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Coordinating Supply Chains with Simple Pricing Schemes: The Role of Vendor-Managed Inventories

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We characterize supply chain settings in which perfect coordination can be achieved with simple wholesale pricing schemes: either retailer-specific constant unit wholesale prices or retailer-specific volume discount schemes. We confine ourselves to two-echelon supply chains with a single supplier servicing a network of retailers who compete with each other by selecting sales quantities. We identify a key sufficient condition, in terms of interdependencies between chain members' operational decisions, under which perfect coordination via simple schemes is feasible, under general cost and demand functions. This condition, which we refer to as *echelon operational autonomy* (EOA), states that the costs incurred by the supplier for a given vector of sales volumes depends only on operational decisions she controls herself. At the same time, the costs incurred by the retailers may depend on operational decisions controlled by the supplier, in which case, the supplier's operational decisions are made to minimize chainwide costs. We show how vendor-managed inventory (VMI) partnerships create EOA and compare the resulting coordinating pricing schemes with those required in a traditional decentralized setting (without EOA). We also discuss compliance issues with the coordinating schemes in view of the Robinson-Patman act and provide remedies to overcome these issues.

Key words: supply chain coordination; pricing; vendor-managed inventory; echelon operational autonomy
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1. Introduction

The literature on decentralized supply chains has focused on two extreme settings: (1) a fully decentralized system in which each retailer chooses all of its policy variables (its sales quantity as well as its replenishment strategy), and (2) a fully centralized system in which all retailer sales quantities and the complete chainwide replenishment strategy are determined by a single decision maker. However, between these extremes, there exist many intermediate or hybrid forms of centralization. For example, in the vendor-managed inventories (VMI) partnership, each retailer determines its sales quantity but delegates the entire replenishment strategy to the supplier. Sometimes, retailers continue to incur the inventory carrying costs. Other, increasingly popular, VMI partnerships transfer all of the carrying costs to the supplier.[1] We refer to the former (latter) as VMI$^-$ (VMI$^+$).

The objective of VMI partnerships is to improve the aggregate performance of the supply chain. For example, the supplier typically sells the same or related items to several independent retail organizations, each of which may have multiple storage facilities. VMI permits the chain to exploit economies of scale in the production and distribution process. Also, by centralizing the replenishment process, inventories can be managed on the basis of actual consumer demands. Indeed, Buzzell and Ortmeyer (1995) report that the introduction of VMI partnerships at Dillard Department Stores, JCPenney, and Wal-Mart has resulted in sales increases of 20% to 25% and 30% inventory turn improvements. VMI is also believed to have resulted in lower retail prices; see e.g., Nelson and Zimmerman (2000).[2]

[1] The capital costs of carrying inventory (usually, the bulk of these costs) are transferred simply by having the retailer pay the supplier only when the items are sold. Andel (1996, p. 55) states: "The Wal-Marts of the world want to get to the point where they don't even own the goods sitting on the shelf. They want their vendors to capture the POS transaction which says: it's been bought, so now you can bill Wal-Mart." Physical warehousing costs are sometimes transferred by suppliers or their third-party logistics providers owning or renting warehouse space in the proximity of the retailers.

[2] This article illustrates the effect VMI partnerships have had on supply chain performance with a description of the highly successful partnership between Kimberly-Clark and various retailers

Nevertheless, VMI partnerships are organizationally challenging and, among other drawbacks, often require major investments in information sharing technology and systems.[3] At the same time, the industrial organization literature has shown that aggregate profits in a supply chain can be maximized to the first-best level, simply by adopting an appropriate pricing scheme for sales to the retailers. Moreover, the coordinating pricing scheme is often based on per unit wholesale prices that are constant or a simple decreasing function of the retailer's order size. If "clever pricing" is all that is needed to achieve supply chain coordination, why do we need initiatives like VMI that are costly to implement?

One reason is that simple pricing schemes, while sometimes sufficient, fail to achieve perfect coordination in other seemingly simple cases. Consider, for example, a supply chain with a single supplier distributing a product to several retailers over an infinite horizon. Demands at each retailer occur at a constant rate that is a decreasing function of this retailer's price. Due to fixed and variable procurement costs, inventories are replenished in batches. In the special case of identical retailers, a single quantity discount scheme based on individual order sizes induces perfect coordination, as shown by Weng (1995). However, Chen et al. (2001) show that in a traditional decentralized setting, no nonlinear order quantity discount scheme achieves perfect coordination when the retailers are not identical.[4] Instead, the authors show that perfect coordination requires an upfront agreement between the chain members to place all orders only at a specific discrete set of epochs and it involves three separate discount schemes, each based on a different retailer attribute: (1) the size of the retailer's orders, (2) the frequency with which orders are placed, and (3) the retailer's cumulative sales volume. Bernstein and Federgruen (2003) extend these results to the case where retailers face price or quantity competition, i.e., each of their demands depends on all retailers' prices. We refer to this model as the *standard inventory model*.

Our main goal is to characterize conditions under which perfect coordination can be achieved with possibly retailer-specific constant wholesale prices or possibly retailer-specific volume discount schemes. (Under a volume discount scheme, the wholesale price is discounted as a function of the retailer's annual sales volume.) We consider two-echelon supply chains with a single supplier servicing a network of retailers who compete with each other by selecting sales quantities.[5] The costs incurred by different chain members typically depend on operational decisions such as their replenishment strategies. Given basic regularity conditions that ensure the existence of Nash equilibria, we show that constant unit wholesale prices achieve perfect coordination for general cost structures, as long as each retailer's operational decisions impact only on its own costs. The supplier's operational decisions, on the other hand, may impact the costs of the entire echelon, i.e., those of the retailers as well as her own. If this is the case, it is required that the supplier's operational decisions be made so as to minimize chainwide costs. This *echelon operational autonomy* (EOA) condition is satisfied in many settings but it fails to apply in the above standard inventory model when operating as a traditional decentralized system: here, the supplier's inventory costs depend on her own replenishment strategy as well as those of the retailers, while both VMI$^+$ and VMI$^-$ partnerships enable perfect coordination via a simple pricing scheme, precisely by establishing EOA. The ability to create EOA may therefore be seen as another reason for companies' investments in VMI partnerships. We show, likewise, that under EOA, perfect coordination is achieved with a simple volume discount scheme. This scheme has two benefits: it is usable even when the regularity conditions required under constant unit wholesale prices fail to apply, and it allows for a continuous range of allocations of the optimal total profits among the firms.

Each retailer's coordinating constant wholesale price equals the marginal cost it imposes on other chain members, plus a markup which depends on the retailer's equilibrium sales volume or market share. The nonlinear pricing scheme, similarly, adds a markup to the per unit average cost this retailer imposes on the other chain members. We give general conditions under which the markups decrease with the retailer's market share. We thus provide an alternative rationale for the prevalent practice of discounting wholesale prices on the basis of annual sales volumes; see e.g., Munson and Rosenblatt (1998). The traditional explanation for why larger retailers get lower wholesale prices is either because the cost of

such as COSTCO. The article quotes Richard B. Berner, chief U.S. economist at Morgan Stanley Dean Witter, as stating that improved supply chain management has played a major role in the continuous decline of retail prices for general merchandise.

[3] Wal-Mart spent billions to develop Retail Link, connecting its stores with participating vendors via dedicated satellite communication systems; see Hornblower (2004). Recently, Wal-Mart has required its 100 top suppliers to tag all of their pallets with the new RFID tags to improve the performance of their VMI partnership; see Barlas (2003). (This requires eight billion tags at a unit cost of 20 cents.) Other companies need to rely on third-party software and consulting services, e.g., PRC Consulting, Simpson, and Knowledge Stores.

[4] No all-unit or incremental discount scheme achieves coordination even when the demand rates and retail prices are exogenously given, irrespective of how many discount price levels are used.

[5] Parallel results apply to the case of price competition; see Bernstein et al. (1999).

doing business with them is lower or because of their larger bargaining power or superior information; see e.g., Schiller and Zellner (1992). Our explanation does not require any of these conditions.

Both schemes involve double marginalization, i.e., a positive margin between the per unit wholesale price for a given retailer and the marginal (or average) cost it imposes on the other chain members, as well as one between the wholesale price and the retail price. We show that the first margin increases with the retailer's "competitive impact," a measure for the degree of competition this retailer presents to the market, and decreases with its market share. The numerical study in Bernstein et al. (1999) reveals that the adoption of a coordinating pricing scheme may increase chain-wide profits by more than 20% when operating under VMI$^+$, and by close to 30% in the case of VMI$^-$, compared to the case in which the supplier selects (constant) wholesale prices so as to maximize her own profits. Thus, a VMI partnership by itself allows only part of the potential for efficiency improvements, but it provides the foundation for maximum efficiency improvements via a simple coordination scheme.

We refer to Aviv (2004), Cheung and Lee (2002), and Netessine and Rudi (2004) for models invoking VMI and the related "drop-shipping" arrangement, and to Cachon (2003) for a survey of coordination mechanisms for a wide range of supply chain models. Ingene and Parry (1995, 2000) consider a supply chain with two nonidentical competing retailers facing linear deterministic demand functions. Each chain member incurs costs proportional to its sales volume. The authors show that perfect coordination cannot be achieved by any uniform constant wholesale price. Instead, they obtain perfect coordination by discounting the (common) wholesale price as a linear function of the retailers' purchase volumes. It is unclear how the scheme can be generalized for industries with more than two retailers or more complex cost and demand structures. Bernstein and Federgruen (2004, 2005) consider systems with an arbitrary number of competing retailers facing random demands. In both models, perfect coordination is achieved with a vector of constant wholesale prices, and in both, EOA holds.

This paper is organized as follows. Section 2 develops the coordination schemes for the general model. In §3, we apply our results to the standard inventory model under VMI$^+$ and VMI$^-$, comparing the coordination schemes with the more complex scheme required in a traditional system (i.e., the standard inventory model without a VMI arrangement).

2. The General Model

Consider a two-echelon supply chain with a supplier distributing a single (or closely substitutable) product(s) to N retailers who in turn sell the product to the consumer market. All demands must be satisfied in their entirety. For $i = 1, \ldots, N$, let p_i be the retail price charged by retailer i, and q_i its annual consumer demand. The two sets of variables are related to each other via general, continuously differentiable, inverse demand functions $p_i = f_i(q_1, \ldots, q_N)$, $i = 1, \ldots, N$. Let $q_{\max}$ denote a nonrestrictively large upper bound for the retailers' annual demand volumes. For substitutable products, each retailer's price decreases when it or any of its competitors increases its targeted sales volume, i.e., $\partial f_i / \partial q_j \le 0$, $i, j = 1, \ldots, N$.

Our main assumption reflects the supply chain members' cost structures. Let σ_i, $i = 0, 1, \ldots, N$, denote the complete set of operational decisions which are controlled by firm i. For example, σ_i may denote a set of capacity decisions, possibly in combination with an infinite horizon replenishment policy. EOA requires that each retailer i's operational decisions σ_i impact its own cost only. However, the supplier's operational decisions σ_0 may impact the costs of all chain members. Under EOA, the chain members' costs depend on the vector q and the operational decisions, and may be described by the following continuous functions:

$$\tilde{h}_0(q, \sigma_0) = \text{cost incurred by the supplier}, \quad (1)$$

$$\tilde{h}_i(q, \sigma_0, \sigma_i) = \text{cost incurred by retailer } i, \quad i = 1, \ldots, N. \quad (2)$$

The supplier charges the retailers for their purchases according to a given pricing scheme and determines her operational decisions σ_0 so as to induce perfect coordination. To do so, σ_0 is chosen to minimize aggregate costs.[6] Decisions are made in the following sequence, at the beginning of the planning horizon, and cannot be revoked thereafter:

Step 1. A wholesale pricing scheme is specified.

Step 2. The retailers simultaneously select their sales volumes.

Step 3. The supplier chooses her operational decisions σ_0.

[6] When each chain member's operational decisions impact its own costs only, as under VMI$^+$, the same σ_0 optimizes the supplier's as well as aggregate costs. Under VMI$^-$, to achieve perfect coordination, it is essential to insist that σ_0 optimize aggregate cost. All VMI$^-$ arrangements we know about either impose this charter on the supplier, or they specify bounds on inventory levels and delivery frequencies, usually determined cooperatively, with chainwide costs as the guiding principle. Often the responsibility of monitoring the replenishment process is delegated to a third party logistics provider. See Bolch (2005) and Buzzell and Ortmeyer (1995, p. 90). Gamble (1994) states, for example, "Vendor-directed inventory management need not be linked to vendor-owned inventory, sometimes called consignment sales. S. C. Johnson & Son Inc. now literally manages the inventory of its products at Kmart, and both parties are smiling. Kmart's in-stock rate of 80% has now reached the upper 90s, and inventory has dropped from enough wax to cover 16 weeks on normal sales to 2–4 weeks."

Step 4. The retailers choose their operational decisions σ_i, $i = 1, \ldots, N$.

Thus, for any given vector q, operational decisions are determined as follows: the supplier selects $\sigma_0^*(q)$, taking into account each retailer i's response $\sigma_i^*(q, \sigma_0)$ to minimize its own cost $\tilde{h}_i$. This gives rise to reduced cost functions $h_0(q)$ and $h_i(q)$, $i = 1, \ldots, N$. Formally, let

$$\sigma_i^*(q, \sigma_0) = \arg\min_{\sigma_i} \tilde{h}_i(q, \sigma_0, \sigma_i)$$

and

$$\sigma_0^*(q) = \arg\min_{\sigma_0} \left\{ \tilde{h}_0(q, \sigma_0) + \sum_{i=1}^{N} \tilde{h}_i(q, \sigma_0, \sigma_i^*(q, \sigma_0)) \right\}.$$

Thus, $h_0(q) = \tilde{h}_0(q, \sigma_0^*(q))$ and $h_i(q) = \tilde{h}_i(q, \sigma_0^*(q), \sigma_i^*(q, \sigma_0^*(q)))$. We assume that $h_i(\cdot)$ is increasing in q_i, $i = 1, \ldots, N$. Let $h^{-i}(q) = h_0(q) + \sum_{j \neq i} h_j(q)$ denote the total cost incurred by all chain members except for retailer i.

EOA arises, for example, in the standard inventory model under VMI, as the supplier takes over the retailers' replenishment decisions, thereby influencing their costs; see §3. Bolch (2005) provides a survey of VMI partnerships both with and without consignment, i.e., VMI$^+$ and VMI$^-$, respectively. This article quotes an estimate that 30% of inventory at the retail level moves through VMI partnerships, especially consumer product goods flowing to mass merchants, grocery stores, drug stores, and some apparel retailers. At the same time, EOA fails to hold in a traditional decentralized setting: effective replenishment strategies for each of the chain members call for batch replenishments at the end of intervals of constant length T_i. In a traditional setting, each chain member chooses its own replenishment interval. Because the supplier's inventory dynamics and costs depend on all of the replenishment intervals, her costs $\tilde{h}_0$ do not just depend on σ_0 but on $\{\sigma_i\colon i = 1, \ldots, N\}$ as well. In contrast, a VMI arrangement empowers the supplier to determine the retailers' replenishment frequencies and epochs, establishing EOA.

The EOA structure also applies to the models in Bernstein and Federgruen (2004, 2005), where retailers face a general system of stochastic demand functions. These models assume that all retailers follow a base-stock policy, while all retailer orders are always filled upon receipt, either from the supplier's inventory or by a back-up source. Thus, the operational cost incurred by each of the channel members only depends on its own operational decisions. The authors show that the chain can be coordinated with constant wholesale prices. As an alternative, consider the case where the supplier does not have access to a back-up source when running out of stock. In this case, the lead time process experienced by each retailer, and hence its inventory cost, depends on the inventory policy adopted by the supplier, while the expected inventory cost of the supplier only depends on her own operational decisions. Thus, EOA is maintained, but because the retailers' costs depend on σ_0, it is now essential that the supplier or a designated third party determine σ_0 to minimize chainwide costs. In Cachon's (2001) two-echelon model, N identical retailers face Poisson demands with a given rate. Because each facility manages its inventory with an (R, Q)-policy, the operational costs of the supplier depend on the retailers' operational decisions and EOA fails to apply. The author shows that the system can be coordinated but only by charging the supplier a penalty for its backorders and the retailers a penalty for consumer backorders, unless a VMI$^-$ arrangement is introduced. The supplier's objective under this arrangement amounts to minimizing chainwide costs.[7] As a final example, consider the one supplier-one retailer model in Cachon and Zipkin (1999). Both firms follow a base-stock policy. To provide an incentive to carry inventory, the supplier is charged a fraction of the backlogging costs due to stockouts at the retailer. In this model, the operational costs of both firms depend on each other's base-stock level, so that EOA fails to apply. Indeed, the proposed coordination mechanism involves period-by-period transfer payments from the supplier to the retailer, which depend on the retailer's inventory level and the backorder size at both firms. Simple pricing schemes fail, even though the lower echelon of the chain consists of a single retailer.

Suppose now that EOA holds. Consider first the centralized solution, where the quantity vector q is chosen from the compact set $\mathcal{Q} = \{q\colon 0 \le q_i \le q_{\max}\}$ to maximize the supply chain wide profit function $\pi_{SC}(q) = \sum_{i=1}^{N} f_i(q) q_i - h_0(q) - \sum_{i=1}^{N} h_i(q)$, which is continuous by the continuity of the demand and cost functions. Thus, π_{SC} achieves its maximum in a vector q^I (and $p^I = f(q^I)$), without loss of generality an interior point of $\mathcal{Q}$. (If q^I is on the boundary, because by assumption $q_i^I < q_{\max}$, $q_i^I = 0$ for some i. Such a retailer i can be eliminated from the system.) Therefore, q^I satisfies the first-order conditions

$$\sum_{j=1}^{N} q_j^I \frac{\partial f_j(q^I)}{\partial q_i} + f_i(q^I) - \sum_{j=0}^{N} \frac{\partial h_j(q^I)}{\partial q_i} = 0, \quad i = 1, \ldots, N. \tag{3}$$

We now analyze the decentralized system. When the supplier charges a vector of constant prices w, retailer i's profit function is given by $\pi_i(q_i \mid q_{-i}, w_i) = q_i(f_i(q) - w_i) - h_i(q)$. The following Nash-Debreu condition is sufficient for the existence of an equilibrium:

[7] This paper discusses another successful VMI$^-$ partnership between Allegiance Healthcare and the Duke University Medical Center, with the supplier chartered to minimize chainwide costs.

(C) The profit functions π_i are quasi-concave in q_i, $i=1,\ldots,N$.

If h_i is linear in q_i, the profit functions are log-concave (and thus quasi-concave), for example, if the inverse demand functions are themselves log-concave in their own arguments, e.g., when they are linear or Cobb-Douglas. The functions $\pi_i(\cdot)$ are concave in q_i if the demand functions are linear and the cost functions h_i are convex in q_i. Concavity is often maintained, even when h_i is concave. Consider, for example, under linear demand functions, the case where $h_i(q)=\gamma_i q_i^{\tau_i}$ and $0<\tau_i\le 1$.[8] Then, (C) holds as long as

$$|\epsilon_i| \le \frac{q_i f_i(q)}{h_i(q_i)} \frac{2}{\tau_i(1-\tau_i)}. \quad (4)$$

Here, ϵ_i is the demand elasticity for retailer i (as calculated from the inverse demand function). Thus, the profit functions are concave as long as

$$\epsilon_i \le 8\frac{q_i f_i(q)}{h_i(q_i)} \le \frac{q_i f_i(q)}{h_i(q_i)} \frac{2}{\tau_i(1-\tau_i)}.$$

This condition is usually satisfied as shown by the empirical data in Bernstein and Federgruen (2003). The equilibrium is unique for twice-differentiable profit functions under (C) and

$$\text{(U)} \qquad -\frac{\partial^2 \pi_i}{(\partial q_i)^2} > \sum_{j\neq i}\left|\frac{\partial^2 \pi_i}{\partial q_i \partial q_j}\right| \quad \text{for all } i=1,\ldots,N.$$

If the centralized optimal sales vector q^I is targeted as an equilibrium in the Cournot retailer game under a vector of constant wholesale prices w^C, the first-order conditions must be satisfied for $q=q^I$, i.e.,

$$\begin{aligned} 0 &= \frac{\partial \pi_i}{\partial q_i} = f_i(q^I) - w_i^C + q_i^I \frac{\partial f_i(q^I)}{\partial q_i} - \frac{\partial h_i}{\partial q_i}, \quad \text{or} \\ w_i^C &= f_i(q^I) + q_i^I \frac{\partial f_i(q^I)}{\partial q_i} - \frac{\partial h_i(q^I)}{\partial q_i} < f_i(q^I) = p_i^I. \end{aligned} \quad (5)$$

The vector q^I satisfies (3), which may be written as

$$f_i(q^I) + q_i^I \frac{\partial f_i(q^I)}{\partial q_i} = \sum_{l=0}^{N} \frac{\partial h_l(q^I)}{\partial q_i} - \sum_{j\neq i} q_j^I \frac{\partial f_j(q^I)}{\partial q_i}.$$

Subsituting this identity into (5), we obtain

$$\begin{aligned} \frac{\partial h^{-i}(q^I)}{\partial q_i} \le w_i^C &= \frac{\partial h^{-i}(q^I)}{\partial q_i} - \sum_{j\neq i} q_j^I \frac{\partial f_j(q^I)}{\partial q_i} \\ &= \frac{\partial h^{-i}(q^I)}{\partial q_i} + \eta_i^M Q^I\left[1 - \frac{q_i^I}{Q^I}\right] \end{aligned} \quad (6)$$

for $i=1,\ldots,N$, where $Q^I=\sum_{j=1}^N q_j^I$ and

$$\eta_i^M = \sum_{j\neq i} \frac{q_j^I}{\sum_{l\neq i} q_l^I}\left(-\frac{\partial f_j(q^I)}{\partial q_i}\right) \ge 0$$

is a weighted average of the marginal impacts a volume increase by retailer i has on its competitor's prices, which we refer to as retailer i's *competitive impact*. Each firm's coordinating wholesale price is thus given by the marginal cost it imposes on all other chain members, plus a markup proportional to the firm's competitive impact. Using Lemma 2 in Milgrom and Roberts (1990), as well as (5) and (6), Theorem 1 below shows that this wholesale price is strictly below the retail price, along with properties of the supplier's markup.

THEOREM 1. *Assume that* (C) *and* (U) *hold.*

(a) *The retailers adopt* q^I *as the unique Nash equilibrium under the constant wholesale prices* w^C *in* (6), *i.e.,* (6) *induces perfect coordination.*

(b) $\partial h^{-i}(q^I)/\partial q_i \le w_i^C < p_i^I$ *for* $i=1,\ldots,N$.

(c) *The supplier's markup* $[w_i^C - \partial h^{-i}(q^I)/\partial q_i]$ *increases with* η_i^M, *retailer i's competitive impact, and, for a given total sales volume* Q^I, *it decreases with this retailer's market share for* $i=1,\ldots,N$.

By part (b), the scheme induces double marginalization. To illustrate part (c), consider a system with two retailers and inverse demand functions $p_1=a_1-bq_1-\beta q_2$ and $p_2=a_2-bq_2-\beta q_1$, with $a_1>a_2$ and $b>\beta$. Note that $\eta_1^M=\eta_2^M=\beta$. If the supplier's cost is linear, i.e., $h_0(q_1,q_2)=c(q_1+q_2)$ and $h_i=0$, then $q_1^I>q_2^I$ and $w_1^C<w_2^C$. The coordination scheme (6) thus provides a rationale for the widely prevalent practice of offering larger discounts to larger retailers (see §1) beyond those that can be justified by economies of scale in the costs. We are not aware of any industrial organization models which justify this practice without relying on asymmetric information or differences in bargaining power.

The second perfect coordination scheme is based on average (incremental) procurement costs incurred for each of the retailers. This scheme applies to general demand and cost functions, even in the absence of condition (C). For $i=1,\ldots,N$, let $\pi_{SC}(q_{-i}^I,q_i) = q_i f_i(q_{-i}^I,q_i)+\sum_{j\neq i} q_j^I f_j(q_{-i}^I,q_i) - \sum_{j=0}^N h_j(q_{-i}^I,q_i)$ denote the "marginal" chainwide profit function when all but retailer i are committed to the sales volume in q^I, and note that it is maximized by $q_i=q_i^I$. The same applies to any increasing affine transformation of π_{SC},

$$\begin{aligned} \tilde{\pi}_{SC}&(q_{-i}^I,q_i) \\ &= \alpha_i\Bigg\{q_i f_i(q_{-i}^I,q_i) + \sum_{j\neq i} q_j^I[f_j(q_{-i}^I,q_i) - f_j(q_{-i}^I,0)] \\ &\qquad - h_i(q_{-i}^I,q_i) - \sum_{j\neq i}[h_j(q_{-i}^I,q_i) - h_j(q_{-i}^I,0)]\Bigg\} \\ &\qquad\qquad \text{for } 0\le\alpha_i\le 1. \end{aligned} \quad (7)$$

When its competitors choose the volumes in q^I, firm i's profit function is given by

$$\pi_i(q_{-i}^I,q_i) = q_i f_i(q_{-i}^I,q_i) - w_i q_i - h_i(q_{-i}^I,q_i). \quad (8)$$

[8] Functions of this type arise in the standard inventory model of §3.

Consider, in particular, the case where $\alpha_i = 1$. The profit functions (7) and (8) coincide (hence share q_i^I as their maximum) if the (nonlinear) per-unit wholesale price is given by

$$\begin{aligned} W_i(q_i) &= \frac{h^{-i}(q_{-i}^I, q_i) - h^{-i}(q_{-i}^I, 0)}{q_i} \\ &\quad - \sum_{j \neq i} q_j^I \frac{f_j(q_{-i}^I, q_i) - f_j(q_{-i}^I, 0)}{q_i} \\ &= \frac{h^{-i}(q_{-i}^I, q_i) - h^{-i}(q_{-i}^I, 0)}{q_i} \\ &\quad + Q^I \eta_i^A(q_i)\left[1 - \frac{q_i^I}{Q^I}\right], \quad \text{where} \qquad (9) \end{aligned}$$

$$\eta_i^A(q_i) = \sum_{j \neq i} \frac{q_j^I}{\sum_{l \neq i} q_l^I} \frac{f_j(q_{-i}^I, 0) - f_j(q_{-i}^I, q_i)}{q_i}.$$

Like η_i^M, $\eta_i^A(q_i)$ is a measure of retailer i's competitive impact, i.e., a weighted average of firm i's impact on its competitors' prices per unit of sales. The scheme, similar to the constant prices in (6), is obtained by replacing marginal costs by average incremental costs and η_i^M by $\eta_i^A(q_i)$. Using a proof similar to that of Theorem 1, we have the following result.

Theorem 2. *The vector q^I arises as an equilibrium in the retailer game induced by scheme* (9). *Thus,* (9) *generates a perfect coordination mechanism, and for all $i = 1, \ldots, N$,*

$$\begin{aligned} &\frac{h^{-i}(q^I) - h^{-i}(q_{-i}^I, 0)}{q_i^I} < W_i(q_i^I) < p_i^I \\ &\quad \Longleftrightarrow \pi_{SC}(q^I) > \sum_{j \neq i} q_j^I f_j(q_{-i}^I, 0) - \sum_{j=0}^{N} h_j(q_{-i}^I, 0), \qquad (10) \end{aligned}$$

i.e., if and only if aggregate first-best profits decrease when retailer i departs from the system.

The nonlinear scheme again involves double marginalization. The supplier's markup for retailer i again increases with the retailer's competitive impact η^A, and decreases with its market share, for a given total sales volume Q^I. By varying α_i, a range of possible coordination schemes arises. Increasing α_i shifts profits from the supplier to the retailers. In contrast, the constant pricing scheme allows only for a single choice. Note that $W_i(q_i)$ is a discount scheme (i.e., $W_i(\cdot)$ is decreasing) if, for example, the h_j-functions are concave in each of their arguments and the inverse demand functions f_j are convex in each of the competitors' sales volumes. (The latter is satisfied, e.g., for all linear and all Cobb-Douglas inverse demand functions.) Finally, Nash equilibria other than q^I may arise under (9). In this case, more coordination is required to steer the channel members to q^I. The equilibrium q^I continues, however, to be the preferred equilibrium. Since maximizing chainwide profits, q^I also allows retailers to achieve the best net profits after inclusion of fixed transfer payments.

Under both the linear and nonlinear schemes, the wholesale prices used for different retailers may be different. In the United States, this may raise concerns about compliance with the Robinson-Patman act. However, except for the markups, all differences are entirely due to differences in the marginal operating cost the retailer imposes on the chain. Such differentials are permitted under the "cost justification" provision in §2(a) of the act. The act provides several other defenses which may justify the remaining differences. Also, the differences in the markups tend to vanish when the number of retailers increases. If the direct price effect $\partial f_j / \partial q_j$ and Q^I remain bounded in N, then all η_i^M-factors and the entire markups decrease to zero.[9] (When the cross-elasticities in the demand functions are zero, so are the η_i^M-factors, resulting in unambiguous compliance with the Robinson-Patman act.) In addition, to the extent that differences in the markups continue to be of concern, the concern applies equally to settings with and without EOA. For example, in the standard inventory model, price differentials are needed to ensure perfect coordination (in a chain with heterogeneous retailers) even in the traditional system in which EOA fails to apply. To address the concern regarding price differentials, one may wish to confine to (an optimally selected) uniform markup, both when EOA applies and when it does not. The numerical study in Bernstein et al. (1999) shows that in the standard inventory model this restriction results in an average chainwide profit loss of no more than 3%. Thus, the main result in the paper continues to apply when restricting to uniform markups: EOA enables close-to-perfect coordination with simple pricing schemes, while, in general, this is not possible in the absence of EOA.

A concern, inherent to almost all pricing schemes, is the possible emergence of "grey" markets, where a retailer with a higher wholesale price orders "via" one with a lower wholesale price. There are several mechanisms to discourage this practice. For example, customers may be required to send the service registration or warranty card with their item's serial number to the supplier. This, or the RFID tags, allows the supplier to check whether the unit is sold by the legitimate or by some other retailer. Finally, the existence

[9] Because a firm's price will generally not go up if all firms decide to increase their sales target by the same amount, we have that $\sum_{i \neq j} |\partial f_j / \partial q_i| < -\partial f_j / \partial q_j$. This implies that the individual terms $|\partial f_j / \partial q_i| \to 0$ as $N \to \infty$ because $-\partial f_j / \partial q_j$ remains bounded as N increases, and so do weighted averages among them.

of point-of-sales information systems, as desired in a VMI partnership, makes it difficult to "hide" grey market shipments.

3. The Standard Inventory Model

In this section, we apply our results to the standard inventory model. In this infinite horizon model, demands at retailer i arise at a constant, but controllable rate q_i. The supplier replenishes her inventory with production runs or orders from an uncapacitated source. From there, the goods are shipped to the retailers. Deliveries to and from the supplier incur fixed and variable costs. Each location incurs carrying costs proportional with the inventory levels. The supplier may incur an annual cost for managing each retailer's needs and transactions, given by a concave function of the retailer's annual sales volume, to reflect economies of scale. Chen et al. (2001) discuss such account management costs. Finally, there may be costs to maintain the product while stocked at the retailer or costs associated with its sales efforts, all a function of the retailer's sales volume. For $i = 1, \ldots, N$,

$K_0 =$ fixed cost incurred for each delivery to the supplier,
$K_i =$ fixed cost incurred for each delivery to retailer i,
$H_0 =$ annual holding cost per unit of inventory at the supplier,
$\bar{H}_i =$ annual holding cost per unit of inventory at retailer i,
$H_i = \bar{H}_i - H_0 \geq 0$, the incremental or echelon holding cost at retailer i,
$c_0 =$ cost per unit delivered to the supplier,
$c_i =$ transportation cost per unit shipped from the supplier to retailer i,
$\psi_i(q_i) =$ annual cost incurred for managing retailer i's account, with $\psi_i(\cdot)$ nondecreasing, concave, and differentiable and $\psi_i(0) = 0$,
$s_i(q_i) =$ annual cost associated with retailer i's sales effort and inventory maintenance needs; $s_i(\cdot)$ is differentiable.

Many marketing studies represent the $s_i(\cdot)$ functions as power functions, i.e., $s_i(q_i) = \gamma_i q_i^{\tau_i}$ with $\gamma_i, \tau_i > 0$; see e.g., Corstjens and Doyle (1981) and Curhan (1973). In a decentralized setting, the fixed cost K_i for a delivery to retailer i may be shared by the supplier and the retailer, with K_i^s and K_i^r their respective parts, $K_i = K_i^s + K_i^r$.

In a traditional decentralized system, σ_i, the operational decisions controlled by firm i include its replenishment strategy, i.e., the epochs and size of its orders. Retailer i's inventory costs depend only on its own σ_i. However, while the input process of the supplier's inventory depends only on her own operational strategy σ_0, her output process depends on $\{\sigma_1, \ldots, \sigma_N\}$. For example, assume that each retailer i places orders every T_i time units; such constant replenishment intervals are, in fact, optimal for the retailer under constant per unit wholesale prices. The supplier's resulting aggregate order stream is, in general, highly nonstationary. It is unknown what her optimal corresponding replenishment policy looks like.

To achieve perfect coordination, one needs to reduce the systemwide annual delivery and inventory cost to its minimum level $C(q)$ in a centralized system. This cost function, too, is hard to characterize, but Roundy (1985) showed that the following function $\bar{C}(q)$ approximates $C(\cdot)$ closely:

$$\bar{C}(q) = \min\{\tilde{C}(q, T) \mid T_i = 2^{m_i} T_b,\ m_i \text{ integer},\ i = 0, 1, \ldots, N\}, \tag{11}$$

where

$$\tilde{C}(q, T) = \sum_{i=1}^{N}(c_0 + c_i)q_i + \frac{K_0}{T_0} + \sum_{i=1}^{N}\left[\frac{K_i}{T_i} + \tfrac{1}{2}\bar{H}_i q_i T_i\right] + \tfrac{1}{2}H_0 \sum_{i=1}^{N} q_i [T_0 - T_i]^+.$$

In fact, $C(q) \leq \bar{C}(q) \leq 1.02C(q)$. The function $\bar{C}(\cdot)$ restricts the T-vectors to power-of-two multiples of a base period T_b. For such power-of-two vectors T, $\tilde{C}(q, T)$ is the cost of a policy under which firm i replenishes its own inventory every T_i time units: the first term denotes the variable procurement costs, the second and third terms the fixed replenishment and retailers' inventory costs, and the last term the supplier's carrying cost. Because $C(q) \leq \bar{C}(q) \leq 1.02C(q)$ and because $\bar{C}(q)$ is the cost of a simple (power-of-two) policy, we use $\bar{C}(\cdot)$ to represent $C(\cdot)$. Let $T^*(q)$ be the vector of power-of-two values that achieves the minimum in (11), and $T^I = T^*(q^I)$.

The supplier's carrying cost, i.e., the last term in $\tilde{C}(q, T)$, depends on all of the T-values, i.e., on all of the chain members' operational strategies. Thus, even when power-of-two policies are used, there is no EOA in the traditional chain. More formally, in the traditional system, $\sigma_i = \{T_i\}$ for $i = 0, \ldots, N$,

$$\tilde{h}_i(q, \sigma_0, \sigma_i) = \tilde{h}_i(q_i, \sigma_i) = c_i q_i + s_i(q_i) + \frac{K_i^r}{T_i} + \tfrac{1}{2} q_i \bar{H}_i T_i,$$

and

$$\tilde{h}_0(q, \sigma_0, \sigma_1, \ldots, \sigma_N) = \frac{K_0}{T_0} + \sum_{i=1}^{N}\left(c_0 q_i + \frac{K_i^s}{T_i}\right) + \tfrac{1}{2}H_0 \sum_{i=1}^{N} q_i [T_0 - T_i]^+ + \sum_{i=1}^{N}\psi_i(q_i).$$

Thus, in the traditional system, (1) and hence EOA fail to apply. Indeed, the example in Theorem 1 in

Chen et al. (2001) shows that no single quantity discount scheme, i.e., no scheme under which the wholesale price w is specified as a decreasing function of the sales volume, achieves coordination even in the special case where the retailers do not compete. The same example can be used to show that no scheme with constant, although possibly retailer dependent, wholesale prices induces perfect coordination:

Example. Let $N = 2$, $K_0 = 100$, $K_1 = K_2 = 10$, $H_0 = H_1 = H_2 = 1$, $c_0 = 10$, $c_1 = c_2 = 1$, $\psi_1(\cdot) = \psi_2(\cdot) = 0$, $s_1(\cdot) = 0$, $s_2(\cdot) = 0$, $q_1(p_1) = 10 - 0.1p_1$, and $q_2(p_2) = 20 - 0.2p_2$. Chen et al. (2001) show that $q_1^I = 4.3$, $q_2^I = 8.6$, $T_0^I = 4$, and $T_1^I = T_2^I = 2$. Assume now that a pair of constant wholesale prices $\{w_1^*, w_2^*\}$ induces the retailers to adopt q_1^I and q_2^I. The retailers will respond by adopting replenishment intervals $\widehat{T}_1 = \sqrt{10/q_1^I} = 1.52$ and $\widehat{T}_2 = \sqrt{10/q_2^I} = 1.07$ or $\overline{T}_1 = 2$ and $\overline{T}_2 = 1 \neq T_2^I$ if confined to power-of-two values.

Bernstein and Federgruen (2003) show that, among other arrangements, coordination requires that the retailers agree up front to choose their replenishment intervals from the set of power-of-two values in (11). In addition, coordination requires a nonlinear multipart discount pricing scheme, where the discounts depend on several retailer attributes. The coordinating pricing scheme takes the form

$$
\begin{aligned}
W_i^D(q_i, T_i) &= W_i^{(1)}(T_i q_i) + W_i^{(2)}(T_i) + W_i^{(3)}(q_i), \quad \text{where} \\
W_i^{(1)}(T_i q_i) &= c_0 + \frac{K_i^s}{T_i q_i}, \qquad (12) \\
W_i^{(2)}(T_i) &= \tfrac{1}{2} h_0 T_0^I - \tfrac{1}{2} h_0 \min\{T_0^I, T_i\}, \\
W_i^{(3)}(q_i) &= \frac{\psi_i(q_i)}{q_i} - \sum_{j \neq i} \frac{\partial f_j(q)}{\partial q_i} q_j^I \\
&= \frac{\psi(q_i)}{q_i} + Q^I \left(1 - \frac{q_i^I}{Q^I}\right) \eta_i^T(q_i), \quad \text{with} \\
Q^I = \sum_{j=1}^N q_j^I \quad &\text{and} \quad \eta_i^T(q_i) = \sum_{j \neq i} \left(-\frac{\partial f_j(q_i, q_{-i}^I)}{\partial q_i}\right) \frac{q_j^I}{\sum_{l \neq i} q_l^I}.
\end{aligned}
$$

Thus, the coordinating wholesale price is the sum of three discount schemes: $W_i^{(1)}$ discounts on the basis of the retailer's replenishment quantity $T_i q_i$, $W_i^{(2)}$ on the basis of the replenishment frequency T_i^{-1}, and $W_i^{(3)}$ on the basis of the retailer's annual sales volume q_i. The term η_i^T is closely related to the competitive impact measures η_i^M and η_i^A in §2. (Under linear demand functions, η_i^T is a simple constant.)

3.1. The Standard Inventory Model Under a VMI Arrangement

Under VMI, the systemwide replenishment strategy is relegated to the supplier, expanding her operational decision set to $\sigma_0 = \{T\}$. Thus, under VMI, the supplier's cost only depends on σ_0, establishing EOA, where it fails to exist in the traditional system. The form of the cost functions $\{h_j(\cdot)\}$ differs between VMI$^+$ and VMI$^-$.

VMI$^+$—In this case, all delivery and inventory carrying costs are borne by the supplier:

$$
\begin{aligned}
h_0^{\mathrm{VMI}^+}(q) &= \bar{C}(q) + \sum_{j=1}^N \psi_j(q_j), \\
h_i^{\mathrm{VMI}^+}(q_i) &= s_i(q_i), \quad i = 1, \ldots, N.
\end{aligned} \tag{13}
$$

VMI$^-$—Without consignment, the supplier continues to select the same systemwide optimal replenishment policy for any given sales vector q, but, as far as inventory carrying costs are concerned, she bears only those associated with her own inventory. For $i = 1, \ldots, N$,

$$
\begin{aligned}
h_0^{\mathrm{VMI}^-}(q) &= \bar{C}(q) + \sum_{j=1}^N \psi_j(q_j) - \sum_{i=1}^N \tfrac{1}{2} \overline{H}_i q_i T_i^*(q), \\
h_i^{\mathrm{VMI}^-}(q) &= s_i(q_i) + \tfrac{1}{2} \overline{H}_i q_i T_i^*(q).
\end{aligned} \tag{14}
$$

The following result follows from Lemma 1 in Bernstein and Federgruen (2003).

Lemma 1. *The chain members' cost functions are all differentiable, both under VMI$^+$ and VMI$^-$, almost everywhere on $\{q\colon 0 \le q_i \le q_{\max},\ i = 1, \ldots, N\}$, i.e., whenever* (11) *has a unique minimum $T^*(q)$, and the derivatives are given by*

$$
\begin{aligned}
\frac{\partial h_0^{VMI^+}}{\partial q_i} &= (c_0 + c_i) + \tfrac{1}{2} H_0 \max\{T_0^*(q), T_i^*(q)\} \\
&\quad + \tfrac{1}{2} H_i T_i^*(q) + \psi_i'(q_i), \\
\frac{\partial h_0^{VMI^-}}{\partial q_i} &= c_0 + c_i + \tfrac{1}{2} H_0 \max\{T_0^*(q), T_i^*(q)\} + \psi_i'(q_i) \\
&= \frac{\partial h_0^{VMI^+}}{\partial q_i} - \tfrac{1}{2} H_i T_i^*(q), \\
\frac{\partial h_i^{VMI^+}}{\partial q_i} &= s_i'(q_i), \qquad \frac{\partial h_i^{VMI^-}}{\partial q_i} = s_i'(q_i) + \tfrac{1}{2} \overline{H}_i T_i^*(q).
\end{aligned}
$$

In sharp contrast to the traditional system, perfect coordination is possible (under conditions (C) and (U)) with constant wholesale prices, both under VMI$^+$ or VMI$^-$. In §2, we show that the conditions hold for broad classes of demand functions and cost functions $h_i^{\mathrm{VMI}^+}$ and $h_i^{\mathrm{VMI}^-}$. (The cost function $h_i^{\mathrm{VMI}^-}$ is obtained from $h_i^{\mathrm{VMI}^+}$ by adding a piecewise linear concave function. To the extent that $h_i^{\mathrm{VMI}^-}$ can be closely approximated by a concave power function, (4) is a sufficient condition for q^I to arise as the equilibrium.) The wholesale prices $\{w_i^{\mathrm{VMI}^-}\}$ and $\{w_i^{\mathrm{VMI}^+}\}$ are given by the following formulas, which result from (6), invoking the derivatives in Lemma 1:

$$w_i^{\mathrm{VMI}^-} = \left[c_0 + c_i + \tfrac{1}{2}H_0[T_0^I - T_i^I]^+ + \psi_i'(q_i^I)\right] + \eta_i^M Q^I\left(1 - \frac{q_i^I}{Q^I}\right), \tag{15}$$

$$\begin{aligned} w_i^{\mathrm{VMI}^+} &= w_i^{\mathrm{VMI}^-} + \tfrac{1}{2}\bar{H}_i T_i^I \\ &= \left[c_0 + c_i + \tfrac{1}{2}\bar{H}_i T_i^I + \tfrac{1}{2}H_0[T_0^I - T_i^I]^+ + \psi_i'(q_i^I)\right] \\ &\quad + \eta_i^M Q^I\left(1 - \frac{q_i^I}{Q^I}\right). \end{aligned} \tag{16}$$

Each of the pricing formulas in (15) and (16) consists of two parts. The expressions within square brackets denote the marginal costs the supplier incurs for retailer i, while the second term is a markup, the magnitude of which depends on the retailer's competitive impact η_i^M. If the retailers do not compete, i.e., when each retailer's demand only depends on its own price, the coordinating wholesale price is simply equal to the marginal costs the supplier incurs on behalf of the retailer; the larger the cross-effects $\{\partial f_j/\partial q_i\colon j \neq i\}$, the larger the markups. The marginal cost components are, of course, different under VMI$^+$ and VMI$^-$, but the difference is a simple expression: $w_i^{\mathrm{VMI}^+} - w_i^{\mathrm{VMI}^-} = \frac{1}{2}\bar{H}_i T_i^I > 0$.

Finally, VMI also permits perfect coordination via a (possibly retailer specific) single volume discount scheme. Let $W_i^{\mathrm{VMI}^+}(q_i)$ and $W_i^{\mathrm{VMI}^-}(q_i)$ denote the scheme for the cases of VMI$^+$ and VMI$^-$, respectively. To define the schemes, let $T_i^*(q_i \mid T_0^I)$ denote the optimal power-of-two replenishment interval for retailer i's inventory, given that the supplier uses T_0^I as her replenishment interval. It only depends on retailer i's own sales volume q_i, and it is piecewise constant and decreasing. (Clearly, $T_i^*(q_i^I \mid T_0^I) = T_i^I$.) In the standard inventory model, the general scheme (9) (which corresponds with the choice $\alpha_i = 1$), is given by

$$\begin{aligned} &W_i^{\mathrm{VMI}^-}(q_i) \\ &= \left[c_0 + c_i + \tfrac{1}{2}H_0[T_0^I - T_i^*(q_i \mid T_0^I)]^+ + \frac{K_i}{q_i T_i^*(q_i \mid T_0^I)} + \frac{\psi_i(q_i)}{q_i}\right] \\ &\quad + \eta_i^A(q_i)Q^I\left[1 - \frac{q_i^I}{Q^I}\right], \end{aligned}$$

$$\begin{aligned} &W_i^{\mathrm{VMI}^+}(q_i) \\ &= \left[c_0 + c_i + \tfrac{1}{2}\bar{H}_i T_i^I + \tfrac{1}{2}H_0[T_0^I - T_i^I]^+ + \frac{K_i}{q_i T_i^I} + \frac{\psi_i(q_i)}{q_i}\right] \\ &\quad + \eta_i^A(q_i)Q^I\left[1 - \frac{q_i^I}{Q^I}\right]. \end{aligned}$$

The nonlinear schemes have again two components. The first, given by the terms in squared brackets in $W_i^{\mathrm{VMI}^-}(q_i)$ and $W_i^{\mathrm{VMI}^+}(q_i)$, is the average cost the supplier incurs for retailer i, as opposed to the marginal cost. The second component is the markup $\eta_i^A(q_i)Q_i^I[1 - q_i^I/Q^I]$ and is identical under each of the schemes W^D, W^{VMI^-}, and W^{VMI^+}. The fact that $\psi_i(q_i)/q_i \geq \psi_i'(q_i)$, because $\psi_i(\cdot)$ is concave with $\psi_i(0) = 0$, permits the following comparisons:

Corollary 1.

$$\begin{aligned} W_i^D(q_i^I, T_i^I) &\leq W_i^{VMI^-}(q_i^I) = W_i^{VMI^+}(q_i^I) - \tfrac{1}{2}\bar{H}_i T_i^I \\ &\leq W_i^{VMI^+}(q_i^I), \end{aligned}$$

$$w_i^{VMI^-} = w_i^{VMI^+} - \tfrac{1}{2}\bar{H}_i T_i^I \leq w_i^{VMI^+},$$

$$W_i^{VMI^-}(q_i^I) = \left[\frac{\psi_i(q_i^I)}{q_i^I} - \psi_i'(q_i^I)\right] + \frac{K_i}{q_i^I T_i^I} + w_i^{VMI^-} \geq w_i^{VMI^-},$$

$$W_i^{VMI^+}(q_i^I) = \left[\frac{\psi_i(q_i^I)}{q_i^I} - \psi_i'(q_i^I)\right] + \frac{K_i}{q_i^I T_i^I} + w_i^{VMI^+} \geq w_i^{VMI^+}.$$

In conclusion, the notion of "echelon operational autonomy" arises as a general sufficient condition (in addition to technical conditions ensuring the existence of Nash equilibria in the retailer competition game) for the ability to coordinate the supply chain with simple pricing schemes. This condition is also necessary under many cost structures, e.g., the standard inventory model. When EOA fails to exist as, for example, under the standard inventory model in a traditional chain, initiatives such as VMI play a fundamental role in creating echelon operational autonomy and thus enabling coordination with simple pricing schemes. We have exhibited the specific form of these coordinating pricing schemes. We have also compared and contrasted the coordination schemes with two possible implementations of the general VMI partnership concept: (i) VMI$^+$, where the supplier incurs all inventory costs, and (ii) VMI$^-$, where the retailers continue to incur the costs associated with their inventory. We have also compared these with the much more complex scheme that is required in a traditional chain without a VMI partnership. Recall that, to achieve perfect coordination, the supplier must choose her replenishment decisions so as to minimize supply-chain-wide replenishment costs which, under VMI$^-$, are different from the actual costs she incurs. This discrepancy provides an argument for implementing VMI with full inventory consignment as in VMI$^+$.

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