

Growth Options and Optimal Default under Liquidity Constraints: The Role of Corporate Cash Balances

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Abstract

In this paper, we develop a structural model that captures the interaction between the cash balance and investment opportunities for a firm that already has some debt outstanding. We consider a firm whose assets produce a stochastic cash flow stream. The firm has an opportunity to expand its operations, which we call a growth option. The exercise cost of the growth option can be financed either by cash or costly equity issuance. In absence of cash, we derive implicit solutions for equity and debt prices when the option is exercised optimally, under both firm value and equity value maximization objectives. We characterize the optimal exercise boundary of the option, and its impact on the optimal capital structure and the debt capacity of the firm. Next, we develop a binomial method to investigate the interaction between cash accumulation and the growth option. In this framework, the firm optimally balances the payout of dividends with the buildup of a cash balance to finance the growth option in the “good states” (i.e., high asset value states), and to provide liquidity in the “bad states” (i.e., low asset value states). We provide a complete characterization of the firm’s strategy in terms of its investment and dividend policy. We find that while the ability to maintain a cash balance does not add significant value to the firm in absence of a growth option, it can be extremely valuable when a growth option is present. Finally, we demonstrate how our method can be extended to firms with multiple growth options.

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1 Introduction

Management of cash plays a crucial role in the profitability and survival of companies. Early literature on corporate debt valuation ignores the importance of cash by assuming that firms can dilute equity at zero cost to finance coupon payments (e.g., Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), and Leland and Toft (1996)). Similarly, standard literature on investment under uncertainty assumes that funds can be raised at no cost to finance the investments (e.g., Brennan and Schwartz (1985) and McDonald and Siegel (1986)). These models have been extended to address numerous issues, such as the effects of strategic debt service, debt renegotiation, and U.S. bankruptcy provisions and their effects on corporate debt valuation, and optimal financing and equity-bond holders' conflict over timing of investments. While we provide a survey of this literature later in the paper, it is important to note at the outset that there has been little work to address important roles of cash and liquidity. In particular, we do not have coherent models of corporate cash balances in a valuation setting, to help us understand the distinct roles played by cash in good and bad states of the world, from a corporate perspective.

In this paper, we develop a structural model that captures two very essential roles of cash: i) to provide liquidity during the time of financial distress, which may lead to bankruptcy and costly liquidation, and ii) to finance future investment and growth opportunities. When a firm is in a bad state (i.e., its asset value is low), its revenue may not be sufficient to cover the coupon payments. In absence of cash, the firm will need to issue additional equity, which may be costly, or declare bankruptcy. However, if the firm has accumulated enough cash reserves, it can use it to postpone equity dilution and delay bankruptcy, or even avoid them altogether if its asset value improves. On the other hand, when the firm is in a good state (i.e., its asset value is high), it may wish to expand its operation by investing in additional assets. The cost of raising capital in the future may be high (due to informational asymmetry, for example), and it may be worthwhile to forego some of the dividend distribution today to save up for future investments. The literature of seasoned equity has documented that there are significant costs to issuing equity.¹ Our framework

¹Corwin (2003) shows that seasoned equity offers were underpriced on average by 2.2% during the 1980s

allows us to characterize an optimal strategy for the firm regarding its dividend distribution, cash holding, timing of investment and declaration of bankruptcy.

1.1 Literature Review

The real options approach to analyzing investment decisions under uncertainty was introduced by Brennan and Schwartz (1985) and McDonald and Siegel (1986). They showed that simple net present value analysis can lead to suboptimal decisions when projects are irreversible. Dixit and Pindyck (1994) provide a good overview of the real options literature. While many papers focus on the optimal time of an *initial* investment, our model considers a firm that is already operational, and is looking for the optimal time to *expand*.

In the classical model of McDonald and Siegel (1986), the firm has an opportunity to invest in a project whose value follows a geometric Brownian motion. They assume that the investment decision is independent of the capital structure, and that the firm has unlimited sources of funding to finance the project. More recent papers use the structural model approach of Merton (1974) to evaluate growth options. The structural approach assumes that the firm's asset value follows a diffusion process, and the equity holders have a call option on this asset. Mauer and Sarkar (2005) solve an investment problem in this setting, as well as the optimal investment boundary and the optimal capital structure to finance the project. In addition, they study the conflict between equity and debt holders, and find that equity holders have a tendency to over-invest (compared to an investment policy that maximizes the firm value), as they benefit from the up-side gain while the down-side lost is shared by the debt holders. Titman and Tsyplakov (2007) further investigate this conflict when the firm has the ability to adjust its capital structure dynamically. In a similar setting, Sundaresan and Wang (2007b) consider an investment problem with strategic debt service. Investment and financing distortions when there are multiple growth options are investigated by Sundaresan and Wang (2007a).

Investment with liquidity constraints has been considered in several papers. Boyle and Guthrie (2003) consider a firm with a cash reserve and existing assets whose market value and 1990s, and that such discounts have increased substantially over time.

is constant. The existing assets produce a stochastic cash flow stream that contributes to the growth of the cash reserve, in addition to the interest earned at a risk-free rate. The firm must finance the investment internally, i.e., from the cash reserve and/or asset sales. Hirth and Uhrig-Homburg (2009) consider a firm with a fixed amount of cash, equity and debt outstanding, awaiting for the optimal time to invest in the production facility. During the waiting period, the firm is not generating any revenue and is diluting equity to make the coupon payments. Consequently, the firm may default even before the investment takes place. If this happens, debt holders seize control, and the firm operates as if it were an all-equity firm. They simultaneously solve for the optimal default and investment boundary. In contrast to these models, our model considers a firm with a production facility already in place, and is waiting to invest in another identical facility. In addition, instead of starting with a fixed amount of cash (i.e., liquid assets), the firm in our model optimally distributes dividends and accumulates cash from its revenue.

There are a few papers that model the cash reserve as a diffusion process instead of the asset value. Jeanblanc-Picqué and Shiryaev (1995) consider a cash process that follows standard Brownian motion and solve for the optimal dividend policy as an optimal control problem. Décamps and Villeneuve (2007) extend this model to incorporate a growth option, which increases the drift of the cash process when exercised. Using a similar setting, Radner and Shepp (1996) solve an investment problem where the firm has finitely many projects to choose from.

1.2 Overview of Major Results

When maintaining a cash reserve is not permitted, we extend the result of Leland (1994) to derive analytical solutions for equity and debt values of a firm with a growth option when equity issuance is costly. Endogenous default and exercise boundaries are characterized by a system of non-linear equations, which can be solved by standard numerical procedures. In contrast to previous investment literature, we incorporate a cost of equity dilution in our analysis. First, we show that the value of the growth option is increasing in both the equity issuance cost and the asset's volatility. While our result regarding volatility is consistent

with the investment literature (see Dixit and Pindyck (1994)), it is different from the results of Décamps and Villeneuve (2007), where they find that volatility has ambiguous effects on the value of a growth option. They discuss how an increase in the volatility can make the option worthless in their model. Consistent with the results of Mauer and Ott (2000), Titman and Tsyplakov (2007) and Moyen (2007), we find that equity value maximization leads to a policy that exercises the option *later* (i.e., the exercise boundary is higher), compared to a policy that maximizes the firm value. This is in contrast with the result of Mauer and Sarkar (2005), where they find that an equity value maximizing policy leads to exercising the option *earlier*. This is because in their setting, the cost of exercise is partially financed through debt issuance. Our result shows that the difference between the boundaries given by the two policies increases as the firm's leverage increases. We also find that having a growth option reduces the debt capacity and the leverage in the optimal capital structure. This is consistent with the reported results in the literature.

In the second part of this paper, we develop a binomial method to determine equity and debt values when the firm optimally distributes dividends and accumulates cash. In addition to using it to pay for an exercise of a growth option, the firm also uses cash for coupon payments whenever current revenue is not sufficient. Our method is robust, and can be easily extended to accommodate different cost structures and multiple growth options. We solve for the optimal strategy, and show that before the option is exercised, the firm's strategy can be completely characterized by three distinct regions, where the firm will either a) exercise the option, b) not exercise and pay no dividend, or c) not exercise and pay dividends. After the growth option is exercised, the strategy reduces to two regions where the firm either pays or does not pay dividends. We find that the region where the firm neither exercises nor pays dividends (case (b) above) is larger when the equity issuance cost or the liquidation cost is higher. We show how the growth option and the ability to hold cash complement one another. The benefit of one is greatly enhanced by the presence of the other, especially when market frictions are significant. We investigate how the optimal levels of cash differ between firms of different characteristics. Under firm value maximization, we find that the optimal level is increasing with the riskiness of the firm, as measured by its default probability. This

result is consistent with empirical evidence that the correlation between the amount of cash held in the firm and its credit spread is positive (e.g., Acharya et al. (2009)). We also study the impact of a growth option and the firm's leverage on its optimal cash level. Finally, we extend our method to evaluate firms with multiple investment options.

The remainder of this paper is organized as follow. We introduce the general setup of the model in Section 2, and provide the analysis of growth option without cash reserves in Section 3. Cash reserves are included in the analysis in Section 4. Section 5 discusses the extension of our results to multiple growth options, and Section 6 concludes the paper.

2 The Model

We assume that the firm's asset value, denoted by V_t , is independent of its capital structure, and follows a diffusion process whose evolution under the risk neutral measure \mathbb{Q} is given by

$$\frac{dV_t}{V_t} = (r - q)dt + \sigma dW_t,$$

where r is the risk-free rate, q is the instantaneous revenue rate, σ is the volatility of the asset value, and W_t is a standard Brownian motion under \mathbb{Q} .²

The instantaneous revenue generated by the firm is given by

$$\delta_t = qV_t$$

At time zero, the firm issues debt with principal P , a continuous coupon rate C , and a maturity T . There is a tax benefit of rate τ associated with coupon payments, such that the effective coupon rate to the firm is $(1 - \tau)C$. The firm uses its revenue δ_t to make the coupon payment, and distribute the surplus as dividends to shareholders at the rate $d_t \leq \delta_t - (1 - \tau)C$. Any remaining revenue is accumulated in a cash reserve, whose current balance is denoted by x_t . The cash reserve is earning interest at a rate $r_x \leq r$, reflecting an

²The value process V_t only represents the value of asset on hand, and does not include the value of the growth option. The option value is embedded in the equity and debt value discussed in Section 3.

agency cost associated with leaving cash inside the firm.

Additionally, the firm has an option to increase its asset size by a factor g , at a fixed cost K . In other words, exercising the option increases the asset value from V_t to $(1 + g)V_t$. At the time of the exercise, if the firm has accumulated enough cash, i.e., if $x_t \geq K$, then it uses cash to pay for the expansion. Otherwise, the firm issues additional equity to make up the difference. We model frictions in the equity market by introducing a cost of equity dilution, γ . Thus, when $x_t < K$, the firm must issue $(K - x_t)/(1 - \gamma)$ dollars worth of equity to exercise the option. The dilution cost, γ , does not only represent the physical cost of issuing equity, such as underwriting and administrative fees, but other frictions in equity issuance, such as agency and asymmetric informational cost as well. As noted by Choe et al. (1993), the proportion of external financing by equity issuance is substantially higher in expansionary phases. They suggest that the firms issue equity in such periods to minimize the costs of adverse selection. Our model explicitly restricts debt issuance to finance the option.

Finally, when the firm declares bankruptcy, it liquidates its asset immediately, incurring a liquidation cost of αV_t , and leaving the debt holders with the remaining $(1 - \alpha)V_t$.

3 Welfare and Pricing in the Presence of a Growth Option

In this section, we provide analytical solutions for the equity and debt values, when the firm is restricted from holding cash, and both the debt and the growth option have infinite maturities.

Since we do not allow the firm to hold a cash balance, it must payout all surplus revenue to shareholders as dividends. Specifically, we restrict the dividend payout rate to be $d_t = (\delta_t - C)^+$.³ We will assume without loss of generality that $\tau = 0$.⁴ We first derive closed-form solutions for equity and debt values when bankruptcy and exercise boundaries are specified exogenously. In other words, we assume that there is a lower boundary on the

³The $+$ operator is defined such that $y^+ = \max(y, 0)$.

⁴This assumption is just for expositional convenience. The role of debt is dependent on the existence of corporate tax benefits in our model.

asset value V_B , below which the firm declares bankruptcy, and an upper boundary V_G , above which the firm exercises the growth option. Upon bankruptcy, the firm liquidates its asset immediately and incurs a liquidation cost of α , receiving $(1 - \alpha)V_B$ as proceeds from the liquidation.

It is well known (e.g., Duffie (1988)) that for any security whose payoff depends on the asset value V_t , its value $f(V_t, t)$, must satisfy the following PDE:

$$\frac{1}{2}\sigma^2 V_t^2 f_{VV} + (r - q)V_t f_V - rf + f_t + g(V_t, t) = 0 \quad (1)$$

where $g(V_t, t)$ is the payout received by the holders of this security.

In the perpetuity case, the security value becomes time-independent, and hence the previous PDE reduces to the ODE:

$$\frac{1}{2}\sigma^2 V^2 f_{VV} + (r - q)V f_V - rf + g(V) = 0 \quad (2)$$

Equation (2) has the general solution

$$f(V) = A_0 + A_1 V + A_2 V^{-Y} + A_3 V^{-X}, \quad (3)$$

where

$$X = \frac{\left(r - q - \frac{\sigma^2}{2}\right) + \sqrt{\left(r - q - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2} \quad (4)$$

$$Y = \frac{\left(r - q - \frac{\sigma^2}{2}\right) - \sqrt{\left(r - q - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2} \quad (5)$$

3.1 Equity Value

Consider a firm that issues perpetual debt, with a continuous coupon paid at the rate C . When the firm's revenue rate exceeds the coupon rate, i.e., $qV \geq C$, equity holders receive dividends at the rate $qV - C$. Otherwise the firm dilutes equity to make the coupon

payment, which is equivalent to receiving negative dividends at the rate $\beta(qV - C)$, where $\beta = 1/(1 - \gamma)$.

Let V_G denote the exercise boundary of the growth option. Therefore, the equity value, E , prior to exercising the option must satisfy:

$$\frac{1}{2}\sigma^2V^2E_{VV} + (r - q)VE_V - rE + \beta(qV - C) = 0 \quad \text{for } V_B \leq V \leq C/q \quad (6)$$

$$\frac{1}{2}\sigma^2V^2E_{VV} + (r - q)VE_V - rE + qV - C = 0 \quad \text{for } C/q \leq V \leq V_G, \quad (7)$$

together with the boundary, continuity and smooth pasting conditions:

$$\begin{aligned} \text{(BC I):} \quad & E(V_B) = 0 \\ \text{(CC):} \quad & E((C/q)^-) = E(C/q) \\ \text{(SP):} \quad & E_V((C/q)^-) = E_V(C/q) \\ \text{(BC II):} \quad & E(V_G) = E^0((1 + g)V_G) - \beta K, \end{aligned} \quad (8)$$

where $E^0(V)$ is the equity value under costly equity dilution in absence of the growth option, as given in Asvanunt, Broadie, and Sundaresan (2009), which is also repeated for reference in Appendix A.

(BC I) follows from the assumption that shareholders get nothing when the firm declares bankruptcy. (CC) and (SP) ensure that the equity value at the equity dilution boundary is continuous and smooth. (BC II) follows from our growth option specification that the firm's asset value increases from V to $(1 + g)V$ at the time of the exercise. And since the firm only has one growth option, its equity value must be the same as that of a firm without a growth option, evaluated at $(1 + g)V_G$, less the cost of exercise βK . The expression for the equity value and its derivation can be found in Appendix B.

3.2 Debt Value

Debt holders receive a constant payout rate of C as long as the firm remains solvent, i.e., while $V \geq V_B$. Therefore the debt value, D , must satisfy:

$$\frac{1}{2}\sigma^2V^2D_{VV} + (r - q)VD_V - rD + C = 0 \quad \text{for } V \geq V_B \quad (9)$$

Equation (9) has a general solution:

$$D(V) = B_0 + B_1V + B_2V^{-Y} + B_3V^{-X}, \quad (10)$$

where X and Y are given by (4) and (5).

Similar to solving for the equity value, we use the fact that $D(V)$ must satisfy the following boundary conditions:

$$\begin{aligned} \text{(BC I):} \quad D(V_B) &= (1 - \alpha)V_B \\ \text{(BC II):} \quad D(V_G) &= D^0((1 + g)V_G), \end{aligned} \quad (11)$$

where $D^0(V)$ is the debt value under costly equity dilution in absence of the growth option as given in Asvanunt, Broadie, and Sundaresan (2009), which is also repeated for reference in Appendix A.

(BC I) follows from the assumption that the firm liquidates its asset immediately upon bankruptcy with liquidation cost α . (BC II) is the debt value when the growth option is exercised. The debt value must be the same as that of a firm without a growth option, evaluated at $(1 + g)V_G$. The cost of the exercise is fully funded by equity, hence there is no cost to the debt holders. The expression for the debt value and its derivation can be found in Appendix B.

3.3 Endogenous Default and Exercise Boundaries

When the bankruptcy and the exercise of a growth option are determined endogenously by the firm, we consider two cases where the firm is acting to maximize the value to the entire firm (both equity and debt holders), and when it acts to maximize the value to the equity holders only.

First we determine the optimal bankruptcy boundary *after* the growth option is exercised by numerically solving $\frac{\partial v^0}{\partial V}\Big|_{V=V_B^0} = 0$ or $\frac{\partial E^0}{\partial V}\Big|_{V=V_B^0} = 0$ for firm value maximization or equity value maximization, respectively.

Prior to exercising the growth option, the optimal bankruptcy and exercise boundaries are determined by similar smooth pasting conditions.

For firm value maximization, the optimal bankruptcy and exercise boundaries must satisfy:

$$\frac{\partial v}{\partial V}\Big|_{V=V_B} = 0 \tag{12}$$

$$\frac{\partial v}{\partial V}\Big|_{V=V_G} = \frac{\partial v_L}{\partial V}\Big|_{V=(1+g)V_G}, \tag{13}$$

where $v(V) = E(V) + D(V)$ is the total firm value.

Similarly, for equity value maximization, the boundaries must satisfy:

$$\frac{\partial E}{\partial V}\Big|_{V=V_B} = 0 \tag{14}$$

$$\frac{\partial E}{\partial V}\Big|_{V=V_G} = \frac{\partial E_L}{\partial V}\Big|_{V=(1+g)V_G} \tag{15}$$

We will sometimes refer to the result of firm value maximization as “first-best,” that of equity value maximization as “second-best.” See Leland (1994) for a discussion of the smooth pasting conditions.

The results presented in Sections 3.4 – 3.5 are computed as follows. First, we simultaneously solve the smooth pasting conditions, (12) – (13) or (14) – (15), for the optimal boundaries, V_B and V_G , numerically. Then, we use these boundaries in the closed-form solutions derived

in this section.

3.4 Optimal Exercise Boundary

In this section, we examine how the optimal exercise boundary changes with various model parameters. We also look at how the results under the first-best and second-best policy differ. Figure 1 plots the optimal exercise boundary for the growth option as a function of γ and σ . We observe that the level of asset value above which the firm exercises the option, V_G , is increasing in both γ and σ . An increase in equity dilution cost effectively increases the cost of exercising the option. Therefore, the firm would require a higher payoff from the option when the dilution cost γ is higher. As a result, firms with higher equity dilution cost tend to wait longer before they exercise. Volatility of the asset value represents the riskiness of the firm. Since the growth option in our model is an expansion of the existing operation, the riskier the firm is, the riskier the new project will be as well. It is well known from the standard growth option literature (see, e.g., Dixit and Pindyck (1994)) that the value of the investment increases with the project's volatility, due to its upside potential. For the very same reason, the firm will wait for the asset value to reach a higher level before exercising the option. Consequently, we observe that the exercise boundary increases with the asset volatility σ .

Figure 2 plots the exercise boundary versus the firm's leverage. We define leverage as a ratio of debt to total firm value. The left panel of Figure 2 shows that the asset value exercise boundary is decreasing with leverage under the first-best policy, but is increasing in leverage under the second-best policy. As the firm's leverage increases, the equity value decreases significantly. Hence, the boundary for an equity value maximizing firm needs to be higher to compensate for the decrease in equity value. At the same time, an increase in leverage has a relatively small impact on the total firm value. Therefore, the boundary under firm value maximization decreases with leverage, as the default probability increases. The right panel of Figure 2 shows that boundary on equity value is decreasing with leverage under both policies.

In all of the cases discussed above, we find that the exercise boundary under the first-best

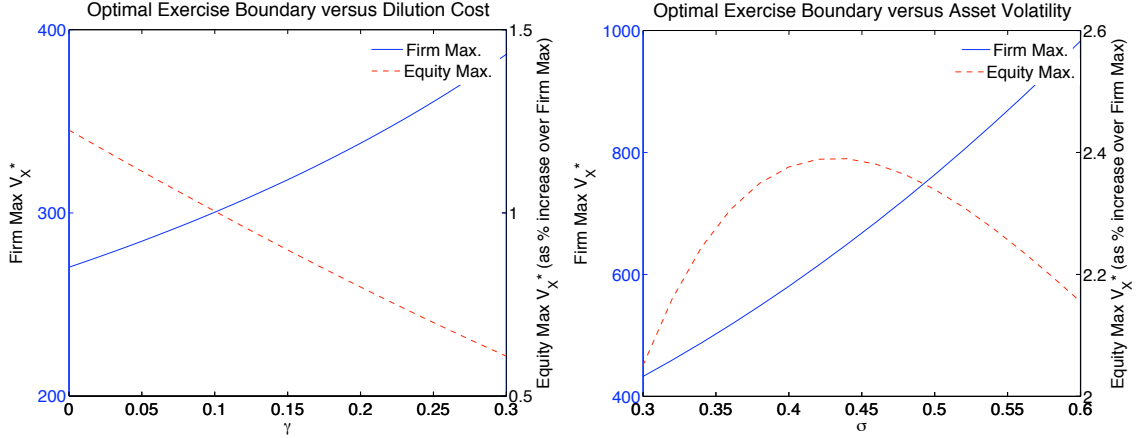


Figure 1: The left panel plots the optimal exercise boundary under first-best policy against equity dilution cost, γ , when the firm has no cash balance. The boundary is increasing with γ . The right panel plots it against the asset volatility, σ . The optimal boundary is also increasing with σ . Both panels also plot the percentage change in exercise boundary if the firm follows second-best policy instead. In all cases, the boundary under first-best policy is always lower than under second-best policy. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$)

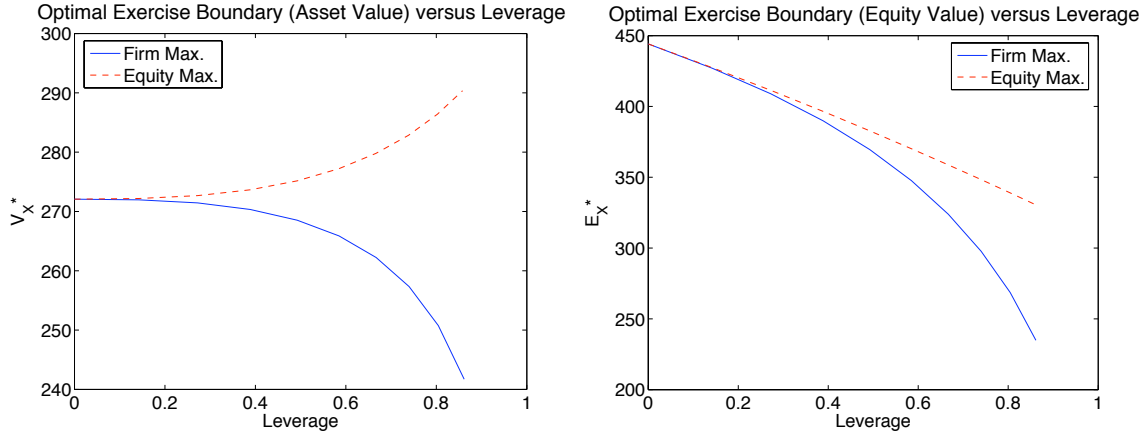


Figure 2: The left panel plots the optimal exercise boundary on asset value against leverage. The boundary is decreasing with leverage under the first-best policy, but is increasing under the second-best policy. The right panel plots the optimal exercise boundary on equity value. The optimal boundary in terms of the equity value is decreasing under both policies. The boundary is always lower for first-best policy than second-best. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$)

policy is always lower than that under the second-best policy. This means that the exercise will occur later under the equity value maximization compared to firm value maximization. The gap between the boundaries of the first-best and the second-best policy is small for firms with low leverage, and rapidly increases as leverage increases. The driving factor behind this result is the fact that the entire cost of investment is financed by the shareholders,

while the benefits are shared with the debt holders as well. As a result, an equity value maximizing firm will demand a higher return from the investment.

3.5 Value of the Growth Option and its Impact on Capital Structure

In this section, we first look at how the value of the growth option varies across firms with different characteristics, and its impact on optimal capital structure. The results under the first-best and the second-best policy are very similar, so we will discuss the results under the former only. Under first-best policy, the value of the growth option is the difference between the firm value with growth option and the firm value without the growth option. The left panel of Figure 3 plots the value of the growth option against the equity dilution cost. As the equity dilution cost increases, the option effectively becomes more expensive and hence we find that its value is decreasing in γ . The right panel of Figure 3 plots the value of the growth option against the firm's asset volatility. As mentioned in Section 3.4, the value of the option is increasing in the volatility of the project. We also find that under both first- and second-best policies, most of the benefit of the growth option goes to the shareholders. Benefit to the debt holders becomes larger as the volatility increases and the firm becomes riskier. This is because the option reduces the probability of default and the dead-weight loss upon liquidation, which is more prominent for riskier firms.

Next we investigate the impact of the growth option on the optimal capital structure of the firm. The left panel of Figure 4 shows that having a growth option reduces the firm's leverage at the optimal capital structure. This is consistent with the empirical observation that firms with many growth opportunities are not as highly leveraged as the others (Billett et al. (2007)). We can also see from this plot that the firm benefits more from the growth option when its leverage is small. This may also be a contributing factor for firms with a growth option to have lower leverage.

Debt capacity is the maximum value of debt that can be sustained by the firm. The right panel of Figure 4 shows the growth option's impact on the debt capacity. We find that the growth option increases the debt capacity of the firm. However, the capacity is attained at a smaller leverage. The benefit of the growth option to the debt holders is increasing

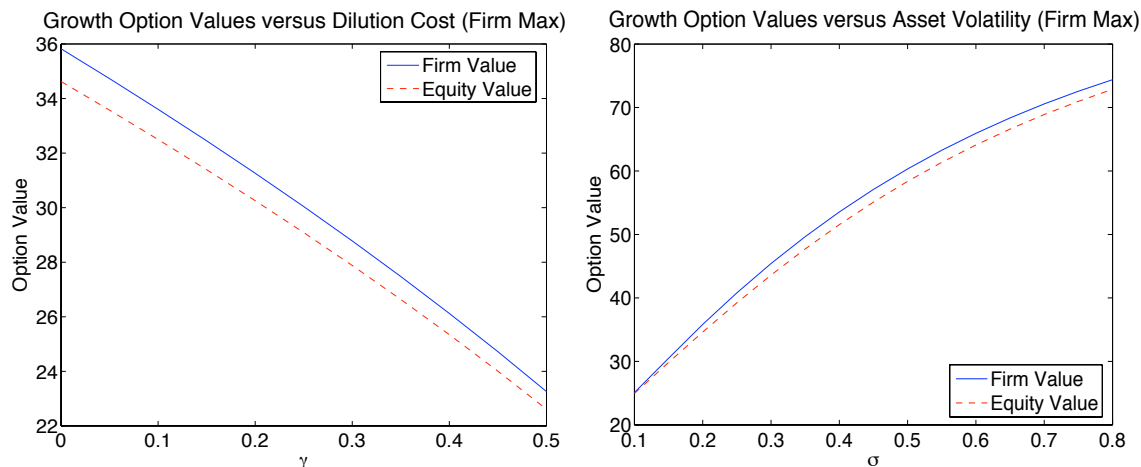


Figure 3: The left panel plots the value of the growth option versus equity dilution cost, γ . The benefit of the growth option to both the firm and equity holders is decreasing in γ . The right panel plots the option values against asset volatility, σ . The benefit to both the firm and equity holders is increasing in σ . Furthermore, the value to the debt holders of a stable firm (small σ) is essentially zero, but increases as the firm becomes more volatile. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$)

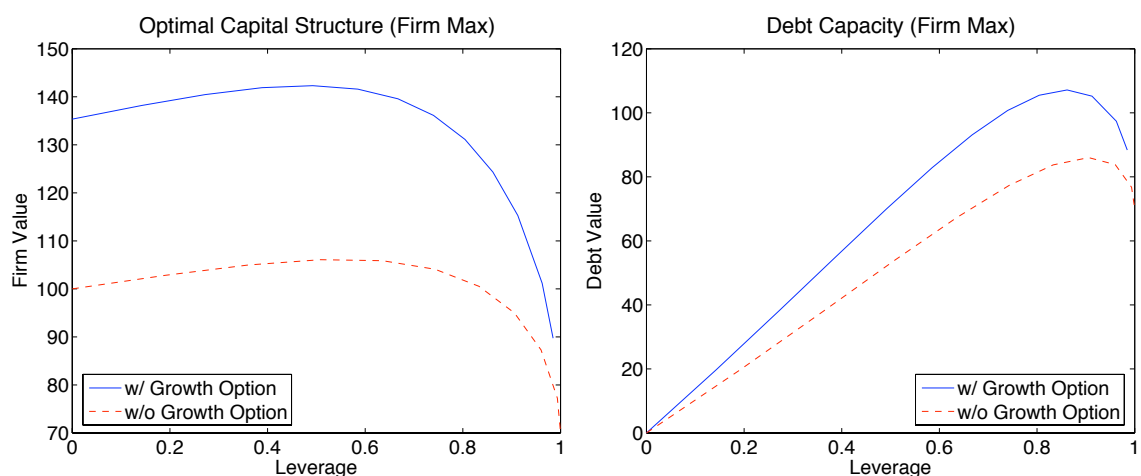


Figure 4: The left panel plots the firm value against leverage for a firm with and without a growth option. It shows that the optimal leverage is lower for the firm with a growth option. The right panel plots the debt value against leverage. The maximum debt value is the debt capacity of the firm. The firm with a growth option has a higher debt capacity. However, the capacity is attained at a lower level of leverage. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$)

in leverage, up to approximately where the debt capacity is attained. This is because exercising the growth option moves the asset value further away from the default boundary, which reduces the default probability and the expected dead-weight loss due to liquidation.

4 Impact of Cash on the Valuation and Exercise of the Growth Option

In this section, we allow the firm to optimally build up a cash reserve by managing dividend distributions to avoid future equity dilution. The firm may choose a dividend payout rate, d , that is less than or equal to the surplus of revenue rate over coupon rate, $(\delta - (1 - \tau)C)^+$, and accumulate cash as discussed in Section 2. We consider both cases where the firm chooses the payout rate d that either maximizes the firm value or the equity value as before.

When equity dilution is not costly, Asvanunt, Broadie, and Sundaresan (2009) and Fan and Sundaresan (2000) show that it is optimal to distribute all surplus revenue as dividends in the absence of a growth option if the firm maximizes equity value. It is not difficult to show that this result remains true in the presence of a growth option. Therefore in our model, the firm is holding cash only when equity dilution is costly, or when it is maximizing the firm value.

In order to determine the optimal policy for the firm that is maintaining a cash reserve, we develop a binomial lattice method similar to that of Asvanunt, Broadie, and Sundaresan (2009). The detailed description and the convergence of the method can be found in Appendix C. The results presented in the remaining of this paper are obtained by the binomial procedure with a maturity of 200 years and 9,600 time steps.

4.1 Optimal Exercise Boundary and Cash Level

When the firm is holding cash, the exercise boundary becomes a function of both the asset value, V , and the cash level, x . Figure 5 shows the optimal exercise boundary for various equity dilution costs. When there is no cost of equity dilution, the amount of cash in the firm has no impact on the exercise boundary since the firm can raise money by diluting equity at no cost. However, with a positive dilution cost, the optimal exercise boundary is decreasing in x , up to $x = K$. For the values of $x > K$, it is no longer necessary for the firm to dilute equity, and hence the exercise boundary is the same as in the case of $\gamma = 0$.

Beyond this point, equity dilution cost becomes irrelevant. This result is very intuitive. When the firm does not have enough cash reserve, it has to undergo costly equity dilution to make up for the shortfall. The more case it needs to raise, the more return it demands from the growth option. Therefore, the boundary decreases as the level of cash balance increases. For the same reason, the exercise boundary is increasing in the dilution cost, γ , for any fixed level of cash, $x \leq K$. As observed in Section 3.4, the boundary is lower under the first-best policy than it is under the second-best policy.

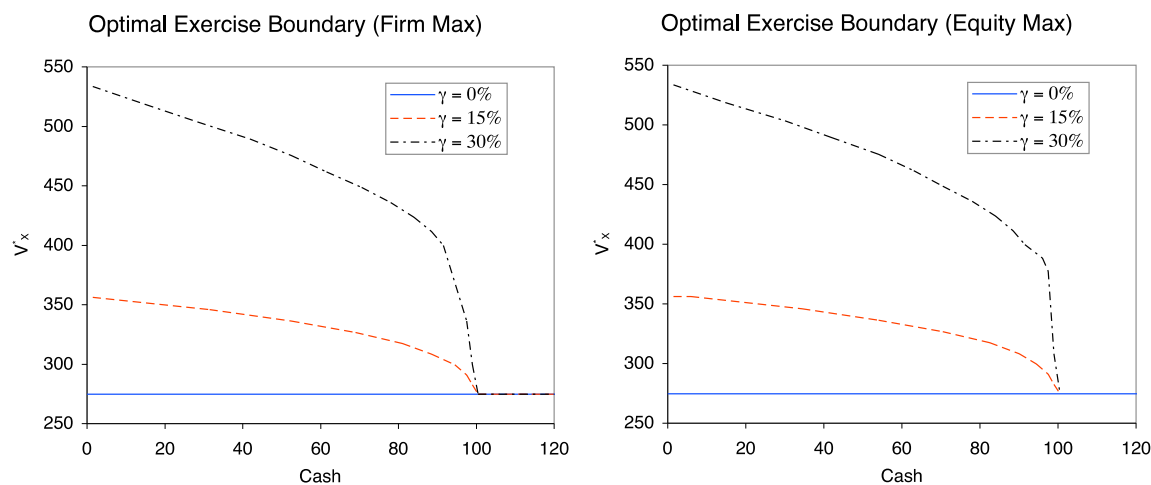


Figure 5: This figure shows the plots of optimal exercise region as a function cash and asset value, under the first- and second-best policies. The optimal exercise boundary is decreasing with cash level, and becomes constant after cash exceeds the cost of the exercise. The boundary is increasing in the equity dilution cost, γ , when the amount of cash is less than the cost of the exercise. The boundary is always lower under first-best policy than under second-best policy. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

Next, we investigate the optimal level of cash, x^* , which also determines the optimal dividend policy. At any given time, if the current cash level, x , is less than the optimal level, x^* , then the firm withholds any revenue surplus until x reaches x^* . On the other hand, if $x > x^*$, then the firm will pay out $x - x^*$ as dividends so that it gets to the optimal level. Figure 6 shows the optimal cash levels with and without the growth option, under the first-best and the second-best policies, when there is no agency cost associated with holding cash inside the firm. First let us consider the first-best policy. Without a growth option, the firm holds cash only to avoid future equity dilution and costly bankruptcy. The optimal level of cash, x^* , is decreasing with the asset value, V . The higher the value of V , the further away the firm is from equity dilution and bankruptcy, so the incentive for

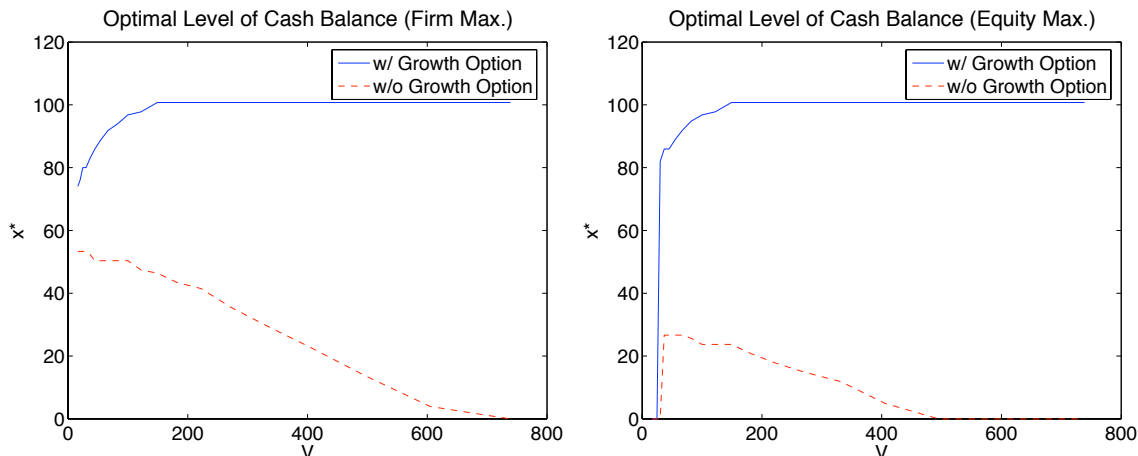


Figure 6: This figure shows the plots of optimal cash level, x^* , versus asset value, V , with and without a growth option under both first- and second-best policies. Under the first-best policy, x^* is decreasing with V when the firm has no growth option, and is increasing with V when the firm has a growth option. Under the second-best policy, the relationship between x^* and V is similar, except that for the values of V that are very close to V_B , the optimal cash level is zero. This is because equity holders will consume the cash before debt holders get it upon liquidation. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

holding cash is less. Under the second-best policy, x^* is also decreasing in V , but only when V is above a certain threshold ($V \geq 25$ in Figure 6). The optimal level of cash below this threshold is zero. This is because when an equity value maximizing firm is very close to bankruptcy, it will pay out all remaining cash as dividends so that the debt holders will not get it upon liquidation. The optimal cash level is also generally lower under the second-best policy for the same reason. With a growth option, the firm will maintain a much higher level of cash in both cases. The optimal cash level is increasing in V , but never exceeds the cost of exercising the option, K . This is because at higher values of V , the firm is more likely to exercise the option so it starts saving toward it. The optimal cash level, x^* , is capped at K because there is no need to maintain more cash than is necessary to exercise the option. The same region where the firm does not keep cash still exists under the second-best policy with a growth option.

Figure 7 shows the impact of equity dilution cost on the optimal cash level when the firm has a growth option and there is an agency cost associated with holding cash. As expected, agency cost reduces the optimal amount of cash held inside the firm. First, note that there is no incentive for holding cash under the second-best policy when there is no equity dilution

cost, as depicted in the right panel of Figure 7. However, under the first-best policy, it is optimal for the firm to maintain a cash reserve even when equity issuance is not costly. The left panel of Figure 7 shows that the optimal cash level is decreasing in V when $\gamma = 0$, as opposed to increasing when $\gamma > 0$. This is because the sole purpose of holding cash is to prolong the life of the firm instead of financing the growth option. By prolonging its life, the firm benefits from receiving more tax deductions and an increase in the probability of the option getting exercised.

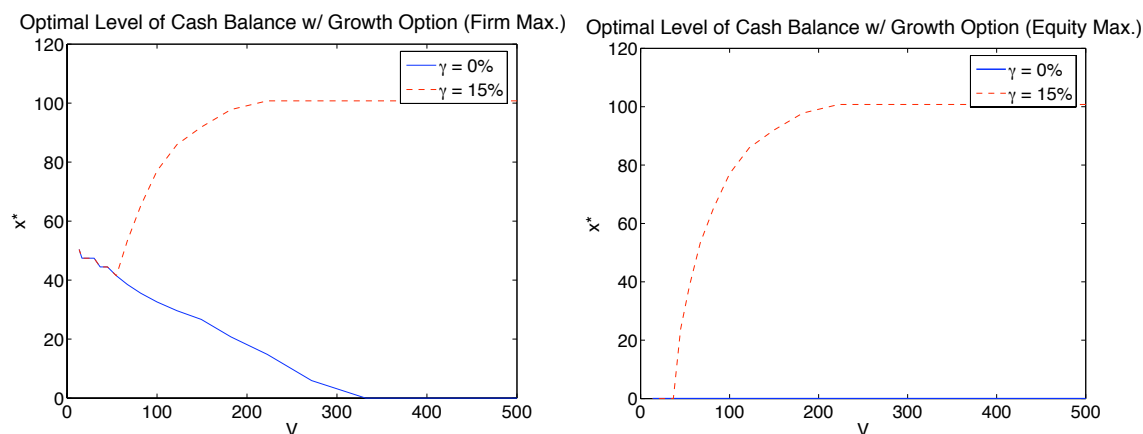


Figure 7: This figure shows the plots of optimal cash level, x^* , versus asset value, V , with and without equity dilution cost, when agency cost is present. In presence in agency cost, it is optimal to hold cash regardless of equity issuance cost under the first-best policy. Under the second-best policy, it is only optimal to hold cash when equity issuance is costly. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.049$)

The optimal exercise boundary shown in Figure 5 and the optimal cash level in Figure 6 completely characterize an optimal strategy for the firm. In Figure 8, we superimpose the two plots onto one another. For simplicity of the discussion, we will refer to x_t as the amount of cash *after* the coupon payment is made. The optimal strategy is as follows. The firm makes two decisions sequentially. First, given its current asset value and cash level, (V_t, x_t) , it decides whether or not to exercise the option. Suppose that the current state (V_t, x_t) is not in the exercise region A in the left panel. Then it has to decide how much of its revenue to save and how much to pay out as dividends. If (V_t, x_t) is in region B, then the firm will do nothing (pay no dividend), and will instantaneously move horizontally to $(V_t, x_t + (qV_t - C)dt)$. Note that depending on the value of V_t , $qV_t - C$ could be either positive or negative. Otherwise, the firm is in region C, and will pay out dividends such

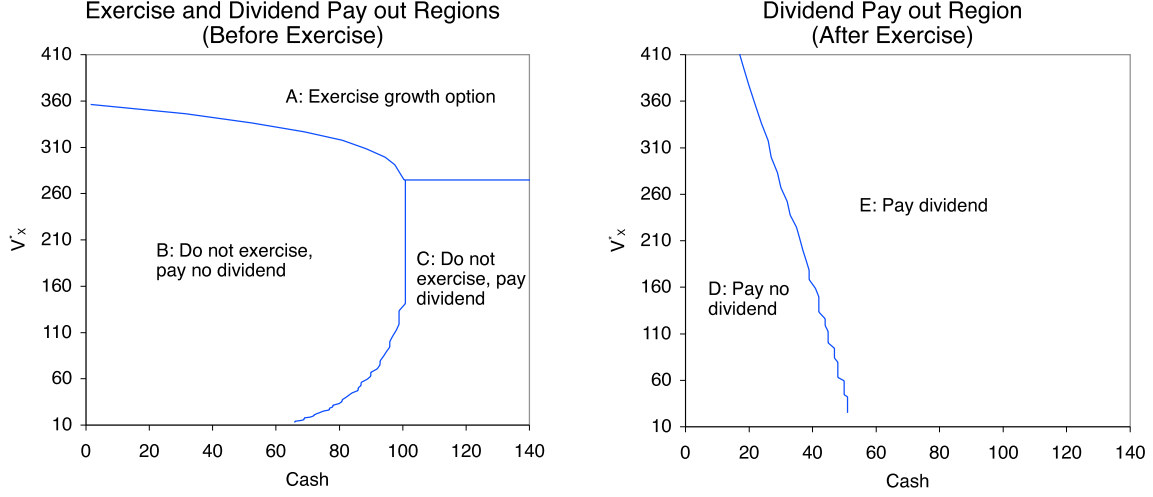


Figure 8: This figure completely characterizes the optimal policy regarding cash and the exercise of the growth option. The left panel corresponds to the decisions before the growth option is exercised. In region A, the firm will exercise the option, and all subsequent decisions are determined in the right panel. In region B, V is not high enough to exercise and x is below the optimal level so the firm will not dividends. In region C, x is above the optimal level so the firm will pay excess above the optimal level as dividend. Once the option is exercised, the decisions in regions D and E and similar to those in regions B and C, respectively. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

that it instantaneously moves to (V_t, x_t^*) . On the other hand, suppose that (V_t, x_t) is in the exercise region A. Then the firm will exercise the option, and instantaneously move to $((1 + g)V_t, (x_t - K)^+)$. Then, the decision on the dividend payout depends on whether its new state is in the region D or E in the right panel. The dividend decision is similar to when the growth option is not yet exercised. Figure 9 shows how the liquidation cost, α , and the equity dilution cost, γ , affect boundaries of these regions. We find that when γ increases from 15% to 30%, region B becomes significantly larger. This is due to a significant increase in the exercise boundary. The dilution cost has little to no impact on the firm's decisions once the option is already exercised. When α decreases from 30% to 0%, we find that region D becomes significantly smaller as the optimal cash level decreases. However, we find that the liquidation cost has no significant impact on the firm's decisions before the option is exercised. This suggests that the dominating incentive for holding cash for a firm with a growth option is to finance the option, rather than to avoid bankruptcy.

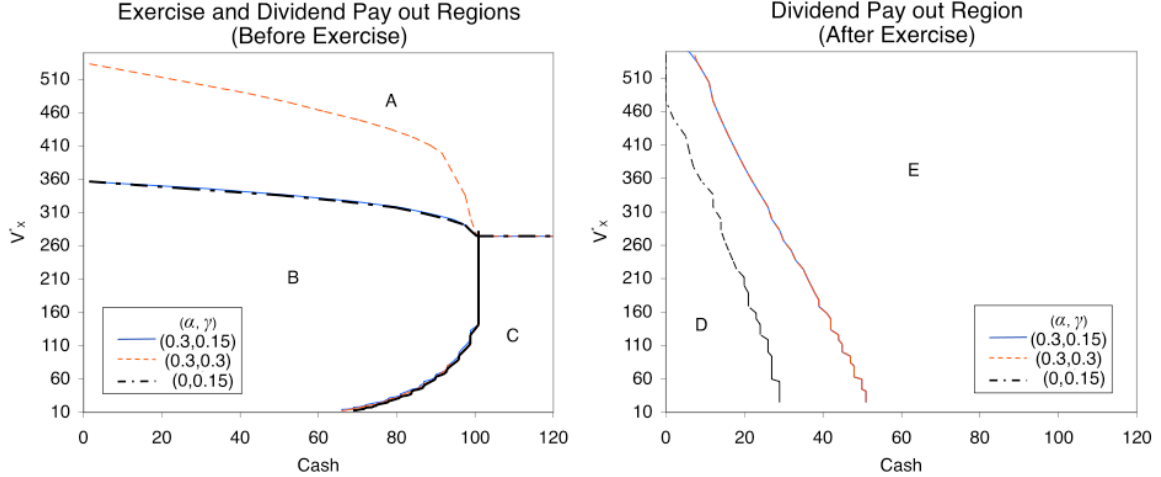


Figure 9: This figure shows how the equity dilution cost, γ , and the liquidation cost, α , affect the various regions of optimal policy. Prior to exercising the option, increasing γ increases region B, where the firm is neither exercising nor paying out dividend. The upper boundary of region B shifts up to the red dotted line when γ increases from 15% to 30%. After exercising the option, decreasing α decreases region D, where the firm pays no dividend. The boundary for region D shifts to the left to the black dash line when α decreases from 30% to 0%. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

4.2 Values of Holding Cash and a Growth Option

In this section, we investigate the interactions between the growth option and the cash balance. Specifically, we compare the value of the growth option to the firm when it can hold cash to when it cannot, and compare the value of holding cash with and without a growth option. The left panel of Figure 10 shows the value of the option as a function of γ . As explained in Section 3.5, the value of the growth option is the difference in the firm (equity) value between having and not having a growth option. We consider only the results of the first-best policy in this section. As before, we find the option value is decreasing in γ , but we find that the decrease is at a much slower rate when the firm is holding a cash reserve. This is because the firm saves cash to reduce the amount of equity dilution required to exercise the option. As observed in the previous section, most of the benefit of the option goes to the shareholders.

The right panel of Figure 10 shows the value of holding cash, with and without a growth option, as a function of γ . Similarly, this is the difference between the firm (equity) value of a firm that optimally accumulates cash reserve and a firm that always pays out dividends in

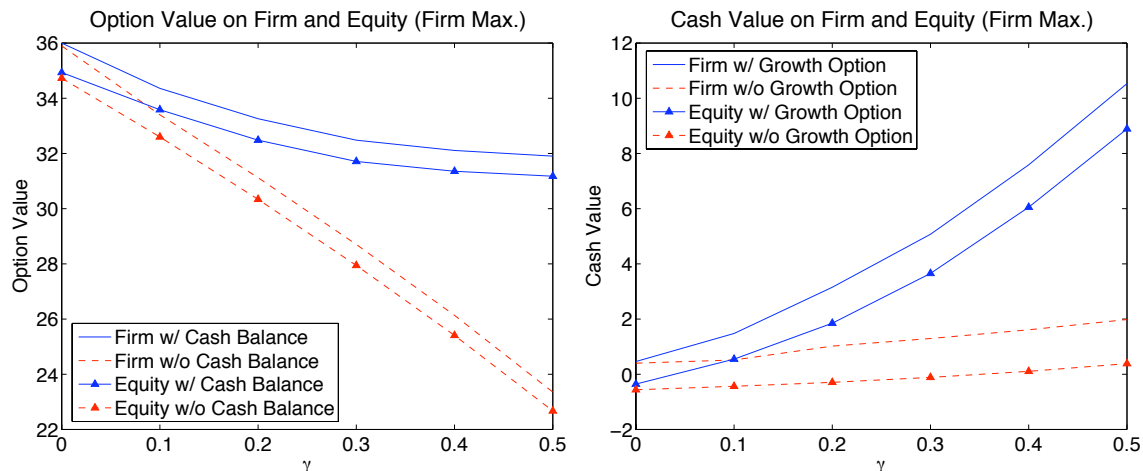


Figure 10: The left panel plots the value of the growth option versus equity dilution cost, γ . The option value is decreasing in γ . It is much less sensitive to γ when the firm is holding cash. The right panel plots the value of holding cash against γ . The cash value is increasing in γ . It is much more sensitive to γ when growth option is present. In all cases, the growth option and cash balance are more valuable to equity holders than they are to debt holders. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

full. The main purpose of cash is to reduce equity dilution cost, and therefore holding cash is more valuable when the dilution cost is higher. The value of holding cash (in both equity and firm value) is very small in the absence of a growth option, but increases significantly when it is present. Under the first-best policy, holding cash may actually have a negative impact on the equity value when γ is small. However, with a growth option present, its impact increases rapidly and becomes positive as γ increases. The results in this section illustrate the importance of cash and the dividend policy for firms with growth opportunities. While dividend policy may not have significant impact for firms without investment opportunities, it can be crucial for start-up firms with substantial growth potentials.

4.3 Cash Reserve Properties across Different Firms

A recent empirical study by Acharya et al. (2009) shows that the correlation between the amount of cash held within a firm and its credit spread is positive. Despite the common belief that firms with large cash reserves are safer, it may be the case that riskier firms need cash to remain liquid in order to avoid default and bankruptcy. We use our model to explore this question. In Figure 11, we reproduce the plot of the optimal cash level (from Figure 6)

in terms of optimal cash-to-asset ratio versus the default probability, which is a good proxy for credit spread. In this figure, we use the probability that the firm will default within the next 200 years (the length of our binomial lattice).⁵ Our model suggests that firms with higher spreads should hold more cash than firms with lower spreads. As explained earlier, this is because riskier firms are more likely to have cash flow shortages, so they hold on to more cash to provide additional liquidity to avoid equity dilution. The presence of a growth option systematically increases the optimal level of the cash-to-asset ratio for all firms.

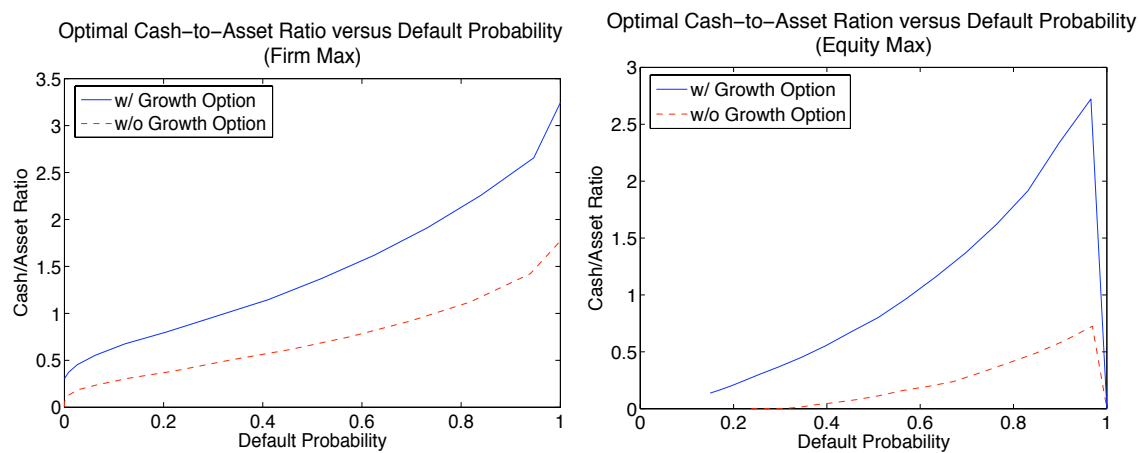


Figure 11: This figure plots the optimal cash-to-asset ratio, under first- and second-best policies, against the default probability. Under the first-best policy, optimal cash-to-asset ratio is increasing with default probability. Under the second-best policy, it is also increasing, but only up to a value of probability close to one. Beyond that, the optimal cash ratio is zero. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$, $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

In the previous results, we looked at the optimal cash ratio for firms with different asset values V , but with the same coupon rate C . What happens if these firms are operating at their optimal capital structure? Figure 12 compares the optimal cash-to-asset ratio under the first-best policy between firms with fixed and optimal coupon. We are plotting cash-to-asset ratio against V instead of the default probability because when C is chosen optimally, the default probability does not change as the value of V changes. Recall that in absence of a growth option, cash is solely used for protection against costly dilution in the bad states. Hence we find that the optimal cash-to-asset ratio is constant with respect to V under the optimal leverage, because C proportionately increases or decreases with V . When a growth

⁵The default probability is used in this case as a measure of the firm's riskiness for comparison purpose only. The level of the probability itself is not intended to represent the actual level of risk of the firm in our discussion.

option is present, however, firms with lower asset values hold more cash than firms whose asset values are higher. This is because the optimal leverage takes into account the future exercise of the growth option. As such, the coupon level is considerably high for the current asset level, and firms with low asset values will hold a large amount of cash to ensure that they survive long enough to exercise the option. Intuitively, the amount of cash these firms hold decreases as V increases. In the fixed coupon case, we choose C to be the optimal coupon for $V = 100$. Consequently, we find that the optimal cash ratio is higher for the fixed coupon case than it is for the optimal coupon case when $V < 100$, and lower when $V > 100$.

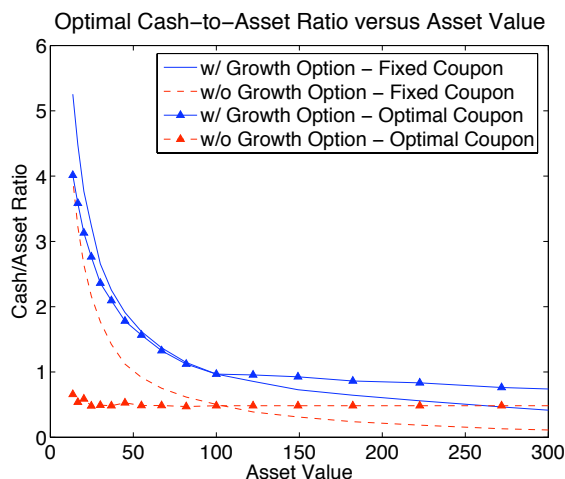


Figure 12: The figure plots the optimal cash-to-asset ratio versus the asset value, V , for a firm with and without a growth option, for both fixed and optimal leverage case. Without a growth option, the optimal cash ratio is decreasing in V when C is fixed. It is constant in V when C is chosen optimally. With a growth option, the optimal cash ratio is decreasing in V for both fixed and optimal C . ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$ (for fixed coupon), $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g = 1$, $K = 100$, $r_x = 0.05$)

5 Extension: Multiple Growth Options

The binomial method described in Appendix C can be easily extended to evaluate firms with multiple growth options of different sizes and cost structures. Appendix D provides a detailed description of the binomial method for multiple options, when their sizes and costs are the same. For a simple illustration, we consider the first-best policy of a firm with a single opportunity to acquire two additional units of its current asset ($g_1 = 2$) at a cost of

$K_1 = 2V_0$, and a firm with two opportunities to acquire a single unit ($g_2 = 1$) at a cost $K_2 = V_0$ each time. Essentially, we are comparing between a firm that must expand its operation in one execution to a firm that can expand its operation by the same amount, but in multiple phases (two in this example). We can also think of these as the same option, where one must be exercised once, while the other can be exercised in two halves. The left panel of Figure 13 shows the optimal exercise boundaries for the two cases. Compared with the boundary of the option with a single exercise, we find that the boundary for the first (of two) exercise is lower, while that for the second (of two) exercise is higher. The right panel of Figure 13 shows the optimal cash level before and after each of the exercises. The optimal cash level is increasing up to \$200, when there are two exercises left, and only up to \$100, when there is only one exercise left. Once all the options are exercised, the optimal cash level for both cases are the same. Figure 14 plots the option value against equity dilution cost, γ . It shows that the option value increases when it can be exercised in two halves, and that the difference is increasing in γ . The option becomes more valuable because the firm does not need to accumulate as much cash for each exercise.

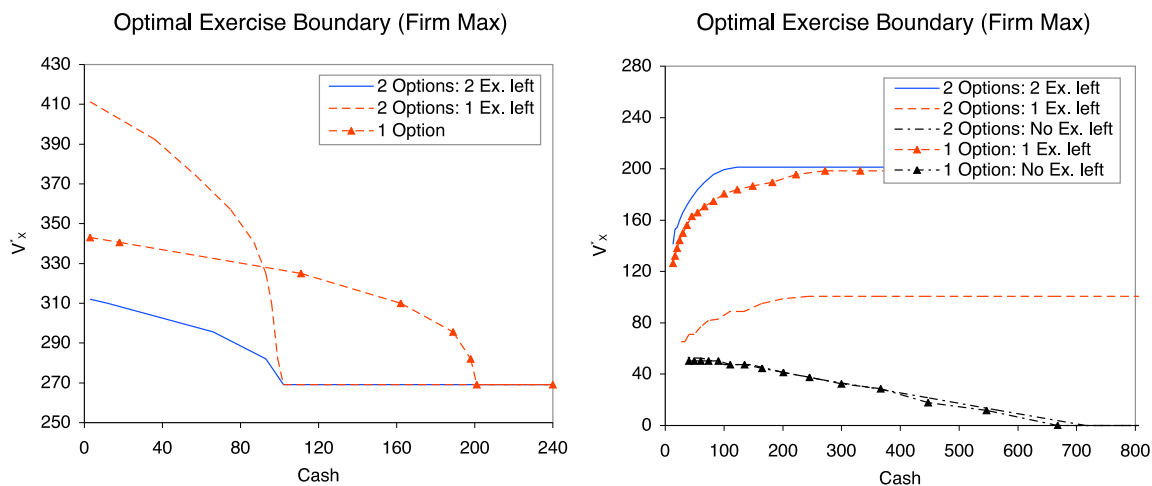


Figure 13: The left panel compares the exercise boundaries for a firm with one and two options. Boundary for the first (of two) exercise is lower than that of a single option, while the boundary for the second (of two) exercise is higher. The right panel shows the optimal cash levels. For the cash of two options, the optimal cash level is increasing up to \$200 before the 1st option is exercised, and up to \$100 after the it is exercised. Once all options are exercised, the optimal cash level is the same as the single option case. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$ (for fixed leverage), $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g_1 = 2$, $g_2 = 1$, $K_1 = 200$, $K_2 = 100$, $r_x = 0.05$)

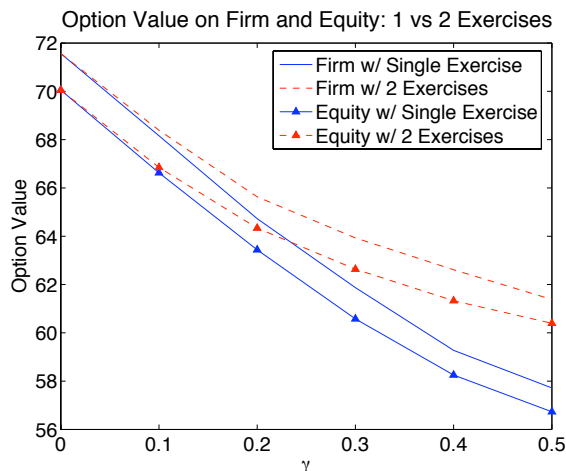


Figure 14: The figure compares the value of growth option between 1 and 2 exercises. The ability to exercise in 2 steps increases the value of the option. The increase is larger when equity dilution is more costly. ($V = 100$, $\sigma = 0.2$, $q = 0.03$, $r = 0.05$, $C = 3$ (for fixed leverage), $\alpha = 0.3$, $\tau = 0.15$, $\gamma = 0.15$, $g_1 = 2$, $g_2 = 1$, $K_1 = 200$, $K_2 = 100$, $r_x = 0.05$)

6 Conclusions

In this paper, we investigate optimal decisions for companies with growth opportunities. First, we provide analytical solutions for the firm’s security values when equity dilution is costly and dividends are paid out in full (i.e., no cash reserve). We show that firms with growth opportunities have lower leverage at the optimal capital structure. We model capital market friction by assuming that equity dilution is costly. A binomial method is developed to determine the optimal dividend policy and the optimal exercise boundary of the growth option. We completely characterize the firm’s optimal strategy by computing the regions where the firm exercises the option, builds up a cash reserve, and distributes dividends. The choice of dividend policy is shown to be extremely important when the firm has investment opportunities, as illustrated by a significant increase in the firm value when it is permitted to hold cash. We also investigate why different firms across different rating categories carry different amounts of cash. We show that the optimal cash-to-asset ratio is increasing with the default probability.

Finally, we illustrate how our binomial method can be extended to evaluate firms with multiple growth opportunities. Through a simple example, we show how the firm’s policy changes if it can implement the expansion in two phases rather than one.

Appendix A

In this Appendix we reproduce the expressions for equity and debt values under costly equity dilution in absence of growth option, derived in Asvanunt, Broadie, and Sundaresan (2009).

Equity Value without Growth Option

Consider the same setup as described in Section 3 without the growth option. The equity value, E^0 , must satisfy:

$$\frac{1}{2}\sigma^2V^2E_{VV}^0 + (r - q)VE_V^0 - rE^0 + \beta(qV - C) = 0 \quad \text{for } V_B \leq V < C/q \quad (16)$$

$$\frac{1}{2}\sigma^2V^2E_{VV}^0 + (r - q)VE_V^0 - rE^0 + qV - C = 0 \quad \text{for } V \geq C/q \quad (17)$$

The general solution (3) becomes:

$$E^0(V) = \begin{cases} A_0 + A_1V + A_2V^{-Y} + A_3V^{-X} & \text{for } V_B \leq V < C/q \\ \tilde{A}_0 + \tilde{A}_1V + \tilde{A}_2V^{-Y} + \tilde{A}_3V^{-X} & \text{for } V \geq C/q \end{cases} \quad (18)$$

To solve for the coefficients in (18), we use the fact that $E^0(V)$ must also satisfy the following boundary, continuity and smooth pasting conditions:

$$\begin{aligned} \text{(BC I):} \quad E^0(V_B) &= 0 \\ \text{(BC II):} \quad \lim_{V \rightarrow \infty} E^0(V) &= V - \frac{C}{r} \\ \text{(CC):} \quad E^0((C/q)^-) &= E^0(C/q) \\ \text{(SP):} \quad E_V^0((C/q)^-) &= E_V^0(C/q) \end{aligned} \quad (19)$$

(BC I) follows from our assumption that the shareholders get nothing when the firm declares bankruptcy. (BC II) is the value of equity when the firm becomes riskless as the asset value approaches infinity. (CC) and (SP) ensures a smooth and continuous transition in and out of equity dilution region. This follows from the fact that the equity value fully anticipates

the future events.

By solving the system of equations above, the equity value is given by:

$$E^0(V) = \begin{cases} \beta \left(V - \frac{C}{r}\right) + A_2 V^{-Y} + A_3 V^{-X} & \text{for } V_B \leq V < C/q \\ \left(V - \frac{C}{r}\right) + \tilde{A}_3 V^{-X} & \text{for } V \geq C/q \end{cases} \quad (20)$$

where

$$\begin{aligned} A_2 &= \frac{(\beta - 1) \left(\frac{C}{q}\right)^Y \left(X \frac{C}{r} - X \frac{C}{q} - \frac{C}{q}\right)}{X - Y} \\ A_3 &= \left[\beta \left(\frac{C}{r} - V_B\right) - A_2 V_B^{-Y} \right] V_B^X \\ \tilde{A}_3 &= \left[(\beta - 1) \left(\frac{C}{q} - \frac{C}{r}\right) V_B^{-X} + \beta \left(\frac{C}{q}\right)^{-X} \left(\frac{C}{r} - V_B\right) \right] V_B^{X+Y} \\ &\quad + \frac{(\beta - 1) \left(X \frac{C}{r} - X \frac{C}{q} - \frac{C}{q}\right) \left(V_B^{-X} - V_B^{-Y} \left(\frac{C}{q}\right)^{Y-X}\right)}{X - Y} V_B^{X+Y} \end{aligned}$$

Debt Value without Growth Option

Similarly, the debt value, D^0 , must satisfy:

$$\frac{1}{2} \sigma^2 V^2 D_{VV}^0 + (r - q) V D_V^0 - r D^0 + C = 0 \quad \text{for } V \geq V_B \quad (21)$$

Equation (21) has a general solution:

$$D^0(V) = B_0 + B_1 V + B_2 V^{-X} \quad (22)$$

where X is given by (4).

Similar to solving for the equity value, we use the fact that $D^0(V)$ must satisfy the following boundary conditions:

$$\begin{aligned} \text{(BC I):} \quad D^0(V_B) &= (1 - \alpha) V_B \\ \text{(BC II):} \quad \lim_{V \rightarrow \infty} D^0(V) &= \frac{C}{r} \end{aligned} \quad (23)$$

(BC I) follows from our assumption on bankruptcy cost, and (BC II) is the value of a perpetual debt as it becomes riskless.

Then we can show that the debt value is given by:

$$D^0(V) = \frac{C}{r} + \left((1 - \alpha)V_B - \frac{C}{r} \right) \left(\frac{V}{V_B} \right)^{-X} \quad \text{for } V \geq V_B \quad (24)$$

Appendix B

Derivation for Equity Value in Section 3.1

Writing the general solution (3) for $E(V)$ as

$$E(V) = \begin{cases} A_0 + A_1V + A_2V^{-Y} + A_3V^{-X} & \text{for } V_B \leq V \leq C/q \\ \tilde{A}_0 + \tilde{A}_1V + \tilde{A}_2V^{-Y} + \tilde{A}_3V^{-X} & \text{for } C/q \leq V \leq V_G, \end{cases} \quad (25)$$

and substitute it in (6), we get

$$\begin{aligned} & \frac{1}{2}\sigma^2 A_2 Y(Y+1)V^{-Y} + \frac{1}{2}\sigma^2 A_3 X(X+1)V^{-X} - (r-q)A_1V - (r-q)A_2YV^{-Y} \\ & - (r-q)A_3XV^{-X} - r(A_0 + A_1V + A_2V^{-Y} + A_3V^{-X}) + \beta(qV - C) = 0 \end{aligned} \quad (26)$$

A_0 and A_1 can be solved by collecting the constant and the coefficient of the linear term in (26) as follows:

$$\begin{aligned} -rA_0 - \beta C &= 0 \Rightarrow A_0 = -\frac{\beta C}{r} \\ (r-q)A_1 - rA_1 + \beta q &= 0 \Rightarrow A_1 = \beta \end{aligned}$$

Similarly, we can substitute (25) in (7) and solve for the coefficients \tilde{A}_0 and \tilde{A}_1 .

$$\begin{aligned} -r\tilde{A}_0 - C &= 0 \Rightarrow \tilde{A}_0 = -\frac{C}{r} \\ (r-q)\tilde{A}_1 - r\tilde{A}_1 + \beta &= 0 \Rightarrow \tilde{A}_1 = 1 \end{aligned}$$

A_2 , A_3 , \tilde{A}_2 and \tilde{A}_3 are determined by solving the system of four linear equations imposed by the conditions in (8), namely

$$A_2 V_B^{-Y} + A_3 V_B^{-X} + \beta V_B - \frac{\beta C}{r} = 0 \quad (27)$$

$$A_2 \left(\frac{C}{q}\right)^{-Y} + A_3 \left(\frac{C}{q}\right)^{-X} + \frac{\beta C}{q} - \frac{\beta C}{r} = \tilde{A}_2 \left(\frac{C}{q}\right)^{-Y} + \tilde{A}_3 \left(\frac{C}{q}\right)^{-X} + \frac{C}{q} - \frac{C}{r} \quad (28)$$

$$-A_2 Y \left(\frac{C}{q}\right)^{-Y-1} - A_3 X \left(\frac{C}{q}\right)^{-X-1} + \beta = -\tilde{A}_2 Y \left(\frac{C}{q}\right)^{-Y-1} - \tilde{A}_3 X \left(\frac{C}{q}\right)^{-X-1} + 1 \quad (29)$$

$$\tilde{A}_2 V_G^{-Y} + \tilde{A}_3 V_G^{-X} + V_G - \frac{C}{r} = (1+G)V_G - \frac{C}{r} - \beta K \quad (30)$$

$$\begin{aligned} &+ \left[(\beta - 1) \left(\frac{C}{q} - \frac{C}{r} \right) (V_B^0)^{-X} + \beta \left(\frac{C}{q} \right)^{-X} \left(\frac{C}{r} - (V_B^0) \right) \right] (V_B^0)^{X+Y} ((1+g)V_G)^{-X} \\ &+ (\beta - 1) \left(X \frac{C}{r} - X \frac{C}{q} - \frac{C}{q} \right) \left((V_B^0)^{-X} - (V_B^0)^{-Y} \left(\frac{C}{q} \right)^{Y-X} \right) \frac{(V_B^0)^{X+Y}}{X-Y} ((1+g)V_G)^{-X}, \end{aligned}$$

where V_B^0 is the bankruptcy boundary of a firm without a growth option.

Hence the equity value before exercising the growth option is given by

$$E(V) = \begin{cases} -\frac{\beta C}{r} + \beta V + A_2 V^{-Y} + A_3 V^{-X} & \text{for } V_B^0 \leq V < C/q \\ -\frac{C}{r} + V + \tilde{A}_2 V^{-Y} + \tilde{A}_3 V^{-X} & \text{for } C/q \leq V \leq V_G, \end{cases}$$

where

$$\begin{aligned}
A_2 &= \frac{1}{qr(Y-X)(V_B^Y V_G^X - V_B^X V_G^Y)} \left(V_B^Y ((g+1)V_G)^{-X} (V_B^0)^{-Y} \left(C \left(\frac{C}{q} \right)^Y (Xr+r-qX)(\beta-1) \right. \right. \\
&\quad V_G^X (V_B^0)^Y ((g+1)V_G)^X + \left. \left(q(X-Y)\beta V_B^X (C-rV_B) - C \left(\frac{C}{q} \right)^X (Yr+r-qY)(\beta-1) \right) \right. \\
&\quad V_G^Y (V_B^0)^Y ((g+1)V_G)^X - gqr(X-Y)V_G^{X+Y+1} (V_B^0)^Y ((g+1)V_G)^X + V_G^{X+Y} \left(-C \left(\frac{C}{q} \right)^Y \right. \\
&\quad \left. \left. (Xr+r-qX)(\beta-1) (V_B^0)^X + \left(C(Yr+r-qY)(\beta-1) \left(\frac{C}{q} \right)^X + Kqr(X-Y)\beta((g+1)V_G)^X \right) \right. \right. \\
&\quad \left. \left. (V_B^0)^Y - Cq(X-Y)\beta (V_B^0)^{X+Y} + qr(X-Y)\beta (V_B^0)^{X+Y+1} \right) \right) \\
A_3 &= \frac{1}{qr(Y-X)(V_B^Y V_G^X - V_B^X V_G^Y)} \left(V_B^X ((g+1)V_G)^{-X} (V_B^0)^{-Y} \left(\left(q(Y-X)\beta V_B^Y (C-rV_B) - C \left(\frac{C}{q} \right)^Y \right. \right. \right. \\
&\quad \left. \left. (Xr+r-qX)(\beta-1) \right) V_G^X (V_B^0)^Y ((g+1)V_G)^X + C \left(\frac{C}{q} \right)^X (Yr+r-qY)(\beta-1) V_G^Y (V_B^0)^Y \right. \\
&\quad \left. ((g+1)V_G)^X + gqr(X-Y)V_G^{X+Y+1} (V_B^0)^Y ((g+1)V_G)^X + V_G^{X+Y} \left(C \left(\frac{C}{q} \right)^Y (Xr+r-qX)(\beta-1) \right. \right. \\
&\quad \left. \left. (V_B^0)^X + \left(-C(Yr+r-qY)(\beta-1) \left(\frac{C}{q} \right)^X - Kqr(X-Y)\beta((g+1)V_G)^X \right) (V_B^0)^Y + Cq(X-Y) \right. \right. \\
&\quad \left. \left. \beta (V_B^0)^{X+Y} - qr(X-Y)\beta (V_B^0)^{X+Y+1} \right) \right) \\
\tilde{A}_2 &= \frac{1}{qr(Y-X)(V_B^Y V_G^X - V_B^X V_G^Y)} \left(V_G^Y ((g+1)V_G)^{-X} (V_B^0)^{-Y} \left(gqr(Y-X)V_B^Y V_G^{X+1} (V_B^0)^Y ((g+1)V_G)^X \right. \right. \\
&\quad \left. \left. + qr(X-Y)\beta V_B^Y (V_B^0)^Y \left((V_B^0)^{X+1} V_G^X + ((g+1)V_G)^X (KV_G^X - V_B^{X+1}) \right) - C \left(V_B^Y V_G^X \right. \right. \right. \\
&\quad \left. \left. \left(-(Yr+r-qY)(\beta-1) (V_B^0)^Y \left(\frac{C}{q} \right)^X + (Xr+r-qX)(\beta-1) (V_B^0)^X \left(\frac{C}{q} \right)^Y + q(X-Y)\beta (V_B^0)^{X+Y} \right) \right. \right. \\
&\quad \left. \left. - \left(-(Yr+r-qY)(\beta-1) V_B^Y \left(\frac{C}{q} \right)^X + (Xr+r-qX)(\beta-1) V_B^X \left(\frac{C}{q} \right)^Y + q(X-Y)\beta V_B^{X+Y} \right) \right. \right. \\
&\quad \left. \left. ((g+1)V_G)^X (V_B^0)^Y \right) \right) \\
\tilde{A}_3 &= \frac{1}{qr(Y-X)(V_B^Y V_G^X - V_B^X V_G^Y)} \left(V_G^X ((g+1)V_G)^{-X} (V_B^0)^{-Y} \left(gqr(X-Y)V_G^{Y+1} ((g+1)V_G)^X (V_B^0)^Y \right. \right. \\
&\quad \left. \left. V_B^X + qr(Y-X)\beta (V_B^0)^Y \left((KV_G^Y - V_B^{Y+1}) ((g+1)V_G)^X + V_G^Y (V_B^0)^{X+1} \right) V_B^X - C \left(V_G^Y \left((Yr+r-qY) \right. \right. \right. \\
&\quad \left. \left. (\beta-1) (V_B^0)^Y \left(\frac{C}{q} \right)^X - (Xr+r-qX)(\beta-1) (V_B^0)^X \left(\frac{C}{q} \right)^Y - q(X-Y)\beta (V_B^0)^{X+Y} \right) V_B^X + \right. \\
&\quad \left. \left(-(Yr+r-qY)(\beta-1) V_B^Y \left(\frac{C}{q} \right)^X + (Xr+r-qX)(\beta-1) V_B^X \left(\frac{C}{q} \right)^Y + q(X-Y)\beta V_B^{X+Y} \right) \right. \\
&\quad \left. \left. ((g+1)V_G)^X (V_B^0)^Y \right) \right)
\end{aligned}$$

Derivation for Debt Value in Section 3.2

Substituting the general solution (22) into (9), we get:

$$\begin{aligned} \frac{1}{2}\sigma^2 B_2 X(X+1)V^{-X} + (r-q)B_1 V - (r-q)B_2 X V^{-X} \\ - r(B_0 + B_1 V + B_2 V^{-X}) + C = 0 \end{aligned} \quad (31)$$

B_0 and B_1 can be solved by collecting the constant and the coefficient of the linear term in (31) as follows:

$$\begin{aligned} -rB_0 + C = 0 &\Rightarrow B_0 = \frac{C}{r} \\ (r-q)B_1 - rB_1 = 0 &\Rightarrow B_1 = 0 \end{aligned}$$

B_2 and B_3 are determined by solving the system of linear equations imposed by the boundary conditions in (23), namely

$$\frac{C}{r} + B_2 V_B^{-Y} + B_3 V_B^{-X} = (1-\alpha)V_B \quad (32)$$

$$\frac{C}{r} + B_2 V_G^{-Y} + B_3 V_G^{-X} = \frac{C}{r} + \left((1-\alpha)V_B^0 - \frac{C}{r} \right) \left(\frac{(1+g)V}{V_B^0} \right)^{-X}, \quad (33)$$

Thus the debt value is given by

$$D(V) = \frac{C}{r} + B_2 V^{-Y} + B_3 V^{-X}, \quad (34)$$

where

$$B_2 = \frac{(V_B V_G)^Y ((1+g)V_G)^{-X}}{(V_B^Y V_G^X - V_B^X V_G^Y)} \left[((1+g)V_G)^X V_B^X \left(\frac{C}{r} - (1-\alpha)V_B \right) - V_G^X (V_B^0)^X \left(\frac{C}{r} - (1-\alpha)V_B^0 \right) \right] \quad (35)$$

$$B_3 = -\frac{(V_B V_G)^X ((1+g)V_G)^{-X}}{(V_B^Y V_G^X - V_B^X V_G^Y)} \left[((1+g)V_G)^X V_B^Y \left(\frac{C}{r} - (1-\alpha)V_B \right) - V_G^Y (V_B^0)^X \left(\frac{C}{r} - (1-\alpha)V_B^0 \right) \right] \quad (36)$$

Appendix C

Binomial Lattice Method for Growth Option with Cash Balance

We extend the binomial lattice method of Broadie and Kaya (2007) and Asvanunt, Broadie, and Sundaresan (2009) to evaluate the securities values in the presence of growth option and cash balances. Following Cox et al. (1979), we divide time into N increments of length $h = T/N$, then we write the evolution of the firm asset value V_t as follows:

$$V_{t+h} = \begin{cases} uV_t & \text{with risk-neutral probability } p \\ dV_t & \text{with risk-neutral probability } 1-p \end{cases}$$

Let $V_t^u = uV_t$ and $V_t^d = dV_t$. By imposing that $u = 1/d$, we have:

$$u = e^{\sigma\sqrt{h}}$$

$$d = e^{-\sigma\sqrt{h}}$$

$$a = e^{(r-q)h}$$

$$p = \frac{a-d}{u-d}$$

In this setting, we approximate revenue during time interval h by:

$$\int_t^{t+h} qV_t dt \approx V_t(e^{qh} - 1) =: \delta_t$$

Similarly, we approximate the coupon payment by:

$$\int_t^{t+h} cPdt \approx P(e^{ch} - 1) =: \zeta$$

Incorporating the tax benefit of rate τ , the effective coupon payment is $(1 - \tau)\zeta$.

We need to construct two lattices, one for the values before the growth option is exercised, and another for the values after the growth option is exercised. Recall that when the firm decides to exercise the growth option, the asset value increases from V to $(1 + g)V$. We will denote the asset values in the exercised lattice by $\tilde{V} = (1 + g)V$, and all other security values by \tilde{E} and \tilde{D} , etc.

Each node in the lattice is a vector of dimension M , where each element in the vector corresponds to a different level of cash balance, x . The values of x are linearly spaced from 0 to x^{\max} .⁶

We distinguish the cash before and cash after revenue and dividend by x and x^+ , respectively.

Let $E_t(V_t, x_t)$ be the equity value at time t when the firm's asset value is V_t , and its cash balance level is x_t . Similarly, $D_t(V_t, x_t)$ is the corresponding debt value. We also let $\mathbb{E}_t[f_{t+h}(V_{t+h}, x_{t+h})]$ be the expected value of the security $f(\cdot)$ at the next time period $t + h$, given the information at time t , where $f(\cdot)$ can represent the equity value, $E(\cdot)$, or the debt value, $D(\cdot)$, namely,

$$\mathbb{E}_t[f_{t+h}(V_{t+h}, x_{t+h})] = pf_{t+h}(V_{t+h}^u, x_{t+h}) + (1 - p)f_{t+h}(V_{t+h}^d, x_{t+h}).$$

Note that x_{t+h} is deterministic because it only depends on the decision at time t . Since we only know the values of $f_{t+h}(V, x)$ for a discrete set of values of x , we use linear interpolation to estimate $f_{t+h}(V_{t+h}^u, x_{t+h})$ and $f_{t+h}(V_{t+h}^d, x_{t+h})$.

Figure 15 illustrates what each node in the two lattices represents.

⁶In our routine, we pick x^{\max} arbitrarily and check if the optimal cash balance, x^* , is attained at x^{\max} . If it does, then we increase x^{\max} and repeat the routine.

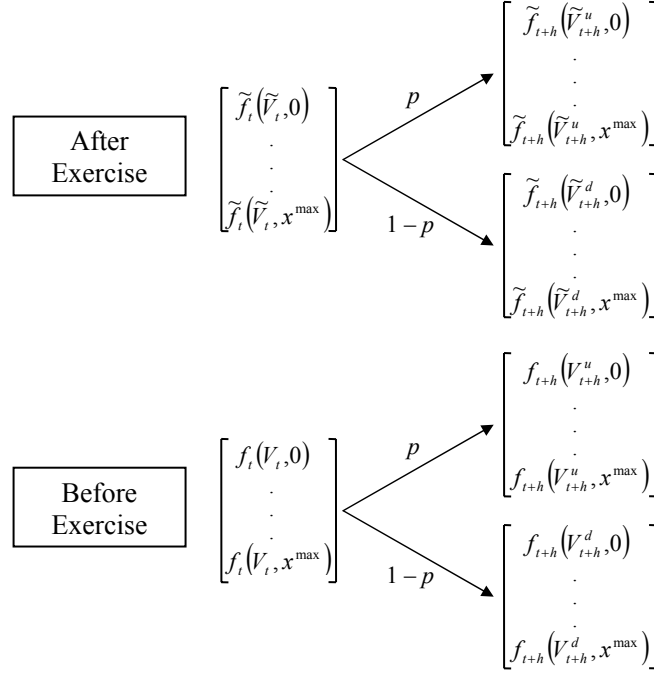


Figure 15: This figure illustrates the vector in each node of the lattices. $f_t(\cdot)$ in the lower lattice represents the security value at each element of the vector before the growth option is exercised. $\tilde{f}_t(\cdot)$ in the upper lattice represents the security value after the growth option is exercised.

First, we construct the exercised lattice. At maturity, the firm returns everything to shareholders after meeting its debt obligations.

If $\tilde{V}_T + x_T + \tilde{\delta}_T \geq (1 - \tau)\zeta + P$:

$$\tilde{E}_T(\tilde{V}_T, x_T) = \tilde{V}_T + x_T + \tilde{\delta}_T - (1 - \tau)\zeta - P$$

$$\tilde{D}_T(\tilde{V}_T, x_T) = \zeta + P$$

If (\tilde{V}_T, x_T) is such that the firm does not have enough value to meet its debt obligation, it declares bankruptcy and liquidates its assets.

If $\tilde{V}_T + x_T + \tilde{\delta}_T < (1 - \tau)\zeta + P$:

$$\tilde{E}_T(\tilde{V}_T, x_T) = 0$$

$$\tilde{D}_T(\tilde{V}_T, x_T) = (1 - \alpha)(\tilde{V}_T + x_T + \tilde{\delta}_T)$$

At other times t , if (\tilde{V}_t, x_t) is such that the firm has enough cash from revenue and cash balance to meet its debt obligations, then it has an option to retain some excess revenue in the cash balance, or pay it out to shareholders as dividend.

1. If $x_t + \tilde{\delta}_t \geq (1 - \tau)\zeta$:

Choose $x_t^+ \in [0, x_t + \tilde{\delta}_t - (1 - \tau)\zeta]$

$$x_{t+h} = e^{rx^h} x_t^+$$

$$\tilde{E}_t(\tilde{V}_t, x_t) = e^{-rh} \mathbb{E}_t \left[\tilde{E}_{t+h}(\tilde{V}_{t+h}, x_{t+h}) \right] + x_t + \tilde{\delta}_t - (1 - \tau)\zeta - x_t^+$$

$$\tilde{D}_t(\tilde{V}_t, x_t) = e^{-rh} \mathbb{E}_t \left[\tilde{D}_{t+h}(\tilde{V}_{t+h}, x_{t+h}) \right] + \zeta$$

x_t^+ is chosen such that the objective value (firm or equity value) is maximized at time t . The optimization is done by performing a search over K discrete points of feasible values of x_t^+ as given above.

If (\tilde{V}_t, x_t) is such that the firm does not have enough from revenue and cash balance, then it has to raise money by diluting equity.

2. If $x_t + \tilde{\delta}_t < (1 - \tau)\zeta$:

If (\tilde{V}_t, x_t) is such that the firm has enough equity to cover the shortfall:

2A. If $(1 - \gamma)e^{-rh} \mathbb{E}_t[\tilde{E}_{t+h}(\tilde{V}_{t+h}, 0)] \geq (1 - \tau)\zeta - x_t - \tilde{\delta}_t$:

$$\tilde{E}_t(\tilde{V}_t, x_t) = e^{-rh} \mathbb{E}_t \left[\tilde{E}_{t+h}(\tilde{V}_{t+h}, 0) \right] + (x_t + \tilde{\delta}_t - (1 - \tau)\zeta)/(1 - \gamma)$$

$$\tilde{D}_t(\tilde{V}_t, x_t) = e^{-rh} \mathbb{E}_t \left[\tilde{D}_{t+h}(\tilde{V}_{t+h}, 0) \right] + \zeta$$

If (\tilde{V}_t, x_t) is such that the firm does not have enough equity value to cover the shortfall, then it declares bankruptcy:

2B. If $(1 - \tau)\zeta - x_t - \tilde{\delta}_t > (1 - \gamma)e^{-rh} \mathbb{E}_t[\tilde{E}_{t+h}(\tilde{V}_{t+h}, 0)]$

$$\tilde{E}_t(\tilde{V}_t, x_t) = 0$$

$$\tilde{D}_t(\tilde{V}_t, x_t) = (1 - \alpha)(\tilde{V}_t + x_t + \tilde{\delta}_t)$$

Once we know the securities values if the option is immediately exercised, we compute its current value by comparing the continuation value with the exercised value. For each node (V_t, x_t) , the continuation values is determined by the exact same routine as the exercised values, with \tilde{V}_t and $\tilde{\delta}_t$ replaced by V_t and δ_t .

Let E_t^X and D_t^X be the equity and debt values if the firm exercises the growth option at time t . These values can be determined as follow.

If (V_t, x_t) is such that the firm has enough cash after making the coupon payments to exercise the option.

1. If $x_t + \delta_t - (1 - \tau)\zeta > K$:

$$E_t^X(V_t, x_t) = \tilde{E}_t(\tilde{V}_t, x_t + \delta_t - (1 - \tau)\zeta - K)$$

$$D_t^X(V_t, x_t) = \tilde{D}_t(\tilde{V}_t, x_t + \delta_t - (1 - \tau)\zeta - K)$$

If (V_t, x_t) is such that the firm does not have enough cash to exercise the option, then it has to raise money by diluting equity.

2. If $x_t + \delta_t - (1 - \tau)\zeta < K$:

$$E_t^X(V_t, x_t) = \tilde{E}_t(\tilde{V}_t, 0) + (x_t + \tilde{\delta}_t - (1 - \tau)\zeta - K)/(1 - \gamma)$$

$$D_t^X(V_t, x_t) = \tilde{D}_t(\tilde{V}_t, 0)$$

Then at each node, we simply replace the values of E_t and D_t by E_t^X and D_t^X if $E_t^X + D_t^X > E_t + D_t$ under the firm value maximization, or if $E_t^X > E_t$ under the equity value maximization. Note that if the firm does not have enough equity to dilute to exercise the option, then E_t^X will be negative and the option will not be exercised.

To determine the rate of convergence of this method, we set $r_x = -\infty$ to force the firm to pay out maximum dividend (not holding any cash). In this case, the results converge to the security values derived in Section 3.

Figure 16 below shows the convergence results of the binomial routine under costly equity dilution, for both equity and firm value maximization. As in Broadie and Kaya (2007), the equity value converges linearly to the true value while the error on the debt value oscillates

sharply.

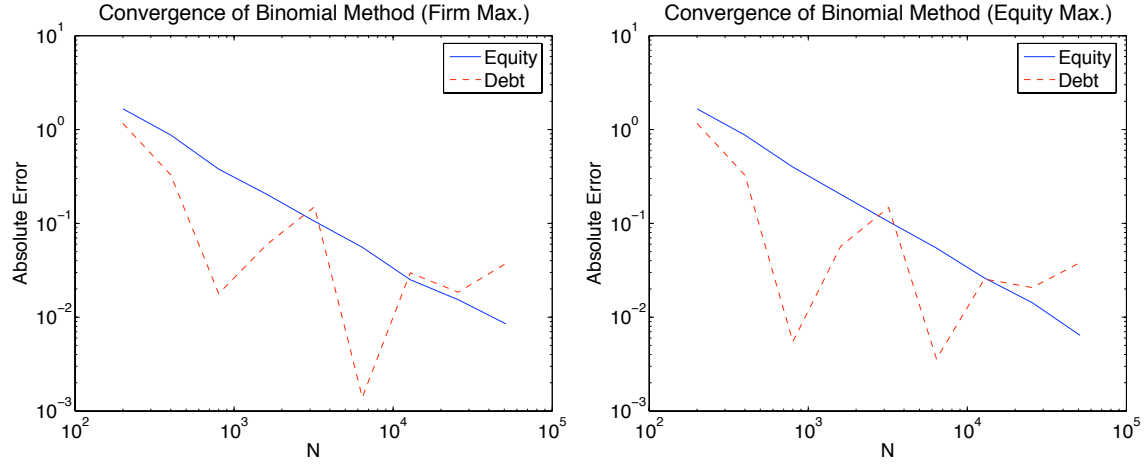


Figure 16: Convergence of Binomial Method for Equity and Debt Values.

Appendix D

Extension of Binomial Method for Multiple Growth Options

The binomial method described in Appendix C can be easily modified to handle multiple options. Let H be the number of options the firm has. Then for each state (V_t, x_t) in the lattice, we add yet another dimension of length $H + 1$. Therefore, each state of the lattice becomes (V_t, x_t, h_t) , where h_t is the number of options left for exercise. In order to keep the size of each option consistent with each other, we assume that asset value increases by $gV_t(0)$, where $V_t(0)$ is the current asset value if the firm has never exercised the option. This is equivalent to having options to acquire g unit of the original asset each time it exercises.

The first step is to compute the continuation values of E_t and D_t at each state (V_t, x_t, h_t) , exactly as described in Appendix C. Then we need to determine whether or not to exercise the option as follow.

For the states with $h_t > 0$, if (V_t, x_t, h_t) is such that the firm has enough cash after making the coupon payments to exercise the option.

1. If $x_t + \delta_t - (1 - \tau)\zeta > K$:

$$E_t^X(V_t, x_t, h_t) = E_t(V_t + gV_t(0), x_t + \delta_t - (1 - \tau)\zeta - K, h_t - 1)$$

$$D_t^X(V_t, x_t, h_t) = D_t(V_t + gV_t(0), x_t + \delta_t - (1 - \tau)\zeta - K, h_t - 1)$$

If (V_t, x_t, h_t) is such that the firm does not have enough cash to exercise the option, then it has to raise money by diluting equity.

$$E_t^X(V_t, x_t, h_t) = E_t(V_t + gV_t(0), 0, h_t - 1) + (x_t + \tilde{\delta}_t - (1 - \tau)\zeta - K)/(1 - \gamma)$$

$$D_t^X(V_t, x_t, h_t) = D_t(V_t + gV_t(0), 0, h_t - 1)$$

Then, we decide whether or not to exercise the option by comparing either $E_t^X + D_t^X$ to $E_t + D_t$ for firm value maximization, or E_t^X to E_t for equity value maximization.

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