is maximized (minimized) when $p_i = p^{\min}(p^{\max})$ and $p_{3-i} = p^{\max}(p^{\min})$. Condition (C2) is equivalent to

$$\frac{b_i\sqrt{2\bar{h}_i K_i^r}}{4} < [d_i(p)]^{3/2}, \quad i = 1, 2,$$

the right-hand side of which is minimized when $d_i(p) = 80$. Thus, because this inequality is satisfied when $d_i(p) = 80$, (C2) is satisfied throughout, ensuring concavity as well as supermodularity of the retailers' profit functions in the continuous game. To show that the retailers' profit function $\hat{\pi}_i(\cdot \mid p_{3-i}, w_i)$ fails to be concave, let $p_{3-i} = 35$ and consider the profit function on the interval $[32, 35]$:

$$\begin{aligned}
&\hat{\pi}_i(34 \mid p_{3-i} = 35, w_i) = 1{,}228\\
&\quad < \frac{1}{3}\hat{\pi}_i(32 \mid p_{3-i} = 35, w_i) + \frac{2}{3}\hat{\pi}_i(35 \mid p_{3-i} = 35, w_i)\\
&\quad = 1{,}234.
\end{aligned}$$

To show that the function fails to be supermodular, note that $\hat{\pi}_i(35 \mid p_{3-i} = 35, w_i) - \hat{\pi}_i(32 \mid p_{3-i} = 35, w_i) = 1{,}235 - 1{,}232 = 3 < \hat{\pi}_i(35 \mid p_{3-i} = 32, w_i) - \hat{\pi}_i(32 \mid p_{3-i} = 32, w_i) = 1{,}103 - 1{,}088 = 15$. Observe that with $p_{3-i} = 35$ and $p_i = 32$ it is optimal to choose $T_i = 2^{-1}$ while for $p_{3-i} = 35$ and $p_i = 35$, $T_i = 1$. It is this discrete jump in the optimal replenishment interval which causes the local violation of concavity and supermodularity.

Note that on the interval of feasible demand rates $d_i(p) \in [80, 290]$, the corresponding optimal power-of-two interval values are either $T_i = 0.5$ or $T_i = 1$. A price vector (p_1^*, p_2^*) is a Nash equilibrium of the restricted game only if it is a Nash equilibrium in one of the four continuous games which arise when restricting the vector (T_1, T_2) to one of the four pairs $(0.5, 0.5), (0.5, 1), (1, 0.5)$, and $(1, 1)$ (and also the prices to four corresponding sets of intervals). Each of these four continuous games has a unique Nash equilibrium giving rise to four price vectors that are candidates for a Nash equilibrium of the restricted game. It is therefore easily verified that the restricted game has exactly *two* Nash equilibria (even though the profit functions fail to be concave or supermodular): $p^{*(1)} = (32.9, 34.7)$ and $p^{*(2)} = (34.7, 32.9)$, the equilibria corresponding with $(T_1, T_2) = (0.5, 1)$ and $(T_1, T_2) = (1, 0.5)$ and profit vector $(\pi_1, \pi_2) = (1{,}231.28, 1{,}144.42)$ and $(\pi_1, \pi_2) = (1{,}144.42, 1{,}231.28)$, respectively. It is clearly impossible to predict which of the two equilibria will be adopted in the market place (if either). The continuous game, on the other hand, has a unique (symmetric) Nash equilibrium $p^* = (33.58, 33.58)$ since satisfying (C2) (see Theorem 2) with a corresponding profit value of 1,294.50 for each retailer. This price vector is a δ-Nash equilibrium in the restricted game with $\delta = 0.0095$, a value obtained by computing $\max_{p_i} \hat{\pi}_i(p_i \mid p_{3-i}^* = 33.58, w_i)$. $\square$

The fraction δ bears close similarity to the optimality gap in (12) and (13). As shown there, δ cannot be uniformly bounded under completely general parameter values. However, it is clear that δ is very small for most product lines and markets, with reasonable gross profit margins. For example, assuming as before that the retailers' gross profits represent at least 32% of the sales and that the annual inventory carrying cost rate is 30% or less of the dollar value of the inventory, a lower bound for the ratio grosspr$_i^*$/cost$_i^*$ varies between 1.85 and 4.24 for the 10 product categories considered in §3. This results in a value of δ varying between 0.08 and 0.02.

In §6, we compare the performance of the supply chain in which the supplier chooses a wholesale price so as to maximize his own profit, with one in which a wholesale price is chosen to optimize supply-chain-wide profits. The comparison is done under a Stackelberg game. Both settings will be compared with the centralized solution. We show that even the "best" linear wholesale pricing scheme fails to result in perfect coordination. Moreover, the optimizing wholesale price vector is hard to compute, in particular when the number of retailers is large. Finally, we have shown that under a linear wholesale pricing scheme, a full equilibrium for the retailers cannot be guaranteed unless the retailers are allowed to choose their replenishment intervals continuously. On the other hand, the absence of an upfront restriction of the replenishment intervals to the set of power-of-two values comes at a severe expense for the supplier and hence for the chain as a whole.

5.1. Perfect Coordination via a Nonlinear Wholesale Pricing Scheme

We now derive a *nonlinear* wholesale pricing scheme which does result in a perfect coordination mechanism. We start with the case where the retailers compete in quantity space. The design of the coordinating pricing scheme is an application of the Groves mechanism (see Groves 1973 and Groves and Loeb 1979). The principle behind the mechanism is to *align*, for each $i = 1, \ldots, N$, the "marginal" supply-chain-wide profit function which arises when all but retailer i are committed to the sales volumes and replenishment intervals that optimize the supply-chain-wide profits, with the profit function retailer i faces in the retailer game in the decentralized chain. We will show that, in our model, the Groves mechanism results in a fairly simple pricing scheme with *three additive* discounts off a constant base price: Each of the discount components depends on a *single* retailer characteristic.

As motivated in the previous subsection, we continue to require that all chain members agree to choose their replenishment intervals from the discrete set of power-of-two values. Let (q^l, T^l) denote a pair of vectors which achieve $\underline{\Pi}_{SC}$. Fix $i = 1, \ldots, N$, and let

$$
\begin{aligned}
&\pi_{SC,i}(q_i, T_i \mid q^l_{-i}, T^l_{-i})\\
&\quad= \Big(\hat{a}_i - \hat{b}_i q_i - \sum_{j\neq i} \hat{\beta}_{ij} q^l_j\Big) q_i\\
&\qquad + \sum_{j\neq i}\Big(\hat{a}_j - \hat{b}_j q^l_j - \sum_{r\neq i,j} \hat{\beta}_{jr} q^l_r - \hat{\beta}_{ji} q_i\Big) q^l_j\\
&\qquad - [(c_0 + c_i) q_i + \psi(q_i)]\\
&\qquad - \sum_{j\neq i}\big[(c_0 + c_j) q^l_j + \psi(q^l_j)\big] - \frac{K_0}{T^l_0}\\
&\qquad - \left[\frac{K_i}{T_i} + \frac{1}{2} h_0 q_i \max\{T^l_0, T_i\} + \frac{1}{2} h_i q_i T_i\right]\\
&\qquad - \sum_{j\neq i}\left[\frac{K_j}{T^l_j} + \frac{1}{2} h_0 q^l_j \max\{T^l_0, T^l_j\} + \frac{1}{2} h_i q^l_j T^l_j\right] \qquad (20)
\end{aligned}
$$

denote the "marginal" supply-chain-wide profit function which arises when all but retailer i are committed to the sales volumes and replenishment intervals in q^l and T^l respectively, and observe that this "marginal" profit function is maximized at $q_i = q^l_i$ and $T_i = T^l_i$. Moreover, (q^l_i, T^l_i) remains a maximizer when this function is shifted in parallel by omitting terms that are constant in (q_i, T_i):

$$
\begin{aligned}
&\tilde{\pi}_{SC,i}(q_i, T_i \mid q^l_{-i}, T^l_{-i})\\
&\quad= \Big(\hat{a}_i - \hat{b}_i q_i - \sum_{j\neq i} \hat{\beta}_{ij} q^l_j\Big) q_i\\
&\qquad - \Big(\sum_{j\neq i} \hat{\beta}_{ji} q^l_j\Big) q_i - [(c_0 + c_i) q_i + \psi(q_i)]\\
&\qquad - \left[\frac{K_i}{T_i} + \frac{1}{2} h_0 q_i \max\{T^l_0, T_i\} + \frac{1}{2} h_i q_i T_i\right]. \qquad (21)
\end{aligned}
$$

Assume now that in the retailer game with a possibly nonlinear wholesale price $w_i(q_i, T_i)$, all but retailer i adopt sales volumes and replenishment intervals in accordance with the vectors q^l and T^l. The profit function for retailer i is then given by

$$
\begin{aligned}
\pi^C_i(q_i, T_i \mid q^l_{-i}, T^l_{-i}, w_i) &= \Big(\hat{a}_i - \hat{b}_i q_i - \sum_{j\neq i} \hat{\beta}_{ij} q^l_j - c_i - w_i\Big) q_i\\
&\quad - \left[\frac{K^r_i}{T_i} + \frac{1}{2} \overline{h}_i q_i T_i\right]. \qquad (22)
\end{aligned}
$$

The profit function π^C_i in the retailer game can thus be aligned *completely* with the function $\tilde{\pi}_{SC,i}$ by specifying the wholesale pricing function $w_i(q_i, T_i)$ as the sum of *three* components, one of which is a decreasing function of the retailer's order quantity, one a decreasing function of his replenishment interval, and one a decreasing function of the annual sales volume. More specifically,

$$
w^D_i(q_i, T_i) = w^{(1)}_i(T_i q_i) + w^{(2)}_i(T_i) + w^{(3)}_i(q_i), \qquad (23)
$$

where

$$
w^{(1)}_i(T_i q_i) = c_0 + \frac{K^s_i}{T_i q_i}, \qquad (24)
$$

$$
w^{(2)}_i(T_i) = \frac{1}{2} h_0 T^l_0 - \frac{1}{2} h_0 \min\{T^l_0, T_i\}, \qquad (25)
$$

$$
\begin{aligned}
w^{(3)}_i(q_i) &= \frac{\psi(q_i)}{q_i} + \sum_{j\neq i} \hat{\beta}_{ji} q^l_j\\
&= \frac{\psi(q_i)}{q_i} + Q^l\left(1 - \frac{q^l_i}{Q^l}\right)\eta_i, \qquad (26)
\end{aligned}
$$

with $Q^l = \sum_{j=1}^N q_j^l =$ total retailer sales (in the centralized solution $\underline{\Pi}_{SC}$), and

$$\eta_i = \sum_{j\neq i} \hat{\beta}_{ji} \frac{q_j^l}{\sum_{r\neq i} q_r^l}.$$

We refer to η_i as the "*competitive impact of retailer i*." It represents a measure for the competitive impact retailer i has on all other retailers, expressed as a weighted average of the marginal impact of an increase of retailer i's sales volume on the prices charged by all other retailers. (The weights are given by the relative magnitudes, in sales volumes, of these retailers.)

THEOREM 4. *Let* (q^l, T^l) *denote a vector of sales volumes and a vector of replenishment intervals under which supply-chain-wide profits equal* $\underline{\Pi}_{SC}$, *i.e., the optimal centralized solution under the power-of-two interval restriction. The pair* (q^l, T^l) *arises as a Nash equilibrium in the retailer game induced by the wholesale pricing scheme* (23). *In other words, the pricing scheme* (23) *generates a perfect coordination mechanism.*

PROOF. Note that for all $i = 1, \ldots, N$ and all (q_i, T_i)

$$\begin{aligned} &\pi_i^C(q_i^l, T_i^l \mid q_{-i}^l, T_{-i}^l, w_i^D) \\ &\quad = \tilde{\pi}_{SC,i}(q_i^l, T_i^l \mid q_{-i}^l, T_{-i}^l) \\ &\quad \geqslant \tilde{\pi}_{SC,i}(q_i, T_i \mid q_{-i}^l, T_{-i}^l) = \pi_i^C(q_i, T_i \mid q_{-i}^l, T_{-i}^l, w_i^D), \end{aligned}$$

where the inequality follows from the fact that (q^l, T^l) achieves $\underline{\Pi}_{SC}$, so that (q_i^l, T_i^l) maximizes $\pi_{SC,i}(\cdot \mid q_{-i}^l, T_{-i}^l)$ and hence $\tilde{\pi}_{SC,i}(\cdot \mid q_{-i}^l, T_{-i}^l)$ because $\pi_{SC,i}$ and $\tilde{\pi}_{SC,i}$ differ by a constant only. □

As mentioned, the first component of the wholesale pricing scheme, $w^{(1)}$, provides an incentive for the retailers to increase their order quantities $\{T_i q_i: i = 1, \ldots, N\}$. The second component $w^{(2)}$ offers a constant discount, h_0, for each additional unit of time the retailer is willing to keep a unit of his item in stock, up to a cap of $A = T_0^l$ time units. Note that the third component in the scheme offers a direct incentive to increase the sales volume. Note also that

$$w_i^D(q_i, T_i) = \left[c_0 + \frac{K_i^s}{T_i q_i} + \frac{\psi(q_i)}{q_i} + \frac{1}{2}h_0 T_0^l - \frac{1}{2}h_0 \min\{T_0^l, T_i\}\right] + Q^l\left(1 - \frac{q_i^l}{Q^l}\right)\eta_i. \tag{27}$$

Here, the term within square brackets represents all cost components (initially) incurred by the supplier that are related to retailer i's sales. These are (i) the variable procurement costs at unit rate c_0, (ii) the supplier's part of the fixed charges for deliveries between the supplier and the retailer, (iii) the cost of carrying units in the supplier's stock which are sold via retailer i, and (iv) the account management costs for retailer i. We refer to these four cost components (i)–(iv) as retailer i's *indirect costs*.

The term within square brackets is identical to the one required in a setting with noncompeting retailers, i.e., one where all cross-elasticities in the demand functions are zero (see Chen et al. 2001). The latter proved that all three discount components are indeed essential in this simpler setting of noncompeting retailers. The authors showed, in particular, that even in the absence of account management costs, no traditional discount scheme based exclusively on order quantities is capable of achieving perfect coordination, regardless of its shape or number of breakpoints. Quantity discounts based on the retailer's replenishment frequency and annual sales volume, as reflected by components $w_i^{(1)}$ and $w_i^{(2)}$ are prevalent in many industries. (See Munson and Rosenblatt 1998 and Brown and Medoff 1990. See Chen et al. 2001 for additional discussion on this issue.)

The second term in (27) represents a markup due to competition. This markup increases with η_i, retailer i's competitive impact. For a given total sales volume Q^l in the market, the supplier's markup for any given retailer decreases with this retailer's market share. The perfect coordination scheme (27) thus provides a rationale for the widely prevalent practice of offering larger discounts to larger retailers (see Brown and Medoff 1990 and Munson and Rosenblatt 1998), beyond those that can be justified by economies of scale in the costs incurred.

Observe that the wholesale pricing scheme itself differentiates between the retailers. Differences between the retailers in the first term of (27) are directly justified by differences in the costs incurred to service the retailers. Such quantity discounts are permitted under §2(a) of the Robinson-Patman Act, the principal federal act governing price discrimination. Differences in the markup, i.e., the second term in (27), fail to be *directly* related to cost differences. As far as this component is concerned, compliance with federal trade regulations is more questionable. On the other hand, the differences in the markup tend to vanish as the number of retailers becomes large (see Corollary 3 in Bernstein et al. 2002 for a more precise asymptotic analysis).

One potential weakness of the coordination mechanism is the fact that while the vector (q^l, T^l), which optimizes supply-chain-wide profits, arises as a Nash equilibrium in the corresponding retailer game, existence of alternative equilibria with suboptimal supply-chain-wide performance cannot be excluded, in general. The following theorem shows, however, that supply-chain-wide optimality is guaranteed for *all* equilibria, as long as the coefficients $[\hat{\beta}_{ij}]$ in the cross terms of the inverse demand functions are symmetric, i.e.,

$$\hat{\beta}_{ij} = \frac{\partial f_i}{\partial q_j} = \frac{\partial f_j}{\partial q_i} = \hat{\beta}_{ji} \quad \text{for all } i \neq j.$$

THEOREM 5. *Assume* $\hat{\beta}_{ij} = \hat{\beta}_{ji}$ *for all* $i \neq j$. *Under the wholesale pricing scheme* w^D, *all Nash equilibria in the retailer game result in supply-chain-wide optimal profits* $\underline{\Pi}_{SC}$.

Proof. Let (q^l, T^l) denote an optimal solution of the centralized system, with the power-of-two interval restriction and let w^D be the wholesale pricing scheme associated with this solution. Let q^0 denote an alternative Nash equilibrium in the retailer game under w^D, and T^0 a corresponding optimal vector of replenishment intervals. Clearly, for all $i = 1, \ldots, N$,

$$\pi_i^C(q_i^0, T_i^0 \mid q_{-i}^0, T_{-i}^0, w_i^D) \geqslant \pi_i^C(q_i^l, T_i^l \mid q_{-i}^0, T_{-i}^0, w_i^D). \quad (28)$$

Recall that for all q_i, T_i

$$\pi_i^C(q_i, T_i \mid q_{-i}^0, T_{-i}^0, w_i^D) = \left(\hat{a}_i - \hat{b}_i q_i - \sum_{j\neq i} \hat{\beta}_{ij} q_j^0 - c_i - w_i^D\right) q_i - \left[\frac{K_i^r}{T_i} + \frac{1}{2}\overline{h}_i q_i T_i\right]. \quad (29)$$

Substituting (23) into (29), we get

$$\pi_i^C(q_i, T_i \mid q_{-i}^0, T_{-i}^0, w_i^D) = \left(\hat{a}_i - \hat{b}_i q_i - \sum_{j\neq i} \hat{\beta}_{ij} q_j^0 - c_i - c_0\right) q_i - \psi(q_i) - \sum_{j\neq i} \hat{\beta}_{ji} q_j^l q_i - \left[\frac{K_i}{T_i} + \frac{1}{2} h_0 q_i \max\{T_0^l, T_i\} + \frac{1}{2} h_i q_i T_i\right]. \quad (30)$$

Next, substituting (30) into (28), we obtain for all $i = 1, \ldots, N$,

$$\begin{aligned}
&\left(\hat{a}_i - \hat{b}_i q_i^0 - \sum_{j\neq i} \hat{\beta}_{ij} q_j^0 - c_i - c_0\right) q_i^0 - \psi(q_i^0) - \sum_{j\neq i} \hat{\beta}_{ji} q_j^l q_i^0 \\
&\quad - \left[\frac{K_i}{T_i^0} + \frac{1}{2} h_0 q_i^0 \max\{T_0^l, T_i^0\} + \frac{1}{2} h_i q_i^0 T_i^0\right] \\
&\geqslant \left(\hat{a}_i - \hat{b}_i q_i^l - \sum_{j\neq i} \hat{\beta}_{ij} q_j^0 - c_i - c_0\right) q_i^l - \psi(q_i^l) - \sum_{j\neq i} \hat{\beta}_{ji} q_j^l q_i^l \\
&\quad - \left[\frac{K_i}{T_i^l} + \frac{1}{2} h_0 q_i^l \max\{T_0^l, T_i^l\} \frac{1}{2} h_i q_i^l T_i^l\right], \\
&= \left(\hat{a}_i - \hat{b}_i q_i^l - \sum_{j\neq i} \hat{\beta}_{ij} q_j^l - c_i - c_0\right) q_i^l - \psi(q_i^l) - \sum_{j\neq i} \hat{\beta}_{ji} q_j^0 q_i^l \\
&\quad - \left[\frac{K_i}{T_i^l} + \frac{1}{2} h_0 q_i^l \max\{T_0^l, T_i^l\} + \frac{1}{2} h_i q_i^l T_i^l\right],
\end{aligned}$$

where the equality follows from $\hat{\beta}_{ji} = \hat{\beta}_{ij}$ for all $i \neq j$. Adding all N inequalities and because $\sum_{i=1}^N \sum_{j\neq i} \hat{\beta}_{ij} q_j^l q_i^0 = \sum_{i=1}^N \sum_{j\neq i} \hat{\beta}_{ji} q_j^0 q_i^l$ (which also follows from $\hat{\beta}_{ji} = \hat{\beta}_{ij}$ for all $i \neq j$), we obtain

$$\begin{aligned}
&\sum_{i=1}^N f_i(q^0) q_i^0 - \sum_{i=1}^N (c_i + c_0) q_i^0 - \sum_{i=1}^N \psi(q_i^0) \\
&\quad - \sum_{i=1}^N \left[\frac{K_i}{T_i^0} + \frac{1}{2} h_0 q_i^0 \max\{T_0^l, T_i^0\} + \frac{1}{2} h_i q_i^0 T_i^0\right] \\
&\geqslant \sum_{i=1}^N f_i(q^l) q_i^l - \sum_{i=1}^N (c_i + c_0) q_i^l - \sum_{i=1}^N \psi(q_i^l) \\
&\quad - \sum_{i=1}^N \left[\frac{K_i}{T_i^l} + \frac{1}{2} h_0 q_i^l \max\{T_0^l, T_i^l\} + \frac{1}{2} h_i q_i^l T_i^l\right].
\end{aligned}$$

Adding the term $-K_0/T_0^l$ to both sides of the inequality, we obtain $\Pi_{SC}(q^0, (T_0^l, T_{-0}^0)) \geqslant \Pi_{SC}(q^l, T^l) = \underline{\Pi}_{SC}$, which proves that q^0 induces optimal supply-chain-wide profits. □

A similar, Groves-based, perfect coordination mechanism can be achieved when the retailers compete in price space. However, the structure of the resulting wholesale pricing scheme is more complex, a direct consequence of the fact that the choice of a retailer's retail price has an impact, not just on his own sales volume, but on that of all other retailers and hence on the indirect costs incurred for them.

Remark. The analysis above assumes that in the decentralized system, the retailers' holding cost rates $\{\bar{h}_i\}$ are independent of the wholesale prices w_i. As discussed in §4, it is often more realistic to assume that $\bar{h}_i$ is an increasing function of w_i, e.g., $\bar{h}_i(w_i) = \bar{h}_i^0 + w_i I$, for some interest rate I. It is easily verfied, along the lines of Chen et al. (2001), that perfect coordination continues to be achievable with a slight modification of the nonlinear pricing scheme (27):

$$\begin{aligned}
w_i^D(q_i, T_i) = c_0 + \left(1 + \frac{1}{2} I T_i\right)^{-1} &\times \left[\sum_{j\neq i} \beta_{ji} q_j^l + \psi(q_i)/q_i + K_i^s/(q_i T_i)\right. \\
&\left. + \frac{1}{2}(h_0 + I c_0) T_0^l - \frac{1}{2}(h_0 + I c_0) \min\{T_0^l, T_i\}\right].
\end{aligned}$$

6. NUMERICAL STUDY

In this section we report on a numerical study comparing the performance of a supply chain under centralized and decentralized management. We also report on an example exhibiting interesting differences in the equilibrium strategies adopted by the retailers when they compete in price or quantity space.

Our first set of problem instances is generated from the following base scenario, with $N = 5$ identical retailers. Their demand function is given by $d_i(p) = a - bp_i + \sum_{j\neq i} p_j$ for all $i = 1, \ldots, 5$, where $a = 90$ and $b = 6$. The cost parameters are as follows: $c_i = 1, \psi(q_i) = 10 + \underline{q_i}$ for $q_i > 0, c_0 = 10, K_0 = 100, h_0 = 5, \ \ h_i = 1$ (i.e., $\bar{h}_i = 6$), $K_i^s = 4$, and $K_i^r = 6$ (i.e., $K_i = 10$) for all $i = 1, \ldots, 5$. In the base scenario, the vector of prices $\tilde{p} = (30, 30, 30, 30, 30)$ corresponds to the vector of quantities $\tilde{q} = (30, 30, 30, 30, 30)$. In addition to the base scenario, we generated 9 additional instances by rotating the demand functions around the point $(\tilde{p}, \tilde{q})$, i.e., we augment a, the intercept of the demand functions, with increments of 10 (i.e., $a_k = 90 + 10k$, $k = 0, 1, \ldots, 9$), and adjust the slope b upwards to ensure that $d(\tilde{p}) = \tilde{q}$. We have computed the optimal centralized solution for each of these instances and compared it with various decentralized systems. We first compute the supply-chain-wide profits under the Stackelberg game with the supplier as the leader and the retailers as the followers, competing in terms of their prices [quantities]. In this case, the supplier selects the wholesale price that maximizes her profits, anticipating the pricing [quantity] and replenishment strategies adopted by the retailers under this wholesale price. We also compute the best supply-chain profits that can be achieved under a linear pricing scheme. We compute the best linear pricing scheme, both when the retailers compete with their prices and when they compete with their sales target levels. We finally compute, for the case of Cournot competition, the supply-chain profits arising when the equilibrium wholesale prices from the *nonlinear* discount scheme $w^D(q^l, T^l)^3$ are specified as a *linear* pricing scheme. (Consider for example the base scenario where $w^D(q^l, T^l) = 20.57$; a rather different equilibrium and associated supply-chain-wide profits are achieved when each retailer is charged a *constant* per-unit cost of 20.57.) Because computation of the *best* linear wholesale pricing scheme is rather tedious, we evaluate this *specific* choice of wholesale prices as a possible heuristic. In all the figures, the horizontal axis describes each scenario corresponding to a value of $k = 0, 1, \ldots, 9$.

Figure 2 exhibits the gaps vis-à-vis the optimal centralized solution of the Stackelberg game solution and the solution under the best linear wholesale pricing scheme, in the cases of both Bertrand and Cournot competition among the retailers. We observe that the gaps of the Stackelberg game solution average 13.8% and 16.0%, respectively, and can be as large as 20.6%, demonstrating the extensive benefits which a supply chain can accrue by implementing an appropriate coordination mechanism. The same set of instances shows that the use of a *nonlinear* discounting scheme, as opposed to the best linear scheme, is important to induce the proper equilibrium behavior, with gaps as large as 4%. Recall that a larger scenario index k is associated with a larger price sensitivity of the demand. Observe that the gap in supply-chain-wide profits incurred under the *best linear* wholesale price *decreases* as the price sensitivity increases. This applies both to the case of Bertrand and that of Cournot competition. No pattern is apparent as far as the gaps of the Stackelberg solutions are concerned.

Similarly, for the Cournot case, the gaps between the centralized solution and the setting in which the supplier charges $w^D(q^l, T^l)$, specified as a *linear* pricing scheme, average 3.1%. On the other hand, when the supplier charges each retailer i, $i = 1, \ldots, 5$, $\hat{w}_i^D(q_i^l, T_i^l) \stackrel{\text{def}}{=} w_i^D(q_i^l, T_i^l) - Q^l(1 - (q_i^l/Q^l))\eta_i^4$ specified as a linear pricing scheme, the gaps vis-a-vis the centralized solution vary between 14% and 23%. Note that $\hat{w}^D(q^l, T^l)_i$ only depends on retailer i's own replenishment interval T_i^l and his annual sales volume q_i^l. This approximation of the coordinating wholesale pricing scheme w^D is therefore of a simpler structure. It is of interest to gauge how closely the simplified scheme (without the externality effect of competitors) approximates the coordinating scheme w^D.

Figure 3 exhibits (1) the wholesale price chosen by the supplier in the Stackelberg game solution, (2) the best *linear* wholesale price value, (3) $w^D(q^l, T^l)$, and (4) $\hat{w}^D(q^l, T^l)$. We observe that in the Stackelberg solution the supplier charges an excessively large wholesale price resulting in unnecessarily large retailer prices and suboptimal sales volumes. The best linear wholesale price is fairly close to the coordinating price w^D, although the former is somewhat higher. Finally, ignoring the term $Q^l(1 - (q_i^l/Q^l))\eta_i$, which represents the externality effect of competition, can result in large changes in the wholesale prices, of up to 44%. The coordinating wholesale price w^D decreases as we move from left to right, i.e., as the price sensitivity of demand increases. The same monotonicity pattern fails to apply (at least locally) for the other wholesale pricing schemes. Note also that $w^D - \hat{w}^D$, the markup applied by the coordinating wholesale pricing scheme, decreases as the price sensitivity of demand is increased. This is to be expected, because η, the com-

Figure 2. Gaps with centralized solution—Bertrand and Cournot.

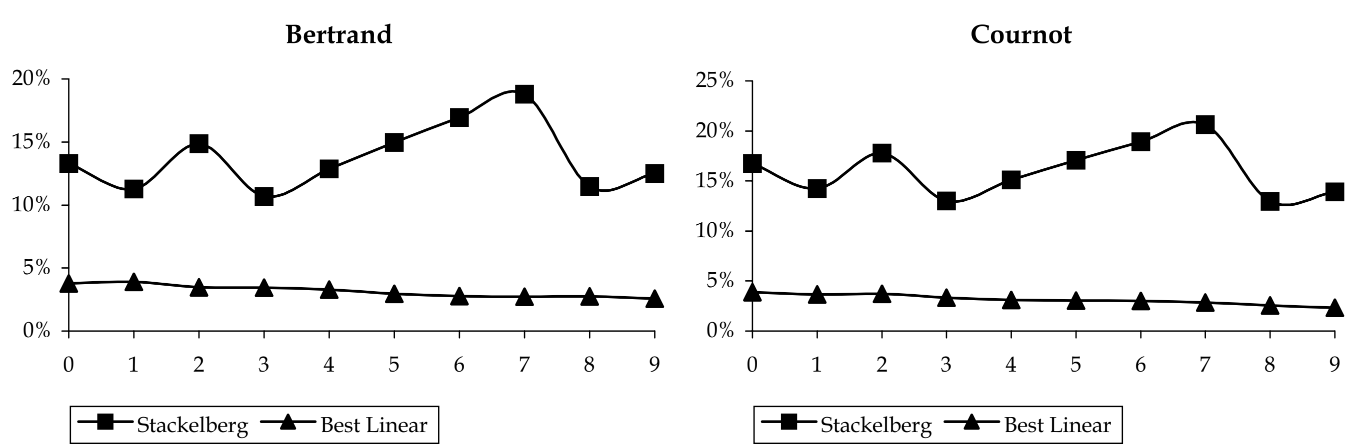
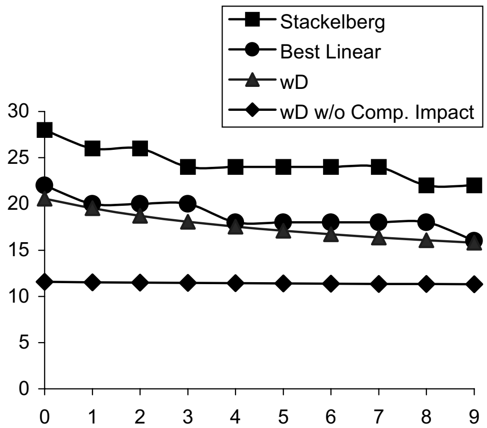


Figure 3. Wholesale prices.



petitive impact of each retailer, is a weighted average of the coefficients $\hat{\beta}$ of the cross terms in the inverse demand functions; the latter decrease rapidly with b, e.g., when going from scenario $k = 1$ to scenario $k = 2$, the coefficients $\hat{\beta}$ drop by 16%.

Finally, consider the following example with $N = 3$ competing retailers, with direct demand functions given by $d_1(p) = 400 - 10p_1 + p_2 + p_3$, $d_2(p) = 150 - 12p_2 + p_1 + 10p_3$, and $d_3(p) = 150 - 12p_3 + p_1 + 10p_2$. The cost parameters are $K_0 = 15$, $K_i^s = 5$, $K_i^r = 20$, $h_0 = 1.2$, $h_i = 0.4w_i$, $c_0 = 3$, and $c_i = 2$ for all $i = 1, 2, 3$. For various values of w_i, we compare the resulting Bertrand and Cournot equilibrium strategies. For example, with wholesale prices $w_i = 25.2$, $i = 1, 2, 3$, we find that the Bertrand equilibrium prices are (37.8, 37.6, 37.6) with corresponding quantities (96.4, 112.8, 112.8). On the other hand, the Cournot equilibrium quantities are given by (106.5, 83.3, 83.3) and corresponding prices (40.0, 53.3, 53.3). Thus, prices are higher in the case of Cournot competition, but quantities are not necessarily lower. We also compute and compare the Stackelberg solution for each setting. In the case in which the retailers face Bertrand competition, the Stackelberg wholesale price is $w = 38.0$, yielding profits of $7,136 for the supplier. In the case in which the retailers engage in Cournot competition, the Stackelberg wholesale price is $w = 36.1$, with profits of $6,061 for the supplier.

7. CONCLUSIONS

In this paper, we have compared the optimal performance of the *centralized* supply chain with that of various *decentralized* supply chains operating under given types of wholesale pricing schemes. While the exact optimal (centralized) strategy is unknown, we have derived an efficiently computable lower bound and upper bound, and we have shown that these bounds are *tight* as long as the gross profit margins of the retailers are not excessively low or the holding cost rate excessively large. The lower bound, for example, represents the profits of a strategy with stationary retailer prices, under the optimal power-of-two replenishment policy for the corresponding vector of sales rates q^l. Both q^l and the vector T^l of replenishment intervals of the corresponding optimal power-of-two policy are easily computable.

When decision making in the supply chain is decentralized, it is easiest to characterize the performance of the chain under a simple *linear* wholesale pricing scheme, characterized by an arbitrary vector of constant wholesale prices w. A Nash equilibrium (of pure strategies) may fail to exist under completely arbitrary parameter combinations, both when retailers engage in price or in quantity competition. However, a Nash equilibrium is guaranteed (under both types of competition) when condition (C1) prevails: Based on empirical data, we have shown that this condition is comfortably satisfied in virtually all retail industries. Under both Bertrand and Cournot competition, the equilibrium is in fact guaranteed to be *unique* under the related condition (C2), which continues to be satisfied in virtually all practical settings.

There are, however, a number of important differences between the equilibrium behavior of the retailers under price and quantity competition. If the retailers compete in quantity space, each adopts a retail price that is *larger* than its equilibrium price under price competition. One might conjecture that, similarly, the vector of equilibrium sales quantities of the retailers under Cournot competition, q^*, is smaller than the vector of sales quantities under Bertrand competition, q^B, but a numerical example in §6 shows that this relationship may fail to hold. Larger sales volumes, under price competition, can only be guaranteed in special cases, e.g., when the retailers are identical. On the other hand, *if* $q^* \leqslant q^B$ *is* satisfied, it follows that each retailer realizes lower profits under price competition than under quantity competition. In the case of price competition, an equilibrium price vector can be found (under (C1)) with the help of the simple iterative tatônnement scheme; in the case of quantity competition, the equilibrium vector q^* can only be found by solving the system of first-order optimality conditions for the N retailers' profit functions. Under Bertrand competition all equilibrium retail prices *increase* as any of the wholesale prices *increase*; the same monotonicity fails to be guaranteed under Cournot competition.

Unfortunately, *perfect* coordination cannot be achieved under *any* linear wholesale pricing scheme. To achieve perfect coordination, a *nonlinear* wholesale pricing scheme is required. We derive such a scheme, which applies *three* additive discounts off a given constant base price: The first discount component represents a *traditional* discount scheme, as it offers discounts *exclusively* as a function of *individual* order sizes. The second discount component offers a constant discount for each additional unit of time that the retailer is willing to keep a unit of his item in stock, up to a given cap of time units. The third and final discount component offers a discount exclusively as a function of the retailer's annual sales volume. Our coordination mechanism therefore provides an economic rationale, within the context of a model with complete information and symmetric bargaining power, for wholesale prices to be discounted on the basis of annual sales volumes. (This type of discount scheme is most prevalent in practice.) The wholesale price charged to retailer i under the coordinating scheme equals the average cost (per unit of sales) of all cost components incurred by the supplier that are directly related to retailer i's sales, plus a *markup*. This markup increases with η_i, retailer i's *competitive impact*, a weighted average of the coefficients of the crossterms in retailer i's inverse demand function. For a given total sales volume in the market, the markup for a given retailer *decreases* with this retailer's market share. If the coefficients of the crossterms in the demand functions, and hence the cross price elasticities of demand, are significantly large, the markups in the coordinating wholesale scheme are essential. Ignoring these, may result in large gaps in the aggregate supply-chain-wide profits.

While linear wholesale pricing schemes fail to achieve *perfect* coordination, they appear to allow for *modest* gaps with respect to the first-best or centralized solution. (The gaps are modest compared to those arising under Stackelberg solutions.) Because it is computationally tedious to identify the *best* linear wholesale pricing scheme, the following appears an effective heuristic: Implement the coordinating wholesale prices, under the vector of prices and replenishment strategies that are optimal for the centralized system, as a *linear* scheme, charging each retailer a *constant* per-unit wholesale price. More extensive numerical work is needed to compare the various wholesale pricing schemes considered in this paper.

APPENDIX

PROOF OF LEMMA 1. $\overline{C}$ is jointly concave as the minimum of a countable number of affine functions in q. In addition, Q can be partitioned into a finite set of regions such that in the interior of each region a single vector $T^l(q)$ achieves the minimum in $\overline{C}(q)$. This proves the lemma for $\overline{C}$. As to $\underline{C}$, introducing auxiliary variables $T_{0i} = \max\{T_0, T_i\}$, (8) may be rewritten as:

$$\text{(P)} \qquad \min_{T_i, T_{0i}} \quad \sum_{i=0}^{N} \frac{K_i}{T_i} + \sum_{i=1}^{N} \frac{1}{2} h_0 q_i T_{0i} + \sum_{i=1}^{N} \frac{1}{2} h_i q_i T_i$$

$$\text{s.t.} \quad T_{0i} \geqslant T_0, \quad i = 1, \dots, N, \tag{31}$$

$$T_{0i} \geqslant T_i, \quad i = 1, \dots, N, \tag{32}$$

$$T_i \geqslant 0, \quad i = 0, 1, \dots, N.$$

This is a convex program and, as such, has a (strong) dual which may be derived as follows. For $i = 1, \dots, N$, let x_i and y_i denote the Lagrange multipliers associated with constraints (31) and (32), respectively. By strong duality, we have that

$$\underline{C}(q) = \max_{x,y\geqslant 0} \min_{T_i \geqslant 0, T_{0i}} \left[\frac{K_0}{T_0} + \sum_{i=1}^{N} \left\{ \frac{K_i}{T_i} + \frac{1}{2} h_0 q_i T_{0i} + \frac{1}{2} h_i q_i T_i \right\} + \sum_{i=1}^{N} x_i (T_0 - T_{0i}) + \sum_{i=1}^{N} y_i (T_i - T_{0i}) \right]$$

$$= \max_{x,y\geqslant 0} \min_{T_i \geqslant 0, T_{0i}} \left[\sum_{i=0}^{N} \frac{K_i}{T_i} + \sum_{i=1}^{N} \left(\frac{1}{2} h_0 q_i - x_i - y_i \right) T_{0i} + \sum_{i=1}^{N} \left(\frac{1}{2} h_i q_i + y_i \right) T_i + \left(\sum_{i=1}^{N} x_i \right) T_0 \right]. \tag{33}$$

Note that for any pair (x, y), if $\frac{1}{2} h_0 q_i - x_i - y_i \neq 0$ for some $i = 1, \dots, N$, the inner minimization problem is unbounded from below; such pairs (x, y) can therefore be excluded from the outer maximization. Introducing auxiliary variables $v_i = \frac{1}{2} h_i q_i + y_i$, $i = 1, \dots, N$, and $v_0 = \sum_{i=1}^{N} x_i$, (33) can be rewritten as

$$\underline{C}(q_1, \dots, q_N) = \max \left[\min_{T_i \geqslant 0} \sum_{i=0}^{N} \left(\frac{K_i}{T_i} + v_i T_i \right) \right]$$

$$= \max 2 \sum_{i=0}^{N} \sqrt{K_i v_i} \tag{34}$$

$$\text{s.t.} \quad x_i + y_i = \frac{1}{2} h_0 q_i, \quad i = 1, \dots, N,$$

$$v_0 = \sum_{i=1}^{N} x_i, \quad i = 1, \dots, N,$$

$$v_i - y_i = \frac{1}{2} h_i q_i, \quad i = 1, \dots, N,$$

$$x_i, y_i \geqslant 0, \quad i = 1, \dots, N,$$

employing the well-known EOQ formula. The dual problem (34) consists of the maximization of a concave objective subject to linear constraints. Using these properties, one easily verifies that the optimum value is jointly concave in q, thus proving part (a) for $\underline{C}$.

Assume now that for a given vector q, the primal problem (P) has a *unique* minimizer $\{T_0^u(q), \dots, T_N^u(q), T_{01}^u(q), \dots, T_{0N}^u(q)\}$. It then follows from Rockafellar (1997) that

$$\frac{\partial \underline{C}(q_1, \dots, q_N)}{\partial q_i} = T_{0i}^u(q) \times \frac{\partial(\frac{1}{2} h_0 q_i)}{\partial q_i} + T_i^u(q) \times \frac{\partial(\frac{1}{2} h_i q_i)}{\partial q_i}$$

$$= \frac{1}{2} h_0 \max\{T_0^u(q), T_i^u(q)\} + \frac{1}{2} h_i T_i^u(q).$$

Finally, it is easily verified from Roundy (1985) that the set of q-vectors for which (P) does *not* have a unique minimum is of measure zero. □

ENDNOTES

1. The product categories are automobiles, furniture, electrical appliances, sporting goods, stationery items, books, men's clothing, women's clothing, footwear, and toys. If $R_0 = \sum_{i=1}^{N} p_i d_i$, I_0 = average inventory at the supplier, and I_i = average inventory at retailer i, the sales-to-inventory ratio for the centralized supply chain is $R_0/(I_0 + \sum_{i=1}^{N} I_i) = 1/(I_0/R_0 + \sum_{i=1}^{N} I_i/R_0) \geqslant 1/(I_0/R_0 + \max\{I_i/p_i d_i,\ i = 1, \dots, N\}) = 1/[1/(R_0/I_0) + 1/(\min\{p_i d_i/I_i,\ i = 1, \dots, N\})]$. A lower bound for the supply-chain-wide sales-to-inventory ratio is thus obtained

by replacing R_0/I_0 and $p_i d_i/I_i$ by the lower quartiles for the wholesale and retailer sectors, respectively.

2. This number is based on Fisher (2001), reporting an average gross profit margin of 32% for Department Stores, 33% for Consumer Electronics and Computer Stores and 36% for Apparel and Accessory Stores.

3. q^l and T^l represent the vector of quantities and replenishment strategies corresponding to the centralized solution.

4. This term corresponds to the externality effect imposed by retailer i's competitors.

REFERENCES

Bernstein, F., F. Chen, A. Federgruen. 2002. Vendor managed inventories and supply chain coordination: The case with one supplier and competing retailers. Working paper, The Fuqua School of Business, Duke University, Durham, NC and Graduate School of Business, Columbia University, New York.

Boyaci, T., G. Gallego. 1997. Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. Working paper, Department of Industrial Engineering and Operations Research, Columbia University, New York.

Brown, C., J. Medoff. 1990. Cheaper by the dozen. Working paper, Harvard University, Cambridge, MA and National Bureau of Economic Research.

Brown, R. 1959. *Statistical Forecasting for Inventory Control*. McGraw-Hill, New York.

Cachon, G. 1999. Competitive supply chain inventory management. S. Tayur, M. Magazine, R. Ganeshan, eds. *Quantitative Models for Supply Chain Management*. Kluwer, Dordrecht, The Netherlands.

——, P. Zipkin. 1999. Competitive and cooperative inventory policies. *Management Sci.* **45** 936–953.

Chen, F. 1999. Decentralized supply chains subject to information delays. *Management Sci.* **45** 1076–1090.

——, A. Federgruen, Y. S. Zheng. 2001. Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Sci.* **47**(5) 693–708.

Clark, A. J., H. E. Scarf. 1960. Optimal policies fo a multi-echelon inventory problem. *Management Sci.* **6** 475–490.

Crowther, J. 1964. Rationale for quantity discounts. *Harvard Bus. Rev.* **42**(2) 121–127.

Dennis, J., R. Schnabel. 1989. A view of unconstrained optimization. G. Nemhauser, A. Rinnooy Kan, M. Todd, eds. *Handbooks in Operations Research and Management Science*, Vol. 1. North-Holland, Amsterdam, The Netherlands.

Dun and Bradstreet, 2000–2001. *Industry Norms and Key Business Ratios*. Dun and Bradstreet Credit Services, New York.

Federgruen, A., P. Zipkin. 1984. Computational issues in an infinite-horizon, multi-echelon inventory model. *Oper. Res.* **32** 818–836.

Fisher, M. 2001. Rocket science retailing and the research opportunities it creates for us. Presentation at INFORMS Miami, November 6, Miami, FL.

Friedman, J. W. 1977. *Oligopoly and the Theory of Games.* North-Holland, Amsterdam, The Netherlands.

Fudenberg, D., J. Tirole. 1991. *Game Theory*. MIT Press, Cambridge, MA.

Groves, T. 1973. Incentives in teams. *Econometrica* **41** 617–631.

——, M. Loeb. 1979. Incentives in a divisionalized firm. *Management Sci.* **25**(3) 221–230.

Harsanyi, J. C., R. Selten. 1988. *A General Theory of Equilibrium Selection in Games.* MIT Press, Cambridge, MA.

Ingene, C. A., M. E. Parry. 1995. Channel coordination when retailers compete. *Marketing Sci.* **14** 360–377.

Jeuland, A. B., S. M. Shugan. 1983. Managing channel profits. *Marketing Sci.* **2**(Summer) 239–272.

Katz, M. L. 1989. Vertical contractual relations. R. Schmalensee, R. D. Willig, eds. *Handbook of Industrial Organization*, Vol. I. Elsevier Science Publishers B.V., Amsterdam, The Netherlands.

Lal, R., R. Staelin. 1984. An approach for developing an optimal discount pricing policy. *Management Sci.* **30** 1524–1539.

Lariviere, M. 1999. Supply chain contracting and coordination with stochastic demand. S. Tayur, M. Magazine, R. Ganeshan, eds. *Quantitative Models for Supply Chain Management*. Kluwer, Dordrecht, The Netherlands.

Lee, H., S. Whang. 1996. Decentralized multi-echelon supply chains: Incentives and information. Working paper, Stanford University, Stanford, CA.

McGuire, T. W., R. Staelin. 1983. An industry equilibrium analyisis of downstream vertical integration. *Marketing Sci.* **2**(Spring) 161–191.

Milgrom, P., J. Roberts. 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica* **58** 1255–1277.

Moorthy, K. S. 1987. Managing channel profits: Comment. *Marketing Sci.* **6**(Fall) 375–379.

Munson, C., M. Rosenblatt. 1998. Theories and realities of quantity discounts: An exploratory study. *Production Oper. Management* **7** 352–369.

Queyranne, M. 1987. Finding 94%-effective policies in linear time for some production/inventory systems. Working paper, University of British Columbia, Vancouver, British Columbia, Canada.

Radner, R. 1980. Collusive behavior in non-cooperative epsilon equilibria of oligopolies with long but finite lives. *J. Econom. Theory* **22** 121–157.

Raju, J., Z. J. Zhang. 1999. Dominant retailer and channel coordination. Working paper, University of Pennsylvania, Philadelphia, PA and Columbia University, New York.

Rockafellar, R. T. 1997. *Convex Analysis*. Princeton University Press, Princeton, NJ.

Roundy, R. 1985. 98% effective integer-ratio lot-sizing for one warehouse multi-retailer systems. *Management Sci.* **31** 1416–1430.

Samuelson, P. 1947. *Foundations of Economic Analysis*. Harvard University Press, Cambridge, MA.

Stein, L., A. El-Ansary. 1992. *Marketing Channels*, 4th ed. Prentice Hall, Englewood Cliffs, NJ.

Tellis, G. J. 1988. The price sensitivity of competitive demand: A meta analysis of sales response models. *J. Marketing Res.* **15**(3) 331–341.

Tirole, J. 1988. *The Theory of Industrial Organization*. MIT Press, Cambridge, MA.

Topkis, D. 1998. *Supermodularity and Complementarity*. Princeton University Press, Princeton, NJ.

Tsay, A., S. Nahmias, N. Agrawal. 1999. Modeling supply chain contracts: A review. S. Tayur, M. Magazine, R. Ganeshan, eds. *Quantitative Models for Supply Chain Management*. Kluwer, Dordrecht, The Netherlands.

Tyagi, R. 1999. On the effects of downstream entry. *Management Sci.* **45** 59–73.

U.S. Census Bureau. 2002. http://www.census.gov/svsd/www/ artstbl.html.

Vives, X. 2000. *Oligopoly Pricing*. MIT Press, Cambridge, MA.

Weng, K. 1995. Channel coordination and quantity discounts. *Management Sci.* **41**(9) 1509–1522.