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# Design and Valuation of Debt Contracts

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*This article studies the design and valuation of debt contracts in a general dynamic setting under uncertainty. We incorporate some insights of the recent corporate finance literature into a valuation framework.*

*The basic framework is an extensive form game determined by the terms of a debt contract and applicable bankruptcy laws. Debtholders and equityholders behave noncooperatively. The firm's reorganization boundary is determined endogenously.*

*Strategic debt service results in significantly higher default premia at even small liquidation costs. Deviations from absolute priority and forced liquidations occur along the equilibrium path. The design tends to stress higher coupons and sinking funds when firms have a higher cash payout ratio.*

This article studies the design and valuation of debt contracts in a general dynamic setting under uncertainty. In doing so we draw together two strands of the finance literature that have developed significantly in recent years, but have done so in large part indepen-

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We have benefited from the comments of seminar participants at the Stockholm School of Economics, ECARE, IGIER, Baruch College, Carnegie-Mellon University, and the ESF Finance Network Symposium in Gerzensee, Switzerland, and from discussion with Hayne Leland, John Parsons, and William Perraudin. We thank Yonghua Pan and Pierre Tychon for computational assistance and comments. This article has been improved by the comments of the referee and the editor, Franklin Allen. This research has been supported through an ESF Network in Financial Markets travel grant. Responsibility for views expressed and for errors is our own.

dently of one another. By incorporating some insights of the recent corporate finance literature into a valuation framework, we obtain a model that seems promising for the empirical study of pricing of risky debt claims and which gives insights into the question of why certain contractual provisions are selected in some situations but not in others. Beyond this we believe our framework suggests a general approach incorporating strategic considerations in a valuation setting which can be usefully explored in future work.

### **Debt valuation theory**

Merton (1974) was the first to apply the valuation insights of Black and Scholes (1973) to the pricing of corporate debt contracts. He takes two key contractual provisions exogenously: first, the lower reorganization boundary (which is the threshold value of the firm at which the control of the firm transfers from the stockholders to the bondholders) is specified. Second, the compensation to be received by creditors upon reaching the lower reorganization boundary is taken as given. Jones et al. (1984) extended the Merton model to coupon and callable debt, however, maintaining the assumption that the lower reorganization boundary is the minimum of the firm's value at maturity and the promised face amount. This implies that there are no enforcements of indentures prior to maturity. When applied to a sample of 305 bonds of various ratings, their model resulted in prices systematically in excess of those observed in the market, that is, their model underestimated observed yield spreads. They interpreted their results as "establishing research priorities in what will be a large and complex task" of applying contingent claims pricing models to risky debt. Kim, Ramaswamy and Sundaresan (1993) pursued this agenda by extending the contingent claims pricing model to incorporate (1) enforcement of bond covenants on intermediate coupon payments and (2) a stochastic term structure. Their results indicated that yield spreads were sensitive to imposing cash flow checks prior to maturity and that they are an increasing function of the bankruptcy costs (parameterized in their model as the recovery rate). Surprisingly, yield spreads were found to be relatively insensitive to the volatility of interest rates. Maloney (1992) has recently extended this approach to allow for more general stochastic processes for the firm cash flows and interest rates.

All these studies take as exogenously given the firm's lower reorganization boundary as well as the payoffs when that boundary is met. These models are also not consistent with the empirical regularities reported in recent studies on financial distress by Asquith, Gertner, and Scharfstein (1991), Franks and Torous (1989, 1993), and Weiss (1990). These regularities are (1) bankruptcies are costly both because of direct costs and because of disruptions of the firm's activities;

(2) bankruptcy procedures give considerable scope for opportunistic behavior by the various parties involved; (3) deviations from absolute priority of claims are common; and (4) despite the incentives to do so, in practice it often proves impossible to renegotiate claims so that formal bankruptcy and liquidation often result. Thus costly liquidations represent a possible source of inefficiency associated with debt contracts.

### **Corporate finance and debt design**

Historically, corporate finance has been concerned with other possible inefficiencies. Jensen and Meckling (1976) emphasized the agency costs of equity finance in the face of managerial moral hazard. In another vein, Myers (1977) identified the agency costs of debt by pointing out the underinvestment problem, that is, the possibility that firms will forego positive net present value investment opportunities because benefits are likely to accrue to debtholders rather than shareholders. Recent theory has reexamined these issues from the perspective of security design. This approach considers how both control rights and cash flow claims may be structured so as to minimize contracting inefficiencies. Aghion and Bolton (1992) and Zender (1991) emphasize that contracts that grant control to one class of agents exclusively may not be efficient because either they fail to give the controlling agent the incentives to make first-best decisions or because contracts sold to outsiders will not be sufficiently valuable to permit raising the required outside finance. They show that contracts with *contingent transfer of control rights* may minimize inefficiencies; this provides a rationale for debt contracts.

The properties of debt contracts have been studied recently in a number of theoretical models. For example, Hart and Moore (1991) examine the problem of designing optimal debt contracts in a dynamic setting under certainty. Bolton and Scharfstein (1993) study the trade-offs involved in making debt more or less easy to renegotiate. A number of other models examine the consequences of alternative bankruptcy models.<sup>1</sup>

### **Synthesis of valuation and corporate finance**

These corporate finance models tend to be very stylized and not truly dynamic. Furthermore, they have *no* valuation consequences or direct testable implications for corporate bond yield spreads. On the other

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<sup>1</sup> Other recent papers concerned with debt include Bergman and Callen (1991), Dewatripont and Tirole (1992), and Franks and Nyborg (1993).

hand, the valuation theory is truly dynamic but fails to endogenize contractual provisions.

Our article attempts to fill the gap by integrating these two strands of research. The basics of our approach are summarized as follows. For us, the terms of a debt contract and bankruptcy law establish the extensive form game in which debtholders and equityholders interact. The allocation of cash flows and the firm's reorganization boundary are determined *endogenously* as the noncooperative equilibrium in this game. Given this solution, we are then able to address the question of the design and the valuation of multiperiod debt contracts under uncertainty.

Two additional recent studies address some of the questions that we do and obtain results that are in some ways similar to our own. Leland (1993) reconsiders the contingent claims framework in continuous time. By assuming that all debt service must be met by issuing new equity he endogenizes the firm's bankruptcy point. Specifically, it is the point where the value of equity is zero. He restricts his study to the case of perpetual debt essentially in order to obtain closed-form solutions permitting the calculation of a variety of interesting comparative statics. His approach is similar to ours in that it endogenizes the lower reorganization boundary of the firm. Otherwise the approaches are quite different. In particular, there is no modeling of the process for resolving financial distress and there is no scope for strategic debt service. Furthermore, since the framework is constrained to using perceptual coupon bonds, it cannot be used to address issues of financial contract design.

Like Leland, Mella-Barral and Perraudin (1993) also work in continuous time and confine their attention to perpetual coupon bonds. They explicitly model the shutdown decision of a firm operating with constant flow costs but with a stochastically varying output price. In this way they are able to study the effect of leverage on the operations of the firm. Their analysis allows for strategic debt service, and in this sense is closer to our model than is any other. However, they do not explicitly model the bankruptcy game. Their analysis is simplified considerably by the fact that they work with perpetual debt. This reduces its relevance to pricing actual bonds with finite maturities and specific (sometimes complicated) contractual features. Furthermore, this means that they cannot address the question of the design of debt contracts.

In the next section we set out our model and derive the equilibrium values of corporate liabilities. Using a simple model of the bankruptcy process, we find the equilibrium often will result in renegotiations with deviations of priority in the favor of equity. In Sec-

tion 2, we compare our results to Merton (1974) and subsequent applications of the contingent claims approach to pricing risky debt. We find our model produces risk premia more in line with levels observed in the market for reasonable parameter values. Section 3 is devoted to the design of debt contracts. We show how high cash payout rates, low leverage, and low liquidation costs lead to higher coupon debt, in general. Sinking fund provisions are generally used to trade off the tax advantages of interest payments with risks of costly liquidations. Section 4 states our conclusions and suggests avenues for future work.

## **1. The Model**

### **1.1 Preliminaries**

The setting we consider is simple: there is one owner-manager who has “access” to a technology and is endowed with a technology-specific human capital. His human capital is imbedded in a project which, if undertaken, will give rise to a stream of rents indefinitely into the future. The project requires financing in the amount of capital  $D$  which the owner-manager does not have. We assume that the financing is to be arranged through the issue of a debt contract, which we take to be characterized by the priority of claims and contingent transfer of control.<sup>2</sup>

We restrict attention to a single homogeneous group of creditors to abstract from holdout problems. The terms of the contract call for a payment of  $CS_t$  in period  $t$  up to the maturity date  $T$ . It is assumed that debt service is met out of cash flows and that asset sales or the issue of new securities would require the explicit agreement of creditors. Finally, we assume that all contracting parties have full information about the states of the nature.

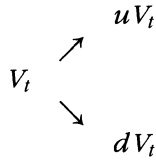
### **1.2 The technology**

The ongoing project is represented as a stochastic process for the value  $V_t$ , which is the present value of current and all future cash flows. The value is assumed to follow a simple binomial process

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<sup>2</sup> This assumption is also used by Aghion and Bolton (1992) and Hart and Moore (1991) among others to study capital structure and security design. In this context, contingent transfer of control through a debt contract is preferred to outside equity since the latter would incur an agency cost from the outset. Later, we will see that the tax deductibility of debt service would provide an additional reason for issuing debt.

as illustrated<sup>3</sup>



where  $d = 1/u$ .  $V_t$  may be interpreted as the cum dividend value of the firm were it to be financed entirely by equity. This is also the value of assets of the firm under alternative financing arrangements. This specification allows us to nest as a special case of Merton's (1974) analysis of zero coupon debt. Cash flows are assumed to be proportional to the value of the project; thus  $f_t = \beta V_t$ , where  $\beta$  is the payout ratio.

The mapping between the cash flows and value processes is based on martingale probabilities  $p$  which are time and state invariant and which are given by  $\frac{r(1-\beta)-d}{u-d}$ .<sup>4</sup> We assume  $u > r(1-\beta) > d$ . It will be noticed that the cash payout ratio  $\beta$  enters the martingale probability of an "up move" with a negative sign. This allows us to model within the same framework "growth firms" (low  $\beta$  projects for which a large proportion of current value is accounted for by expected future growth) and "cash cows" (high  $\beta$  projects for which a large proportion of current value is accounted for by near-term cash flows). Our main restriction is that in taking the cash flow process as exogenous we are not able to study explicitly the possible feedback of the financial contracting onto future production decisions.

Once underway, control of the project can be transferred only at a cost. This may in part take a direct form, such as legal costs, that reflect costly verification of collateral values. In part this may reflect a loss of project-specific human capital: after the transfer of control, it takes time and effort for the creditor to find another management team that will be able to produce the cash flows from the technology at its full efficiency. In what follows we assume these costs are all summarized in constant liquidation cost of  $K$  so that the collateral value equals  $V - K$ . The model could easily accommodate other functional forms for collateral values possibly at the cost of greater complexity. In other respects the markets are assumed to be frictionless. We examine later a variant of the model that allows for corporate income taxation.

<sup>3</sup> Our approach is easily applied to more general two-state branching processes with state-dependent probabilities. For expositional clarity, we are illustrating the model with a simple binomial lattice.

<sup>4</sup> This can be derived by solving for  $p$  such that  $V_t$  is the sum of the current cash flow plus the expected discounted value of  $V_{t+1}$ . This solution is unique. The Appendix provides a proof of these assertions.

All cash flows from the project are paid out in the form of dividends, debt service, or to cover bankruptcy costs. Thus the model satisfies the value preservation property,  $V = E + B + L$ , where  $E$  is the value of equity,  $B$  is the value of debt, and  $L$  is the present expected value of future bankruptcy costs. Thus the value of the levered firm  $V_l$ , given by the value of its financial claims, is the asset value of the firm less the expected bankruptcy costs,  $V_l = V - L$ . We now proceed to discuss the interaction of the owner-manager and the creditor taking the financial contract as given. The optimal design of financial contracts is discussed below.

### 1.3 The game

The technology, the provisions of the debt contract, and the law establish the environment within which the owner and the creditor interact. We represent this with an extensive form game which, though very simple, has interesting dynamic implications. Our general approach can readily accommodate modifications to this game form.

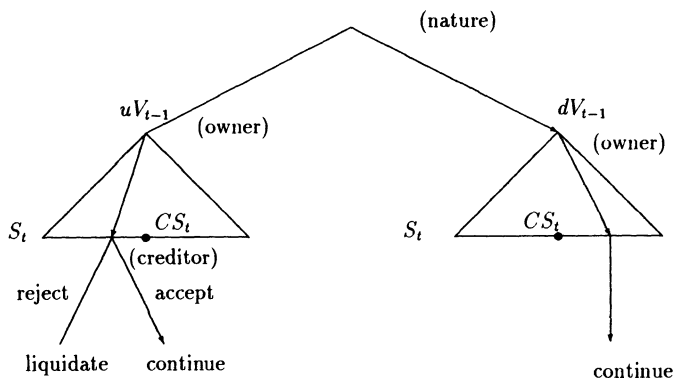
At any given time while the project is ongoing, there will be a realization of the cash flow,  $f_t$ . Given this, the owner chooses a level of debt service,  $S_t \in [0, f_t]$ . If the debt service is equal to or greater than the contracted amount,  $CS_t$ , the game continues to the next period. If it is less than the contracted amount, the creditor has the choice of not initiating legal action (i.e., accepting the service) or initiating legal action (i.e., rejecting the debt service). If the debt service is accepted the project continues to the next date. If the debt service is rejected, the project is liquidated, leaving the creditor with the cum dividend value of the project less the liquidation cost.

The subgame originating from a state  $V_{t-1}$  is depicted in Figure 1.

At each node the player taking the action is indicated in parentheses. (A random variable is viewed as an action taken by nature.) The arrows indicate a decision. Thus in the figure the event is a “down move” to  $dV_{t-1}$ . From that point the owner chooses a debt service that lies within the range indicated by the base of the triangle. Here he is shown as offering a debt service greater than the contracted service  $CS_t$ , which leads to a continuation to date  $t + 1$ . Had there been an “up move” to  $uV_{t-1}$  the owner is shown as offering a debt service below the contracted amount. Since a covenant of the debt contract is violated, this creates a decision node for the creditor. The figure indicates that he accepts the service rather than electing to liquidate the project. In Figure 1 the indicated actions of the owner and creditor are not necessarily their best responses. Below we will discuss the *equilibrium* in the game.

In our formulation we have modified the modeling of the firm bankruptcy process fundamentally as compared to the approach of





**Figure 1**  
**The subgame at  $t$**

The arrows in Figure 1 indicate a decision. The event illustrated is a “down move” to  $dV_{t-1}$ . From that point the owner chooses a debt service that lies within the range indicated by the base of the triangle. Here he is shown as offering a debt service greater than the contracted service  $CS_t$ , which leads to a continuation to date  $t+1$ .

Merton where bankruptcy is determined simply by the state of the nature and the contract, leaving no scope for either the owner or the creditor to take any initiative. Our model gives the owner the scope for choosing to underperform his debt contract, even when the health of the project would enable him to fully meet his obligations. We will see that the costliness of liquidation means that the owner can exploit this opportunity to his own advantage. At the same time our model gives the creditor the scope for choosing how to approach a potential bankruptcy. He need not force bankruptcy when it is not in his interests to do so. This can be viewed as a simple representation of a negotiation between the owner and the creditor that arises whenever a contractual provision is violated. Admittedly this is very stylized and gives a first-mover advantage to the owner. Other ways of modeling bankruptcies could be considered that might allow more elaborate interactions, possibly to the creditor's advantage.

In our formulation an important loan indenture is meeting the currently scheduled payment of interest and principal. This is a basic and widespread feature of debt contracts. It should be noted, however, that violating this does not automatically throw the firm into bankruptcy. It merely creates a decision node for the creditor in which he must decide whether or not to initiate legal action against the owner-manager.

Requiring debt service to lie in the interval  $[0, f_t]$  is important and requires some comment. First, it implicitly assumes that the owner cannot issue additional debt once the project is underway. Loan indentures very frequently do forbid the issue of additional debt with

priority equal or superior to that of the original issue. Here we also exclude the issue of junior debt since the incorporation of subordinated debt valuation is a complication best approached after our basic model is fully understood. Similarly, we do not allow debt service to be funded through asset sales or equity issues. Loan indentures commonly do restrict asset liquidation since this might be exploited by owner-managers as a means of extracting value at the expense of undermining collateral values. Furthermore, the assets in place may be discrete.

The assumption that no new securities are issued to service existing debt simplifies our analysis and arguably comes close to reality in many circumstances. There may be fixed costs associated with security issues that may be higher for a firm in financial distress than for a healthy firm. Furthermore, adding new classes of claimants may make subsequent renegotiations more difficult so that the firm liquidation may simply be delayed but not prevented. Therefore, we believe working with a model where new security issues are excluded is a good starting point for the analysis of strategic debt service. Nevertheless, we would view enriching the game form to allow for securities issues in *some* circumstances to be an interesting avenue for future work.

#### 1.4 Equilibrium

We assume that, once the debt contract has been established, the owner and the creditor each choose actions in their own self-interest. We assume complete information in the sense that the game as just described, as well as the payoffs to the two sides, are both common knowledge. As a result our attention is restricted to the subgame perfect equilibria of the game. That is to say that an equilibrium is constructed under the assumption that at each possible decision point, including those that are never actually observed in equilibrium, the agent makes a maximizing decision. As is typical in such settings, we construct the equilibrium recursively.

At maturity  $T$ ,  $V_T$  is observed, and the owner selects a debt service  $S_T$ . If this is not less than the contracted amount,  $S_T \geq CS_T$ , the game ends. Otherwise,  $S_T < CS_T$ , the creditor must decide whether to accept or reject. If debt service fulfills the contract or if the creditor accepts, the payoffs to the debtholder and the owner, respectively, are

$$(S_T, V_T - S_T),$$

whereas if a debt service is rejected the payoffs are

$$(\max(V_T - K, 0), 0).$$

The assumption that the liquidation value of the firm cannot be negative reflects the fact that liquidation costs are deducted from remaining firm value.

Equilibrium in this subgame is formed by the decisions rules of the creditor and owner that constitute the best responses in light of the payoffs. Given underperformance,  $S_T < CS_T$ , the best response of the creditor is to accept if  $S_T \geq \max(V_T - K, 0)$  and to reject otherwise. The best response of the owner is to set  $S_T = CS_T$  if  $V_T - K > CS_T$ . Otherwise he sets  $S_T = \max(V_T - K, 0)$ . Thus if the value of the firm is relatively high so that the liquidation value exceeds the contracted debt service, the owner is best off simply honoring the contract. For relatively low values of the firm, the owner is best off by making the minimum debt service payment which just leaves the creditor indifferent between accepting or liquidating the firm.

The payoffs for the creditor and owner conditional on the realization of  $V_T$  are given by the equilibrium values which are

$$B(V_T) = \min(CS_T, \max(V_T - K, 0)) \quad (1)$$

for debt, while the value of the equity is

$$E(V_T) = V_T - B(V_T). \quad (2)$$

Notice that these payoffs reflect the possibility of strategic debt service: in terminal states where contracted payment is less than the project value but greater than its liquidation value, the owner-manager underperforms the debt contract, but by an amount insufficient to provoke the creditor to take legal action.

For periods prior to the term of the debt contract, the best responses of the agents will be based on similar reasoning. That is, the owner-manager in some states will try to reduce debt service to the point where the creditor is just indifferent between accepting the service and taking legal action. The important complication in these earlier periods is that, in calculating payoffs, the agents must take into account the values of the continuation subgames. Since these depend on the future realizations of the project values, they are uncertain. Under the assumption that the markets are dynamically complete, both the debtholder and the owner-manager will evaluate the continuation payoffs using the same martingale probabilities [see Harrison and Kreps (1978)]. This preserves the complete information character of the game.

Suppose at time  $t$  the project is ongoing, and there has been a realization  $V_t$ . The owner selects a debt service  $S_t$ . If this at least equals the contracted amount,  $CS_t$ , the game continues. If this underperforms the contract, the creditor can reject the payment, which results in the

liquidation value

$$\max(V_t - K, 0).$$

Alternatively, the creditor can accept, which results in the payoff

$$S_t + \frac{pB(uV_t) + (1 - p)B(dV_t)}{r}$$

where  $r$  is one plus the risk-free interest rate per period. Thus, in the face of underperformance, the best response of the creditor amounts to selecting the greater of these two values. The owner determines his best action in light of this. For relatively high realized values of the firm,  $(V_t - K) > CS_t$ , the owner simply meets the contractual debt service. For relatively lower values of the firm, he elects to underperform the contract to the point that leaves the creditor indifferent between liquidating or not.

To find the equilibrium value of the debt contract, note that the owner will try to reduce the debt service but seeks to avoid provoking the creditor to liquidate the firm. If liquidation does not occur, then in state  $V_t^j$  the level of debt service is

$$S(V_t) = \min \left( CS_t, \max \left( 0, \max(V_t - K, 0) - \frac{pB(uV_t) + (1 - p)B(dV_t)}{r} \right) \right). \quad (3)$$

The value of debt thus is

$$B(V_t) = S(V_t) + \frac{pB(uV_t) + (1 - p)B(dV_t)}{r}. \quad (4)$$

The corresponding value of equity is

$$E(V_t) = f_t - S(V_t) + \frac{pE(uV_t) + (1 - p)E(dV_t)}{r}. \quad (5)$$

In some states, however, cash flows will be insufficient to pay an amount acceptable to the creditor. Since in this framework asset sales and new security issues are excluded, liquidation will result. Thus, forced liquidation occurs if

$$S(V_t) > f_t.$$

In the case of forced liquidation, the value of debt is

$$B(V_t) = \max(0, \min(V_t - K, CS_t + P_t)), \quad (6)$$

where  $P_t$  is the principal of loan outstanding at time  $t$ . The value of

equity is

$$E(V_t) = V_t - K - B(V_t). \quad (7)$$

This completes the construction of a subgame perfect equilibrium. This is a unique equilibrium if the liquidation costs are strictly positive. For in that case, even though  $S_t$  is chosen to leave the creditor indifferent between accepting and rejecting the service, rejecting never occurs in equilibrium. The owner's best action in anticipation of rejection is to slightly raise  $S_t$ , which would make the creditor strictly prefer acceptance.

This equilibrium merits a few comments. First, in contrast with the traditional approach in contingent claims valuation, the states in which contract default occurs have been determined endogenously from the primitives of the model. Furthermore, control of the project is transferred to creditors only in a subset of default states where the firm is illiquid. Second, the equilibrium may involve strategic debt service, which is consistent with the observed opportunism of contracting parties under many bankruptcy procedures. Our strategic debt service can be interpreted as the outcome of a negotiation process with a deviation from absolute priority in favor of equity. The size of the deviation depends on the health of the asset in place and the costliness of liquidation. Third, costly forced liquidations can occur in equilibrium since asset sales are infeasible. This kind of *ex post* inefficiency implies that the debt capacity of the project, that is, the maximum amount that can be raised to finance the project may be less than its full asset value. This implies a possible *ex ante* inefficiency as well: positive net present value projects may not be undertaken if the debt capacity is less than the financing requirement of the owner-manager. Thus the model reflects an underinvestment problem that can be interpreted as an agency cost of debt. However, it should be recognized that since we do not allow feedback onto future investment decisions we do not incorporate the form of agency cost originally identified by Myers (1977).<sup>5</sup> Fourth, our model has been set out in discrete time. This has the advantage of making the modeling of the bankruptcy process transparent. Our particular model has the feature of giving a first-mover advantage to the owner-manager. A number of alternative models of the bankruptcy process that have been suggested in the literature could be incorporated as rather direct extensions of our framework. In contrast, modeling noncooperative games in continuous time can present subtle difficulties. Indeed, Fudenberg and Tirole

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<sup>5</sup> See Mello and Parsons (1992) for an attempt to measure debt agency costs arising both from lack of investment commitment by shareholders and from bankruptcy costs. However, this study does not endogenize the lower reorganization boundary of the firm, nor does it accommodate strategic behavior in bankruptcy.

[1985] have convincingly argued that for continuous time games is preferable to find the equilibrium in a discretized version of the game and then take limits as the time interval goes to zero. Fifth, the equilibrium holds for quite general stochastic processes. Furthermore, it is consistent with risk averse utility maximizing owners and creditors.

## **2. Valuation**

In this section we use our model to evaluate straight debt contracts, that is, contracts offering nominally a fixed coupon and full principal reimbursement at maturity. This involves a promised payment of a coupon interest rate of  $c$  per period and a principal  $P$  to be reimbursed at the maturity date  $T$ . Thus we have  $CS_t = cP$  for  $t < T$  and  $CS_T = (c + 1)P$ . We first consider the case of a zero coupon (pure discount) debt to serve as a benchmark for comparison with Merton (1974). Later we consider the more important case of coupon bonds.

### **2.1 Discount debt**

A useful benchmark for our model is Merton's (1974) analysis of zero coupon, risky bonds. Our model in the special case of a zero coupon ( $c = 0$ ) and zero cost of liquidation ( $K = 0$ ) is a discretized version of Merton's. Strategic considerations in the bankruptcy decision only enter into play for strictly positive bankruptcy costs ( $K > 0$ ). Table 1 presents the risk premium of debt (i.e., the difference between the internal rate of return on the risky bond,  $R$ , and the risk-free rate,  $r$ ) for zero coupon bonds under a variety of assumptions about the remaining parameters. For ease of comparison we have parameterized our model as in Merton with respect to the volatility of value process,  $\sigma^2$ , the firm's quasi-debt firm value ratio,  $d = Pe^{-rT}/V_0$ , and have assumed  $\beta = 0$ .

The results for  $K = 0$  correspond to those reported by Merton (1974) in his Table 1. The slight discrepancies between our results and his are due to our working with a discrete version of his continuous time model. Thus we see that, in the case of zero bankruptcy costs, the premium is increasing in the leverage,  $d$ , and the volatility of the underlying asset,  $\sigma^2$ . The effect with respect to time to maturity depends crucially on the degree of the firm's leverage. For example, for  $d = 0.5$  the premia for  $T = 2$  are less than those when  $T = 10$ , whereas the reverse holds for  $d = 1.5$ . The interpretation is that, for firms with a low degree of leverage, default will occur only if the firm value declines substantially, a prospect that is more likely for long maturities than for short maturities. For highly leveraged firms, default will be avoided only if the firm value improves significantly, a prospect that is more likely for higher maturities.

**Table 1**  
**Discount debt**

<b>(a) Time to maturity (<math>T</math>) = 2.00 years</b> <b><math>V = 1</math>, <math>\beta = 0.00</math>, <math>c = 0.00</math>, <math>r = 1.05^1</math></b>				
Risk premium, $R - r$ (%) <sup>2</sup>				
$\sigma^2$	$d$	$K = 0$	$K = 0.1$	$K = 0.2$
0.030	0.200	0.000	0.000	0.000
0.030	0.500	0.012	0.106	0.513
0.030	1.000	5.129	8.396	12.734
0.030	1.500	20.578	25.556	31.429
0.100	0.200	0.002	0.050	0.378
0.100	0.500	0.817	2.031	4.138
0.100	1.000	9.738	13.365	17.779
0.100	1.500	23.022	27.581	32.845
0.200	0.200	0.118	0.796	2.702
0.200	0.500	3.087	5.682	9.358
0.200	1.000	14.266	18.404	23.310
0.200	1.500	26.601	31.232	36.345

<b>(b) Time to maturity (<math>T</math>) = 10.00 years</b>				
0.030	0.200	0.004	0.031	0.113
0.030	0.500	0.376	0.629	0.994
0.030	1.000	2.430	2.988	3.680
0.030	1.500	4.969	5.678	6.568
0.100	0.200	0.474	1.049	1.956
0.100	0.500	2.113	2.860	3.799
0.100	1.000	4.826	5.622	6.568
0.100	1.500	7.116	7.919	8.869
0.200	0.200	1.872	3.435	5.522
0.200	0.500	4.379	5.695	7.356
0.200	1.000	7.351	8.494	9.878
0.200	1.500	9.546	10.609	11.867

<sup>1</sup>  $V$  refers to the initial value of the firm,  $d$  is the quasi debt to value ratio,  $c$  is the coupon,  $\beta$  is the cash payout ratio (cash flow divided by the value of the firm),  $r$  is the riskless rate,  $K$  is the fixed liquidation cost, and  $T$  is the time to maturity in years.

<sup>2</sup>  $R - r$  is the risk premium. The results are based on discrete time binomial valuation procedure in which the time to maturity is divided into 1,000 time steps.

The effect of positive bankruptcy costs is indicated in Table 1 by the yield spreads calculated under the assumption that  $K = 0.1, 0.2$ . For example, for a firm with  $\sigma^2 = 0.03$  and  $d = 0.5$ , 2-year debt requires an insignificant spread (about 1 basis point) if liquidation is costless, whereas it carries spreads of 10 basis points and 51 basis points, respectively, when liquidation costs are 10 percent and 20 percent of value.<sup>6</sup>

<sup>6</sup> It should be stressed that  $K = 0.2$  is used to reflect an upper bound on the direct and indirect costs of liquidation.

Thus Table 1 demonstrates an important property of our model—properly accounting for costly bankruptcy can better explain observed premia on traded risky debt than does the standard contingent claims framework.<sup>7</sup> Much of the increase in calculated spreads is accounted for by strategic debt service in our model. To see this we have also computed the implied spreads if Merton's (1974) model is modified by deducting liquidation costs  $K$  whenever the exogenous reorganization boundary  $\min(V_T, P)$  is met. For the case of  $T = 2$ ,  $\sigma^2 = 0.03$ ,  $d = 0.5$ , and  $K = 0.2$  this ad hoc adjustment results in a yield spread of only 8 basis points. Thus most of the 51 basis point spread found in our model reflects strategic debt service in states where  $V_T \geq P$ . In order to get additional insights on the impact of strategic debt service in our model, we provide Figure 2 in which the liquidation costs are varied from 0 to 0.20 and the default premia implied by our model are compared with those implied by Merton with ad hoc adjustment. Note that for a lower debt level of  $d = 0.2$  the possibility of strategic debt service contributes much more to the default premia even when the liquidation costs are relatively low. To see why, note that in the range of states  $V_T \in (P, P + K)$  our model involves strategic debt service, whereas the modified Merton model repays debt in full. In the range  $V_T < P$ , debt payments in the two models are identical. For initially low leverage firms the former range is relatively more likely; therefore, a higher proportion of yield spreads are accounted for by strategic debt service.

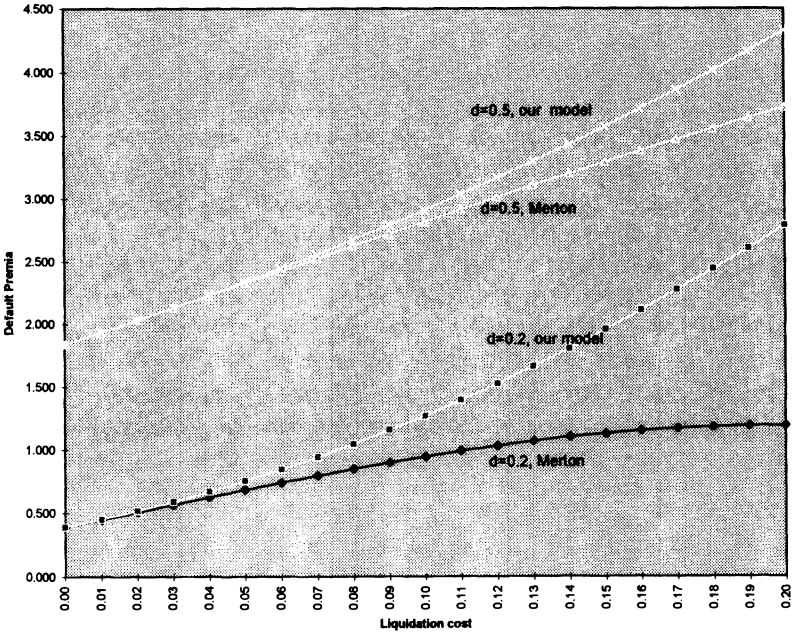
Careful inspection of Table 1 reveals that the amount that bankruptcy costs increase the yield spreads depends systematically on the degree of leverage and the volatility of the underlying asset. This is shown clearly in Figures 3 and 4, which plot yield spreads on zero coupon debt in the Merton model ( $K = 0$ ) and in our model with  $K = 0.1$  with respect to  $d$  and  $\sigma^2$ .

In Figure 3 we see that yield spreads in the face of costly bankruptcy exceed those with costless bankruptcy for all degrees of leverage. The difference in yield spreads grows with the degree of leverage, particularly in the case of short-term debt ( $T = 2$ ). Similarly in Figure 4 we see that the premia for bonds with costly bankruptcy are greater than premia without bankruptcy cost by an amount that is an increasing function of the underlying asset volatility. Increasing the volatility increases the chances that asset values will erode sufficiently after 2 years so that shareholders will default in the payment of principal outstanding either opportunistically or through forced liquidation.

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<sup>7</sup> Below we make an explicit comparison to observed spreads with reference to coupon bearing debt.





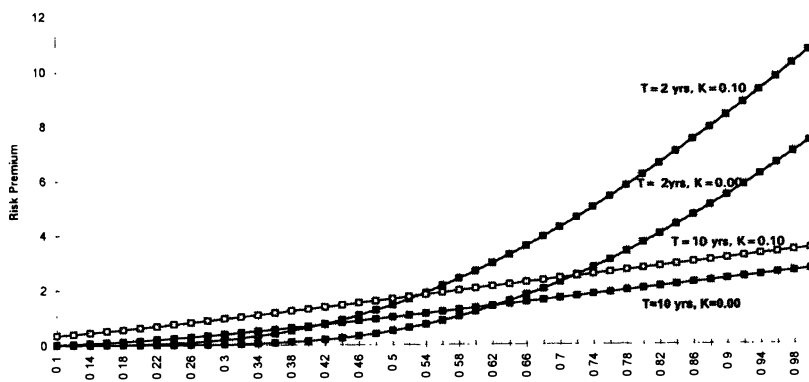
**Figure 2**  
**Effect of strategic debt service on default premia**

Starting firm value is assumed to be 1. The debt has a time to maturity of 10 years. A zero coupon bond, with a firm beta of 0.04 and a variance of 10 percent is considered. Riskless rate is assumed to be 5 percent.

Having established some of the properties of our model for the case of pure discount instruments we now turn to the empirically more important case of coupon paying debt.

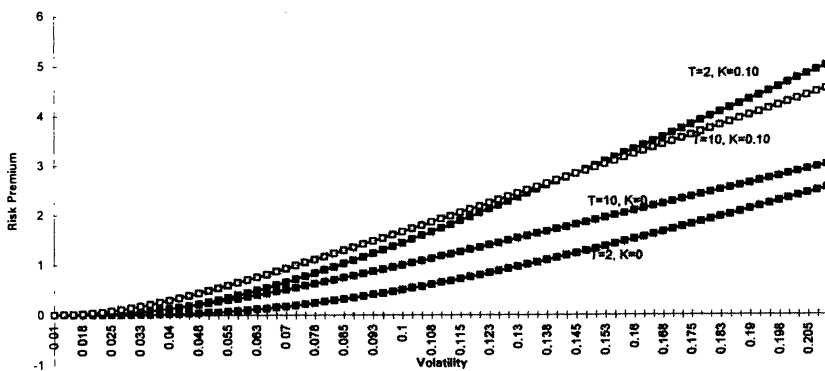
## 2.2 Coupon paying debt

A widely observed feature of risky debt contracts such as corporate bonds or syndicated bank loans is that loan indentures specify the regular payment of interest. Violation of this covenant can lead to bankruptcy and possible liquidation of the firm. This common practice reflects the fact that such flow payments are more readily verifiable than are firm values. Our model incorporates this feature when we specify a strictly positive coupon payment ( $c > 0$ ). When liquidation costs are zero ( $K = 0$ ), equilibrium debt service involves the payment of the scheduled coupon whenever the firm is able to pay ( $\beta V_t \geq cP$ ); otherwise, the firm is liquidated. This is a direct extension of



**Figure 3**  
**Effect of debt ( $d$ ) on risk premium**

This figure shows the effect of debt level on risk premium. A zero coupon bond, with a firm beta of 0 and a variance of 3 percent is considered. Riskless rate is assumed to be 5 percent.



**Figure 4**  
**Effect of volatility on risk premium**

The effect of volatility on risk premium is plotted for debt contracts with 2 years and 10 years maturities. Two liquidation costs (0 and 10 percent) are used. We have assumed a zero coupon bond, with a firm beta of 0. Riskless rate is assumed to be 5 percent.

the Merton (1974) model.<sup>8</sup> When  $K > 0$  equilibrium debt service in our model may fall short of scheduled coupon payment without provoking liquidation.

Results pertaining to coupon paying debt are presented in Table 2.

<sup>8</sup> Kim, Ramaswamy, and Sundaresan (1993) extend the Merton model to coupon debt and allow for interest rate uncertainty. Our model with  $c > 0$  and  $K = 0$  is a discretized version of their model in the case of zero interest rate volatility.

**Table 2**  
**Coupon debt**

Time to maturity ( $T$ ) = 2.00 years $V = 1$ , $\beta = 0.10$ , $c = 0.10$ , $r = 1.05$							
Risk premium, $R - r$ (%) and bond price, $B$							
$\sigma^2$	$P$	$K = 0$		$K = 0.1$		$K = 0.2$	
		$R - r$	$B$	$R - r$	$B$	$R - r$	$B$
0.030	0.200	0.0000	0.2190	0.0000	0.2190	0.0033	0.2190
0.030	0.400	0.0005	0.4381	0.0467	0.4377	0.3931	0.4349
0.030	0.600	0.1154	0.6557	1.1265	0.6437	3.2311	0.6196
0.030	0.800	1.3613	0.8547	4.9593	0.8006	9.4939	0.7376
0.030	1.000	5.2333	0.9958	11.0074	0.8972	17.5217	0.7983
0.100	0.200	0.0006	0.2190	0.1135	0.2186	0.8058	0.2158
0.100	0.400	0.1188	0.4371	1.4226	0.4269	3.7347	0.4093
0.100	0.600	1.0055	0.6452	4.3400	0.6072	8.4881	0.5633
0.100	0.800	2.5711	0.8361	7.5786	0.7635	13.2828	0.6890
0.100	1.000	5.5684	0.9897	11.3698	0.8913	17.9093	0.7928
0.200	0.200	0.0430	0.2189	1.1839	0.2144	3.9532	0.2038
0.200	0.400	0.4974	0.4341	3.9863	0.4074	8.5708	0.3750
0.200	0.600	1.7529	0.6364	6.8005	0.5807	12.6833	0.5223
0.200	0.800	3.7057	0.8190	9.3993	0.7388	15.8690	0.6578
0.200	1.000	5.8894	0.9840	11.7317	0.8856	18.3205	0.7871
Time to maturity ( $T$ ) = 10.00 years							
0.030	0.200	0.0321	0.2780	0.3602	0.2718	1.1849	0.2569
0.030	0.400	0.5155	0.5378	1.4209	0.5056	2.6057	0.4670
0.030	0.600	1.6407	0.7471	2.9029	0.6868	4.3726	0.6243
0.030	0.800	3.1564	0.9006	4.6492	0.8178	6.3662	0.7344
0.030	1.000	5.0813	0.9946	6.7689	0.8957	8.7301	0.7968
0.100	0.200	0.4765	0.2696	2.1688	0.2404	4.2609	0.2096
0.100	0.400	1.4857	0.5033	3.2769	0.4468	5.4490	0.3887
0.100	0.600	2.7824	0.6923	4.5286	0.6181	6.5894	0.5433
0.100	0.800	4.0296	0.8508	5.7473	0.7631	7.7546	0.6752
0.100	1.000	5.2222	0.9857	6.9302	0.8869	8.9183	0.7881
0.200	0.200	1.1050	0.2582	4.2667	0.2095	7.4505	0.1719
0.200	0.400	2.3809	0.4740	4.8206	0.4045	7.9177	0.3343
0.200	0.600	3.4482	0.6627	5.5296	0.5802	8.0542	0.4975
0.200	0.800	4.1523	0.8442	5.9937	0.7515	8.1684	0.6588
0.200	1.000	5.5106	0.9681	7.2566	0.8696	9.2954	0.7711

Here we have taken the face value of the debt,  $P$ , to be the indicator of leverage.<sup>9</sup> The table reports results for a 10 percent per annum coupon bond to be paid continuously. The payout ratio,  $\beta$ , is also set 10 percent per annum. Other parameters are set as in the case of zero coupon debt.

The results are reported with respect to three levels of liquidation costs,  $K = 0, 0.1, 0.2$ . Required yield spreads are increasing in  $K$ , as expected. In fact, the premia appear quite sensitive to changes in liquidation costs. For example, for the parameters  $\sigma^2 = 0.03$ ,  $P = 0.6$ ,

<sup>9</sup> Note that since this differs from the quasi-debt ratio  $d$ , Tables 1 and 2 are not directly comparable.

and  $T = 2$  the spread ranges from 11 basis points to 323 basis points for  $K = 0$  to  $K = 0.2$ , respectively. The sensitivity of coupon debt is the result of the fact that the frequent cash flow checks implied by coupon debt increase the scope for opportunistic debt service.

The results for coupon debt are more directly comparable to risky debt traded in the market place. In the United States yield spreads on long-term corporate bonds of firms rated AAA or AA have averaged approximately 125 basis points in recent years [see Salomon Brothers (1992)]. The median debt-to-capital ratio for firms in this category is approximately 20 percent [see Standard and Poors (1982)]. For a firm with this degree of leverage an asset variance of 10 percent corresponds to an equity volatility that is consistent with this firm being rated AAA or AA.<sup>10</sup>

From the results for the case of  $P = 0.2$ ,  $\sigma^2 = 0.1$ , and  $T = 10$ , we see that these levels of yield spreads are produced in our model with  $K = 0.045$ . Thus, with coupon debt observed spreads are consistent with low levels of asset volatility and bankruptcy costs of less than 5 percent of initial values.

The results in Table 2 reveal that the sensitivities with respect to the basic parameters are similar to those found in our model for discount debt. Yield spreads increase with increases of volatility ( $\sigma^2$ ). They are almost always increasing in the degree of leverage used. The only exception appears in the case of  $T = 10$ ,  $\sigma^2 = 0.2$ , and  $K = 0.2$ , where an increase in principal from 0.2 to 0.4 entails a decrease in the yield spread. The reason that increasing leverage can at times be beneficial to bondholders in our context is that it can make forced liquidations more frequent, which in turn can result in a higher net payoff to bondholders than if they were exposed to opportunistic debt service subsequently. The sensitivity with respect to time to maturity depends on the degree of leverage. The spread is increasing in  $T$  if leverage is low and decreasing if leverage is high.

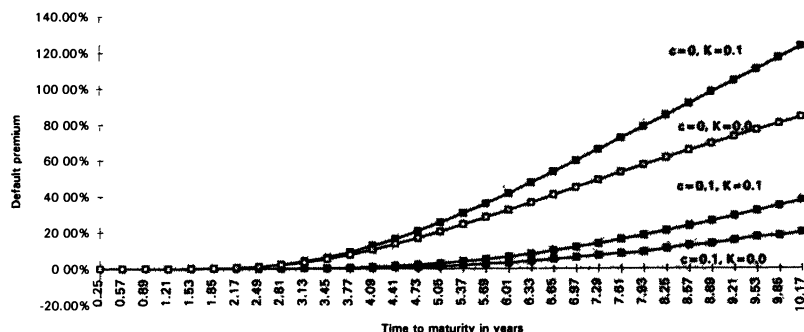
Figure 5 presents the yield spreads as a function of time to maturity for bonds with face value of 0.3 when the underlying project has a payout rate of 10 percent per year and a volatility of 0.03. Four cases are plotted, corresponding to two values of liquidation costs ( $K = 0, 0.1$ ) and two coupons ( $c = 0, 0.1$ ). In each case the yield spread is increasing in the time to maturity, reflecting the low leverage involved. Furthermore, spreads are higher when the liquidation costs

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<sup>10</sup> The volatility of equity as related to the value of the firm is given as

$$\sigma_e = \sigma E_V V/E \leq \sigma V/E,$$

where  $E_V$  is the partial derivative of equity with respect to underlying value. Thus  $\sigma^2 = 0.1$  and  $P = 0.2$  give an upper bound for  $\sigma_e$  of 0.395.



**Figure 5**  
**Risk structure of interest rates**

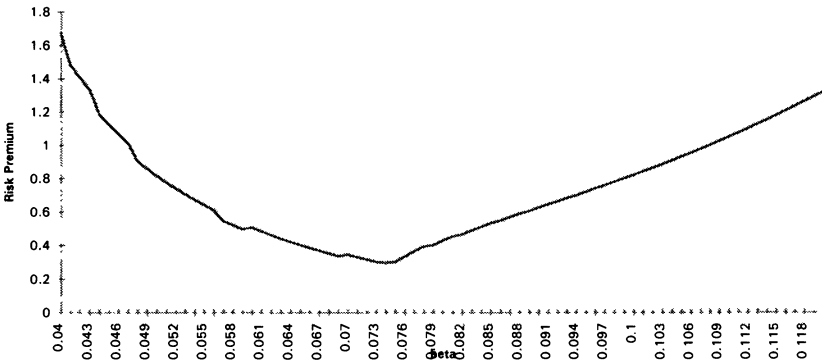
Starting firm value is assumed to be 1. The risk structure is plotted when the firm beta is 0.1, face is 0.3, and the variance is 3 percent. Riskless rate is assumed to be 5 percent.

are positive. For the case presented, spreads are higher for the zero coupon bond than for the coupon bearing bond, holding liquidation costs constant. We explore this effect more explicitly in what follows.

Further subtleties of bond valuation in the face of opportunistic debt service are illustrated in Figures 6 and 7. Figure 6 depicts the relationship between the yield spread and the cash payout ratio,  $\beta$ . It is striking that yield spreads are initially decreasing and then increasing in this payout rate. Sufficiently low levels of  $\beta$  combined with a positive coupon interest lead to a quick forced liquidation. Even if liquidation costs are not so high as to prevent the bondholder from fully recovering the principal, this means the bond carries a risk premium, since the risk-free bond would be valued more than its face value when coupons exceed the risk-free rate. Thus an increase in  $\beta$  from low levels reduces the chances of this occurring and thus reduces required yield spreads. However, increases in  $\beta$  also imply that the growth prospects of the project are worsened.<sup>11</sup> Thus a high  $\beta$  project will not be cash constrained initially; however, it has a relatively high possibility of falling in value, pushing the project into the range where owner-managers will engage in opportunistic violation of the terms of the debt contract. In other words, extending high levels of long-term, fixed interest credit to today's cash cows is a risky business that should command a significant yield spread.

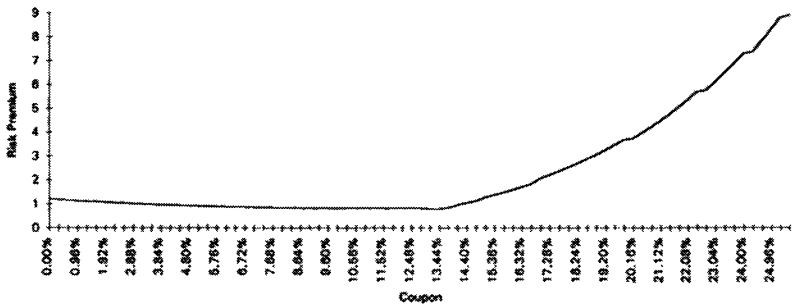
Figure 7 represents the relationship between required spreads and coupon level for debt with given principal and maturity. Again yield

<sup>11</sup> The probability of a "down" move is  $1 - p = \frac{u - r(1 - \beta)}{u - d}$  which is increasing in  $\beta$ .



**Figure 6**  
**Effect of beta on risk premium**

The effect of cash payout ratio (beta) on risk premium is illustrated for  $T = 10$  years, face = 0.30, and coupon = 10 percent. The liquidation costs is assumed to be 10 percent and the variance is 3 percent. Riskless rate is assumed to be 5 percent.



**Figure 7**  
**Coupon effect on risk premium**

This figure examines the effect of increasing the coupon rate on risk premium. We assume that the debt has 10 years to maturity, beta = 0.10, and face = 0.30. The liquidation costs is assumed to be 10 percent and the variance is 3 percent. Riskless rate is assumed to be 5 percent.

spreads at first decrease and later increase as the coupon is increased from 0 to 25 percent. This reflects two effects that operate in different directions. First, increasing the coupon increases the range of states in which a violation of an indenture is likely to occur. However, instead of unambiguously working in favor of the bondholder, the violation of the covenant presents the bondholder with a difficult choice. Either he initiates legal action and thereby incurs a deadweight loss, or he accepts the debt service the owner is willing or able to pay. Since accepting is often the better choice, the owner can take advantage of

this to default opportunistically, that is, by increasing the range of outcomes where the debt will not be fully serviced. The second, opposite effect is that a positive coupon in some states results in bankruptcy for reasons of illiquidity. These forced liquidations in part can help the bondholder in cutting off a possible future of opportunistic debt service by the owner-manager. In effect the positive coupons combined with a no-asset-sale clause serves as a credible commitment by bondholders to require debt service.

To summarize, in comparison to the previous models in the valuation literature surveyed in the introductory section, our model produces higher risk premia due to the opportunistic debt service and the resulting deviations from the absolute priority rule. Empirically observed yield spreads can be replicated with lower levels of bankruptcy costs and asset volatilities in our model.

Our discussion has underlined the ways that a costly bankruptcy process can directly and indirectly affect bond valuations. One general thread that runs through this is that a more costly bankruptcy process provides greater scope for opportunistic debt service since the bondholder will hesitate to initiate legal action. The corollary of this is that increasing liquidation costs which depress bond values can increase equity values. Of course, this effect depends crucially on the incidence of bankruptcy costs which in our model fall on the bondholder. The effect is also tempered to the extent there is a positive probability of forced liquidation. We explore these issues below in our treatment of the design of contract terms. Before doing so, however, we consider how valuation is affected by the introduction of corporate taxes.

### 2.3 Corporate taxes

We have already noted that the framework that we have introduced could be modified without changing the essential valuation methodology. In particular, recent literature on financial distress provides a rich array of one-shot bankruptcy models that could substitute for that in Section 2. We view understanding the dynamic implications of such models a very worthwhile line of inquiry, but we will not attempt this here. Instead, we consider only a slight modification of our model. In particular, we model an extremely simple tax code where corporate earnings are taxed at a flat rate of  $\tau$  but where all interest payments are tax deductible. In this context, equity valuation in the absence of forced liquidation involves replacing Equation (5) with

$$E(V_t) = (f_t - S(V_t))(1 - \tau) + \frac{pE(uV_t) + (1 - p)E(dV_{t+1})}{r} \quad (8)$$

for  $t < T$ . At maturity,  $E_T = (1 - \beta)V_T + (f_T - s_T cP)(1 - \tau) - s_T P$  where  $s_T = S(V_T)/(1 + c)P$ . In the case of forced liquidation, tax

liabilities on current earnings are deducted before calculating liquidation values. This is the only way bond valuations are modified. Since this tax liability  $\tau\beta V_t$  is small relative to  $V_t$ , introducing taxes will have little effect on equilibrium debt service strategies. Taxes will mean, however, that modifications of the terms of the debt contract can have different impacts on equity values depending on the tax rates applied. Thus taxes change bond valuation little but can be an important consideration in determining optimal contract terms.

### **3. Design of Debt Contracts**

A problem of security design emerges when an inappropriate choice of contract features can result in inefficiencies, that is, a loss of aggregate value for the contracting parties. An optimal financial contract minimizes these inefficiencies subject possibly to certain constraints. The literature on security design has considered several alternative sources of contracting inefficiency. Examples include incomplete risk sharing [Allen and Gale (1988)], inefficient information acquisition [Boot and Thakor (1993)], and inefficient allocation of control rights [Aghion and Bolton (1992)].

In our framework, the security design problem originates from inefficiencies in the process for managing financial distress. Specifically, they arise through costly liquidations which we have seen may be possible outcomes in equilibrium. This feature of our model is similar to other treatments emphasizing control rights [see, in particular, Aghion and Bolton (1992) and Hart and Moore (1991)]. In this related literature and in our model, debt is chosen despite its possible inefficiencies because the alternative forms of financial contracting such as all outside equity, will imply inefficiencies or agency costs of their own which may exceed those of debt. However, unlike these models, our framework allows us to explore in detail the choice of the terms of the debt contract, taking into account their full dynamic implications in a stochastic setting. Consequently, we are closer to the practical design problem faced by companies issuing debt.

Formally, a change in the contract specification affects the game form faced by the contracting parties. The problem is to choose the contractual features so as to minimize the inefficiencies that emerge in the associated equilibrium. With many periods and possible states of the world, the design problem is likely to admit a great multiplicity of solutions unless some structure is placed on the set of possible contracts.<sup>12</sup> We consider contracts where contractual debt service is

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<sup>12</sup> Hart and Moore (1991) working in a certainty setting find multiple solutions, which leads them to introduce some restrictions on contract forms.



time dependent, but not state dependent, and all control rights remain with the owner-manager unless there is a default on contractual terms that induces the debtholders to proceed with liquidation. Further, we focus on four features: (1) coupon, (2) maturity, (3) face amount, and (4) the amortization schedule. These features allow us to model a large variety of contract forms encountered in practice. However, the treatment of convertible and callable bonds is left for future work.

Contract terms are modeled as follows. Let  $P_t$  be the outstanding principal amount of the bond in period  $t$ ,  $T$  be the maturity date of the bond,  $c$  be the coupon,  $A_t$  be the principal amount that is amortized in period  $t$  and  $g$  be the grace period in years, that is, the number of years before amortization is included in scheduled debt service. The index  $t$  refers to the number of years elapsed since the issue date. Then the contracted payments to the creditors are  $CS_t = cP_t + A_t$ , where  $P_t = P_{t-1} - A_{t-1}$ . The amortization is computed as follows:  $A_t = 0$ , if  $t \leq g$  and  $A_t = P/(T - g)$ , otherwise. By setting  $g = T - 1$ , we get straight coupon debt, which has zero coupon debt as a special case. Generally, this specification permits sinking fund schedules with different grace periods and purely amortizing debt.<sup>13</sup>

In the design problem the owner-manager selects the vector  $(c, T, P, g)$  so as to maximize the value of the equity subject to constraint that the value of the debt be at least equal to the funding requirement for undertaking the project. This choice will be guided by the primitives of the firm which are (1) the payout factor  $\beta$ , (2) the liquidation costs  $K$ , (3) the amount of money that the firm needs to raise in the debt market  $D$ , (4) the volatility of the firm  $\sigma^2$ , and (5) the tax rate  $\tau$ . Thus the problem is

$$\max_{c, T, P, g} E(V_0; \sigma^2, \beta, r, K, \tau)$$

such that

$$D \leq B(V_0; \sigma^2, \beta, r, K, \tau).$$

Here both  $E$  and  $B$  are given by the equilibrium values in the game implied by the contract. As seen above, the equilibrium generally will not be first-best efficient because it will typically result in forced liquidations in some future states. The solution to the security design problem may also be *ex ante* inefficient in the sense that positive

<sup>13</sup> A term annuity is taken as the limiting case as principal tends toward zero and coupon is adjusted to maintain  $cP$  constant.

net present value projects may not be undertaken. This will occur when it is not possible to write a debt issue valuable enough to meet the funding requirement. The solution will be second-best optimal when the corporate tax rate is zero. This is because the model has a basic value preservation property so that the value of equity equals the asset value of the firm less the value of debt less the value of expected future liquidation costs plus the value of future tax shields. Thus absent taxes, the problem reduces to minimizing liquidation costs subject to a funding constraint.

This design problem does not admit of analytical solutions in non-trivial cases, and its numerical solution is itself rather time-consuming.<sup>14</sup> We report here the results of two sets of design experiments. The first restricts the contracts to be nonamortizing bonds and varies the payout rate,  $\beta$ ; the liquidation costs,  $K$ ; and the debt ratio,  $D$ . The second focuses on the relationship between the speed of amortization and the remaining parameters.

The results of the first experiment are summarized in Table 3. Each row reports the optimal contract for a given combination of  $\beta$ ,  $K$ ,  $D$ , and  $T$ . We describe the optimal contract by the contractual interest payment expressed as a percent of the value of debt and report the associated value of equity.

The most striking pattern in Table 3 is that as the payout rate  $\beta$  increases, the contractual interest increases and the value of equity decreases. To interpret this it is important to recall that the payout rate determines the expected growth rate of the asset value of the firm: the higher  $\beta$  the slower growth in the asset value of the firm. Fast-growing, that is, low  $\beta$ , firms favor relatively low interest burdens because this reduces the chances of an early forced liquidation that would deny the owner the benefits of future growth. In doing so they forego the tax shield benefits that would be associated with higher contractual interest. As we consider progressively slower growing firms, balancing the trade-off leads to higher and higher interest payments. The cash cow (the high  $\beta$  firm with poor growth prospects) chooses a relatively higher interest burden because the value of the near-term tax shield is substantial and the prospect of a near-term illiquidity default is less daunting because cash is relatively abundant and the future cut off by such a default is less bright. The value of equity with the optimal contract is a decreasing function of  $\beta$  because most of the benefits of a high growth rate accrue to equity.

The effect of leverage on contract design is seen by comparing the

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<sup>14</sup> Our program written in C produces the results for Table 4 in about 72 hours on a RISC 6000 machine.

**Table 3**  
**Optimal design of debt contracts**

<b><math>V = 1, \tau = 0.10, r = 1.05</math></b>			
<i>Debt ratio</i>	$\beta$	<i>Coupon (%)</i>	<i>Equity</i>
<b>Time to maturity = 5 years</b>			
<i>Liquidation cost = 0.1</i>			
0.250	0.040	3.69	0.66470
0.250	0.060	5.25	0.65648
0.250	0.080	7.69	0.64992
0.250	0.100	7.80	0.64380
0.500	0.040	2.58	0.42695
0.500	0.060	4.44	0.41670
0.500	0.080	5.44	0.41174
0.500	0.100	5.60	0.40583
<i>Liquidation cost = 0.2</i>			
0.250	0.040	3.71	0.61797
0.250	0.060	5.67	0.60799
0.250	0.080	7.71	0.60086
0.250	0.100	9.67	0.59649
0.500	0.040	3.00	0.39859
0.500	0.060	0.43	0.38694
0.500	0.080	5.43	0.38262
0.500	0.100	7.36	0.38149
<b>Time to maturity = 10 years</b>			
<i>Liquidation cost = 0.1</i>			
0.250	0.040	2.55	0.67233
0.250	0.060	3.93	0.66082
0.250	0.080	4.73	0.65229
0.250	0.100	5.94	0.64701
0.500	0.040	4.85	0.43718
0.500	0.060	6.59	0.41544
0.500	0.080	3.17	0.40767
0.500	0.100	8.99	0.39768
<i>Liquidation cost = 0.2</i>			
0.250	0.040	3.32	0.64787
0.250	0.060	5.17	0.63919
0.250	0.080	7.12	0.63675
0.250	0.100	8.85	0.63431
0.500	0.040	2.09	0.41478
0.500	0.060	3.78	0.40842
0.500	0.080	5.24	0.40157
0.500	0.100	8.00	0.35585

cases of  $D = 0.25$  and  $D = 0.5$ , holding other parameters constant. In most cases the firm with the higher leverage selects a lower level of contractual interest. The reason is that higher leverage tends to make forced liquidation in advance of maturity more likely. The firm can offset this by decreasing contractual interest payments. This appears to be the dominant effect, but as is often the case in the comparative statics of bond valuation, other effects are present that can sometimes dominate. In particular, for  $T = 10$  and  $K = 0.1$  higher leverage is

**Table 4**  
**Optimal design of debt contracts**

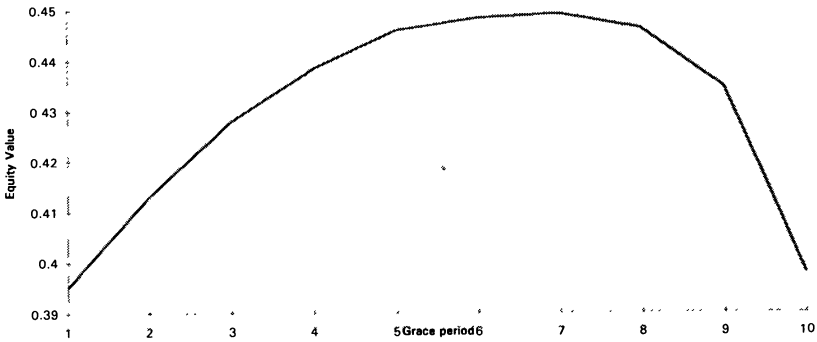
$\tau = 0.1, K = 0.1$					
$T$	$\beta$	$D$	$g$	Contractual interest (%)	Equity
5	0.04	0.25	5	3.69	0.66470
5	0.04	0.50	5	2.58	0.42695
5	0.10	0.25	4	3.45	0.64814
5	0.10	0.50	4	0.98	0.44243
10	0.04	0.25	9	2.42	0.67808
10	0.04	0.50	8	1.77	0.45896
10	0.10	0.25	9	0.81	0.66795
10	0.10	0.50	7	3.51	0.44917

associated with higher interest. By raising interest payments the firm is able to increase the tax shield benefits which are substantial for the highly levered firm. This plus the reduced probability of default at maturity are sufficient to compensate the heightened prospect of an early illiquidity default.

From Table 3 we also see that the value of equity with the optimal design is decreasing in the cost of liquidation. This might at first seem surprising because our game form allowed equity holders to extract a surplus by reducing debt service by an amount determined by liquidation costs. Here we see that some of the burden of *realized* liquidation costs falls on equity. The reason is that such liquidations sometimes occur when liquidation values exceed the face value of debt so that equity holders benefit from smaller liquidation costs.

The last point to be made with reference to Table 3 is that equity is higher for  $T = 10$  than for  $T = 5$  for most of the cases. Here firms tend to prefer to issue longer-term debt rather than shorter-term debt, because with nonamortizing debt there is relatively high chance of liquidation at maturity. The longer this is forestalled the better. Again this is not the only effect at work, as can be seen in the case of higher leverage, higher  $\beta$  firms with lower liquidation costs. For a sufficiently high  $\beta$  the firm is expected to shrink with time. If so, repayment of principal is more problematic with long-term debt than short-term debt.

In this first experiment the sensitivities with respect to several parameters reflected two considerations, namely, the implications for the tax shield and for the probability of liquidation. For example, raising coupon interest brings carry tax benefits, but its effect on default is ambiguous. By reducing the principal needed to produce a debt of given value, higher coupon reduces the chances of difficulties in repaying the loan at maturity. But it raises the chances of provoking an illiquidity default prior to maturity. This suggests that there may be



**Figure 8**  
**Optimal design of debt contracts**

This figure examines the effect of sinking fund provisions on equity values. We assume that the debt has 10 years to maturity,  $\beta = 0.10$ , and the debt level  $d = 0.50$ . The optimal grace period is 7 years.

some advantages to considering designs that involve some amortization of principal or of a sinking fund. We explore this in a second design experiment in which we consider various possible values of the grace period before starting a straight-line amortization of principal. Other parameters considered are  $T$ ,  $\beta$ , and  $D$ .

The general result that emerges from this experiment is that optimal debt contracts will typically require partial repayments of principal prior to maturity, but that it is often beneficial to delay the start of repayments of principal. This is depicted clearly in Figure 8. Here, as the grace period is increased, we see the highest value of equity that is attainable by varying coupon and principal so as to produce debt that meets the funding requirement. It can be seen that the equity increases until  $g = 7$ , beyond which it declines. The reason for this is that by setting grace too low, amortization will start too early, reducing the tax shield that comes exclusively from the payments of interest. On the other hand, by allowing too long a period of grace, the debt service payments toward the end of the life of the bond are relatively heavy. This runs a relatively great risk of provoking a liquidation. The optimal grace period balances these two considerations.

The second experiment is further reported in Table 4, which lists the optimal grace and contractual interest as well as the associated value of equity attained. Making the grace period an object of design choice of course has implications for the optimal contractual interest, and this alters significantly some of the patterns seen in Table 3. In particular, in most cases we see that the higher values of  $\beta$  are associated with lower values of contractual interest. This result emerges because in these cases the higher  $\beta$  firms select contracts with shorter grace periods. That is, slower growth firms choose to begin repaying principal earlier

in order to reduce the chances of realizing costly bankruptcies as the project matures. This may be desirable even if it implies that coupon levels must be kept moderate so that tax shield benefits are sacrificed.

In our view, our findings on amortizing debt provide a rationale for the widely observed use of sinking funds in corporate bond issues. We see that the optimal sinking fund feature balances off tax shield benefits with potential costs of financial distress that can fall on shareholders. Furthermore, despite the fact that we have not explicitly dealt with monitored financial contracts, our model helps explain the frequent use of grace periods in bank loans. Again, borrowers delay principal payments so that they can reap the tax benefits of interest payments. However, beyond a certain point amortization must begin in order to control the risks of costly liquidations.

#### **4. Conclusions**

We have presented an internally consistent, arbitrage-free framework for valuing risky debt contracts which incorporates a number of the features emphasized in recent contributions to corporate finance. The model allows for strategic debt service because the costliness of formal bankruptcy may induce creditors to accept deviations from contractual payments. When the framework is applied to the problem of valuing standard debt contracts using plausible parameters, we find yield spreads over Treasuries that correspond closely to observed levels even if costs of bankruptcy are relatively low. This suggests that the framework offers promise for the econometric modeling of spreads of corporate bonds and other risky debt contracts.

Our framework is flexible enough to allow altering assumptions about contractual features, bankruptcy procedures, or other aspects of the economic environment. We examined a model variant with corporate taxes. Other bankruptcy games seen in the static theory of corporate finance could be introduced into our dynamic framework at the cost of adding parameters and computational complexity.

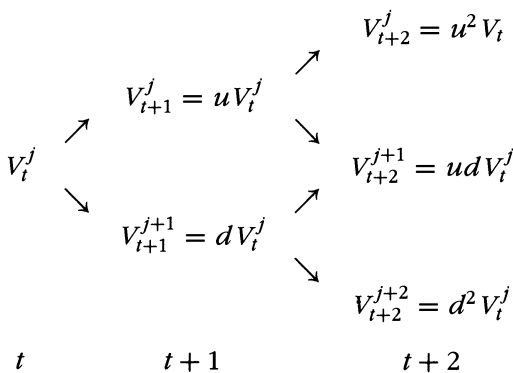
The framework is used to address the question of debt contract design, that is, to find the combination of contractual features that minimize the combined tax burden and expected bankruptcy costs. Our results show that high growth (low  $\beta$ ) firms in general tend to use a low coupon debt contract. On the other hand, low growth firms tend to use a high coupon debt contract. Although not reported here, as the tax rate increases, firms tend to increase the coupons to take advantage of the tax shield from interest payments. Highly levered firms tend to use low coupon debt. In general, sinking fund provisions are used so as to better match the cash payout from the assets: here the trade-offs are between tax shields and forced liquidations.

In our view, there is considerable opportunity for additional work along the lines of the framework we have developed here. In particular, the extension of the model to callable and convertible debt and the incorporation of a stochastic term structure would be very desirable. Second, it is clear that the modeling of the bankruptcy game could have an important impact on the results. The game form that we have adopted gives a first-mover advantage to the owner-manager with the result that he has the most scope for opportunism. Other models of bankruptcy might alter the distribution of the gains from renegotiation.<sup>15</sup> Changing the bankruptcy model is very likely to have important implications for optimal security design. For example, in our model strategic debt service tends to reduce the frequency of illiquidity default. Models with less scope for renegotiation may result in lower coupons or greater reliance on sinking funds. Finally, we believe that the valuation of subordinated debt and the issue of traded debt versus bank loans could be usefully studied within a framework similar to the one we have used here.

## Appendix

This appendix shows that the martingale probability which makes the cash flow process internally consistent with the cum cash flow value process of the firm is given by  $p = \frac{(1-\beta)r-d}{u-d}$ . The proof is provided for two time steps: it is easy to verify that the arguments go through for any number of future time periods.

The cum cash flow value process is specified below. The associated cash flow process is obtained by simply multiplying the cum cash flow values at each node by the factor  $\beta$ .



<sup>15</sup> Interesting alternative simplified models can be found in Aghion, Hart, and Moore (1992), Bergman and Callen (1991), Hart and Moore (1991), and Kahn and Huberman (1989).

Let  $p$  be martingale probability. Then,

$$V_{t+1}^j = \frac{pV_{t+2}^j + (1-p)V_{t+2}^{j+1}}{r} + \beta V_{t+1}^j.$$

Recognizing that  $V_{t+2}^j = uV_{t+1}^j$  and  $V_{t+2}^{j+1} = dV_{t+1}^j$  and substituting these relationships in the previous equation we get

$$r(1-\beta) = pu + (1-p)d.$$

Solving, we get

$$p = \frac{r(1-\beta) - d}{u - d}.$$

Note that the same relationship holds for  $p$  in each step of the recursion. The martingale probability  $p$  is also state independent. It is useful to point out that  $\frac{\partial p}{\partial \beta} = -\frac{r}{u-d}$ . This implies that as the cash payout increases, the martingale probability  $p$  falls: this will have the effect of giving greater weight to cash flows in relatively “poorer” states of the world and lesser weight to “richer” states of the world in the discounting process. This occurs in our model so as to keep the cum cash flow value of the firm the same, *irrespective of the payout*. Note that this effect becomes less important as  $\sigma^2$  increases: this is due to the fact that as  $\sigma^2$  increases  $(u-d)$  increases as well, causing  $p$  to depend less on the payout factor  $\beta$ . Thus, in our model, as the volatility of the firm increases, the payout factor  $\beta$  becomes less important in affecting the martingale probability. These factors play a role in the valuation of alternate debt contracts.

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