Inflation Ambiguity and the Term Structure of U.S. Government Bonds

Maxim Ulrich[a],[*]

[a] Columbia Business School, 3022 Broadway, New York, NY 10027, USA

Abstract

Variations in trend inflation are the main driver for variations in the nominal yield curve. According to empirical data, investors observe a set of empirical models that could all have generated the time-series for trend inflation. This set has been large and volatile during the 1970s and early 1980s and small during the 1990s. I show that log utility together with model uncertainty about trend inflation can explain the term premium in U.S. Government bonds. The equilibrium has two inflation premiums, an inflation risk premium and an inflation ambiguity premium.

1. Introduction

Model uncertainty about trend inflation explains the upward sloping term premium of nominal U.S. Government bond yields. This result holds for an investor with logarithmic utility and low ambiguity aversion. The same model without ambiguity aversion fails to explain the upward sloping term premium.

The importance of trend inflation for modeling nominal yields is well established (Ang et al. (2008a) and Gürkaynak et al. (2005)). I proxy trend inflation with the median one quarter ahead inflation forecast from the Survey of Professional Forecasters (SPF). This real-time proxy is an unbiased forecast for realized inflation with an R^2 of 65%. I assume this median forecast to be the reference (benchmark) model for trend inflation. Under rational expectations this has two implications. First, this reference model coincides with the data generating process for trend inflation. Second, investors fully trust the inflation model and require only an inflation risk premium for unpredictable inflation innovations that correlate negatively with consumption growth (Buraschi and Jiltsov (2005), Piazzesi and Schneider (2006)).

I slightly deviate from rational expectations in the following way. The investor does not fully trust the reference model for trend inflation. I use SPF data to quantify the trust-worthiness of the reference inflation model. The investor monitors a set of inflation models that could all have generated the observed inflation and trend inflation data. Whenever the investor observes new inflation data, he applies a likelihood ratio test to quantify the trust-worthiness of the reference model. This paper shows that ambiguity about the underlying inflation model induces an inflation ambiguity premium on top of the traditional inflation risk premium.[1]

I assume that the worst expected instantaneous change in the trustworthiness coincides with a constant multiple times the cross-sectional variance of real-time SPF inflation fore-casts. This variance is high when investors disagree strongly about the model for trend inflation. On the other hand, it is zero, if all investors use the same model to forecast in-

---

[1] This distinction between risk vs. uncertainty is in line with Knight (1921).

¹ flation.[2] I estimate the multiple with a panel of bond yield and macro data. According

² to the empirical study of Patton and Timmermann (2010)), model uncertainty is the most

³ plausible economic reason for the observed dispersion in macro forecasts.

⁴

⁵ I analyze the effect of inflation ambiguity in a simple representative agent asset pricing

⁶ model with the following key assumptions. First, trend inflation correlates negatively with

⁷ trend consumption growth. This is supported in my data set and consistent with Piazzesi

⁸ and Schneider (2006). Second, the investor faces inflation ambiguity. This implies that he

⁹ does not fully trust his reference model. Both assumptions together imply that inflation am-

¹⁰ biguity induces also ambiguity about trend consumption growth. Depending on the absolute

¹¹ magnitude of the correlation, the latter can be big or small. Min-max preferences make the

¹² investor want to find a worst-case inflation model within the set of potential models.[3].

¹³

¹⁴ In equilibrium, an unexpected *increase* of trend inflation leads to an unexpected *fall* of

¹⁵ (i) the real value of nominal bonds, (ii) the trustworthiness of the reference model, (iii) the

¹⁶ outlook on trend consumption growth. Implication (i) and (ii) make nominal bond yields

¹⁷ carry a positive inflation ambiguity premium. This premium is new in the literature and the

¹⁸ focus of my study. [4]

¹⁹

²⁰ I estimate the model with a rich panel of macroeconomic and bond data. The likeli-

---

[2] The cross-sectional variance of real-time inflation forecasts explains 12% of variations in the first principal component of nominal yields and 8% of variations in the slope of the nominal yield curve. The empirical section contains more details on the data.

[3] The worst-case is defined in terms of minimal expected life-time utility.

[4] Goodfriend and King (2005) argue that rising long-term nominal bond yields during the monetary policy experimentation of the late 1970s and early 1980s were created by "inflation scares". These are periods where investors are not sure about the future inflation scenarios, and instead observe a wide range of future inflation models. One can interpret my multiple prior inflation model as a general equilibrium asset pricing model which supports that idea. Periods of incomplete central bank credibility in Goodfriend and King (2005) can be interpreted as periods of high inflation ambiguity in my model.

hood based estimation is set-up such that the degrees of freedom that model uncertainty adds to the econometrician are restricted dramatically. The estimated unconditional inflation ambiguity premium is 1.75% for a nine year bond and 0.3% for a two year bond. In contrast, the inflation risk premium is flat at around zero, as expected for a low risk aversion economy. The reason for the inflation ambiguity premium to be so steep is rooted in the high persistence of trend inflation. Robust distortions to trend inflation die out only slowly, which results in a steep accumulated ambiguity premium. The time variation of the inflation ambiguity premium matches with the time variation of the dispersion among SPF inflation forecasts. Times of increased dispersion characterize times of increased model mistrust. Inflation ambiguity tends to spike at the beginning of recessions. The ten year inflation ambiguity premium climbed to 2.6% in the mid 1970s and the beginning of 1981.

The steep ambiguity premium provides an intuitive explanation for the shape of the yield curve during the monetary policy experimentation. The term structure of inflation expectations (reference model) was strongly downward sloping, while the nominal yield curve was rather flat. The model attributes this to the high amount of model mistrust. Downward sloping inflation expectations were offset by an upward sloping inflation ambiguity premium. This means that although investors expected inflation to mean revert over the next years (reference model), investors priced bonds with a steep inflation ambiguity premium. The latter was required because investors mistrusted their reference inflation model. The implied detection error probability is 47.7%. This says that given the data on trend inflation and trend consumption growth, a likelihood ratio test would pick the wrong model with probability 47.7%. Together with the steep inflation ambiguity premium this implies that a small amount of aversion against inflation ambiguity is sufficient to generate an upward sloping nominal yield curve.

Inflation ambiguity has a marginal effect on the real side of the economy. The unconditional quarterly trend consumption growth rate is 0.65% under the reference model and 0.64999% under the worst-case model. In contrast, the unconditional quarterly trend inflation rate is 0.96% percent under the reference model, while it is 1.0675% under the distorted model. This indicates that inflation ambiguity can account for the term premium in nominal bonds, while leaving the real side of the economy practically unaffected.

The paper is structured as follows. Chapter 2 presents the model, chapter 3 focuses on asset pricing implications, chapter 4 states the data and econometric methodology. Empirical results are summarized in chapter 5 and robustness and empirical fit are discussed in chapter 6. Chapter 7 compares the model with the related literature and chapter 8 concludes.

2. The Model

Time is continuous over $t \in [0, ..., \infty)$. The complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, Q^0)$ describes the benchmark (approximate, reference) model for the economy. I denote expectations under the benchmark model as $E[.]$ instead of E^{Q^0}. The distorted probability measure (worst-case measure) will be determined endogenously. I denote that measure as Q^h and indicate expectations with regard to that measure as $E^h[.]$. All mathematical conditions from Chen and Epstein (2002) are assumed to be fulfilled and all Brownian motions are pairwise orthogonal.

2.1. Benchmark Model for the Economy

I work with an endowment economy where the consumption process solves

$$d \ln c_t = (c_0 + z_t)dt + \sqrt{\sigma_{0c} + \sigma_{1c}u_t}dW_t^c, \tag{1}$$

where $c_0, \sigma_{0c}, \sigma_{1c}$ are positive scalars. The process z is the time-varying trend component of consumption growth and u captures heteroscedasticity in conditional consumption growth. Both processes follow pair-wise orthogonal continuous-time $AR(1)$ processes

$$dz_t = \kappa_z z_t dt + \sigma_{1z} dW_t^z + \sigma_{2z} dW_t^w \tag{2}$$

$$du_t = \kappa_u u_t dt + \sigma_u dW_t^u, \tag{3}$$

where $\kappa_u, \kappa_z, \sigma_{2z}$ are negative scalars and σ_u, σ_{1z} are positive scalars. Trend consumption growth, z, is subject to two pairwise orthogonal Brownian shocks, dW^z and dW^w. Shocks dW^z are unique to z, while shocks dW^w also affect trend inflation.

I assume an exogenous process for the aggregate price level p. Its predictable component is driven by trend inflation w

$$d\ln p_t = (p_0 + w_t)dt + \sqrt{\sigma_{0p} + \sigma_{1p} v_t} dW_t^p + \rho_{pc}\sqrt{\sigma_{0c} + \sigma_{1c} u_t} dW_t^c, \tag{4}$$

where dW^p and dW^c are two orthogonal Gaussian shocks. Note that $p_0, \sigma_{0p}, \sigma_{1p}$ are positive scalars, while ρ_{pc} can be positive or negative. Trend inflation and inflation volatility follow continuous-time $AR(1)$ processes

$$dw_t = \kappa_w w_t dt + \sigma_w dW_t^w \tag{5}$$

$$dv_t = \kappa_v v_t dt + \sigma_v dW_t^v, \tag{6}$$

where $\sigma_w > 0, \sigma_v > 0$ and $\kappa_w < 0, \kappa_v < 0$. A small but negative σ_{2z}, as observed in the data, captures a negative instantaneous correlation between trend consumption growth and trend inflation. An unpredictable increase in trend inflation lowers trend consumption growth below its expectation. This is supported in my data set and in Piazzesi and Schneider (2006).

Risk associated with the consumption stream is evaluated through a logarithmic utility function $u(c_t) = \ln(c_t)$, i.e.

$$U(c_t) = E_t \left[\int_t^\infty e^{-\rho s} \ln c_s ds \right], \tag{7}$$

where $\rho > 0$ is the subjective time discount factor.

2.2. Model Misspecification Doubts

The beauty of the rational expectations assumption is that the investor knows the unique model that generates the exogenous processes in the economy. Relaxing this assumption exposes the investor to model uncertainty. Under model uncertainty, the investor is confronted with a set of several models, which all could be the data generating process of the exogenous processes in the economy.

The investor copes with model uncertainty by comparing all potential models via likelihood ratio tests. Such a likelihood ratio has a very convenient analytical characterization in my model. Let a_T denote the likelihood ratio between the worst-case measure Q^h and the benchmark measure Q^0

$$a_T := \frac{dQ_T^h}{dQ_T^0} = \exp\left(-\frac{1}{2} \int_0^T h_t^2 dt + \int_0^T h_t dW_t^w \right), \quad a_0 \equiv 1, \tag{8}$$

where the process h characterizes the conditional expected value and conditional variance of the likelihood ratio.

Such a likelihood ratio quantifies the statistical distance between both models. I therefore call a_T the amount of relative entropy between the worst-case model Q^h and the reference

1 model Q^0. Finding endogenously the worst-case model form a set of potential models is

2 equivalent to finding the worst-case distortion h within a set of potential distortions. The

3 next subsection explains how the investor finds the worst-case distortion h.

4

The growth rate of relative entropy is a martingale under the empirical measure

$$\frac{da_t}{a_t} = h_t dW_t^w. \tag{9}$$

5 This implies that the investor does not expect to learn which model is correct. Changes

6 in aggregate uncertainty coincide with changes in relative entropy. These changes are due

7 to unpredictable innovations in trend inflation, dW^w. An unpredictable increase in trend

8 inflation coincides with an unpredictable increase in aggregate uncertainty. Said differently,

9 the investor perceives the real and nominal economy as more ambiguous in times when trend

10 inflation is higher than anticipated.

11

An ambiguity averse investor has min-max preferences. As an equilibrium outcome, he

will evaluate his expected life-time utility under a worst-case measure Q^h

$$E_0^h \left[\int_0^\infty e^{-\rho t} \ln c_t dt \right]. \tag{10}$$

12 The next subsection explains how the min operator determines endogenously Q^h.

13

The last equation can be rewritten under the benchmark (empirical) measure

$$U^h(c_0, a_0) := E_0 \left[\int_0^\infty a_t \cdot e^{-\rho t} \ln c_t dt \right], \tag{11}$$

14 where a_t can be interpreted as a Radon-Nikodym derivative between the worst-case and the

benchmark probability measure. The time-varying change in inflation ambiguity a_t can be regarded as an endogenously restricted multiplicative preference shock (Hansen and Sargent (2008) p. 54).

2.2.1. Restricting Ambiguity

The stochastic nature of the economy implies that the empirical log-likelihood ratio varies stochastically. The investor is averse against unpredictable variations in the empirical log-likelihood ratio. I assume that the expected worst-case change in the one period ahead log-likelihood ratio is smaller than a stochastic upper bound, $UB_t > 0$.

The investor pessimistically expects (worst-case) that the log-likelihood ratio changes over the next instant by less than $UB_t dt$. If the realized change is larger, the investor mistrusts his reference model even more. This means model uncertainty is more severe than expected. On the other hand, if the realized change is smaller than $UB_t dt$, the investor trusts his reference model more. The aggregate uncertainty would fall in the economy.

I assume UB_t has a time-varying component η^2 and a scaling factor A. I assume η follows a continuous-time $AR(1)$ process

$$UB_t := A\eta_t^2 \tag{12}$$

$$d\eta_t = (a_\eta + \kappa_\eta \eta_t)dt + \sigma_\eta dW_t^\eta, \tag{13}$$

where A, a_η, σ_η are positive scalars, while κ_η is a negative scalar.

Formally, the investor seeks for each period t a distortion h_t to the instantaneous trend inflation shock dW^w that minimizes his expected life-time utility and that induces an expected

growth in the log-likelihood ratio that is not larger than UB_t

$$\min_{\{h_t\}_t} E_0^h \left[\int_0^\infty e^{-\rho t} \ln c_t dt \right] \tag{14}$$

$$s.t. \quad \frac{1}{2}h_t^2 dt \leq A\eta_t^2 dt \tag{15}$$

$$s.t. \quad d\eta_t = (a_\eta + \kappa_\eta \eta_t)dt + \sigma_\eta dW_t^\eta \tag{16}$$

$$s.t. \quad d\ln c_t = (c_0 + z_t)dt + \sqrt{\sigma_{0c} + \sigma_{1c}u_t}dW_t^c \tag{17}$$

$$s.t. \quad dz_t = \kappa_z z_t dt + \sigma_{1z}dW_t^z + \sigma_{2z}(dW_t^{w,h} + h_t dt), \tag{18}$$

1 where $\frac{1}{2}h_t^2 dt$ coincides with the expected instantaneous change (under the worst-case model)

2 of the log-likelihood ratio between the worst-case and the approximate model, i.e. $E_t^h \left[d\ln \left(\frac{dQ_t^h}{dQ_t^0} \right) \right]$.[5]

3 The constraint in equation (**??**) is called growth rate constraint of relative entropy between

4 the potentially correct trend inflation models and its reference model. This constraint quan-

5 tifies the worst expected change in the log-likelihood ratio that the investor is prepared to

6 observe with the arrival of the next data point.

7

8   A negative conditional correlation between trend consumption growth and trend infla-

9 tion (i.e. $\sigma_{2z} < 0$) implies three things. First, the constraint on the entropy growth rate is

10 binding in each period. This means that the investor chooses in t an instantaneous distor-

11 tion to trend inflation shocks that are expected to produce an entropy growth rate of $A\eta_t^2$.

12 The investor implements this by choosing the optimal distortion h to be $h_t = \sqrt{2A}\eta_t$. For

13 convenience, I define $m_h \equiv \sqrt{2A}$. Second, the optimal distortion h is positive, implying that

14 the unconditional expected value of trend inflation is positive under the worst-case measure,

---

[5]

$$\ln \left(\frac{dQ_T^h}{dQ_T^0} \right) = -\frac{1}{2} \int_0^T h_t^2 dt + \int_0^T h_t dW_t^w$$
$$= \frac{1}{2} \int_0^T h_t^2 dt + \int_0^T h_t dW_t^{w,h}.$$

while it is zero under the approximate measure. Third, unconditional trend growth in consumption under the worst-case measure is negative, while it is zero under the approximate measure.

I stress three economic mechanisms that are crucial for understanding how a positive shock to trend inflation affects the inflation ambiguity premium. Assume $dW_t^w > 0$. First, the realized relative change of the log-likelihood ratio, $\frac{da_t}{a_t}$, is higher than anticipated. This warns the investor that the realized data in $t+1$ gives more statistical weight to the worst-case model, compared to the investor's reference model. As a result, the investor mistrusts more his reference model for trend inflation and trend consumption growth. In mathematical terms this means, $\frac{da_t}{a_t} - E_t\left[\frac{da_t}{a_t}\right] = h_t dW_t^w > 0$. Second, realized trend consumption growth is lower than anticipated, i.e. $dz_t - E_t[dz_t] = \sigma_{2z} dW_t^w < 0$. Third, nominal bonds do not hedge the increase in model mistrust. The real payoff of a nominal bond is lower than anticipated if $dW_t^w > 0$. To sum up, a nominal bond pays out less than expected (in real units), while the amount of model mistrust is higher than anticipated. An ambiguity averse investor requires an inflation ambiguity premium for holding nominal bonds.

3.   Asset Pricing

Define $M_{t,t+\Delta}$ to denote the intertemporal marginal rate of substitution (MRS). It is formally defined as

$$M_{t,t+\Delta} := e^{-\rho\Delta}\frac{U_c^h(c_{t+\Delta}, a_{t+\Delta})}{U_c^h(c_t, a_t)}, \tag{19}$$

where U_c^h denotes the partial derivative of U^h with regard to c. The MRS in the economy is affected by changes in consumption growth and changes in inflation ambiguity

$$M_{t,t+\Delta} = e^{-\rho\Delta} \left(\frac{c_{t+\Delta}}{c_t} \right)^{-1} \frac{a_{t+\Delta}}{a_t}. \tag{20}$$

1

2

The second term in the previous equation represents the log-utility consumption risk kernel. The third term denotes the inflation ambiguity kernel. The MRS depends on (u_t, z_t, h_t), and it is driven by risk innovations dW_t^c (consumption risk) and ambiguity innovations dW^w (inflation ambiguity). This makes dW^c the priced risk factor and dW^w the priced uncertainty factor.

$$M_{t,t+\Delta} = e^{-(\rho+c_0)\Delta - \int_t^{t+\Delta} z_m dm - \int_t^{t+\Delta} \sqrt{\sigma_{0c}+\sigma_{1c}u_m}dW_m^c} \times e^{-\frac{1}{2}\int_t^{t+\Delta} h_m^2 dm + \int_t^{t+\Delta} h_m dW_m^w}. \tag{21}$$

Note, that I specify (z, u) exogenously, while h is determined endogenously as the solution to (??) to (14). The real interest rate r is defined as $r_t dt := E_t \left[-\frac{dM_{0,t}}{M_{0,t}} \right]$ and coincides with

$$r_t = \rho + c_0 + z_t - 0.5(\sigma_{0c} + \sigma_{1c}u_t). \tag{22}$$

3

4

The nominal pricing kernel depends on the MRS and the aggregate price level. It is formally defined as $M_{t,t+\Delta}^\$:= M_{t,t+\Delta}\frac{p_t}{p_{t+\Delta}}$. The model implies that the nominal SDF depends multiplicatively on the consumption risk kernel, inflation and the inflation ambiguity kernel

$$M_{t,t+\Delta}^\$ = e^{-\rho\Delta} \left(\frac{c_{t+\Delta}}{c_t} \right)^{-1} \left(\frac{p_{t+\Delta}}{p_t} \right)^{-1} \frac{a_{t+\Delta}}{a_t}. \tag{23}$$

The nominal interest rate R is defined as $R_t dt := E_t \left[-\frac{dM_{0,t}^{\$}}{M_{0,t}^{\$}} \right]$ and coincides with

$$R_t = r_t + (p_0 + w_t) - \frac{1}{2} \left(\rho_{pc}^2 (\sigma_{0c} + \sigma_{1c} u_t) + \sigma_{0p} + \sigma_{1p} v_t \right) - \rho_{pc} (\sigma_{0c} + \sigma_{1c} u_t). \qquad (24)$$

The first inner parenthesis on the right hand side coincides with expected inflation, the second denotes precautionary savings and the last one presents the inflation risk premium. Inflation ambiguity does not increase precautionary savings.[6] It also does not affect the spot nominal interest rate. Both results apply because neither consumption nor inflation shocks are ambiguous. Ambiguity about trend inflation enters the term structure of nominal bonds through the investor's worst-case expectation of what inflation will be in the future. This is consistent with the notion that long-term bond yields are risk and ambiguity adjusted averages of expected future short rates.

Marginal utility has several attractive properties. First, trend inflation is a priced uncertainty factor. The investor therefore, requires an inflation ambiguity premium for unpredictable changes in trend inflation (relative entropy). Second, Cochrane and Hansen (1992) point out that the low correlation between the real return of assets and consumption growth is an important reason for many empirical asset pricing puzzles. Accounting for inflation ambiguity breaks the tight link between consumption growth and marginal utility, which would otherwise exist in standard CRRA models. Third, the real return on nominal assets varies if consumption changes, inflation changes, or if aggregate inflation uncertainty changes.

---

[6] Maenhout (2004), Liu et al. (2005), Leippold et al. (2008), Kleshecheslski and Vincent (2009), Gagliardini et al. (2009) have a model uncertainty set-up where risk and uncertainty increase precautionary savings.

3.1. Term Premium

Let $N_t(\tau)$ denote the price of a τ-maturity nominal bond. It is formally defined as $N_t(\tau) := E_t\left[M_{t,t+\tau}^{\$}\$1\right]$. The bond price contains an inflation premium which accounts for inflation risk and for inflation ambiguity, i.e.

$$N_t(\tau) = E_t\left[M_{t,t+\tau}\right] E_t\left[\frac{p_t}{p_{t+\tau}}\$1\right] + cov_t\left(M_{t,t+\tau}, \frac{p_t}{p_{t+\tau}}\$1\right). \tag{25}$$

The inflation risk premium is the traditional explanation for the term premium (Piazzesi and Schneider (2006), Wachter (2006), Buraschi and Jiltsov (2005)). It is positive if inflation is bad news for future consumption.[7] It is known that a frictionless log-utility economy cannot reconcile the upward sloping term premium. Introducing inflation ambiguity provides an additional economic reason for a positive term premium

$$cov_t\left(M_{t,t+\tau}, \frac{p_t}{p_{t+\tau}}\$1\right) = e^{-\rho\tau} cov_t\left(\frac{c_t}{c_{t+\tau}}\frac{a_{t+\tau}}{a_t}, \frac{p_t}{p_{t+\tau}}\$1\right). \tag{26}$$

The inflation ambiguity premium is required in equilibrium because changes in aggregate ambiguity correlate with changes in the real value of a nominal bond. This means that an unpredictable positive shock to trend inflation reduces the real payout of a nominal bond at a time when the investor becomes more pessimistic about the quality of his reference inflation and consumption model. The inflation ambiguity premium is orthogonal to the inflation risk premium. The inflation ambiguity premium can be large, even if the inflation risk premium is zero.

---

[7] There are mixed empirical results on the empirical magnitude of the inflation risk premium. Hördahl and Tristani (2010) find evidence for a positive but low inflation risk premium. An inflation ambiguity premium could still explain why the nominal term premium is significantly positive and upward sloping.

The inflation premium in bond prices from equation (26) simplifies to

$$e^{-\rho\tau} cov_t \left(\bar{A}(\tau) e^{\int_t^{t+\tau} h_m dW_m^w}, \$1 \bar{B}(\tau) e^{-\int_t^{t+\tau} \left(\kappa_w \int_t^m w_k dk + \sigma_w \int_t^m dW_k^w \right) dm} \right) \tag{27}$$

$$\bar{A}(\tau) \equiv e^{-c_0\tau - \int_t^{t+\tau} z_s ds + \int_t^{t+\tau} \sqrt{\sigma_{0c} + \sigma_{1c} u(s)} dW_s^c - \frac{1}{2} \int_t^{t+\tau} h_s^2 ds} \tag{28}$$

$$\bar{B}(\tau) \equiv \exp\left(-(p_0 + w_t) \right) \tau. \tag{29}$$

1 The term $\int_t^{t+\tau} h_m dW_m^w$ stands for unpredictable changes in inflation ambiguity and $-\sigma_w \int_t^m dW_k^w$

2 stands for unpredictable changes of the real value of a nominal bond. Shocks to trend in-

3 flation drive both components. Their correlation is negative. As a result, nominal bond

4 prices are lower compared to a world without inflation ambiguity. Said differently, inflation

5 ambiguity increases the yields of nominal bonds.

6

7   The endogenous distortion to trend inflation shocks, $h_t = m_h \eta_t$, governs the time-series

8 behavior of the inflation ambiguity premium. The slope of the term premium depends on the

9 persistence of trend inflation, κ_w. Trend inflation is persistent in the data. Small instanta-

10 neous distortions, $\sigma_w \cdot m_h \cdot \eta_t$, to trend inflation affect also long-term inflation forecasts. An

11 increase in persistence leads to a steepening of the inflation ambiguity premium. In contrast,

12 the inflation risk premium depends on the persistence of trend consumption growth and the

13 magnitude of the correlation between inflation and consumption.

14

15 *3.2.   Term Structure of Real and Nominal Bonds*

The exponentially affine MRS implies an affine term structure for real yields

$$y_t^r(\tau) = -\frac{1}{\tau} \left(A^r(\tau) + B^{r'}(\tau) S_t \right), \quad S_t \equiv (u_t \ z_t \ h_t)', \tag{30}$$

1 where $A^r(\tau)$ and $B^r(\tau)$ are deterministic functions of the structural parameters. The ana-

2 lytical solution follows directly from Duffie et al. (2000).

3

4 Real yields depend on trend consumption growth z, consumption volatility u and the

5 worst-case distortion to trend inflation shocks h. Ambiguity about trend inflation has hardly

6 any impact on the term premium of real bonds. This can be seen from $cov_t\left(\frac{c_t}{c_{t+\tau}}, \frac{a_{t+\tau}}{a_t}\right)$. The

7 conditional covariance depends on σ_{2z} and the persistence of trend consumption growth.

8 Although σ_{2z} is negative in the data, its magnitude is small. I later show that my empirical

9 estimates suggest that trend consumption growth under the distorted measure is only mar-

10 ginally smaller than trend consumption growth under the benchmark measure.

11

The nominal pricing kernel $M^{\$}$ is also exponentially affine. Nominal bond yields are

affine in the state variables

$$y_t^{\$}(\tau) = -\frac{1}{\tau}\left(A^{\$}(\tau) + B^{\$\prime}(\tau)X_t\right), \quad X_t \equiv (u_t\ v_t\ w_t\ z_t\ h_t)', \tag{31}$$

12 where $A^{\$}(\tau)$ and $B^{\$}(\tau)$ are deterministic functions of model parameters. The analytical

13 solution follows directly from Duffie et al. (2000).

14

15 The model is expected to explain the nominal term premium because changes in inflation

16 ambiguity, da_t, covary negatively with the real return on nominal bonds.

17 **4. Data and Econometric Methodology**

18 I estimate the equilibrium term structure model with a panel of macro and bond yield

19 data. The estimation methodology is set up such that it ties the hand of the econometrician.

20 First, the ambiguity process is restricted to follow an observable continuous-time $AR(1)$

process. Second, I do not allow for other inflation shocks, besides shocks to trend inflation, to be priced. Third, I restrict the macro parameters of the observable processes to not be tweaked by the large bond yield panel. The last point is explained below in more detail.

The estimation frequency is quarterly and the estimation length is first quarter 1972 to second quarter of 2009. The macro panel consists of the Federal funds rate, realized GDP growth, GDP implicit price deflator, one quarter ahead forecast on GDP growth and one quarter ahead forecast on inflation.[8]

Real GDP growth, the GDP implicit price deflator and the Federal funds rate are from the St. Louis Fed database (FRED). The quarterly forecasts on GDP growth and inflation coincide with the corresponding median forecast from the Survey of Professional Forecasters (SPF). For each quarter, I determine the dispersion in inflation forecasts, η^2, as the cross-sectional variance among SPF's inflation forecasts.

The data panel also contains for each time point ten continuously compounded U.S. government bond yields of maturities one year to ten years. This data is from the first quarter 1972 to the second quarter 2009. I also use continuously compounded yields from U.S. Treasury Inflation Protected Securities (TIPS) with maturities of five years to ten years. This data is from the first quarter 2003 to the second quarter 2009. All bond data is from the Board of Governors of the Federal Reserve System.

I discretize the Gaussian state processes, and realized inflation and GDP growth by an Euler-Marujama scheme. The quarterly transition density of theses discretized processes is

---

[8]The SPF publishes forecasts for GDP growth but not for consumption growth. I therefore work with GDP data.

Gaussian. The resulting estimation is a Quasi Maximum Likelihood (QML) method.

I match w with the demeaned median one quarter ahead inflation forecast. Ang et al. (2007) find that the median SPF inflation forecast constitutes the best out-of-sample predictor for inflation. Accordingly, I match z with the demeaned GDP growth forecast. I fix c_0 to coincide with the sample mean of GDP growth and p_0 to coincide with the sample mean of inflation.

For each quarter, I identify the upper boundary on the expected entropy growth rate, $A\eta_t^2$, by the dispersion of inflation forecasts. The dispersion corresponds to the cross-sectional variance among one quarter ahead inflation forecasts scaled by a positive constant A. The process η^2 is fixed throughout the estimation, while A is estimated as an ambiguity preference parameter during the QML estimation. An observed increase in η_t^2 maps directly into an increase in inflation ambiguity. My modeling choice is consistent with Anderson et al. (2009) and Patton and Timmermann (2010). The latter use a term structure of GDP growth and inflation forecasts to show that dispersion in forecasts do probably not arise from heterogeneity in information, but rather from heterogeneity in models.

A preliminary look at the data reveals that η^2 explains 12% of variations in the first principal component of nominal yields and 8% of variations in the slope of the nominal yield curve. This provides first evidence that changes in the cross-sectional variation of inflation forecasts explains variations in the term structure of yields. An additional first look at the data reveals that trend inflation explains 60% of variations in the first principal component of nominal yields. Trend inflation is a persistent process. Ambiguity about a persistent process of such high importance for the yield curve will probably have a significant impact on the yield curve itself.

Before I run a QML estimation, I first perform ols regressions on the observed macro-economic processes. This includes the time-series of (w, z, η). These estimations provide parameter estimates on $(\kappa_w, \kappa_z, \kappa_\eta, a_\eta)$ and $(\sigma_w, \sigma_{1z}, \sigma_{2z}, \sigma_\eta)$ and their empirical confidence intervals. For the QML estimation that follows, I fix a_η to coincide with its estimated sample counterpart and constrain the other parameters to lie within the 99% empirical confidence interval. This procedure ties the hands of the econometrician because it does not allow the large bond panel to tweak these macro parameters.

The heart of the estimation is a QML estimation. There are 18 parameters to be estimated and 22 identifying likelihood restrictions that come from the five state equations, a panel of eight nominal yields, six real yields, Federal funds rate, realized inflation and realized GDP growth. For a given parameter vector θ, I invert the affine nominal yield relationship for the one year and ten year yield to obtain u_t and v_t. This is consistent with a huge body of empirical term structure models (Ang and Piazzesi (2003), Chen and Scott (1993)).

5.   Empirical Results

The equilibrium term structure model is estimated for the entire period of 1972 to 2009. I use sample averages of the state variables to infer implied average yield curves and implied average term premiums for particular subsamples.

5.1.   Inflation Ambiguity explains the Term Premium

The model provides the first empirical evidence in the literature that model uncertainty together with log-utility can account for the nominal term premium. Figure (1) and Table (3) present the estimated term structure of the inflation ambiguity premium. The aver-

age ambiguity premium is strongly upward sloping. The average premium for ambiguity on trend inflation has been 0.3% for the two year yield and 1.75% for the nine-year yield. This premium has been higher during the monetary policy experimentation (1979-1983) and lower during the Great Moderation (1984-2007). The model captures this result through a stochastic aversion against inflation ambiguity.

The stochastic upper bound of the expected change of relative entropy has been higher during the period of the monetary policy experimentation. This leads to higher equilibrium distortions, $\sigma_w \cdot m_h \cdot \eta_t$, to trend inflation. With all else equal, this leads to a more negative conditional covariance between the uncertainty kernel and the real return on nominal bonds. Trend inflation is persistent (Table (1)). This explains why pessimistic distortions to trend inflation accumulate to sizeable inflation ambiguity premiums for long-term nominal bonds. This equilibrium channel for the term premium distinguishes the model from stochastic risk aversion, like habit formation (Buraschi and Jiltsov (2007)) and Wachter (2006), or models with recursive preferences and learning about a single inflation prior (Piazzesi and Schneider (2006)).

The inflation premium has two components. An upward sloping inflation ambiguity premium and a flat inflation risk premium. Table (3) shows that the inflation ambiguity premium accounts for the entire upward sloping term premium. Other components of the nominal yield curve are trend inflation and the real yield curve. One year inflation expectations are 3.8% and 3.4% for a nine year forecast horizon under the reference model (empirical measure). The real yield curve is flat at 2.2%.

Accounting for inflation ambiguity provides an explanation for high long-term nominal bond yields and strongly downward sloping inflation expectations during the early 1980s.

The lower panel of Table (3) shows that during the monetary policy experimentation, inflation expectations (empirical measure) were on average 10.8% for a two year forecast horizon and 8% for a nine year forecast horizon. The nominal yield curve has been only slightly inverted during this period. My model attributes this to a strongly upward sloping inflation ambiguity premium. The set of potential inflation models, or equivalently, the amount of model mistrust, has been high during this period. An ambiguity averse investor distorts his subjective forecast for trend inflation and trend consumption growth more pessimistically. The high persistence of trend inflation makes instantaneous positive distortions to trend inflation die out slowly over time. The result is an upward sloping inflation ambiguity premium in nominal bond yields.

Piazzesi and Schneider (2006) use recursive preferences and parameter uncertainty to amplify the inflation risk premium to account for that period. My methodology, on the other hand, provides a framework on how fear of inflation misspecification gives rise to a steep inflation ambiguity premium. The inflation ambiguity premium is present in equilibrium because the payoff profile of long-term nominal bonds makes these assets the worst possible hedge instrument against inflation ambiguity. Periods of increased model mistrust are periods in which nominal bonds pay out less in real terms.

The time-variation of the inflation ambiguity premium stems from the time-varying dispersion of inflation forecasts. This dispersion has been volatile during the 1970s, which induces a volatile inflation ambiguity premium. Figure (2) confirms the intuition that the set of potential inflation models has been strongly time varying during the monetary policy experimentation. The model mistrust for trend inflation and subsequently the inflation ambiguity premium have increased at the beginning of the last four NBER recessions. Both fall sharply at the middle of recessions. In the mid 1970s and the beginning of the 1981 and

1991 recession, the one year inflation ambiguity premium peaked at 0.45%. The ten year ambiguity premium peaked at the same time at 2.6%.

The negative correlation between trend inflation and trend consumption growth is supported in the data and important for the upward sloping inflation premium. Piazzesi and Schneider (2006) point this out for the inflation risk premium. Figure (4) makes the same point for the inflation ambiguity premium. Keeping all estimated parameters and state variables fixed and changing only the sign of σ_{zz} shows that the yield curve would slope downwards because of a downward sloping inflation ambiguity premium. Although this would not happen in a full estimation, it provides intuition for the importance of the negative correlation.[9]

6.   Robustness of Empirical Results

My empirical results show that model misspecification doubts about trend inflation can account for the steep slope in nominal U.S. Government bond yields. Analogously to risk aversion and risk premiums it is crucial to know how much uncertainty is necessary to generate the term premium. Anderson et al. (2003) suggest the usage of detection error probabilities in order to quantify the amount of model uncertainty.

6.1.   Detection Error Probabilities

Detection error probabilities examine whether after seeing the entire sample of the state vector, an econometrician can tell whether the state variables follow the distorted (worst-case) or the reference (empirical benchmark) dynamic. More formally, an econometrician

---

[9]Unreported estimation results show that if an econometrician estimates the model with the counterfactual restriction $\sigma_{2z} > 0$, the inflation ambiguity premium would be close to zero, because a downward sloping premium produces large pricing errors for bond yields, without adding anything positive to the overall model fit. The numerical optimizer would prefer to rather work without the ambiguity state variable than to generate big pricing errors.

looks at the ratio between the likelihood that the state vector has been generated by the worst-case model compared to the likelihood that it was generated by the reference model.

My explanation of detection error probabilities is related to page 215 in Hansen and Sargent (2008). I denote L_A to be the likelihood that the observed sample of trend inflation has been generated by the worst-case model. In contrast, L_B stands for the likelihood that the observed realization of trend inflation was sampled from the approximate model. The log-likelihood ratio is defined as $\ln \frac{L_A}{L_B}$. A likelihood ratio test selects model A when $\ln \frac{L_A}{L_B} > 0$ and vice versa.

When model A (worst-case) generated the state vector, the likelihood of a model detection error is $p_A = Prob\left(\ln \frac{L_A}{L_B} < 0 | A\right)$. When model B (approximate) generated the data, the probability of a model detection error is $p_B = Prob\left(\ln \frac{L_A}{L_B} > 0 | B\right)$. The magnitude of both probabilities depends on the estimated $m_h = \sqrt{2A}$. The investor puts the same prior weight on both detection errors and determines the probability of detection error by

$$p(m_h) = \frac{1}{2}(p_A + p_B). \tag{32}$$

Hansen and Sargent (2008) suggest to set $p(m_h)$ to a number bigger than 0.1 or 0.2 and then to invert $p(m_h)$ to find a plausible m_h. I deviate from that suggestion by directly estimating m_h in the QML with the panel of bond and macro data. Having estimated m_h, I plug the estimate into $p(m_h)$ and find that the corresponding probability of detection error is 47.7%. This says that the econometrician faces a 47.7% chance of choosing the wrong model for trend inflation and trend consumption growth. Said differently, there is a huge amount of ambiguity about whether trend inflation and trend consumption growth follow the worst-case or the reference model.

My model implies a stochastic log-likelihood ratio between the worst-case and the approximate model

$$\ln\left(\frac{L_A}{L_B}\right) = \ln\left(\frac{dQ_T^h}{dQ_T^0}\right) = -\frac{1}{2}m_h^2\int_0^T \eta_t^2 dt + m_h\int_0^T \eta_t dW_t^w, \tag{33}$$

where I plugged in the equilibrium outcome $h_t = m_h\eta_t$.

The previous equation reveals that the magnitude of the detection error probability depends on the scaling parameter $m_h = \sqrt{2A}$, the observed process η^2, and the realized Brownian shocks to trend inflation dW^w. The latter shock is the reason for realized uncertainty (realized relative entropy) to deviate from the expected uncertainty (expected relative entropy). The derivation of the detection error probability for the model follows Anderson et al. (2003) and Maenhout (2006).

There are two reasons for the estimated detection error probability to be so high. First, the observed time-series of trend inflation under the worst-case and under the approximate model are close to each other. Figure (3) visualizes this. This says that on a quarterly horizon, realized worst-case inflation expectations are only marginally higher than inflation expectations under the reference model. Second, trend inflation is persistent. The estimated parameter that controls the speed of mean reversion, κ_w, is -0.07 (Table (1)). Besides inflation volatility, trend inflation is the most persistent macro variable in the sample. The result is that although expected future instantaneous distortions are small (high detection probability error), since past distortions die out slowly, the accumulated distortions add up to a sizeable ambiguity spread.

6.2.   Estimation Fit

The average pricing error across all nine fitted nominal yields is five basis points per quarter. Table (2) shows that the average pricing error on real bonds is fourteen basis points per quarter. I proxy real bond yields with yields on Treasury Inflation Protected Securities (TIPS). Liquid data on TIPS is only available since the early 2000. The mean fitting error per quarter is 4.4 basis points for consumption growth and 0.7 basis points for inflation. The empirical identification strategy implies a perfect fit to trend inflation and trend consumption growth.

The model estimates imply an unconditional quarterly trend inflation rate of 0.96% under the reference model and 1.0675% under the worst-case model. The analog for the unconditional quarterly trend GDP growth rate is 0.65% for the approximate model and 0.64999% for the worst-case model. These statistics show that ambiguity about trend inflation has a negative but tiny effect on the real side of the economy. The effect on the nominal economy is sizeable. In steady state, the inflation forecast under the worst-case model is 0.1075% higher per quarter.[10]

7.   Literature

My paper is closest related to a set of equilibrium models that try to explain why long-term nominal bond yields are higher than the short-term counterpart. Current research focuses nearly exclusively on the inflation risk premium.[11] Piazzesi and Schneider (2006) and Buraschi and Jiltsov (2005) share the insight that a meaningful inflation risk premium

---

[10]The unconditional expected value for quarterly inflation under the worst-case measure is $p_0 + \lim_{t \to \infty} E^h[w(t)]$ and analogously $c_0 + \lim_{t \to \infty} E^h[z(t)]$ for expected consumption growth.

[11]Piazzesi and Schneider (2006) use a recursive utility investor with a unique inflation prior who faces parameter uncertainty. Their set-up amplifies the inflation risk premium to realistic values. Buraschi and Jiltsov (2005) show that a log utility framework with taxes can account for a meaningful inflation risk premium.

requires inflation to be a predictive carrier of future bad consumption news. Buraschi and Jiltsov (2007), Wachter (2006) and Rudebusch and Swanson (2008) show that habit formation can be useful in amplifying the inflation risk premium. In contrast to this work, my model shows that a log-utility investor can account for the nominal term premium in U.S. Government bonds, once one controls for the empirically observed model misspecification doubts on inflation. The key driver of the term premium is an inflation ambiguity premium instead of an inflation risk premium. This has the advantage that dispersion in professional inflation forecasts provide a useful proxy for the amount of inflation ambiguity that an investor faces. My model maps the observed dispersion into an ambiguity premium. Moreover, recent empirical evidence suggests that the inflation risk premium might be rather small (Hördahl and Tristani (2010)).

It is surprising that ambiguity about the inflation model has been overlooked in the literature. Previous research has collected mounting evidence on the importance of trend inflation for the modeling of bond yields. Furthermore, the SPF reveals that macroeconometric experts differ on their preferred model for trend inflation. Ang and Piazzesi (2003), Ang et al. (2006), Duffee (2007) and Joslin et al. (2009) show the importance of inflation and business cycle factors for the nominal yield curve. These papers assume that the investor observes a unique prior. This is a simplifying assumption. In the data, as the SPF illustrates, investors are confronted with a set of potentially correct inflation models. Ang et al. (2007) find that the median forecast for inflation, published by the SPF, is the best out-of-sample inflation forecast. This cross-sectional dispersion in these SPF forecasts is time-varying over the business cycle. Patton and Timmermann (2010) analyze the term structure of dispersion in forecasts of inflation and GDP growth and provide evidence that dispersion is generated by model heterogeneity.

There is no theoretical model in the literature that provides a pricing framework for an investor who does not observe the underlying data generating process for trend inflation, but who instead observes a set of multiple inflation models. My model closes this gap in the literature. I propose an equilibrium term structure model for real and nominal bonds that takes into account that investors face model uncertainty about trend inflation. One can regard uncertainty about trend inflation as a result of an imperfectly understood conduct of monetary policy. Evidence for the latter is provided by Ang et al. (2009), Clarida et al. (2000), Ang et al. (2008b), Cogley and Sargent (2002), and Goodfriend and King (2005).

Early contributions on multiple prior general equilibrium models are Gilboa and Schmeidler (1989), Epstein and Wang (1994), Epstein and Schneider (2003), Chen and Epstein (2002), Epstein and Miao (2003). All of these papers focus on a set of multiple priors on consumption risk.[12] Inflation expectations and a multiple prior for trend inflation has not been analyzed so far. My paper focuses explicitly on the modeling of inflation ambiguity and its impact on the nominal term premium. This also extends current research on model uncertainty, which traditionally focuses on equity markets.[13]

8. Conclusion

This paper finds that a small concern for inflation ambiguity explains the upward sloping term premium in nominal U.S. Government bond yields. Nominal bonds are not only a bad hedge against consumption risk, they are also a bad hedge against inflation ambiguity. Periods in which the investor looses trust in his reference inflation and consumption model

---

[12] An alternative tool for handling model uncertainty is Anderson et al. (2003), Cagetti et al. (2002), Hansen and Sargent (2008), Hansen and Sargent (2005), Hansen et al. (2005), Maenhout (2004), and Maenhout (2006).

[13] Some recent research is Garlappi et al. (2007), Uppal and Wang (2003), Liu et al. (2005), Drechsler (2009), Sbuelz and Trojani (2002), Maenhout (2004), Leippold et al. (2008), Dow and Werlang (1992), Trojani and Vanini (2004), Cao et al. (2005), Miao and Wang (2009), and Chen et al. (2009). Klesecheslski and Vincent (2009) and Gagliardini et al. (2009) study the impact of dividend uncertainty on real bonds.

coincide with periods in which nominal bonds pay out less in real units.

It is an equilibrium result, that the investor requires a positive inflation ambiguity premium for holding nominal bonds. This premium is time varying and high during periods of high uncertainty. It has increased during the monetary policy experimentation and subsequently fallen during the Great Moderation. The estimated detection error probability of 47.7% shows that the likelihood ratio between the worst-case and the reference inflation model is so high that an econometrician cannot judge which model has generated the observed time-series of inflation and consumption.

The high persistence of trend inflation makes small instantaneous differences in the inflation models accumulate to a sizeable difference for long horizon forecasts. The impact on the real economy is negative and small in terms of economic magnitude. The analysis concludes that accounting for inflation ambiguity explains why long-term nominal bond yields are higher than their short-term counterpart.

References

Anderson, E., Ghysels, E., Juergens, J., 2009. The impact of risk and uncertainty on expected returns. Journal of Financial Economics 94, 233–263.

Anderson, E., Hansen, L., Sargent, T., 2003. A quartet of semigroups for model specification, robustness, prices of risk, and model detection. Journal of the European Economic Association 1, 68–123.

Ang, A., Bekaert, G., Wei, M., 2007. Do macro variables, asset markets or surveys forecast inflation better? Journal of Monetary Economics 54, 1163–1212.

Ang, A., Bekaert, G., Wei, M., 2008a. The term structure of real rates and expected inflation. Journal of Finance forthcoming.

Ang, A., Boivin, J., Dong, S., Loo-Kung, R., 2009. Monetary policy shifts and the term structure. Working Paper.

Ang, A., Dong, S., Piazzesi, M., 2008b. No-arbitrage Taylor rules. Working Paper.

Ang, A., Piazzesi, M., 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics 50, 745–787.

Ang, A., Piazzesi, M., Wei, M., 2006. What does the yield curve tell us about GDP growth? Journal of Econometrics 131, 359–403.

Buraschi, A., Jiltsov, A., 2005. Inflation risk premia and the expectation hypothesis. Journal of Financial Economics 75, 429–490.

Buraschi, A., Jiltsov, A., 2007. Term structure of interest rates implications of habit persistence. Journal of Finance 62(6).

Cagetti, M., Hansen, L., Sargent, T., Williams, N., 2002. Robustness and pricing with uncertain growth. Review of Finanical Studies 15, 363–404.

Cao, H., Wang, T., Zhang, H., 2005. Model uncertainty, limited market participation and asset prices. Review of Financial Studies 18, 1219–1251.

Chen, H., Ju, N., Miao, J., 2009. Dynamic asset allocation with ambiguous return predictability. Working Paper.

Chen, R., Scott, L., 1993. Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates. Journal of Fixed Income 3, 14–31.

Chen, Z., Epstein, L., 2002. Ambiguity, risk and asset returns in continuous time. Econometrica 70, 1403–1443.

Clarida, R., Gali, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: Evidence and some theory. Quarterly Journal of Economics 115, 147–180.

Cochrane, J., Hansen, L., 1992. Asset pricing explorations for macroeconomics. NBER Macroeconomics Annual, 115–165.

Cogley, T., Sargent, T., 2002. Drifts and volatilities: Monetary policies and outcomes in the post WWII U.S. Working Paper.

Dow, J., Werlang, R., 1992. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. Econometrica 60, 197–204.

Drechsler, I., 2009. Uncertainty, time-varying fear, and asset prices. Working Paper.

Duffee, G., June 2007. Are variations in term premia related to the macroeconomy? Working Paper.

Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. Econometrica 68, 1343–1376.

Epstein, L., Miao, J., 2003. A two-person dynamic equilibrium under ambiguity. Journal of Economic Dynamics and Control 27, 1253–1288.

Epstein, L., Schneider, M., 2003. Recursive multiple-priors. Journal of Economic Theory 113, 1–31.

Epstein, L., Wang, T., 1994. Intertemporal asset pricing under Knightian uncertainty. Econometrica 62, 283–322.

Gagliardini, P., Porchia, P., Trojani, F., 2009. Ambiguity aversion and the term structure of interest rates. Review of Financial Studies 22, 4157–4188.

Garlappi, L., Uppal, R., Wang, T., 2007. Portfolio selection with parameter and model uncertainty: A muli-prior approach. The Review of Financial Studies 20, 41–81.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. Journal of Mathematical Economics 18, 141–153.

Goodfriend, M., King, R., 2005. The incredible Volcker disinflation. Journal of Monetary Economics 52(5), 981–1015.

Gürkaynak, R., Sack, B., Swanson, E., 2005. The excess sensitivity of long-term interest rates: Evidence and implications for macroeconomic models. American Economic Review 95, 425–436.

Hansen, L., Sargent, T., 2005. Robust estimation and control under commitment. Journal of Economic Theory 124, 258–301.

Hansen, L., Sargent, T., 2008. Robustness. Princeton University Press.

Hansen, L., Sargent, T., Turmuhambetova, G., Williams, N., September 2005. Robust control and misspecification. Working Paper.

Hördahl, P., Tristani, O., 2010. Inflation risk premia in the US and the Euro area. Working Paper.

Joslin, S., Priebsch, M., Singleton, K., 2009. Risk premium accounting in macro-dynamics term structure models. Working Paper.

Kleschecheslski, I., Vincent, N., 2009. Robust equilibrium yield curves. Working Paper.

Knight, F., 1921. Risk, uncertainty, and profit. Boston: Houghton Mifflin.

Leippold, M., Trojani, F., Vanini, P., 2008. Learning and asset pricing under uncertainty. Review of Financial Studies 21(6), 2565–2597.

Liu, J., Pan, J., Wang, T., 2005. An equilibrium model of rare-event premia and its implication for option smirks. Review of Financial Studies 18, 131Ű164.

Maenhout, P., 2004. Robust portfolio rules and asset pricing. Review of Finanical Studies 17, 951–983.

Maenhout, P., 2006. Robust portfolio rules and detection-error probabilities for a mean-reverting risk premium. Journal of Economic Theory 128, Nř1, 136–163.

Miao, J., Wang, N., 2009. Risk, uncertainty, and option exercise. Working Paper.

Patton, A., Timmermann, A., 2010. Why do forecasters disagree? Lessons from the term structure of cross-sectional dispersion. Journal of Monetary Economics forthcoming.

Piazzesi, M., Schneider, M., 2006. Equilibrium yield curves. 2006 NBER Macroeconomics Annual forthcoming.

Rudebusch, G., Swanson, E., 2008. Examining the bond premium puzzle with a DSGE model. Journal of Monetary Theory 55, 111–126.

Sbuelz, A., Trojani, F., 2002. Equilibrium asset pricing with time-varying pessimism. Working Paper.

Trojani, F., Vanini, P., 2004. Robustness and ambiguity aversion in general equilibrium. Review of Finance 2, 279–324.

Uppal, R., Wang, T., 2003. Model misspecification and underdiversification. Journal of Finance 58.6, 2465–2486.

Wachter, J., 2006. A consumption-based model of the term structure of interest rates. Journal of Financial Economics 79, 365–399.

Table 1: PARAMETER ESTIMATES (Standard Errors)

Panel A: State Variables

	Drift, κ	Volatility, σ
u_t	-0.127 (0.004)	0.082 (0.006)
v_t	-0.012 (0.0005)	0.083 (0.01)
w_t	-0.070 (0.001)	0.003 0.00006)
z_t	-0.269 (0.00007)	0.004 (0.0004) -7.14$e-7$ (< 0.00001)
η_t	-0.300 (<0.00001)	0.006 (0.0006)

Panel B: Growth and Inflation

c_0	0.0065 (fixed)
p_0	0.0096 (fixed)
a_η	0.0013 (fixed)
ρ	0.001 (fixed)
σ_{0c}	1.059e-8 (0.00004)
σ_{1c}	0.023 (0.0005)
σ_{0p}	1.21e-8 (0.00008)
σ_{1p}	0.036 (0.0002)
ρ_{pc}	0.994 (0.02)
m_h	5.85 (0.1)

Note: The table presents parameter estimates and their standard error (in parenthesis). The asymptotic standard errors are determined based on the score of the log likelihood. The second column that corresponds to row z_t represents the estimate for σ_{1z}, the estimate for σ_{2z} is given in the third column.

Table 2: Yield Curve, in %, per quarter

Nominal Yields

Maturity	Model	Data
R	1.5383	1.6
4	1.5822	1.5822
8	1.5679	1.6391
12	1.5962	1.6819
16	1.6247	1.7159
20	1.6643	1.7453
24	1.7081	1.7717
28	1.7583	1.795
32	1.8099	1.8154
36	1.8463	1.8327
40	1.8484	1.8484

Real Yields

Maturity	Model	Data
20	0.5999	0.3782
24	0.5931	0.4109
28	0.5879	0.4400
32	0.5838	0.4650
36	0.5805	0.4859
40	0.5778	0.5014

Note: The table compares data and model implied continuously compounded government bond yields. Numbers are in % and per quarter. The upper panel is devoted to nominal bond yields and the Federal funds rate R. The estimation contains nominal bond yields from 1972.I to 2009.II. The lower panel focuses on Treasury Inflation Protected bond yields (TIPS). The estimation contains real bond yields from 2003.I to 2009.II.

Table 3: Components of Nominal Yield Curve, in %, annualized

1972-2009					
Maturity	r	$E[\pi]$	IAP	$1.0e+3 \cdot IRP$	$y^{\$}$
8	2.2080	3.7844	0.2793	0.0293	6.2716
12	2.2083	3.7313	0.4453	0.0613	6.3850
16	2.2087	3.6767	0.6243	0.0747	6.5098
20	2.2089	3.6229	0.8253	0.0720	6.6572
24	2.2091	3.5704	1.0530	0.1013	6.8325
28	2.2092	3.5194	1.3045	0.0933	7.0332
32	2.2094	3.4698	1.5604	0.1387	7.2396
36	2.2095	3.4212	1.7545	0.1547	7.3853

$1979-1983$					
Maturity	r	$E[\pi]$	IAP	IRP	$y^{\$}$
8	1.6343	10.8476	0.3502	0.4451	13.2773
12	1.8132	10.2297	0.5211	0.5456	13.1097
16	1.9155	9.7144	0.7089	0.6061	12.9449
20	1.9784	9.2758	0.9223	0.6424	12.8189
24	2.0197	8.8956	1.1659	0.6643	12.7454
28	2.0485	8.5602	1.4378	0.6775	12.7241
32	2.0697	8.2606	1.7192	0.6854	12.7349
36	2.0858	7.9896	1.9454	0.6902	12.7109

Note: The table decomposes the model implied nominal yield curve into its components. The upper panel focuses on 1972-2009. The lower panel focuses on the monetary policy experimentation (1979-1983). Column maturity is in quarters. The abbreviations mean: r (real interest rate), $E[\pi]$ (trend inflation (reference model)), IRP (inflation risk premium), IAP (inflation ambiguity premium), $y^{\$}$ (model implied nominal yield). The last column is the sum of column 2 to column 5. The data is annualized and in percent.

Figure 1: **Inflation Ambiguity Premium in Different Sup-Periods**
This figure presents the inflation ambiguity premium in nominal bond yields for several time-periods. The model is estimated over the entire sample 1972 to 2009 and sample averages are used to construct the premium for different periods. The different sample periods are the monetary policy experimentation (1979-1983), the Great Moderation (1984-2007), and the entire sample 1972-2009. The x-axis presents bond maturities in years. The y-axis is in percent and annualized.
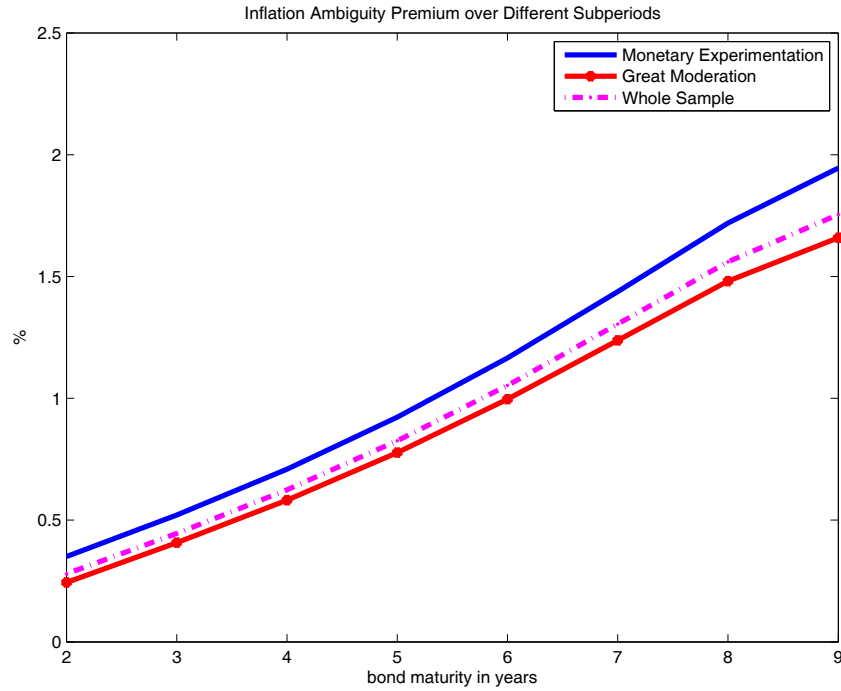
Figure 2: **Cross-Sectional Dispersion in Inflation Forecasts and Inflation Ambiguity Premium in Nominal Bond Yields, 1972.I - 2009.II**
This figure shows the time-series of the model implied ten-year and one-year inflation ambiguity premium, together with the data implied cross-sectional standard deviation of SPF inflation forecasts. The model is estimated with macro and bond yield data from 1972 to 2009. The x-axis presents the time-horizon. The y-axis is in percent and annualized.
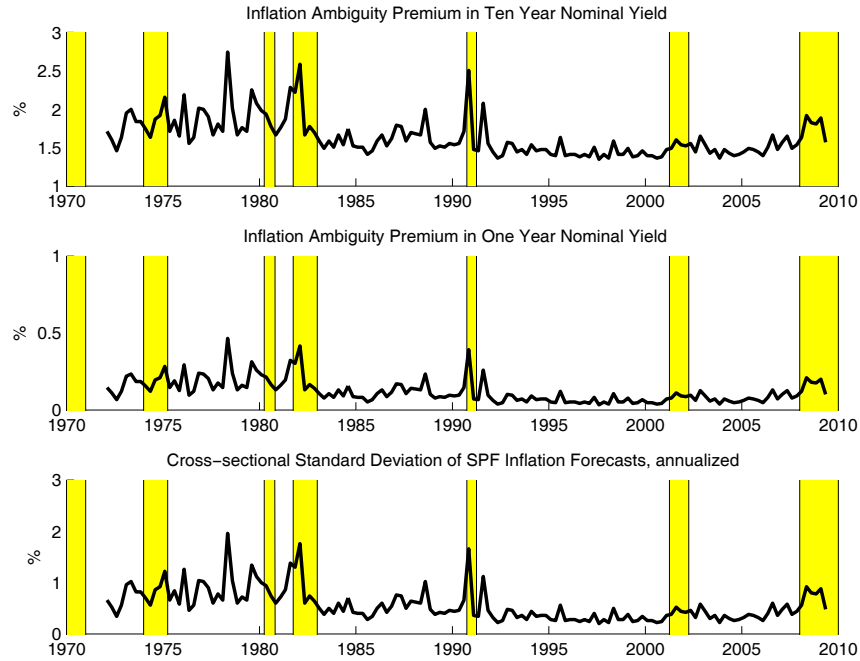
Figure 3: **Empirical vs. Robust One-Quarter Ahead Inflation Forecast, 1972.I - 2009.II**
This figure plots the time-series of quarterly trend inflation for two different models. The solid blue line coincides with the reference model, while the red ∗ line corresponds to the worst-case model. The x-axis presents the time-horizon. The y-axis is in percent and annualized.
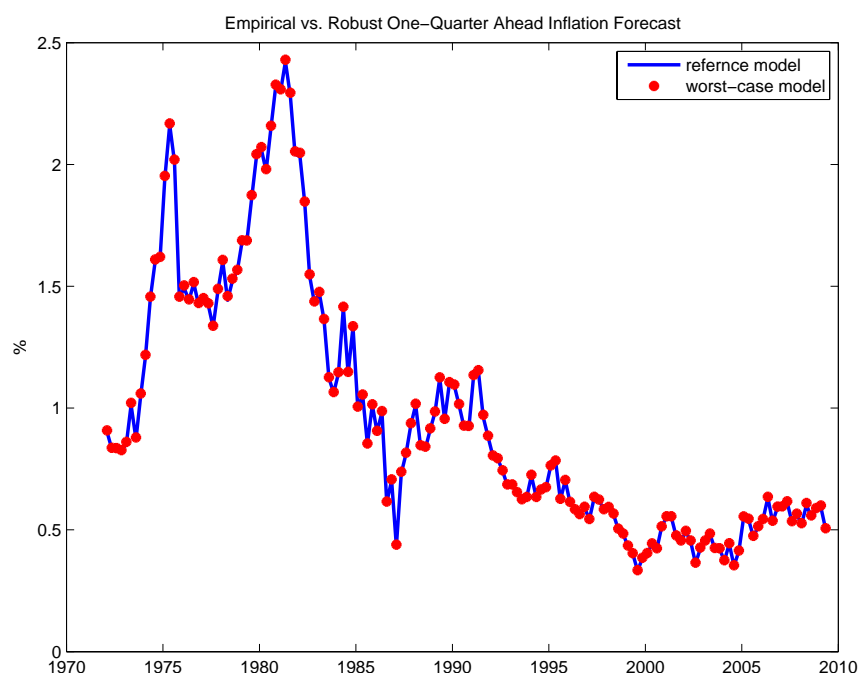
Figure 4: **Average Nominal Yield Curve for Different Values of Inflation Non-Neutrality, 1972.I - 2009.II**

This figure shows that different assumptions on the non-neutrality of inflation can change the sign of the slope of the yield curve. The estimated model implied term structure of nominal bond yields corresponds to the upward sloping curve. The underlying correlation between trend inflation and trend consumption growth, σ_{2z} is negative. All estimates are fixed and only σ_{2z} is varied. If it is set to zero, the corresponding nominal yield curve would be slightly downward sloping. On the other hand, if $\sigma_{2z} > 0$, the resulting yield curve would be strongly downward sloping. The model is not re-estimated. The x-axis presents bond maturities in years. The y-axis is in percent and annualized.