Comments on a Blood-Bank Inventory Model of Pegels and Jelmert

A paper by C. CARL PEGELS AND ANDREW E. JELMERT [*Opns. Res.* **18:** 1087–1098 (1970)], entitled "An Evaluation of Blood-Inventory Policies: A Markov Chain Approach," has drawn two comments.

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PEGELS AND JELMERT present a Markov model of blood-bank inventories that appears, on the surface, to be an interesting application. On closer examination, however, the usefulness of the model is unclear.

Essentially, they have modeled the twenty-one-day life of a single unit of human blood in a blood bank. Each day, the unit may be transfused or it may age another day. On its last day of shelf-life it may be transfused or become outdated. Given the conditional probabilities that form the model, they show how one may calculate such measures as the fraction of units entering the blood bank that outdate, the average age of the blood at the time it is transfused, the average inventory level, and the average shortage that can be expected.

It must be emphasized that use of the model requires the specification of the probability that a unit of blood is transfused at each of its allowable ages in inventory. However, PEGELS AND JELMERT address themselves only briefly to the question of determining values for these probabilities. In their discussion, they imply that such values may be specified by blood-bank administrators as a matter of policy, and they present several guides for assuring that the probability values are not inconsistent. Unfortunately, they give no indication of how these probabilities, once selected, might be implemented. Further, they give no indication of how the selection process relates to the primary policy decisions that must be routinely made by blood bank administrators—namely, the number of units of blood to carry in inventory, the ordering rules, and the procedures for selecting units from the inventory for crossmatch. In practice, it is these policies, together with the pattern of physicians' requests for and transfusion of the blood, that determine the probability values in question. The implication that relevant policies can be determined from the probabilities is completely inverted.

Lacking a means of creating blood-handling policies from the probabilities, the question naturally arises as to whether the probabilities that correspond to a given set of policies can be determined. Unfortunately, this can be done only by observing the policies in use in a real-world situation or in an accurate simulation. In such circumstances, however, all the quantities that can be predicted with the Markov model can also be directly observed.

The reader who is interested in further research on blood-bank inventories may wish to refer to a review article by ELSTON,[1] or to the blood-bank simulation studies of ELSTON AND PICKREL[2] AND JENNINGS.[3]

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IN ESSENCE, Pegels and Jelmert propose to use Markov chain theory to calculate the probabilities of expiration and the average age of blood at transfusion. Our purpose here is to comment briefly on some aspects of the problem that are not treated fully in their paper.

To begin, we must define the Markov chain used by Pegels and Jelmert. The definition is missing in their paper, but I believe that the following is consistent with what they had in mind. For simplicity we deal with a single blood type and the first of their models in which blood is either in inventory (unassigned), expired, or transfused. Let X_n denote the state of a particular pint of blood at the start of day n, $n=0, 1, 2, \cdots$. By a particular pint of blood we mean, for example, "the pint of blood donated by Mr. Jones on the 23rd of August 1970." The state space is, as Pegels and Jelmert give it, $\{0, 1, 2, \cdots, 20, \mathrm{I}, \mathrm{II}\}$, where the arabic numbers represent the age of the blood in days, roman I represents transfused blood, and roman II expired blood. Blood expires if it is not transfused before it reaches an age of 21 days. Clearly I and II are absorbing states and $0, 1, 2, \cdots, 20$ are transient states. We use these terms loosely, since it is by no means clear that X_n is a Markov chain. For simplicity let $X_0=0$.

Given an issuing policy, say LIFO or FIFO or one more complicated, some random mechanisms of demand and supply, and a complete description of the status of inventory in the system when our pint of blood arrives, we can speak of a stochastic process X_n and determine the probability law of the transitions of X_n. Perhaps the simplest case is when (a) daily demand is a sequence of independent, identically distributed random variables with d_j the probability that demand is j pints, (b) daily supply is likewise a sequence of independent, identically distributed random variables with s_j the probability that supply is j pints, and (c) either LIFO or FIFO is employed as the issuing policy. Let Y_n be the number of units in inventory younger than our unit and O_n the number of units in inventory older than our unit. Then, for a FIFO policy, we must have

$$P\{X_{n+1}=i+1|X_n=i,\, O_n=k\} = \textstyle\sum_{j=0}^{j=k} d_j \qquad (i=0, 1, \cdots, 19)$$

$$P\{X_{n+1}=\mathrm{I}|X_n=i,\, O_n=k\} = 1-\textstyle\sum_{j=0}^{j=k} d_j, \qquad (i=0, 1, \cdots, 20)$$

$$P\{X_{n+1}=\mathrm{II}|X_n=20,\ O_n=k\} = \sum_{j=0}^{j=k} d_j.$$

Similarly, for a LIFO policy, assuming for simplicity that daily supply arrives before daily demand, we have

$$P\{X_{n+1}=i+1|X_n=i,\ Y_n=k\} = a_k \qquad (i=0, 1, \cdots, 19)$$

$$P\{X_{n+1}=\mathrm{I}|X_n=i,\ Y_n=k\} = 1 - a_k \qquad (i=0, 1, \cdots, 20)$$

$$P\{X_{n+1} = \mathrm{II}|\ X_n = i, Y_n = k\} = a_k$$

$$a_k = \sum_{j=0}^{j=\infty} \sum_{r=0}^{k+j} d_r\, s_j.$$

The fact that these probabilities depend on the distribution of the inventory on hand makes my first point: X_n *is not a Markov chain.* Further, the quantity q_t that Pegels and Jelmert call the probability of transfusion is not independent of the inventory, and so it is not too surprising that they speak of trial-and-error calculations for the q_t (*see* page 1097). There may exist proportions q_t having the properties the authors postulate, but it is misleading to speak of a Markov chain or of probabilities. However, all is not dark. While X_n is not a Markov chain, there are more complicated Markov chains inbedded in the blood-bank problem, such as the vector process $\{X_{0n}, X_{1n}, \cdots, X_{20n}\}$, where X_{in} is the number of pints of blood of age i days available at the start of day n. This chain is large, but by using a computer it might be possible to calculate expiration probabilities, age at transfusion, etc., either by matrix methods (much as Pegels and Jelmert employ), or by a simple simulation. Clever condensation of the state space is possible, depending on the issuing policy used and the information desired from the analysis. It is even conceivable that one could formulate and solve the problem of determining optimal issuing policies using a Markov optimization model in the style of HOWARD.[3,5,6,7]

This brings us to the last point. It is laudable to present mathematically the dichotomy between risk of expiration on the one hand (use FIFO) and age at transfusion on the other (use LIFO). But we must ask if average age at transfusion is the appropriate function. If all blood of age 20 days or less were equally valuable, our only concern would be with minimizing the probability of expiration. Their concern suggests that this is clearly not so, which suggests the existence of a function $v(i)$, the 'value' of transfusing blood of age i. If we could estimate this function, we could consider a more satisfactory problem: maximizing the value transfused with bounds on the probability of expiration. It would be helpful even if we only knew something of the shape of $v(i)$—whether it is convex or concave, for example. The literature on inventory-depletion management suggests possibilities.[1,2,5] It would seem to be not only necessary to keep track of the age distribution of blood in inventory for an adequate formal description of the behavior of the system, but also desirable from a practical point of view, as good issuing policies should depend on this information. This would be the case even if one were to use average age at transfusion as a criterion.

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Comments on a Letter by Woolsey

A LETTER by R. E. D. Woolsey [*Opns. Res.* **20,** 729–737 (1972)] entitled "Operations Research and Management Science Today, or, Does an Education in Checkers Really Prepare One for a Life of Chess?" has drawn some responses.

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I should like to enquire why Robert Woolsey's superb paper has been relegated to the position of a Letter to the Editor. It is after all one of the best things we have had for a long time.

This is the kind of trivial rubbish that ought to count as a Letter to the Editor.

A. Charnes, *University of Texas, Austin, Texas;* **W. W. Cooper,** *Carnegie-Mellon University, Pittsburgh, Pennsylvania; and* **B. Mellon,** *Palo Alto, California*

One begins to be bewildered by R. E. D. Woolsey's repetitions on a theme. In the process of repetition, he has now pushed to an extreme where others who are also interested in applications may not wish to follow him.

In his repeated specifications of "actual work experience" of a "hands-on variety," he appears to believe that he may have found a foolproof desideratum for operations research in education and practice. There may, however, be conditions where it is unwise or even impossible to proceed in this manner. This was true in the earliest days when we were experimenting with these approaches to refinery operations and other types of chemical-engineering-management-OR combinations designed to alter the then prevalent practices. We found that team approaches involving a mix of persons were necessary. We have continued to find this kind of team approach desirable in other areas, such as advertising and marketing, account-