

# Shimer Meets the Production Based Asset Pricing Crowd: Labor Search and Asset Returns\*

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## Abstract

THIS IS THE LATEST ONE. Beginning with Shimer (2005) and Hall (2005), a recent branch of the business cycle literature has explored the role of wage rigidity in accounting for the statistical characteristics of key labor market variables over the business cycle; in particular, high vacancy and unemployment volatility and a high negative correlation between the two. As a further exploration, we extend the Mortensen-Pissarides structure of period-by-period Nash wage bargaining to an environment where there is labor force heterogeneity (permanently employed "insiders" and outsiders subject to separations) and limited participation in the financial asset markets. We show that a reasonable calibration of the resulting model satisfactorily accounts not only for aggregate fluctuations in unemployment and vacancies and their cross-correlations but also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours). The model also achieves a satisfactory replication of the major financial return phenomena; namely, a low risk-free rate, a high equity premium, and an upward sloping term structure. The key to these results is the variable income insurance effectively provided by shareholders and given to workers arising from the interaction of Nash wage bargaining superimposed on the incomplete financial market structure. We refer to the variable income insurance as (income) distribution risk.

*Keywords:* Nash bargaining; business cycles; equity premium puzzle; limited participation

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# 1 Introduction

A recent body of research (e.g., Hall (2005) and Shimer (2005)) argues that the conventional search model of employment dynamics due to Mortensen (1992) and Pissarides (1988, 1990) (MP hereafter) cannot account for key cyclical movements in labor market variables when superimposed on standard real business cycle paradigms. In particular, the high cyclical volatility of vacancies and unemployment as well as their negative correlation at business cycle frequencies are statistical realities that are difficult to replicate in DSGE models. The consensus perspective on this anomaly has been that the MP mechanism for wage determination accommodates too much wage flexibility. This excessive wage flexibility in turn dampens the cyclical movements in firms' incentives to hire and keeps vacancy and unemployment volatilities counterfactually low.

In this paper we revisit these issues by adopting an expanded labor market modeling perspective: while we retain the basic structure of labor market search cum period-by-period Nash wage bargaining, we extend the MP model to a fully dynamic environment where the asset markets are incomplete and perfect risk-sharing between capital owners and workers is not guaranteed. More specifically we develop Nash wage bargaining between capitalists and workers within a macro model with limited stock market participation, and emphasize the interactions of the labor and financial markets in a manner unique to the DSGE literature. As a consequence, we are able to extend the ability of a basic DSGE construct to explain not only the stylized facts of the business cycle and labor markets (especially those aspects emphasized by Shimer (2005) and Hall (2005)), but also the basic stylized financial facts as well.

The standard real business cycle model with a single persistent productivity shock and capital adjustment costs is the foundation on which we build. As noted, there are two types of agents: insider-stockholders and outsider-worker-non-stockholders. The former have full access to financial markets, namely the stock and bond markets. In contrast, the latter group, who comprise the majority of households, do not participate in the stock market but trade only in the risk free bond market. Default free bonds are thus available to all households. The assumption of limited asset market participation is empirically appropriate: it is well documented that more than two thirds of US households held no stock prior to the 1990s, and that households in the top 20% of the wealth distribution alone owned more than 98% of stocks during the 1990s, despite the stock market participation rate having increased substantially during this period (Mankiw and Zeldes (1991) and Poterba (2000)).

What emerges in this setting is a Nash wage bargaining outcome between capital owners and workers in which vacancy postings and unemployment levels are substantially influenced by the pattern of capital market participation. Both mechanisms by which firm owners and workers interact reinforce one another to reduce wage volatility. In particular, *ceteris paribus*, restricted capital market participation has the equilibrium consequence of shareholders providing workers with partial insurance against their labor income variation (see also Danthine and Donaldson (2002) and Guvenen (2003, 2009)). This insurance

is manifest as countercyclical variation in the income shares of workers in the presence of low wage volatility. More specifically, a high productivity realization coincides with the situations where wage bills rise less than output in the short run. Conversely, a lower productivity realization coincides with situations where the wage bill falls less than output in the short run. Since, *ceteris paribus*, Nash bargaining wage outcomes also lead to a counter-cyclical wage income share, these effects reinforce one another to produce a very sluggish response of wages to productivity shocks. This sluggish response of wage income to output variation over the business cycle we entitle the ‘operating leverage effect’ because, like financial leverage, it has the consequence of increasing the income risk to shareholders with implications for the equity premium and other financial quantities.

At the same time, stockholders are hindered from smoothing their consumption in two ways: first, capital adjustment costs discourage consumption smoothing via investment variation and, second, the frictional cost of wage variation due to the income insurance mechanism discourages adjustments along the wage dimension as well. As a result, shareholders attempt to smooth their own consumption by adjusting employment at the extensive margin: high productivity shock realizations dramatically increase job vacancies and employment, while low productivity shocks substantially decrease them. This set of events resolves the unemployment and vacancy volatility puzzles raised by Shimer (2005), as well as reproducing their negative correlation. In fact, the model formulation presented here gives rise to much greater vacancy and unemployment volatility than is found in the seminal models of Andolfatto (1996), Merz (1995), and Gertler and Trigari (2009), which effectively assume a complete asset market structure.

Shareholder income variation arising from the partial insurance they provide to workers due to the incomplete asset market structure significantly affects the Nash wage bargaining position of the firm acting on their behalf. Accordingly, we view this income distribution risk as akin to Shimer’s (2005) hypothesized ad hoc Nash bargaining power shock, and, as such, our model can also be viewed as suggesting micro-foundations for that device. More specifically, we may interpret our model as indirectly providing an answer to the question posed by Shimer: “It seems plausible that a model with a combination of wage and labor productivity shocks could generate the observed behavior of unemployment, vacancies and real wages. . . the unanswered question is what exactly a wage shock is” (Shimer (2005, p. 42)). In our framework that shock represents wage income variation arising from market incompleteness, and the (partial) income insurance provided to workers by stockholders.

In summary, the principal contribution of this paper is to propose a reasonable and tractable mechanism that resolves the unemployment and vacancy volatility puzzles emphasized by Shimer (2005) and Hall (2005), while, at the same time, enabling the model to achieve a satisfactory resolution of long-standing major financial asset pricing puzzles. More specifically, we postulate Nash wage bargaining in an environment where there is limited participation in the financial asset markets. What emerges from these considerations is a fully endogenous Nash bargaining power shock, which we will identify with

(income) distribution risk, and which plays the key role in generating the operating leverage effect in our context. We then demonstrate that a reasonable calibration of the resulting model accounts not only for aggregate fluctuation in unemployment and vacancies but also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours) over the business cycle. The model is also highly general in that its replication of the full range of labor market statistics does not compromise its performance on the financial front, or with respect to the major macroeconomic aggregates.

The structure of the paper is as follows. Section two presents the model and the definition of equilibrium. Section 3 presents the basic results along the aggregates, labor market and financial dimensions. Section 4 decomposes the model by attributing the overall pattern of results to those individual model features principally responsible for them. It assesses, for example, the effects of various alternative preference specifications on the full range of results. Section 5 compares our results with those arising from existing prominent models in the allied literature. Section 6 concludes.

## 2 The Model

We consider a discrete-time infinite horizon economy with two distinct infinitely lived agent types, "insider-stockholders" and "outsider-nonstockholders." The continuum of "insider-stockholders" is distributed on a set of Lebesgue measure  $\mu_s$  while the continuum of "outsider-nonstockholders" is indexed on a set of Lebesgue measure 1.

### 2.1 Insider-stockholder

Following Guvenen (2003) the insider-stockholder, endowed with one unit of time, supplies labor services to the (representative) firm and trades securities— both equity claims to the firm's net income stream, and a one-period risk-free real bond. What distinguishes our model from the Guvenen (2003) model, however, is that the insider-stockholder trades his labor services exclusively in the segmented labor market for insider-stockholders. This market is characterized by employment adjusting along the intensive margin only; i.e., the labor income risk of the insider-stockholder entirely originates from fluctuations in hours worked, not in total employment. This environment implies that the firm and insider-stockholders have a permanent relationship. As Barro (1997) points out, wages are thus not allocational. The environment also can be viewed as nesting in a Lucas (1978b) span of control setup or a Rosen (1982) hierarchy, where workers are assigned to managerial, production, and non-market tasks based on their comparative advantage.

Given his information set  $\Omega_t^s$ , the representative insider-stockholder  $s$  maximizes his lifetime expected utility as given by:

$$V^s(\Omega_0^s) = \max_{\{h_t^s, c_t^s, e_{t+1}^s, b_{t+1}^s\}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t^s - X_t, h_t^s)] \quad (1)$$

s.t.

$$c_t^s + p_t^e e_{t+1}^s + p_t^f b_{t+1}^s \leq w_t^s h_t^s + (p_t^e + d_t) e_t^s + p_t^f b_t^s \quad (2)$$

where  $u$  denotes his period utility function,  $c_t^s$  his period  $t$  consumption, and  $h_t^s$  his period  $t$  labor hours. The variable  $X_t$  represents the exogenous habit stock; it evolves according to

$$X_t = \xi X_{t-1} + (1 - \xi) \chi \bar{c}_{t-1}^s$$

where  $\chi$  is the habit parameter of the insider-stockholder group, and  $\bar{c}_{t-1}^s$  is the average consumption level of the entire insider-stockholder group in the previous period:

$$\bar{c}_{t-1}^s \equiv \frac{1}{\mu_s} \int c_{t-1}^s d\mathcal{X}$$

with  $\mathcal{X}$  standing for the measure of insider-stockholders. In addition,  $d_t$  denotes the aggregate period  $t$  dividend payment by the firm to its stockholders and  $e_t^s$  and  $b_t^s$ , respectively, his period  $t$  stock and bond holdings. The corresponding period  $t$  prices of these securities are  $p_t^e$  and  $p_t^f$ . Lastly,  $w_t^s$  is the insider-stockholder's wage rate, exogenous from his perspective while  $E_t^s \equiv E(\cdot | \Omega_t^s)$  denotes his expectations operator conditional on his information set  $\Omega_t^s$ . The parameter  $\beta$  is the economy-wide subjective discount factor.

We adopt a variation of GHH preference for the insider-stockholder:

$$u(c_t^s - X_t, h_t^s) = u(c_t^s - X_t - H(h_t^s))$$

where  $H(\cdot)$  is his disutility of labor hours. This specification of the period utility function combines the standard GHH preference with a special form of external habit formation or "catching up with the Joneses" (see Abel (1990)). By neglecting the lagged average consumption level of the whole insider-stockholder group ( $\chi = 0$ ), the preference function specified above is reduced to the standard GHH utility function widely employed in the investment-shock literature (Greenwood, Hercowitz, and Huffman (1988)). It is well known that the GHH class of preferences has an extremely weak short-run wealth effect on the labor supply. More specifically, the Hicksian wealth effect of a real wage increase on hours worked is zero for this class of preferences.<sup>1</sup> Knowledge of this fact helps to define the representative insider-stockholder correctly; otherwise, the representative insider-stockholder will decrease his labor

<sup>1</sup>For more detail, see Jaimovich and Rebelo (2009).

supply in response to a positive productivity shock because of the short-run wealth effect.

Moreover, the GHH class of preferences features a marginal rate of substitution between consumption and labor supply that depends only on the labor supply itself. That is, the labor supply is determined independently of intertemporal consumption-savings choice and thus the effect of intertemporal consumption substitution on the labor supply is completely eliminated. Indeed, the marginal rate of substitution between consumption and labor supply in this model economy reads as:

$$-\frac{u_h(c_t^s - X_t, h_t^s)}{u_c(c_t^s - X_t, h_t^s)} = H_1(h_t^s). \quad (3)$$

Conditional upon his information set  $\Omega_t^s$ , the recursive formulation of the insider-stockholder's problem is represented as:

$$V^s(\Omega_t^s) = \max_{\{c_t^s, h_t^s, e_{t+1}^s, b_{t+1}^s\}} \left[ \begin{array}{c} u(c_t^s - X_t, h_t^s) \\ + \lambda_t^s [w_t^s h_t^s + (p_t^e + d_t)e_t^s + p_t^f b_t^s - c_t^s - p_t^e e_{t+1}^s - p_t^f b_{t+1}^s] \\ + \beta E(V^s(\Omega_{t+1}^s) | \Omega_t^s) \end{array} \right] \quad (4)$$

where  $\lambda_t^s$  is the Lagrange multiplier associated with the insider-stockholder's budget constraint (2).

The solution to the above recursive problem (4) is characterized by the customary necessary and sufficient first order conditions

$$w_t^s = H_1(h_t^s) \quad (5)$$

$$p_t^e = \beta E_t[\Lambda_{t,t+1}^s (p_{t+1}^e + d_{t+1})] \quad (6)$$

$$p_t^f = \beta E(\Lambda_{t,t+1}^s | \Omega_t^s) \quad (7)$$

where  $\Lambda_{t,t+1}^s$  denotes the insider-stockholder's intertemporal marginal rate of substitution (IMRS).

## 2.2 Outsider-nonstockholder

We also postulate a continuum of infinitely-lived outsider-nonstockholders, uniformly distributed on a set of Lebesgue measure 1, who supply labor services via a Nash bargaining wage contract in their segmented labor market (to be specified). These agents differ from insider-stockholders in their investment opportunity sets, job opportunity sets and consumption-smoothing motives. First, the outsider-nonstockholder group is restricted from participating in the equity market, although they can freely trade one-period risk-free bonds. This limited participation creates an asymmetry in consumption-smoothing opportunities; outsider-nonstockholders have to rely exclusively on the bond market, whereas insider-stockholders have the additional tool of (indirectly) adjusting their physical capital holdings in response to productiv-

ity shocks. Second, we adopt heterogeneity in the preference specification (Hornstein and Uhlig (1999)) for the baseline model: while capital owners (insider-stockholders) are subject to the "habit formation" feature noted earlier, outsider-nonstockholder "at-will" workers are not. As Hornstein and Uhlig (2000) suggests, this can be viewed as modelling the result of self-selection: agents who easily become accustomed to a high consumption level, i.e. have habit formation preferences, may, over time, be more likely to build up a large capital stock (physical and human) than agents who do not. It is therefore natural to identify this group more closely with firm ownership. In Section 4, we show, however, that the habit formation feature of capitalists has a negligible effect on the relatively volatile behavior of labor market activity over the business cycles. In other words, the operating leverage effect, which we emphasized in the introduction and upon which our results crucially depend, is independent of the habit formation of capitalists. Habit formation will still play an important role, however, in replicating the stylized financial statistics.<sup>2</sup>

The third distinction is that outsider-nonstockholders trade their labor services exclusively in a segmented labor market for outsider-nonstockholders with its own special characteristics. Unlike the insider's labor market, the outsider's labor market is characterized by the variation in employment at the extensive as well as the intensive margins. Another feature of this labor market arrangement is that firms and outsider-nonstockholders Nash bargain over wages in a context of search and matching frictions. Since the model allows for heterogeneous agents, this wage bargaining is endogenously modified to reflect the environment where the workers (outsider-nonstockholder) bargain over wages with the capital owners (insider-stockholders). The resulting Nash bargaining wage is a hybrid of the standard Nash bargaining wage of the representative agent model and a risk-sharing labor contract as in Danthine and Donaldson (2002). The modified bargaining wage is renegotiated on a period-by-period basis. This additional labor income risk due to the variation at the extensive margin and the contractual nature of this bargaining wage further weakens the ability of stockholders, who have a strong consumption-smoothing motive, to smooth their consumption.

Following Merz (1995), each outsider-nonstockholder is viewed as a large extended family which contains a continuum of family members uniformly distributed on a set of Lebesgue measure 1. Each family consists of employed and unemployed outsiders, who pool their financial and labor incomes before choosing per-capita consumption and (risk-free) asset holdings. Accordingly, given his information set  $\Omega_0^n$ , the representative outsider-nonstockholder solves<sup>3</sup> :

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<sup>2</sup>In particular, the adoption of the habit formation preference makes the aggregate EIS implied from the model consistent with Hall's empirical findings: Hall (1988) estimates the aggregate EIS close to zero. Indeed, the aggregate EIS in our model economy is 0.0307. This low EIS seems to be consistent with an upward (real) term structure. The same intuition is found in Binsbergen et al. (2008); in their estimated DSGE model with fully specified Epstein-Zin preferences, they find that a low elasticity of intertemporal substitution (around 0.06) is estimated from upward-sloping (nominal) yield curve data and macro data. We discuss the implied EIS in Appendix 2 as part of a broader model evaluation.

<sup>3</sup>More "structural" form of the contemporaneous utility is to introduce search effort per worker seeking employment:

$$v(c_t^n - n_t L(h_t^n) - (1 - n_t)L(e))$$

$$V^n(\Omega_0^n) = \max_{\{h_t^n, c_t^n, b_{t+1}^n\}} E_0 \sum_{t=0}^{\infty} \beta^t [v(c_t^n - n_t L(h_t^n)) - (1 - n_t) L(0)] \quad (8)$$

s. t.

$$c_t^n + p_t^f b_{t+1}^n \leq w_t^n h_t^n n_t + b(1 - n_t) + b_t^n + T_t. \quad (9)$$

$$n_{t+1} = (1 - \rho) n_t + s_t(1 - n_t) \quad (10)$$

In the above problem,  $v(\cdot)$  denotes the outsider-non-stockholder's period utility function,  $L(\cdot)$  his disutility of labor function, and  $h_t^n$  his period  $t$  labor hours supplied when employed. Either employed or unemployed, outsider-nonstockholders are perfectly insured within the family; thus  $c_t^n$  denotes the consumption level common to all nonstockholder outsiders.<sup>4</sup> The expression  $b_t^n$  denotes the family's period  $t$  bond holdings;  $w_t^n$  is the outsider-nonstockholder's wage determined through the contracting process in the labor market for outsider-nonstockholders while  $b$  represents unemployment benefits and  $T_t$  is lump sum transfers from the government. The  $n_t$  term represents the fraction of available outsider-nonstockholders actually at work in period  $t$ , and  $E_t \equiv E(\cdot | \Omega_t^n)$  is the expectation operator conditional on his information set  $\Omega_t^n$ . Equation (10) describes the evolution of the fraction of workers who are employed, as a function of the exogenous separation rate  $\rho$  and,  $s_t$ , the (exogenous from the non-shareholder worker's perspective) probability that an unemployed worker is matched to the firm in period  $t$ .

Again, we adopt a special form of GHH preference for the representative outsider-nonstockholder's period utility. Conditional upon his information set  $\Omega_t^n$ , the recursive formulation of the outsider-nonstockholder's problem is represented as:

where  $e$  is search effort. However, empirical studies show that search effort is negligible. Therefore, without loss of generality, we assume that  $L(e) = L(0) = 0$ .

<sup>4</sup>Formulation (8) - (10) may be rewritten to distinguish more sharply between the consumption of the employed and unemployed which we denote respectively as  $c_t^{n,e}$  and  $c_t^{n,u}$ . Accordingly the non-stockholder family optimization problem becomes:

$$V^n(\Omega_0^n) = \max_{\{h_t^n, c_t^{n,e}, c_t^{n,u}, b_{t+1}^n\}} E_0 \left( \sum_{t=0}^{\infty} \beta^t [n_t v(c_t^{n,e} - L(h_t^n)) + (1 - n_t) v(c_t^{n,u} - L(0))] \right)$$

s. t. 
$$n_t c_t^{n,e} + (1 - n_t) c_t^{n,u} + p_t^f b_{t+1}^n \leq w_t^n h_t^n n_t + b(1 - n_t) + b_t^n + T_t, \text{ and}$$

$$n_{t+1} \leq (1 - \rho) n_t + s_t(1 - n_t).$$

For this formulation, in equilibrium,  $v_1(c_t^{n,e} - L(h_t^n)) = v_1(c_t^{n,u})$ , so that  $c_t^{n,e} - L(h_t^n) = c_t^{n,u}$  by GHH preferences. Average consumption within the family,  $c_t^n$  (as above), is defined by:

$$c_t^n = n_t c_t^{n,e} + (1 - n_t) c_t^{n,u} = c_t^{n,u} + n_t L(h_t^n).$$

Therefore,  $c_t^{n,e} - L(h_t^n) = c_t^{n,u} = c_t^n - n_t L(h_t^n)$  and  $v_1(c_t^{n,e} - L(h_t^n)) = v_1(c_t^{n,u}) = v_1(c_t^n - n_t L(h_t^n))$ .

This latter identification means that the intertemporal asset pricing relationship and intra-temporal labor supply decisions under either formulation are identical (the necessary and sufficient first-order conditions coincide). Under GHH preferences cum optimal risk sharing, the average utility of employed and unemployed workers, taking full account of their relative consumption benefits and hours obligation, coincides with the utility of average consumption less average hours disutility.



$$V^n(\Omega_t^n) = \max_{\{c_t^n, b_{t+1}^n, h_t^n\}} \left[ \begin{array}{c} v(c_t^n - n_t L(h_t^n)) \\ + \lambda_t^n (b_t^n + w_t^n h_t^n n_t + b(1 - n_t) - p_t^f b_{t+1}^n - c_t^n) \\ + \beta E(V^n(\Omega_{t+1}^n) | \Omega_t^n) \end{array} \right]^5 \quad (11)$$

where  $\lambda_t^n$  is the Lagrange multiplier associated with the outsider-nonstockholder's budget constraint (9). The solution to the above recursive problem (11) is characterized by the necessary and sufficient first order conditions:

$$v_c(c_t^n - n_t L(h_t^n)) = \lambda_t^n \quad (12)$$

$$w_t^n = L_1(h_t^n) \quad (13)$$

$$p_t^f = \beta E\left(\frac{v_1(c_{t+1}^n - n_{t+1} L(h_{t+1}^n))}{v_1(c_t^n - n_t L(h_t^n))} \mid \Omega_t^n\right). \quad (14)$$

Note that outsider-nonstockholders' hours are supplied under the condition that the (hourly) wage equals the marginal rate of substitution of consumption for leisure.

We next describe the functioning of this labor market and its wage determination process.

### 2.3 Search in the labor market for outsider-nonstockholders

There is one infinitely lived representative firm that behaves competitively.<sup>6</sup> The firm hires  $n_t$  outsider-nonstockholders from the outsider's labor market in period  $t$ . The firm also posts  $\nu_t$  vacancies in order to attract new outsiders for its period  $t + 1$  production. The total number of unemployed outsiders who search for a job in period  $t$ ,  $u_t$ , is given by:

$$u_t \equiv 1 - n_t.$$

Based on the Mortensen and Pissarides search theory, we postulate that the following matching technology exists in the labor market for outsiders in period  $t$ . The exponents  $\sigma$  and  $(1 - \sigma)$  describe, respectively, the elasticity of matches with respect to vacancies and unemployment.

$$M(\nu_t, 1 - n_t) = \sigma_m \nu_t^\sigma (1 - n_t)^{1-\sigma},$$

where  $\sigma_m$  is a scale parameter, and  $m_t \equiv M(\nu_t, 1 - n_t)$  represents "matches," the number of newly hired outsiders.

<sup>5</sup>  $\Omega_t^n = \{w_t, n_t, s_t, b_t, P_t^f\}$ . In addition, there is no multiplier for equation (10) as it contains no decision variables.

<sup>6</sup> Equivalently, it can be assumed that there is a continuum of infinitely lived identical competitive firms distributed on the unit interval  $[0, 1]$ .

The probability that the firm fills a vacancy in period  $t$ ,  $q_t$ , is given by

$$q_t = \frac{M(\nu_t, 1 - n_t)}{v_t} = \frac{m_t}{v_t},$$

while the probability that a searching outsider finds a job in period  $t$ ,  $s_t$ , is given by

$$s_t = \frac{M(\nu_t, 1 - n_t)}{1 - n_t} = \frac{m_t}{u_t}.$$

Both  $q_t$  and  $s_t$  are assumed exogenous from the perspectives of both the firm and the outsider-nonstockholders. The tightness of the labor market,  $\theta_t$ , is measured by  $\theta_t = \frac{v_t}{u_t}$ .

Employment relationships between the firm and outsiders may dissolve for exogenous reasons in each period  $t$ : this is represented as the invariant probability of separation  $\rho$ . The specification that the job separation rate is constant while the job finding probability is endogenous is consistent with evidence presented in Hall (2005) and Shimer (2005); they report that while the job finding probability is indeed cyclical, the separation rate is substantially less so. Outsiders who lose a job in period  $t$  ( $\rho n_t$  outsiders) are not allowed to search until period  $t + 1$ . Therefore,

$$u_{t+1} = \rho n_t + (1 - n_t) - m_t = 1 - n_{t+1}.$$

## 2.4 Firm

Each period, the firm produces output  $y_t$  according to the following aggregate production function:

$$y_t = f(k_t, \mu_s h_t^s, h_t^n n_t) z_t$$

where  $z_t$ ,  $k_t$ ,  $h_t^s$ , and  $h_t^n n_t$  denote, respectively, the aggregate productivity shock, capital stock in period  $t$ , aggregate labor (hours) supplied by the insider-stockholders, and the aggregate labor hours supplied by the outsider-nonstockholders. With respect to the latter quantity, more specifically,  $n_t$  represents the number of outsiders employed from the matching labor market for outsiders at the end of period  $t - 1$  and  $h_t^n$  is outsiders' labor hours demanded per outsider by the firm in period  $t$ ;  $n_t$  thus evolves according to the following law of motion:

$$n_{t+1} = (1 - \rho)n_t + q_t \nu_t.$$

Each period,  $\rho n_t$  outsiders separate exogenously from the firm's employment pool, which is, in turn, augmented by posting vacancies  $\nu_t$  and hiring new outsiders  $q_t \nu_t = m_t$ . The firm owns the (physical) capital stock,  $k_t$ . Each period the capital stock depreciates at the rate of  $\delta$  and is supplemented by new investment  $i_t$ .

Two costs of adjusting the firm's capital stock and the labor force of outsider-nonstockholders are next introduced. Merz and Yashiv (2007) report that the simultaneous introduction of these two adjustment costs empirically affects the market value of the firm; ignoring either cost does not match with their empirical evidence.<sup>7</sup>

Capital adjustment costs have a long tradition in the investment theory literature. Such costs form a wedge between the shadow price of capital installed within the firm and the price of an additional unit of capital. We replace the standard capital-accumulation technology with the specification employed in Jermann (1998):

$$k_{t+1} = (1 - \delta)k_t + G\left(\frac{i_t}{k_t}\right)k_t$$

where the adjustment cost function  $G(\cdot)$  is given by

$$G\left(\frac{i_t}{k_t}\right) = \frac{a_1}{1 - \frac{1}{\xi}} \left(\frac{i_t}{k_t}\right)^{1 - \frac{1}{\xi}} + a_2$$

and  $a_1$  and  $a_2$  are chosen so that  $G(\delta) = \delta$ , and  $G'(\delta) = 1$ . With these identifications, the elasticity parameter  $\xi \equiv -\frac{1}{G''(\delta)\delta} > 0$  is independent of the determination of the model's steady-state equilibrium, i.e. the steady state is not affected by the positive value  $\xi$ ;  $\xi = \infty$  corresponds to the benchmark case of no adjustment costs. This specification enables Tobin's  $q$  to vary by differentiating between the (shadow) prices of the installed capital and the new investment good prices.

Second, we introduce a cost of adjusting employment. These costs influence the rate at which the firm adds new workers to its existing labor force. We replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs, as in Gertler and Trigari (2009). Defining the hiring rate  $x_t$  as the ratio of new hires  $q_t\nu_t$  to the existing workforce of outsider-nonstockholders, the quadratic adjustment costs of the employment size of outsider-nonstockholders is given by

$$\frac{\kappa}{2}x_t^2n_t$$

where  $x_t \equiv \frac{q_t\nu_t}{n_t} = \frac{\text{new hires}}{\text{existing workforce}} \equiv$  hiring rate and  $\kappa$  is a constant vacancy cost.

The (financial) capital structure of the representative firm consists of one perfectly divisible equity share and one-period risk-free bonds: the firm is not only equity-financed but also financed by the issuance of one period default free (risk free) corporate bonds at price  $p_t^f$ . The total supply of corporate bonds is constant over time and equals a fraction  $\varphi$  of the average capital stock owned by the firm as in Danthine and Donaldson (2002). In each period, the firm makes net interest payments  $(\varphi\bar{k} - p_t^f\varphi\bar{k})$  to bondholders. Since the Modigliani-Miller theorem holds true in this framework, the existence of leverage has no effect on real allocations<sup>8</sup>.

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<sup>7</sup>[New footnote here]

<sup>8</sup>We can verify this property by solving the model with and without leverage: real allocations are identical. The

The firm's decision problem is to maximize its pre-dividend stock market value  $d_t + p_t^e$  on a period-by-period basis given its information set  $\Omega_t^f = \Omega^f(k_t, \lambda_t, q_t, n_t)$ :

$$\max_{\{i_t, h_t^s, x_t\}} d_t + p_t^e \equiv d_t + E(\beta \Lambda_{t,t+1}^s (p_{t+1}^e + d_{t+1}) \mid \Omega_t^f) \quad (15)$$

$$\begin{aligned} \text{s.t. } d_t &\equiv f(k_t, \mu_s h_t^s, h_t^n n_t) z_t - i_t - w_t^s \mu_s h_t^s - w_t^n h_t^n n_t - \frac{\kappa}{2} x_t^2 n_t - \varphi \bar{k} + p_t^f \varphi \bar{k} \\ k_{t+1} &= (1 - \delta) k_t + G\left(\frac{i_t}{k_t}\right) k_t \\ n_{t+1} &= (1 - \rho) n_t + q_t \nu_t. \quad ^9 \end{aligned}$$

In the above problem,  $\Lambda_{t,t+1}^s$  is the marginal rate of substitution of the insider-stockholders,  $w_t^s$  is their competitive wage and  $w_t^n$  is the Nash bargaining wage for outsider-nonstockholders (specified later).

Letting  $V^f(\Omega_t^f) \equiv d_t + p_t^s$ , the recursive representation of the firm's problem is written as:

$$V^f(\Omega_t^f) = d_t + \beta E(\Lambda_{t,t+1}^s V^f(\Omega_{t+1}^f) \mid \Omega_t^f).$$

The necessary and sufficient first-order condition for the firm's optimal investment decision is given by:

$$i_t : (-1) + \beta E(\Lambda_{t,t+1}^s V_{k_{t+1}}^f \mid \Omega_t^f) \frac{\partial k_{t+1}}{\partial i_t} = 0.$$

By the envelope theorem,

$$k_t : \frac{\partial V^f(\Omega_t^f)}{\partial k_t} = f_1(k_t, \mu_s h_t^s, h_t^n n_t) z_t + \beta E(\Lambda_{t,t+1}^s V_{k_{t+1}}^f \mid \Omega_t^f) \frac{\partial k_{t+1}}{\partial k_t} = 0.$$

The investment Euler equation is thus represented as:

$$1 = \beta E(\Lambda_{t,t+1}^s G'\left(\frac{i_t}{k_t}\right) [f_1(k_{t+1}, \mu_s h_{t+1}^s, h_{t+1}^n n_{t+1}) z_{t+1} + \frac{(1 - \delta) + G\left(\frac{i_{t+1}}{k_{t+1}}\right)}{G'\left(\frac{i_{t+1}}{k_{t+1}}\right)} - \frac{i_{t+1}}{k_{t+1}}] \mid \Omega_t^f). \quad (16)$$

The first-order condition for the firm's optimal hiring decision of insiders is given by

$$h_t^s : w_t^s = f_2(k_t, \mu_s h_t^s, h_t^n n_t) z_t, \quad (17)$$

fundamental reason behind this neutral Modigliani-Miller outcome is that the firm's crucial intertemporal decisions are all in accord with the intertemporal marginal rate of substitution of the insider-stockholders; i.e. there is no agency problem between firm owners and managers in this environment. It turns out that the absence of corporate governance problems is important to deriving Nash wage bargaining between capitalists and workers.

<sup>9</sup>Note that to choose the hiring rate  $x_t$  is to choose the number of vacancies  $\nu_t$ .

while the first-order condition for the firm's optimal hiring rate for outsiders is given by

$$x_t : \kappa x_t = \beta E_t \Lambda_{t,t+1}^s J_{t+1} \quad (18)$$

where  $J_t \equiv \frac{\partial V^f(\Omega_t)}{\partial n_t}$  is the firm's shadow value of one additional outsider hired.

## 2.5 Characterizing the Nash bargaining problem<sup>10</sup>

In this section, we formalize the Nash wage bargaining process between the firm and the outsider-nonstockholders. In this environment, there exists a wedge between capital owners' intertemporal marginal rate of substitution (IMRS) and workers' IMRS: the firm is the representative of the capital owners (insider-nonstockholders), not workers. Nevertheless, we show that the Nash wage bargaining solution can be constructed in a tractable way. In other words, the firm's matching surplus and the outsider-nonstockholder's employment and unemployment values can be defined in terms of current consumption so as to make them consistent with the firm's shadow value of one added worker and the outsider-nonstockholder's value of becoming employed, respectively. What emerges from this representation of the Nash bargaining problem in terms of current consumption is a tractable form of Nash bargaining which nests, as the special case, the standard Nash bargaining wage in the representative agent analogue.

**Firm's shadow value of hiring one outsider** Presuming that the firm's decision variables are chosen optimally, the firm's pre-dividend stock market value  $V^f(\Omega_t^f) \equiv V_t^f \equiv d_t + p_t^e$  can be represented recursively as follows:

$$\begin{aligned} V_t^f &= d_t + p_t^e \\ &= d_t + \beta E(\Lambda_{t,t+1}^s (p_{t+1}^e + d_{t+1}) \mid \Omega_t^f) \\ &= d_t + \beta E(\Lambda_{t,t+1}^s V_{t+1}^f \mid \Omega_t^f) \end{aligned}$$

Let us be more specific about the structure of  $J_t = \frac{\partial V^f(\Omega_t^f)}{\partial n_t}$ , the per-capita value to the firm of hiring one outsider in period  $t$ :

$$J_t = h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t - w_t^n h_t^n + \frac{\kappa}{2} x_t^2 + (1 - \rho) \beta E_t \Lambda_{t,t+1}^s J_{t+1}$$

where  $h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t$  defines the "extensive marginal product of outsiders' labor."<sup>11</sup>

<sup>10</sup>This model is essentially the same as one in which shareholder-workers directly manage the firm, as though it were a private company. As firm owners, they trade bonds with their workers. There is no explicit stock market under this formulation, and no MRSs to be conveyed by the shareholders to the firm as in the present "more realistic" formulation. Accordingly, the return on equity is measured as the marginal product of capital. The present formulation is more consistent with the recent literature.

<sup>11</sup>In the matching labour market for outsiders, we distinguish between the "extensive marginal product of outsiders' labour" and the "intensively marginal product of insiders' labour." Similarly, the intensive marginal product of labour,

The first-order condition for the hiring rate equates the marginal cost of adding an outsider with discounted marginal benefit:

$$\kappa x_t = \beta E_t \Lambda_{t,t+1}^s J_{t+1}. \quad (19)$$

Note that condition (19) is identical to the firm's optimal hiring decision for outsiders (18).

Using the definition of  $J_t$ , we have the following equivalent optimality condition:

$$\kappa x_t = \beta E_t \Lambda_{t,t+1} [h_{t+1}^n f_3(k_{t+1}, \mu_s h_{t+1}^s, h_{t+1}^n n_{t+1}) z_{t+1} - w_{t+1}^n h_{t+1}^n + \frac{\kappa}{2} x_{t+1}^2 + (1 - \rho) \kappa x_{t+1}].$$

**Distribution risk** In equilibrium, the extent of partial risk sharing that results from insider-stockholders and outsider-nonstockholders interacting in the bond market will influence the outcome of the Nash wage bargaining process and will in turn be affected by it. To measure the cumulative effect we introduce the ratio between the insider-stockholder's marginal utility and the outsider-nonstockholder's marginal utility:

$$\phi_t \equiv \frac{u_c((c_t^s - \chi c_{t-1}^s - H(h_t^s))}{v_c(c_t^n - n_t L(h_t^n))} = \frac{\lambda_t^s}{\lambda_t^n}. \quad (20)$$

as characterizing the extent of risk-sharing between these two groups. If  $\phi_t$  is constant across time and in all states, the relation (20) coincides with the efficient risk-sharing condition. Alternatively, suppose that  $\phi_t$  is constant across period  $t$  states for each  $t$  but time-varying.<sup>12</sup> A larger  $\phi_t$  is evidence of a greater share of aggregate income to workers while a smaller  $\phi_t$  suggests a greater share to capital owners (shareholders). Suppose, in addition, that  $\phi_t$  is time-varying and countercyclical over the business cycle. This countercyclicity means that when a high-productivity state is realized, a smaller  $\phi_t$  is realized and insider-stockholders (capital owners) reap most of the benefits from that high productivity state; in comparison, when a low-productivity state is realized, a greater share of aggregate income goes to outsider-nonstockholders, i.e. the normally low payment to capital owners is further reduced by labor's priority claim on output. Accordingly, the countercyclicity of  $\phi_t$  captures the idea that the shares of income going to labor and capital are not equally risky and that insider-stockholders, via the institution of the firm, are partially insuring the outsider-nonstockholders. This "distribution risk" (variation in  $\phi_t$ ) is largely borne by the firm and its owners.<sup>13, 14</sup>

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$MPL_{h_t^n}$ , is defined as  $\frac{\partial y_t}{\partial h_t^n} = n_t z_t f_3(k_t, \mu_s h_t^s, 1, h_t^n \cdot n_t)$ .

<sup>12</sup>Here the *optimal* contract is not necessarily optimal in the Pareto sense. In this case, relation (20) is reduced to the optimality condition of the Boldrin-Horvath (1995) type *optimal* contract.

<sup>13</sup>Empirically, labour's share is much less risky than the share going to capital; labour's claim on output is largely fixed and negotiated prior to the actual realization of the output.

<sup>14</sup>In an earlier paper, Danthine and Donaldson (2002) posit that the observed variations in factor income shares are the result of exogenous changes in this ratio  $\phi_t$  which they refer to as *distribution risk* (hereafter we call the ratio  $\phi_t$  distribution risk). This risk is assumed to be uninsurable. They view  $\phi_t$  as capturing the relative bargaining power of the two parties at the time the contract is negotiated. The assumed countercyclicity of this distribution risk guarantees that labour's share is much less risky than the share going to capital. In comparison, our endogenous distribution risk measure

We make no *a priori* assumption either about the cyclicity of distribution risk or about the source of this risk; rather, distribution risk in this economy is generated entirely endogenously in equilibrium: our economy features one source of uncertainty resulting from systemic risk (the economy-wide productivity shock). It turns out, however, that distribution risk (defined as per (20)) is indeed *countercyclical* over the business cycle in the present model. Furthermore, our Nash bargaining wage contract between capitalists (insider-stockholders) and laborers (outsider-nonstockholders) precisely identifies distribution risk  $\phi_t$  with the balance of "bargaining power" between capitalists and laborers. As a result, we provide a structural specification of the source of distribution risk.

**Outsider-nonstockholder's shadow value** The present discounted value to an outsider of employment in terms of current consumption in period  $t$ ,  $W_t$ , is defined recursively as

$$W_t = w_t^n h_t^n + (1 - \rho)\beta E_t \Lambda_{t,t+1}^n W_{t+1} + \rho\beta E_t \Lambda_{t,t+1}^n U_{t+1}$$

where  $\Lambda_{t,t+1}^n \equiv \frac{\lambda_{t+1}^n}{\lambda_t^n}$  is the outsider-nonstockholder's IMRS.

We recursively define  $U_t$  as the present discounted value to an outsider of unemployment in terms of current consumption in period  $t$ :

$$U_t = L(h_t^n) + b + s_t\beta E_t \Lambda_{t,t+1}^n W_{t+1} + (1 - s_t)\beta E_t \Lambda_{t,t+1}^n U_{t+1}.$$

Here, the value of being unemployed depends upon the outsider's current disutility of supplying hours  $L(h_t^n)$  (measured in units of final good consumption), his unemployment benefits  $b$ , and the likelihood of his being employed or unemployed next period; an unemployed outsider has a chance of finding a new job,  $s_t$ .

The outsider-nonstockholder's matching shadow value in terms of final good consumption,  $S_t^n$ , is therefore defined as the difference between the employment value and the unemployment value:

$$\begin{aligned} S_t^n &\equiv W_t - U_t \\ &= (w_t^n h_t^n - L(h_t^n) - b) + (1 - \rho - s_t)\beta E_t \Lambda_{t,t+1}^n S_{t+1}^n. \end{aligned} \tag{21}$$

Alternatively, the matching shadow value  $S_t^n$  can also be derived from the marginal benefit of a outsider-nonstockholder family from having an additional family member employed. The recursive is very different.

representation of the outsider-nonstockholder's problem is:

$$\begin{aligned}
V_t^n &\equiv V^n(\Omega_t^n) = \max_{\{b_{t+1}^n, h_t^n\}} \left[ \begin{aligned} &v(c_t^n - n_t L(h_t^n) - (1 - n_t)L(0)) \\ &+ \lambda_t^n (w_t^n h_t^n n_t + (1 - n_t)b + b_t^n - p_t^f b_{t+1}^n - c_t^n) \\ &+ \beta E(V^n(\Omega_{t+1}^n) \mid \Omega_t^n) \end{aligned} \right] \quad (22) \\
&\text{s.t.} \\
n_{t+1} &= (1 - \rho)n_t + s_t(1 - n_t).
\end{aligned}$$

The marginal benefit of one hired worker,  $V_{n_t}^n \equiv \frac{\partial V_t^n}{\partial n_t}$ , can be obtained by applying the Envelope theorem to representation (22):

$$\frac{\partial V_t^n}{\partial n_t} = w_t^n h_t^n \lambda_t^n - (L(h_t^n) + b)\lambda_t^n + \beta E_t \frac{\partial V_{t+1}^n}{\partial n_{t+1}} \frac{\partial n_{t+1}}{\partial n_t}$$

where  $\frac{\partial n_{t+1}}{\partial n_t} = (1 - \rho - s_t)$ .

Define the outsider-nonstockholder's shadow value to the firm of one hired worker,  $S_t^n$ , as

$$\begin{aligned}
S_t^n &\equiv \frac{1}{\lambda_t^n} \frac{\partial V_t^n}{\partial n_t} \quad (23) \\
&= (w_t^n h_t^n - L(h_t^n) - b) + (1 - \rho - s_t)\beta E_t \frac{\lambda_{t+1}^n}{\lambda_t^n} S_{t+1}^n.
\end{aligned}$$

It follows immediately that the above shadow value,  $S_t^n$  in (23), exactly coincides with the outsider-nonstockholder's matching shadow value (21).

**Nash wage bargaining** Before formalizing the Nash bargaining wage contract between insider-stockholders and outsider-nonstockholders, first note that the firm's intertemporal decisions are all in accord with the intertemporal marginal rate of substitution of the insider-stockholders: there is no agency problem between firm owners and managers in this environment. Accordingly, the firm's matching surplus can thus be identified with the marginal benefit to the representative shareholder of adding one outsider-nonstockholder worker. In other words, the firm's matching surplus, denoted  $V_{n_t}^s$ , can be formulated as:

$$V_{n_t}^s \equiv \frac{\partial V_t^s}{\partial n_t}$$

where  $V_t^s \equiv V^s(\Omega_t^s)$  is the value function of insider-stockholders.

As shown in the previous section, the outsider-nonstockholder's matching surplus,  $V_{n_t}^n$ , can be readily identified with the marginal benefit (to the family) of one additional worker being hired:

$$V_{n_t}^n \equiv \frac{\partial V_t^n}{\partial n_t}.$$



Identifying each matching surplus with its marginal benefit is appropriate in the situation where two heterogeneous agents with different attitudes toward risk bargain over the wage. Indeed, the existing game theory literature holds that the division of a joint bargaining surplus can be significantly affected by heterogeneity in the agents' risk aversion coefficients<sup>15</sup>. We therefore argue that the matching surplus in this environment should be defined in terms of marginal benefits in a manner that captures the nontrivial effect of risk aversion on bargaining.

Based on previous reasoning, the Nash wage bargaining problem between capitalists and workers can be formulated in the following way:

$$\max_{w_t^n} (V_{nt}^s)^{1-\eta} \cdot (V_{nt}^n)^\eta \quad (24)$$

where  $\eta$  is the bargaining power parameter of the outsider-nonstockholder group viewed, for the moment, as exogenously given.

The optimization above takes into account that in each period, outsiders' hours worked is set according to the following condition:

$$MRS_{c,l}^n = w_t^n \quad (25)$$

where  $MRS_{c,l}^n$  represents the outsider-nonstockholder's marginal rate of substitution for leisure vs. consumption.

The advantage of condition (25) is that the determination of hours worked is independent of any definition of the joint matching surplus corresponding to the Nash wage bargaining. The more popular specification, namely, the efficient bargaining contract, requires that

$$h_t^n \in \arg \max M_t \equiv V_{nt}^s + V_{nt}^n$$

when the joint matching surplus,  $M_t$ , is defined as  $V_{nt}^s + V_{nt}^n$ . As Nash (1950) showed, however, the joint matching surplus  $M_t$  can be any (convex and compact) subset of the sum  $V_{nt}^s + V_{nt}^n$ , i.e.  $M_t \subset V_{nt}^s + V_{nt}^n$ . Hence we argue that the condition (25) does not sacrifice much generality.

This mechanism for determining outsider's hours worked, proposed by Christiano et al (2007), can be viewed as capturing the idea that outsiders are monopolistic suppliers of labor hours as favored by New Keynesian models. Alternatively, the same specification can be viewed as describing the situation where a generic agency problem between the firm and workers cannot be ignored; the firm cannot completely induce workers' efforts (hours worked) since hours worked are in the nature of "hidden action." Indeed, condition (25) is strengthened by the observed fact that the hours worked per employee is rarely the object of bargaining agreements.

The wage  $w_t^n$  which solves the bargaining problem (24) must satisfy the following optimality condi-

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<sup>15</sup>For greater detail, see Roth and Rothblum (1982).

tion<sup>16</sup>:

$$\eta V_{n_t}^s = (1 - \eta) V_{n_t}^n. \quad (26)$$

Condition (26) can be rewritten as:

$$\eta \lambda_t^s J_t = (1 - \eta) \lambda_t^n (W_t - U_t). \quad (27)$$

using the substitutions  $V_{n_t}^s = \lambda_t^s J_t$  and  $V_{n_t}^n = \lambda_t^n (W_t - U_t)$ . A standard calculation based on the condition (27) guarantees that the Nash bargaining wage between two heterogeneous groups is given by

$$w_t^n = \frac{(1 - \eta) \frac{1}{\phi_t}}{(1 - \eta) \frac{1}{\phi_t} + \eta} \frac{[L(h_t^n) + b - F_t^n]}{h_t^n} + \frac{\eta}{(1 - \eta) \frac{1}{\phi_t} + \eta} \frac{[h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t + \frac{\kappa}{2} x_t^2 + F_t^s]}{h_t^n} \quad (28)$$

where  $F_t^n \equiv \beta(1 - \rho - s_t) E_t \frac{\lambda_{t+1}^n}{\lambda_t^n} (W_{t+1} - U_{t+1})$  and  $F_t^s \equiv \beta(1 - \rho) E_t \frac{\lambda_{t+1}^s}{\lambda_t^s} J_{t+1}$  denote, respectively, the future net expected welfare benefits to the outsider-nonstockholders and to the firm (insider-stockholders) from one additional employed worker. By the very presence of the term  $\phi_t$  in expression (28) it is apparent that the financial market structure influences Nash bargaining wage determination.

Letting  $\eta_t \equiv \frac{\eta}{(1 - \eta) \frac{1}{\phi_t} + \eta}$ , the solution (28) can be rewritten as:

$$w_t^n = (1 - \eta_t) \frac{[L(h_t^n) + b - F_t^n]}{h_t^n} + \eta_t \frac{[h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t + \frac{\kappa}{2} x_t^2 + F_t^s]}{h_t^n}. \quad (29)$$

This Nash bargained wage (29) is seen to nest the standard bargaining wage under the representative agent regime as a special case. In the case of the representative-agent construct, markets are complete so that  $\phi_t$  is equal to 1, and the solution (29) is reduced to the standard Nash bargaining solution ( $\eta_t = \eta$ ). This observation highlights the significant role of limited asset market participation in generating variable distribution risk  $\phi_t$ , and thus variable  $n_t$ .

More important, it can be shown that up to a first-order approximation,

$$\hat{\eta}_t = (\text{constant}) \cdot \hat{\phi}_t. \quad (17)$$

In other words, the notion of distribution risk can be identified with a Nash bargaining power shock up to a first-order approximation. Later it will be shown that distribution risk in this sense is countercyclical over the business cycle. Indeed, the countercyclicity of distribution risk in this model will play the key role in generating the unemployment fluctuations over the business cycle with the coveted properties: the countercyclicity of the distribution risk creates excessively smooth wages that induce a fixed wage income effect (the operating leverage effect), which encourages the observed volatility of key labor market

<sup>16</sup>This condition is called the *constant surplus sharing rule*.

<sup>17</sup>A  $\hat{\cdot}$  on a variable denotes log deviations from the corresponding steady-state value.

variables of interest. Our sense of distribution risk is thus exactly the same as the Nash bargaining power shock Shimer took into account without invoking its source (Shimer, 2005). In fact, our work may be viewed as providing microfoundations for the Shimer's *ad hoc* Nash bargaining power shock. Note that the only exogenous driving force in our economy is an aggregate productivity shock which induces the countercyclicality of our distribution risk. This may be seen as a direct answer to Shimer's unanswered question, as stated in Shimer (2005): "*It seems plausible that a model with a combination of wages and labor productivity shocks could generate the observed behavior of unemployment, vacancies, and real wages... the answered question is what exactly a wage shock is.*" Our model is a particular instance of what Shimer seeks. It also provides micro foundations for the exogenous distribution risk assumed in Danthine and Donaldson (2002).

## 2.6 Equilibrium

In this economy, market clearing requires that for all  $t$ ,

$$\begin{aligned} e_t &= \int e_t^s d\boldsymbol{\varkappa} = 1, \\ \phi\bar{k} &= \int b_t^s d\boldsymbol{\varkappa} + \int b_t^n d\boldsymbol{\omega}, \\ c_t &= \int c_t^s d\boldsymbol{\varkappa} + \int c_t^n d\boldsymbol{\omega}, \\ y_t &= c_t + i_t + \frac{\kappa}{2} x_t^2 n_t, \end{aligned}$$

where  $\boldsymbol{\varkappa}$  and  $\boldsymbol{\omega}$  respectively stand for the measure of insider-stockholders and the measure of outsider-nonstockholders. Lump sum transfers are taxed to balance the government budget constraint:

$$T_t + (1 - n_t)b = 0.$$

We define the equilibrium as follows:

**Definition 1** *Under the above market-clearing conditions, a decentralized stationary recursive equilibrium is defined as: a set of decision rules  $\{c_t^s(\cdot), c_t^n(\cdot); h_t^s(\cdot), h_t^n(\cdot); e_{t+1}(\cdot); i_t(\cdot), h_t(\cdot); \nu_t(\cdot)\}$  and a set of wage and price functions  $\{w_t^s(\cdot), w_t^n(\cdot); p_t^e(\cdot), p_t^f, d_t(\cdot)\}$  given the information set of aggregate states  $\Omega = \{k_t, n_t; \lambda_t\}$  such that (i)  $\{c_t^s(\cdot), h_t^s(\cdot); e_{t+1}(\cdot), b_{t+1}^s(\cdot)\}$  solves the intertemporal problem (1) given the information set  $\Omega_t^s$  (ii)  $\{c_t^n(\cdot), h_t^n(\cdot); b_{t+1}^n\}$  solves the outsider-nonstockholder's intertemporal problem (8) given his information set  $\Omega_t^n$  (iii)  $\{w_t^n(\cdot)\}$  satisfies the optimality condition (27) (iv)  $\{i_t(\cdot), x_t(\cdot)\}$  solves the firm's intertemporal problem given the information set  $\Omega^f$  (15) (v)  $w_t^s(\cdot)$  satisfies the condition (17) (vii)  $\{p_t^e(\cdot), d_t(\cdot)\}$  satisfies the Lucas asset pricing equations (6), while  $\{p_t^f(\cdot)\}$  satisfies the*

equations (7) and (14) (ix) The economy follows two laws of motion:  $k_{t+1} = (1 - \delta)k_t + G(\frac{i_t}{k_t})k_t$  and  $n_{t+1} = (1 - \rho)n_t + q_tv_t$ . Rational expectations are assumed for all agents.

## 2.7 Asset Pricing

Under the decentralized stationary recursive equilibrium defined in Section 3.8, it is possible to define and compute equilibrium asset prices and returns. Using the dividend series, the conditional price  $p^e(\Omega_t)$  of an equity security is recursively computed according to the Lucas' (1978a) asset pricing equation:

$$p^e(\Omega_t) = \beta E\left(\frac{\lambda_{t+1}^s}{\lambda_t^s} [p^e(\Omega_{t+1}) + d(\Omega_{t+1})] \mid \Omega_t\right),$$

where  $\Omega_t = \{k_t, n_t, z_t\}$  is the aggregate state of economy and  $\lambda_t^s = u_c(c^s(\Omega_t), h^s(\Omega_t))$  is the shareholder-worker's equilibrium marginal utility.

Using these prices, the time series of equity returns is computed in the conventional way:

$$R_{t,t+1}^e = \frac{p^e(\Omega_{t+1}) + d(\Omega_{t+1})}{p^e(\Omega_t)} - 1.$$

In a similar fashion, the price of a one-period risk-free real bond is given by

$$p^f(\Omega_t) = \beta E\left(\frac{\lambda_{t+1}}{\lambda_t} \mid \Omega_t\right)$$

where  $\lambda_t = u_c(c^s(\Omega_t), h^s(\Omega_t))$  or  $\lambda_t = v_c(c^n(\Omega_t), h^n(\Omega_t))$ . Note that the risk free bond is available to all households. The one period risk-free rate of return,  $R_t^f$ , is then computed using

$$R_t^f = \frac{1}{p^f(\Omega_t)} - 1.$$

Given the aggregate state  $\Omega_t = \{k_t, n_t, z_t\}$ , the conditional term structure  $\{R_{t,n}^f\}$  can also be derived. Let  $p_n^f(\Omega_t) = \beta^n E\left(\frac{\lambda_{t+n}}{\lambda_t} \mid \Omega_t\right)$  denote the price of a risk free discount bond in period  $t$  that pays one unit of consumption in period  $t + n$ . Then  $\{R_{t,n}^f\}$  is defined according to

$$R_{t,n}^f = \left[\frac{1}{p_n^f(\Omega_t)}\right]^{1/n} - 1,$$

Appendix 1 details the strategy for computing these various rates.

### 3 Calibration

In this paper, the business cycle is characterized as deviations from a Hodrick-Prescott filtered trend. The time unit of the model is three months. To match the US Solow residual we calibrate the process for aggregate productivity shocks to match the quarterly AR(1) process found by Cooley and Prescott (1995). The productivity shock  $z_t$  thus evolves according to the law of motion:

$$\log z_{t+1} = 0.95 \log z_t + \epsilon_{t+1}$$

where  $\epsilon$  is distributed normally, with mean zero and standard deviation  $\sigma_\epsilon$ ; in what follows, the standard deviation of technology shock  $\sigma_\epsilon$  will be chosen by a procedure of "hyperparameter search."

For all simulation runs, the production function employed is the customary Cobb-Douglas function

$$z_t f(k_t, h_t^s \cdot 1, h_t^n \cdot n_t) = z_t M k_t^\alpha ((\mu_s h_t^s \cdot 1)^\mu (h_t^n \cdot n_t)^{1-\mu})^{1-\alpha}$$

where  $\mu \equiv \frac{\mu_s}{1+\mu_s}$ .

The parameter  $M$  serves as a scale parameter, while  $\mu = \frac{\mu_s}{1+\mu_s}$  and  $1 - \mu$  are, respectively, the normalized measures of insider-stockholders and the outsider-nonstockholders. To allow for debt-financing while imposing the constraint that corporate debt is risk-free, we scale our production technology by setting  $M = 1.25$ . This makes the average output high enough to guarantee a uniformly positive dividend in all states of nature for empirically relevant calibrations of the firm's debt level. Following Guvenen (2003), the stock market participation rate,  $\mu_s$ , is set to be 25 percent, so that  $\mu$  equals 0.20.

The parameter  $\alpha$  is typically calibrated to reproduce the observed share of capital in total value added. We adopt the most commonly used value, 0.36. The subjective discount factor  $\beta$  is fixed at  $\beta = 0.99$ , corresponding to a steady state return on capital of 4%. Following Kydland and Prescott (1982), the quarterly capital depreciation rate  $\delta$  is 0.020.

The model economy assumes that search and matching frictions characterize the labor market only for outsider-nonstockholders. Therefore, we calibrate the labor market for outsider-nonstockholders using standard parameters for labor market search and matching.

The empirical literature provides several estimates of the US worker separation rate. We follow Davis, Haltiwanger and Schuh (1996) and fix the quarterly separation rate  $\rho$  at 8 percent. According to Petronglo and Pissarides (2001), the elasticity of matches to unemployment of outsiders  $1 - \sigma$  falls within the range of plausible values of 0.5 to 0.7. We set  $1 - \sigma$  to be 0.5. The mean quarterly unemployment rate of the model economy is set to 6%, which is customary in the literature (e.g. Merz (1995) and Christoffel and Kuester (2008)). Following, e.g. Cooley and Quadrini (1999), the steady state value of the vacancy-filling probability  $\bar{q}$  is set to be 0.7. The existing literature mostly suggests that the bargaining power parameter  $\eta$  is equal to 0.5; we follow suit.

The choice of the unemployment benefit  $b$  is controversial. In Shimer (2005), the unemployment benefit  $b$  is set to 0.40 so that average "replacement rate," i.e., the ratio of benefits to average wages, is 0.41. This value implies that the matching model cannot account for the observed fluctuations of unemployment. In Hagedon and Manovskii (2006), the unemployment benefit  $b$  is set to 0.95 so that the average replacement rates are 0.98, which contributes to the *opposite* conclusion that the standard search model *is* consistent with the data. The main reason behind these conflicting conclusions is that higher unemployment benefits  $b$  make workers indifferent to the substitution between working and not working. This indifference significantly dampens the variations of the standard period-by-period Nash bargaining wage over the business cycle. In short, Shimer's critique is not extremely robust to the choice of the parameter  $b$ .

To avoid the above controversy, we put more restrictions on the choice of  $b$ . The OECD (1996), in particular, computes the average replacement rates across countries, and finds that average replacement rates are at most 0.20 in the United States (Hornstein et al, 2005). For this reason, we choose  $b$  to be consistent with this empirical evidence;  $\frac{b}{\bar{w}^n \bar{h}^n} = 0.20$ . Table 1 summarizes the prior discussion.

Table 1: Unemployment benefit as a fraction of the average wage income of outsider-nonstockholders: various estimates

	US Data	This paper	Shimer	Hagedon & Manovskii	Hall
$\frac{b}{\bar{w}^n \bar{h}^n}$	0.2	0.20	0.41	0.98	0.41

The vacancy cost  $\kappa$  is chosen endogenously so that the steady state ratio of adjustment costs to output  $\frac{\kappa}{2} \frac{\bar{x}^2 \bar{n}}{\bar{y}}$  is 0.01. This ratio is a widely accepted upper bound in the business cycle literature. The period utility function of the representative insider-stockholder is postulated as

$$u(c_t^s - X_t - H(h_t^s)) = \frac{(c_t^s - X_t - B_s(h_t^s)^{\psi_s})^{1-\gamma_s} - 1}{1 - \gamma_s}$$

where  $\gamma_s$  is the coefficient of insider-stockholder's relative risk aversion,  $\psi_s$  is the insider-stockholder's disutility parameter of labor hours, which controls the Frisch elasticity of labor, and  $H(h_t^s) = B_s(h_t^s)^{\psi_s}$ . The disutility parameter  $B_s$  is obtained from the steady state calculation.<sup>18</sup>

The preference of the representative outsider-nonstockholder is postulated as

$$v(c_t^n - n_t L(h_t^n) - (1 - n_t)L(0)) = \frac{(c_t^n - n_t B_n(h_t^n)^{\psi_n})^{1-\gamma_n} - 1}{1 - \gamma_n}$$

where  $\gamma_n$  is the coefficient of outsider-nonstockholder's relative risk aversion,  $\psi_n$  is the outsider-nonstockholder's disutility parameter of labor hours and  $L(h_t^n) = B_n(h_t^n)^{\psi_n}$ . The disutility parameter  $B_n$  is also obtained

<sup>18</sup>  $B_s$  (and  $B_n$  below) are chosen so that  $\bar{h}^s = \bar{h}^n = 1/3$ .

from the steady state calculation. We assume that  $\gamma_s$  is equal to  $\gamma_n$  and  $\psi_s$  is equal to  $\psi_n$  with  $\gamma$  denoting the economy-wide coefficient of relative risk aversion (i.e.  $\gamma_s = \gamma_n \equiv \gamma$ ) and  $\psi$  as the economy-wide disutility-of-labor parameter (i.e.  $\psi_s = \psi_n \equiv \psi$ ). With these identifications, none of the results cited below can be attributed to differential risk aversion.<sup>19</sup>

It is well known that empirical studies do not offer much precise guidance when it comes to calibrating the habit formation parameter  $\chi$ , the capital adjustment cost  $\xi$  and the coefficients of relative risk aversion  $\gamma$ . It is also widely known that the standard deviation of the technology shock innovation,  $\sigma_\epsilon$ , is difficult to measure from available data since this number, usually identified with the direct estimate of the volatility of Solow residual for the post war period, is significantly affected by measurement error. Furthermore, a high value of  $\sigma_\epsilon$  suggests a probability of technological regress that is implausibly large. Lastly, we add the disutility-of-labor parameter  $\psi$  to our list of free parameters. Although it is believed to be less than 0.5 (e.g., McCurdy (1981)), the estimate of the Frisch elasticity of labor supply is not conclusive. Indeed, Imai and Kean (2004) recently estimated the Frisch elasticity of labor supply as 3.8, which is much higher than what is generally believed.

The lack of clarity in parameter determination leads us to conduct a "hyperparameter search" for the parameters that are free at this point ( $\chi, \xi, \gamma, \psi, \sigma_\epsilon$ ) to match a set of empirical targets of interest. This amounts to minimizing an equally weighted quadratic criterion function written in the deviation from each empirical target in the manner of Jermann (1998). For the baseline calibration, we choose the free parameters ( $\chi, \xi, \gamma, \psi, \sigma_\epsilon$ ) to match four empirical targets: (i) the relative standard deviation of unemployment (a ratio of unemployment volatility to output volatility) (ii) the risk-free rate volatility (iii) the mean risk-free rate and (iv) the equity premium. Practically, we restrict our hyperparameter search to a grid of values for  $\chi \in [0, 0.9]$ ,  $\xi \in [0.23, \infty)$ ,  $\sigma_\epsilon \in [0.0037, 0.00712]$ ,  $\psi \in [1, 2]$  and  $\gamma \in [1, 7]$ . These intervals encompass most estimates from the literature. For the baseline calibration, the minimum is achieved for  $\sigma_\epsilon = 0.006$ ,  $\chi = 0.9$ ,  $\xi = 0.23$ ,  $\psi = 1.4$  and  $\gamma = 3.6$ . A value of  $\psi = 1.4$  implies that the Frisch elasticity of labor supply in this economy is  $\frac{1}{1.4-1} = 2.5$  as in Jaimovich and Rebelo (2008). Our Frisch elasticity of labor supply is thus higher than its traditional estimate but is less than the Imai-Kean estimate of 3.8. At 0.6%, the value of the innovation standard deviation is much smaller than the values used by other macro-asset pricing models, e.g., Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), and Guvenen (2003). These models value the innovation standard deviation per quarter at close to 2%. For instance, Boldrin, Christiano and Fisher (2001) use permanent shocks with a standard deviation of 1.8% per quarter. Indeed, our value is even smaller than the direct estimate of the volatility of Solow residuals for the post war period, which is about 0.7%. We view a reduced reliance on large technology disturbances as a favorable attribute of the model. The model is then solved using the log-linearization methods widely employed in the business cycle literature. Log-normal formulae are

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<sup>19</sup>This being said, we recognize that habit formation makes the insider-stockholder effectively more risk averse than the outsider-nonstockholder.

applied to price the relevant asset returns (see e.g. Uhlig (1999) or Jermann (1998) and Appendix 1).<sup>20</sup>,

21

## 4 Results

### 4.1 Model Results

**Reassessing Shimer's critique:** Before reporting the quantitative results for the baseline model, we raise several issues as to how Shimer's critique might be best represented in (real) business cycle models with labor-market search, and modify it accordingly. In his seminal paper, Shimer claims that the incorporation of the standard search model into a real business cycle framework with intertemporal substitution of leisure, capital accumulation, and other extensions such as the Merz (1995) or Andolfatto (1996) models does not invalidate his critique. In his words, "Neither paper can match the negative correlation between unemployment and vacancies, and both papers generate real wages that are too flexible in response to productivity shocks" (p.45). Indeed, the Andolfatto model does not pass the litmus test for the unemployment volatility puzzle Shimer raises: the model allows for a real wage that is too flexible in response to productivity shocks with the result that the volatility of job vacancies is too low to match its empirical counterpart. The Merz model, however, is hard to reject on this basis alone. Table 2 in her paper shows that the model with fixed search intensity can replicate, quite well, the basic stylized facts of labor market volatility; the wage is indeed rigid in terms of its relative standard deviation ( $\frac{\sigma_w}{\sigma_y} = 0.34$ ) and the job vacancies are reasonably volatile ( $\frac{\sigma_v}{\sigma_y} = 6.38$ ). Both models generate the negative correlation between unemployment and vacancies, although that correlation is only weakly negative. Furthermore, it can be shown, up to a first-order approximation, that the Merz model with fixed search intensity is isomorphic to the Andolfatto model with inelastic labor supply of hours. The relative success of the Merz model (with fixed search intensity) in generating realistic labor market statistics rides not only on wage stickiness, however, but also on the absence of variations at the intensive margin. If the Merz model were to allow for variations at the intensive margin, its ability to explain labor market volatility might be significantly compromised; the representative firm now could substitute between hours per incumbent and hiring new workers. This substitution effect is not negligible over the business cycle, and explains why the Andolfatto model performs so poorly on the dimensions of the labor market business cycles: it allows both variations. Accordingly, a DSGE model's ability to resolve the unemployment volatility puzzle may depend upon the extent to which the labor

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<sup>20</sup>Log-normal formulae can be found in the Appendix 1.

<sup>21</sup>Given the generally accepted parameter choices from earlier macro studies and the parameters arising from the hyperparameter search, we solve for all the steady state variables under the added assumption that  $\bar{n} = .90$ ,  $\bar{u} = .10$  (unemployment),  $\bar{q} = .7$ , and  $\bar{h}^s = \bar{h}^n = 1/3$ . These latter choices, commonplace in the literature, in turn determine  $B_s$ ,  $B_n$ ,  $\sigma_n$ , etc.



supply of hours is elastic. To see if (quarterly) business cycle models with labor-market search can pass a litmus test for the resolution of the unemployment volatility puzzle, a consideration of variations at both the intensive margin and at the extensive margin is required.

We propose the following expansion of Shimer’s critique: (i) a quarterly business cycle model with labor-market search must generate the absolute amplitude of the standard deviations of key variables in the labor market activities as well as their relative magnitude vis-a-vis the standard deviation of output; (ii) the model must allow for variations at the intensive margin and at the extensive margin simultaneously; and (iii) the negative correlation between unemployment and vacancies must be substantially consistent with the data.<sup>22</sup> The present model possesses all of these features.

Table 2 reports the second moments of endogenous aggregate variables as implied by the model, namely unconditional standard deviations, and their contemporaneous correlation with output, alongside the moments implied by the data. Table 4 reports the associated financial statistics implied by the model alongside the financial statistics implied by the data (Mehra and Prescott, 1985). These results are discussed below.

Table 2: Aggregate business cycle statistics: the baseline model

<b>Business Cycle Statistics</b>							
Variable	Meaning	Std		Std. to $\sigma_y$		Corr. with $y$	
		Data <sup>(i)</sup>	Model	Data	Model	Data	Model
$y$	output	1.59	1.47	-	-	-	-
$c$	consumption	1.23	1.39	0.77	0.95	0.83	0.94
$i$	investment	4.87	2.22	3.06	1.51	0.91	0.86
$h^{total}$	total hours <sup>(i)</sup>	1.51	1.34	0.95	0.91	0.92	0.90
$h$	hours per worker <sup>(ii)</sup>	0.69	0.65	0.43	0.44	0.62	0.90
$h^s$	hours per insider	-	1.05	-	0.71	-	1.00
$h^n$	hours per outsider	-	0.56	-	0.38	-	0.87
$w$	wage <sup>(iii)</sup>	0.70	0.37	0.44	0.25	0.68	0.88
$w^s$	wage per insider	-	0.42	-	0.29	-	1.00
$w^n$	wage per outsider	-	0.23	-	0.16	-	0.87
$n$	employment	1.02	0.90	0.64	0.61	0.78	0.98
$u$	unemployment	11.01	10.36	6.92	7.05	-0.87	-0.84
$\nu$	vacancy	13.15	13.42	8.27	9.13	0.91	1.00
$\theta$	tightness	21.66	22.52	13.62	15.32	0.90	0.98

$$^{(i)} h_t^{tot} = \mu_s h_t^s + n_t h_t^n$$

$$^{(ii)} h_t = h_t^{tot} / n_t + \mu_s$$

$$^{(iii)} w_t = \mu_s w_t^s + n_t + w_t^n$$

<sup>22</sup>The Merz model (with fixed search intensity) cannot pass Shimer’s (2005) litmus test for the resolution of the unemployment volatility puzzle. For instance, the amplitude of the standard deviation of vacancies is 6.85% while its empirical counterpart is around 13.15%; it also violates the condition (iii); the correlation between unemployment and vacancies (-0.15) falls short of its realism (-0.89); and the Merz model allows only for variations at the extensive margin.

**Labor market volatility:** The model reproduces the substantial fluctuations in the key variables of labor market activity found in the data and emphasized by Shimer (2005) and Hall (2005). In particular, in terms of the (absolute) volatility, the model comes remarkably close to the (absolute) volatilities of the key labor market variables including unemployment  $u$ , vacancies  $\nu$ , and the market tightness measure  $\theta \equiv \frac{\nu}{u}$ . This indicates that the propagation mechanism in this model economy is quite powerful since the standard deviation of the productivity shock required to produce the observed variations in the labor market variables of interest is 0.006, which is smaller than the direct estimate of the volatility of Solow residuals from the post war data (about 0.007).

A distinguishing feature of our analysis is that we can disentangle the variations at the intensive margin from the variations at the extensive margin. Fortunately, the model comes close to matching precisely both the relative volatility of total hours (0.91 versus 0.95 in the data) and hours per worker (0.44 versus 0.43 in the data). Although the correlation of hours per worker with output is too procyclical, the model nevertheless captures the basic reality of the labor market as displayed in the data.<sup>23</sup> As a consequence, the statistical behavior of employment also comes reasonably close to its empirical counterpart.

Along the wage dimensions, however, the model somewhat overstates or understates the empirical analogues: the real hourly wage is insufficiently volatile and the contemporaneous correlation of hourly wage with output is too procyclical. The departure of hourly wage volatility from its empirical magnitude is in a way predictable. The Nash bargaining wage (wage per outsider) in this model economy is significantly affected by the countercyclicality of endogenous distribution risk or Nash bargaining power shock. This effect dampens the variations in the Nash bargaining wage over the business cycle. Indeed, the endogenous distribution risk is both highly volatile and strongly countercyclical, and thus the equilibrium wage is less volatile over the business cycle. Nevertheless, the correlation of the wage per outsider with output is still procyclical. The wage per insider is also less volatile, but its root cause is quite different: it is determined by the marginal product of labor. This mechanism for wage determination usually results in low volatility and strong procyclicality. In the indivisible RBC model of Hansen (1985), where the wage coincides with the marginal product of labor, for example, the relative standard deviation of the real wage is 0.28 and the correlation of the wage with output is 0.88.

Additional insight into the resolution of the unemployment volatility puzzle can be obtained by examining the model's impulse response functions to estimate how a positive 1% productivity shock

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<sup>23</sup>For the U.S. historical period 1964:1 - 2002:1, Cheron and Langot (2004) report that  $\text{corr}(w, y) = .28$ , a much lower value than we report in Table 2 ( $\text{corr}(w, y) = .68$ ). In order to achieve a wage-output contemporaneous correlation this low these authors employ a Rogerson and Wright (1988) utility specification of the form

$$\frac{(c_t^n - n_t L(h_t^n))^{1-\gamma}}{1-\gamma} + ac_t^n, a > 0.$$

They work, however, with a representative agent formulation similar to Andolfatto (1996). We suspect that this modification of worker preferences would, in our context, work towards the same goal. It has the added feature that if the constant  $a > 0$  is properly chosen the utility of the non-shareholder workers who are employed will exceed that of their unemployed family members.

affects the key decision variables in the benchmark model. Using the method of undetermined coefficients proposed by Campbell (1994), the key detrended endogenous variables are expressed as a linear function of the state variables (in logs). For instance, consumption in the baseline model can be expressed as:

$$\hat{c}_t = \eta_{cz}\hat{z}_t + \vec{\eta}_{cs} \cdot \vec{s}_t.$$

Here  $\eta_{xy}$  denotes the elasticity of endogenous variable "x" with respect to state variable "y",  $\vec{s}_t$  is the vector of state variables itself and  $\vec{\eta}_{cs}$  is the corresponding vector of the elasticities of endogenous variable "x" with respect to the vector  $\vec{s}_t$ . Table 3 summarizes the elasticities of the endogenous variables of interest with respect to productivity shock  $z$ .

Table 3: Equilibrium elasticities for the baseline model

	$\eta_{yz}$	$\eta_{\lambda^s z}$	$\eta_{wz}$	$\eta_{w^n z}$	$\eta_{hz}$	$\eta_{\nu z}$	$\eta_{wiz}$	$\eta_{lsz}$	$\eta_{dz}$	$\eta_{\phi z}$
Model										
Baseline	1.07%	-19.84%	-0.001%	-0.02%	0.12%	11.706%	-0.07%	-1.08%	3.78%	-10.23

In response to a positive 1% productivity shock, Nash wage bargaining between capital owners (insiders) and workers (outsider-nonstockholders) makes it possible for capital owners to provide workers with limited insurance against their labor income variations. Indeed, the distribution risk, namely workers' bargaining power,  $\phi_t$ , immediately drops down by 10.23%, signalling a dramatic decrease of workers' bargaining power. This in turn dampens the volatility of Nash bargaining wage (outsider-nonstockholder's wage)  $w_t^n$  and thus the bargaining wage only decreases slightly (by 0.02%). In turn, the overall average wage in this economy is almost acyclical; in response to a positive 1% productivity shock, the average wage  $w_t$  varies by -0.001%. Note that 80 percent of the workforce in this economy comes from the outsider-nonstockholders. Since outsider's labor hours supplied are dependent on the determination of Nash bargaining wage, they too are dampened and the overall hours supplied merely increase by 0.12%. As a result, aggregate wage income,  $wi_t \equiv \mu_s w_t^s h_t^s + w_t^n h_t^n n_t$ , is nearly acyclical; i.e. it drops by 0.07% while the labor income share drops significantly by 1.08%. This fixed wage effect (via the operating leverage mechanism) amplifies the firm's free cash flow: indeed, the dividends increase by a substantial 3.78%.

To smooth their consumption, capital owners (insider-stockholders), who already face both a high level of volatility of marginal rates of substitution ( $\eta_{\lambda^s z} = -19.84\%$ ) and the frictional reallocation of capital due to the *a priori* specified cost of adjusting capital, now must deal with the additional frictions of reallocating labor inputs due to the distribution risk and the resulting bargaining wage for any given magnitude of workers (outsider-nonstockholders) employed. Therefore, in the last resort, capital owners

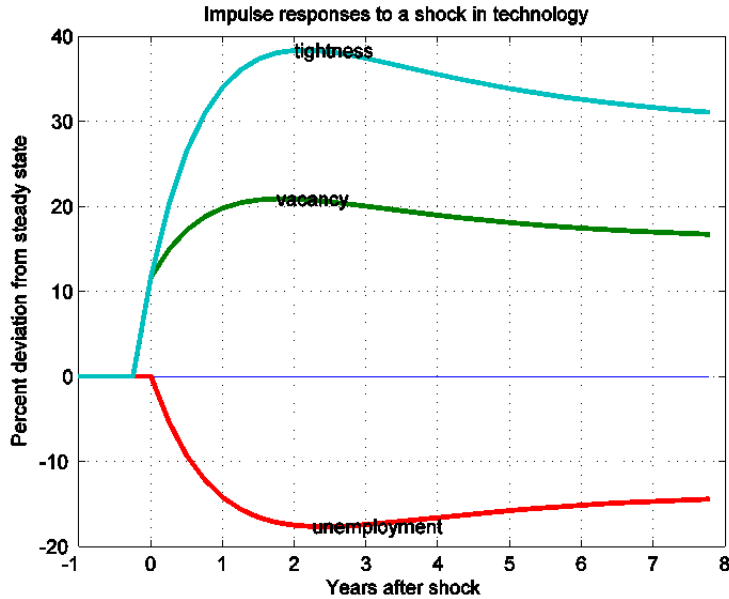


Figure 1: Employment fluctuations: baseline model

end up seeking to increase employment in the next period,  $n_{t+1}$ , by enormously increasing job vacancy postings; in other words, expecting trading frictions due to imperfect job matches in the labor market for outsider-nonstockholders, capital owners (firms) increase job vacancies by 11.706%. As they build up the employment level of workers in the following period, market tightness also increases dramatically while the unemployment decreases persistently (See Figure 1). As capital owners build up the labor stock of workers, however, wage income gets more risky than output, and, after one year, the rise of wage income exceeds that of output; in other words, the operating leverage effect or the fixed wage income effect is completely destroyed after one year. We conclude that our operating leverage channel is a short-run mechanism for shifting workers' labor income risk on to the capital owners.

In sum, we argue that the short-run operating leverage channel is the key mechanism for resolving the unemployment puzzle. Distribution risk plays a key role in generating this short-run operating leverage channel: the countercyclical distribution risk (workers' bargaining power) dampens the resulting equilibrium bargaining wages significantly, creating the rigid wage income effect.

**Aggregate volatilities:** Qualitatively, the model respects the basic business cycle stylized facts quite well: investment volatility exceeds that of output which, in turn, exceeds that of consumption. Aggregate hours volatility is only slightly less than output, as in the data. As Table 2 shows, however, there is a downside: the absolute volatilities of consumption and investment depart from their empirical counterparts. Total consumption has 95 percent of the volatility of output and investment is only

one and half times as volatile as output. These results may be laid at the feet of our model's capital adjustment costs. As well documented in Jermann (1998), capital adjustment costs make it more costly to smooth consumption through changing the capital stock, resulting in a lower volatility of investment. Consumers end up taking more consumption risk (higher volatility of total consumption).

**Financial statistics: the equity premium:** For the basic return statistics related to the equity premium – the short rate (or one-period risk free rate), the return on equity, and the premium itself (all averages) – the model provides a quite reasonable match of theory to data (see Table 4). Return volatilities are especially close to their empirical counterparts (the Mehra and Prescott (1985) statistics). As is typical of this style of model, the mean risk free rate is a bit too high and the mean equity return about one and one-half percentage points too low relative to the period (1889-1978) studied by Mehra and Prescott (1985). For the expanded period 1871-1993, however, Campbell and Cochrane (1999) report a U.S. equity premium of 3.9% which is very similar to its model generated counterpart.

Table 4: Financial statistics: the baseline model

<b>Financial Statistics</b>					
Variable	Meaning	Mean		Std	
		Data	Model	Data	Model
$R^f$	risk-free bond return	0.80	1.24	5.67	6.48
$R^e$	equity return	6.98	5.48	16.54	17.71
$R^P$	equity premium	6.18	4.28	16.67	17.40

**Financial statistics: the term structure:** It is well documented that standard RBC models (e.g., the indivisible labor model of Hansen (1985)) with persistent technology shocks generate a downward-sloping average real term structure, as shown in the 4th column of Table 5 (the "RBC model"). In contrast, the average real term structure generated by the present model is upward sloping with volatility declining with a longer time to maturity (Benchmark calibration in Table 2). Based on return data for U.S. traded TIPS, McCullough's web page reports that the average real term structure for the U.S. is also upward sloping, a fact confirmed by Sinha (2010) using an expanded TIPS data set (Table 5). Using much more extensive U.K. inflation indexed security returns Sinha (2010) also confirms an upward sloping real term structure for that nation as well. Mishkin's (1990) conclusion that the real and nominal term structures move together also argues for an upward sloping real term structure. While the present model does well regarding the replication of this particular term structure shape, and the declining volatilities, the absolute level of returns and return volatilities remain somewhat excessive vis-a-vis the data.

The intuitive explanation behind the positive slope is as follows: Although our preference specifica-

tion does not belong to a class of "generalized expected utility" preferences, the baseline model conveys the sense that there is strong preference for late resolution of uncertainty among agents in the model economy: in other words,  $CRRA = \gamma = 3.6 \ll \frac{1}{EIS} = \frac{1}{0.0307} = 25.25$ . Here the EIS is understood as the model-implied aggregate EIS, computed as per Appendix 2. When there is a preference for a late resolution of uncertainty, agents prefer to buy short maturity bonds and roll them over instead of buying long maturity bonds, which pay off only in a single distant time period. As a result, the demand for short term bonds is high while the demand for long term bonds is low; consequently, the prices of short term bonds are high relative to the prices of long term bonds.

The same intuition is found in Binsbergen et al. (2008); in their estimated DSGE model with fully specified Epstein-Zin preferences, they estimate a low elasticity of intertemporal substitution (around 0.06) from upward-sloping (nominal) yield curve data and macro data. Their estimates also satisfy the condition that  $CRRA \ll \frac{1}{EIS}$ , which further supports the hypothesis that there is strong preference for a late resolution of uncertainty.

Table 5: Term structure: the baseline model<sup>(i)</sup>

Maturity	US Data				Model			
	Nominal Data		McCulloch Data		Benchmark (this paper)		Hansen	
	Mean	Std	Mean	Std	Mean	SD	Mean	SD
4	5.60	2.93	1.06	1.61	1.39	6.32	4.0387	0.31
8	5.81	2.89	1.39	1.37	1.76	5.72	4.0375	0.27
12	5.98	2.82	1.69	1.23	2.10	4.98	4.0370	0.25
16	6.11	2.79	1.95	1.15	2.37	4.30	4.0368	0.23
20	6.19	2.74	2.16	1.09	2.57	3.72	4.0366	0.22

<sup>(i)</sup> Maturity in quarters. Nominal data is from Wachter (2006).  
McCulloch data is drawn from Piazzesi and Schneider (2006).

**Shimer's Calibration:** The baseline model also replicates the observed fluctuations in unemployment and job vacancies at the business cycle frequencies computed in Shimer (2005). In constructing his statistics, Shimer chooses a much smoother trend component, corresponding to an HP filter smoothing parameter of  $10^5$ . Accordingly, we do the same for both data and model generated statistics. Since a  $\lambda = 10,000$  gives rise in the data to a  $\sigma_y = 2\%$ , the model's  $\sigma_\epsilon$  must be correspondingly increased to match this figure for the model's detrended output. A choice of  $\sigma_\epsilon = .812\%$  accomplishes this goal; otherwise, all the parameter choices are taken from the benchmark parameter parameterization.

Table 6 replicates the results of this exercise. As would be expected, all volatilities are higher under this new decomposition. Nevertheless, the match of model statistics to data continues to be very close.

## 4.2 Attribution of Results

Table 6: Labor market volatility: the baseline model with Shimer’s detrending parameter

	$u$	$\nu$	$\theta$
Standard deviation	17.3 (19.0)	22.3 (20.2)	37.6 (38.2)
Quartely autocorrelation	0.94 (0.936)	0.88 (0.940)	0.94 (0.941)
Correlation matrix	$u$	1	-0.80 (-0.894)
	$\nu$	-	1
	$\theta$	-	-
			1

In this section we seek to measure the relative contributions of the baseline model’s principal features to its overall performance. These features include (1) search and matching frictions for outsider-nonshareholders, (2) Nash wage bargaining for determining the wage of the insider-nonstockholders, and (3) "distribution risk" arising from incomplete financial markets and the resulting provision of income insurance to the outsider-nonstockholders by the insider-stockholders. Initially the discussion will focus on macro quantities and then move to the determinants of the equity premium.

To do this we analyze three simplified versions of the benchmark model and contrast the results obtained with those of the benchmark itself. They are: Model A – a standard (no search, no Nash bargaining, complete asset markets) representative agent RBC model with competitive labor markets for both insiders and outsiders; Model B – a model which is otherwise identical to that in Model A but with search and matching frictions in the market for the labor of outsiders; asset markets are also complete; and Model C – a model that is in every way identical to the benchmark but which allows both insiders and outsiders to trade the same subset of securities (bonds and stocks only). Model B is essentially the model of Andolfatto (1996); Model C is described more fully in Appendix 2 (as are all versions). For business cycle related aggregate volatilities, the results of this exercise can be found in Table 7. In Table 8, the current benchmark model is also analyzed under the added requirement of no-habit formation; i.e.,  $\chi = 0$ .

Model A’s results noticeably fail to replicate the stylized facts of the business cycle: consumption is more volatile than output, while investment is much less so. In a standard RBC formulation otherwise to Model A (but with CRRA utility-of-consumption and no COA for capital), one would expect to see an extremely smooth consumption with  $\sigma_c$  much less than  $\sigma_y$  and  $\sigma_i > \sigma_y$ . This follows from the fact that Model A admits three tools for consumption smoothing:  $\{h_t^s\}$ ,  $\{h_t^n\}$  and  $\{i_t\}$  all may be freely adjusted to stabilize consumption.

Under the present formulation of Model A, these smoothing effects are much weaker for two principal reasons. First, under GHH preferences, there is no wealth effect influence on labor supply with the equilibrium consequences of substantial hours variation (restrained only by the convexity of labor disutility). with the high procyclicality of both hours and wages, the result is high labor income volatility. Since dividend income, in magnitude, is small relative to labor income, total income volatility is high for both

Table 7: Relative aggregate volatilities for a variety of related models

Aggregate Statistics															
	Data			Model A			Model B			Model C			Model 4 (baseline)		
	(a)	(b)	(c)	(a)	(b)	(c)	a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$y$	1.59			1.59									1.47		
$c$	1.23	.77	.83	1.70	1.07	1.00							1.39		
$i$	4.87	3.06	.91	1.26	.79	1.00							2.22		
$k$															
$h^{tot}$	.69	.05	.92										1.34		
$h^s$				1.13	.71	–							1.05		
$h^n$				1.13	.71	–							.56		
$w$	.70	.44	.68			1.00							.37		
$w^s$				.46	.29	–							.42		
$w^n$				.46	.29	–							.23		
$n$	1.02	.64	.78		–	–							.90		
$u$	11.01	6.92	–.87		–	–							10.36		
$v$	13.15	8.27	.91		–	–							13.42		
$\theta$	21.66	13.62	.90		–	–							22.52		
$\phi$					–	–									
$\epsilon$	.712			.668											

Financial Statistics					
	Data	Model A	Model B	Model C	Model 4
$E[R_t^c]$	6.98	3.80			5.48
$\sigma[R_t^c]$	16.54	2.91			17.71
$E[R_t^f]$	.80	3.59			1.24
$\sigma[R_t^f]$	5.67	.96			6.48
$E[R_t^p]$	6.18	.21			4.28
$\sigma[R_t^p]$	16.67				17.40

(i) (a) standard deviation, (b) s.d./ $\sigma_y$ , (c)  $\text{corr}(x, y)$  for all aggregates "x"

(ii) For all cases  $\gamma = 3.6$ ,  $\beta = .99$ ,  $\alpha = .36$ ,  $\mu_s = .20$ ,  $\psi = 1.4$ ,  $\xi = .23$ ,  $\sigma = .5$ ,  $b/\bar{w}^n \bar{h}^n = .9$ ,  $\rho = .08$ ,  $n = .5$ ,  $\bar{q} = .7$ , etc. as applicable.

agent groups. With savings being undertaken exclusively by the firm under our decentralization scheme, consumption volatility for both agents is high relative to output.<sup>24</sup> Second, the COA on capital makes it expensive for the planner to smooth consumption via investment variation. Discouraged from doing so he selects a smooth investment series. While high hours volatility would naturally go in tandem with low wage volatility (as observed), wage variation is counterfactually low.<sup>25</sup> Finally, with full employment at all times, there is no unemployment as vacancy volatility. All in all, Model A can be judged to fail

<sup>24</sup>Note that for Model A,  $\gamma$ ,  $\psi$ ,  $B_n$  and  $B_s$  are the same for both agents. Under complete financial markets, there is perfect income insurance and  $c_t^n = c_t^s = c_t$  with competitive labor markets and identical utility parameters, wages and the wage bill are perfectly correlated with output and proportional to one another. With perfect risk sharing,  $\phi_t \equiv 1$ , and displays no volatility.

<sup>25</sup>This low wage volatility, however, holds promise that GHH preferences will be a significant determinant of any resolution to Shimer's puzzle.



along the aggregate dimensions, especially those of special interest in the present paper.

The same comment applies to the financial dimensions of the model. While consumption volatility is high on a relative basis, it is still low in an absolute sense with the result that both agents view the equity and debt securities as pretty much equivalently risky. Accordingly, the premium is a miserable .21% and all return volatilities are dramatically too low.

Table 8: Robustness: business cycles

	$\sigma_u/\sigma_y$	$\sigma_\nu/\sigma_y$	$\sigma_\theta/\sigma_y$	$\rho(u, \nu)$	$\sigma_{th}/\sigma_y$	$\sigma_h/\sigma_y$	$\sigma_w/\sigma_y$	$\sigma_\phi/\sigma_y$	$\rho(\phi, y)$	$\rho(ls, y)$
Data	6.92	8.27	13.62	-0.88	0.95	0.43	0.44	-	-	-0.053*
Model										
	Limited participation models									
Baseline	7.05	9.13	15.32	-0.79	0.91	0.44	0.25	5.38	-0.89	-0.03
No habit	5.82	7.57	12.66	-0.78	0.75	0.37	0.20	4.77	-0.95	-0.39
	Full participation models									
SH model	0.675	2.58	3.01	-0.55	0.69	0.67	0.45	0	-	0.98
Andolfatto										
Hansen	-	-	-	-	0.77	0.77	0.28	0	-	-

Table 9: Robustness: financial statistics for a variety of models

	$E[R_t^f]$	$\sigma[R_t^f]$	$E[R_{t,t+1}^e]$	$\sigma[R_{t,t+1}^e]$	$E[R_{t,t+1}^e - R_t^f]$	$\sigma[R_{t,t+1}^e - R_t^f]$
Data	0.80	5.67	6.98	16.54	6.18	16.67
Model						
	Limited participation models					
Baseline	1.24	6.48	5.48	17.71	4.28	17.40
Optimized	0.11	5.98	4.96	17.47	4.89	17.00
No habit	2.205	2.706	3.82	8.33	1.61	7.77
Danthine-Donaldson	2.46	4.05	5.92	22.20	3.46	22.34
	Full participation models					
SH model	1.93	5.04	4.65	13.93	2.76	14.44
Andolfatto						
Hansen	4.10	0.37	4.11	0.52	0.01	-

To analyze the effect of our distribution risk and the interaction between our distribution risk and limited asset market participation, we compare our benchmark model with a business cycle model with search but which allows all to have the full access to the financial market, i.e. in which there is no limited asset market participation, although segmented labor markets are retained. This is Model 3. This search model is the representative agent (family) model that shares with our baseline model the preference specification of agents (a hybrid of GHH preference and external habit formation), the presence of

two segmented labor markets including search-matching labor market and the capital accumulation technology with adjustment cost *except the limited asset market participation*. The model also can generate a sizable equity premium. Therefore, the model is a good reference point to analyze the effect of limited asset market participation in our context<sup>26</sup>.

The contribution of habit formation to model performance can be inferred from the data presented in Tables 8 and 9. Comparing the "baseline" and "no habit" lines of Table 8, we see that habit formation contributes significantly to model volatility, particularly as regards the labor market: unemployment, tightness, vacancies, and wage volatility all decline with the habit feature, and it alone, is removed. With the exception of  $\rho(\ell^s, y)$ , the correlation structure, however, is not much changed.

This result has its origins in the interaction of the Nash-bargaining wage determination with the partial income insurance mechanism arising from the asymmetrical security trading opportunities. Under habit formation, the insider-shareholders display a much-heightened desire to stabilize their marginal utility of consumption. In response the firm acts in a manner to stabilize its aggregate wage bill by reducing the volatility of vacancies, wages, and hours.<sup>27</sup> Reduced volatility in employment and labor market tightness follows as an equilibrium consequence. With both insider-shareholder and outsider-nonshareholder income volatility reduced, the volatility of the distribution risk measure simultaneously declines. Accordingly, there is less need for implicit income insurance; the volume of bond trading declines (not shown) and the share of income to worker nonstockholders increases.

### 4.3 Relative Model Performance: Comparisons with the Literature

**Unemployment volatility puzzle** We begin by comparing the results of our benchmark model with those of two other leading business cycle models with search and matching frictions, namely, Gertler and Trigari (2005) and Christoffel and Kuester (2008). We also compare our model with the existing benchmark business cycle models with search such as Andolfatto (1996) and Merz (1995) and the standard RBC model of Hansen (1985).

Table 10 shows that our model and the two models with staggered Nash bargaining wage can account well for the observed volatility in the key labor market variables emphasized by Shimer (2005) and Hall (2005). In addition, our model and the Christoffel-Kuester (2008) model capture variations at the intensive margin as well as at the extensive margin. These latter models account especially well for the amplitude of the volatilities of the key labor market indicators including unemployment and vacancies.

Nevertheless, Table 10 could reasonably support the assertion that the model of this paper best replicates the stylized facts of the U.S. labor market; at least as regards the critical volatilities and

<sup>26</sup>It deserves being mentioned that a direct representation of the baseline model without limited asset market participation is not trivial; this results from the definition of Nash bargaining wage between capitalists and workers, which requires that the firm's crucial intertemporal decisions be all in accord with the intertemporal marginal rate of substitution of the capitalists only.

<sup>27</sup>Because of capital costs of adjustments it is more costly to stabilize consumption by allowing more investment volatility.

Table 10: Comparison: labor market volatility

	$\sigma_y$	$\sigma_u/\sigma_y$	$\sigma_\nu/\sigma_y$	$\sigma_\theta/\sigma_y$	$\sigma_n/\sigma_y$	$\sigma_{h^{total}}/\sigma_y$	$\sigma_h/\sigma_y$	$\sigma_w/\sigma_y$	$\rho(w, y)$	$\rho(u, \nu)$
Data	1.59	6.92	8.27	13.62	0.64	0.95	0.43	0.44	0.68	-0.88
This paper	1.47	7.05	9.13	15.32	0.61	0.91	0.44	0.25	0.88	-0.79
Gertler-Trigari	-	5.68	7.28	12.52	0.44	-	-	0.48	0.55	-
Christoffel-Kuester	1.91	5.74	7.23	-	-	1.09	0.78	0.22	0.09	-
Merz	1.07	4.63	6.38	1.67	-	-	0.51	-	0.95	-0.15
Andolfatto	1.45	0.68*	3.20*	2.64	0.51	0.59	0.22	0.39	0.95	-0.19
Hansen	1.76	-	-	-	-	0.77	0.77	0.28	0.87	-

Notes: The statistics from Gertler and Trigari (2005) are from their model with the staggeredness  $\lambda = 11/12$ , i.e 4 quarters. The statistics from Merz (1995) are from her model with fixed search intensity. \*numbers in the Andolfatto model are the ones reproduced by Costain and Reiter (2008).

correlations. There are exceptions, however; e.g., Christoffel and Kuester (2008) dominate on the relative wage volatility dimension,  $\frac{\sigma_w}{\sigma_y}$ . On the basis of labor market related quantities, the evidence supports our assertion that limited asset market participation is an important ingredient for understanding observed labor market behavior.

Gertler and Trigari (2009) embed the standard Nash bargaining wage contracting into the framework of (Calvo-type) staggered multiperiod wage contracting while retaining a setting of complete financial markets. Their wage contract ends up taking the form of a fixed wage over an exogenously given horizon. The Gertler-Trigari (2009) model is quite successful in accounting for the overall volatility in the data when average wage contract length is assumed to be four quarters (the fourth row of Table 10). The model, however, is silent about how variations at the intensive margin affects its quantitative validity; in other words, the model completely abstracts from variable labor hours. As observed in the Andolfatto (1996) and Merz (1995) models, we cannot exclude the possibility that the output of the Gertler-Trigari (2009) model is extremely sensitive to variations of labor hours.

Christoffel and Kuester (2008) incorporate search frictions in the labor market into a New Keynesian framework characterized by price rigidities in the goods market. Building on the concept of right-to-manage Nash wage bargaining proposed by Trigari (2006), the Christoffel-Kuester model can account for the observed variations of key indicators of labor market activity, including vacancies and unemployment, as well as the variations at the intensive margin (fifth row of Table 10). To reproduce the empirically pronounced fluctuations in the level of unemployment over the business cycle, the model must rely on (i) multiple shocks including productivity shocks, monetary policy shocks, government spending shocks, and a risk premium shock, (ii) an exogenously specified duration of the wage contract (five months) and (iii) exogenously specified fixed costs of maintaining an existing job which amplify profit fluctuations for any given degree of wage fluctuations. A notable feature of the Christoffel-Kuester (2008) model is

that it requires the same operating leverage channel to give a satisfactory replication of the pronounced fluctuations of unemployment. Without an exogenous risk premium shock or *ad hoc* fixed costs of job maintenance, profit fluctuations for any given degree of wage fluctuations are completely destroyed, i.e. the operating leverage is completely absent. In contrast, our model completely endogenizes the operating leverage channel. Table 11 summarizes the features of the Gertler and Trigari (2009) and Christoffel and Kuester (2008) models as compared to our benchmark formulation.

Table 11: Comparative model features

	This paper	Gertler-Trigari	Christoffel-Kuester
Productivity shock	Yes	Yes	Yes
Government spending shock	No	No	Yes
Monetary policy shock	No	No	Yes
Risk premium shock	No (endogenous risk premium)	No	Yes
Duration of Nash bargaining wage contract	one quarter	3-4 quarters	5 months
Extensive margin	Yes	Yes	Yes
Intensive margin	Yes	No	Yes
Model type	RBC model with real rigidities	RBC model	NK model

From the perspective of model parsimony, Table 11 certainly suggests that the model formulation of the present paper dominates Christoffel and Kuester (2008). With respect to hours variation at the intensive margin, it is quite possible that the addition of this feature will compromise the results in Gertler and Trigari (2009). Nevertheless, the features of this latter model by which the Nash bargained wage is renegotiated approximately at annual frequencies is admittedly more realistic than the present model's quarterly scenario. Unfortunately, none of the alternative models discussed thus far provides financial statistics. The financial performance of the model must thus be judged under a different set of benchmarks.

**Equity premium puzzle** In this section, we begin by comparing the output of our model with that of several leading macro-asset pricing models proposed in the existing literature; in particular, Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), Givonen (2003) and Jermann (1998).

To maximize the model's ability to match stylized financial statistics, we now set up a hyperparameter search slightly different from the baseline calibration. Assuming that  $\chi = 0.9$  and  $\xi = 0.23$ , we conduct a hyperparameter search for the reduced number of free parameters  $(\gamma, \psi, \sigma_\epsilon)$  to match a smaller number of financial statistics of interest. Specifically, we choose the free parameters  $(\gamma, \psi, \sigma_\epsilon)$  to match three empirical targets: (i) a risk-free rate volatility (iii) the mean risk-free rate and (iv) the equity premium. Practically, we restrict our hyperparameter search to a grid of values for  $\sigma_\epsilon \in [0.0037, 0.00712]$ ,  $\psi \in [1, 2]$  and  $\gamma \in [1, 7]$ . We then minimize a weighted quadratic criterion function written in the deviation from

each empirical target with common weight of 1 with the exception of a weight of 10 for risk-free rate volatility. The minimum is achieved for  $\sigma_\epsilon = 0.0052$ ,  $\psi = 1.26$  and  $\gamma = 4.5$ . The coefficient of relative risk aversion jumps to 4.5 from the benchmark case of 3.6;  $\psi = 1.26$  implies that the Frisch elasticity of labor supply in this economy is  $\frac{1}{1.26-1} = 3.846$  which is similar to the Imai-Kean estimate of 3.8. Our value of the innovation standard deviation,  $\sigma_\epsilon$ , is 0.52%, which is even smaller than the value of our benchmark model, 0.6%.

Tables 12 and 13 display the statistics from the simulated models along with their empirical counterparts from US data. The model optimized by the above method of hyperparameter search (hereafter the "optimized model") generates an equity premium of 4.89%, which, not surprisingly, is higher than the baseline model. The standard deviation of excess returns is 17.00%, which is broadly consistent with the empirical magnitude of the standard deviation of excess returns found in the US data—the standard deviation of excess returns is estimated to be 16.67%. The volatility of the average risk-free rate is 5.98%, which achieves a satisfactory replication of its empirical counterpart, 5.67%. The average risk-free rate, however, is 0.11%, which is too low vis-a-vis its empirical target of 0.80% in the US data.

Table 12: Model comparison: financial statistics

	$E[R_t^f]$	$\sigma[R_t^f]$	$E[R_{t,t+1}^e]$	$\sigma[R_{t,t+1}^e]$	$E[R_{t,t+1}^e - R_t^f]$	$\sigma[R_{t,t+1}^e - R_t^f]$
Data	.80	5.67	6.98	16.54	6.18	16.67
Model						
Baseline	1.24	6.48	5.48	17.71	4.28	17.40
Optimized (Shimer's parameters)	0.11	5.98	4.96	17.47	4.89	17.00
Danthine-Donaldson	2.46	4.05	5.92	22.20	3.46	22.34
Guvenen (2003)	1.98	5.73	5.30	14.10	3.32	14.70
Boldrin-Christiano-Fisher	1.20	24.6	7.83	18.4	6.63	–
Jermann	0.82	11.64	7.00	19.86	6.18	–
Hansen	4.05	0.46	4.04	0.48	0.01	.0013

Most finance-cum-production models require that the capital owner display a strong desire to smooth his consumption intertemporally while simultaneously acting in a context that makes it difficult to reallocate labor or capital to that same end. The frictions attendant to reallocating labor or capital are key to generating a high equity premium. Essentially, they substitute for some form of market incompleteness: in either case, agents are prevented from smoothing their consumption across states and dates. Both our baseline model and its optimized cousin are in this tradition: capital owners, who already face a high level of MRS volatility and the costly reallocation of capital due to the capital adjustment costs, now must also deal with the additional friction of reallocating labor inputs.

A distinguishing feature of our mechanism is that the model still can achieve a satisfactory replication of the stylized financial statistics despite allowing for variations at both the intensive and the extensive

Table 13: Model comparison: aggregate volatilities

	$\sigma_y$	$\sigma_c/\sigma_y$	$\sigma_i/\sigma_y$	$\sigma_{h^{tot}}/\sigma_y$	$\sigma_h/\sigma_y$	$\rho(c, y)$	$\rho(i, y)$	$\rho(th, y)$	$\rho(h, y)$
Data	1.59	0.77	3.09	0.95	0.43	0.83	0.91	0.92	0.62
Model									
Baseline	1.47	0.95	1.51	0.91	0.44	0.94	0.86	0.90	0.94
Optimized	1.28	0.96	1.73	1.02	0.23	0.89	0.82	0.88	0.86
Danthine-Donaldson	1.77	0.82	1.72	–	–	0.96	0.93	–	–
Guvenen (2003)	2.40	0.96	1.13	–	–	–	–	–	–
Boldrin-Christiano-Fisher	1.97	0.69	1.67	0.51	0.51	0.95	0.97	0.86	0.86
Jermann	1.76	0.49	2.64	–	–	–	–	–	–
Hansen	1.76	0.29	3.24	0.77	0.77	0.87	0.99	0.98	0.98

margins. Most macro-asset-pricing models assume that the labor supply is inelastic or, when endogenous labor supply choice is incorporated, the models usually display low volatility of total hours worked, which comes close to the observed volatility of hours per worker rather than that of total hours. Therefore, variations at the extensive margin are frequently ignored in this class of the models.

The essence of our mechanism for generating a substantial equity premium, partial risk-sharing manifest as endogenous distribution risk, generates less risky shares of income going to labor in the short run: wage bills vary less than output, falling proportionately less in recessions and increasing less during expansions relative to an uninsured scenario. As a result, the risk in the firm’s free cash flow and derived dividends increases substantially due to the semi-fixed wage bill.

As in the previous section, additional insight as to the source of the equity premium can be obtained by examining the model’s impulse response functions. In what follows, we emphasize that the operating leverage effect is the key to generating a high equity premium. Table 14 summarizes the elasticities of the endogenous variables of interest with respect to a productivity shock  $z$ . As reference points, we summarize the elasticities of the endogenous variables from the benchmark model and the standard RBC model of Hansen (1985).

Table 14: Comparative elasticities

	$\eta_{yz}$	$\eta_{\lambda^*z}$	$\eta_{wz}$	$\eta_{w^nz}$	$\eta_{hz}$	$\eta_{\nu z}$	$\eta_{wiz}$	$\eta_{lsz}$	$\eta_{dz}$	$\eta_{\phi z}$
Model										
Baseline	1.07%	–19.84%	–0.001%	–0.02%	0.12%	11.706%	–0.07%	–1.08%	3.78%	–10.23%
Optimized	0.96%	–27.05%	–0.05%	–0.07%	–0.05%	17.05%	–0.34%	–1.21%	0.33%	–20.07%
Hansen	1.94%		0.47%	–	1.37%	–	1.94%	–	–8.62%	–

Figure 2 presents the impulse response functions from the optimized model. As in the benchmark

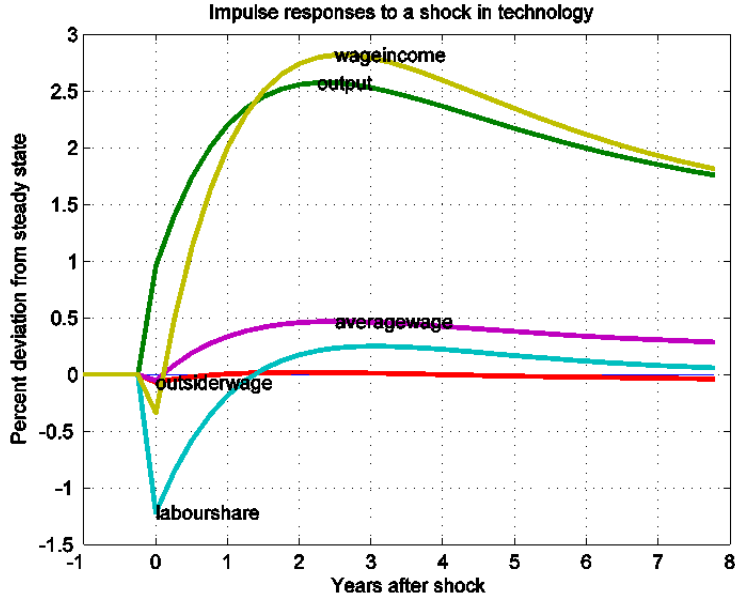


Figure 2: Impulse response of macroaggregates to a one time positive productivity disturbance: optimized model

model, the optimized version highlights the significance of the operating leverage effect. In response to a positive 1% productivity shock, the optimized model displays a 20.07% decrease of the distribution risk  $\phi_t$  sensitivity to a positive productivity shock,  $\eta_{\phi z}$ , thereby dampening wage income sensitivity by 0.34%. Furthermore, as Figure 2 clearly indicates, this operating leverage effect is more persistent in the "optimized model" than in the benchmark case: after approximately one and half years, the wage income is more affected than output (the increase in wage income exceeds the increase in output).

Intuitively, high-productivity shocks coincide with the situations where the bargaining power of capitalists is high and the bargaining power of workers is low. As a result, the high residual payment to capitalists due to the high-productivity realization is further magnified by the decrease of the labor income share. Accordingly, the firm's free cash flow and derived dividends increase during the boom. Conversely, low productivity shocks reduce further the already low residual payment to capitalists. We have argued that the persistent procyclicality of the residual payments to capitalists can generate a sizable equity premium.

Let us summarize these remarks. There are several main channels to achieving a high equity premium cited in the existing literature.<sup>28</sup> First, there is the operating leverage channel with idiosyncratic distribution risk, as advocated by Danthine and Donaldson (2002) and the present model, which postulates

<sup>28</sup>As noted earlier, these leading macro-asset-pricing models abstract from the wedge between variations at the intensive margin and at the extensive margin. In other words, variations at the extensive margin are ignored in this class of the models.

that capital owners effectively provide nonshareholder-workers with insurance against the latter group's income variation as a byproduct of security trading. This risk-sharing mechanism, *ceteris paribus*, creates a high level of volatility of shareholder consumption. This occurs in a setting where the shareholders can themselves mitigate high consumption volatility via the posting of vacancies and the management of the capital stock. Yet, they are discouraged from doing so by high adjustment costs at each margin. As a result, the risk properties of the residual payments to firm owners, in equilibrium, are magnified and a substantial risk premium is achieved.

This channel originates from limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution in consumption. Insider-shareholders participate in both stock and bond markets while more risk-averse outsider-nonshareholders trade only bonds. Since bond trading is their only mechanism for consumption smoothing, nonshareholders bid up bond prices, resulting in a low risk-free rate. In equilibrium, insider-shareholders end up insuring non-shareholders by increasing their own debt holdings exactly when a low-productivity realization reduces both agents' income. As such, bond market events serve to smooth the consumption of non-stockholders and amplify the volatility of shareholder consumption. As a result, stockholders demand a large premium for holding aggregate risk (Güvener (2003)).

Second, the magnitude of the equity premium is enhanced by the strong consumption smoothing motives of the insider-shareholders arising from habit formation preferences. This feature has nothing to do with outsider-nonshareholder income insurance, but reinforces the effects of it to increase further the volatility of the insider-shareholder's marginal rate of substitution.

Third, given the strong insider-shareholder motives for consumption smoothing arising from channels one and two above, these same agents are discouraged from doing so by the costs of adjusting capital and labor (the latter in the form of the cost of adjusting vacancies).

## 5 Concluding Remarks

In response to the unsatisfying empirical performance of the conventional model of unemployment dynamics due to Mortensen and Pissarides, a recent body of studies (Merz (1993), Andolfatto (1996) and Gertler and Trigari (2009)) has emphasized the importance of the degree of wage rigidity in accounting for observed volatility in variables characterizing labor market activity over the business cycle. In this paper, we extend the Mortensen and Pissarides model with Nash wage bargaining to an environment where the asset market is incomplete and perfect risk-sharing between capital owners and workers cannot be guaranteed. We develop period-by-period Nash wage bargaining between capitalists and workers in a macro model with two key features: limited participation in the stock market and labor-force heterogeneity (permanently employed insiders and outsiders subject to employment variation).



What emerges from these considerations is a short-run operating leverage effect which substantively assists in overcoming the unemployment volatility puzzle emphasized by Shimer (2005) and Hall (2005). Specifically, the operating leverage effect may be interpreted as serving the role of an endogenous Nash bargaining power shock perfectly analogous to the exogenous Nash bargaining power shock proposed by Shimer (2005). As such, our model can be viewed as an attempt to provide microfoundation for Shimer's proposed Nash bargaining power shock. This operating leverage effect simultaneously provides a mechanism for generating a sizable equity premium. In summary, a reasonable calibration of the resulting model, which successfully replicates the basic financial statistics, also accounts well for not only for aggregate fluctuations in unemployment and vacancies and their negative correlation, but also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours) over the business cycle. In contrast to existing leading macro-asset-pricing models, the model is unique in the sense that without compromising the overall performance on the financial front, it can pretty much fully reproduce the stylized business cycle facts of the labor market activity.

Many years of research on the properties of DSGE models suggest that it is the allocation of risks across the various economic participants that determines the ability of models to explain jointly the financial stylized facts and the basic properties of macro aggregates. With respect to the latter, the replication of labor market related statistics has historically proven to be generally the most challenging. Clearly, the assignment of risks cannot be separate from the financial market structure confronting a model's economic agents. It is in this spirit that we have elected to impose upon a DSGE model with well understood labor market features (search and matching cum Nash wage bargaining) an empirically realistic incomplete financial market structure.

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## Appendix 1: Model Solution: Asset Pricing

**Asset Pricing** Let  $1 + R_{t+1}$  be the gross return on an asset held from period  $t$  to period  $t + 1$ . If the price and the cash flow of the asset in period  $t$  are denoted by  $P_t$  and  $F_t$ , respectively, then

$$1 + R_{t+1} = \frac{P_{t+1} + F_{t+1}}{P_t}. \quad (30)$$

The Arrow-Lucas-Rubinstein asset pricing equation requires that any asset with (30) must satisfy

$$1 = E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} (1 + R_{t+1})\right]. \quad (31)$$

Equivalently, we can rewrite (31) as follows:

$$0 = \log \beta + \log E_t[\exp(\hat{\lambda}_{t+1} - \hat{\lambda}_t + r_{t+1})] \quad (32)$$

where  $\hat{\lambda}_t$  is the log-deviation of marginal utility of consumption from its steady state value and  $r_{t+1} \equiv \log(1 + R_{t+1})$ . Assuming that  $\lambda_{t+1}$  and  $1 + R_{t+1}$  are jointly lognormally distributed and using the standard formula for the expectation of lognormally distributed variables, equation (32) can be written as:

$$0 = \log \beta + E_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t] + E_t[r_{t+1}] + \frac{1}{2}[\sigma_{\lambda_t}^2 + \sigma_{r_t}^2 + 2\rho_{\lambda r t} \sigma_{\lambda_t} \sigma_{r_t}] \quad (33)$$

where  $\sigma_{\lambda_t}^2 \equiv \text{Var}_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t] = E_t[(\hat{\lambda}_{t+1} - E_t \hat{\lambda}_{t+1})^2]$ ,  $\sigma_{r_t}^2 \equiv \text{Var}_t[r_{t+1}] = E_t[(r_{t+1} - E_t r_{t+1})^2]$ , and  $\rho_{\lambda r t}$  is the conditional correlation, i.e.  $\rho_{\lambda r t} \sigma_{\lambda_t} \sigma_{r_t} \equiv \text{Cov}_t[(\hat{\lambda}_{t+1} - \hat{\lambda}_t), r_{t+1}]$ .

**Risk-free Rate** A risk-free asset (one quarter real bond) with the risk-free rate  $r_t^f \equiv \log(1 + R_t^f)$  can be priced in much simpler way. Since  $\sigma_{r_t}^2 = E_t[(r_{t+1}^f - E_t r_{t+1}^f)^2] = 0$ , we have

$$r_t^f = -\log \beta - E_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t] - \frac{1}{2} \sigma_{\lambda_t}^2. \quad (34)$$

Then the simple risk-free rate is given by

$$1 + R_t^f = \exp r_t^f.$$

The unconditional moments of the simple risk-free rate can be calculated using the log-normal formula for the unconditional expectation:

$$\begin{aligned} E[1 + R_t^f] &= \exp(E[r_t^f] + \frac{1}{2} \text{Var}[r_t^f]) \\ \text{Var}[R_t^f] &= \text{Var}[1 + R_t^f] = \exp(2E[r_t^f] + 2\text{Var}[r_t^f]) - \exp(2E[r_t^f] + \text{Var}[r_t^f]). \end{aligned}$$

The unconditional moments of  $r_t^f$  are given by

$$\begin{aligned} E[r_t^f] &= -\log \beta - \frac{1}{2} \text{Var}[\hat{\lambda}_{t+1} - E_t \hat{\lambda}_{t+1}] \\ \text{Var}[r_{t,n}^f] &= \text{Var}[E_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t]]. \end{aligned}$$

**Term Structure** For the calculation of the conditional term structure  $\{R_{t,n}^f\}$ , we can apply the same log-linear and log-normal framework as the risk-free rate case. Note that the conditional term structure  $\{R_{t,n}^f\}$  can be represented by

$$1 + R_{t,n}^f = [\beta^n E_t[\frac{\lambda_{t+n}}{\lambda_t}]]^{-1/n}.$$

Denote the quarterly continuously-compounded yield of a  $n$ -period real bond by  $r_{t,n}^f \equiv \log 1 + R_{t,n}^f$ . Again using the standard formula for the expectation of lognormally distributed variables, we have

$$\begin{aligned} r_{t,n}^f &= -\frac{1}{n} \log[\beta^n E_t[\frac{\lambda_{t+n}}{\lambda_t}]] \\ &= -\log \beta - \frac{1}{n} \log E_t[\exp(\hat{\lambda}_{t+n} - \hat{\lambda}_t)] \\ &= -\log \beta - \frac{1}{n} [E_t[\hat{\lambda}_{t+n} - \hat{\lambda}_t] + \frac{1}{2} \sigma_{\lambda_t}^2]. \end{aligned}$$

The simple bond yield is given by

$$1 + R_{t,n}^f = \exp r_{t,n}^f.$$

The unconditional moments of the simple bond yield can be calculated using the log-normal formula for the unconditional expectation:

$$\begin{aligned} E[1 + R_{t,n}^f] &= \exp(E[r_{t,n}^f] + \frac{1}{2} \text{Var}[r_{t,n}^f]) \\ \text{Var}[R_{t,n}^f] &= \text{Var}[1 + R_{t,n}^f] = \exp(2E[r_{t,n}^f] + 2\text{Var}[r_{t,n}^f]) - \exp(2E[r_{t,n}^f] + \text{Var}[r_{t,n}^f]). \end{aligned}$$

The unconditional moments of  $r_{t,n}^f$  are given by

$$\begin{aligned} E[r_{t,n}^f] &= -\log \beta - \frac{1}{2n} \text{Var}[\hat{\lambda}_{t+n} - E_t \hat{\lambda}_{t+n}] \\ \text{Var}[r_{t,n}^f] &= \frac{1}{n^2} \text{Var}[E_t[\hat{\lambda}_{t+n} - \hat{\lambda}_t]]. \end{aligned}$$

**Equity** To calculate the equity returns, we adopt a slightly different strategy. The Arrow-Lucas-Rubinstein asset pricing equation tells us that the period  $t$  equity price  $p_t^e$  must equal the present value

of all future dividends discounted by the pricing kernel:

$$p_t^e = E_t\left[\sum_{k=1}^{\infty} \beta^k \frac{\lambda_{t+k}}{\lambda_t} d_{t+k}\right]$$

where  $\frac{\lambda_{t+k}}{\lambda_t}$  is the stochastic discount factor of insider-stockholders due to the presumed limited participation in the stock market.

Note that equivalently, the period  $t$  equity price  $p_t^e$  can be written as:

$$\begin{aligned} p_t^e &= \sum_{k=1}^{\infty} E_t\left[\beta^k \frac{\lambda_{t+k}}{\lambda_t} d_{t+k}\right] \\ &= \sum_{k=1}^{\infty} E_t\left[\beta^k \frac{\lambda_{t+k}}{\lambda_t} \frac{d_{t+k}}{d_t} d_t\right] \\ &= \sum_{k=1}^{\infty} E_t\left[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k} - \hat{d}_t) d_t\right] \end{aligned} \quad (35)$$

where  $\hat{d}_t$  is the log-deviation of dividend from its steady state value.

Using equation (35), the simple quarterly equity return is given by

$$\begin{aligned} 1 + R_{t,t+1}^e &= \frac{p_{t+1}^e + d_{t+1}}{p_t^e} \\ &= \frac{\left[\sum_{k=1}^{\infty} E_{t+1}\left[\beta^k \exp(\hat{\lambda}_{t+1+k} - \hat{\lambda}_{t+1} + \hat{d}_{t+1+k} - \hat{d}_{t+1}) d_{t+1}\right]\right] + d_{t+1}}{\sum_{k=1}^{\infty} E_t\left[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k} - \hat{d}_t) d_t\right]} \\ &= \frac{\left[\sum_{k=1}^{\infty} E_{t+1}\left[\beta^k \exp(\hat{\lambda}_{t+1+k} - \hat{\lambda}_{t+1} + \hat{d}_{t+1+k})\right]\right] + \exp(\hat{d}_{t+1})}{\sum_{k=1}^{\infty} E_t\left[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k})\right]} \end{aligned}$$

Applying the standard log-normal formula to the random variables  $\{\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}\}_{k=1}^{\infty}$ , each conditional expectation term can be written as:

$$\begin{aligned} &E_t[\exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k})] \\ &= \exp\left[E_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}] + \frac{1}{2} \text{Var}_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}]\right]. \end{aligned}$$

Both terms,  $E_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}]$  and  $\text{Var}_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}]$ , respectively can be computed and then



we approximate  $1 + R_{t,t+1}^e$  by

$$\frac{[\sum_{k=1}^n E_{t+1}[\beta^k \exp(\hat{\lambda}_{t+1+k} - \hat{\lambda}_{t+1} + \hat{d}_{t+1+k})]] + \exp(\hat{d}_{t+1})}{\sum_{k=1}^n E_t[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k})]}$$

for sufficiently large number  $n$ .

## Appendix 2: A Further Check on Model Legitimacy: Elasticity of Intertemporal Substitution

To see how the elasticity of intertemporal substitution (EIS) is identified in our preference specifications, first note that the equations (7) and (14) can be rewritten as

$$\frac{1}{1 + R_t^f} = \beta E\left(\frac{\lambda_{t+1}^s}{\lambda_t^s} \mid \Omega_t\right) \quad (36)$$

$$\frac{1}{1 + R_t^f} = \beta E\left(\frac{\lambda_{t+1}^n}{\lambda_t^n} \mid \Omega_t\right) \quad (37)$$

where  $\lambda_t^s = u_c(c^s(\Omega_t), h^s(\Omega_t))$ ,  $\lambda_t^n = v_c(c^n(\Omega_t), h^n(\Omega_t))$  and  $\frac{1}{1+R_t^f} = p_t^f$ .

The period utility function of the representative insider-stockholder is postulated as

$$u(c_t^s - X_t - H(h_t^s)) = \frac{(c_t^s - X_t - H(h_t^s))^{1-\gamma_s} - 1}{1 - \gamma_s}$$

while the preference of the representative outsider-nonstockholder is postulated as

$$v(c_t^n - L(h_t^n)) = \frac{(c_t^n - L(h_t^n))^{1-\gamma_n} - 1}{1 - \gamma_n}.$$

Here  $\gamma_s$  and  $\gamma_n$  are the insider-stockholder's coefficient of risk aversion and the outsider-nonstockholder's coefficient of risk aversion, respectively;  $X_t$  is the exogenous habit stock, evolving according to

$$X_t = \phi X_{t-1} + (1 - \phi)\chi c_{t-1}^s$$

where  $c_{t-1}^s$  denotes the aggregate average level of the insider-stockholder group's consumption last period,  $\chi$  is the habit parameter of the insider-stockholder group and  $\phi = 0$ .

Under the above specifications of the each agent's preference, we log-linearize and rearrange the equations (36) and (37) in order to obtain the each agent's EIS (the bar "-" represents the steady state

value of variables):

$$\frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s} \hat{r}_t^f = E_t \left[ \log \frac{c_{t+1}^s}{c_t^s} + [\text{remainder terms}] \right] \quad (38)$$

$$\frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n} \hat{r}_t^f = E_t \left[ \log \frac{c_{t+1}^n}{c_t^n} + [\text{remainder terms}] \right]. \quad (39)$$

From the equations (38) and (39), we identify the insider-stockholder's EIS with

$$\frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s}$$

while the outsider-nonstockholder's EIS can be identified with

$$\frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n}.$$

Note also that

$$\begin{aligned} \log \frac{c_{t+1}}{c_t} &= \log \left( 1 + \frac{\mu_s(c_{t+1}^s - c_t^s) + (c_{t+1}^n - c_t^n)}{c_t} \right) \\ &\approx \mu_s \frac{(c_{t+1}^s - c_t^s)}{c_t} + \frac{(c_{t+1}^n - c_t^n)}{c_t} \\ &\approx \mu_s \log \frac{c_{t+1}^s}{c_t^s} \cdot \frac{c_t^s}{c_t} + \log \frac{c_{t+1}^n}{c_t^n} \cdot \frac{c_t^n}{c_t}. \end{aligned} \quad (40)$$

Therefore, taking the expectation operator  $E_t$  in (40) and using the equations (38) and (39), we derive the formula:

$$\begin{aligned} &E_t \left[ \log \frac{c_{t+1}^s}{c_t^s} + [\text{remainder terms}] \right] \\ &= \left[ \mu_s \frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s} E_t \frac{c_t^s}{c_t} + \frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n} E_t \frac{c_t^n}{c_t} \right] \hat{r}_t^f. \end{aligned}$$

We identify the aggregate EIS with

$$\mu_s \frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s} E_t \frac{c_t^s}{c_t} + \frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n} E_t \frac{c_t^n}{c_t}.$$

Abstracting from uncertainty our identified EIS is reduced to

$$\mu_s \frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}} + \frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}} \quad (41)$$

which must be the true value of the aggregate EIS in our economy when estimated from the generated data. Under our benchmark calibration, the aggregate EIS is predicted to be close to zero (0.0307).

This predicted number is consistent with Hall's findings: consumption growth is completely insensitive to changes in interest rates and thus EIS is close to zero. It is also consistent with the conclusion found in Binsbergen et al. (2008); in their estimated DSGE model, they find that a low elasticity of intertemporal substitution (around 0.06) is estimated from upward-sloping (nominal) yield curve data and macro data.

### Appendix 3: Background Models to Table 7

1. Model A: A standard RBC model with competitive wage determination for both insiders and outsiders (no search and matching), and complete financial markets.

The notation and all functional forms (i.e., utility functions, the production function) coincide with those of Section 2 with the sole exception that there is no habit formation stock ( $X_t \equiv 0$ ) in the insider-stockholder's utility representation. Because of the competitive factor markets and complete asset markets the competitive equilibrium allocation can be obtained by solving the associated Pareto problem:

$$\begin{aligned} & \max_{\{c_t^n, c_t^s, h_t^s, h_t^n, i_t\}} E \left( \sum_{t=0}^{\infty} \beta^t [\mu_s u(c_t^s - H(h_t^s)) + v(c_t^n - L(h_t^n))] \right) \\ \text{s.t.} \quad & \mu_s c_t^s + c_t^n + i_t = f(k_t, \mu_s h_t^s, h_t^n) \tilde{z}_t \\ & k_{t+1} = (1 - \delta) k_t + G\left(\frac{i_t}{k_t}\right) k_t; k_0 = k_{ss} \\ & \log \tilde{z}_{t+1} = \Delta \log z_t + \tilde{\epsilon}_{t+1}. \end{aligned}$$

In Table 7 we list the statistical summary of this model's output under "Model 1."

The dividend is defined by:

$$d_t = y_t - i_t - \mu_s w_t^s h_t^s - w_t^n h_t^n - r_f \phi \bar{k}.$$

2. Model B: This model is identical to the one described above, but with search and matching in the labor market for outsider-nonstockholders. A Pareto formulation may also be analyzed:

$$\begin{aligned} & \max_{\{c_t^s, c_t^n, h_t^s, h_t^n, v_t\}} E \left( \sum_{t=0}^{\infty} \beta^t [\mu_s u(c_t^s - H(h_t^s)) + v(c_t^n - L(h_t^n) n_t - (1 - n_t) L(0))] \right) \\ \text{s.t.} \quad & \mu_s c_t^s + c_t^n + \frac{K}{2} \frac{(m_t)^2}{n_t} + i_t = f(k_t, \mu_s h_t^s, n_t h_t^n) \tilde{z}_t \\ & n_{t+1} = (1 - \rho) n_t + m_t \\ & m_t = \sigma_m v_t^\sigma (1 - n_t)^{1-\sigma} \\ & k_{t+1} = (1 - \delta) k_t + G\left(\frac{n_t}{k_t}\right) k_t \\ & \log \tilde{z}_{t+1} = \Delta \log z_t + \tilde{\epsilon}_{t+1}. \end{aligned}$$

In Table 7, the statistical summary of this model's output is found under the heading "Model 2."

The dividend is defined by:

$$d_t = y_t - i_t - \left(\frac{K}{2}\right) \frac{m_t^2}{n_t} - \mu_s h_t^s w_t^s - h_t^n w_t^n - r_t^f \phi \bar{k}$$

Model B is very similar to that of Andolfatto (1996).

3. Model C: As noted in the main text, this model considers a representative agent (family) that shares with the baseline model the preference specification of agents (a hybrid of GHH preference and external habit formation). There are two segmented labor markets including a search-matching labor market and a capital accumulation technology with adjustment cost. These elements are shared with Models A and B. Financial markets are not strictly complete but all agents trade the same two securities: there is no restricted participation.

C.1 The shareholder-worker's problem is essentially identical to that detailed in Section 2.1; that is, equations (1) and (2). For convenience, we replicate it here:

$$V^s(\Omega_0^s) = \max_{\{h_t^s, c_t^s, e_{t+1}^s, b_{t+1}^s\}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t^s - L(h_t^s))] \\ \text{s.t.} \\ c_t^s + p_t^e e_{t+1}^s + p_t^f b_{t+1}^s \leq w_t^s h_t^s + (p_t^e + d_t) e_t^s + p_t^f b_t^s$$

## C.2 Households.

Following Merz (1995), non-shareholders may be viewed as members of a large extended family which contains a continuum of family members uniformly distributed on a set of Lebesgue measure 1. Employment status within the family, employed or unemployed, can vary according to a matching friction, but family members are perfectly insured against their idiosyncratic employment experiences. The family pools its financial and labor income before choosing per-capita consumption and asset holdings. Accordingly, given its information set  $\Omega_t^h = \{z_t, n_t, k_t, s_t\}$ , the representative family solves:

$$V^h(\Omega_0^h) = \max_{\{c_t^n, n_t, h_t^n, b_{t+1}^n, e_{t+1}^n\}} E_0 \sum_{t=0}^{\infty} \beta^t [v(c_t^n - n_t L(h_t^n) - (1 - n_t)L(0))] \quad (42) \\ \text{s.t.} \\ c_t^n + p_t^e e_{t+1}^n + p_t^f b_{t+1}^n \leq (p_t^e + d_t) e_t^n + w_t^n h_t^n n_t + b(1 - n_t^w) + b_t + T_t. \\ n_{t+1} = (1 - \rho)n_t + s_t(1 - n_t)$$

In the above problem,  $v(\cdot)$  denotes their period utility function,  $L(\cdot)$  is the disutility of labor function, while  $h_t^n$  denotes the period  $t$  labor hours supplied by those employed. The period utility function  $v$  is

given by

$$v(c_t - n_t L(h_t^n) - (1 - n_t)L(0)) = \frac{(c_t - n_t B_n(h_t^n)^\psi)^{1-\gamma} - 1}{1-\gamma}, \text{ since } L(0) \equiv 0.$$

We eschew, as in Models A and B, any habit formation representations. Problem (42) is identical to formulation (8), (9), (10) except that the class of admissible assets now includes the equity of the firm in which the workers are employed.

### C.3 The firm

The firm's problem is identical to the baseline model except some small changes of notation. Given its information set  $\Omega_t^f = \{z_t, k_t, n_t^d\}$ , the firm's problem reads as:

$$\max_{\{i_t, h_t^s, x_t\}} d_t + p_t^e \equiv d_t + E(\beta \Lambda_{t,t+1}^s (p_{t+1}^e + d_{t+1}) \mid \Omega_t^f) \quad (43)$$

$$\begin{aligned} \text{s.t. } d_t &\equiv f(k_t, \mu_s h_t^s, h_t^n n_t^d) z_t - i_t - \mu_s w_t^s h_t^s - w_t^n h_t^n n_t^d - \frac{\kappa}{2} x_t^2 n_t^d - \varphi \bar{k} + p_t^f \varphi \bar{k} \\ k_{t+1} &= (1 - \delta) k_t + G\left(\frac{i_t}{k_t}\right) k_t \\ n_{t+1}^d &= (1 - \rho) n_t^d + q_t \nu_t \end{aligned}$$

where  $\Lambda_{t,t+1}^s \equiv \frac{\lambda_{t+1}^s}{\lambda_t^s}$  is the marginal rate of substitution of the representative family. In equilibrium,  $n_t = n_t^d$

### 3.4 Nash bargaining

The Nash wage bargaining between the firm and the outsiders can be formulated similarly to what has been proposed earlier. Note that the firm's crucial intertemporal decisions are all in accord with the intertemporal marginal rate of substitution of the representative family. In this environment where there are no corporate governance problems, we can again define the Nash bargaining problem in the same way as in the baseline model:

$$\max_{w_t^s} (V_{n_t}^s)^{1-\eta} \cdot (V_{n_t}^n)^\eta$$

where  $\eta$  is the bargaining power parameter of outsider-nonshareholders viewed as exogenously given. The expressions  $V_{n_t}^s$  and  $V_{n_t}^n$  are given by

$$\begin{aligned} V_{n_t}^s &\equiv \frac{\partial V_t^h}{\partial n_t^d} \\ V_{n_t}^n &\equiv \frac{\partial V_t^h}{\partial n_t^w} \end{aligned}$$

where  $V_t^h \equiv V^h(\Omega_t^h)$ .

The bargained hours of outsiders are determined by the condition (25). The firm's shadow value  $J_t$

and the outsider's shadow value  $W_t - U_t$  are exactly the same as those in the baseline model. It can be readily shown that

$$\begin{aligned} V_{n_t}^s &= \lambda_t^h J_t \\ V_{n_t}^n &= \lambda_t^h (W_t - U_t) \end{aligned}$$

This implies that the Nash bargaining wage solution for Model 3 is given by

$$w_t^n = (1 - \eta) \frac{[L(h_t^n) + b - F_t^n]}{h_t^n} + \eta \frac{[h_t^n f_3(k_t, h_t^p, h_t^n n_t^d) z_t + \frac{\kappa}{2} x_t^2 + F_t^s]}{h_t^n}.$$

Note that the distribution risk  $\phi_t$  completely disappears under this formulation since there is no financial market limited participation.

**Equilibrium** The equilibrium concept of this model is identical to that of the baseline model and includes

$$n_t^w = n_t^d.$$

In Table 7, we present the statistical summary of this model's output under "Model C."