Efficient Channel Contracting
for Vertically Differentiated Products

Abstract

We describe research on a supply chain contracting problem that was sponsored by a major semi-conductor manufacturer. The manufacturer sells products (semiconductor parts) with varying quality levels through a network of distributors to end consumers (independent computer shops, system configurators, hobbyists, etc.) who have heterogeneous valuations for quality. Since production costs for semi-conductors are essentially independent of quality levels (within a part family), the manufacturer earns much more selling higher quality parts. However, the firm's distributors traditionally care more about total unit volume since their margins are similar on all products. As a result, the economics of the two parties have historically not been well aligned. The manufacturer wanted to better understand this incentive problem and develop new strategies to improve its own and distributors profits. The analysis presented here supported this effort.

To analyze this problem, we consider the simple case in which the product family consists of two parts with high and low quality. We first identify the distortions inherent in the firm's status quo wholesale pricing contracts. We then investigate alternative mechanisms that may better coordinate their channel. We start our analysis with the single-distributor case and show that revenue sharing coordinates the channel and can be designed to arbitrarily allocate profits. We study potential *asymmetric* contracts which create different incentives for different parts within a given product line and show which ones can potentially improve channel performance. Taking into account the fact that in many regions the manufacturer has a network of distributors, we then extend our results to the case of Cournot competition. We first characterize the distributor's strategic interaction and then analyze a similar set of coordinating mechanisms. We show that the efficiency of the wholesale pricing contract improves as the number of competitors increases. Lastly, we discuss some of the challenges and roadblocks encountered as we tried to implement a revenue sharing program at our sponsor and why other contracts were not good candidates for implementation. The experience sheds some light on the challenges transforming supply chain contracting concepts into workable real-world programs.

Keywords: Channel Coordination; Supply Chain Contracting; Vertical Differentiation

1. Introduction

This paper is based on a sponsored research project with a leading semi-conductor manufacturer. The firm, one of the world's great engineering companies, produces leading edge products (mainly CPUs) and has successfully used product innovation to lead the semi-conductor industry for decades. Yet as computer technology has advanced, it is becoming increasingly difficult to impress customers with faster technology alone and the value they place on more processing power is reaching saturation levels. The competitive landscape is also changing and today competitors offer alternative processors at attractive price points. As a result, status quo practices were called into question and senior management wanted to reevaluate the core components of their business, including pricing practices and market segmentation strategies, many of which had been in place since the earliest days of the PC revolution. They sought a deeper understanding of the strengths and weaknesses of their current pricing process and wanted to explore innovative strategies that could potentially improve profits. We were engaged to help in this effort.

1.1 Business description

The firm designs, produces and sells several product families of CPUs (processors) and chip sets. These products are targeted for desktop, server and mobile businesses. Each product family consists of CPUs whose performance (roughly speed and cache size) ranges from highest to lowest. We will refer to performance also as "quality" and use both words interchangeably. To give an example, a typical product family, named C-FAM, is depicted in Figure 1. It has three quality offerings in terms of performance, referred to as "Performance", "Value" and "Legacy".

The firm has two primary customer segments:

1) **OEMs** - large original equipment manufacturers like Dell, HP, IBM

2) **The channel** - the name given to a diverse set of distributors and sub-distributors who in turn supply independent computer shops, system configurators, resellers and hobbyists.
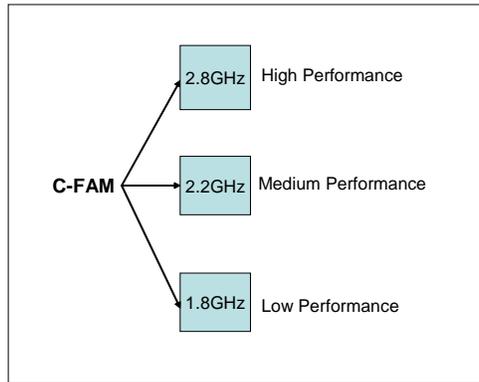
Figure 1: A representative product family

Traditionally most of the firm's business is with OEMs, but a significant percentage of sales in certain product lines, (e.g. their desktop business) comes from the channel. The channel is represented schematically in Figure 2.

The channel has special strategic importance to the firm as it represents a competitive counterbalance to large OEMs. In particular, a strong channel prevents two or three very large OEMs from dominating the entire market and ultimately being in a position to dictate business terms and prices to parts manufacturers. The channel also enables the firm to more easily penetrate emerging markets and sell products near the end of their life cycles.

Even though the channel is strategically important, historically it has survived on very low margins and has been relatively unprofitable for distributors. Indeed, distributors often carry the manufacturer's parts primarily as "loss leaders" to help sell a more profitable bill of materials for a complete system. A myriad of incentive programs and bonus structures currently keep distributors "alive" financially. Furthermore, since OEM pricing is negotiation-based rather than list-price-based, often OEM's end up getting the best available pricing. This leaves the channel at a competitive disadvantage. Considering all these factors, our sponsor wanted to focus our research efforts on better understanding and improving its channel business, specifically targeting the trading relationship between the firm and its distributors.
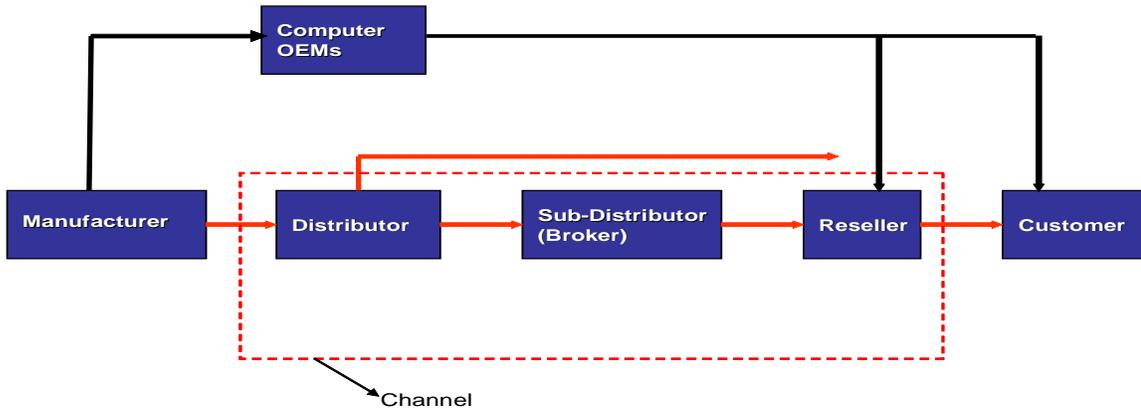
Figure 2: Channel business

1.2 Distributors and pricing practices

Distributors (internally referred as "Distis") range in size and have a heavy presence in certain regional markets (e.g. Americas, Europe-Middle East and Asia-Pacific). They have varying capabilities. Some serve mainly as order fulfillers while others are more value-added distributors, providing training to their sales team to help customers design and configure systems using the manufacturer's products.

In terms of pricing practices, most distributors operate on a cost-plus model (fixed mark-up over their cost), though they are free to charge any price they wish. The manufacturer cannot dictate suggested retail prices (MSRPs). Pricing practices across distributors are diverse. However, there are some select programs such as back-end rebates (BERs) that provide payments to a distributor after a product is sold in order to ensure distributors make a certain minimum margin on selected products. Over the life cycle of the product, the manufacturer itself follows a "waterfall strategy" - with product occupying certain price points that are moved down the structure as new products are introduced. Price protection is provided during product transitions to ensure minimum distributor profits on current inventory as new generations of products are introduced.

4

We had to take into account several challenges in the channel business environment. One particularly vexing one was the existence of a gray market (open market is another term), which are manufacturer's products resold through unauthorized retailers. These are genuine products, but diverted for sale mostly from lower priced countries or regions. The firm had little control over this gray market activity and dealt with it largely reactively, e.g., by randomly sampling products and trying to trace them to identify distributors that buy or sell on the gray market. Still, the gray market is not illegal per se and almost every distributor was free to source from the gray market at any point depending on the price.

1.3   Improving channel pricing practices

There are two broad areas for improvement in channel pricing practices: i) pricing of initial product technology and its evolution throughout its life cycle, and ii) pricing different quality levels within a product family. We chose to focus on the latter for this project because our sponsored believed pricing of products within a part family offered more scope for improvement. The dominant concern was that customers generally choose the lowest performing CPU in a family. They felt that the poor incentives for distributors to sell high-performance parts under the current pricing structure was a significant contributing factor. In the companies parlance, they were not achieving potential *"sell-up"* within the product family.

To better understand the problem, it is worth examining the product and cost structure more closely. Within a family, different performance levels are produced as a result of a "binning" process during production, which results in a random (but predictable) yield of parts that are able to run at different clock speeds. Hence, the cost of producing different quality levels is essentially the same, yet the firm is able to charge significantly higher prices for high quality parts. For example, a 2.8GHz CPU may be priced 100% higher than a 1.8GHz CPU from the same family. As a result, margins at the higher end of the product line are much larger. But since the firm attempts to capture this value through a simple wholesale pricing scheme, distributors face significantly higher costs for high quality parts and do not enjoy nearly the same margin advantage selling them. Indeed, as noted, they typically mark up parts a fixed percentage above cost. Hence the economics of the two parties are poorly aligned.

In this environment, while the manufacturer's incentive is to maximize its gross

margin and encourage selling up in the product line without sacrificing too much from overall volume, the distributor's incentive is to sell as many units as possible. To summarize, using the firm's terminology, the manufacturer's incentive is to "sell-up" and "sell-through", while the distributor's incentive is just to "sell fast". This tension between the manufacturer and the distributor creates distortion and incentive conflicts in the channel which was the main focus of our research project and the topic of this paper. We sought to understand these conflicts and find mechanisms that would better align the economics of firm and its distributors.

1.4 Overview of analysis and results

Based on our discussions with company staff, we developed and analyzed a model of the channel where the manufacturer makes a high (H) and low (L) quality product which are pre-determined. The manufacturer prices and sells these products to a distributor which in turn prices and sells these to a market with consumers that have heterogeneous valuations for quality.

In this environment, we identify distortion in two dimensions. The first one is the well-known "double marginalization" phenomenon, which basically undermines the "sell through" objective of the manufacturer by causing retail prices to be higher overall than is channel optimal. The second distortion we identify is more novel; it is that the price gap between high and low quality products increases even further when products are sold through an intermediary. This second distortion is ultimately what degrades the "sell-up" incentive in the channel.

While the manufacturer could potentially help alleviate this problem by changing the quality of its product line, e.g., increase or decrease the quality gap between H and L products, we do not consider this option here, mainly because the part quality is largely fixed by the characteristics of the semi-conductor manufacturing process. Choosing quality levels is well studied in the vertical differentiation literature for firms selling directly. Because in our case quality differences are essentially fixed, we focus on ways to restore the distortions through potential contractual arrangements and other mechanisms.

We start our analysis with a single distributor setting. We show that a form of revenue sharing coordinates the channel can be designed to arbitrarily allocate profits. We study potential *asymmetric* contracts as well, which create different incentives for

different parts within a given product line and show which ones can potentially align the economics of the firm and its distributors and improve channel performance. Since the manufacturer has a large network of distributors both within the same geography and in different geographies, we then extended this model to multiple distributors to understand the effect of competition among distributors. We find that some of the contracts we study in the single-distributor case coordinate the channel in the case of multiple distributors with different contract parameters. However, we also observe that as the number of distributors increases, the manufacturer can capture most of the channel profit through simple wholesale pricing. This result suggests the manufacturer may do well under simple wholesale pricing in regions where there are many competing distributors and that more complicated coordinating contracts are potentially more effective with larger distributors that have more market power. We discuss this and related implementation issues and challenges in Section 6.

While our analysis is rooted in the specific sponsor firm and industry, the issue we address is generic. Today, many firms use some kind of product versioning strategy with products offered at different quality levels and also sell through a channel of intermediaries. Even Dell, which has been the icon for the direct-sales model, is now considering selling through distributors. On a theoretical level, our work makes unique contributions. In general, the coordination literature has not addressed selling in a vertical differentiation setting, and economic models of vertical differentiation almost all assume direct selling without any intermediary. Our work tries to fill this gap in the literature. Lastly, to the best of our knowledge, there are not that many documented contract implementation initiatives at a company, and we believe the practical insights gained in our project offer a contribution to the existing literature on contracting and supply chain coordination.

2.   Literature Review

Vertical differentiation has been an important research area in the field of economics and marketing. Researchers have investigated the manner in which products of different quality levels compete in the marketplace. Likewise, supply chain and distribution channel coordination has attracted the attention of researchers in the field of both operations and marketing for a long time. We will review the related literature and position our work relative to these two research streams.

7

In the vertical differentiation literature, "quality" generally refers to the level of some attribute in which higher level is always preferred to a lower level. For example, everything else being equal, a higher resolution camera is preferable to a lower resolution one, a faster processor is preferable to a slower one, etc.. This is in contrast to markets with horizontally differentiated products where there is no ordering with respect to the level of attribute. For example, not everyone would prefer a red over a blue shirt.

Mussa and Rossen (1978) were the first to consider a monopolist choosing quality positions to serve a market of heterogeneous customers. Moorthy (1984) investigates the same problem with a different model, emphasizing the fact that consumers self-select the product they purchase; if lower quality products are sufficiently attractive, higher end consumers may find it beneficial to buy the lower quality product rather than buying the higher quality one targeted at them. Therefore, while the firm provides the top valuation segment with its preferred quality, it distorts the quality of the lower level segment.

These basic models have been extended to consider oligopolies competing on quality. Gabszewicz and Thisse (1979) looks into the effect of competition in differentiated industries. Similarly, Gal-Or (1983) investigates the impact of increased competition on the quality levels and on the prices of the products when firms choose both the quantity and the quality of their products. Shaked and Sutton (1982) consider an oligopolistic market where each firm chooses both the quality and the price of its product. They analyze the problem at three stages where in the first stage each firm observes which firms have entered and which have not. In the second stage, each firm chooses the quality of its product and in final stage they choose prices. Moorthy (1988) investigates product and price competition in a duopoly and as in earlier papers find out that each firm should differentiate its product from its competitor.

One distinction among the papers in this research stream has been the assumption on cost structures. Mussa and Rossen (1978), Gabszewicz and Thisse (1979), Moorthy (1984), Ronnen (1991) all assume that variable cost of production is independent of quality (There is fixed cost that increases with quality). Moorthy (1988) is the first to explicitly include variable cost that is increasing and convex in quality. Desai (2001) and Rhee (1996) use similar models.

Some of the other papers in this area look into the same problem by adding an

attribute other than quality to the product. For example, Rhee (1996) investigates the effect of heterogeneity along an unobservable attribute (such as brand) on both quality and price equilibrium in a two-stage game framework and find that firms may offer products of identical qualities in equilibrium. Vondenbosch and Weinberg (1995) also extend one dimensional vertical differention to two dimensions and analyze product and price differentiation. A significant finding of theirs is that, unlike the one-dimensional vertical differentiation model, firms do not tend toward maximum differentiation; they tend to choose positions that represent maximum differentiation in one dimension and minimum differentiation on the other dimension.

Another extension in this area has been the research on damaged goods where a lower quality product is manufactured by damaging the main higher quality product. Laser printers that are provided in fast and slow speeds that are identical otherwise, software with different levels of functionalities etc. are common examples of such products. Since the cost of producing these quality levels are almost the same, this is a special case of earlier research with a specialized cost structure. Deneckere and McAffee (1996) identify conditions under which introducing a damaged lower quality product is profitable for a monopolist under two different market segmentation assumptions. All this literature assumes that the firms sell these vertically differentiated parts directly to the market and do not identify any incentive conflicts that could arise as a result of any intermediaries. Villas-Boas (1998) considers an intermediary when selling vertically differentiated goods, but their focus is on quality selection within a product line and they do not analyze coordination issues and potential competition scenarios.

After Spengler (1950)'s seminal paper on double marginalization, there has been sequence of papers in the economics, marketing and operations literature on channel coordination. Channel coordination, in essence, involves optimizing the joint performance of the supply chain and then allocating the gains among the various parties.

Jeuland and Shugun (1983) study coordination issues in a bilateral monopoly and derives an optimal discount pricing policy. McGuire and Staelin (1983) investigate the optimality of forward integration in a duopolistic retail market. This literature concentrates on deriving the terms of trade that generates channel coordination.

In a newsvendor setting with fixed prices, Pasternack (1985) explores the role of returns in the context of perishable commodities. He shows that a properly chosen wholesale price and a return rebate coordinates the channel. Lee et al. (2000) show

that price protection program, a form of rebate provided by the manufacturer to the retailer as the price drops during the life cycle of the PC, achieves channel coordination. Lariviere and Porteus (2001) explore a price-only contract. Quantity-flexibility contracts (Tsay and Lovejoy (1999)) and sales-rebate contracts (Taylor 2002) have also been shown to coordinate the channel in this setting. In a price-setting newsvendor model, Bernstein and Federgruen (2005) study a price-discount contract and demonstrate that it is a coordinating contract. Emmons and Gilbert (1998) study a model that incorporates price sensitive end consumer demand in a one period return model. In a similar setting, Cachon and Lariviere (2005) and Dana and Spier (2001) study coordinating revenue sharing contracts and show its advantages and limitations compared to other contracts. Lariviere (1998), Corbett and Tang (1998), Tsay et al. (1998), Cachon (2003) provide excellent reviews on this literature.

While the vertical differentiation literature does not focus on incentive issues and coordination problems, the contracting literature in operations and marketing do not consider scenarios where the firm could be selling a vertically differentiated product line and potential economic distortions arising from that. Our main goal with this paper is to address this gap. On the other hand, there are only a few contracting papers that address on how to actually implement such contracts (Cachon and Lariviere (2001)). Hence, a subsidiary goal of ours is to help fill this gap by explaining some of the challenges and roadblocks when undertaking such an initiative in practice.

3.   Model

Our model is modest but it has sufficient detail to study the potential economic distortions in selling vertically differentiated products and how to mitigate them with contractual arrangements. We consider a manufacturer and a distributor which sells the manufacturer's products. We take one product family and two parts that belong to that family. Throughout the paper, we will use High (H) and Low (L) to denote the quality levels of these two parts. We assume that the distributor faces a market with heterogeneous customers. The manufacturer was not really sure how much the customer valued the brand vs. the individual performance of the CPU together with the price the distributor charges. In order to reflect that in the model, we assume that the consumer valuation for the products has 2 components:

1. Brand/Family valuation (R) which is the same for both products

2. Quality valuation (v) which increases linearly with the quality of the product

Normalizing the quality for the H-product to 1 and the L-product to γ, consumer's overall valuation would be $R + v$ and $R + \gamma v$ for H and L-products respectively. First, we introduce the following notation and then describe the sequence of events:

p_i: Selling price at the consumer market per unit for product $i = H, L$

w_i: Manufacturer's price to the distributor per unit for product $i = H, L$

c_i: Manufacturer's production cost per unit for product $i = H, L$

N: Total market size for this family of products

d_i: Demand generated by the distributor for product $i = H, L$

v: Consumer valuation for quality which is assumed to have Uniform Distribution over $(0, 1)$

All market parameters are assumed to be known. The sequence of events is as follows: The Manufacturer announces the wholesale prices for both the H and the L product. The distributor decides how to price these two products which in turn determines the demand d_H and d_L for both H and L products respectively. This demand is built and shipped by the manufacturer which has no capacity constraint.

A comment on the assumptions so far is in order. Most of the supply chain co-ordination literature in operations is motivated by single products whose demand is stochastic, which makes the ordering decision and the associated inventory cost the key concerns in those models. Our work is primarily motivated by high-tech supply chains which have three important features: First, semiconductor manufacturers tend to make an aggregate forecast at the product family level rather than at the part level since family level forecasts are generally very accurate. They plan and position their supply chain according to these forecasts and build the products as part-level demand is realized. That part level demand is what we are referring to by d_H and d_L which can be met quickly because of the nature of this forecasting and positioning process. Secondly, most manufacturers provide parts to their downstream supply chain partners in a consignment agreement so that the downstream partner (the distributor in our case) is not overly concerned with inventory holding costs. Indeed, our sponsor provides regular inventory assistance programs to help reduce holding costs for the distributors. As for the manufacturer's capacity, capacity in such manufacturing environments is generally allocated in advance for a product family based on its aggregate

forecast which, as mentioned, is quite accurate. Therefore capacity for individual parts generally does not become a problem as the difference between the production time of a high and a low performance part is negligible.

Consistent with this high-tech supply chain structure and to maintain our focus on segmentation of the market and the associated dynamics and problems of planning and selling two vertically differentiated products, we assume all demand generated by the distributor for individual parts can fully be satisfied and sold.

The distributor needs to set prices p_H and p_L for both H and L products which will in turn determine demand d_H and d_L. If we call v_H and v_L the valuation of threshold customers, the prices p_H and p_L need to be set such that the type v_H will be indifferent between buying H and L products and v_L will be indifferent between buying the L product and not buying. Assuming the utility of not buying is zero:

$$p_L = R + \gamma v_L \tag{1}$$

$$R + v_H - p_H = R + \gamma v_H - p_L \tag{2}$$

Equation 1 says that a v_L type customer would gain zero utility by buying the L-product and equation 2 says that a v_H type customer would gain the same utility if he had switched to the L product and paid p_L. From 2, $p_H = R + v_H - \gamma(v_H - v_L)$

Now, the above price setting problem can be viewed as finding the threshold customer types. Due to uniform distribution assumption, the demand generated as a result is:

$d_H = N(1 - v_H)$
$d_L = N(v_H - v_L)$

Let $d = d_H + d_L$

Our assumptions are:

A1. $0 < c_L \leq c_H < R$
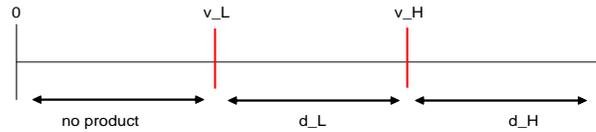
A2. $c_H - c_L < 1 - \gamma$

A3. $R - c_L < \gamma$

Figure 3: Demand for High and Low Performance Products

The first assumption says that producing H type products is more expensive than producing the L types, and the brand value alone is higher than either production cost, so there is some profit to be gained by selling both types. The second assumption says that the increase in production cost from H to L types should not be more than the increase in quality valuation of these two products, which ensures that the manufacturer makes a higher margin on the H product. The third assumption ensures some of the market is uncovered. Under these assumptions which generally reflect our sponsor's business environment, it is profitable for the integrated channel to sell both H and L products in positive quantities.

4.   Single Distributor

In this section, we assume there is a single distributor for the manufacturer and it operates in a monopolistic environment. While this is not true in reality of course, starting with a single distributor for the initial analysis helps isolate and understand the key phenomenon. Furthermore, it roughly approximates a geography where there exists a major distributor with market power.

We first look at the performance of the centralized solution in section 4.1 to give us

the first best outcome as a baseline for comparison. We then proceed to analyze the status quo wholesale pricing mechanism under decentralization in section 4.2. Observing the inefficiency of wholesale pricing, we then analyze several alternative channel coordinating mechanisms is section 4.3. Channel coordination is achieved when the performance of the integrated channel is replicated by the decentralized supply chain. To achieve this, the terms of the contract must be specified to induce the distributor to behave in the way that is optimal for the integrated channel. We want to understand how channel coordination can be achieved. When coordination is achieved, we are interested in how total profit is allocated between the two parties, which is an indicator of whether or not coordination can be feasibly implemented.

4.1 Centralized Solution

We begin by examining the scenario in which the manufacturer and the distributor are under the same ownership. The performance of this "centralized solution" will serve as a benchmark against which we compare the performance of the decentralized system where the distributor is independent.

Let superscript C represent the values associated with the centralized solution and let Π be the profit of the system. As a result, the profit maximization problem for the centralized system is:

$$\Pi^C(v_H, v_L) = \max_{v_H, v_L} \left\{ (R + v_H - \gamma(v_H - v_L) - c_H)N(1 - v_H) + (R + \gamma v_L - c_L)N(v_H - v_L) \right\}$$

Solving for the above equation, we get the demand generated for both products together with the total profit of the centralized solution:

$$v_H^C = \frac{(1-\gamma)+c_H-c_L}{2(1-\gamma)}; v_L^C = \frac{\gamma-R+c_L}{2\gamma}$$
$$d_H^C = N(\frac{1}{2} - \frac{c_H-c_L}{2(1-\gamma)}); d_L^C = N(\frac{R(1-\gamma)+c_H\gamma-c_L}{2\gamma(1-\gamma)})$$
$$\Pi^C(v_H, v_L) = \frac{N}{4} \left\{ \frac{(R+1-c_H)\gamma(1-\gamma-c_H+c_L)}{\gamma(1-\gamma)} + \frac{(R+1-c_L)(R(1-\gamma)+c_H\gamma-c_L)}{\gamma(1-\gamma)} \right\}$$

4.2 Decentralized Solution

Assuming that the manufacturer is the Stackelberg leader, we analyze the wholesale pricing game between the manufacturer and the distributor. In this section, we will use the superscript D to represent the values for the decentralized solution and the subscript d and m to represent the distributor and the manufacturer respectively. Here, we assume that the manufacturer announces the wholesale prices first. In response, the distributor prices both products in the market and generates demand d_H^D and d_L^D which is satisfied by the unconstrained manufacturer. The manufacturer optimizes its own system i.e. decides on wholesale prices w_H and w_L knowing that it will in return get d_H^D and d_L^D.

In this decentralized system, the distributor's problem is the same as the integrated channel except that the production cost c_H and c_L are replaced with w_H and w_L:

$$v_H^D = \frac{(1-\gamma)+w_H-w_L}{2(1-\gamma)}; v_L^D = \frac{\gamma-R+w_L}{2\gamma}$$
$$d_H^D = N(\frac{1}{2} - \frac{w_H-w_L}{2(1-\gamma)}); d_L^D = \frac{R(1-\gamma)+w_H\gamma-w_L}{2\gamma(1-\gamma)}$$
$$\pi_d^D = \frac{N}{16}\left\{\frac{(R+1-w_H)\gamma(1-\gamma-w_H+w_L)}{\gamma(1-\gamma)} + \frac{(R+1-w_L)(R(1-\gamma)+w_H\gamma-w_L)}{\gamma(1-\gamma)}\right\}$$

In order for the distributor to sell both products and generate positive demand, we assume that the difference between wholesale prices satisfies $w_H - w_L < 1 - \gamma$. Imposing this constraints gives us the manufacturer's problem:

$$\pi_m^D(v_H, v_L) = \max_{w_H, w_L}\left\{(w_H - c_H)N(\frac{1}{2} - \frac{w_H - w_L}{2(1-\gamma)}) + (w_L - c_L)N(\frac{R}{2\gamma} + \frac{w_H\gamma - w_L}{2\gamma(1-\gamma)})\right\}$$

s.t.

$$w_H - w_L < 1 - \gamma$$

$$w_L > R - \gamma$$

Solving for the manufacturer's problem we get: $w_H^* = \frac{R+1}{2} + \frac{c_H}{2}; w_L^* = \frac{R+\gamma}{2} + \frac{c_L}{2}$

Our first result is on the inefficiency due to decentralization (All proofs are in the appendix.):

Proposition 1   a) $p_H^D > p_H^C$ and $p_L^D > p_L^C$

b) $p_H^D - p_L^D > p_H^C - p_L^C$

c) Total demand for the channel is determined by the L-product wholesale price w_L

Part a) of the above proposition shows that the classical double marginalization result holds in the case of pricing vertically differentiated products; that is, retail prices of both products are higher than channel optimal. Part b) shows the adverse affect on the "sell up" objective of the manufacturer. It confirms the existence of what we call *vertical double marginalization*, which leads to an inflated difference in the price of the low and high quality products; this kind of channel distortion is specific to this problem. The combined effect of these two distortions causes a decrease in not only total demand but also the "sell-up" achieved by the distributor. Finally, part c) says that it is really the L-product wholesale price that determines the total demand while w_H determines how much of that total demand is for the H-product. A manufacturer concerned with just increasing its market share would focus more on pricing the low quality part. In general, what we observe here is that the economics of the two parties are not well coordinated. In the next section, we analyze coordinating mechanisms that eliminate these distortions.

4.3   Coordinating Contracts

This section first considers channel coordination with revenue sharing contract. We then analyze other types of potential contracts which our company sponsor asked us to investigate.

4.3.1   Revenue Sharing Contracts

A well-known implementation of revenue sharing is the case of Blockbuster Inc.(See Cachon and Lariviere (2005) for details.) In a revenue sharing arrangement, the distributor keeps a certain portion of the total revenue and gives the remainder back to the manufacturer. In return, manufacturers provide products at a variable cost that is closer to their manufacturing cost. In Blockbuster's case, the main motivation behind revenue sharing was increasing product availability. Our focus, in contrast, is ensuring that downstream channel partners have the same sell-through and sell-up incentive as the upstream manufacturer.

We assume that before the distributor decides on selling prices p_H and p_L, the manufacturer and the distributor agree on a revenue sharing contract with three parameters. The first two are the wholesale prices w_H and w_L per unit that the distributor

will pay. The second, λ, is the distributor's share of revenue generated from each unit, the remaining 1-λ going to the manufacturer. Hence, we can write the profit functions for the problem as:

$$\pi_d(v_H, v_L) = \lambda \left\{ (R + v_H - \gamma(v_H - v_L) - c_H)N(1 - v_H) + (R + \gamma v_L - c_L)N(v_H - v_L) \right\} -$$

$$w_H N(1 - v_H) - w_L N(v_H - v_L)$$

$$\pi_m(w_H, w_L) = (1 - \lambda) \left\{ (R + v_H - \gamma(v_H - v_L) - c_H)N(1 - v_H) + (R + \gamma v_L - c_L)N(v_H - v_L) \right\} +$$

$$w_H N(1 - v_H) + w_L N(v_H - v_L) - c_H N(1 - v_H) - c_L N(v_H - v_L)$$

$$\Pi(v_H, v_L) = \pi_m + \pi_d$$

Recall, $\left\{ v_H^C, v_L^C \right\}$ are the maximizers of Π i.e. the total channel profit if it was managed centrally. We then have:

Theorem 1 *Consider a revenue sharing contract with parameters $w_H = \lambda c_H$, $w_L = \lambda c_L$ and $\lambda \in (0, 1]$. Then, $\left\{ v_H^C, v_L^C \right\}$ are the optimal threshold valuations for the distributor, and the contract coordinates the channel and allocates the profit according to λ*

In the wholesale pricing scheme we analyzed in section 4.2, the manufacturer adds a margin to the production cost of both products when selling to the distributor. Similarly, the distributor adds its own margin (which is a fixed percentage in several industries) and determines a market price for both products. This upward pressure on the prices as the products move downstream is the main reason for channel distortions. If the manufacturer provided both products at unit production cost this would make the distributor take the same action as in a centralized system; however this can basically be viewed as transferring the company to the distributor since all the profit will stay with the downstream partner. Revenue sharing spans these two extremes. It allows the downstream partner, the distributor in our case, to be the λ percent owner of the entire channel – paying for λ percent of the production cost and keeping λ percent of the total revenue generated. Hence, it is in distributor's best interest to increase the total profit. This is how revenue sharing ensures that the distributor has the exact same sell-up and sell-through incentive as the centralized system.
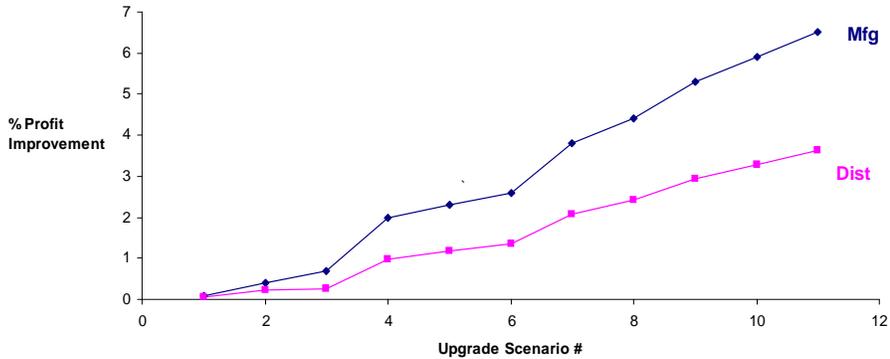
Figure 4: Percent profit improvement under *wholesale pricing* for both the manufacturer and the distributor as amount of upgrade increases

To get a more realistic sense of the impact of revenue sharing, we took a sample from our sponsor's quarterly data to compare its current wholesale pricing with a potential revenue sharing arrangement. We took four products in a family whose quality ranged from lowest to highest and obtained some base demand levels. We then came up with different scenarios in which the demand figures are changed by assuming a percentage of a lower performing part demand is upgraded to the next high performing part in the family. We created several such upgrade scenarios and calculated the percent profit improvement for both the manufacturer and the distributor. The graph in figure 4 shows the profits for the current process of wholesale pricing, where the horizontal axis represents the upgrade scenario and the amount of upgrading increases along this axis. Obviously, the profit increases for both as there is more upgrading, but the figure confirms that the manufacturer has a greater incentive to sell-up (which is an upgrade on the consumer side) since that has a much greater impact on its profit compared to that of the distributor. The second graph in figure 5 is created using the exact same scenarios, but with a revenue sharing arrangement; note that the manufacturer's and the distributor's incentives are perfectly aligned in this case.

18

Every contract design is evaluated based also on its practical feasibility. One important parameter in a revenue sharing environment is λ. As the outside opportunity cost for the distributor under consideration increases, this percentage will likely have to increase. In other words, it should be set such that the distributor will not deviate from what is best for the channel. We discuss implementation challenges associated with revenue sharing and other contracts in more detail in section 6.
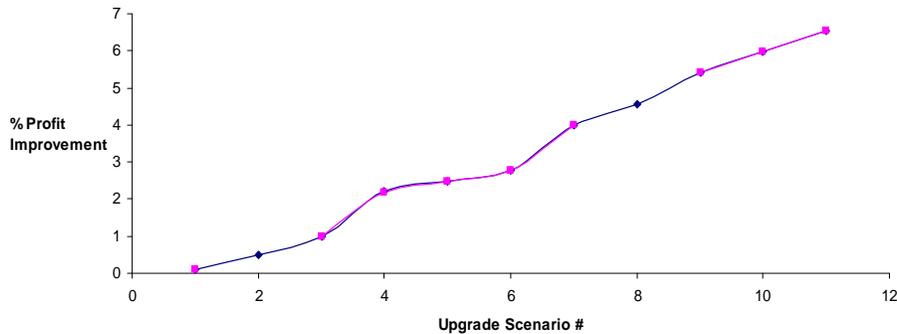

Figure 5: Percent profit improvement under *revenue sharing* for both the manufacturer and the distributor as amount of upgrade increases

4.3.2 Average Selling Price (ASP)

One of the mechanisms we were asked to study was how an average selling price (ASP) based sales would affect the distributor incentives. Under this mechanism, the manufacturer keeps track of the average selling price of the distributor for a given quarter for a certain product family. At the beginning of the quarter, a target ASP a is set and announced to the distributor. At the end of the quarter, the realized ASP (r) is checked and if it is greater than the target ASP, the manufacturer gives away a fixed percentage (λ) of the revenue realized from this difference back to the distributor. Our main interest here was whether or not such a mechanism could motivate the distributor

19

to sell up in the market without sacrificing from volume. Writing the profit function for the distributor as:

$$\pi_d(v_H, v_L) = \begin{cases} \{R(v_H, v_L) - C(v_H, v_L) + \lambda(r - a)N(1 - v_L)\} & r \geq a \\ \{R(v_H, v_L) - C(v_H, v_L)\} & r < a \end{cases}$$

where $R(v_H, v_L) = (R + v_H - \gamma(v_H - v_L))N(1 - v_H) + (R + \gamma v_L)N(v_H - v_L)$ and $C(v_H, v_L) = w_H N(1 - v_H) + w_L N(v_H - v_L)$ and $r = \frac{R(v_H, v_L)}{N(1 - v_L)}$

We can then say that for a fixed λ:

Proposition 2 *a) The total demand d is non-increasing in a.*

b) The target ASP has no effect on d_H

We studied ASP based sales to understand its effect on the two main objectives of the manufacturer. The first part of the proposition says that the ASP mechanism degrades the sell-through effect and the second part of the proposition, contrary to intuition, tells us that setting an ASP for the distributor does not really create a sell-up incentive. The main reason is that increasing the realized ASP on the distributor's side can be done by decreasing the overall volume and increasing the prices p_H and p_L and that is exactly what the above mechanism does. This really does not help acheive the main objectives of the manufacturer.

4.3.3 Other Contracts

In this section, we briefly examine other contracts our sponsor either used in the past or suggested as possible coordinating mechanisms. The first such contract is a *selective target rebate.*

In this type of contract, a rebate is offered selectively on the product line. In order to achieve the sell-up objective, we design it such that the manufacturer offers a rebate r only for the H product if its demand exceeds a threshold t. The L product is sold at a wholesale price that supports the sell-through objective of the manufacturer. This is based on the observation made above that w_L determines total demand.

In this setting, the two transfer payments from the distributor to the manufacturer are:

$$T_H = \begin{cases} \{w_H N(1 - v_H) - (N(1 - v_H) - t)r\} & N(1 - v_H) > t \\ w_H(1 - v_H) & N(1 - v_H) \leq t \end{cases}$$

$T_L = \{w_L N(v_H - v_L)\}$

Writing the profit functions with these transfer payments, we have:

Theorem 2 *Consider the selective target rebate contract with rebate r and the set of parameters:*

$w_H = r + c_H$; $w_L = c_L$ *where*

$t \leq t_0$. *The profit allocations are as:*

$\pi_d = \Pi^C - tr$

$\pi_m = tr$ *and*

$t_0 = N \left[\frac{1}{2} - \frac{r}{4(1-\gamma)} - \frac{c_H - c_L}{2(1-\gamma)} \right]$ *and* $r < (1-\gamma) - (c_H - c_L)$

and the channel is coordinated under this contract.

Intuitively, the manufacturer is making margin r on the t units and giving away the volume above t at production cost c_H. However, as the manufacturer increases the margin r, the threshold t has to decrease, otherwise it becomes unprofitable for the distributor to use the rebate option. That sets an upper bound on t based on the opportunity profit of simply purchasing at the wholesale prices. In other words, the manufacturer's incentive to place a threshold only on the H product is to push the distributor toward selling up. However, this offer stops providing an incentive once the threshold is too high.

Another observation we made about this contract is that the profits can not be allocated completely arbitrarily as in revenue sharing contract. Actually, there we argued that the λ must be set considering the opportunity profit of the supplier $\bar{\pi}$. When we consider the same argument, the upper bound on the threshold t needs to be changed to

$$\min \left\{ \frac{\pi^C - \bar{\pi}}{r}, N \left[\frac{1}{2} - \frac{r}{4(1-\gamma)} - \frac{c_h - c_L}{2(1-\gamma)} \right] \right\}.$$

Let's define $1 - \gamma$ as the *quality gap* between H and L products. If we define the efficiency of the contract as $\frac{\pi_m(v_H, v_L)}{\Pi^C(v_H, v_L)}$, this efficiency is only 50% for simple wholesale price contract under any quality gap i.e. the contract efficiency is independent of the product line design. Based on our numerical study, we observe that the efficiency of the selective target rebate contract is around 85% when the quality gap is as high as 95%. However this efficiency significantly drops as the quality gap decreases and reaches the wholesale price contract efficiency when the quality gap is 55%. Even though the selective target rebate is a coordinating contract, the manufacturer would prefer to

implement it for a product family where the quality gap between the parts is wide. Normally, the L part cannibalizes sales of the H part when the quality of these parts is close and therefore the distributor decreases the quantity of H parts it sells. In that case, the manufacturer needs to provide more incentive to the distributor. However, as the difference in quality between these two parts increases and the cannibalization effect decreases, the distributor sells the H part on its own because it now has its "own" natural market. This means that there is less need for sell-up incentives from the manufacturer. Hence, when $1 - \gamma$ is high, the manufacturer can afford to set a higher threshold (t_o) as well as a higher rebate r.

Another contract we analyzed is the *quantity discount* contract that have been studied in several other contexts in prior literature. We focus on how to modify and apply this kind of contract to our problem of channel coordination with vertically differentiated products. Based on similar observations we made for the selective target rebate contract, we again narrow down our focus and assume that the quantity discount only be used for the H product, while wholesale pricing is used for the L product. Specifying $w_H = W - w d_H$, the profit functions are:

$$\pi_d = (R + v_H(1-\gamma) + \gamma v_L - W + wN(1-v_H))N(1-v_H) + (R + \gamma v_L - w_L)N(v_H - v_L)$$

$$\pi_m = (W - wN(1-v_H) - c_H)N(1-v_H) + (w_L - c_L)N(v_H - v_L)$$

We then have:

Theorem 3 *Consider the quantity discount contract with the set of parameters:*

$w_H = c_H + m - wN(1 - v_H)$

$w_L = c_L$ *and* $w = \frac{m}{N(1 - \frac{c_H - c_L}{1 - \gamma})}$

where m is the margin on the H-product. The profit allocations are:

$\pi_d = \Pi^C - \frac{mN}{4}(1 - \frac{c_H - c_L}{1 - \gamma})$

$\pi_m = \frac{mN}{4}(1 - \frac{c_H - c_L}{1 - \gamma})$

and the channel is coordinated under this contract.

The wholesale price for the lower quality product set at the production cost ensures the same sell-through objective, while the discounted wholesale price for the H product targets the sell-up incentive. The margin m needs to be bounded such that the distributor profit is greater than its opportunity profit $\bar{\pi}$. The design of this contract

is quite similar to the selective target rebate contract. However, the profit allocation of the quantity discount contract is much more favorable for the manufacturer. We again observe that the manufacturer can afford to provide less incentive for the distributor as the quality gap increases. While the degree of incentive was measured by the threshold t_o in the target rebate contract was increasing in the quality gap $1 - \gamma$, here the measure is the discount term w decreases as the quality gap widens, i.e. the manufacturer could provide less of a discount for a product family containing parts whose quality range is quite different. However, in the case of a small quality gap, even though the discount term has to be higher, the manufacturer can still get most of the channel profit by counterbalancing this effect with an increased margin m. This was not possible in the target rebate contract where the margin had to be bounded by an expression which was decreasing in the quality gap.

The next contract we study is a *bundling contract*, in which the manufacturer sells a mix of products in a bundle at a single price. With two vertically differentiated products as in our case, the manufacturer can bundle x units of the H product and y units of the L product and sell is at a bundle price w_B. When the distributor buys Q bundles, it will have Qx units of the H product and Qy units of the L product to sell. If the manufacturer's objective is to push the market toward selling higher quality parts, it can design the bundle such that the distributor ends up getting, and therefore selling, more H products than it would if they were sold separately.

The manufacturer first needs to determine the bundle design, i.e. x and y together with a wholesale price w_B, such that when the distributor orders Q units of the bundle, the channel reaches the same H and L product sales in the market as the centralized system.

Theorem 4 *There is a coordinating bundling contract (x^*, y^*) with*
$$w_B^* = min(\frac{R(x+y)+x+(1-\gamma)y-xc_H-yc_L}{2(x^2+2(1-\gamma)xy+(1-\gamma)y^2)}, \eta, \beta) \text{ where the distributor orders } Q = \frac{(1+R)x+(1-\gamma+R)y-w}{2(x^2+y^2+2xy(1-\gamma))}$$
with $\eta = Ry + Rx + x + (1-\gamma)y$ and $\beta = Ry + y(1-\gamma)(d_L^C + 1 - 2d_H^C)$

However, the bundling contract does not let the manufacturer allocate profits in its best interest, leaving most of the profit with the distributor, which is clearly undesirable from the manufacturer's standpoint.

5. Competing Distributors

In actuality, our manufacturer had to sell its products through several distributors and resellers that were roughly similar, at least within the same geography. Therefore, they were interested in understanding the effects of competition among distributors and how it might impact the contractual arrangements we analyzed. In this section, we assume that the manufacturer is selling its product line to multiple competing identical distributors. We also will assume that the products are not further differentiated at the distributors and that any cost they incur is normalized to zero.
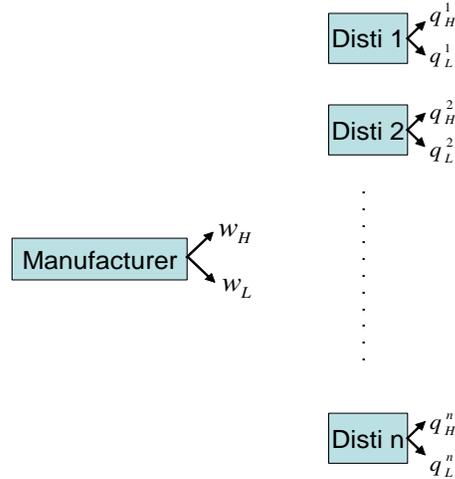

Figure 6: Competitive Model

In this setting, if the distributors enter into Bertrand price competition, then at the equilibrium they will each end up selling the products at a price equal to their cost (i.e. the wholesale price), leaving them with zero profit. This level of competition is arguably too extreme. Therefore, we will instead analyze Cournot quantity competition where each distributor i orders q_H^i and q_L^i of H and L products respectively, which collectively determine the prices p_H and p_L in the market. Let $(\bar{q}_H, \bar{q}_L) = [(q_H^1, q_L^1), ...(q_H^n, q_L^n)]$ be the vector of quantities and $\pi_d^i(q_H^i, q_L^i)$ the distributor i's profit where i=1...n.

In a competitive setting the collection of all (q_H^i, q_L^i) determine p_H and p_L. Nor-

malizing the market size to one, we have:

$$p_H = R + 1 - q_H^i - \gamma q_L^i - \sum_{j \neq i} q_H^j - \gamma \sum_{j \neq i} q_L^j$$

$$p_L = R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j)$$

First, we will analyze the distributor's Cournot game under a general linear whole-sale pricing scheme (w_H, w_L). With this we have:

$$\pi_d^i(q_H^i, q_L^i) = p_H q_H^i + p_L q_L^i - w_H q_H^i - w_L q_L^i$$

Since we assumed that the products are not further differentiated and that the costs incurred by the distributors are the same, we will characterize the symmetric equilibrium for this game. Define $\mu_i(q_H^1, q_L^1, q_H^2, q_L^2, ... q_H^N, q_L^N) = \frac{\partial \Pi_d^i}{\partial q_H^i}$ and $\mu_i(q_H^1, q_L^1, q_H^2, q_L^2, ... q_H^N, q_L^N) = \frac{\partial \Pi_d^i}{\partial q_L^i}$. We assume:

A4. There is a compact set K of R_+^{2N} such that for $(q_H^1, q_L^1, q_H^2, q_L^2, ... q_H^N, q_L^N) \in R_+^{2N} \setminus K$, $\mu_i^H(q_H^1, q_L^1, q_H^2, q_L^2, ... q_H^N, q_L^N) < 0$, $\mu_i^L(q_H^1, q_L^1, q_H^2, q_L^2, ... q_H^N, q_L^N) < 0$ $\forall i$ meaning that the industry output is bounded.

A5-a. $w_H - w_L < 1 - \gamma$

A5-b. $w_L - \gamma w_H < R(1 - \gamma)$

Assumptions A5-a and A5-b guarantee a non-degenerate Cournot equilibrium.

Theorem 5 *a) In the distribution game under Cournot competition, there exists Nash Equilibrium*

b) The Nash equilibrium $(\bar{q_H}^*, \bar{q_L}^*)$ is unique

c) The unique Nash equilibrium $(\bar{q_H}^*, \bar{q_L}^*)$ is locally stable

If the system was managed centrally and $\Pi_d^i(q_H^i, q_L^i)$ is the profit from this centrally managed distributer i, then the total system profit is:

$$\Pi^C(\bar{q_H}, \bar{q_L}) = \sum_{i=1}^N \Pi_d^i(q_H^i, q_L^i)$$

where:

$$\Pi_d^i = (R+1-q_H^i-\gamma q_L^i-\sum_{j\neq i}q_H^j-\gamma\sum_{j\neq i}q_L^j)q_H^i+(R+\gamma(1-q_H^i-q_L^i-\sum_{j\neq i}q_H^j-\sum_{j\neq i}q_L^j))q_L^i-c_Hq_H^i-c_Lq_L^i$$

Let $(\bar{q_H^o}, \bar{q_L^o}) = [(q_H^{1o}, q_L^{1o})...(q_H^{io}, q_L^{io})...(q_H^{1n}, q_L^{1n})$ be the optimal quantities for the above system. In the following section, we analyze some of the potential mechanisms that achieve the performance of the centrally managed channel.

5.1 Coordinating Contracts

In the case of a monopoly distributor, we showed that revenue sharing was a coordinating contract and helped the manufacturer achieve its sell-up and sell-through objectives. The same is true when there are several competing distributors:

Theorem 6 *The revenue sharing contract with the following set of parameters:*
$$w_H^i = \lambda_i(c_H + Q_H^{io} + \gamma Q_L^{io})$$
$$w_L^i = \lambda_i(c_L + \gamma(Q_H^{io} + Q_L^{io}))$$

where $Q_H^{io} = \sum_{j\neq i}q_H^{jo}$ and $Q_L^{io} = \sum_{j\neq i}q_L^{jo}$ coordinates the channel with profit allocations:

$$\pi_d^i = \lambda_i(\Pi_d^i(\bar{q_H^o}, \bar{q_L^o}) + \pi_e^i) \ and$$
$$\pi_m = \sum_{i=1}^n(1-\lambda_i)(\Pi_d^i(\bar{q_H^o}, \bar{q_L^o}) + \pi_e^i) - \pi_e^i \ with \ \pi_e^i = (Q_H^{io} + \gamma Q_L^{io})q_H^{io} + \gamma(Q_H^{io} + Q_L^{io})q_L^{io}$$

The percentage share λ needs to be bounded by a term $\frac{\Pi_d^i(\bar{q_H^o}, \bar{q_L^o})}{\Pi_d^i(\bar{q_H^o}, \bar{q_L^o})+\pi_e^i}$ to make sure that the manufacturer's profit is greater than zero. These λs can be made equal across the distributors by taking into account these different bounds. Note that the above coordinating contract parameters require different values for different distributors when the centrally coordinated channel has a non-symmetric solution. However, with equal (q_H^{io}, q_L^{io}) for all distributors, which is the expected outcome when distributors are identical, the wholesale prices offered will be identical which makes its implementation across the distribution channel feasible and in conformance with antitrust laws. Otherwise, due to the uniform pricing strategy of the sponsor, this would not be a policy that they could implement.

The selective target rebate is coordinating in the case of a single distributor. With multiple competing distributors, it is again a coordinating contract with the right

contract parameters:

Theorem 7 *The selective target rebate contract with rebate r and the following set of parameters:*

$$w_H^i = (c_H + r + Q_H^{io} + \gamma Q_L^{io})$$
$$w_L^i = (c_L + \gamma(Q_H^{io} + Q_L^{io}))$$

where the threshold $t_i \leq t_{io}$ coordinates the channel with profit allocations:

$$\pi_d^i = \Pi_d^i(\bar{q}_H^o, \bar{q}_L^o) - \pi_r^i - t_i r$$
$$\pi_m = \sum_{i=1}^n (\pi_r^i + t_i r) \ with$$

$$\pi_r^i = (r + Q_H^{io} + \gamma Q_L^{io})q_H^{io} + \gamma(Q_H^{io} + Q_L^{io})q_L^{io} \ and$$
$$t_{io} = 2(q_H^{io} - r) + q_L^{io} - (Q_H^{io} + \gamma Q_L^{io}) - c_H + (R + 1 - q_H^{io} - \gamma q_L^{io} - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo})$$

Similar to the revenue sharing contract, the above contract parameters are identical for a centrally managed channel with equal (q_H^{io}, q_L^{io}) which is what we would expect with symmetric distributors.

Lastly, we consider the quantity discount contract which was again shown to be coordinating in the monopoly case with appropriate parameters, can be designed to be coordinating with competing distributors as well:

Theorem 8 *The quantity discount contract with discount term w and the following set of parameters:*

$$w_H^i = W_i - w_i q_H^i \ where \ W_i = (c_H + m + Q_H^{io} + \gamma Q_L^{io})$$
$$w_L^i = c_L + \gamma(Q_H^{io} + Q_L^{io})$$

where the discount contract term $w_i = \frac{m}{2q_H^{io}}$, coordinates the channel with profit allocations:

$$\pi_d^i = \Pi_d^i(\bar{q}_H^o, \bar{q}_L^o) - \pi_Q^i$$
$$\pi_m = \sum_{i=1}^n \pi_Q^i \ with$$

$$\pi_Q^i = (\frac{m}{2} + Q_H^{io} + \gamma Q_L^{io})q_H^{io} + \gamma(Q_H^{io} + Q_L^{io})q_L^{io}$$

With all these three contracts studied under the competitive case, the common problem is that they could have non-identical parameters across distributors, which would not be an acceptable policy for the manufacturer. Hence, we wanted to understand the effect of competition under their current wholesale pricing scheme and then evaluate all potential mechanisms.

5.2 Wholesale Pricing

In this section, we will explore how competition affects the overall supply chain efficiency and the manufacturer's profit when the manufacturer offers just a wholesale price contract with w_H and w_L, which was also studied in the single distributor case. For this wholesale pricing game, the quantities ordered by the distributor are:

$$q_H^i = \frac{1}{n+1} - \frac{w_H - w_L}{(1-\gamma)(n+1)}$$

$$q_L^i = \frac{R}{\gamma(n+1)} + \frac{w_H}{(1-\gamma)(n+1)} - \frac{w_L}{\gamma(n+1)(1-\gamma)}$$

$i = 1...n$

The manufacturer's problem will then be:

$$\pi_m(w_H, w_L) = \max_{w_H, w_L} \left\{ (w_H - c_H)\frac{n}{n+1}(1 - \frac{w_H - w_L}{1-\gamma}) + (w_L - c_L)\frac{n}{n+1}(\frac{R}{\gamma} + \frac{w_H}{1-\gamma} - \frac{w_L}{\gamma(1-\gamma)}) \right\}$$

The manufacturer will offer wholesale prices:

$$w_H^* = \frac{R+1}{2} + \frac{c_H}{2}$$

$$w_L^* = \frac{R+\gamma}{2} + \frac{c_L}{2}$$

These are surprisingly the same wholesale prices as in monopoly case. Furthermore, they are independent of n which is consistent with previous literature (Tyagi (1999)). If we denote the total demand generated by all distributors by d_H and d_L, we capture the efficiency results in the below proposition:

Proposition 3 *a)* $\lim_{n\to\infty} d_H = d_H^C$; $\lim_{n\to\infty} d_L = d_L^C$ *where* d_H^C *and* d_L^C *are the H and L demand in centralized solution*

b) $\lim_{n\to\infty} \pi_m(w_H, w_L) = \Pi^C$

This proposition shows that as the number of Cournot competitors increases, the efficiency of the system improves, leaving most of the profit with the manufacturer. This result suggests it may be a better strategy to use wholesale pricing in geographies where there are many small competing distributors since that would lead naturally to the coordinating outcome. More complex contracts, such as revenue sharing, are consequently more advantageous in markets where there is a large dominant distributor with market power.

6.   Implementation Challenges

Our objective was to design a contract that i) would align the economics of our sponsor and its distributors and ii) could be feasibly implemented. In this section, we evaluate the contracts we studied from a practical standpoint and summarize the reactions and concerns of company executives. We start with revenue sharing:

1. Revenue Sharing: This contract was presented as a "profit partnership program" during our formal and informal discussions throughout the company. The analysis and our recommendations were taken quite favorably by the sponsor and it generated the greatest internal support. However, we heard many concerns from different departments within the company as we communicated our ideas. These included:

i. *Gray market impact*: As explained earlier, the manufacturer does not have much control over the gray (open) market and they were concerned that any kind of new program could exacerbate the problem. Since our recommendation involved lowering wholesale prices significantly, their main concern was that this would encourage distributors to divert high end parts to the gray market. We suggested as a possible enforcement mechanism that they offer the program with a "trigger strategy" threat to withdraw it if diversion was detected. Still, there was significant doubt as to how such an arrangement could affect open market dynamics.

ii. *High administrative cost*: This is typical concern for all revenue sharing implementations and was true for our case as well. The manufacturer currently did not have visibility into sales at its distributors. While requiring some development, this was among the more minor concerns since it was seen as something that they could resolve with modest effort and investment.

iii. *"Best terms" contractual arrangements*: Our sponsor had contracts with major OEMs which required them to provide the best available pricing at any point in time. There was concern that a revenue sharing arrangement would imply lower pricing to distributors than OEM pricing under certain conditions. For some in the finance department, this was a "show-stopper". Others felt that if the contract offered the right incentives, it should be made available to OEMs as well.

iv. *Forecasting accounts receivable*: If a program like profit partnership was implemented, the credit department believed that forecasting accounts receivable would become a major issue since the revenue the sponsor would collect would have two components, one of which was paid at the time of sale and the other portion coming from the revenue the distributor would make at the end of the quarter. This would make it harder to forecast sales and receivables.

v. *Credit risk*: Charing a small wholesale price up front and waiting to receive a revenue share after parts were sold created a significant credit risk. Since the wholesale prices was lowered significantly under the program, this meant that the riskier distributors would become even more risky. While a concern, it was felt that appropriate limits on lines of credit could be used to manage this risk.

According to theorem 1, to achieve full coordination, wholesale prices have to be even lower than the production cost. We realized that we had to modify the contract design to mitigate the business risks identified above. Ultimately, we recommended that there be two payments by the distributor:

1. An advance payment (same for all parts within the family): Covers production cost and a portion of the revenue share.

2. An after-sales payment: Remaining portion of the revenue share at the time of sales

or in 90 days whichever comes first.

These modifications to the original contract eased some of the credit related concerns of our sponsor.

2. Other Contracts:

The ASP mechanism was ruled after we observed that it did not lead to centralized outcome. Furthermore, it has many similarities with what the manufacturer was already doing and it could not be fully adopted across all distributors.

The selective target rebate contract is an asymmetric contract with different terms for different parts within the product line. Moreover, the targets had to be adjusted depending on the characteristics of each distributor. Moreover, the fact that it resembled several other bonus programs already tried was another disadvantage, since several managers felt it would be seen as 'just another sales gimmick' at the distributor level.

The quantity discount contract, on the other hand, creates an incentive for the distributor to buy at volume discounts then sell on the gray market. As a result, it was again not a strong candidate in terms of implementation.

Among all the alternatives, revenue sharing ultimately proved to be the most promising and generated the greatest internal interest due to its simplicity and the fact that it directly addressed the core incentive issues. It was also robust and didn't require detailed knowledge of the distributors' cost and demand information. Positioning it as a profit partnership program helped communicate the concept internally and to distributors. Initial discussions with a key distributor were also positive; they were eager to participate in a pilot implementation. Despite this promising feedback and and many proposal, studies and internal meetings to flesh out the practical details of the concept, ultimately senior managers opted not to pursue a pilot program. Concerns about the gray market impact, the potential treat to OEM relationships, the financial risk and the implementation complexity and cost, collectively, were simply too great to give executives sufficient comfort about prototyping the program. In addition, the firm's sales growth and margins were improving due to a new generation of products, so the organization overall felt less of a need to modify its long-standing pricing and trading practices. In short, revenue sharing was simply too much of a radical change from the status quo and the business risks were judged to outweigh the potential rewards.

7.   Conclusions

We started with the aim of an actual contract implementation that would align the incentives of the manufacturer's supply chain and improve overall profit. In the end, we could not achieve this objective, despite a successful research effort to identify and analyze coordinating contracts and significant efforts to translate this theory into a practical distributor contracting program. Our theoretical results contribute to our understanding of contracting when selling vertically differentiated products. And the lessons learned from the implementation efforts provide new and interesting research directions. For example, it may be that wholesale pricing is the only feasible pricing strategy for certain industries and business environments due to factors that lie beyond the models considered in the theoretical literature, such as gray markets and credit risk. We also learned lessons about the organizational challenges involved in implementing such radically different terms of trade, which impact many functional areas of the firm. Understanding environments where certain contracts can and cannot be implemented is a topic we're currently investigating.

8. APPENDIX

PROOF OF PROPOSITION 1

a) From equations 1 and 2, we know that $p_H = R + v_H - \gamma(v_H - v_L)$ and $p_L = R + \gamma v_L$. In the centralized system, we have: $v_H^C = \frac{(1-\gamma)+c_H-c_L}{2(1-\gamma)}; v_L^C = \frac{\gamma-R+c_L}{2\gamma}$. In the decentralized system, we have: $v_H^D = \frac{(1-\gamma)+w_H-w_L}{2(1-\gamma)}; v_L^D = \frac{\gamma-R+w_L}{2\gamma}$. Plugging in w_H^* and w_L^* in these decentralized values, we get: $p_H^D = \frac{3}{4}(R+1) + \frac{c_H}{4}$ and $p_L^D = \frac{3}{4}(R+\gamma) + \frac{c_L}{4}$. Similarly, $p_H^C = \frac{1+R}{2} + \frac{c_H}{2}$ and $p_L^C = \frac{R+\gamma}{2} + \frac{c_L}{2}$. The first part follows by assumption 1.

b) $p_H^D - p_L^D > p_H^C - p_L^C$ by assumption 2 which says that the value gain due to quality difference is more than the difference in cost of manufacturing.

c) Total Demand $= N(1 - v_L) = N(\frac{1}{2} - \frac{(R-w_L)}{2\gamma})$. Hence, total demand is not affected by w_H.

PROOF OF THEOREM 1

With the contract parameters as given and revenue shared as explained:

$\pi_d^D = \lambda\{(R + v_H - \gamma(v_H - v_L))N(1 - v_H) + (R + \gamma v_L)N(v_H - v_L)\}$ - $\lambda c_H N(1-v_H) - \lambda c_L N(v_H - v_L) = \lambda\{(R + v_H - \gamma(v_H - v_L) - c_H)N(1 - v_H) + (R + \gamma v_L - c_L)N(v_H - v_L)\} = \lambda \Pi^C$

As a result, (v_H^C, v_L^C) would maximize π_d^D as well and the profit is arbitrarily allocated according to λ.

PROOF OF PROPOSITION 2

For a very small a and λ, from the FOC of the $\pi_d(v_H, v_L)$ which is concave in both v_H and v_L, we have:

$v_H = \frac{(1-\gamma)(1+\lambda)+w_H-w_L}{2(1-\gamma)(1+\lambda)}$ and $v_L = \frac{(1+\lambda)(\gamma-R)+w_L+\lambda a}{2\gamma(1+\lambda)}$

$d_H = N(\frac{1}{2} - \frac{w_H-w_L}{2(1-\gamma)(1+\lambda)})$ and total demand $d = N(\frac{1}{2} + \frac{R}{2\gamma} - \frac{w_L+\lambda a}{2\gamma(1+\lambda)})$

From d_H and d, both parts of the proposition follow.

PROOF OF THEOREM 2

$\pi_d(v_H, v_L) = (R + v_H(1-\gamma) + \gamma v_L)N(1 - v_H) + (R + \gamma v_L)N(v_H - v_L) - T_H - T_L$ where T_H and T_L are the transfer payments:

$T_H = \{w_H N(1 - v_H) - N(1 - v_H - t)r\} \quad N(1 - v_H) > t$

$\qquad w_H(1 - v_H) \qquad\qquad\qquad N(1 - v_H) < t$

$$T_L = \{w_L N(v_H - v_L)\}$$

$$\pi_s(v_H, v_L) = T_H + T_L - c_H N(1 - v_H) - c_L N(v_H - v_L)$$

Define $K(t) = \pi_d^r - \pi_d^n$ where π_d^n is the profit from not using the rebate option. Suppose that $(v_H^n, v_L^n) = argmax(\pi_d^n)$ and $(v_H^r, v_L^r) = argmax(\pi_d^r)$. $K(t)$ is continuous and decreasing. Hence a threshold $t_0 \; \exists$ where $K(t_0) = 0$ with $t_0 \in (d_H^r, d_H^n)$ where $d_H^r = N(1 - v_H^r)$ and similar for d_H^n Consider the FOC with $t \geq t_0$:

$$\frac{\partial \pi_d(v_H, v_L)}{\partial v_H} = (1 - \gamma)N(1 - v_H) - N(R + v_H(1 - \gamma) + \gamma v_L) + N(R + \gamma v_L - c_L) - \frac{\partial T_H}{\partial v_H}$$
$$\frac{\partial \pi_d(v_H, v_L)}{\partial v_L} = \gamma N(1 - v_H) + \gamma(V_H - v_L) - (R + \gamma v_L - w_L) - \frac{\partial T_L}{\partial v_L}$$

Doing the algebra, we have $v_H^* = v_H^C$ and $v_L^* = v_L^C$ if and only if $w_H = r + c_H$ and $w_L = c_L$. With this, we have $\pi_d = \Pi^C - tr$. If the distributor does not use the rebate, we have $v_H = \frac{1}{2} + \frac{r + c_H - c_L}{2(1 - \gamma)}$. $r < (1 - \gamma) - (c_H - c_L)$ ensures non-negative d_H, We also find that $t_0 = N\left[\frac{1}{2} - \frac{r}{4(1 - \gamma)} - \frac{c_H - c_L}{2(1 - \gamma)}\right]$. If we say π_m^r and π_m^n is manufacturer's profit under rebate and no-rebate option respectively, we know that $\pi_m^r = rt_0 > (w_H - c_H)d_H^n = r(\frac{1}{2} - \frac{c_H - c_L}{2(1 - \gamma)} - \frac{r}{2(1 - \gamma)})$ Therefore, with these parameters, the distributor and the manufacturer is better off using the target rebate contract which coordinates the channel as well.

PROOF OF THEOREM 3

$\pi_d(v_H, v_L) = (R + v_H(1 - \gamma) + \gamma v_L - W + w(1 - v_H)N)(1 - v_H)N + (R + \gamma v_L - c_L)N(v_H - v_L)$

Setting $W = c_H + m$ where $m > 0$ and based on concavity of the profit function:

$\frac{\delta \pi_d(v_H, v_L)}{\delta v_H} = (1 - \gamma - wN)(1 - v_H)N - (R + v_H(1 - \gamma) + \gamma v_L - c_H - m + w(1 - v_H)N)N + (R + \gamma v_L - c_L)N$

$\frac{\delta \pi_d(v_H, v_L)}{\delta v_L} = \gamma(1 - v_H)N + \gamma N(v_H - v_L) - (R + \gamma v_L - c_L)N$

Setting $w = \frac{m}{(1 - \frac{c_H - c_L}{1 - \gamma})N}$ (> 0 with assumption (2)) and $w_L = c_L$ and rearranging the terms leads to: $v_H^* = v_H^C$ and $v_L^* = v_L^C$. With these parameters, the quantity discount contract coordinates the channel with:

$\pi_d(v_H^C, v_L^C) = \Pi^C - m(1 - v_H) + w(1 - v_H)^2 N = \Pi^C - \frac{mN}{4}(1 - \frac{c_H - c_L}{1 - \gamma})$ and

$\pi_m(v_H^C, v_L^C) = \frac{mN}{4}(1 - \frac{c_H - c_L}{1 - \gamma})$

PROOF OF THEOREM 4

We know that if q represents the quantity sold, $p_H = 1 - q_H - (1-\gamma)q_L$ and $p_L = (1-\gamma)(1 - q_H - q_L)$. When Q units is ordered, we have $q_H = Qx$ and $q_L = Qy$. Therefore:

$\pi_d(Q|x,y) = (R + 1 - Qx - (1-\gamma)Qy)Qx + (R + (1-\gamma)(1 - Qx - Qy))Qy - wQ$ is concave in Q which gives us:

$Q^* = \frac{(R+1)x + (1-\gamma+R)y - w}{2(x^2 + 2(1-\gamma)xy + (1-\gamma)y^2)}$

(1) $Q^*x = \frac{(R+1)x^2 + (1-\gamma+R)yx - wx}{2(x^2 + 2(1-\gamma)xy + (1-\gamma)y^2)} = N(\frac{1}{2} - \frac{c_H - c_L}{2(1-\gamma)})$

(2) $Q^*y = \frac{Rxy + Ry^2 + yx + (1-\gamma)y^2 - wy}{2(x^2 + 2(1-\gamma)xy + (1-\gamma)y^2)} = N(\frac{c_H - c_L}{2(1-\gamma)} - \frac{R - c_L}{2\gamma})$

which places the condition on wholesale price $w_B \le Ry + Rx + x + (1-\gamma)y = \eta$. Combining (1) and (2), gives us an equation of the form $Ax^2 + Bx + C$ for a given $y = \bar{y}$ with $A = R + 1 - 2d_H^C - 2d_L^C \ge 0$ where d_H^C and d_L^C are RHS of equations (1) and (2) respectively. $B = (2d_L^C(1-\gamma) - (1-\gamma) - 1)\bar{y} - w < 0$ since $b < \frac{1}{2}$ and $\bar{y} > 0$ and $C = w\bar{y} - R\bar{y}^2 - \bar{y}^2(1-\gamma)(d_L^C + 1 - 2d_H^C)$

In order to have x that meets (1) and (2), we need $C \le 0$ which places the second condition for a given $\bar{y}$: $w_B \le R\bar{y} + \bar{y}(1-\gamma)(d_L^C + 1 - 2d_H^C) = \beta$. On the other hand, manufacturer's problem for a given x and y is:

$\pi_m(w_B) = w_B Q - Qxc_H - Qyc_L$

Together with the constraints, we have: $w_B^* = \min(\frac{R(x+y) + x + (1-\gamma)y - xc_H - yc_L}{2(x^2 + 2(1-\gamma)xy + (1-\gamma)y^2)}, \eta, \beta)$

PROOF OF THEOREM 5

a) Based on Freidman (1977), the existence of NE is guaranteed under concave $\pi_d(q_H^i, q_L^i)$ and assumption A4. $\frac{\partial^2 \Pi_d^i(q_H^i, q_L^i)}{\partial q_H^i 2} = -2 < 0$ and $\frac{\partial^2 \Pi_d^i(q_H^i, q_L^i)}{\partial q_L^i 2} = -2\gamma < 0$.

b) Under given wholesale prices, the best response functions $r_H^i(q_H^{-i}, q_L^{-i})$, $r_L^i(q_H^{-i}, q_L^{-i})$ for each distributor i is:

$r_H^i(q_H^{-i}, q_L^{-i}) = \frac{1 - w_H - 2\gamma q_L^i - \gamma \sum_{j \ne i} q_L^j - \sum_{j \ne i} q_H^j + R}{2}$

$r_L^i(q_H^{-i}, q_L^{-i}) = \frac{\gamma - 2\gamma q_H^i - \gamma \sum_{j \ne i} q_H^j - \gamma \sum_{j \ne i} q_L^j - w_L + R}{2\gamma}$

Since we are interested in symmetric equilibrium, with the above $2n$ equations we get a system of 2 equations with 2 unknowns with a unique outcome $(\bar{q}_H^*, \bar{q}_L^*)$ as the unique NE.

c) In an "n" firm single product Cournot competition, define $a_i = \frac{\partial \pi_i}{\partial x_i^2}$ and $b_i = \frac{\partial \pi_i}{\partial x_i x_j}$ with following assumptions:

(i) Costs and demand are twice continuously differentable

(ii) Industry output is bounded

(iii) For all i, π_i is pseodo-concave wrt. own output

(iv) All equilibrium is non-degenerate

(v) $\forall i$, $P' < C_i''(x_i^*)$ where $P(X)$ is the inverse demand function and C_i is cost function.

In Dastidar (2000) Proposition 2 states that under conditions (i) to (v) a regular, Cournot equilibrium is always locally stable if for all i, either $b_i \leq 0$ or for all i $b_i > 0$. If we define J as:

$$
\begin{bmatrix}
a_1 & b_1 & b_1..... & b_1 \\
b_2 & a_2 & b_2..... & b_2 \\
b_3 & b_3 & a_3..... & b_3 \\
... & ... & ... & ... \\
b_n & b_n... & b_n & a_n
\end{bmatrix}
$$

Negative trace $\sum(s_i a_i)$ (where $s_i > 0$ is adjustment speed) is necessary and positive $|J|$ is sufficient condition for local stability of unique Cournot equilibrium and the conditions in the proposition in Dastidar (2000) ensure that. For our model, if we define:

$$a_i^H = \frac{\partial^2 \pi_i}{\partial^2 q_H^i 2}; \ a_i^L = \frac{\partial^2 \pi_i}{\partial q_L^i 2}$$
$$b_i^H = \frac{\partial^2 \pi_i}{\partial q_H^i \partial q_H^j}; \ b_i^L = \frac{\partial^2 \pi_i}{\partial q_H^i \partial q_L^j}; \ c_i^L = \frac{\partial^2 \pi_i}{\partial q_L^i \partial q_L^j}$$

where $a_i^H = -2$; $a_i^L = -2\gamma$; $b_i^H = -1$; $b_i^L = -\gamma$; $c_i^L = -\gamma$ which makes the marginal profit matrix $JJ=$

$$
\begin{bmatrix}
A & B \\
B^T & C
\end{bmatrix}
$$

where the matrix $A =$

$$\begin{bmatrix} a_1^H & b_1^H & b_1^H \ldots & b_1^H \\ b_2^H & a_2^H & b_2^H \ldots & b_2^H \\ b_3^H & b_3^H & a_3^H \ldots & b_3^H \\ \ldots & \ldots & \ldots & \ldots \\ b_n^H & b_n^H \ldots & b_n^H & a_n^H \end{bmatrix}$$

and the matrix $B=$

$$\begin{bmatrix} b_1^L & b_1^L & b_1^L \ldots & b_1^L \\ b_2^L & b_2^L & b_2^L \ldots & b_2^L \\ b_3^L & b_3^L & b_3^L \ldots & b_3^L \\ \ldots & \ldots & \ldots & \ldots \\ b_n^L & b_n^L \ldots & b_n^L & b_n^L \end{bmatrix}$$

with matrix $C=$

$$\begin{bmatrix} a_1^L & c_1^L & c_1^L \ldots & c_1^L \\ c_2^L & a_2^L & c_2^L \ldots & c_2^L \\ c_3^L & c_3^L & a_3^L \ldots & c_3^L \\ \ldots & \ldots & \ldots & \ldots \\ c_n^L & c_n^L \ldots & c_n^L & a_n^L \end{bmatrix}$$

By row and column transformations and because $c_i^L = b_i^L$ in our case, the marginal profit matrix JJ simplifies to JJ':

$$\begin{bmatrix} AA & 0 \\ 0 & I \end{bmatrix}$$

where AA is a new symmetric matrix obtained by row transformation from JJ; I is an identity matrix and 0 is the zero matrix. We know that $|JJ'|=|JJ|=|AA|$ in such a system. In Dastidar (2000), the associated results were obtained based on the J matrix which has the same structure with our A and hence AA matrix where all $b_i < 0$ with $(\gamma < 1)$. Therefore, same results carry over to our model which proves local stability of the unique Cournot equilibrium.

PROOF OF THEOREM 6

$\Pi^C(\bar{q_H}, \bar{q_L}) = \sum_{i=1}^{N} \Pi_d^i(q_H^i, q_L^i)$. If $(q_H^{\bar{o}}, q_L^{\bar{o}})$ is the optimal solution for $\Pi^C(\bar{q_H}, \bar{q_L})$ with (q_H^{-io}, q_L^{-io}) representing the part of the solution for all but the i'th distributor, the FOC would be:

$\frac{\partial \Pi^C(q_H^{-io}, q_L^{-io})}{\partial q_H^i} = (R+1-2q_H^i-2\gamma q_L^i - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo}) - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo} - c_H$

$\frac{\partial \Pi^C(q_H^{-io}, q_L^{-io})}{\partial q_L^i} = (R+\gamma(1-2q_H^i-2q_L^i - \sum_{j \neq i} q_H^{jo} - \sum_{j \neq i} q_L^{jo}) - \gamma \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo} - c_L$

With the wholesale prices as given, the FOC for the ith distributor given that the others have (q_H^{-io}, q_L^{-io}) is:

$$\frac{\partial \pi_d^i}{\partial q_H^i} = \lambda[R + 1 - 2q_H^i - 2\gamma q_L^i - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo} - c_H - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo}]$$

$$\frac{\partial \pi_d^i}{\partial q_L^i} = \lambda[R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j) - c_L - \gamma q_H^i - \gamma q_L^i - \gamma \sum_{j \neq i} q_H^{jo} - $$
$\gamma \sum_{j \neq i} q_L^{jo}]$ which gives (q_H^{io}, q_L^{io}) as the unique solution i.e. $(q_H^{\bar{o}}, q_L^{\bar{o}})$ coordinates the channel. Working on the algebra with these values, the profits are:

$\pi_d^i = \lambda_i(\Pi_d^i(q_H^{\bar{o}}, q_L^{\bar{o}}) + (Q_H^{io} + \gamma Q_L^{io})q_H^{io} + \gamma(Q_H^{io} + Q_L^{io})q_L^{io})$

$\pi_m = \sum_{i=1}^{n}(1-\lambda_i)(\Pi_d^i(q_H^{\bar{o}}, q_L^{\bar{o}}) + (Q_H^{io} + \gamma Q_L^{io})q_H^{io} + \gamma(Q_H^{io} + Q_L^{io})q_L^{io}) - ((Q_H^{io} + \gamma Q_L^{io})q_H^{io} + \gamma(Q_H^{io} + Q_L^{io})q_L^{io})$ where $Q_H^{io} = \sum_{j \neq i} q_H^{jo}$ and $Q_L^{io} = \sum_{j \neq i} q_L^{jo}$ which also gives the bound on λ.

PROOF OF THEOREM 7

$\pi_d^i(q_H^i, q_L^i) = (R + 1 - q_H^i - \gamma q_L^i - \sum_{j \neq i} q_H^j - \gamma \sum_{j \neq i} q_L^j)q_H^i + (R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j))q_L^i - T_H - T_L$ where T_H^i and T_L^i are the transfer payments for disti i:

$T_H^i = \{w_H^i q_H^i - (q_H^i - t)r\} \quad q_H^i > t$
$\qquad w_H^i q_H^i \qquad\qquad\qquad q_H^i \leq t$

$T_L^i = \{w_L^i q_L^i\}$

$\pi_s(q_H^i, q_L^i) = \sum_{i=1}^{n}(T_H^i + T_L^i - c_H q_H^i - c_L q_L^i)$

$\Pi^C(\bar{q_H}, \bar{q_L}) = \sum_{i=1}^{N} \Pi_d^i(q_H^i, q_L^i)$. As expressed earlier, let $(q_H^{\bar{o}}, q_L^{\bar{o}})$ be the optimal solution for $\Pi^C(\bar{q_H}, \bar{q_L})$ with (q_H^{-io}, q_L^{-io}) representing the part of the solution for all but the i'th distributor. Define $K_i(t_i) = \pi_d^{ir} - \pi_d^{in}$ where π_d^{in} is the profit from not using the rebate option. Suppose that $(q_H^{in}, q_L^{in}) = argmax(\pi_d^{in})$ and $(q_H^{ir}, q_L^{ir}) = argmax(\pi_d^{ir})$. $K_i(t_i)$ is continuous and decreasing. Hence a threshold $t_{io} \exists$ where $K_i(t_{io}) = 0$ with

$t_{io} \in (q_H^{ir}, q_H^{in})$. Consider the FOC with $t \geq t_{i0}$:

$$\frac{\partial \pi_d^i}{\partial q_H^i} = \lambda[R + 1 - 2q_H^i - 2\gamma q_L^i - \sum_{j \neq i} q_H^j - \gamma \sum_{j \neq i} q_L^j - w_H + r]$$

$$\frac{\partial \pi_d^i}{\partial q_L^i} = \lambda[R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j) - w_L]$$

With the wholesale prices as given, the FOC for the ith distributor given that the others have (q_H^{-io}, q_L^{-io}) is:

$$\frac{\partial \pi_d^i}{\partial q_H^i} = \lambda[R + 1 - 2q_H^i - 2\gamma q_L^i - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo} - c_H - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo}]$$

$$\frac{\partial \pi_d^i}{\partial q_L^i} = \lambda[R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j) - c_L - \gamma q_H^i - \gamma q_L^i - \gamma \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo}]$$ which gives (q_H^{io}, q_L^{io}) as the unique solution i.e. $(\bar{q}_H^o, \bar{q}_L^o)$ coordinates the channel. If the distributor does not use the rebate option with these wholesale prices, it is easy to see that the ordering quantity in this case $q_H^{in} = q_H^{io} - r$ and $q_L^{in} = q_L^{io}$. With these values, we would get:

$\pi_d^{in} = \Pi_i(q_H^{io}, q_L^{io}) - \pi_r^i - r[2(q_H^{io} - r) + q_L^{io} - (Q_H^{io} + \gamma Q_L^{io}) - c_H + R + 1 - q_H^{io} - \gamma q_L^{io} - \sum_{j \neq i} q_H^j - \gamma \sum_{j \neq i} q_L^j]$ where $\pi_r^i = q_H^{io}(Q_H^{io} + \gamma Q_L^{io}) - \gamma q_L^{io}(Q_H^{io} + \gamma Q_L^{io})$ with Q_H^{io} and Q_L^{io} as defined which would give us $t_{io} = 2(q_H^{io} - r) + q_L^{io} - (Q_H^{io} + \gamma Q_L^{io}) - c_H + (R + 1 - q_H^{io} - \gamma q_L^{io} - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo})$

PROOF OF THEOREM 8

With the $w_H = W - wq_H$ type contract and $W = c_H + m$ where $m > 0$:

$\pi_d^i(q_H^i, q_L^i) = (R + 1 - q_H^i - \gamma q_L^i - \sum_{j \neq i} q_H^j - \gamma \sum_{j \neq i} q_L^j - W + wq_H^i)q_H^i + (R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j) - w_L)q_L^i$. As we did earlier, let $(\bar{q}_H^o, \bar{q}_L^o)$ be the optimal solution for $\Pi^C(\bar{q}_H, \bar{q}_L)$ with (q_H^{-io}, q_L^{-io}) representing the part of the solution for all but the i'th distributor. Then:

$$\frac{\partial \pi_d^i(q_H^i, q_L^i | q_H^{-io}, q_L^{-io})}{\partial q_H^i} = [R + 1 - 2q_H^i - 2\gamma q_L^i - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo} - c_H - \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo} - m + 2wq_H^i]$$

$$\frac{\partial \pi_d^i(q_H^i, q_L^i | q_H^{-io}, q_L^{-io})}{\partial q_L^i} = [R + \gamma(1 - q_H^i - q_L^i - \sum_{j \neq i} q_H^j - \sum_{j \neq i} q_L^j) - c_L - \gamma q_H^i - \gamma q_L^i -$$

$$\gamma \sum_{j \neq i} q_H^{jo} - \gamma \sum_{j \neq i} q_L^{jo}]$$

would give (q_H^{io}, q_L^{io}) as the unique solution and would achieve channel coordination with $w_i = \frac{m}{2q_H^{io}}$

PROOF OF PROPOSITION 3

a)$d_H = \sum_{i=1}^{n} q_H^i = \frac{n}{n+1}(1 - \frac{w_H - w_L}{1-\gamma})$

$d_L = \sum_{i=1}^{n} q_L^i = \frac{n}{n+1}(\frac{R}{\gamma} + \frac{w_H}{1-\gamma} - \frac{w_L}{\gamma(1-\gamma)})$

Plugging in w_H^* and w_L*, we would get $d_H = \frac{n}{n+1}(\frac{1}{2} - \frac{c_H - c_L}{2(1-\gamma)})$ and $d_L = \frac{n}{n+1}(\frac{R}{2\gamma} + \frac{\gamma c_H - c_L}{2\gamma(1-\gamma)})$

$\lim_{n \to \infty} d_H = (\frac{1}{2} - \frac{c_H - c_L}{2(1-\gamma)}) = d_H^C$

$\lim_{n \to \infty} d_L = (\frac{R}{2\gamma} + \frac{\gamma c_H - c_L}{2\gamma(1-\gamma)}) = d_L^C$

b) $\pi_m(w_H^*, w_L^*) = \frac{n}{n+1} \left\{ \frac{R+1-c_H}{2}(\frac{1}{2} - \frac{c_H - c_L}{2(1-\gamma)}) + \frac{R+\gamma-c_L}{2}(\frac{R}{2\gamma} + \frac{\gamma c_H - c_L}{2\gamma(1-\gamma)}) \right\}$

$\pi_m(w_H^*, w_L^*) = \frac{n}{n+1}\Pi^C$ which gives us the result.

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