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Determining the Relation between Fire Engine Travel Times and Travel Distances in New York City

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A simple physical model of the way fire engines travel leads to the hypothesis that T, the average fire engine travel time, depends on D, the distance travelled according to $T(D)=2(D/a)^{1/2}$ if $D \leqq d$ and $T(D)=v_c/a+D/v_c$ if $D>d$. The parameter a can be interpreted as an acceleration and v_c as a cruising velocity. A field experiment was run, and the above model validated and the parameters estimated, for New York City. It was also found that regional traffic conditions and hour of day appear to have only minor effects on average response velocities.

THE TIME it takes for municipal emergency vehicles, such as fire engines, police cars, and ambulances, to respond to calls for service is an important and widely used indicator of the performance of emergency service agencies. Most of the models developed for analysis of the deployment of emergency vehicles implicitly or explicitly include travel time at one stage or another.[1,3,6] Yet, little is known about actual travel times and how they vary with distance, time of day, weather, etc. In our particular case the New York City Fire Department planned to base important decisions relating to the deployment of fire companies on mathematical models that depended on predictions of travel times and travel velocities. For some examples of how travel time data have been used in New York City Fire Department planning problems, see references 3 and 4.

No empirical data on travel times and travel speed were then available, and we failed to find reports of such data in the literature. So, during the summer of 1971, the Department carried out an experiment to measure the travel times, distances, and speeds of selected fire companies. Selected units measured their travel times with stopwatches and their response distances with vehicle odometers. The resulting data (over 2000 observations) were statistically analyzed at The New York City-Rand Institute to answer a number of questions:

1. How does travel time depend on distance? Most analysts make the assumption that travel time = travel distance/average response velocity, i.e., that it takes twice as long to travel a mile as it takes to travel half a mile. We found, on the contrary, that in most parts of New York City travel time increases with the square root of distance for short runs, and linearly only for long runs.

2. How do response velocities vary by time of day? Both we and the Department had presumed that velocities, and hence travel times, varied considerably by time of day as a result of differences in traffic conditions, street lighting, etc. We found, surprisingly, that although differences do exist, they are considerably less than expected and can be ignored for many planning purposes.

3. How do travel velocities vary among regions of the city? Only small variations among regions were detected in the travel time/distance relation. This suggests that the average velocity for a given travel distance is almost constant throughout the city.

Details of the experimental results are presented below. Before proceeding to them, we outline briefly how the experiment was carried out.

Mechanics of the Experiment

Fifteen units participated in the experiment: thirteen ladder companies and two battalion chiefs' cars. Each unit had an odometer that read in tenths of miles, so that reasonably accurate distance records could be produced. Only moderately busy companies were selected, since the process of data collection would have been unduly burdensome if the units were very busy, and it would have taken months to gather data on an adequate number of responses with units that were not busy enough. Each unit was provided with a stopwatch and copies of a form to keep a record of all responses made from quarters. Responses made while returning from an earlier run or from a position in the field were not included, because of the difficulty of accurately recording times, distances, and locations at time of dispatch.

Consideration was given to obtaining a good geographical spread of participating companies, but the need for odometers that recorded in tenths of miles was a limiting factor: only ladder companies (responsible for rescuing people and ventilating the fire) and battalion chiefs (responsible for directing operations at the fire) participated, since no engines (responsible for delivering water on the fire) had such odometers. (Our failure to include engines in the experiment could introduce an element of bias in the results, since engines are generally smaller than ladders and are able to maneuver more easily in traffic and narrow streets, so that they may travel slightly faster. This should be remembered in applying the results of this experiment.)

In order to encourage cooperation in collecting data, the recording forms were kept simple and impersonal. Consequently, such information as date, identity of the officer recording the data, whether the company was the first due to respond, and special circumstances (such as weather or road conditions) were not recorded.

Of the thirteen ladder companies selected for the experiment, eight were located in Manhattan, two in Brookyln, two in Queens, and one in the Bronx. One Battalion Chief each in Manhattan and the Bronx also participated. Locations of participating companies are shown in Fig. 1, and summary statistics resulting from the experiment are given in Table I.

The methods of data collection and data analysis associated with this experiment are general and can be used by other fire departments to determine the response characteristics of their fire-fighting units. The computer program that was used to analyze the data is documented, and a user's manual has been published.[2] The program has already been used to analyze data collected in similar experiments conducted by fire departments in Yonkers, New York; Wilmington, Delaware; Jersey City, New Jersey; and Denver, Colorado.

Editing the Data

The raw data were edited to eliminate obviously erroneous records. We used a number of consistency checks in this process. For example, we eliminated records for which the average velocities attained were higher than 60 mph. In addition, observations for runs to the same alarm box were grouped and, if distances varied by more than $\frac{1}{4}$ mile, an independent check of the possibility of such readings was made. Less than 5 percent of the original data were eliminated by this process.

1. THE RELATION BETWEEN TRAVEL TIME AND RESPONSE DISTANCE

Other things being equal, the farther a fire engine travels, the longer it takes to make the trip. The time/distance relation normally employed assumes that a unit makes an entire trip at a constant velocity and, therefore, that travel time increases proportionally with the distance traveled. In this study, we attempt to determine whether this relation is valid, or if some other, more complicated model should be used.

We hypothesized the following: suppose that, for short runs, a unit never reaches a cruising velocity, but rather increases its speed for the first half of the trip, as it accelerates, gets onto main thoroughfares, etc., and then decelerates for the last half of the trip, as it approaches its destination, gets off main thoroughfares, etc. Suppose further that, for longer runs, there is a similar initial 'acceleration' phase, but that the unit then runs at cruising speed for some distance before decelerating as it nears its destination.

Fig. 1. Location of ladder companies participating in the experiment.

TABLE I

SUMMARY OF RESPONSE CHARACTERISTICS

Company	Response velocity (MPH)						Response distance (mi)						Travel time (min)						Number of runs		
	1st-due boxes		2nd-due boxes		All boxes		1st-due boxes		2nd-due boxes		All boxes		1st-due boxes		2nd-due boxes		All boxes		1st-due	2nd-due	Total
	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St Dev.	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.			
Manhattan																					
Ladder 1	16.0	7.1	17.3	7.3	16.7	7.2	0.45	0.23	0.62	0.26	0.54	0.26	1.81	0.88	2.36	0.88	2.11	0.96	149	158	329
Ladder 2	18.3	6.5	24.6	6.4	19.9	6.9	0.56	0.26	0.91	0.26	0.68	0.30	1.78	0.57	2.35	0.81	2.05	0.72	12	8	23
Ladder 3	16.7	6.7	19.1	5.6	18.2	6.3	0.48	0.23	0.83	0.28	0.67	0.31	1.78	0.72	2.62	0.52	2.23	0.77	85	86	185
Ladder 4	18.1	7.1	18.7	7.6	18.1	7.2	0.46	0.20	0.71	0.28	0.54	0.26	1.67	0.79	2.48	0.97	1.93	0.97	84	32	119
Ladder 11	14.1	6.9	15.8	5.2	14.5	6.6	0.40	0.21	0.57	0.19	0.43	0.22	1.84	0.85	2.23	0.59	1.92	0.82	128	34	162
Ladder 12	15.2	5.1	15.5	5.6	15.4	5.3	0.45	0.21	0.74	0.27	0.59	0.29	1.80	0.63	2.96	0.79	2.36	0.92	100	87	197
Ladder 15	16.8	8.7	20.0	9.2	17.9	9.1	0.45	0.30	0.80	0.35	0.54	0.37	1.67	0.77	2.58	0.81	1.87	0.89	79	22	109
Ladder 25	19.1	6.7	—	—	19.6	6.5	0.51	0.31	—	—	0.77	0.50	1.66	0.98	—	—	2.41	1.59	66	—	124
All Manhattan Ladders	16.3	7.0	17.6	6.8	17.0	7.0	0.45	0.24	0.71	0.28	0.58	0.32	1.76	0.81	2.54	0.81	2.13	0.95	703	—	1241
Brooklyn																					
Ladder 110	17.5	5.8	17.0	5.2	17.3	5.5	0.67	0.27	0.95	0.39	0.79	0.35	2.36	0.73	3.32	0.69	2.78	0.85	59	46	105
Ladder 122	18.4	5.4	21.5	3.8	19.6	5.0	0.62	0.24	0.96	0.22	0.79	0.37	2.09	0.78	2.67	0.47	2.45	1.04	49	29	81
Bronx																					
Ladder 42	16.8	5.4	19.2	6.9	17.9	6.0	0.43	0.17	0.88	0.30	0.60	0.31	1.60	0.53	2.88	0.95	2.03	0.92	78	40	133
All ladders except Queens	16.5	6.8	17.8	6.6	17.2	6.8	0.47	0.25	0.75	0.30	0.61	0.33	1.81	0.80	2.64	0.83	2.18	0.96	889	542	1560
Queens																					
Ladder 155	23.7	6.0	23.5	6.0	23.6	6.0	0.81	0.37	1.37	0.35	1.08	0.54	2.04	0.79	3.58	0.82	2.76	1.27	42	26	74
Ladder 162	25.7	6.0	29.5	5.4	27.4	6.0	1.94	1.13	2.07	0.73	2.00	0.96	4.41	2.17	4.22	1.30	4.33	1.80	73	61	138
Battalion 1	21.4	7.4	—	—	—	—	0.77	0.42	—	—	—	—	2.24	1.14	—	—	—	—	138	—	—
Battalion 45	25.6	7.0	—	—	—	—	1.45	0.70	—	—	—	—	3.44	1.85	—	—	—	—	96	—	—

These hypotheses can be expressed mathematically as follows:

Let: a = acceleration, D = length of the run, d_c = distance required to achieve cruising velocity, v_c = cruising velocity, and T = travel time. Then, using basic mathematical relations, and assuming constant acceleration and deceleration during the initial and final phases of travel and a constant cruising velocity, v_c, during the middle phase, we derive for travel time as a function of distance,

$$T(D)=\begin{cases}2(D/a)^{1/2}, & \text{if } D\leqq 2d_c,\\ v_c/a+D/v_c, & \text{if } D>2d_c.\end{cases} \tag{1}$$

The simple linear relation traditionally assumed is $T(D)=D/\bar{v}(D)$. A generalization of this relation is $T(D)=a+bD$.

With these hypotheses in mind, we proceeded to examine the experimental data. Least squares fits were made of relation (1) as well as of the functions:

$$T(D)=cD^{1/2}, \tag{2}$$

$$T(D)=a+bD. \tag{3}$$

Analyses were done separately for each participating company. In addition, separate fits of the models were done (1) for runs (responses) to alarms to which the company was the closest ladder, (2) for runs to alarms to which it was the second closest ladder, and (3) for all runs, including runs made to more distant alarms. The purpose of these separate analyses was to determine how the travel-time patterns varied among companies, and how they differed, if at all, for short runs and for longer runs.

All least squares fits were done using the average travel time at each possible measured distance as the observations. Since odometers used in the experiment read only to the nearest tenth of a mile, the measured distances are at tenth-of-a-mile intervals. However, the observations were weighted so that the fits were equivalent to having used the individual travel time observations.

The results of our analyses are presented in some detail elsewhere.[5] Here we summarize those findings:

1. In regions of the city where average response distances are short (about ½ mile or less), $T(D)=cD$ provides a better fit than (3) to the data. Generally, response distances are short in most regions, except in eastern Queens, Staten Island, some parts of Brooklyn, and the north Bronx. For the longer runs that are typical in the latter regions, the relation between distance and time looks linear; but the square-root fit is still very good, since the slope of the square-root function changes slowly at such distances (see Fig. 2).

2. In regions of the city in which average response distances are longer, $T(D)=a+bD$ is a better time/distance model than (2). These are the regions where the average distance to first-due alarm boxes is more than ½ mile.

3. Although the parameter values for different companies within each region exhibit statistically significant variations, these differences are not very large and, for many purposes, a single function can be used for all companies, within each type of region. For example, see Figs. 2, 3, and 4, which show data and fitted square

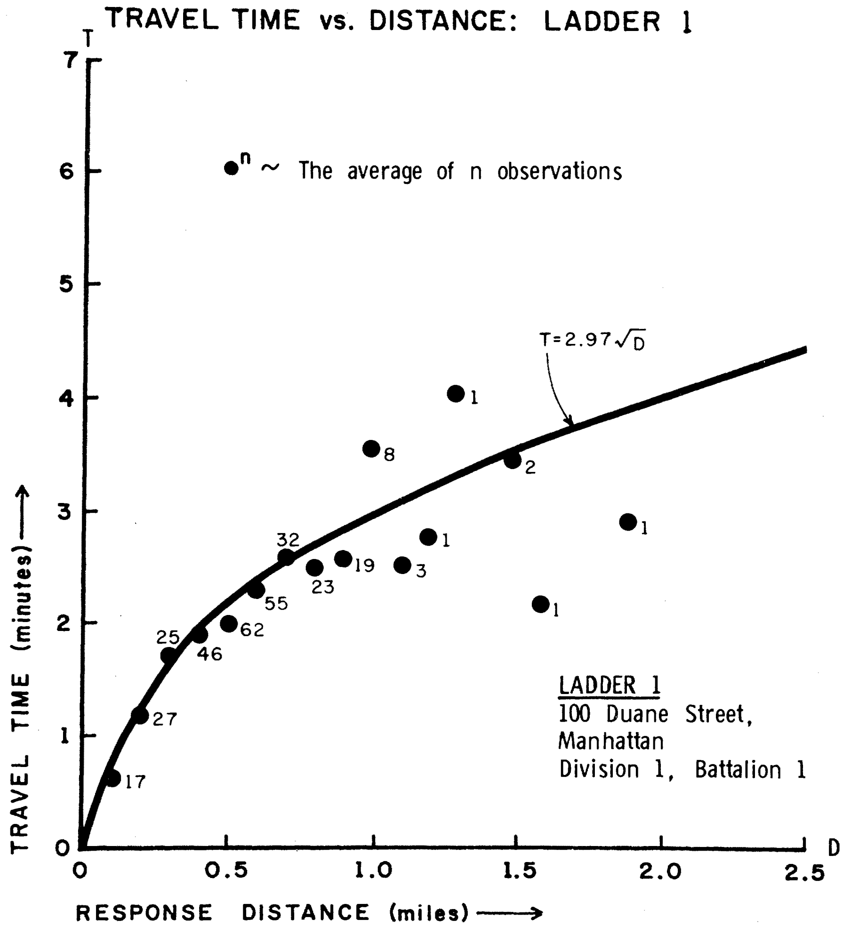

Figure 2

root functions for companies from three different boroughs of the city. These results are typical.

4. There is little practical difference in the travel time vs. distance function between first-due runs and second-due runs. (See Table I.)

5. A single continuous function that is piece-wise square-root and linear (as in equation (1) above) produces good estimates of average travel times for all regions of the city. (See Fig. 5.)

We make a few remarks to explain point 5. An iterative method was developed for performing the constrained nonlinear regression needed to fit relation (1). (See Appendix.) The results of our analysis using data for

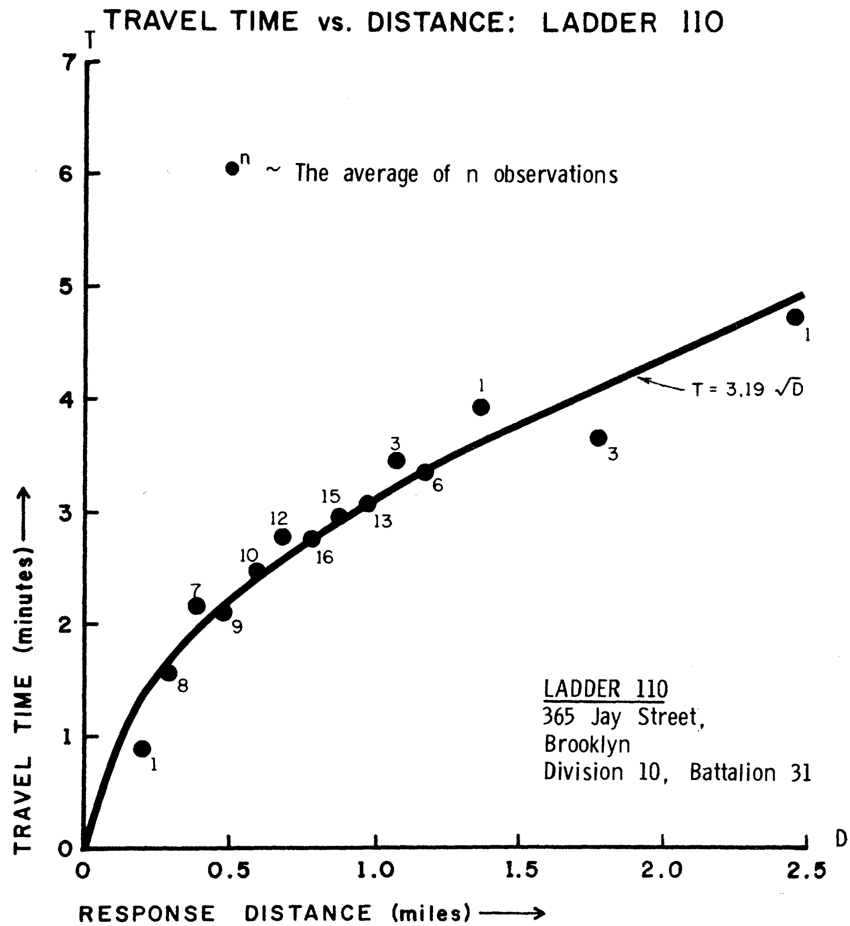

Figure 3

all responses are shown in Fig. 5. The fitted function is $T=2.88D^{1/2}$ if $D \leqq 0.88$ miles, and $T=1.35+1.53D$ if $D>0.88$ miles.

With the notation used above, the acceleration cutoff distance, d_c, is 0.44 miles; the cruising velocity, v_c, is 39.2 mi/hr.; and the acceleration, a, is 29.0 mi/hr/min. Although we do not document it here, we should remark that the 'goodness of fit' is largely insensitive to the choice of d_c in the range from 0.3 to 0.6 miles. Further, for different values of d_c in this range, the values of the parameters are relatively stable. The reason for this stability is the near linearity of the square-root function in this range. Table II presents a summary of the data used for these fits. Included in the average travel time for each distance are data collected on all runs by all the participating ladder companies.

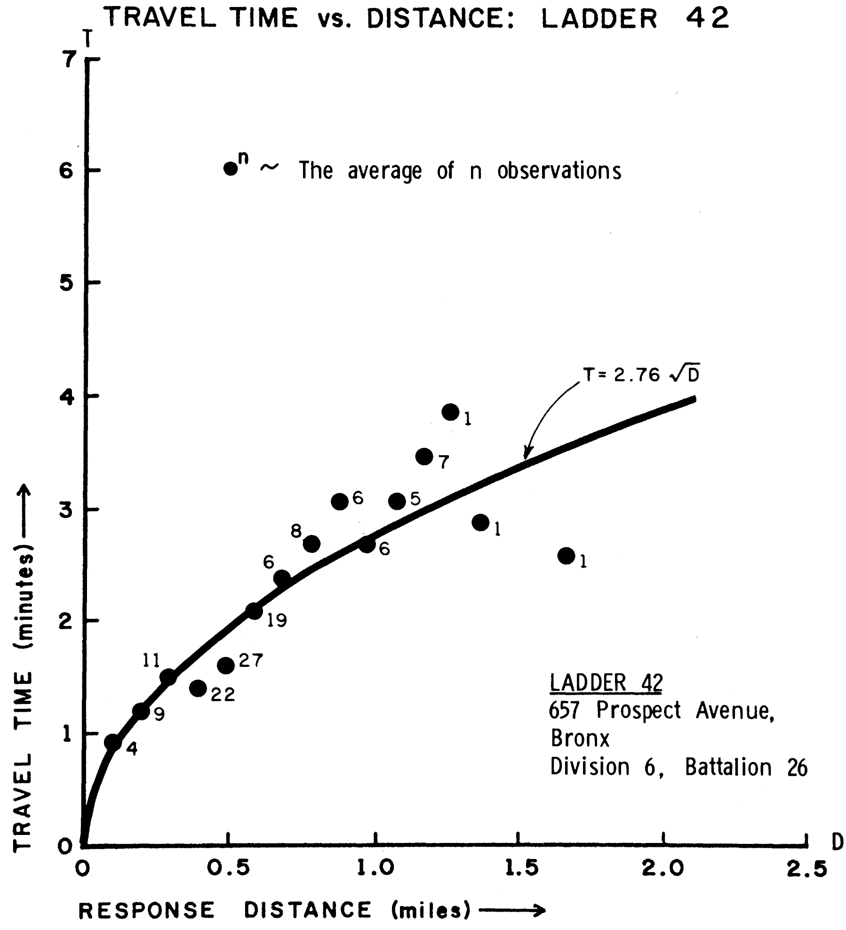

Figure 4

2. VARIATION IN RESPONSE VELOCITIES BY TIME OF DAY

Since the New York City Fire Department has been considering varying the number of companies on duty and the number of engines dispatched to an alarm at different hours of the day, an understanding of how response velocities vary by hour of day was of particular interest.

1. Do fire engines travel faster or slower in daylight then in the dark?
2. How much slower do fire engines travel during rush hours?

The results of our analysis are simple and surprising. First, there is no practical difference between travel velocities under conditions of daylight and darkness. Second, while velocities are lower during rush hours, they are not as much lower as we or the Department expected. The reduction in average velocity (of about 20 percent) is greatest during the 8 A.M.–9 A.M. period. (One qualification needs to be made about these observations.

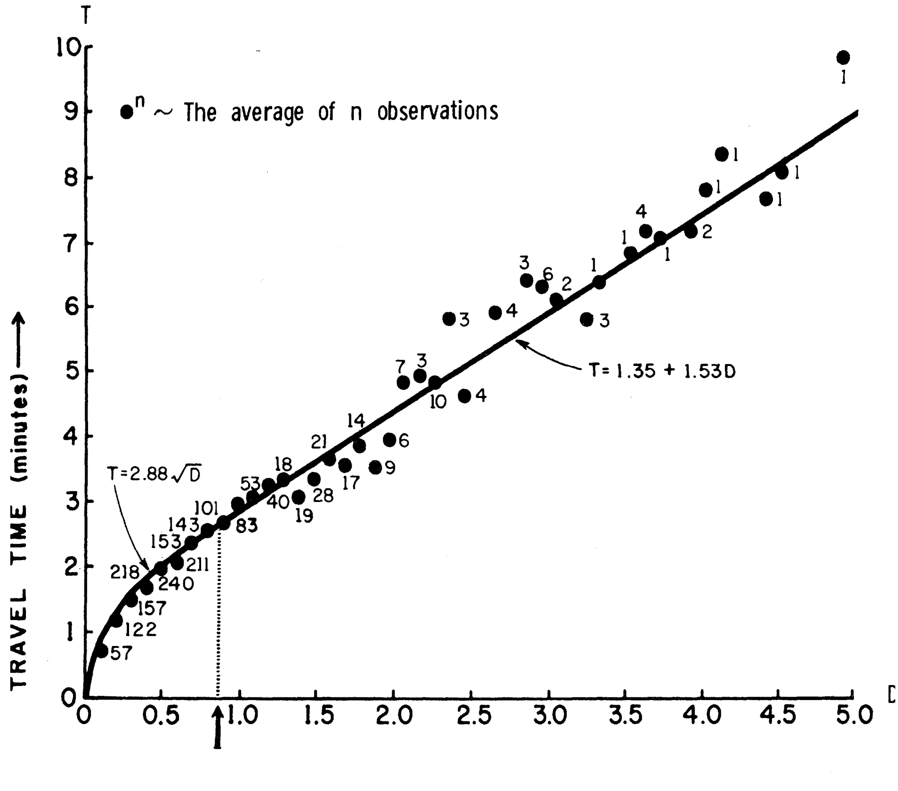

Fig. 5. Square root—linear model; all responses; all companies.

Since dates were not recorded, we could not separate weekends and weekdays. A reasonable assumption is that the rush-hour effect would be stronger on weekdays. Although most of our observations came from weekdays, we were unable to sort out the weekend effect.) The data in Table III support these conclusions. It shows the average and standard deviation of velocity for runs by all ladder companies grouped by two-hour intervals and by division of the day into the following four periods: 5 A.M. to 8 P.M., excluding the 'rush hours' (these are taken to be daylight hours); 8 P.M. to 5 A.M. (these are taken to be hours of darkness); and two rush-

TABLE II

Summary of Travel Characteristics: All Runs for All Participating Ladder Companies

Travel distance (miles)	Number of observations	Average travel time (minutes)	Standard deviation of travel time (minutes)
0.10	57.0	0.75	0.42
0.20	122.0	1.22	0.62
0.30	157.0	1.52	0.60
0.40	218.0	1.73	0.68
0.50	240.0	2.08	0.72
0.60	211.0	2.19	0.63
0.70	153.0	2.47	0.65
0.80	143.0	2.65	0.68
0.90	101.0	2.79	0.72
1.00	83.0	3.06	0.86
1.10	53.0	3.11	0.88
1.20	40.0	3.31	0.79
1.30	18.0	3.40	0.75
1.40	19.0	3.19	0.68
1.50	28.0	3.48	0.96
1.60	21.0	3.75	0.77
1.70	17.0	3.65	0.63
1.80	14.0	4.00	0.96
1.90	9.0	3.62	0.61
2.00	6.0	4.03	0.69
2.10	7.0	4.94	0.91
2.20	3.0	5.09	1.81
2.30	10.0	4.99	1.11
2.40	3.0	5.97	1.28
2.50	4.0	4.76	0.64
2.70	4.0	6.04	1.46
2.80	3.0	4.58	0.58
2.90	6.0	6.52	0.35
3.00	2.0	6.46	0.65
3.10	3.0	6.29	0.73
3.20	1.0	4.52	0.00
3.30	1.0	5.92	0.00
3.40	4.0	6.57	0.83
3.60	1.0	7.00	0.00
3.70	2.0	7.37	1.24
3.80	1.0	7.25	0.00
4.00	1.0	7.32	0.00
4.10	1.0	8.00	0.00
4.20	1.0	8.50	0.00
4.50	1.0	7.83	0.00
4.60	2.0	8.21	2.89
4.90	1.0	10.00	0.00

hour periods, 8 A.M. to 9 A.M. and 4:30 P.M. to 5:30 P.M. The combined results are typical of those obtained for individual companies and companies grouped by region; and they indicate that, although there are time-of-day effects, they are not strong. For more details see reference 5.

We note in passing that the number of observations in each two-hour period listed in Table III illustrates the dramatic difference in the demand on the Fire Department by hour of day. The peak of 239 calls in the period

TABLE III

SUMMARY OF RESPONSE VELOCITIES BY TIME OF DAY ALL RUNS FOR ALL PARTICIPATING LADDER COMPANIES

Hours	Average velocity	Standard deviation of velocity	No. of Observation
0000–0200	19.2	7.3	136
0200–0400	17.7	6.8	104
0400–0600	18.0	7.7	62
0600–0800	17.7	7.1	45
0800–1000	16.2	6.7	61
1000–1200	18.3	7.6	117
1200–1400	18.9	6.9	165
1400–1600	18.4	7.3	205
1600–1800	17.0	7.6	200
1800–2000	18.9	7.3	216
2000–2200	18.5	7.5	239
2200–2400	18.8	7.2	272
1500–2000 (rush hours excluded)	18.2	7.3	904
2000–0500 (dark)	18.6	7.3	742
0800–0900 (morning rush hour)	14.3	5.2	27
1630–1730 (evening rush hour)	18.2	7.6	99
All hours	18.3	7.3	1772

2000 to 2200 is more than 5 times the 45 calls received during the period 0600 to 0800.

APPENDIX

FITTING THE PIECEWISE TRAVEL-TIME FUNCTION

The problem of fitting a continuous piecewise square root-linear travel-time curve to the experimental data can be expressed as follows:

Given N sets of observations (T_i, D_i, M_i), $i=1, 2, \cdots, N$ where T_i denotes the average travel time (in minutes) of the M_i responses having a response distance of D_i (in miles), find values of the parameters a, b, c, and d to

$$\text{minimize} \sum_{i=1}^{i=N} M_i[T_i - f(D_i)]^2, \tag{5}$$

subject to

$$f(D)=\begin{cases} cD^{1/2}, & \text{if } D \leqq d, \\ a+bD, & \text{if } D>d, \end{cases}$$

and

$$a+bd=cd^{1/2}, \quad b=c/2d^{1/2}.$$

The first two constraints specify the form of the piecewise function to be fitted, and the second two specify that the two pieces of the curve are to be tangent at the break point d (they must meet and have the same slope at d). After eliminating a and c by solving for them in terms of b and d, we can write the problem as:

find b and d to

$$\text{minimize } q(b,d)=\sum_{i=1}^{i=N_d} M_i(T_i-2b\sqrt{dD_i})^2+\sum_{i=N_d+1}^{i=N} M_i(T_i-bd-bD_i)^2 \quad (6)$$

where, under the assumption that the sets of observations are ordered by increasing value of D_i, N_d is the largest value of i such that $D_i \leqq d$. If we fix a value of d (and, hence, of N_d), we can determine $b^*(d)$, the optimal value of b for that value of d, by differentiating (5) with respect to b and equating the derivative to zero. The result is

$$b^*(d)=[2d^{1/2}\sum_{i=1}^{i=N_d} M_iT_iD_i^{1/2}+\sum_{i=N_d+1}^{i=N} M_iT_i(d+D_i)]/$$
$$\cdot[4d\sum_{i=1}^{i=N_d} M_iD_i+\sum_{i=N_d+1}^{i=N} M_i(d+D_i)^2].$$

Then, by varying d, we can determine an optimal pair of values b^* and d^*. We performed this search for d^*, using an interactive computer program that, for a given d, computed $b^*(d)$ and $q(d)$. Using this program, we mapped out $q[b^*(d)]$ as a function of d in the range of interest.

NOTE

Jack Hausner is currently at the Institute for Law and Social Research, Washington, D.C.

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