Structural Estimation of a Large-Scale Procurement Combinatorial Auction

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Combinatorial auctions (CAs) are multi-unit auction mechanisms in which bidders can bid on combinations of items or *packages*. CAs have received considerable attention from academia, and their practical use has increased significantly over recent years especially in procurement projects (Cramton et al. 2006). Units in procurement projects often exhibit cost synergies, and using a CA can be particularly advantageous to the auctioneer because it enables bidders to express their cost synergies in the bidding process. If the units were auctioned separately, bidders face a risk of winning only some of the units, known as *exposure risk*, which prevents them from expressing the synergies as much as they could have in a CA. Indeed, under suitable conditions, allowing combination bids is necessary for an efficient and optimal mechanism in the presence of synergies (Levin 1997).

Allowing package bids, on the contrary, can also hurt the efficiency and optimality of the allocation. Bidders may place discounted package bids even when they do not have cost synergies among the units that form the combination. Indeed, firms may have incentives to bundle units where they have a cost advantage with other units where they do not have an advantage. This practice, which we refer to as *strategic bundling*, may result in a CA that offers a less efficient and more expensive allocation than a set of separate auctions for each unit (Cantillon and Pesendorfer 2006). This raises an important design issue: how to allow combination bids to allow bidders' to express their synergies while suppressing the negative effects of strategic bundling. In general, the auctioneer would like to permit package bids among units if and only if those units exhibit cost synergies. However, in most practical settings the precise nature of firms' cost synergies is their private information.

With this motivation, in this work we propose an estimation method that identifies important properties of the bidders' cost structure using bidding data. An important challenge is identifying costs separately from mark-ups, which also determine bids. To do so, we use a *structural* estimation method in which we pose a model of bidders' behavior. In a nutshell, the structural model imposes restrictions on how mark-ups are determined, and thereby enabling us to identify the cost structure of firms. This can then be used to inform auction design decisions such as which package bids to allow. We believe our method and results can bring substantial impacts on enhancing efficiency of CAs.

Despite the practical usefulness and the extensive literature in the topic, little is known about bidders' behavior of CAs. There are several papers conducting empirical studies to estimate cost synergies in CAs using a *reduced form* approach (Ausubel et al. 1997, Olivares et al. 2009). While those papers offer useful methodologies to identify the factors of synergies and estimate the values of those factors, such approach cannot be used to back out mark-up information directly. The paper that is most related to ours is Cantillon and Pesendorfer (2006). They present a structural model for a small size CA of London buses routes. Unfortunately, their method cannot be generalized to large scale CAs like the one we study here, which are relatively common in practice. These issues motivate our work of developing a structural estimation approach for more general CAs.

We perform our analysis using the data obtained from the Chilean school meal project. The Chilean government spends around half a billion dollars a year to feed around 2.5 million children in its public schools. In 1997, the government replaced the old allocation system based on subjective criteria with a CA. In this environment, synergies are expected to arise from *economies of scale* in food inputs and from sharing operational and transportation assets/facilities, which we refer to as *economies of density*. Since 1997, the CA has been successfully used and it provides us with a rich set of data. We collected data on all the bids for every auction in 1999-2009 and also on the information about participating firms and the schools served.

Our approach allows us to identify key aspects of the firms' cost structure such as the extent of economies of density and scale. This information can be used by the Chilean government to improve the auction design. For example, if economies of density are predominant it may be preferable to only allow package bids among units that are nearby.

One of the key features of our work is that we develop a methodology to effectively reduce the dimensionality of the model. In principle, we have a decision model that aims to maximize the firm's expected profit over all possible package bids given the firm's cost structure and given the knowledge of the competitors' bids distribution. The resulting optimization problem is highly dimensional making the analysis computationally challenging. Since the winning probabilities of each combination cannot be earned analytically in general, we are forced to compute most of the values numerically via simulation. The huge number of the possible combinations and bids submitted increases the computational burden and makes the solution very imprecise, requiring the need for a reasonable approach to reduce the dimensionality of the problem. To overcome such computational issues, we propose a restricted model, where each mark-up is set as a function of a small number of parameters. We show that our model can still capture a fairly rich mark-up structure. Then, we use the first order condition of the firms' optimization problem as a basis for estimation. The reduction in the problem's dimensionality reduces substantially the computational burden, making the

estimation feasible.

We have performed numerous computational experiments on small examples and we were able to recover the cost structures and mark-ups with reasonable accuracy. We are currently applying our method to uncover the cost structure of firms in the Chilean school meal auction and use it to suggest improvement in the mechanism design of this procurement auction.

We believe that an important contribution of this paper is that we develop a structural estimation methodology that is applicable to a big set of large scale CAs. Although our study is based on a specific auction data, our modeling and identification methodology is fairly general and could be used in other contexts of CAs. Using our method, the auction designers can identify important aspects of the firms' cost structure and their bidding behavior in a CA. With this information a better auction design can be proposed. We believe our work provides a significant step forward on understanding bidders' behavior and as a consequence give us insights in improving the design of CAs.

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