

## Discriminatory versus uniform Treasury auctions: Evidence from when-issued transactions

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### Abstract

We use when-issued transactions data to assess the Treasury's current experiment with uniform auctions. When-issued volume is higher under uniform as compared to discriminatory auctions, suggesting a higher information release, which should reduce pre-auction uncertainty and the winner's curse. Under uniform auctions, when-issued volatility falls after the auction and again after the outcome announcement. The pattern is the opposite for discriminatory auctions. This is further evidence that uniform auctions increase pre-auction information and lower the short squeeze. A direct comparison of markups in uniform and discriminatory auctions yields mixed results.

*Key words:* Winner's curse; Short squeeze; Markups; When-issued markets; Strategic behavior.

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## 1. Introduction

The U.S. Treasury regularly schedules auctions for Treasury debt of various maturities. Currently, three-month and six-month T-bills are auctioned weekly, 52-week T-bills are auctioned monthly, three-year and ten-year Treasury notes are auctioned quarterly, and the thirty-year Treasury bond is auctioned twice a year. These auctions are multiple price, sealed bid discriminatory auctions. Since September 1992, the Treasury has been experimenting with uniform price, sealed bid auctions for the monthly auctions of two-year and five-year Treasury notes. Given the current auction cycle, there are 126 discriminatory auctions and 24 uniform price auctions per year. The contemporaneous use of both types of auctions provides an opportunity to test how these two auction mechanisms have performed.

Uniform auctions differ from discriminatory auctions in that the winning competitive bidders pay the same price, equal to the lowest winning bid, rather than their bid price. In either format, bidders can submit multiple competitive and noncompetitive bids. Competitive bids specify yield and quantity, while noncompetitive bids specify quantity only. The amount of the auctioned issue sold via competitive bids is equal to the total quantity auctioned less the amount demanded by noncompetitive tenders. Thus, under the discriminatory auction, the U.S. Treasury essentially acts as a perfectly discriminating monopolist by awarding the security to the highest competitive bidder and working down through the competitive bids until the entire amount is sold. At the highest winning yield, bids are allocated on a pro rata basis. With the uniform auction, however, the Treasury essentially awards securities at the common market clearing yield. Under both formats, noncompetitive bids are awarded in full at the quantity weighted average yield paid by winning competitive bidders.

While the ability to price discriminate should increase revenue, Milton Friedman, among others, has suggested that the Treasury could increase revenue by selling its securities using uniform price auctions. The thrust of Friedman's argument, taken from his correspondence to Henry Goldstein and reported in Goldstein (1962), is that the discriminatory auction '... makes secondary distribution more important than the alternative would [since it favors dealers and large banks active in the money market who are able to bid more shrewdly than investors more removed from the latest market developments]. But this means that more resources are used in distributing the securities than otherwise. These resources must be being paid. Hence the average return to the Treasury is less.' Friedman (1991) argues that the Treasury might save about 75 basis points by switching to uniform price auctions.

Chari and Weber (1992) make a similar point. They argue that the incentives to collect information are larger with discriminatory auctions than with uniform price auctions. These incentives arise from the fact that in discriminatory auctions the most aggressive bidders tend to pay more than others, which does

not occur in uniform price auctions. Although Chari and Weber argue that pre-auction information collection is costly and may reduce the revenue to the Treasury, their basic insight suggests that the uniform auction should increase revenue.

A conventional auction theoretic view is that a higher degree of information among bidders tends to reduce the winner's curse and thereby increases the revenue to the Treasury (see, e.g., Milgrom and Weber, 1982). Other commentators employ analogies to single unit auctions to argue in favor of the uniform auction. Auction theory shows, under the common value assumption, that when a single unit is sold through a second price auction, the expected selling price is higher than when the unit is sold through a first price auction (Milgrom and Weber, 1982). By analogy, Bikhchandani and Huang (1993) and Milgrom (1989) argue that the uniform auction would have a higher expected selling price than the discriminatory auction, because the winner's curse leads bidders to submit lower demand schedules under the discriminatory auction than under the uniform auction. The few theoretical papers that explicitly model common value multiple unit (or share) auctions reach dramatically different conclusions. Wilson (1979) argues that the two auction formats will lead to the same revenue, Bikhchandani and Huang (1989) argue in favor of the uniform auction, and Back and Zender (1993) argue in favor of the discriminatory auction. Umlauf (1993) studies uniform and discriminatory Mexican T-bill auctions over the period 1986–1991 and finds that uniform auctions raise more revenue than discriminatory auctions. The primary purpose of our paper is to provide the first empirical comparison of the performance of the two auction formats for the U.S. Treasury markets.

Irrespective of the auction mechanism, Treasury auctions are preceded by forward trading in markets known as *when-issued markets*. When-issued markets and their implications for markups and strategic behavior are explored by Bikhchandani, Edsparr, and Huang (1994) and Simon (1994a). Markups are measured as the difference between the average winning auction yield and the when-issued yield around the time of the auction. We also examine the when-issued market but our paper differs from earlier contributions in that our data set consists of *all* transactions in the when-issued market that were executed by Garban, one of the four most active interdealer brokers in the U.S. Treasury market.<sup>1</sup> This data enables us to use transaction prices to reexamine the results of previous papers on the measurement of markups by, for example, examining

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<sup>1</sup>Simon (1994a) uses intraday quotes from Cantor Fitzgerald for Treasury coupon issues for the period January 1990 to September 1991. Bikhchandani et al. collect data for the period February 24, 1986 to November 28, 1988, but their data set is more restricted than Simon's in that they only have 1:00 p.m. and 3:30 p.m. when-issued quotes, and their study only covers 13-week and 26-week T-bills.

the role of the depth of the when-issued market as represented by the size and frequency of trades. The depth of the market also allows us to examine the presence of strategic behavior in the Treasury auctions more precisely than in previous studies. Much emphasis has been placed by previous work on 1:00 p.m. quotes to measure the markups and to infer the presence of strategic behavior. We show that trading at exactly 1:00 p.m. (the bidding time) is quite sparse. In addition, we show that the actual release times (when the Treasury announces the results of the auction) have a range of more than one hour during the 1991–1994 period, often coming well before the scheduled time of 2:00 p.m. Previous studies, notably Bikhchandani et al. (1994), rely on quotes at precisely 3:30 p.m. (the scheduled release time during the time period that they study) to draw inferences about the presence of strategic behavior. In view of our finding, evidence from such papers should be viewed with caution.

Perhaps more important, we examine the sample period July 1992 to August 1993, a period in which both uniform price and discriminatory auctions were used by the Treasury. This allows us to provide some preliminary evidence on the impact of auction design on when-issued volume and volatility and the magnitude of markups. Our evidence suggests that the markups (measured by the average of the differences between the auction rate and when-issued rates during the 30 minutes of trading prior to bidding) are positive and statistically significant for the three-month and six-month T-bill discriminatory auctions. The markups for other discriminatory auctions are generally positive, but given the small number of auctions in the sample, we cannot draw any reliable inference. On the other hand, the markups differ a great deal for uniform price auctions depending on when they are measured. The two-year and five-year T-notes are the only class of securities for which we have data on *both* auction formats. A direct comparison of the mean markups in discriminatory and uniform price auctions leads to mixed results, even though the volumes auctioned via the two formats are comparable on a per auction basis.

Finally, we are able to shed some light on the informativeness of the when-issued market and the potential for short squeezes under the two auction regimes. Chari and Weber (1992) argue that when-issued volume should be higher under the discriminatory format because it is by trading in this market that dealers obtain information. However, their argument that private information is more valuable under the discriminatory auction could also imply that there should be less pre-auction trading under that format, since a dealer who engages in a trade could be viewed as releasing private information. We show that under discriminatory auctions both the post-bidding period and the post-announcement period experience high volatility of when-issued rates relative to the pre-auction period. In contrast, under uniform auctions, when-issued rates are more volatile prior to bidding. This suggests that the uniform auction format leads to more pre-auction information release than the discriminatory auction, something that should lower the winner's curse and lead to a higher selling price

for the Treasury. However, our findings also suggest that there should be less of a short squeeze with the uniform as compared to the discriminatory auction, which might tend to reduce the revenue to the Treasury.

The next section discusses the when-issued market in more detail. The data and some preliminary evidence on the characteristics of the when-issued market are described in Section 3. Section 4 examines markups and tests for differences in mean markups between uniform and discriminatory auctions. Section 5 examines patterns in volatility for evidence on strategic bidding behavior in auctions. Section 6 concludes.

## 2. When-issued market

When-issued trading occurs during the period between the auction announcement date and the actual issue date of the security. Prior to the Treasury's scheduled auction date for a given security, dealers and investors actively participate in the when-issued market. In this market, dealers and investors may either take long positions or short positions in the Treasury security to be auctioned. Settlement takes place on the issue date. Trading in the when-issued market is done in terms of the yields at which the security is expected to be issued. As noted in the *Joint Report on the Government Securities Market* (1992), primary dealers account for 95% of the total awards in the competitive sector of the auction. Of these, about 20% represent bids that the dealers submit for their customers. More importantly, the short position of dealers in the when-issued market when they enter the auction amounts to 40% of their awards. Thus bidders enter into the auction with significant short positions in the when-issued market. At 1:00 p.m. on the auction date, bids are submitted. The Treasury announces the auction awards and the coupon of the issue is set around 2:00 p.m., after all the bids are received in the auction. After the auction date, the security trades on a price basis.

When dealers sell short in the when-issued market, they sell securities that they do not own on the assumption that they can acquire them through aggressive bidding in the auction. But since the auctions are by sealed bid, dealers face significant uncertainty about other dealers' bidding strategies and the quantity they are likely to be awarded in the auction. Since when-issued contracts specify physical delivery, any deficit between the when-issued short positions and the auction awards must be covered in the post-auction when-issued, repo, and cash markets. This fact has rather important implications for both the yield behavior in the post-bidding (after 1:00 p.m.) market and for the yield behavior following the auction award announcements (after 2:00 p.m.).

Potential bidders in Treasury auctions use the when-issued market to learn about the demand for the security to be auctioned. As the trading progresses

towards the auction date, this market generates and aggregates more information about the depth of the auction (in terms of the strength of the demand) and about the diversity of the participants. This price discovery role of the when-issued market is clearly vital to the process of selling Treasury securities. Note that dealers can acquire private information prior to the auction: if a dealer receives a few large orders from some institutional customers, then only that dealer is privy to that information. Additionally, institutions often place bids in the auction through dealers, who consequently obtain private information about the aggressiveness of bidding (see, e.g., Sirri, 1993). Interdealer trades are unlikely to convey all information about the aggregate short position in the market as the bidding time approaches. Sundaresan (1992) suggests that the introduction of futures trading on the auctioned security prior to bidding might reveal the open interest and hence improve bidding efficiency.

The when-issued market also provides price and quantity insurance: buyers in this market know that they will get the requisite amount of the securities at known prices on the issue date. This possibility is much more remote in the auction or even in the secondary market. As we show later, the when-issued market is quite liquid prior to the auction so that buyers motivated by liquidity considerations can use the when-issued market to obtain the requisite insurance more effectively. The price discovery role of the when-issued market can come at a cost: periodically, by being relatively marginally more aggressive in the sealed bid auction, some bidders will be in a position to squeeze others who are net short in the when-issued market. While the seller (the Treasury) may benefit from this aggressive bidding by a subset of dealers, the short squeeze arises because all short positions in the when-issued market can only be covered by using the newly auctioned issue. The cost of insurance provided by the when-issued market may reflect a 'squeeze premium' in the when-issued price, anticipating the expected costs of a short squeeze in the post-auction period.

There may be other informational costs associated with trading in the when-issued market. As Bikhchandani et al. (1994) and Simon (1994a) argue, bidders with private information may not wish to trade in the pre-auction when-issued market so as to protect their information for use in the auction. After the auction at 1:00 p.m., such bidders might trade in the when-issued market to augment their bidding strategies. This suggests that private information may have a tendency to be released after bidding at 1:00 p.m. Additional information will be revealed when the auction results are announced around 2:00 p.m. We examine whether the timing of information release differs under the two auction formats.

To sum up, the activity in the when-issued market prior to bidding, the markups, and the pattern of release of information may all depend on the auction format. We investigate actual when-issued volumes, markups, and the pattern of information release using transactions data under the discriminatory and uniform auction formats.

Table 1  
Auction breakdown

Number of discriminatory and uniform auctions in the period July 1992 to August 1993 for which we have when-issued transactions data.

Benchmark maturity	Number of discriminatory auctions (D)	Number of uniform price auctions (UP)	Sample period
3 months	38	0	January 1993 to June 1993 July 1992, August 1992, and December 1992
6 months	38	0	January 1993 to June 1993 July 1992, August 1992, and December 1992
1 year	6	0	January 1993 to August 1993 <sup>a</sup>
2 years	2		<i>Discriminatory</i> July 1992 and August 1992
		8	<i>Uniform Price</i> January 1993 to August 1993
3 years	3		February 1993 to August 1993
5 years	2		<i>Discriminatory</i> July 1992 and August 1992
		7	<i>Uniform Price</i> January 1993 to August 1993 <sup>b</sup>
30 years	2	0	February 1993 and August 1993
Total number of auctions	91	15	

<sup>a</sup>We do not have the data for the April 1993 and July 1993 one-year auction.

<sup>b</sup>We do not have the data for the April 1993 five-year auctions.

### 3. Empirical evidence

This section provides some evidence on the characteristics of the when-issued market. The depth of the when-issued market is first examined in terms of the frequency of trades and then in terms of the size distribution of trades.

#### 3.1. Data set

The data are for the period July 1992 to August 1993. Table 1 shows the breakdown of the auctions across different maturities in this period. By the nature of the auction cycle, the data is heavily weighted towards discriminatory auctions. Since Treasury bills are auctioned every week, they constitute a dominant part of our sample. The sample covers a total of 91 discriminatory auctions

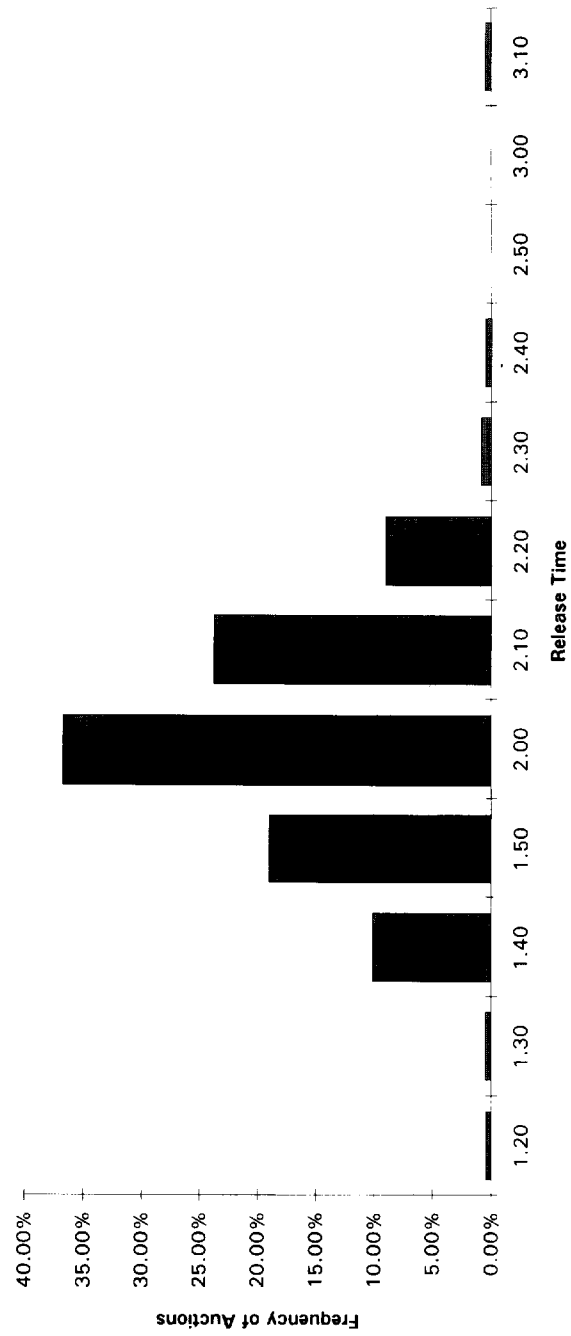


Fig. 1. Release times of auctions (1991-1994); mean: 1:56 p.m., standard deviation: 12.42 minutes (total 271 auctions).



Table 2  
When-issued transactions size breakdown

Calculations are based on all when-issued transactions executed by Garban for the auctions listed in Table 1 in the period July 1992 to August 1993. *Mean size* is the average daily transaction volume. *Maximum size* is the largest daily transaction volume. *Minimum size* is the smallest daily transaction volume. [Transaction sizes are given in millions of principal]

Benchmark maturity	Mean size	Maximum size	Minimum size	Standard error
3 months	\$427.16	\$4,716.00	\$5.00	\$29.25
6 months	\$473.05	\$2,970.00	\$8.00	\$23.81
1 year	\$1,149.94	\$4,962.00	\$102.00	\$116.19
2 years (UP)	\$1,325.98	\$6,078.00	\$117.00	\$195.04
2 years (D)	\$472.36	\$1,076.00	\$43.00	\$79.03
3 years	\$1,918.50	\$4,318.00	\$248.00	\$453.49
5 years (UP)	\$1,330.52	\$5,254.00	\$64.00	\$127.49
5 years (D)	\$795.25	\$1,598.00	\$73.00	\$128.22
30 years	\$228.38	\$735.00	\$27.00	\$91.00

and 15 uniform auctions. For each of the auctions covered in the study, we have collected the quantity weighted average winning auction yields in the discriminatory auctions and the stop out or market clearing yields in the uniform auctions from the U.S. Treasury, as well as the actual release times (RTs) when the auction results are announced on electronic screens (telerate). Fig. 1 provides summary statistics as well as the frequency distribution of release times of auctions conducted during the period July 1, 1991 to May 31, 1994. The data cover 271 auctions during this period. The mean release time is 1:56 p.m. with a standard deviation of 12.42 minutes. The range is 1 hour and 50 minutes. In the rest of the paper, we use actual release times to track the pattern of information release in the when-issued market.

### 3.2. Evidence on when-issued trading

To get a perspective on the nature of the when-issued market, we provide some summary information on the *size* of the trades as well as the *frequency of trading*. The distribution of the size of when-issued trades is shown in Table 2.

For three-month and six-month T-bills, daily volumes have a mean range of \$400–500 million (par amount). For the 30-year bonds the mean is \$228 million. This experience could differ significantly from one interdealer broker to another: for example, Cantor–Fitzgerald is reputed to have a stronger flow in the longer maturity sector. Their size experience is likely to be different from Garban. The most striking pattern about the data is that the uniform price auctions in the

Table 3  
When-issued days and transactions

Number of when-issued trading days and number of when-issued transactions in our dataset for all when-issued trades executed by Garban for the auctions listed in Table 1 in the period July 1992 to August 1993.

Benchmark maturity	Number of trading days	Number of total transactions	Mean number of transactions per day	Std. error of number of transactions per day
3 months (D)	247	3,660	14.82	10.44
6 months (D)	247	4,419	17.89	11.58
1 year (D)	50	2,665	53.30	30.12
2 years (D)	14	738	52.71	29.61
2 years (UP)	45	5,377	119.49	15.24
3 years (D)	8	1,456	182.00	92.97
5 years (UP)	65	10,907	167.80	15.85
5 years (D)	14	1,464	104.57	59.65
30 years (D)	8	293	36.63	31.11

two-year and five-year maturities have much bigger trading volume: the average size is over \$1.3 billion, well in excess of all the other benchmark maturities (the three-year maturity is an exception but it has a larger standard error). We can reject the hypothesis that the average size of when-issued trading under discriminatory auctions is the same as the average size of when-issued trading under uniform price auctions (the *t*-statistics are 2.4 and 1.9 respectively for the two-year and five-year maturities).

The breakdown of the overall number of trading days and the total number of trades is given in Table 3. We have a total of 30,979 trades, of which 14,695 transactions are drawn from discriminatory auctions and 16,284 trades are drawn from uniform price auctions. On a per day basis, uniform price auctions have about 148 transactions and the discriminatory auctions have about 25 transactions. By this measure, it is quite clear that when-issued trading is much more active under uniform price auctions. For the two-year auctions, there were 45 transaction days under the uniform price regime and 14 transaction days under the discriminatory regime; for five-year auctions there were 65 transaction days under the uniform price regime and 14 transaction days under the discriminatory regime.

The preliminary results in Tables 2 and 3 indicate that when-issued market activity could be a function of the auction mechanism. It appears that the absence of price discrimination in uniform auctions encourages dealers and customers to place more orders in the when-issued market. Perhaps the lowering of the winner's curse effected by the introduction of the uniform auction

increases the willingness of dealers to go short in the when-issued market. This, in turn, may promote more active participation in the when-issued market. Conversations with a few traders familiar with bidding confirm this general intuition. We further explore the strategic implications of our findings in Section 5.

Alternatively, there may be a learning period over which the bidders who are already familiar with the when-issued market can fine-tune their bidding strategies in uniform price auctions, which are relatively new to them. Under this view, participation in bidding in the uniform price auction should increase over time and the reliance on when-issued trading should decrease over time. However, the bid-to-cover ratios, which measure the ratio of demand to supply, do not show any increasing trend during the period September 1992 to September 1994 for the uniform price auctions. Most of the increased participation appears to be in the when-issued market.

#### **4. Markups in auctions**

The U.S. Treasury is interested in examining whether the uniform auction will provide a higher revenue than the discriminatory auction. In both auction formats, bidders will attempt to minimize the winner's curse by shading down their bids in the auction relative to their assessment of the true value of the security. This shading will move the bid closer to the market consensus. The extent of shading will depend to a great degree on the perceived informational differences between bidders. The difference between the 'true value' of the security and the auction average is frequently regarded as a measure of the winner's curse.

##### *4.1. Measures of markups*

Several measures of the 'true value' of the security may be constructed from the when-issued yields at different times: (i) pre-market when-issued yield (ten-minute trading period just prior to bidding), (ii) contemporaneous market when-issued yield (at the time of bidding), (iii) the interim or early aftermarket when-issued yield (ten-minute period just after bidding), (iv) the late aftermarket when-issued yield (last ten-minute period on the auction day), and (v) an estimate of the security's value as given by a model of the yield curve that adjusts for liquidity and tax effects present in the yield curve.

All these measures have problems. The first two measures use when-issued rates which may be directly affected by the use of a specific auction format. Parsons and Raviv (1985) show, in a different context, that the pre-auction strategy will be jointly determined with the strategy in the auction. We may therefore expect the when-issued rates under the uniform price format to differ

from the when-issued rates under the discriminatory price format. Markups may also differ. After bidding, incentives for releasing private information may increase. As a result, the early aftermarket measure may reflect the 'true value' better than the first two measures. However, since the auction results are not known yet, there still may be some noise in the third measure. After the release of auction results, bidders may make some *ex post* trades to coordinate their when-issued positions with their auction awards. This process should impound all relevant information into the late aftermarket when-issued yield. This was the intuition behind the use of closing prices of substitutes in the secondary market on the auction day by Cammack (1991). The last measure, which requires an estimate based on the yield curve, is prone to errors in estimation – typically, the estimation procedure will require a model or a spline fitting procedure. In addition, liquidity and taxation factors have to be modeled in providing the estimate. Since the markups are of the order of a few basis points, this method is not practical. In order to provide a perspective, we review the available existing evidence on markups and provide estimates of markups using when-issued yields at various times.

The possibility of a squeeze premium creates a problem with respect to interpreting our evidence on markups in terms of revenue to the Treasury. Any squeeze premium will be impounded in the when-issued yields as well as in the auction yield. Thus, the markup will not capture any increase or decrease in revenue due to a change in the intensity of the short squeeze. The evidence on markups, therefore, must be interpreted with caution.

#### 4.2. Previous evidence on markups

A number of papers have attempted to measure the markups in discriminatory auctions. They differ in the segments of the Treasury market covered. The sample periods and data sets also vary. The markup is typically measured as the auction average rate minus the when-issued rate at the time of the auction. The available evidence is summarized in Table 4.

Note that the evidence has markups varying from about four basis points to less than one basis point. As noted in Table 4, researchers have employed several different benchmarks to assess the success of a particular auction format. Cammack (1991) uses the difference between the auction average and the average of the current on-the-run quotes at the end of the auction day. More recently, Simon (1994a) takes the difference in yields using the quoted when-issued bid at the time of the auction. The motivation behind this measure comes from common value, single unit auction theory. If the when-issued market aggregated information perfectly, the when-issued yield *just prior to bidding* should ideally be equal to the auction yield. However, in comparison to the typical size of a Treasury auction, the bids and asks in the when-issued market are valid only for small quantities. To a potential bidder who has sold

Table 4  
 Markups in auctions: Summary of previous evidence

The *markup* is the difference between the quantity weighted average yield (or rate) of winning bids and a benchmark yield (or rate) which is typically based on the when-issued yield (or rate) at a particular point in time.

Author	Data and sample	Measure of markups	Size of markup
Cammack (1991)	T-bills (1973–1984)	Auction average minus average of current on-the-run at close on auction date	4 basis points
Spindt and Stolz (1992)	T-bills (1982–1988)	Auction average minus bid of WI at 30 minutes before auction	1.3 basis points
Bikhchandani et al. (1994)	T-bills (1986–1988)	Auction average minus bid of WI at 1:00 p.m.	1 basis points
Simon (1994a)	Coupon (1990–1991)	Auction average minus bid of WI at 1:00 p.m.	3–8th of a basis point

a total of  $x$  billion before 1:00 p.m. at a weighted average when-issued rate of  $y\%$ , the break-even bidding yield at the auction is  $y\%$ . The 1:00 p.m. when-issued quote (which is valid for a small amount) is important to the extent that it represents the new yield at which a large quantity ( $x$  billion) may be purchased.

#### 4.3. Liquidity around bidding time

Typically, bidders bid for large quantities (on the order of several billions of dollars) in the auction. In our data set, however, transactions around 1 p.m. are rare and are typically only on the order of a few million dollars. A significant proportion of when-issued trading occurs well before the bidding time. So, comparing the when-issued price at 1:00 p.m. with the auction average is not necessarily most informative. The when-issued prices at 1:00 p.m. may not convey much information about where the bids may be placed in the auctions. The critical issue is whether the when-issued market is liquid around the bidding time. To shed some evidence on this we turn to the transactions data on the depth of the market around 1:00 p.m. summarized in Tables 5a through 5c. The lack of market clearings by way of trades around the ten-minute period 12:55 p.m. to 1:04 p.m. is best illustrated by examining the three-month T-bill auctions. During this ten-minute interval, there were only 28 trades. This is to be viewed in the context of the fact that transactions data from 38 auctions were scanned during the ten-minute interval to generate the T-bill statistics in Table 5a. The results for other auctions are qualitatively very similar. For the

six-month T-bills, there were just 31 trades during this ten-minute period. This averages to less than one trade per auction.

The lack of active trading around 1:00 p.m. raises serious doubts about previous studies which have relied on when-issued quotes at precisely 1:00 p.m. and other selected points in time. Such quotes may not be very informative about the yields at which large quantities could be transacted.

Table 5a  
Markups and depth at different time periods

The markup is defined as the quantity weighted average of winning auction bids in yield (or rate) minus the when-issued yield (or rate) at the indicated time.

Benchmark maturity	Time window	Number of trades	Mean markups	Std. dev.	Std. error	Min.	Max.
3 months	12:50 to 12:54	10	0.0030	0.0042	0.0013	-0.0050	0.0100
	12:55 to 12:59	13	0.0054	0.0048	0.0013	-0.0000	0.0100
	12:59:59		0.0100				
	13:00:04		-0.0050				
	13:00 to 13:04	15	0.0027	0.0056	0.0015	-0.0050	0.0150
	13:05 to 13:09	9	0.0033	0.0087	0.0029	-0.0100	0.0200
	Release Time (RT)	3	-0.0017	0.0029	0.0017	-0.0050	0.0000
	RT to RT + 4	9	0.0033	0.0066	0.0022	-0.0100	0.0150
	RT + 5 to RT + 9	10	0.0100	0.0053	0.0017	0.0000	0.0150
6 months	12:50 to 12:54	10	0.0030	0.0063	0.0020	-0.0100	0.0100
	12:55 to 12:59	20	0.0013	0.0039	0.0009	-0.0005	0.0100
	12:59:58		0.0005				
	13:00:35		0.0000				
	13:00 to 13:04	11	0.0027	0.0041	0.0012	0.0000	0.0100
	13:05 to 13:09	5	0.0000	0.0000	0.0000	0.0000	0.0000
	Release Time (RT)	4	0.0050	0.0091	0.0046	-0.0050	0.0150
	RT to RT + 4	12	0.0029	0.0084	0.0024	-0.0005	0.0200
	RT + 5 to RT + 9	37	0.0062	0.0086	0.0014	-0.0200	0.0200
1 year	12:00 to 12:54	9	0.0067	0.0050	0.0017	0.0000	0.0100
	12:55 to 12:59	20	0.0064	0.0047	0.0014	0.0000	0.0100
	12:58:52		0.0000				
	13:00:48		0.0050				
	13:00 to 13:04	6	0.0054	0.0040	0.0016	0.0000	0.0100
	13:05 to 13:09	3	0.0050	0.0043	0.0025	0.0000	0.0075
	Release Time (RT)						
	RT to RT + 4	6	-0.0058	0.0120	0.0049	-0.0200	0.0100
	RT + 5 to RT + 9	6	-0.0042	0.0116	0.0047	-0.0150	0.0150

Table 5b

Markups and depth at different time periods

The markup is defined as the quantity weighted average of winning auction bids in yield minus the when-issued yield at the indicated time.

Benchmark maturity	Time window	Number of trades	Mean markups	Std. dev.	Std. error	Min.	Max.
3 years	12:50 to 12:54	5	0.0070	0.0011	0.0005	0.0050	0.0075
	12:55 to 12:59	3	0.0083	0.0029	0.0017	0.0050	0.0100
	12:56:44		0.0100				
	13:00:23		0.0050				
	13:00 to 13:04	8	0.0072	0.0884	0.0003	0.0050	0.0075
	13:05 to 13:09	8	0.0081	0.0073	0.0026	0.0025	0.0250
	Release Time (RT)						
	RT to RT + 4	9	0.0189	0.0022	0.0007	0.0150	0.0200
	RT + 5 to RT + 9	2	0.0175	0.0035	0.0025	0.0150	0.0200
30 years	12:50 to 12:54	4	0.0113	0.0025	0.0013	0.0100	0.0150
	12:55 to 12:59	1	0.0050			0.0050	0.0050
	12:58:05		0.0050				
	13:01:37		0.0100				
	13:00 to 13:04	2	0.0100	0.0000	0.0000	0.0100	0.0100
	13:05 to 13:09	5	0.0060	0.0078	0.0035	0.0025	0.0200
	Release Time (RT)						
	RT to RT + 4	2	0.0400	0.0000	0.0000	0.0100	0.0100
	RT + 5 to RT + 9	1	0.0450			0.0450	0.0450

Table 5c

Markups and depth at different time periods

For discriminatory auctions, the markup is defined as the quantity weighted average of winning auction bids in yield minus the when-issued yield at the indicated time. For uniform auctions, the markup is defined as the stop out yield minus the when-issued yield at the indicated time.

Benchmark maturity	Time window	Number of trades	Mean markups	Std. dev.	Std. error	Min.	Max.
2 years (D)	12:50 to 12:54						
	12:55 to 12:59						
	12:47:33		0.0021				
	13:00:39		0.0046				
	13:00 to 13:04	3	0.0047	0.0002	0.0001	0.0046	0.0050
	13:05 to 13:09	4	0.0029	0.0045	0.0022	- 0.0029	0.0075
	Release Time (RT)						
	RT to RT + 4						
	RT + 5 to RT + 9	3	0.0107	0.0204	0.0118	- 0.0129	0.0225

Table 5c (continued)

Benchmark maturity	Time window	Number of trades	Mean markups	Std. dev.	Std. error	Min.	Max.
5 years (D)	12:50 to 12:54						
	12:55 to 12:59	4	0.0032	0.0041	0.0021	0.0011	0.0093
	12:58:15		0.0011				
	13:01:48		0.0111				
	13:00 to 13:04	8	0.0098	0.0000	0.0003	0.0093	0.0111
	Release	2	0.0177	0.0164	0.0116	0.0061	0.0293
	Time (RT)						
	RT to RT + 4	4	0.0090	0.0121	0.0061	- 0.0039	0.0193
	RT + 5 to RT + 9	6	0.0171	0.0106	0.0043	- 0.0039	0.0243
2 years (UP)	12:50 to 12:54	21	- 0.0104	0.0117	0.0025	- 0.0350	0.0150
	12:55 to 12:59	17	0.0022	0.0216	0.0052	- 0.0350	0.0150
	12:59:20		0.0350				
	13:00:38		0.0350				
	13:00 to 13:04	18	0.0035	0.0215	0.0051	- 0.0375	0.0150
	13:05 to 13:09	5	0.0110	0.0219	0.0098	- 0.0350	0.0050
	Release	2	- 0.0025	0.0035	0.0025	0.0000	0.0050
	Time (RT)						
	RT to RT + 4	15	0.0090	0.0089	0.0023	- 0.0250	0.0000
	RT + 5 to RT + 9	31	0.0127	0.0097	0.0017	- 0.0250	0.0025
5 years (UP)	12:50 to 12:54	19	0.0049	0.0207	0.0047	- 0.0250	0.0150
	12:55 to 12:59	21	- 0.0015	0.0143	0.0031	- 0.0300	0.0150
	12:59:56		0.0000				
	13:00:05		0.0000				
	13:00 to 13:04	28	- 0.0082	0.0181	0.0034	- 0.0300	0.0650
	13:05 to 13:09	22	- 0.0051	0.0178	0.0038	- 0.0300	0.0150
	Release	4	0.0038	0.0118	0.0059	- 0.0050	0.0200
	Time (RT)						
	RT to RT + 4	22	0.0101	0.0095	0.0020	- 0.0150	0.0200
	RT + 5 to RT + 9	34	0.0046	0.0123	0.0021	- 0.0050	0.0250

#### 4.4. Markups from transactions data

To provide a perspective on markups, Tables 5a through 5c also calculate markups using transactions data at different time periods on the auction date. These time periods correspond to the ten-minute period prior to bidding, the ten-minute interval after bidding, the release time, and the ten-minute interval after the release time. We also provide markups on trades that occurred just prior to 1:00 p.m. and just after 1:00 p.m.



Table 6  
Statistical tests of markups (30-minute intervals)

Mean markups are calculated using all transaction in the 30-minute interval: (i) before 1:00 p.m. (the auction time), (ii) after the release time, and (iii) before the end of the trading day. Three- and six-month auctions are discriminatory while two- and five-year auctions are uniform.

Benchmark maturity	Mean markups	Std. error	t-statistic
<i>Discriminatory auctions</i>			
3 months (1:00 p.m.)	0.003545	0.000670	5.2909
3 months (Release)	0.005000	0.002559	1.9541
3 months (end)	0.007333	0.001503	4.8577
6 months (1:00 p.m.)	0.001807	0.000622	2.9078
6 months (Release)	0.001452	0.001506	0.9637
6 months (end)	0.004156	0.001369	3.0353
<i>Uniform auctions</i>			
2 years (1:00 p.m.)	- 0.002090	0.002235	- 0.9343
2 years (Release)	0.004951	0.002327	2.1277
2 years (end)	- 0.018920	0.004991	- 3.7909
5 years (1:00 p.m.)	0.003909	0.002704	1.4456
5 years (Release)	- 0.01054	0.003192	- 3.3007
5 years (end)	0.053984	0.008718	6.1922

Since T-bills have the largest number of transactions in discriminatory auctions, we focus on this segment first. Notice from Table 5a that the markups for three-month T-bills vary from - 0.005 to 0.01. Mean markups prior to bidding and after the release time are positive. Based on half-hourly trades (see Table 6) the markups are also significant. For six-month T-bills, markups range from zero to 0.0062. Table 6 shows the statistical significance of mean markups based on trades in half-hour intervals before bidding, after the release time, and before the end of the trading day. From Tables 5a, 5b, and 5c, we note that mean markups in discriminatory auctions prior to bidding are always positive. After the release time, mean markups tend to remain positive for discriminatory auctions, excepting a small number of cases for three-month and one-year T-bills.

Mean markups in discriminatory auctions are relatively small and of the order of magnitude reported in previous studies. Mean markups are positive and statistically significant for three-month T-bills for *all the three time periods*. For six-month T-bills, the markups are significant for all time periods except the release time. On the other hand, mean markups for uniform price auctions vary significantly depending on the time period used. They are statistically significantly negative in the two-year auctions at the end of the day and in the five-year

auctions around the release time. We use the three-month and six-month T-bills as the benchmark in discriminatory auctions to be consistent with much of the previous work. The evidence in Table 6 confirms the view that the mean markups are nonnegative in the T-bill auctions that we have considered while they are highly variable in the uniform auctions.

Given that T-bills constitute the majority of the sample, our conclusions are strongly influenced by the experience of Treasury bill auctions. Table 5a shows that the mean markups are, by and large, statistically significantly different from zero for T-bills. (When the transactions are grouped into half-hourly intervals, the markups are statistically significant and positive for three-month and six-month bills prior to and after bidding.) For three-year notes and 30-year bonds, we do not have enough trades to draw reliable inferences. In contrast, note from Table 5c that for the 15 uniform price auctions there is much greater liquidity. Note also that the markups fluctuate quite a bit. In fact, prior to bidding the average markups are statistically indistinguishable from zero for both two-year and five-year uniform auctions. This is due to the fact that the markups fluctuate a great deal in uniform price auctions prior to bidding, indicating greater liquidity in the when-issued market. In the two-year uniform price auctions, there were 38 trades in the ten-minutes period prior to bidding. For the five-year notes, there were a total of 40 trades in the same period. Contrast the corresponding results from discriminatory auctions for two-year and five-year notes: the depth is poor and the mean markups are uniformly positive. However, even for uniform auctions, in the ten-minute period after the release time, markups become positive and significant. This may be due to bidders covering their net short positions after receiving the auction awards.

To examine this issue further, we investigated the markups during the last 15 minutes of trading activity on the auction day for all benchmark maturities.<sup>2</sup> The results are reported in Table 7. As we can see from Table 7, mean markups for three- and six-month Treasury bills are always positive. Mean markups for two-year uniform price auctions are not statistically different from zero. For the five-year uniform price auctions, the results vary dramatically from one five-minute period to another: the markups are positive in two five-minute intervals and indistinguishable from zero in the last five minutes. The markups also show a wide range: -8 to 13.5 basis points. The range suggests that the markups measured at any single point in time should be interpreted with caution.

<sup>2</sup>In discriminatory auctions for two-year and five-year notes, trading was thin in the last 15 minutes, so we identify the last trade of the day and using this as the benchmark, we collect data on all trades within five minutes of the last trade. We repeat this procedure for the other two five-minute intervals. The same rule is used for all auctions.

Table 7

Markups and depth during the last 15 minutes of trading on auction day – presented in 5-minute intervals

For discriminatory auctions, the markup is defined as the quantity weighted average of winning auction yield minus the when-issued yield at the indicated time. For uniform auctions, the markup is defined as the stop-out yield minus the when-issued yield at the indicated time.

Benchmark maturity	Number of trades	Mean markups	Std. dev.	Std. error	Min.	Max.
3 months	65	0.006769	0.014126	0.001752	– 0.04	0.03
	51	0.00578	0.014709	0.002059	– 0.04	0.03
	58	0.008276	0.01506	0.001978	– 0.035	0.035
6 months	69	0.005000	0.010254	0.001234	– 0.02	0.025
	67	0.003358	0.012259	0.001497	– 0.02	0.025
	58	0.004052	0.012228	0.001606	– 0.015	0.025
1 year	13	– 0.01211	0.035146	0.009747	– 0.080	0.03
	11	– 0.027500	0.040727	0.012279	– 0.075	0.035
	13	– 0.02019	0.034873	0.009672	– 0.075	0.035
3 years	6	0.014166	0.014200	0.005797	– 0.0025	0.03
	8	0.017500	0.014516	0.005132	– 0.0025	0.03
	6	0.00000	0.006124	0.0025	– 0.0025	0.0125
30 years	2	0.01250	0.074246	0.05250	– 0.041	0.0650
	5	0.047500	0.039131	0.01750	– 0.0225	0.0650
	3	0.03750	0.04763	0.02750	– 0.0175	0.0650
2 years (D)	2	0.01500	0.00707	0.00500	0.010	0.020
	3	0.00250	0.012990	0.00750	– 0.0125	0.01
	3	– 0.0333	0.013769	0.0079	– 0.0125	0.01250
5 years (D)	4	– 0.057500	– 0.081675	0.04083	– 0.1000	0.0650
	2	– 0.03500	0.035355	0.02500	– 0.0600	– 0.0100
	2	– 0.0325	0.04596	0.03250	– 0.0650	0.000
2 years (UP)	21	– 0.01595	0.031912	0.006963	– 0.050	0.05
	17	– 0.0094	0.039524	0.00958	– 0.065	0.05
	14	– 0.01232	0.040233	0.010753	– 0.065	0.0475
5 years (UP)	13	0.027884	0.046096	0.012784	– 0.0250	0.0875
	21	0.033809	0.066959	0.014611	– 0.08	0.135
	10	0.01625	0.068122	0.021542	– 0.08	0.135

#### 4.5. A direct test of the difference in mean markups

Table 8 presents a direct test of the difference between the mean markups for discriminatory and uniform price auctions. There are very few trades in the one-hour period prior to the end of the day in the discriminatory auctions: the trades that occur during this period do not increase much

Table 8  
Differences in mean markups (60-minute intervals)

Null hypothesis: difference in means equal zero.  $t$  (crit.) is the critical  $t$ -statistic for a one-sided test at the 5% level of significance. The data is sampled in 60-minute intervals prior to 1:00 p.m.

Benchmark maturity	Mean markups	Variance	Number of trades	$t$ -statistic
2 years (D) (1:00 p.m.)	0.005269	1.33e-05	13	
2 years (UP) (1:00 p.m.)	- 0.00048	0.00040	142	
				$t = 2.9296$
				$t$ (crit.) = 1.65697
2 years (D) (Release)	- 0.00030	4.81e-05	21	
2 years (UP) (Release)	0.00695	0.00035	101	
				$t = - 3.02721$
				$t$ (crit.) = 1.66255
5 years (D) (1:00 p.m.)	0.004441	2.22e-05	29	
5 years (UP) (1:00 p.m.)	0.006987	0.00823	200	
				$t = - 1.1527$
				$t$ (crit.) = 1.65165
5 years (D) (Release)	0.015243	0.000129	21	
5 years (UP) (Release)	- 0.00601	0.00036	106	
				$t = 6.876747$
				$t$ (crit.) = 1.67865

from the ones reported in Table 7. Hence we have not included this period in Table 8.

As seen from Table 8, a direct test of the difference in the mean markups between uniform and discriminatory auctions yields inconclusive results. In the case of two-year auctions, the hypothesis that the mean difference is zero is rejected at the 5% level of significance for both the 1:00 p.m. period and the release time. However, uniform price auctions tend to have a lower mean markup than discriminatory auctions for the 1:00 p.m. period, whereas discriminatory auctions have a lower mean markup than uniform price auctions around the release time.

For the five-year auctions, the results are strongly in favor of the hypothesis that the mean markups are lower in uniform price auctions when the release time period is considered: here the null hypothesis is rejected at 1% level of significance. However, the null hypothesis is not rejected for the pre-auction time period. Our findings seem to differ from Simon (1994b), who investigates the Treasury's experiment with uniform price auctions in the 1970s. Simon concludes that the uniform price auctions raised the issuing cost of Treasury by roughly 0.75% of the issuing rate.

## 5. Evidence on strategic behavior

According to auction theory, the expected selling price increases as uncertainty about the true value of the auctioned object decreases. The stated objective of the when-issued market is to serve a price discovery role. It is an empirical question how well the when-issued market aggregates information. Several indicators of the information efficiency of the when-issued market can be constructed:

1. *The markup*: In a perfectly efficient market, the markup should be close to zero or should only reflect a compensation for the quantity risk that is borne. The intuition derives from the theory of common value, single unit auctions. In the context of share auctions, Wilson (1979) and Back and Zender (1993) argue that due to implicit collusion between bidders, even when all bidders have perfect information about the true value of the object, a bidder will disperse his bids and the average selling price will be less than the true value. This conclusion, however, is shown by Nyborg (1994) not to be robust to small changes in the model formulation.
2. *Volatility before and after auction*: Under the hypothesis that the when-issued market aggregates information perfectly, there is little reason to expect volatility of when-issued yields to increase after the auction is held or after the announcement of the auction results.
3. *Dispersion of bids*: If the when-issued market aggregates information perfectly, bidders will not have private information entering the auction and should, therefore, submit bids with identical yields. (Since we do not have access to the actual bids submitted, we cannot examine this third measure.)

However, because of the fact that there is no information about open interest in the when-issued market and because of strategic trading, we may observe deviations from the perfectly efficient benchmark measures. There are two principal strategic issues discussed in the literature: (1) aggressive bidding in the auction in an attempt to set up a short squeeze, and (2) not revealing private information in the when-issued market before the auction so as not to be informationally disadvantaged in the auction (or, to have an information advantage in the auction). In both of these cases, strategic trading would involve masking information (intentions) prior to the auction. Thus, with respect to the second measure of information efficiency, a volatility increase after the auction would be evidence of traders starting to act on their information in the when-issued market immediately after the auction. Hence, it also would be evidence against the pre-auction when-issued market serving its price discovery role perfectly. The Treasury is concerned about short squeezes (see the *Joint Report*).

Under the possibility of a short squeeze, the when-issued yield after announcement of the auction results may significantly depend on the allocations received by bidders. The squeeze possibilities are likely to be large when dealers receive large demands from their clients and, as a result, end up with large short positions which they have to cover in the auction, or failing that, in the aftermarket. Since this demand from clients would constitute a major part of dealers' private information, it is difficult to separate the strategic issues of squeeze and nonsqueeze information masking.

### *5.1. Summary of the existing evidence*

Simon (1994a) tests for the presence of strategic trading by regressing the change in the interim when-issued yield (the change in the when-issued yield from the time of the auction to just prior to the auction result announcement) on the markup (the weighted average of winning yields in the auction less the when-issued yield just prior to the auction) and duration. He finds a positive relation between the interim change and the markup and interprets this as evidence of strategic trading. To further distinguish between the two types of strategic trading listed above, Simon runs the regression without the auctions for which there was a negative markup. The coefficient on the markup is then no longer significant. Simon interprets this as meaning that strategic trading consists primarily of strategic squeezing rather than information masking. The idea is that if when-issued yields only move systematically when the bidding in the auction is surprisingly aggressive, there is an asymmetry which cannot be explained by informational advantages per se. If systematic changes were due to dealers sharing information after the auction, we should see a positive relation between the markup and when-issued yield changes on the subset of auctions for which the markup was positive. The fact that this is not the case leads Simon to conclude that the nature of strategic trading is primarily squeeze-related.

Simon's conclusion is critiqued by Bikhchandani et al. (1994). They argue that Simon's finding is consistent with informationally motivated strategic trading, even if there were no attempt to squeeze the market. Their argument is that dealers with private information regarding the value of the to-be-auctioned security must employ dramatically different strategies to profit from their information depending on whether they think when-issued yields are too high or too low. In the former case, informed dealers will hold back in the when-issued market and bid aggressively in the auction; negative markups would result and interim when-issued yields would tend towards the auction yield as dealers' private information are impounded into when-issued yields after the auction has been held. Thus, a strong association between the markup and the interim change in when-issued yields would be observed when markups are negative. This contrasts with the case in which dealers think that when-issued yields are too low. Now, informed dealers can only profit by shorting in the when-issued

market since it is not possible to short in the auction. The claim is that this weakens the link between the markup and changes in the interim when-issued yield when the markup is positive. However, there are a number of reasons why the issue is not quite so clear cut.

Although a dealer's optimal trading strategy may depend on whether he assesses pre-auction when-issued yields to be too high or too low, dealers are not endowed with identical information and will, therefore, have different views on whether when-issued yields are high or low. Moreover, a trader shorting heavily in the when-issued market may cause when-issued yields to increase. Thus, a dealer may choose to reveal only part of his private information through pre-auction when-issued trading. Dealers who assess when-issued yields to be too low will also take into account that by shorting heavily, they may set themselves up to be squeezed. In contrast, when bidders have relatively good information, they would profit by buying aggressively and would thus be able to squeeze rather than be squeezed. Simon's result may possibly be explained with reference to this asymmetry. However, as noted above, the difference between masking information in order to gain an advantage in a strategic squeeze and masking information for pure informational reasons is tenuous since the squeeze possibilities are likely to be large when demand for the new issue is relatively high. As pointed out by Cornell (1993), it is hard to visualize private information in Treasury auctions that is not squeeze-related, given the fact that there are many close substitutes traded actively in the secondary market. The potential for squeeze is high in general since, as noted earlier, dealers on average enter auctions with short positions of close to 40% of the auctioned amount and dealers also account for 95% of the successful auction awards. We therefore think that it is difficult to examine for the presence of private information that is not related to squeeze.

Notwithstanding how one interprets the economic contents of Simon's (1994a) regressions, one problem with his evidence is that the release time of the auction results is assumed to be at 2 p.m. However, as seen in Fig. 1, there is considerable variation in actual release times. Another problem is the lack of liquidity in the when-issued market at 1 p.m. We run Simon's (1994a) regressions using our transactions data and actual auction release times. The results are reported in Tables 9a and 9b. Let  $y_i = w_{RT_i} - w_{AT_i}$ , where  $w_{RT_i}$  is the when-issued yield five minutes prior to the release time and  $w_{AT_i}$  is the when-issued yield at the time of auction  $i$ . Let  $x_i$  be the markup at 1:00 p.m. in auction  $i$ . Finally, let  $D$  be the duration of the security. Consider the following regression:

$$y_i = \alpha + \beta x_i + \gamma D + \varepsilon_i. \quad (1)$$

This regression is run for the entire sample of discriminatory auctions. It is then re-run after excluding all auctions with negative markups.

It is evident in the case of discriminatory auctions that there is a weak association between markups and the interim changes in the when-issued yields

Table 9a

OLS regression results,  $y_i = \alpha + \beta x_i + \gamma D + \varepsilon_i$ , discriminatory auctions

Regressions of changes in when-issued rates from the time of bidding to five minutes prior to the release time ( $y_i$ ) on the aggressiveness of auctions as measured by markups or the spread between auction average rates and the contemporaneous when-issued rates ( $x_i$ ) and duration ( $D$ ). Ordinary least squares regressions with White's (1980) correction for heteroskedasticity have been used. Standard errors are provided in parentheses.

Maturity	Intercept $\alpha$	Markups $\beta$	Duration $\gamma$	$R^2$
All discriminatory auctions	- 0.001002 (0.00319)	0.621484 (0.51536)	- 0.001458 (0.000862)	7.88%
Discriminatory auctions with positive markups	0.009446 (0.00299)	0.084913 (0.5209)	0.001727 (0.00072)	42.67%

Table 9b

OLS regression results,  $y_i = \alpha + \beta x_i + \gamma D + \varepsilon_i$ , uniform auctions

Regressions of changes in when-issued rates from the time of bidding to five minutes prior to the release time ( $y_i$ ) on the aggressiveness of auctions as measured by markups or the spread between stop-out rates and the contemporaneous when-issued rates ( $x_i$ ) and duration ( $D$ ). Ordinary least squares regressions with White's (1980) correction for heteroskedasticity have been used. Standard errors are provided in parentheses.

Maturity	Intercept $\alpha$	Markups $\beta$	Duration $\gamma$	$R^2$
All uniform auctions	- 0.00802 (0.00805)	0.116784 (0.1183)	0.002791 (0.00236)	15.56%
Uniform auctions with positive markups	0.00780 (0.00548)	0.056479 (0.0615)	0.000311 (0.00145)	14.62%

prior to the release time. This is consistent with Simon's finding, although the effect is weaker. When auctions with negative markups are eliminated, then even this weak association disappears. For uniform auctions, there is *no* association between the markups and the interim when-issued yield changes. If we follow Simon's (1994a) interpretation of these regressions, our evidence suggests that informational advantages (whether they are squeeze-related or not) are not present in uniform price auctions.

## 5.2. Evidence on the pattern of volatility

Many models of market microstructure imply that the release of new information results in increased volatility. Such models, in which the price process is typically a martingale, imply that the release of private or new information

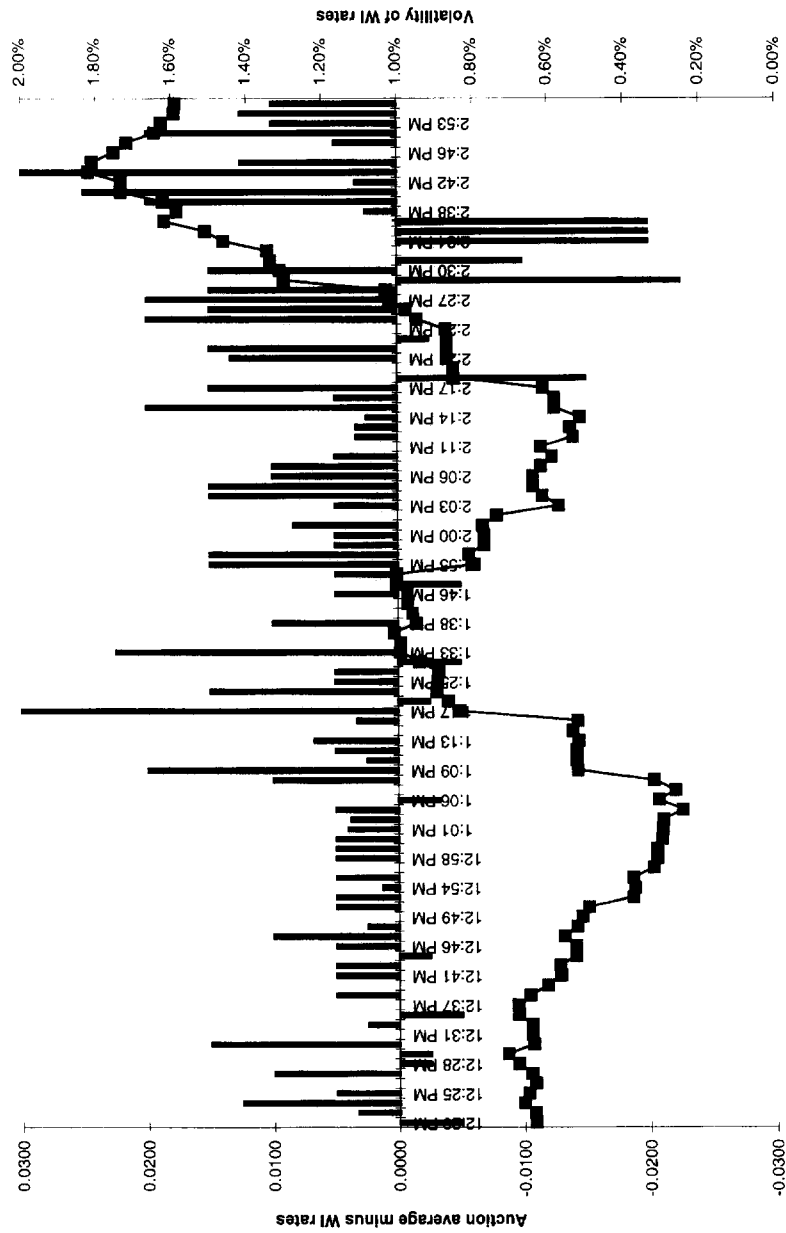


should lead to an increase in volatility (see, e.g., Admati and Pfleiderer, 1988). Since the bid–offer spreads in the when-issued markets are small, increased volatility could not be attributed to high frequency jumps between the bid and the ask. So we must conclude that in this market an increase in volatility should be intimately related to the release of new (private) information. Therefore, we focus on the general question of strategic behavior by examining the volatility of when-issued yields before and after the auction, as suggested earlier by our second measure of informational efficiency. To motivate the strategic issues, we focus on three distinct time windows on the auction date. The first window is the period 12:00 to 12:59 before the bidding. The second window is the period after the bids have been submitted but before the auction results have been announced. This window corresponds to the period 1:00 p.m. to the release time, RT. The third window refers to the one-hour period after the auction results announcement, RT to RT + 1. Prior to the first window, the bidders are subject to the quantity restrictions and it has been argued that they might have incentives not to ‘show their hands’ at least in the context of discriminatory auctions. During the second window, they might wish to complement their bidding strategies in the when-issued market. Evidence from Simon (1994a) suggests that at least 50% of the information is released during this window. In the third and final window, results are known and bidders may cover any deviations between their positions in the when-issued markets and the amount that was awarded to them in the auction. We take up each auction mechanism in turn.

### 5.2.1. Discriminatory auctions

Figs. 2 through 10 plot the markup and the volatility of the markup for *each* benchmark maturity on the auction date in one-minute intervals. The markup at time  $t$  is defined as the when-issued yield at time  $t$  less the auction yield (quantity weighted winning auction yield for discriminatory auctions or stop out yield for uniform auctions). Volatilities are based on rolling estimates with 15 transactions starting with the 12:00 transaction as the first trading observation. One of the most striking features of the volatility plot in the case of discriminatory auctions (Figs. 2 through 8) is the increase in the volatility of the when-issued yields (just after the first time window) at the time of bidding as well as at the announcement of the results of the auction. In fact, the time series plots in Figs. 2 through 8 illustrate a generally upward-sloping volatility behavior. For the three-year auctions, illustrated in Fig. 6, there is a dramatic increase in volatility after 1:00 p.m. which subsides a little around 2:00 p.m. only to increase again.

For concreteness, let us consider the two-year note shown in Fig. 5. It depicts the series of yield differences from 60 minutes prior to the auction to 60 minutes after the announcement of the auction results. At first, the series is fairly smooth. At 1:00 p.m. the volatility begins to increase slightly. Around the time of the



Vertical bars refer to the average markups (the difference between auction average and the when-issued rates) across all 3 month auctions at the indicated times on the auction day. Dark squares connected by the solid line plots the volatility of markups.

Fig. 2. Three-month auctions (auction average minus WI rates and volatility of WI rates).

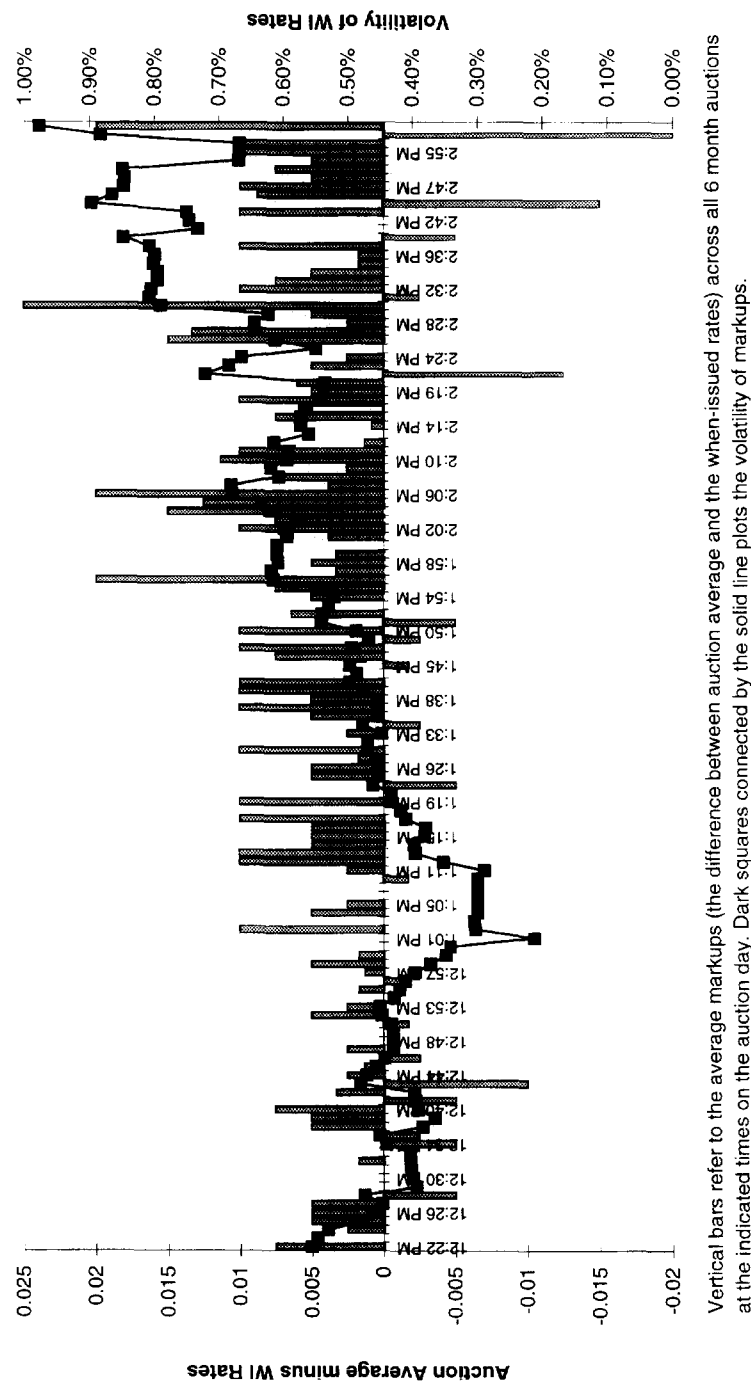
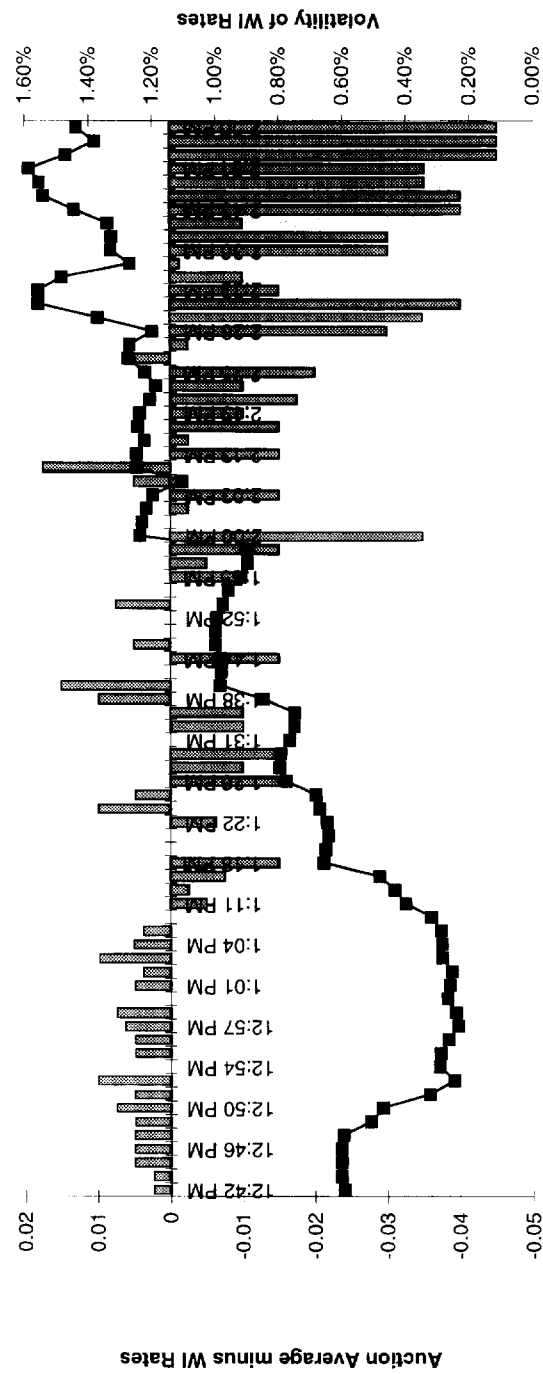


Fig. 3. Six-month auctions (auction average minus WI rates and volatility of WI rates).



Vertical bars refer to the average markups (the difference between auction average and when-issued rates) across all 1 year auctions at the indicated times on the auction day. Dark squares connected by the solid line plots the volatility of markups.

Fig. 4. One-year auctions (auction average minus WI rates and volatility of WI rates).

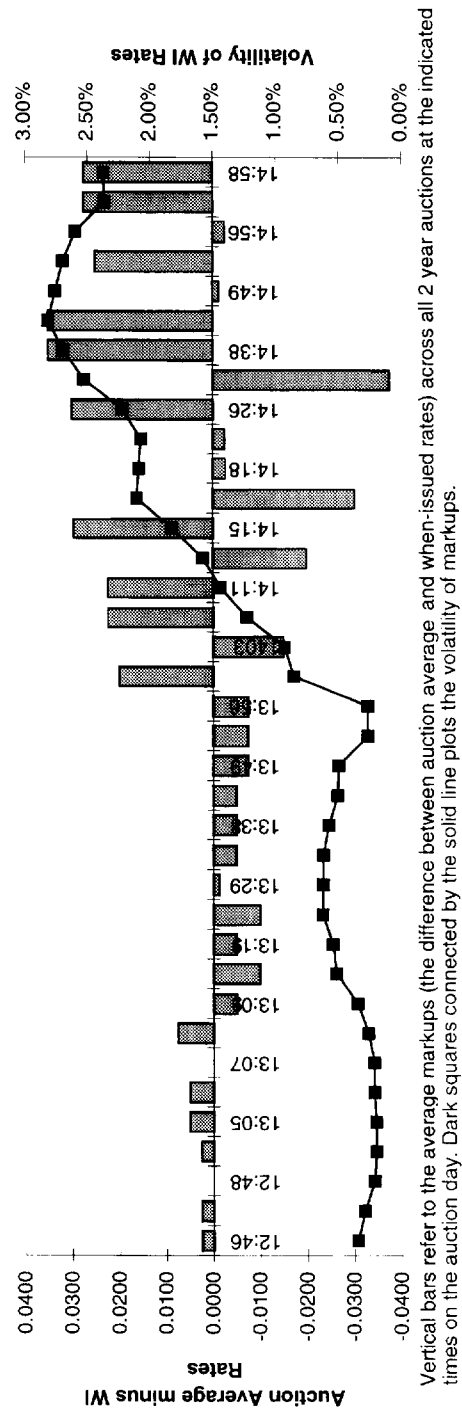


Fig. 5. Two-year discriminatory auctions (auction average minus WI rates and volatility of WI rates).

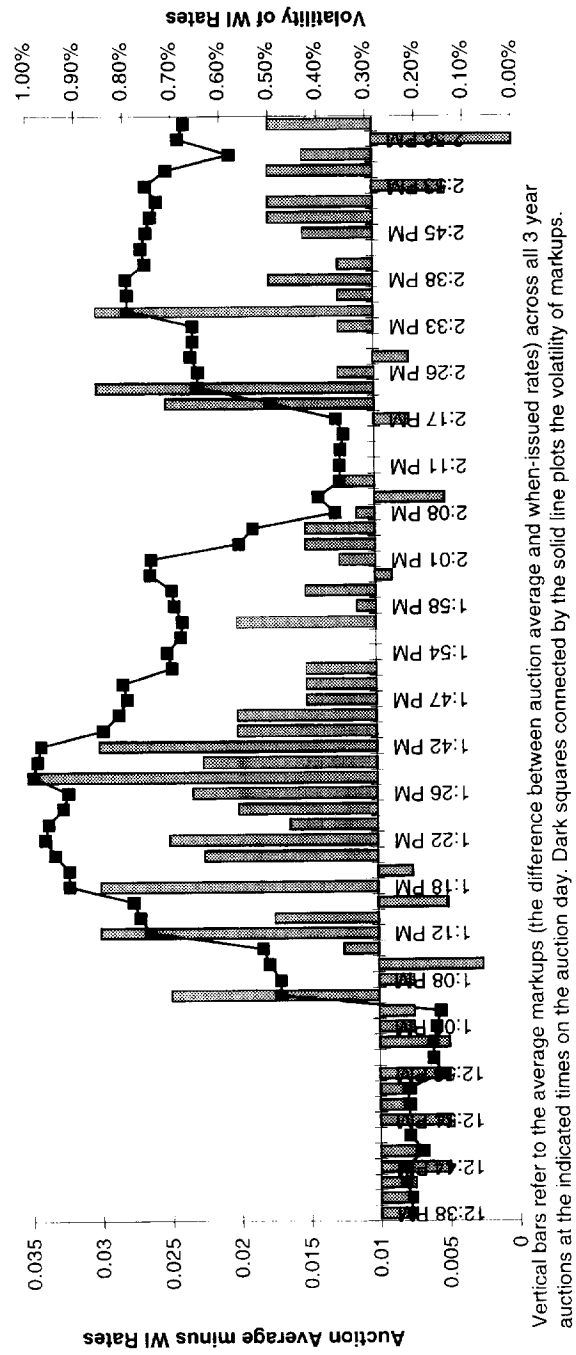
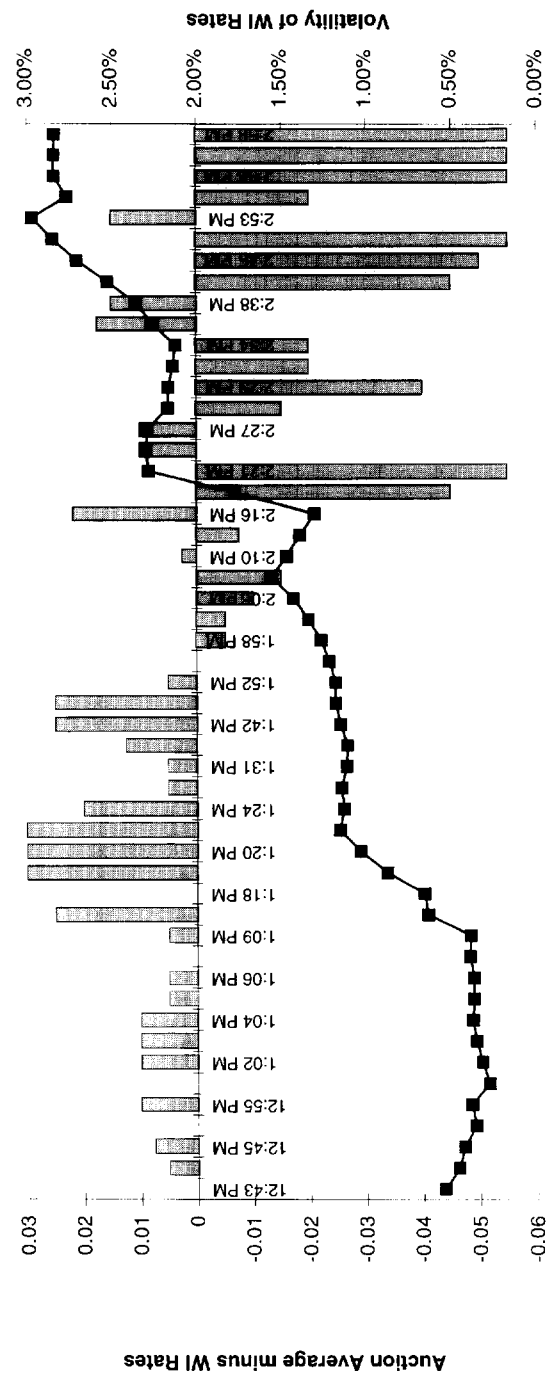


Fig. 6. Three-year auctions (auction average minus WI rates and volatility of WI rates).



Vertical bars refer to the average markups (the difference between auction average and when-issued rates) across all 5 year auctions at the indicated times on the auction day. Dark squares connected by the solid line plots the volatility of markups.

Fig. 7. Five-year discriminatory auctions (auction average minus WI rates and volatility of WI rates).

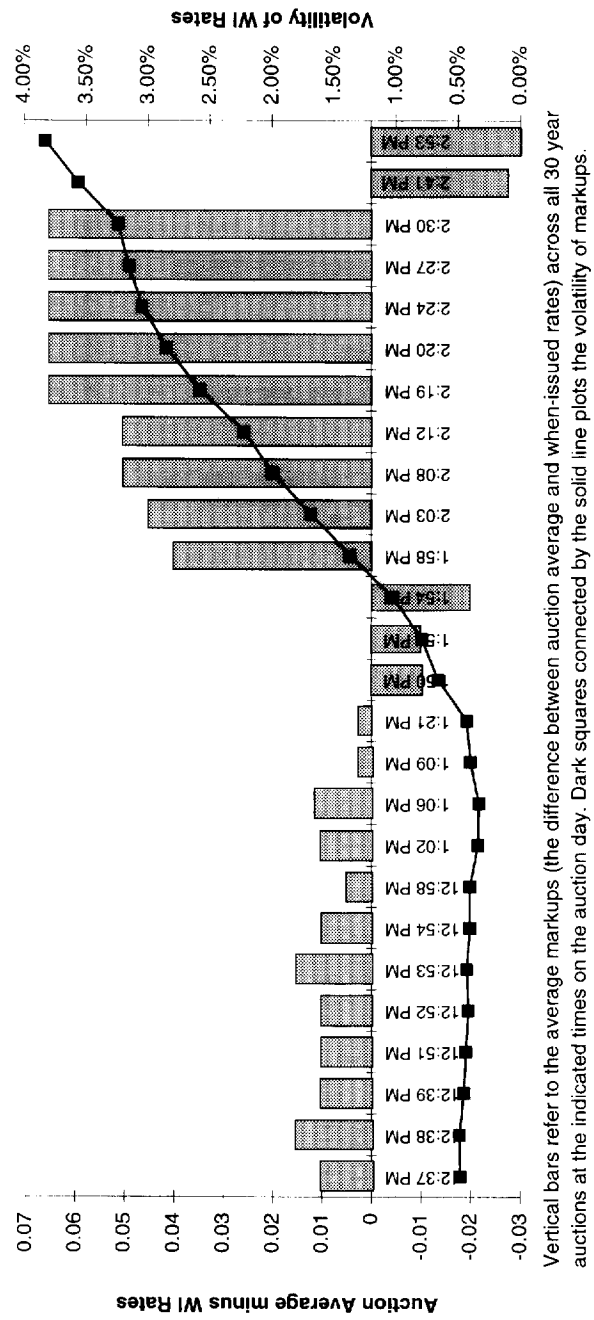


Fig. 8. Thirty-year auctions (auction average minus WI rates and volatility of WI rates).



Table 10  
 Evidence on strategic behavior in discriminatory auctions  
 Volatility is calculated as the variance of markups in the indicated time windows.

Benchmark maturity	Time window	Volatility	Average size of trades (millions of \$)	Number of trades
3 months	12:00 to 12:59	0.57%	41.04	101
	1:00 to RT	1.38%	28.40	94
	RT to RT + 1 hr	1.06%	30.63	64
6 months	12:00 to 12:59	0.64%	61.32	127
	1:00 to RT	1.07%	24.29	199
	RT to RT + 1 hr	0.62%	19.36	76
1 year	12:00 to 12:59	0.51%	49.13	54
	1:00 to RT	0.91%	11.97	37
	RT to RT + 1 hr	1.73%	27.57	68
2 years	12:00 to 12:59	0.36%	20.69	13
	1:00 to RT	0.69%	6.05	21
	RT to RT + 1 hr	2.69%	7.72	28
3 years	12:00 to 12:59	0.23%	18.17	47
	1:00 to RT	1.09%	10.93	41
	RT to RT + 1 hr	0.58%	10.16	63
5 years	12:00 to 12:59	0.47%	14.93	29
	1:00 to RT	1.02%	7.77	30
	RT to RT + 1 hr	2.89%	13.23	45
30 years	12:00 to 12:59	0.46%	18.33	24
	1:00 to RT	0.64%	7.88	8
	RT to RT + 1 hr	3.23%	6.63	16

announcement of the auction results, 2:00 p.m. volatility increases in a significant manner. As reported in Figs. 2 through 8, this pattern is repeated across all the discriminatory auction regimes. In Table 10, we have summarized some key features of the volatility behavior and the depth of the market under the discriminatory auction scheme. Note that all the information in this table pertains to the respective time windows indicated. In Table 10, volatility for each time window is estimated using all the transactions in that window.

Since the pattern is strikingly similar for each benchmark maturity, let us take the three-month auction to illustrate the main regularity. The average volatility for the 60 minute pre-auction period is 0.57%. In the interim period, the 60 minutes between the auction and the auction results announcement, the average volatility increases to 1.38%. After the results are announced the volatility drops but remains at a level of 1.06%, which is more than twice the pre-bidding level. Examination of time series plots in Figs. 2 through 8 shows that the changes in

volatility occur around the bidding time and the announcement time. It should be noted that the bid–offer spreads could be changing in these time windows as noted in Simon (1994a).

The pattern of volatility established in these figures is consistent with the view that dealers trade strategically, masking their private information prior to the auction. Once the auction has taken place, however, dealers will trade on their private information, thus inducing a higher volatility in the when-issued yields. The lifting on quantity restrictions may also contribute to a higher volatility. The additional increase in volatility after the announcement of the auction results can be attributed to the high quantity uncertainty faced by bidders in a discriminatory auction. This implies that there will be considerable surprise with respect to the allocations won by bidders in the auction, inducing additional volatility which is augmented by the accompanying ‘squeeze play,’ whereby some traders attempt to squeeze those who must cover their short positions (see, e.g., Sundaresan, 1992; Jegadeesh, 1993).

Note that the depth of the market summarized by the average size of the trades during these windows illustrates that the pre-bidding period is generally when large trades are done. The depth varies in the second window (after bidding but prior to the announcement of the results). For note auctions, the liquidity picks up again after the results are announced.

#### 5.2.2. Uniform price auctions

Under the uniform regime, the volatility of when-issued yields also changes at the time of the auction and the outcome announcement. However, in contrast with the discriminatory regime, the uniform auction produces a pattern of volatility that is *decreasing* over time. This can be seen in Fig. 9, which depicts yield differences and volatility for the two-year note under the uniform auction from about 60 minutes prior to the auction to 60 minutes after the announcement of the auction results. The figure shows a high volatility in the first 60 minutes relative to the 60-minute interim period. The volatility drops sharply around the announcement of the auction results, 2:00 p.m., and the series stays relatively smooth thereafter. Table 11 summarizes the evidence for uniform price auctions.

While the pattern of volatility for the two-year note under the uniform regime contrasts dramatically with that under the discriminatory regime, the evidence appears a bit ambiguous for the five-year note as shown in Fig. 10. But examination of Table 11 reveals that in both maturities, the volatility is *non-increasing* with time which is in sharp contrast to the evidence for discriminatory auctions presented in Table 10. The average size of the trades falls over time much like the evidence for discriminatory auctions. The depth in terms of the average size is highest before bidding. The depth of the when-issued market in terms of the number of transactions is highest after the announcement of the auction results in terms of the number of trades that are executed. By this

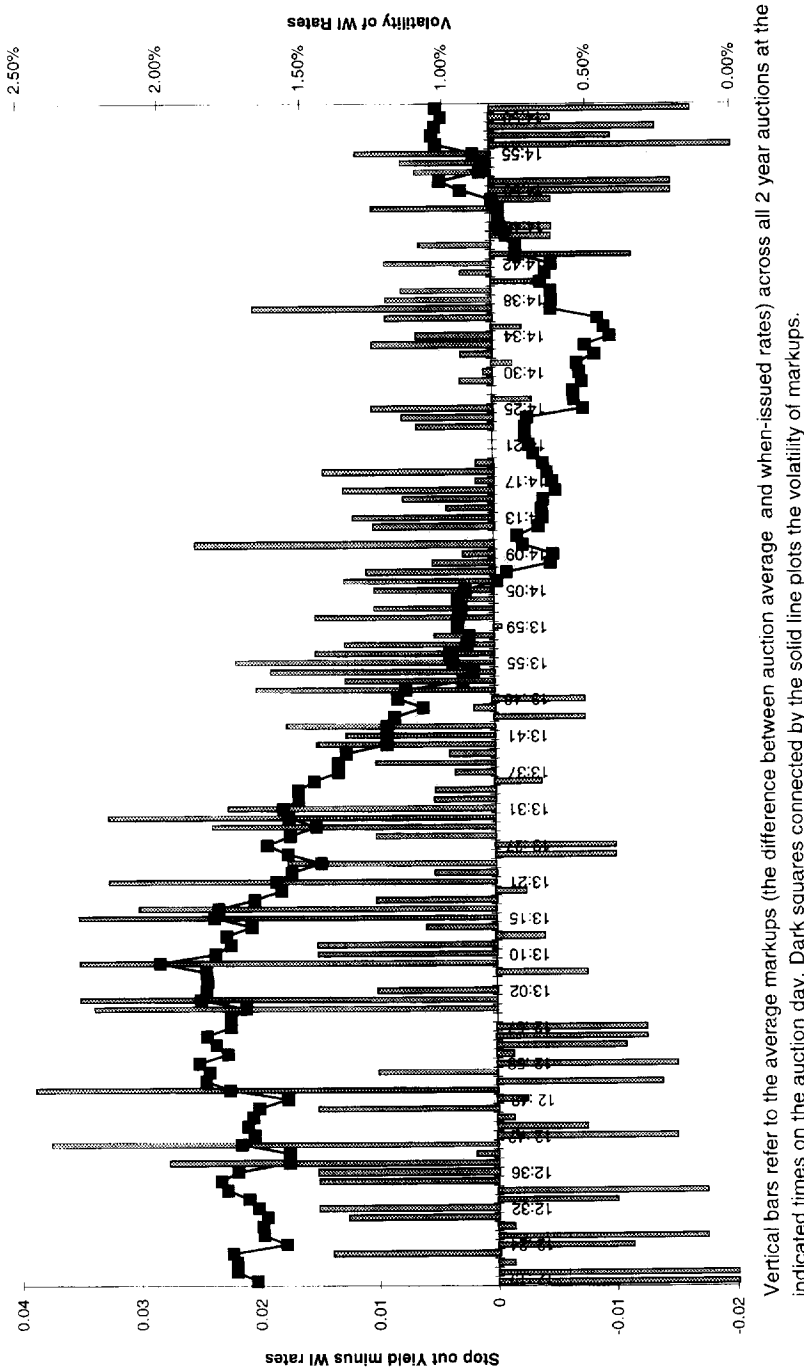
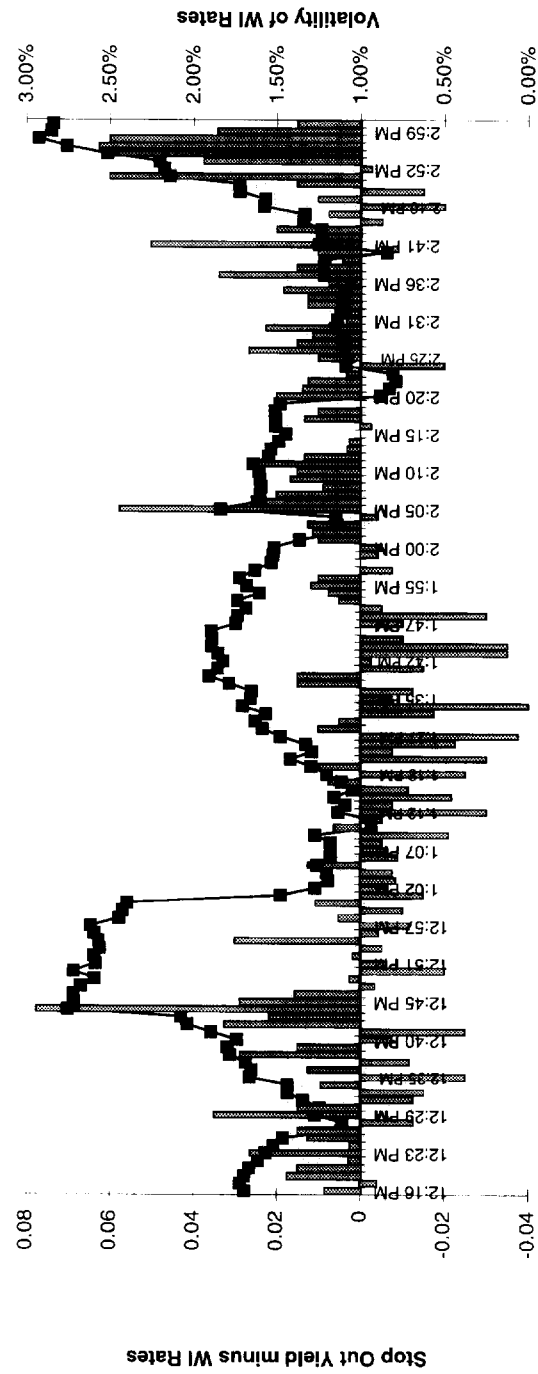


Fig. 9. Two-year uniform price auction (stop out yield minus W1 rates and volatility of W1 rates).



Vertical bars refer to the average markups (the difference between stop out and the when-issued rates) across all 5 year auctions at the indicated times on the auction day. Dark squares connected by the solid line plots the volatility of markups.

Fig. 10. Five-year uniform price auctions (stop out yield minus WI rates and volatility of WI rates).

Table 11  
 Evidence on strategic behavior in uniform price auctions  
 Volatility is calculated as the variance of markups in the indicated time windows.

Benchmark maturity	Time window	Volatility	Average size of trades (millions of \$)	Number of trades
2 years	12:00 to 12:59	2.00%	19.75	142
	1:00 to RT	1.94%	13.79	108
	RT to RT + 1 hr	1.38%	14.96	232
5 years	12:00 to 12:59	2.87%	11.59	200
	1:00 to RT	2.01%	8.49	118
	RT to RT + 1 hr	2.02%	8.44	263

measure, the depth is lowest in the second time window which is after bidding but before the results are announced.

Tables 12a and 12b report the results of a formal test of whether the differences in volatilities for discriminatory and uniform auctions in the three distinct time windows are statistically significant. In what follows,  $\sigma_i^2$  denotes the volatility in time window  $i$  where  $i = 1, 2, 3$ . Our null hypotheses are  $H_0: \sigma_1^2 = \sigma_2^2$  and  $H_0: \sigma_2^2 = \sigma_3^2$ . In discriminatory auctions, we test the following alternative hypotheses: (i) the volatility in the first time window is less than the volatility in the second time window, and (ii) the volatility in the second time window is less than the volatility in the third time window. As Table 12a illustrates, with few exceptions, we can reject the hypothesis that the volatility prior to the bidding in the auction is the same as the volatility after the bidding. We can also reject the hypothesis that the volatility prior to the announcement of the results is the same as the volatility after the results have been announced. By and large, the alternative hypotheses cannot be rejected by the data.

From Table 12b for two-year uniform auctions, we cannot reject the hypothesis that the volatility during the first period is lower than the volatility in the second time window. The volatility in the second time window, however, is larger than the volatility in the post-announcement period. For five-year notes, the volatility in the second time window is statistically significantly smaller than the volatility in the first time window, but not different from the volatility in the third time window.

The time pattern of decreasing volatility under the uniform regime is consistent with the view that under the uniform auction, higher rents can be extracted from the use of private information in the when-issued market than in the auction itself since the auction yield is relatively insensitive to a single dealer's bid. This means that there will be a high release of private information under the uniform regime prior to the auction, hence the high volatility. This association

Table 12a

Time pattern of volatility in discriminatory auctions

For each benchmark maturity, we perform two tests: (1) the volatility in the one-hour period prior to 1 p.m. is equal to the volatility between 1 p.m. and the release time, and (2) the volatility between 1 p.m. and the release time is equal to the volatility in the one-hour period after the release time.

Benchmark maturity	$H_0: \sigma_1^2 = \sigma_2^2$ $H_A: \sigma_1^2 < \sigma_2^2$	$H_0: \sigma_2^2 = \sigma_3^2$ $H_A: \sigma_2^2 < \sigma_3^2$
3 months	$F = 0.1711$ $F_{\text{critical}} = 0.7701$	$F = 1.6830$ $F_{\text{critical}} = 0.6769$
6 months	$F = 0.3534$ $F_{\text{critical}} = 0.8100$	$F = 2.9649$ $F_{\text{critical}} = 0.7188$
1 year	$F = 0.3155$ $F_{\text{critical}} = 0.6810$	$F = 0.2752$ $F_{\text{critical}} = 0.6745$
2 years	$F = 0.2762$ $F_{\text{critical}} = 0.4855$	$F = 0.0664$ $F_{\text{critical}} = 0.5711$
3 years	$F = 0.0454$ $F_{\text{critical}} = 0.6761$	$F = 3.5625$ $F_{\text{critical}} = 0.6297$
5 years	$F = 0.2146$ $F_{\text{critical}} = 0.6136$	$F = 0.1241$ $F_{\text{critical}} = 0.6357$
30 years	$F = 0.5263$ $F_{\text{critical}} = 0.5012$	$F = 0.0391$ $F_{\text{critical}} = 0.3799$

Table 12b

Time pattern of volatility in uniform price auctions

For each benchmark maturity, we perform two tests: (1) the volatility in the one-hour period prior to 1 p.m. is equal to the volatility between 1 p.m. and the release time, and (2) the volatility between 1 p.m. and the release time is equal to the volatility in the one-hour period after the release time.

Benchmark maturity	$H_0: \sigma_1^2 = \sigma_2^2$ $H_A: \sigma_1^2 > \sigma_2^2$	$H_0: \sigma_2^2 = \sigma_3^2$ $H_A: \sigma_2^2 > \sigma_3^2$
2 years	$F = 1.0619$ $F_{\text{critical}} = 1.3539$	$F = 1.9748$ $F_{\text{critical}} = 1.3035$
5 years	$F = 2.0328$ $F_{\text{critical}} = 1.3197$	$F = 1.0059$ $F_{\text{critical}} = 1.2310$

between high volatility and the strategic release of private information has been highlighted in the market microstructure literature (e.g., Admati and Pfleiderer, 1988). The increased number of transactions under uniform auctions relative to discriminatory auctions confirms this intuition (see Tables 10 and 11).

The decrease in volatility after the outcome announcement can be seen partly as reflecting the low quantity uncertainty faced by bidders in a uniform auction. This contrasts with the discriminatory auction in which the quantity uncertainty is relatively high. But the evidence also suggests that the uniform auction's quantity uncertainty is further lowered by the large volume of pre-auction trading. Since dealers are more willing to trade on their private information in the when-issued market under the uniform auction, they will enter the auction with more balanced positions than under the discriminatory auction because the when-issued interdealer market permits dealers to lay off risks that are initially taken up through dealing with pension funds and other nonstrategic, or liquidity, traders. Pension funds and other institutions approach dealers prior to bidding to ensure that they receive a fixed quantity of the to-be-auctioned issue at a fixed price. Dealers' private information consists primarily of such non-strategic traders' privately placed orders. This gives dealers a signal of the total demand for the new issue. The cost of this information acquisition is represented by the risks associated with the accumulated short position. In addition to the usual risks of being short a forward contract, a dealer who is short in the when-issued market also carries the risk of being squeezed in the post-auction market as a result of not obtaining enough securities to cover the short position. The when-issued market provides a market in which these risks can be reduced. The extent to which dealers engage in transactions to reduce large short positions varies between the two auction formats. As argued above, under the discriminatory auction, private information is relatively more valuable. As a result, less of the short position risk will be laid off under the discriminatory regime than under the uniform auction regime. This implies more balanced portfolios under the uniform regime, and consequently less squeeze play following the outcome announcement.

In conclusion, the uniform auction serves to lower post-auction volatility through two effects: (1) an intrinsic lower quantity risk with a uniform auction as compared to a discriminatory auction, and (2) higher pre-auction information revelation under the uniform auction, which also results in more balanced positions among dealers. This also reduces quantity uncertainty, especially with respect to the squeeze play.

### *5.2. Welfare implications*

Auction theory based on single unit auctions suggests that the uniform auction should have a higher expected selling price than the discriminatory auction because the winner's curse is mitigated. A recent paper by Chatterjea and Jarrow (1994), which explicitly models the when-issued market in the context of discriminatory and uniform auction formats, concludes that in the long run, uniform auctions yield a greater revenue than discriminatory auctions. The volatility and volume evidence presented in this paper suggests that uniform

and discriminatory Treasury auctions will have different expected selling prices because of the different incentives under these regimes to trade on private information in the when-issued market prior to the auction. Our evidence suggests a greater release of information under the uniform auction regime. This tends to reduce the markups and increase the expected selling price. This information-sharing effect augments the inherent advantage of the lower winner's curse of the uniform auction.

However, there are two additional effects which actually may favor the discriminatory auction: (1) the squeeze premium and (2) collusion. The low markups we find suggest a low degree of collusion, we therefore focus on the squeeze premium effect (see Bikhchandani and Huang, 1993, for a general discussion of collusion; see also Back and Zender, 1993). We argue above that the larger volume and volatility in the run up to the auction under the uniform regime suggest that this environment is more conducive to laying off liquidity risks, and hence enhances information sharing as well as risk sharing. Better risk sharing would suggest that the squeeze premium is lower under the uniform auction. This is good news for pension funds and other liquidity traders, who ultimately would have to bear squeeze associated costs, but bad news for the Treasury in the sense that a lower squeeze premium implies a lower expected selling price. On the other hand, persistent short squeezing may lead to some players dropping out of the market. Thus while short squeezing may enhance revenue in the short run, it may be harmful in the long run. If the objective of a good auction design is to sell an issue at a price as close as possible to its intrinsic value, then the uniform auction seems to score better than the discriminatory auction. But if the objective is to obtain as high as possible a price for the issue, the discriminatory auction may do better in that it induces a bigger squeeze value. This line of argument assumes that the higher squeeze premium under the discriminatory auction does not affect liquidity traders' demand for trading in the when-issued market. If this were not the case, it might be that the discriminatory auction would merely discourage liquidity trading and thus lower the squeeze premium. If this lower liquidity trade were reduced sufficiently, the squeeze premium could end up being equally large under the two auction regimes.

## 6. Conclusions

Using transactions data, we have shed some light on two critical issues pertaining to auctions theory. First, we show that uniform price auctions result in highly variable mean markups. In sharp contrast, the markups for three-month and six-month discriminatory auctions are significantly different from zero. Second, we show that there is evidence of strategic behavior in discriminatory auctions which appears to produce increasing volatility in when-issued



yields subsequent to the bidding, with a further increase subsequent to the announcement of auction awards. By contrast, uniform price auctions for two-year notes (and to a lesser extent the five-year auctions) reflect a decreasing volatility pattern. We also characterize the when-issued market. This market appears to have a varying depth on the auction day. The depth at 1:00 p.m. is quite poor. It appears that when-issued trading is much more active for uniform price auctions in comparison with discriminatory auctions. This is consistent with greater liquidity trading which might account for lowered markups.

Our results are based on the transactions experience of Garban, which is one of the four major interdealer brokers in the U.S. Treasury market. It is possible that the trading experience of other interdealer brokers such as Cantor–Fitzgerald is somewhat different from that of Garban, especially in the volume of trading in different benchmark maturities. We do not think that the transaction prices could have been qualitatively very different for other broker dealers. Moreover, we doubt that the jump in when-issued trading under uniform price auctions is idiosyncratic to Garban. Since all the trades used in this study are from the interdealer broker market, it measures the activity between dealers and *not* between dealers and customers. The trading experience of Cantor–Fitzgerald might reflect more customer-driven transactions.

## References

- Admati, Anat R. and Paul Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 3–40.
- Back, Kerry and Jaime F. Zender, 1993, Auctions of divisible goods: On the rationale for the Treasury experiment, *Review of Financial Studies* 6, 733–764.
- Bikhchandani, Sushil and Chi-Fu Huang, 1989, Auctions with resale markets: A model of Treasury bill auctions, *Review of Financial Studies* 2, 311–340.
- Bikhchandani, Sushil and Chi-Fu Huang, 1993, The economics of Treasury securities markets, *Journal of Economic Perspectives* 7, 117–134.
- Bikhchandani, Sushil, Patrik L. Edsparr, and Chi-Fu Huang, 1994, The Treasury bill auction and the when-issued market: Some evidence, Working paper (Sloan School of Management, MIT, Cambridge, MA).
- Cammack, Elizabeth, 1991, Evidence of bidding strategies and the information in Treasury bill auctions, *Journal of Political Economy* 99, 100–130.
- Chari, V.V. and R. Weber, 1992, How the U.S. Treasury should auction its debt, *Quarterly Review*, Federal Reserve Bank of Minneapolis, Fall, 3–12.
- Chatterjea, Arkadev and Robert Jarrow, 1994, Market manipulation and a model of the United States Treasury securities auction market, Working paper (Cornell University, Ithaca, NY).
- Cornell, Bradford, 1993, Adverse selection, squeezes, and the bid–ask spread on Treasury securities, *Journal of Fixed Income* 3, 39–47.
- Friedman, Milton, 1991, How to sell government securities, *Wall Street Journal*, August 28.
- Goldstein, Henry, 1962, The Friedman proposal for auctioning Treasury bills, *Journal of Political Economy* 70, 386–392.
- Jegadeesh, Narasimhan, 1993, Treasury auction bids and the Salomon squeeze, *Journal of Finance* 48, 1403–1419.

- Joint report on the government securities market. January 1992 (Department of Treasury, SEC, and Board of Governors of the Federal Reserve System, Washington, DC).
- Milgrom, Paul. 1989. Auctions and bidding: A primer. *Journal of Economic Perspectives* 3, 3: 22.
- Milgrom, Paul and Robert J. Weber. 1982. A theory of auctions and competitive bidding. *Econometrica* 50, 1089–1122.
- Nyborg, Kjell G., 1994. On implicit collusion in share auctions. Manuscript (London Business School, London).
- Parsons, John E. and Artur Raviv. 1985. Underpricing of seasoned issues. *Journal of Financial Economics* 14, 377–397.
- Simon, David P., 1994a. Markups, quantity risk, and bidding strategies at Treasury coupon auctions. *Journal of Financial Economics* 35, 43–62.
- Simon, David P., 1994b. The Treasury's experiment with single price auctions in the mid-1970s: Winner's or taxpayer's curse?. *Review of Economics and Statistics* 76, 754–760.
- Sirri, Erik R., 1993. Fischer Francis Trees & Watts. HBS case 9-293-056 (Harvard Business School, Boston, MA).
- Spindt, Paul A. and Richard W. Stolz. 1992. Are US Treasury bills underpriced in the primary market?. *Journal of Banking and Finance* 16, 891–908.
- Sundaresan, Suresh, 1992. Pre-auction markets and post-auction efficiency: The case for cash settled futures on on-the-run Treasuries. Working paper (Columbia University, New York, NY).
- Sundaresan, Suresh. 1994. An empirical analysis of U.S. Treasury auctions: Implications for auction and term structure theories. *Journal of Fixed Income* 4, 35–50.
- Umlauf, Stephen, 1993. An empirical study of the Mexican Treasury bill auction. *Journal of Financial Economics* 33, 313–340.
- Wilson, Robert, 1979. Auctions of shares. *Quarterly Journal of Economics* 93, 675–689.