Investment, Tobin's q, and Interest Rates*

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Abstract

To study the impact of stochastic interest rates and capital illiquidity on investment and firm value, we incorporate a widely-used arbitrage-free term structure model of interest rates into a standard q-theoretic framework. Our generalized q model informs us to use corporate credit-risk information to predict investments when empirical measurement issues of Tobin's average q are significant (e.g., equity is much more likely to be mis-priced than debt) as in Philippon (2009). Consistent with our theory, we find that credit spreads and bond q have significant predictive powers on micro-level and aggregate investments corroborating the recent empirical work of Gilchrist and Zakrajšek (2012). We also show that the quantitative effects of the stochastic interest rates and capital illiquidity on investment, Tobin's average q, the duration and user cost of capital, as well as the value of growth opportunities are substantial. These findings are particularly important in today's low interest-rate environment.

Keywords: term structure of interest rates; capital adjustment costs; average q; marginal q; duration; assets in place; growth opportunities; bond q;

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1 Introduction

One widely-held conventional wisdom in macroeconomics is that investment should respond negatively to interest rates. Various macroeconomic models rely on this negative relation. The neoclassical q theory of investment explicitly incorporates productivity shocks and capital adjustment costs into a dynamic optimizing framework and generates predictions between investment and interest rates.[1] However, almost all q-theoretic models assume that the interest rate is constant over time, which by construction rules out the impact of the interest rate risk and dynamics on investments. Moreover, there is limited empirical evidence in support of the widely-used investment/interest rate relation and the q-theory of investment.[2] Philippon (2009) demonstrates that interest rates measured by bond yields have significant predictive power for aggregate investment even in the Modigliani-Miller (MM) world. He argues that the superior performance of bond prices over standard total-firm-value-based measures (e.g., Tobin's average q) for investment regressions can be plausibly attributed to mis-pricing, in that equity being the levered claim on the firm is more likely to be mis-priced than bonds making bond prices more informative for investment[3] or a potential disconnect (even in a rational model) between current capital investments and future growth options.[4]

In terms of the theory, we recognize the importance of stochastic interest rates on investment and the value of capital by incorporating a widely-used term structure model of interest rates (Cox, Ingersoll, and Ross, 1985) into a neoclassical q-theoretic model of investment (Hayashi, 1982).[5] We show that investment decreases with interest rates, and

[1] Lucas and Prescott (1971) and Abel (1979) study investment dynamics under uncertainty with convex adjustment costs. Hayashi (1982) provides homogeneity conditions under which the firm's marginal q is equal to its average q.

[2] Abel and Blanchard (1986) show that marginal q, constructed as the expected present value of marginal profits, still leaves unexplained large and serially correlated residuals in the investment regressions.

[3] Gilchrist, Himmelberg, and Huberman (2005) show that dispersion in investor beliefs and short-selling constraints can give rise to mis-pricing in the stock market and a weak link between investment and the market.

[4] For example, when growth options differ significantly from existing operations and near-term investment decisions are primarily driven by physical capital accumulation, bond prices are naturally more informative for investments than the firm's total value, as the equity value portion of the firm's value is mostly determined by the perceived value of growth options, which is largely uncorrelated with the value of capital stock.

[5] Abel and Eberly (1994) develop a unified neoclassical q theory of investment with constant interest rates. McDonald and Siegel (1986) and Dixit and Pindyck (1994) develop the real options approach of investment also with constant interest rates. The q theory and the real options framework are two complementary value-maximizing approaches of modeling investment. These two approaches focus on different but closely related real investment frictions (i.e., capital adjustment costs versus irreversibility, respectively.)

moreover, the term structure of interest rates has a first-order and highly nonlinear effects on investment and Tobin's average q, and therefore a firm ignoring the interest rate risk and dynamics will significantly distort its investment and reduce its value. Moreover, at a low interest-rate environment such as today's, capital illiquidity, measured by the capital adjustment costs as in the standard q theory, has very large effects on corporate investment, Tobin's q, the user cost of capital, and the value of growth opportunities. Given the wide range of parameter estimates for capital adjustment costs, which is often premised on the constant interest rate assumption, in the literature,[6] our analysis highlights the importance of explicitly incorporating risk-adjusted interest rate dynamics via an arbitrage-free term structure and re-estimating capital illiquidity/adjustment cost parameters. As physical capital is long lived subject to depreciation, the duration, Tobin's q, and the value of the firm's growth opportunities are all quite sensitive to capital adjustment costs especially when interest rates are low.

We further generalize our q theory with stochastic interest rates to incorporate leverage by building on Philippon (2009). This generalization is important for our empirical analyses because it motivates us to use credit risk information to predict corporate investment and also to avoid standard investment-opportunity measures, e.g., Tobin's q, which often have significant measurement issues. The premise of our analysis that Tobin's q can be poorly measured is well recognized in the investment literature. In an important paper, Erickson and Whited (2000) show that despite its simple structure, a standard neoclassic q-theory without any financial imperfection has good explanatory power once empirical measurement error issues are properly addressed, e.g., via method of moments.[7]

Consistent with our theory, we find that the relative bond prices *positively* and credit spreads *negatively* predict investment at both the firm- and the aggregate levels. Moreover, the predictive power of credit-risk-based measures for investment remains strong and robust after controlling for well-known predictors. Our empirical findings are consistent with the recent work in the literature. For example, Gilchrist and Zakrajšek (2007) report that increasing the user cost of capital by 100 basis points is associated with a reduction of in-

[6]See Gilchrist and Himmelberg (1995), Hall (2004), Cooper and Haltiwanger (2006), and Eberly, Rebelo, and Vincent (2012) for a wide range of estimates. We provide more detailed discussions in Section 3.

[7]Gomes (2001) makes a related point that financial constraints are neither necessary nor sufficient in generating investment-cash flow sensitivity by simulating a quantitative q-model with financial frictions.

vestment around 50 to 75 basis points and a one-percent reduction in the capital stock in the long run. Philippon (2009) shows that aggregate corporate bond yields predict aggregate investment substantially better than the stock-market-based measures, e.g., Tobin's q. Gilchrist and Zakrajšek (2012) show that their constructed corporate bond yield index has considerable predictive power for aggregate economic variables. In summary, our aggregate and firm-level results corroborate these existing studies and provide additional support for the q theory of investment.

The remainder of the paper proceeds as follows. Section 2 presents our q-theory of investment with term structure of interest rates. Section 3 provides the model's solution and discuss the quantitative results. Section 4 provides the empirical evidence for the model's predictions at both the firm-level and aggregate data. Section 5 concludes. Appendices contain technical details related to the main results in the paper and also a few generalizations of our baseline model. In particular, Appendix C contains our model's generalizations including asymmetric adjustment costs, price wedge of capital, fixed costs, and irreversibility.

2 Model

First, we generalize the neoclassic q theory of investment to incorporate the effects of stochastic interest rates and then introduce leverage in an MM setting with the objective of linking our model's prediction to bond data as in Philippon (2009).

2.1 Economic Environment

Stochastic interest rates. While much work in the q theory context assumes constant interest rates, empirically, there are substantial variations of interest rates over time. Additionally, corporate investment payoffs are often long term and are sensitive to the expected change and volatility of interest rates.

Researchers often analyze effects of interest rates via comparative statics (by using the solution from a dynamic model with a constant interest rate). However, comparative static analyses miss the expectation effect by ignoring the dynamics and the risk premium of interest rates. By explicitly incorporating a term structure of interest rates, we analyze the persistence, volatility, and risk premium effects of interest rates on investment and firm value in a fully specified dynamic stochastic framework.

We choose the widely-used CIR term structure model where the short rate r follows

$$dr_t = \mu(r_t)dt + \sigma(r_t)d\mathbb{B}_t, \quad t \geq 0, \tag{1}$$

where $\mathbb{B}$ is the standard Brownian motion under the risk-neutral measure, and the risk-neutral drift $\mu(r)$ and volatility $\sigma(r)$ are respectively given by

$$\mu(r) = \kappa(\xi - r), \tag{2}$$

$$\sigma(r) = \nu\sqrt{r}. \tag{3}$$

Note that both the conditional mean $\mu(r)$ and the conditional variance $\sigma^2(r)$ are linear in r. The parameter κ measures mean reversion of interest rates. The implied first-order autoregressive coefficient in the corresponding discrete-time model is $e^{-\kappa}$. The higher κ, the more mean-reverting the interest rate process. The parameter ξ is the long-run mean of interest rates. The CIR model captures the mean-reversion and conditional heteroskedasticity (stochastic volatility) of interest rates belonging to the widely-used affine models of interest rates.[8] In Section 2.3, we explicitly specify the risk premium process for the interest rate. Next we turn to the production technology.

Production and investment technology. A firm uses its capital to produce output.[9] Let K and I denote its capital stock and gross investment rate, respectively. Capital accumulation is standard in that

$$dK_t = (I_t - \delta K_t)\, dt, \quad t \geq 0, \tag{4}$$

where $\delta \geq 0$ is the rate of depreciation for capital stock.

The firm's operating revenue over time period $(t, t+dt)$ is proportional to its time-t capital stock K_t, and is given by $K_t dX_t$, where dX_t is the firm's productivity shock over the same

[8]Vasicek (1977) is the other well known one-factor model. However, this process is less desirable because it implies conditionally homoskedastic (normally distributed) shocks and allow interest rates to be unbounded from below. Vasicek and CIR models belong to the "affine" class of models. See Duffie and Kan (1996) for multi-factor affine term-structure models and Dai and Singleton (2000) for estimation of three-factor affine models. Piazzesi (2010) provides a survey on affine term structure models.

[9]The firm may use both capital and labor as factors of production. As a simple example, we may embed a static labor demand problem within our dynamic optimization. We will have an effective revenue function with optimal labor demand. The remaining dynamic optimality will be the same as in the standard q theory.

time period $(t,\ t + dt)$. After incorporating the systematic risk for the firm's productivity shock, we may write the productivity shock dX_t under the risk-neutral measure[10] as follows,

$$dX_t = \pi dt + \epsilon d\mathbb{Z}_t, \quad t \geq 0, \tag{5}$$

where $\mathbb{Z}$ is a standard Brownian motion. The productivity shock dX_t specified in (5) is independently and identically distributed (i.i.d). The constant parameters π and $\epsilon > 0$ give the corresponding (risk-adjusted) productivity mean and volatility per unit of time.

The firm's operating profit dY_t over the same period $(t, t + dt)$ is given by

$$dY_t = K_t dX_t - C(I_t, K_t)dt, \quad t \geq 0, \tag{6}$$

where $C(I, K)$ is the total cost of the investment including both the purchase cost of the capital goods and the additional adjustment costs of changing capital stock. The firm may sometimes find it optimal to divest and sell its capital, $I < 0$. Importantly, capital adjustment costs make installed capital more valuable than new investment goods. The ratio between the market value of capital and its replacement cost, often referred to as Tobin's q, provides a measure of rents accrued to installed capital. The capital adjustment cost function $C(I, K)$ plays a critical role in the neoclassical q theory of investment. In this section, we assume that $C(I, K)$ satisfies $C_I > 0$ and $C_{II} > 0$. Additionally, for simplicity, we assume that $C(I, K)$ is homogeneous with degree one in I and K, in that $C(I, K) = c(i)K$ where $i = I/K$. Note that $c'(i) > 0$ and $c''(i) > 0$ are implied by the monotonicity and convexity properties of $C(I, K)$ in I. In Appendix C, we generalize our baseline model to allow a much richer specification for $C(I, K)$ by incorporating asymmetric adjustment costs, price wedge, and fixed costs.

For simplicity, we assume that interest rate risk and the productivity shock are uncorrelated, i.e. the correlation coefficient between the Brownian motion $\mathbb{B}$ driving the interest rate process (1) and the Brownian motion $\mathbb{Z}$ driving the productivity process (5) is zero.

Liquidation option. Capital often has an alternative use if deployed elsewhere. Empirically, there are significant reallocation activities between firms as well as between sectors.[11]

[10] The risk-neutral measure incorporates the impact of the interest rate risk on investment and firm value. By directly specifying dynamics under the risk-neutral measure, we avoid the complication of specifying the risk premium at the stage. In Section 2.3, we explicitly state the risk premium and then infer the implied dynamics under the physical measure.

[11] See Eisfeldt and Rampini (2006) for equilibrium capital reallocation.

We assume that the firm has an option to liquidate its capital stock at any time; doing so allows the firm to recover ℓ per unit of capital where $0 < \ell < 1$ is a constant. Let τ_L denote the firm's stochastic liquidation time. This optionality significantly influences firm investment and the value of capital.

2.2 Tobin's q, Investment, and Liquidation

While our model features stochastic interest rates and real frictions, i.e., capital adjustment costs, there are no financial frictions and hence the Modigliani-Miller theorem holds. The firm chooses investment I and liquidation time τ_L to maximize its value defined below:

$$\mathbb{E}\left[\int_0^{\tau_L} e^{-\int_0^t r_v dv} dY_t + e^{-\int_0^{\tau_L} r_v dv} \ell K_{\tau_L}\right]. \tag{7}$$

While the discount rate in (7) is the risk-free rate, the risk-free rate r and the cumulative net profits Y are both under the risk-neutral measure. Therefore, the firm's well diversified investors earn an expected return in excess of the risk-free rate r and the implied risk premium can be inferred.

Let $V(K, r)$ denote firm value. Using the standard principle of optimality, we have the following Hamilton-Jacobi-Bellman (HJB) equation,

$$rV(K,r) = \max_I \ (\pi K - C(I,K)) + (I - \delta K)\, V_K(K,r) + \mu(r) V_r(K,r) + \frac{\sigma^2(r)}{2} V_{rr}(K,r). \tag{8}$$

The first term on the right side of (8) gives the firm's risk-adjusted expected cash flows. The second term gives the effect of net investment on firm value. The last two terms give the drift and volatility effects of interest rate changes on $V(K,r)$. The firm optimally chooses investment I by demanding that its risk-adjusted expected return equals r at optimality, which implies that the two sides of (8) are equal.

Let $q(K,r)$ denote the marginal value of capital, which is also known as the marginal q, $q(K,r) \equiv V_K(K,r)$. The first-order condition (FOC) for investment I is

$$q(K,r) \equiv V_K(K,r) = C_I(I,K)\,, \tag{9}$$

which equates $q(K,r)$ with the marginal cost of investing $C_I(I,K)$. With convex adjustment costs, the second-order condition (SOC) is satisfied, and hence the FOC characterizes investment optimality.

We show that the optimal liquidation policy is described by a threshold policy with the endogenously determined cutoff level r^*, in that if $r \geq r^*$, the firm optimally liquidates its capital stock but otherwise continues its operations. At the moment of liquidation τ_L, $V(K, r^*) = \ell K$ holds as an accounting identity: the firm's value upon liquidation equals ℓ per unit of K. Additionally, the optimal liquidation policy must satisfy the smooth pasting condition $V_r(K, r^*) = 0$, which is the FOC for τ_L. Intuitively, as the firm's liquidation value ℓK is independent of r (by assumption, which can be relaxed in a more general model), the firm's value just before liquidation must also be insensitive to r, in that $V_r(K, r^*) = 0$.

Capital K and interest rate r are the two state variables in our model. It shows that the firm's value is proportional to its contemporaneous capital stock K, in that

$$V(K, r) = K \cdot q(r), \tag{10}$$

where $q(r)$ is both Tobin's average and marginal q independent of K. Note that the equality between the two qs in our model is due to the homogeneity property as in Hayashi (1982).

Next, we summarize the main results in the following proposition.

Proposition 1 *In the region $r < r^*$, where r^* is the endogenously determined liquidation threshold, Tobin's average q, $q(r)$, solves the following ODE:*

$$rq(r) = \pi - c(i(r)) + (i(r) - \delta)q(r) + \mu(r)q'(r) + \frac{\sigma^2(r)}{2}q''(r), \quad r < r^*, \tag{11}$$

where the optimal investment $i(r)$ is monotonically increasing in $q(r)$, in that

$$c'(i(r)) = q(r), \tag{12}$$

as implied by $c''(\cdot) > 0$. The firm optimally liquidates its capital stock when $r \geq r^*$ and the optimal threshold r^* satisfies the following value-matching and smooth-pasting conditions:

$$q(r^*) = \ell \quad \text{and} \quad q'(r^*) = 0. \tag{13}$$

2.3 Risk premia

As in CIR, we assume that the interest rate risk premium is given by $\lambda\sqrt{r}$, where λ is a constant that measures the sensitivity of risk premium with respect to r. By the no-arbitrage principle, we have the following dynamics for the interest rate under the physical measure,[12]

$$dr_t = \mu^{\mathbb{P}}(r_t)dt + \sigma(r_t)d\mathbb{B}_t^{\mathbb{P}}\,, \tag{14}$$

where $\mathbb{B}^{\mathbb{P}}$ is the standard Brownian motion under the physical measure $\mathbb{P}$, the drift $\mu^{\mathbb{P}}(r)$ is

$$\mu^{\mathbb{P}}(r) = \kappa\,(\xi - r) + \nu\lambda r = \kappa^{\mathbb{P}}(\xi^{\mathbb{P}} - r)\,, \tag{15}$$

and

$$\kappa^{\mathbb{P}} \;=\; \kappa - \lambda\nu\,, \tag{16}$$

$$\xi^{\mathbb{P}} \;=\; \frac{\kappa\xi}{\kappa - \lambda\nu}\,. \tag{17}$$

The parameter $\kappa^{\mathbb{P}}$ given in (16) measures the speed of mean reversion under the physical measure. The higher $\kappa^{\mathbb{P}}$, the more mean-reverting. We require $\kappa^{\mathbb{P}} > 0$ to ensure stationarity. The parameter $\xi^{\mathbb{P}}$ given in (17) measures the long-run mean of r under the physical measure $\mathbb{P}$. The volatility function under $\mathbb{P}$ is given by (3), which is the same as that under the risk-neutral measure implied by the diffusion invariance theorem.[13]

We now specify the risk premium associated with the productivity shock. Let ω denote the correlation coefficient between the firm's productivity shock and the aggregate productivity shock. Write the firm's productivity shock dX_t under the physical measure as follows,

$$dX_t = \pi^{\mathbb{P}}dt + \epsilon d\mathbb{Z}_t^{\mathbb{P}}, \tag{18}$$

where $\mathbb{Z}_t^{\mathbb{P}}$ is a standard Brownian motion driving X under the physical measure. The drift for X under the physical measure, $\pi^{\mathbb{P}}$, is linked to the risk-neutral drift π as follows, $\pi^{\mathbb{P}} = \pi + \omega\eta\epsilon$, where η captures the aggregate risk premium per unit of volatility.[14]

2.4 Incorporating Leverage under MM

An immediate and important empirically testable implication of (12) is that Tobin's average q should be the sufficient statistic for investment. However, it is well known the

[12]Using the Girsanov theorem, we relate the Brownian motion under the physical measure $\mathbb{P}$, $\mathbb{B}^{\mathbb{P}}$, to the Brownian motion under the risk-neutral measure, $\mathbb{B}$, by $d\mathbb{B}_t = d\mathbb{B}_t^{\mathbb{P}} + \lambda\sqrt{r_t}dt$. See Duffie (2002).

[13]Because of the square-root volatility function, the CIR interest rate process under both measures is also referred to as a square-root process.

[14]As for the interest rate analysis, we apply the Girsanov theorem to link the Brownian motions for the productivity shocks under the risk-neutral and physical measures via $d\mathbb{Z}_t = d\mathbb{Z}_t^{\mathbb{P}} + \omega\eta dt$.

empirical evidence is disappointing.[15] As equity is subordinate to debt, and moreover, debt has more predictable cash flows than equity, any potential mis-pricing of the firm's total value implies that its equity value as the levered claim on the firm is even more mis-priced.[16] How to avoid using equity market information but effectively use bond price information to forecast corporate investments? By building on the insight that equity is a call option on the firm's total value and incorporating the pricing framework developed in Black and Scholes (1973) and Merton (1974), into the neoclassic q framework, Philippon (2009) constructs an alternative measure capturing the firm's investment opportunities by using bond prices.

Let B_t denote the market value of the firm's debt outstanding at time t and let E_t denote the market value of the firm's all outstanding common equity. For simplicity, we assume that the firm has only debt and common equity. Let $b_t = B_t/K_t$ and $e_t = E_t/K_t$. Obviously, the accounting identity $V_t = B_t + E_t$ implies $q_t = b_t + e_t$. For simplicity, we assume that the MM theorem holds. Our main argument that bond q is a better empirical proxy for investment opportunities than Tobin's average q remains valid even in settings where the MM theorem does not hold due to conflicts of interest, informational frictions, or tax distortions.[17]

As in Philippon (2009), we refer to b_t as the bond's q and use it to measure the firm's investment opportunity in a setting with a constant book-leverage policy.[18] The firm continuously issues and retires multiple units of bonds. Each unit of the newly issued bond has a principal normalized to one. For outstanding bonds issued at any date, a fixed fraction α of them (in terms of their principals) is continuously called back at par. The firm pays coupons at the rate of ρ on all bonds' outstanding principals prior to default.

Let Ψ_t denote the total principal (face value) of all outstanding bonds at time t. Before liquidation, i.e. $t < \tau_L$ and over $(t, t+dt)$, its bondholders receive total cash flows $(\rho+\alpha)\Psi_t dt$, where $\rho\Psi_t$ is the total coupon rate and $\alpha\Psi_t$ is the total bond buyback rate. Given the time-$(t + dt)$ total principal on all outstanding bonds is Ψ_{t+dt}, the new issuance over $(t, t + dt)$ must have a principal of $\Psi_{t+dt} - (1 - \alpha dt)\Psi_t$. At the liquidation time τ_L, bonds are treated *pari passu* and receive their share of liquidation proceeds proportional to its outstanding

[15]See Summers (1981) and Fazzari, Hubbard, and Petersen (1988) for early contributions and Caballero (1999) for a survey.

[16]See Gilchrist and Himmelberg (1995) and Erickson and Whited (2000) for example.

[17]See Appendix A.6 for example.

[18]This construction is similar to the modeling of debt maturity in Leland (1994, 1998) and the subsequent dynamic capital structure models.

principal. The book leverage is defined as Ψ_t/K_t.

Assumption 1 *The firm's book leverage is constant over time in that* $\psi_t \equiv \Psi_t/K_t = \psi$ *for* $t < \tau_L$, *where* ψ *is the target (constant) book leverage.*

The following proposition characterizes debt pricing. Appendix A.5 provides the details.

Proposition 2 *The scaled value of corporate debt,* $b(r)$, *solves:*

$$rb(r) = \rho\psi + \alpha(\psi - b(r)) + \mu(r)b'(r) + \frac{\sigma^2(r)}{2}b''(r)\,, \quad r < r^*\,, \tag{19}$$

subject to the following boundary condition:

$$b(r^*) = \min\{\psi, \ell\}\,, \tag{20}$$

where r^* *is the the firm's endogenous liquidation threshold given in Proposition 1.*

Let $b^{free}(r)$ denote the value of a risk-free bond that pays coupons indefinitely and has the same call-back and coupon policies as described above. For the risk-free bond, we use the same pricing equation (19) for $b^{free}(r)$ but change the boundary condition (20) to $\lim_{r\to\infty} b^{free}(r) = 0$.

3 Solution

We first calibrate the model, then provide a quantitative analysis of the effects of stochastic interest rates on investment and firm value, and finally analyze the model's predictions for firms with leverage.

3.1 Parameter choices

For the interest-rate process parameters, we use estimates reported in Downing, Jaffee and Wallace (2009).[19] Their annual estimates are: the persistence parameter $\kappa^{\mathbb{P}} = 0.1313$, the long-run mean $\xi^{\mathbb{P}} = 0.0574$, the volatility parameter is $\nu = 0.0604$, and the risk premium parameter $\lambda = -1.2555$. Negative interest rate premium ($\lambda < 0$) implies that the interest

[19] They use the methodology of Pearson and Sun (1994) and daily data on constant maturity 3-month and 10-year Treasury rates for the period 1968-2006.

rate is more persistent ($\kappa < \kappa^{\mathbb{P}}$) and is higher on average ($\xi > \xi^{\mathbb{P}}$) after risk adjustments. Under the risk-neutral measure, we have the persistence parameter $\kappa = 0.0555$, the long-run mean $\xi = 0.1359$, and the volatility parameter $\nu = 0.0604$. No arbitrage/equilibrium implies that the volatility parameter remains unchanged.

We choose the annual capital depreciation rate $\delta = 0.09$. The annual mean and volatility of the risk-adjusted productivity are $\pi = 0.18$ and $\epsilon = 0.09$, respectively, which are in line with the estimates of Eberly, Rebelo, and Vincent (2012) for large US firms. We set the liquidation value per unit capital is $\ell = 0.9$ (as suggested in Hennessy and Whited, 2007). For our numerical exercise, we normalize the purchase price of capital to one and choose a quadratic adjustment cost function:

$$c(i) = i + \frac{\theta}{2} i^2 \,. \tag{21}$$

We consider three levels for the annual adjustment cost parameter, $\theta = 2, 5, 20$, which span the range of empirical estimates in the literature.[20]

3.2 Investment and Tobin's average q

Panel A of Figure 1 plots the optimal $i(r)$ with respect to r for $\theta = 2, 5, 20$. As one may expect, $i(r)$ decreases in r. Less obviously but importantly, at a low interest-rate environment such as today's, investment is very sensitive to capital illiquidity. For example, as we increase θ from 2 to 5, near $r = 0$ the firm's investment drops significantly by 79% from 0.49 to 0.10 demonstrating very strong effects of r on investment. Also, investment responds more with respect to changes in r when capital is more liquid, i.e., a lower θ. When interest rates are high, large discounting implies that firm value is mostly driven by its existing capital stock. Therefore, a firm with more illiquid capital optimally chooses to divest less, *ceteris paribus*, which implies the single-crossing feature of $i(r)$ for two levels of θ.

[20] The estimates of the adjustment cost parameter vary significantly in the literature. Procedures based on neoclassic (homogeneity-based) q theory of investment (e.g. Hayashi (1982)) and aggregate data on Tobin's q and investment typically give a high estimate for the adjustment cost parameter θ. Gilchrist and Himmelberg (1995) estimate the parameter to be around 3 using unconstrained subsamples of firms with bond rating. Hall (2004) specifies quadratic adjustment costs for both labour and capital, and finds a low average (across industries) value of $\theta = 1$ for capital. Whited (1992) estimates the adjustment cost parameter to be 1.5 in a q model with financial constraints. Cooper and Haltiwanger (2006) estimate a value of the adjustment cost parameter lower than 1 in a model with fixed costs and decreasing returns to scale. Eberly, Rebelo, and Vincent (2012) estimate a value θ around 7 for large US firms in a homogeneous stochastic framework of Hayashi (1982) with regime-switching productivity shocks.

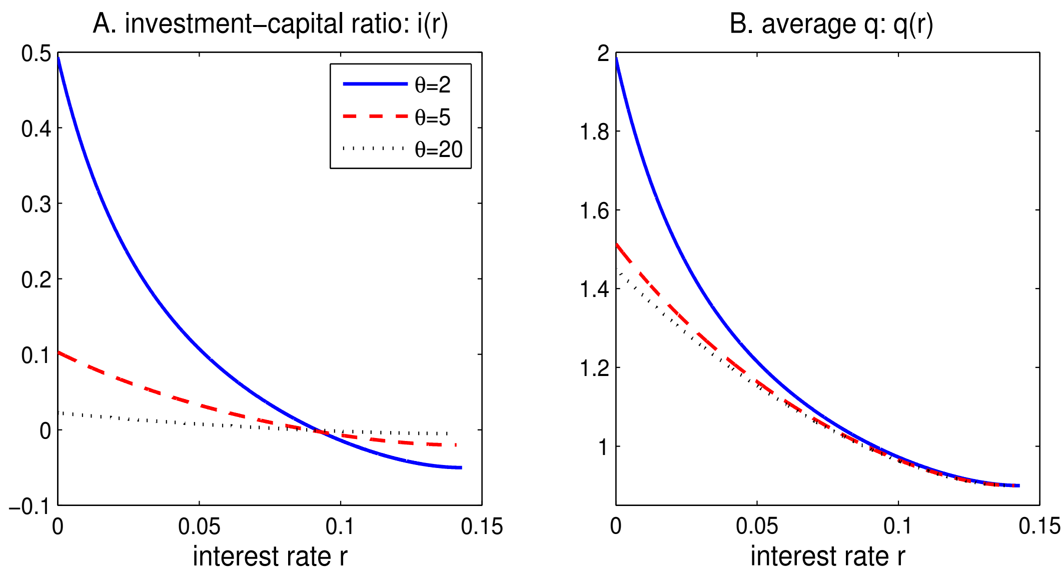

Figure 1: **The investment-capital ratio $i(r)$ and Tobin's average q, $q(r)$**

Panel B of Figure 1 plots Tobin's q for $\theta = 2, 5, 20$. First, the lower the capital adjustment cost θ, the higher Tobin's $q(r)$. Second, $q(r)$ is decreasing and convex in r. Importantly, at a low interest-rate environment such as today's, firm value is very sensitive to capital illiquidity. For example, as we increase θ from 2 to 5, near $r = 0$ Tobin's q drops significantly by 24% from 1.99 to 1.51. With $\theta = 2$, Tobin's q at $r = 0$ is $q(0) = 1.99$, which is 71% higher than $q(\xi^{\mathbb{P}}) = 1.16$ at its long-run mean, $\xi^{\mathbb{P}} = 0.0574$. In summary, our analyses demonstrate that firm value is quite sensitive to capital illiquidity θ and stochastic interest rates r.

As physical capital is a long-lived asset subject to depreciation, we propose a measure that is analogous to the concept of duration for fixed-income securities, which allows us to quantify the interest-rate sensitivity of the value of capital. We also generalize the widely used concept of the user cost of capital developed by Jorgenson (1963) and Hall and Jorgenson (1967) to our q-theoretic setting with term structure of interest rates.

3.3 Firm duration and the user cost of capital

Duration. By analogy to bond pricing, we next define duration for firm value as follows,

$$D(r) = -\frac{1}{V(K,r)}\frac{dV(K,r)}{dr} = -\frac{q'(r)}{q(r)}\,, \tag{22}$$

where the last equality follows from the homogeneity property, $V(K,r) = q(r)K$. Panel A of

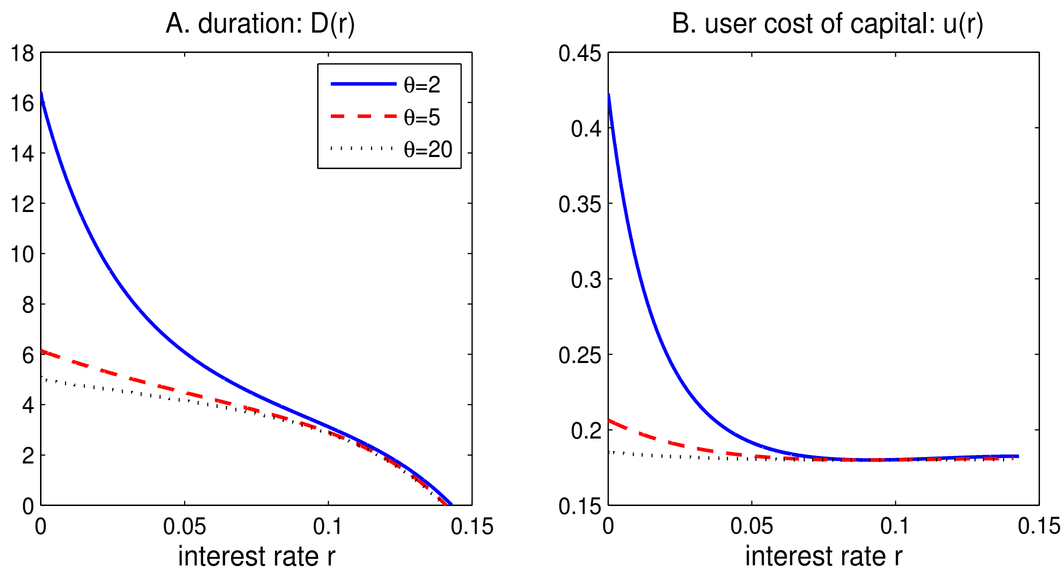

Figure 2: **Duration for firm value $D(r)$ and the user cost of capital $u(r)$**

Figure 2 plots duration for firm value, $D(r)$, as a function of r for $\theta = 2, 5, 20$. Intuitively, the higher the interest rate, the lower the duration. Additionally, at low interest rates such as today's environment, duration is very sensitive to the level of capital adjustment costs. For example, as we increase θ from 2 to 5, near the zero interest rate level, the firm's duration is significantly reduced from 16.43 to 6.17. Overall, the quantitative effects of r on duration are quite significant.

User cost of capital. Jorgenson (1963) and Hall and Jorgenson (1967) introduce the user (rental) cost of capital in their neoclassical framework of investment with no capital adjustment costs. Abel (1990) shows how to calculate the user cost of capital in deterministic q models with capital adjustment costs. We extend Abel (1990) to our q-theoretic setting with stochastic interest rates and term premia. Let $u(\cdot)$ denote the user cost of capital.

Consider a capital's owner who decides to rent her capital out. For each unit of time, the owner collects rents u from the user of her capital, anticipates a risk-adjusted expected change of value $\mathcal{H}q(r)$, where $\mathcal{H}q(r)$ is given by

$$\mathcal{H}q(r) = \mu(r)q'(r) + \frac{\sigma^2(r)}{2}q''(r)\,, \tag{23}$$

and additionally expects a value loss $\delta q(r)$ due to capital depreciation. In equilibrium and after risk adjustments, the owner earns the risk-free rate of return r on the value of capital, $q(r)$. That is, $rq(r) = u(r) + \mathcal{H}q(r) - \delta q(r)$, which can be written as

$$u(r) = (r + \delta)\, q(r) - \mathcal{H}q(r)\,. \tag{24}$$

Importantly, $\mathcal{H}q(r)$ uses the risk-neutral drift $\mu(r)$, not the physical drift $\mu^{\mathbb{P}}(r)$, so that we account for risk premia when calculating the user cost of capital.

By substituting the valuation equation (11) for Tobin's average q and the FOC for investment (12) into (24), we obtain:

$$u(r) = \pi - (c(i) - c'(i)i(r)) > \pi\,, \tag{25}$$

where the inequality in (25) follows from the monotonicity and convexity of the capital adjustment cost $c(\,\cdot\,)$ and $c(0) = 0$. We can also express $u(r) = \pi - C_K(I, K)$. Intuitively speaking, the user's marginal benefit per unit of capital equal the sum of its risk-adjusted expected productivity π and the marginal benefit of reducing capital adjustment costs, i.e. $-C_K > 0$. For a quadratic capital adjustment cost given in (21), $u(r) = \pi + \frac{(q(r)-1)^2}{2\theta}$.

Panel B of Figure 2 plots $u(r)$ for $\theta = 2, 5, 20$. First, $u(r)$ is greater than the risk-adjusted productivity, i.e., $u(r) \geq \pi = 18\%$ for all r. Second, $u(r)$ is highly non-linear in r. At a low interest rate environment such as today's, the user cost of capital is very sensitive to the level of capital adjustment costs. With a moderate level of adjustment cost $\theta = 2$, $u(0) = 0.423$, which implies that the benefit of reducing capital adjustment costs, $-C_K(I, K) = 0.243$, is the majority part of the user cost of capital. As we increase θ from 2 to 5, near the zero interest rate level, the firm's $u(r)$ is significantly reduced from 0.432 to 0.206.

While the standard Jorgensonian user cost of capital $u(r)$ equals $r+\delta$ (with perfect capital liquidity and constant price for the capital good), we show that the $u(r)$ is non-monotonic in r when capital is illiquid and subject to adjustment costs. Indeed, $u(r)$ *decreases* with r in the empirically relevant range of r as we see from Figure 2. To understand the intuition behind this result, we use the formula for the user cost of capital $u(r)$ given in (25), which implies $u'(r) = c''(i)i'(r)i(r)$. As capital adjustment cost is convex, i.e., $c''(i) > 0$, and investment decreases with r, i.e., $i'(r) < 0$, $u(r)$ is decreasing in r as long as the firm's gross investment is positive, i.e., $i(r) > 0$. That is, under the normal circumstances when the firm's gross investment is positive, we expect that the user cost of capital $u(r)$ *decreases* with the interest rate r.

Additionally, the higher the adjustment cost θ, the less sensitive $u(r)$ with respect to r because the reduction of the marginal adjustment cost, $-C_K(I, K)$, is smaller. Intuitively, with infinite adjustment costs, i.e., $\theta \to \infty$, there is no capital accumulation and hence $u(r)$ is simply equal to the risk-adjusted productivity $\pi = 18\%$. Overall, the quantitative effects of r and the capital adjustment cost θ on the user cost of capital $u(r)$ are quite significant. We next provide a decomposition of firm value.

3.4 Leverage

As in Philippon (2009), we define the relative bond price, denoted by $b^{relative}(r)$, as the ratio between the value of corporate bonds and the value of risk-free bonds:

$$b^{relative}(r) = \frac{b(r)}{b^{free}(r)}\,, \tag{26}$$

where the formulas for $b(r)$ and $b^{free}(r)$ are reported in Proposition 2 in Section 2.4. Let $y(r)$ denote the yield spread:

$$y(r) = \frac{b^{free}(r)}{b(r)} - 1 = \frac{1}{b^{relative}(r)} - 1\,. \tag{27}$$

The firm's average q can be expressed as:

$$q(r) = \frac{b(r)}{Lev(r)} = \frac{b^{free}(r)}{Lev(r)} b^{relative}(r) = \frac{b^{free}(r)}{Lev(r)} \frac{1}{1 + y(r)}\,, \tag{28}$$

where $y(r)$ is the credit spread given in (27) and Lev is the market leverage:

$$Lev(r) = \frac{b(r)}{b(r) + e(r)} = \frac{b(r)}{q(r)}\,. \tag{29}$$

In Section 4, we use the implications of (28) to conduct our empirical analyses. Unlike Philippon (2009), we focus on the interest rate shocks.

Figure 3 plots $b(r)$, the model-implied bond q. The bond q decreases almost linearly in r suggesting that the inverse of $b(r)$ is a good approximation of r, which motivates us to use the bond q to construct proxies for interest rates. As r increases, the firm eventually gets liquidated and $b(r)$ approaches to the book leverage ψ.

Figure 4 shows that the relative bond price $b^{relative}(r)$ and yield spreads $y(r)$ positively and negatively predict investments in Panels A and B, respectively. This figure generates

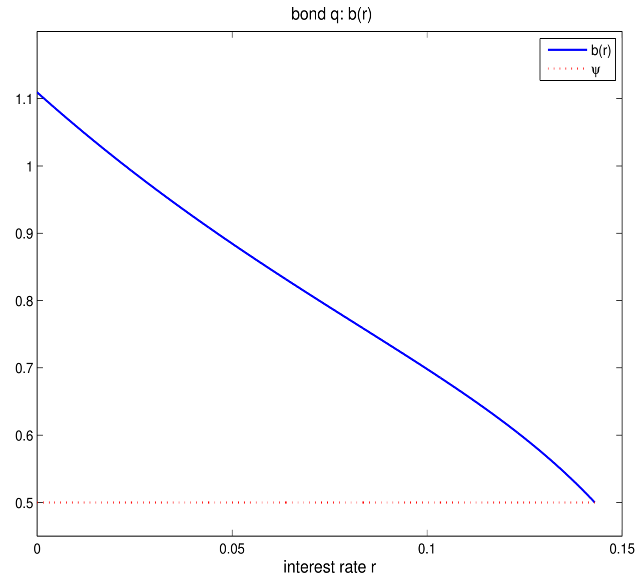

Figure 3: **The bond q, $b(r)$.** Parameter values: $\psi = 0.5$, $\ell = 0.9$, $\alpha = 0.1$, and $\rho = 0.2$.

testable implications and motivates our empirical design in the next section. We use bond-value-based measure for investment opportunities as they are more reliable and less subject to measurement issues, as pointed out by Philippon (2009). Note that the predictive relations are not necessarily driven by the time-varying premium in the stock market and can be solely driven by the time-varying interest rates. Next, we turn to these empirical predictions.

4 Empirical Analyses

Our model implies that Tobin's q is a sufficient statistic to predict investment and also the interest rate negatively predicts investment via its impact on Tobin's q. However, it is well known that the empirical predictive power of Tobin's q for investment is weak. Also the empirical relation between interest rates and investment in the literature is ambiguous. These empirical results seem to challenge the validity of the standard q theory of investment. But it is worth noting that some recent empirical work yields more promising results.

In this section, we first use our theory to guide the construction of our empirical proxies and then test our model's predictions. Using the first-order approximation of Tobin's average

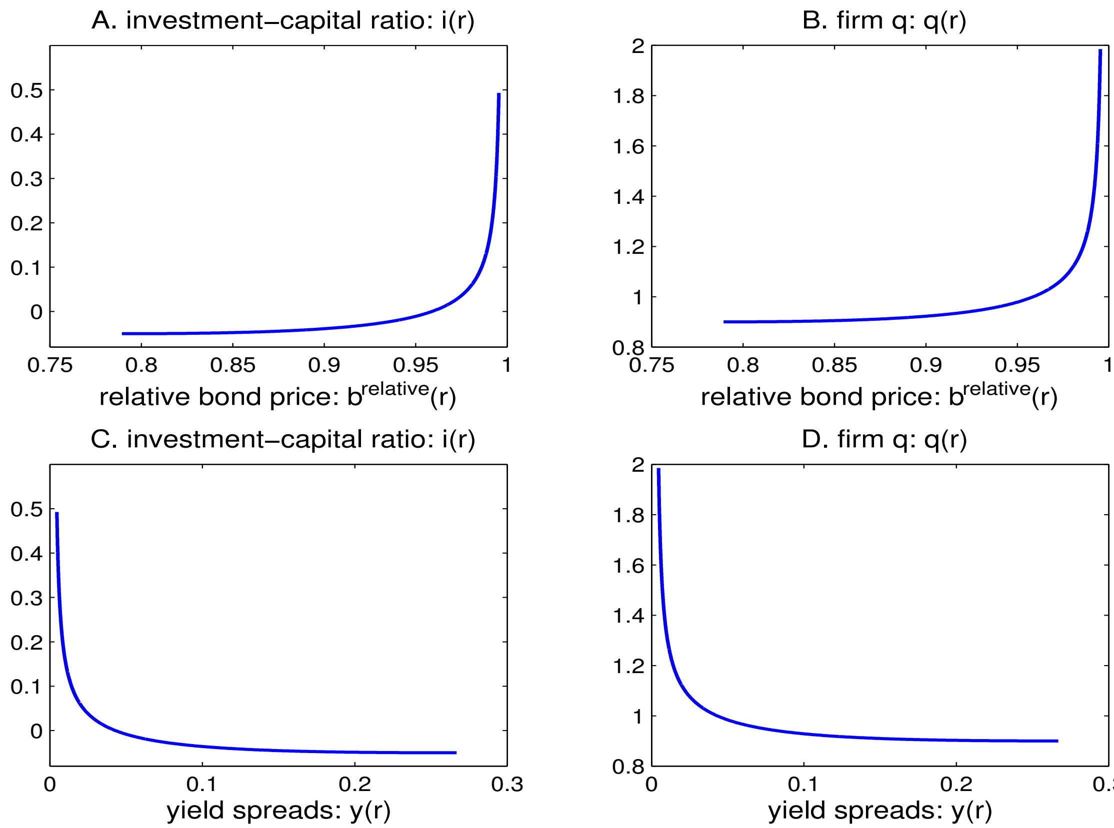

Figure 4: **The effects of relative bond price and yield spreads on $i(r)$ and firm Tobin's q, $q(r)$.** Parameter values: $\psi = 0.5$, $\ell = 0.9$, $\alpha = 0.1$, $\rho = 0.2$.

q around unity, we obtain the following approximate relations for the Tobin's average q:

$$q - 1 \approx \ln[1 + (q-1)] = \ln b^{free} - \ln Lev + \ln b^{relative}, \tag{30}$$

$$= \ln b^{free} - \ln Lev - \ln(1+y), \tag{31}$$

where the two equalities follow from the identities given in (28). That is, after controlling for the risk-free rate information embedded in the logarithmic risk-free bond price $\ln b^{free}$ and firm leverage measured by $\ln Lev$, the logarithmic relative bond price $\ln b^{relative}$ or the corporate credit spread $\ln(1+y)$ can be used to effectively back out Tobin's average q. Empirically, this is highly desirable as Tobin's average q heavily depends on the equity price which is much more subject to mis-pricing than corporate debt, the measurement-error argument in Philippon (2009).

Equations (30) and (31) motivate us to control for the risk-free rate information and the firm's leverage and consider the following three empirical measures of credit risk: 1) the relative bond price (*Rela BP*) as the ratio between the 10-year Treasury rate and the Baa corporate bond yield, i.e., $\frac{0.1+\text{10-year Treasury rate}}{0.1+Baa \text{ bond yield}}$ as in Philippon (2009);[21] 2) Moody's Baa corporate yield in excess of the 10-year Treasury rate (*Baa-Tb10y*); 3) Moody's Baa corporate bond yields in excess of Aaa corporate bond yields (*Baa-Aaa*).

We find significant predictive powers by all three credit risk proxies. That is, at both the firm's and aggregate level, we provide empirical support for (1) the negative relation between investment and credit spreads and (2) the positive relation between investment and the value of capital. Our results complement recent work by Philippon (2009) and Gilchrist and Zakrajšek (2012), who find that bond yields (prices) are informative of investment.

Before discussing our empirical results in detail, we summarize the data information.

4.1 Data

Our empirical analyses use both the aggregate and firm-level data from 1963 to 2014. Next we provide the summary statistics and the construction of various variables. Appendix E provides additional details.

Aggregate Data. Aggregate investment is the private non-residential fixed investment and the corresponding stock of capital is the private non-residential fixed assets from National Income and Product Accounts (NIPA.) Treasury interest rates and the Moody's Baa and Aaa corporate bond yields are from the Federal Reserve Bank of St. Louis. Book leverage is the total liabilities of the nonfinancial corporate business sector from the Flow of Funds scaled by the stock of capital from NIPA.[22]

Panel A of Table 1 reports the summary statistics of the aggregate variables. Investment rate (*IK*) has a mean of 11% and volatility of 1% per annum. Relative bond price (*Rela BP*) has a mean of 0.89 and volatility of 0.045 per annum. *Baa-Tb10y* and *Baa-Aaa* have a mean of 2.03%, and 1.03%, respectively, and volatility of 0.74% and 0.42%, respectively.

To control for the impact of idiosyncratic volatility, we construct a measure of idiosyncratic volatility (denoted as *IdioV*) by calculating the cross-sectional volatility of the monthly

[21] Following Philippon (2009), we add 0.1 to both the 10-year Treasury rate in the numerator and the Baa bond yield in the denominator to reflect that the average maturity of corporate bonds is 10 years. In our model, this is achieved by setting $\alpha = 0.1$.

[22] We use book leverage for our empirical analyses but all our results remain valid with market leverage.

stock returns as in Philippon (2009) and use *IdioV* in our empirical analysis. The mean of *IdioV* is 0.56 per annum. The mean and volatility of book leverage (*BLev*) are 0.46 and 0.06, respectively. To control for the impact of the time-varying risk premium on investment, we calculate the price-to-dividend ratio (denoted as *PD*) of the S&P composite stock price index, which has been shown to predict the expected stock market returns.[23] The mean and volatility of PD are 37.28 and 16.80, respectively.

Firm-level Data. Monthly market values of equities are from CRSP and accounting information is from the CRSP/Compustat Merged Industrial Files. The sample includes firms with common shares (shrcd= 10 and 11) and firms traded on NYSE, AMEX, and NASDAQ (exchcd= $1, 2$, and 3). We omit firms whose primary standard industry classification (SIC) codes are between 4900 and 4999 (utility firms) or between 6000 and 6999 (financial firms). We correct for the delisting bias following the approach in Shumway (1997).

Panel B of Table 1 reports the summary statistics of the annual firm-level variables. The firm-level investment-capital ratio (*IK*) has a mean of 0.29 and volatility of 25% per annum. Firms' book leverage (*BLev*) has a mean of 0.29, and an annual volatility of 0.24. The moments of return on assets (*ROA*), tangibility (*Tang*) and the logarithm of sales (*Sales*) are within the range of estimates in the literature.

Next, we provide empirical evidence in support of the predictability of corporate investments by our theory guided credit-risk-based measures.

4.2 Predicting firms' investments

We specify our baseline investment regression as follows:

$$(IK)_{j,t+1} = \beta x_t + \gamma Z'_{j,t} + \varphi_j + \varepsilon_{j,t+1}, \tag{32}$$

where $(IK)_{j,t+1}$ denotes the investment rate of firm j in period $t+1$, x_t is the key aggregate predictive variable, φ_j is the firm-specific fixed effect, and $Z_{j,t}$ denotes a vector of control variables for firm j in period t including *BLev*, *ROA*, *Tang*, and *Sales*.[24] Table 2 reports the

[23] Campbell and Shiller (1988) and Hodrick (1992), among others, have shown that dividend yields predict stock market returns.

[24] We do not include a time dummy because x_t is an aggregate variable. Our results remain robust after controlling for the aggregate price-to-dividend ratio which proxies for the time-varying risk premium. We also do not include the Tobin's Q as a control because our main regressors capture the effects of Q as seen from equations (30) and (31).

regression results. We cluster the standard errors by firm and time. Our aggregate predictive variable x corresponds to the aggregate relative bond price (*Rela BP*) in Specifications 1 and 4, Baa corporate yield in excess of the 10-year Treasury rate (*Baa-Tb10y*) in Specifications 2 and 5, and Baa corporate bond yields in excess of Aaa corporate bond yields (*Baa-Aaa*) in Specifications 3 and 6, respectively.

Consistent with the model, the aggregate relative bond price (*Rela BP*) positively forecasts firm-level investment rates with a slope of 0.9 and a t-statistic of 9.3 in the univariate regression (i.e., Specification 1.) Importantly, this predictability result remains significant in the multivariate regression (i.e., Specification 4) with various controls introduced earlier. The point estimate is 0.39 with a t-statistic at 5.2.

In our other univariate regressions, (i.e. Specifications 2 and 3) we show that the credit risk measures, i.e., *Baa-Tb10y* and *Baa-Aaa* negatively predict firm-level investment rates with an OLS coefficient of -4.3 and -2.6, respectively. These estimates are also highly significant with a t-statistic of -6.7, and -1.9, respectively. In summary, all three univariate regression results are consistent with the theory. We also report multivariate regressions for *Baa-Tb10y* and *Baa-Aaa* with the various control variables defined earlier in specification 5 and 6, respectively. Both two measures remain significant.

Finally, the predictability of bond-value-based measures for firms' investments are also economically significant. For example, a one-standard-deviation increase of *Baa-Tb10y* is associated with a 1.6 percentage decrease of the firm's investment.

As the aggregate credit spreads are plausibly exogenous to firms, our findings suggest that micro-level corporate investments respond negatively to aggregate interest rates, consistent with our model's key prediction.

4.3 Predicting aggregate investments

Having shown that credit-risk-based measures predict firm-level investments, we now examine the time-series predictability of the aggregate relative bond price and credit spreads for future aggregate investment (Predictive variables are lagged by one year as in Section 4.2.) The first specification of Table 3 shows that the relative bond price (*Rela BP*) positively forecasts aggregate investment with a slope of 0.11, which is significant with a t-statistic of 4.4. This prediction is consistent with our model as Tobin's average q measured by using bond data instead of market equity data contains information about firms' future investment, as argued by Philippon (2009).

The other two measures, e.g., *Baa-Tb10y* and *Baa-Aaa*, all negatively predict future aggregate investment, with slopes of -0.7 and -1.2, respectively (Specifications 2 and 3). They are also highly significant with a t-statistic of -4.8 and -2.7, respectively.

Specifications 4 to 6 present the multivariate regressions with various controls. These regressions show that *Rela BP*, *Baa-Tb10y*, and *Baa-Aaa*, still predict aggregate investments after we control for the 3-month Treasury bill rate (*Tb3m*), idiosyncratic volatility ($IdioV$), book leverage (*BLev*),[25] and the price-to-dividend ratio (*PD*).[26]

In summary, our empirical findings are economically and statistically significant, and are consistent with our model's theoretical predictions on the relation between credit-risk-based measures and investments at both the firm level and the aggregate level.

5 Conclusion

We recognize the importance of stochastic interest rates and incorporate a widely-used term structure model of interest rates into a neoclassic q-theory model of investment. We show that the term structure of interest rates significantly alter both the qualitative and quantitative effects of interest rates on investment and the value of capital. Empirically, we show that our theory-guided bond-information-based measures of firms' investment opportunities have strong predictive powers for both the firm-level and aggregate investments complementing Gilchrist and Zakrajšek (2007, 2012) and Philippon (2009) by providing additional empirical support for the q theory.

For simplicity, we have chosen a one-factor model for the term structure of interest rates. Much empirical work has shown that multi-factor term-structure models fit the yield curve much better.[27] As a result, different factors contributing differently to various interest rates should also have different effects on investments and firm value. For example, a multi-factor term structure model allows us to analyze the different effects of long-term and short-term interest rates on investments.

[25]The results are also significant after controlling for market leverage.

[26]Note that the predictability of aggregate investment by interest rates remains significant after we control for the time-varying aggregate risk premium, which is proxied by the aggregate price-to-dividend ratio.

[27]See Piazzesi (2010) for a survey.

Also extending our model to incorporate financial constraints allows us to analyze how the term structure of interest rates influences a firm's interdependent investment and financing (e.g., cash holdings, leverage, and risk management) policies.[28] Finally, structurally estimating our model with both term structure and corporate investment information may allow us to generate additional insights, provide quantitative predictions, and better understand the "what-if" counterfactuals.[29]

[28]See Cooley and Quadrini (2001), Gomes (2001), and Whited (1992), among others, for quantitative assessments of financial frictions on corporate investment. See Gourio and Michaux (2011) on the effects of stochastic volatility on corporate investment under imperfect capital markets.

[29]Cooper and Haltiwanger (2006) esimate both convex and nonconvex adjustment costs parameters in a dynamic neoclassical investment model but with constant interest rates. See also Strebulaev and Whited (2012) for a review of the recent research development in dynamic models of finance and investment, and structural estimation in corporate finance.

Appendices

A Technical Details

A.1 Unlevered firm under stochastic interest rates

We use the homogeneity property of the firm's value function in capital stock K to simplify our analysis. Specifically, we may write the firm's value function as follows:

$$V(K,r) = K \cdot q(r), \tag{A.1}$$

and Proposition 1 characterizes the solution for $q(r)$. Below we briefly sketch out a proof for Proposition 1.

Substituting (A.1), and various implied relations into the PDE (8) for $V(K,r)$ and simplifying, we obtain the ODE (11). The FOC for investment I given in (9) implies that the optimal i satisfies (12). Next, we turn to the boundary conditions. Upon the liquidation of capital at τ_L, the firm collects its liquidation value ℓK_{τ_L} and hence the value-matching condition $V(K_{\tau_L}, r^*) = \ell K_{\tau_L}$. Also the optimal liquidation decision gives the smooth-pasting condition: $V_r(K_{\tau_L}, r^*) = 0$. Simplifying these two conditions, we obtain (13).

Finally, we report the natural boundary condition at $r = 0$. Equation (8) implies the following condition at $r = 0$: $\max_I \pi K - C(I,K) + (I - \delta K) V_K(K,0) + \kappa\xi V_r(K,0) = 0$.

A.2 The benchmark with constant interest rates

Next, we provide closed-form solutions for i and Tobin's q when $r_t \equiv r$ for all t. This special case is Hayashi (1982) with i.i.d. productivity shocks. Next, we summarize the main results with constant interest rates. The ODE (11) is simplified to

$$rq = \max_i \; (\pi - c(i)) + (i - \delta)\, q, \tag{A.2}$$

where i satisfies $c'(i) = q$. Equivalently, we may write the average q under optimal i as

$$q = \max_i \frac{\pi - c(i)}{r + \delta - i}, \tag{A.3}$$

provided that the following condition holds:

$$\pi < c(r + \delta). \tag{A.4}$$

Equation (A.3) ensures that firm value is finite. Let $\widehat{r}$ denote the interest rate level where the firm is indifferent between liquidating and operating as a going concern, in that Tobin's average q satisfies $q(\widehat{r}) = \ell$ where $q(\,\cdot\,)$ is given by (A.3).

The case with quadratic adjustment costs. When $c(i)$ is quadratic and given in (21), the convergence condition (A.4) takes the following explicit expression:

$$(r+\delta)^2 - 2\left(\pi - (r+\delta)\right)/\theta > 0. \tag{A.5}$$

If $r > \widehat{r}$, the firm liquidates itself and its value is $V = \ell K$. If $r \leq \widehat{r}$, $V = qK$, where

$$q = 1 + \theta i\,, \tag{A.6}$$

and the optimal investment-capital ratio $i = I/K$ is constant and given by

$$i = r + \delta - \sqrt{(r+\delta)^2 - \frac{2}{\theta}\left(\pi - (r+\delta)\right)}\,. \tag{A.7}$$

The cutoff $\widehat{r}$ at which the firm is indifferent between liquidation and continuation satisfies:

$$\frac{\ell - 1}{\theta} = \widehat{r} + \delta - \sqrt{(\widehat{r}+\delta)^2 - \frac{2}{\theta}\left(\pi - (\widehat{r}+\delta)\right)}\,. \tag{A.8}$$

A.3 The user cost of capital

Incorporating risk premia into Abel (1990), we define the user cost of capital u via the following present value formula:

$$q_t = \mathbb{E}_t\left[\int_t^\infty e^{-\int_t^s (r_v+\delta)dv} u_s ds\right]\,. \tag{A.9}$$

Equation (A.9) states that time-t marginal q equals the risk-adjusted present value of the stream of marginal cash flows attributable to a unit of capital installed at time t. Because capital depreciates at the rate of δ, a unit of capital purchased at time t only is worth $e^{-\delta(s-t)}$ unit at time s explaining $(r+\delta)$ in the exponent in (A.9). Note that our definition of user cost of capital is after the risk adjustment in that the expectation operator $\mathbb{E}_t\left[\,\cdot\,\right]$ in (A.9) is under the risk-neutral measure, which incorporates the effects of risk premia for interest rate and productivity shocks.

A.4 Decomposition: Assets in place and growth opportunities

We separate the impact of interest rates on assets in place and growth opportunities, and quantify their separate contributions to firm value.

Assets in place. Let $A(K,r)$ denote the value of assets in place, which is the present discounted value of future cash flows generated by existing capital stock without any further investment/divestment in the future by permanently setting $I=0$. We use the following standard HJB equation for $A(K,r)$:

$$rA(K,r) = \pi K - \delta K A_K(K,r) + \mu(r)A_r(K,r) + \frac{\sigma^2(r)}{2}A_{rr}(K,r)\,. \tag{A.10}$$

Using the homogeneity property $A(K,r) = K \cdot a(r)$ and substituting it into (A.10), we obtain the following ODE (A.11) for $a(r)$:

$$(r+\delta)\,a(r) = \pi + \mu(r)a'(r) + \frac{\sigma^2(r)}{2}a''(r)\,. \tag{A.11}$$

The value of assets in place $A(K,r)$ vanishes as $r\to\infty$, i.e. $\lim_{r\to\infty} A(K,r)=0$, which implies $\lim_{r\to\infty} a(r)=0$. Finally, (A.11) implies that the natural boundary condition at $r=0$ can be simplified as $\pi - \delta a(0) + \kappa\xi a'(0) = 0$.

Intuitively, the value of assets in place (per unit of capital) for an infinitely-lived firm can be viewed as a perpetual bond with a discount rate given by $(r+\delta)$, the sum of interest rate r and capital depreciation rate δ. Using the perpetual bond interpretation, the "effective" coupon for this asset in place is the firm's constant expected productivity π after the risk adjustment (i.e. under the risk-neutral probability).

Panel A of Figure 5 plots the value of assets in place, $a(r)$. By definition, $a(r)$ is independent of growth and the adjustment cost parameter θ. By the perpetual bond interpretation, we know that $a(r)$ is decreasing and convex in r. Quantitatively, $a(r)$ accounts for a significant fraction of firm value. For example, at its long-run mean $\xi^{\mathbb{P}} = 0.0574$, $a(\xi^{\mathbb{P}}) = 1.117$, which accounts for about 96% of total firm value, i.e. $a(\xi^{\mathbb{P}})/q(\xi^{\mathbb{P}}) = 0.96$ for $\theta = 2$.

The value of assets in place generally is not equal to the "book" value or replacement costs of capital, contrary to the conventional wisdom. The value of assets in place is $A(K,r) = a(r)K$, while the book value of capital is K. In general, $a(r)\neq 1$. However, the value of assets in place does not account for growth opportunities, to which we now turn.

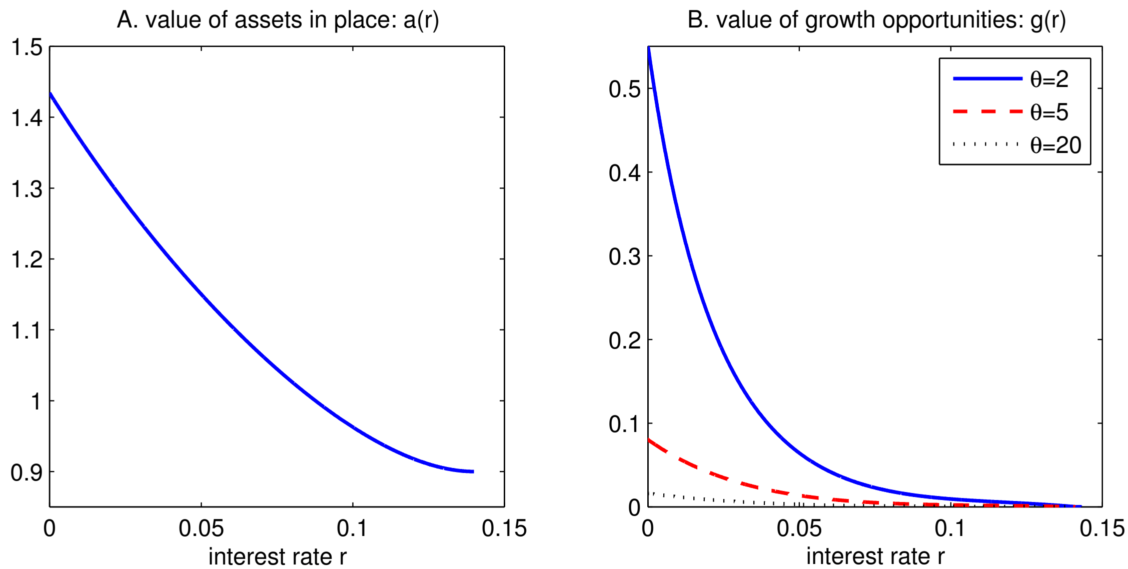

Figure 5: **Valuing assets in place $a(r)$ and growth opportunities $g(r)$**

Growth opportunities. The value of growth opportunities, $G(K,r)$ given by $G(K,r) = V(K,r) - A(K,r)$, accounts for the value of optimally adjusting investment in response to changes in interest rates. The scaled value, $g(r) = G(K,r)/K$ is given by

$$g(r) = q(r) - a(r). \tag{A.12}$$

Panel B of Figure 5 plots $g(r)$ for $\theta = 2, 5, 20$. The quantitative effects of interest rates and capital illiquidity on $g(r)$ are strong. At a low interest rate environment such as today's, the value of growth opportunities is very sensitive to the level of adjustment cost θ and interest rates. With a moderate value of $\theta = 2$, the value of growth opportunities is about 55.2% of the existing capital stock, i.e., $g(0) = 0.552$. As we increase the interest rate from 0 to its long-run mean $\xi^{\mathbb{P}} = 0.0574$, the value of growth opportunities per unit of capital stock drops by more than 90% from 0.552 to 0.047. As we increase θ from 2 to 5, at $r = 0$, the value of growth opportunities decreases by 0.472 from 0.552 to 0.080. In summary, both interest rates and capital illiquidity have first-order effects on the value of growth opportunities.

A.5 Levered firm under stochastic interest rates

For simplicity, we assume that the investment decision I and the liquidation time τ_L are chosen to maximize the firm's total value. That is, we assume that the MM theorem holds and leverage simply requires us to do valuation given the leverage policy. Therefore, the firm's (investment and liquidation) decisions and Tobin's q are given in Proposition 1. We thus only need to report the bond pricing results given corporate policies.

Bond pricing. Given investment and liquidation decisions in Proposition 1, the firm's total debt/bond value $B(K, r)$ satisfies the following pricing equation when $r < r^*$:

$$rB(K,r) = \rho\Psi - \alpha(B - \Psi) - (I - \delta K)\psi\frac{B}{\Psi} + (I - \delta K)B_K + \mu(r)B_r + \frac{\sigma^2(r)}{2}B_{rr}\,. \quad \text{(A.13)}$$

The first term on the right side of (A.13) gives the total coupon payments. The second term reflects the net gains/losses due to the rollover of the existing bonds Ψ_t and the new bond issue at the market price B_t. By fixing the firm's book leverage Ψ_t/K_t at a constant level ψ, the net change of the face value for the bond is $d\Psi_t = \psi dK_t = (I_t - \delta K_t)\psi dt$, and thus the shareholders collect $B_t d\Psi_t/\Psi_t = (I_t - \delta K_t)\psi B_t dt/\Psi_t$ by adjusting the outstanding debt amount, which is captured by the third term. Finally, the last three terms illustrate the effects of the physical capital stock K and interest rate r on $B(K, r)$. Upon liquidation, given the debt holders' seniority over equity investors, we must have

$$B(K_{\tau_L}, r^*) = \min\{\Psi_{\tau_L}, \ell K_{\tau_L}\} = \min\{\psi, \ell\}K_{\tau_L}\,. \quad \text{(A.14)}$$

Using the homogeneity property $b_t = B_t/K_t$ and the pricing equation (A.13), we obtain the ODE (19) for b. Equation (A.14) implies (20). Equity pricing is given by $e(r) = q(r) - b(r)$ where $q(r)$ and $b(r)$ are given by Propositions 1 and 2, respectively.

A.6 A non-MM model

As in Leland (1994), we can generalize our model for levered firms by allowing the firm to choose its default policy with the objective of maximize its equity value. By doing so, equity investors face a time-inconsistency problem as incentives before and after debt issue differ. The pricing formulae for equity and debt will thus be different, but the key idea that bond's q is still a more robust measure than the Tobin's q for firms' investment opportunities remains valid. Next, we summarize the main results when investment and default decisions are chosen sequentially by the firm's equity holders in the following proposition.

Proposition 3 *The scaled bond value $b(r)$, equity value $e(r)$, and investment $i(r)$ jointly satisfy the following equations in the continuation region where $r < r^*$:*

$$\begin{aligned} rb(r) &= \rho\psi + \alpha(\psi - b(r)) + \mu(r)b'(r) + \frac{\sigma^2(r)}{2}b''(r)\,, && \text{(A.15)} \\ (r+\delta-i(r))e(r) &= \pi - c(i(r)) - \rho\psi + \alpha(b(r)-\psi) + (i(r)-\delta)b(r) \\ &\quad +\mu(r)e'(r) + \frac{\sigma^2(r)e''(r)}{2}\,, && \text{(A.16)} \\ c'(i(r)) &= b(r) + e(r)\,, && \text{(A.17)} \end{aligned}$$

subject to the following boundary conditions:

$$b(r^*) = \min\{\psi, \ell\}\,, \quad e(r^*) = max\{0, \ell - \psi\}\,, \quad \textit{and } \; e'(r^*) = 0\,. \tag{A.18}$$

And the firm's Tobin's q is given by $q(r) = b(r) + e(r)$.

Equation (A.17) reflects that the firm's investment policy is chosen to maximize Tobin's q. The optimal default decision is chosen to maximize equity value, as one see from $e'(r^*) = 0$.

B Stationary Distributions of r and $q(r)$

We now report the stationary distributions of the interest rate r and Tobin's q. Recall that the stationary distributions of the interest rate in the CIR model under both the physical and risk-neutral measures are the Gamma distribution with different parameter values. The probability density function (pdf) under the risk-neutral measure, $f_r(r;\kappa,\xi)$, is given by

$$f_r(r;\kappa,\xi) = \frac{1}{\Gamma\left(2\kappa\xi/\nu^2\right)}\left(2\kappa/\nu^2\right)^{2\kappa\xi/\nu^2} r^{2\kappa\xi/\nu^2-1}e^{-2\kappa r/\nu^2}\,, \tag{B.1}$$

where $\Gamma(\,\cdot\,)$ is the Gamma function. The pdf for r under the physical measure is then $f_r(r;\kappa^{\mathbb{P}},\xi^{\mathbb{P}})$.

Applying the standard probability density transformation technique, we have the following probability density function for Tobin's q under a given measure,

$$f_q(q) = \frac{f_r(r)}{|q'(r)|}\,. \tag{B.2}$$

Intuitively, the pdf $f_q(q)$ depends on the pdf $f_r(r)$ for the interest rate and inversely depends on the sensitivity of Tobin's q with respect to r. We plot the stationary distributions for

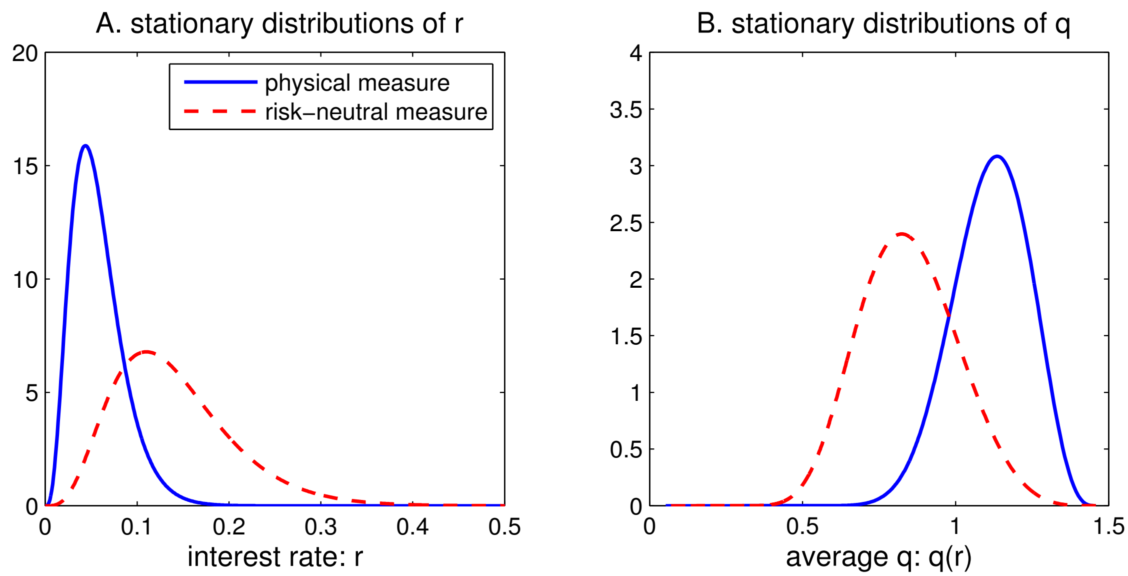

Figure 6: **Stationary distributions for r and Tobin's average q**

Tobin's q under both measures in Panel B of Figure 6. We see that the distribution of r after risk adjustments shifts to the right as interest rates are on average higher and more transitory under the risk-neutral measure than under the physical measure. As a result, the distribution of q after risk adjustments shifts to the left due to the risk premium.

C Asymmetry, Price Wedge, and Fixed Costs

C.1 Model

We extend the convex adjustment cost $C(I, K)$ in our baseline model along three important dimensions. Empirically, downward adjustments of capital stock are often more costly than upward adjustments. We capture this feature by assuming that the firm incurs *asymmetric* convex adjustment costs in investment ($I > 0$) and divestment ($I < 0$) regions. Hall (2001) uses the asymmetric adjustment cost in his study of aggregate market valuation of capital and investment. Zhang (2005) uses this asymmetric adjustment cost in studying investment-based cross-sectional asset pricing.

Second, as in Abel and Eberly (1994), we assume a wedge between the purchase and sale prices of capital, for example due to capital specificity and illiquidity premium. There is much empirical work documenting the size of the wedge between the purchase and sale prices. Arrow (1968) stated that "there will be many situations in which the sale of capital goods cannot be accomplished at the same price as their purchase." The wedge naturally depends on the business cycles and market conditions.[30] Let p_+ and p_- denote the respective purchase and sale prices of capital. An economically sensible assumption is $p_+ \geq p_- \geq 0$ with an implied wedge $p_+ - p_-$.

Third, investment often incurs fixed costs. Fixed costs may capture investment indivisibilities, increasing returns to the installation of new capital, and organizational restructuring during periods of intensive investment. Additionally, fixed costs significantly improve the empirical fit of the model with the micro data. Inaction becomes optimal in certain regions. To ensure that the firm does not grow out of fixed costs, we assume that the fixed cost is proportional to its capital stock. See Hall (2004), Cooper and Haltiwanger (2006), and Riddick and Whited (2009) for the same size-dependent fixed cost assumption.

Following Abel and Eberly (1994), we write the region-dependent function $c(i)$ as follows,

$$c(i) = \begin{cases} 0, & \text{if} \quad i = 0\,, \\ \phi_+ + p_+ i + \frac{\theta_+}{2} i^2, & \text{if} \quad i > 0\,, \\ \phi_- + p_- i + \frac{\theta_-}{2} i^2, & \text{if} \quad i < 0\,, \end{cases} \tag{C.1}$$

where ϕ_+ an ϕ_- parameterize the fixed costs of investing and divesting, p_+ and p_- are the respective price of purchasing and selling capital, and θ_+ and θ_- are the asymmetric convex adjustment cost parameters. For $i > 0$, $c(i)$ is convex in i. For $i < 0$, $c(i)$ is also convex. Panels A and B of Figure 7 plots $c(i)$ given in (C.1), and the marginal cost of investing $c'(i)$, respectively. Note that $c(i)$ is not continuous at $i = 0$ and hence $c'(i)$ is not defined at the origin ($i = 0$). For illustrative simplicity, we set the liquidation value to zero, i.e., $\ell = 0$.

C.2 Solution

In general, the model solution has three distinct regions: (positive) investment, inaction, and divestment regions. We use $q_+(r)$, $q_0(r)$ and $q_-(r)$ to denote Tobin's q in these three regions, respectively. The following proposition summarizes the main results.

[30] The estimates range from 0.6 to 1, depending on data sources, estimation methods, and model specifications. See Pulvino (1998), Hennessy and Whited (2005), Cooper and Haltiwanger (2006), and Warusawitharana (2008), for example.

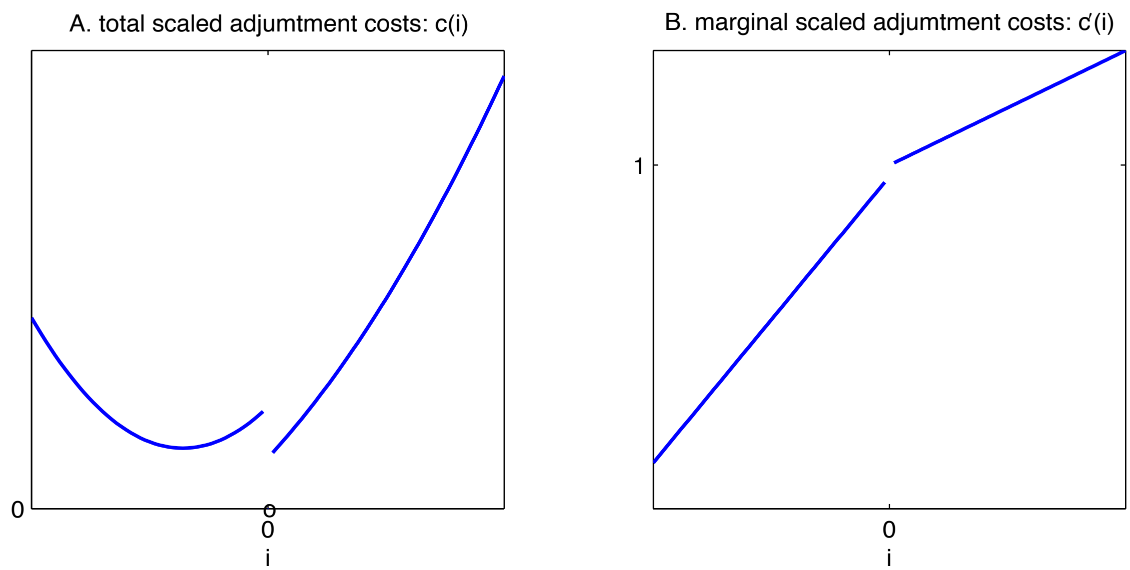

Figure 7: **The cost of investing $c(i)$ and marginal cost of investing $c'(i)$**

Proposition 4 *Tobin's q in investment, inaction, and divestment regions, $q_+(r)$, $q_0(r)$, and $q_-(r)$, respectively, solve the following three linked ODEs,*

$$(r+\delta)\,q_+(r) = \pi - \phi_+ + \frac{(q_+(r)-p_+)^2}{2\theta_+} + \mu(r)q_+'(r) + \frac{\sigma^2(r)}{2}q_+''(r), \quad \text{if } r < \underline{r}, \tag{C.2}$$

$$(r+\delta)q_0(r) = \pi + \mu(r)q_0'(r) + \frac{\sigma^2(r)}{2}q_0''(r), \quad \text{if } \underline{r} < r < \overline{r}, \tag{C.3}$$

$$(r+\delta)\,q_-(r) = \pi - \phi_- + \frac{(q_-(r)-p_-)^2}{2\theta_-} + \mu(r)q_-'(r) + \frac{\sigma^2(r)}{2}q_-''(r), \quad \text{if } r > \overline{r}\,. \tag{C.4}$$

The endogenously determined cutoff interest rate levels for these three regions, $\underline{r}$ and $\overline{r}$, satisfy the following boundary conditions,

$$\pi - \phi_+ - \delta q_+(0) + \frac{(q_+(0)-p_+)^2}{2\theta_+} + \kappa\xi q_+'(0) = 0\,, \tag{C.5}$$

$$q_+(\underline{r}) = q_0(\underline{r}), \qquad q_0(\overline{r}) = q_-(\overline{r})\,, \tag{C.6}$$

$$q_+'(\underline{r}) = q_0'(\underline{r}), \qquad q_0'(\overline{r}) = q_-'(\overline{r})\,, \tag{C.7}$$

$$q_+''(\underline{r}) = q_0''(\underline{r}), \qquad q_0''(\overline{r}) = q_-''(\overline{r})\,, \tag{C.8}$$

$$\lim_{r\to\infty} q_-(r) = 0\,. \tag{C.9}$$

The optimal investment-capital ratios, denoted as $i_+(r)$, $i_0(r)$, and $i_-(r)$, are given by

$$i_+(r) = \frac{q_+(r) - p_+}{\theta_+}, \quad \text{if} \quad r < \underline{r}, \tag{C.10}$$

$$i_0(r) = 0, \quad \text{if} \quad \underline{r} \leq r \leq \overline{r}, \tag{C.11}$$

$$i_-(r) = -\frac{p_- - q_-(r)}{\theta_-}, \quad \text{if} \quad r > \overline{r}. \tag{C.12}$$

When r is sufficiently low ($r < \underline{r}$), the firm optimally chooses to invest, $I > 0$. Investment is proportional to $q_+(r) - p_+$, the wedge between Tobin's q and purchase price of capital, p_+. Tobin's q in this region, $q_+(r)$, solves the ODE (C.2). Condition (C.5) gives the firm behavior at $r = 0$. The right boundary $\underline{r}$ is endogenous. Tobin's q at $\underline{r}$, $q_+(\underline{r})$, satisfies the first set of conditions in (C.6)-(C.8), i.e. $q(r)$ is twice continuously differentiable at $\underline{r}$.

Similarly, when r is sufficiently high ($r > \overline{r}$), the firm divests, $I < 0$. Divestment is proportional to $p_- - q_-(r)$, the wedge between the sale price of capital, p_-, and Tobin's q. Tobin's q in the divestment region, $q_-(r)$, solves the ODE (C.4). Condition (C.9) states that the firm is worthless as $r \to \infty$, the right boundary condition. The left boundary for the divestment region $\overline{r}$ is endogenous. Tobin's q at $\overline{r}$, $q_-(\overline{r})$, satisfies the second set of the conditions in (C.6)-(C.8), i.e. $q(r)$ is twice continuously differentiable at $\overline{r}$.

For r in the intermediate range ($\underline{r} \leq r \leq \overline{r}$), the firm optimally chooses to be inactive, $i(r) = 0$, and hence incurs no adjustment costs. Tobin's q in this region thus behaves likes assets in place and solves the linear ODE (C.3). The optimal thresholds $\underline{r}$ and $\overline{r}$ are endogenously determined by conditions (C.6)-(C.8), as we discussed previously.

Proposition 4 focuses on the case where all three regions exist, i.e. $0 < \underline{r} < \overline{r}$.

C.3 Three special cases

We next study the impact of each friction on investment and Tobin's q. For the baseline case, we set $\theta_+ = \theta_- = 2$ (symmetric convex costs), $p_+ = p_- = 1$ (no price wedge) and $\phi_+ = \phi_- = 0$ (no fixed costs). For each special case, we only change the key parameter of interest and keep all other parameters the same as in the baseline case just described.

Asymmetric convex adjustment costs. Much empirical evidence suggests that divestment is generally more costly than investment, i.e. $\theta_- > \theta_+$. We set the adjustment cost parameter $\theta_+ = 2$ for investment ($I > 0$) and $\theta_- = 2, 5, 20$ for divestment ($I < 0$).

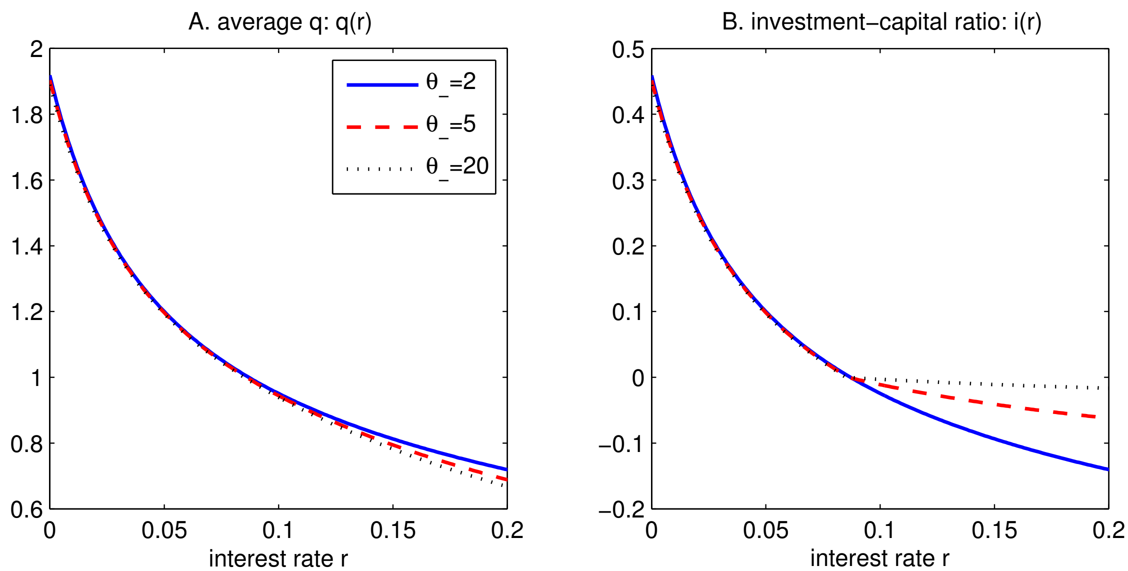

Figure 8: **Tobin's average q and $i(r)$ with asymmetric convex adjustment costs**

Figure 8 shows that the divestment adjustment cost parameter θ_- has strong impact on Tobin's q and $i(r)$ in the divestment region (high r), but almost no impact on $q(r)$ and $i(r)$ in the positive investment region. When r is sufficiently high, the firm divests, and changing θ_- has first-order effects on divestment. The higher the value of θ_-, the more costly divestment and the less divestment activity. With $\theta_- = 20$, the firm is close to facing an irreversible investment option, and hence the optimal divestment level is close to zero. When r is sufficiently low, it is optimal to invest. The divestment option is far out of the money and thus changing θ_- has negligible effects on valuation and investment.

The wedge between purchase and sale prices of capital. We now turn to the effects of price wedge. We normalize the purchase price at $p_+ = 1$ and consider two sale prices, $p_- = 0.8, 0.9$, with implied wedge being 0.2 and 0.1, respectively. We also plot the baseline case with no price wedge as a reference.

Figure 9 plots Tobin's q and the investment-capital ratio $i(r)$ for a firm facing a price wedge. The price wedge leads to three distinct investment regions: investment ($I > 0$), inaction (zero), and divestment ($I < 0$). With low interest rates, the firm invests for growth and the asset sales option is sufficiently out of the money. Hence, price wedge has negligible effects on Tobin's q and investment. However, with high interest rates, the asset sales option

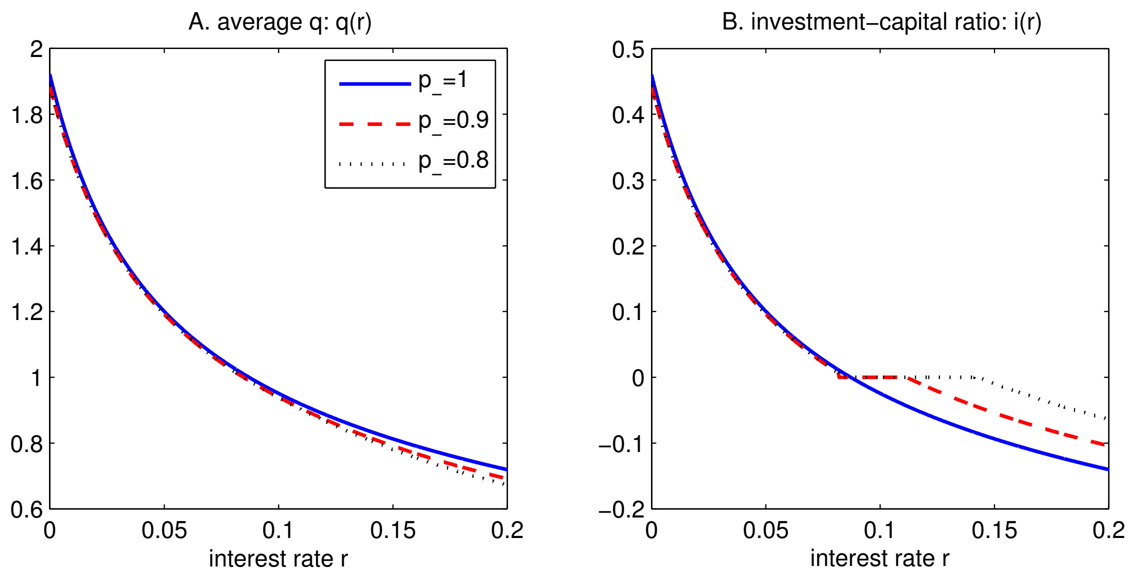

Figure 9: **Tobin's q and the investment-capital ratio $i(r)$ with price wedges**

becomes in the money and divestment is optimal. The price wedge thus has significant effects on divestment and value. With wedge being $p_+ - p_- = 0.2$, the firm invests when $r \leq 0.082$ and divests when $r \geq 0.141$. For intermediate values of r ($0.082 \leq r \leq 0.141$), inaction is optimal. In this range, the marginal cost of investment/divestment justifies neither purchasing nor selling capital due to the price wedge. Note that inaction is generated here by the price wedge, not fixed costs. Finally, we note that investment/divestment activities and inaction significantly depend on the price wedge. For example, the inaction region narrows from (0.082, 0.141) to (0.082, 0.109) when the price wedge decreases from 20% to 10%.

Fixed costs and optimal inaction. We now study two settings with fixed costs: (a) fixed costs for divestment only ($\phi_+ = 0$, $\phi_- = 0.01$), and (b) symmetric fixed costs for both investment and divestment ($\phi_+ = \phi_- = 0.01$). We also plot the case with no fixed costs ($\phi_+ = \phi_- = 0$) as a reference.

Figure 10 plots Tobin's average q and the investment-capital ratio $i(r)$ under fixed costs. With fixed costs for divestment, $\phi_- > 0$, we have three regions for $i(r)$. For sufficiently low interest rates ($r \leq 0.082$), optimal investment is positive and is almost unaffected by ϕ_-. For sufficiently high r ($r \geq 0.142$), divestment is optimal. The firm divests more aggressively with fixed costs of divestment than without. Intuitively, the firm's more aggressive divestment

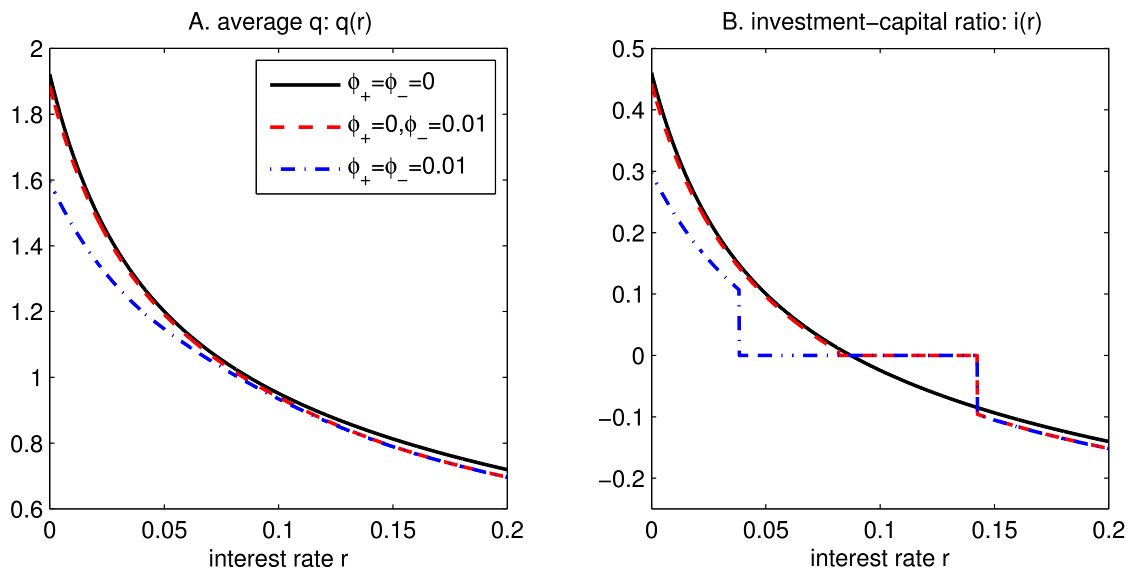

Figure 10: **Tobin's q and $i(r)$ with fixed adjustment costs**

strategy economizes fixed costs of divestment. Additionally, fixed costs generate an inaction region, $0.082 \leq r \leq 0.142$. The impact of fixed costs of divestment is more significant on Tobin's q in medium to high r region than in the low r region.

Now we incrementally introduce fixed costs for investment by changing ϕ_+ from 0 to 0.01, while holding $\phi_- = 0.01$. We have three distinct regions for $i(r)$. For high r, $r \geq 0.142$, the firm divests. Tobin's q and $i(r)$ in this region remain almost unchanged by ϕ_+. For low r, $r \leq 0.038$, the firm invests less with $\phi_+ = 0.01$ than with $\phi_+ = 0$.

Introducing the fixed costs ϕ_+ discourages investment, lowers Tobin's q, shifts the inaction region to the left, and widens the inaction region. The lower the interest rate, the stronger the effects of ϕ_+ on Tobin's q, investment, and the inaction region.

C.4 Irreversibility

Investment is often irreversible, or at least costly to reverse after capital is installed. There is much work motivated by the irreversibility of capital investment. Arrow (1968) is a pioneering study in a deterministic environment. Our model generates irreversible investment as a special case. We have three ways to deliver irreversibility within our general framework. Intuitively, they all work to make divestment very costly. We may set the re-sale price

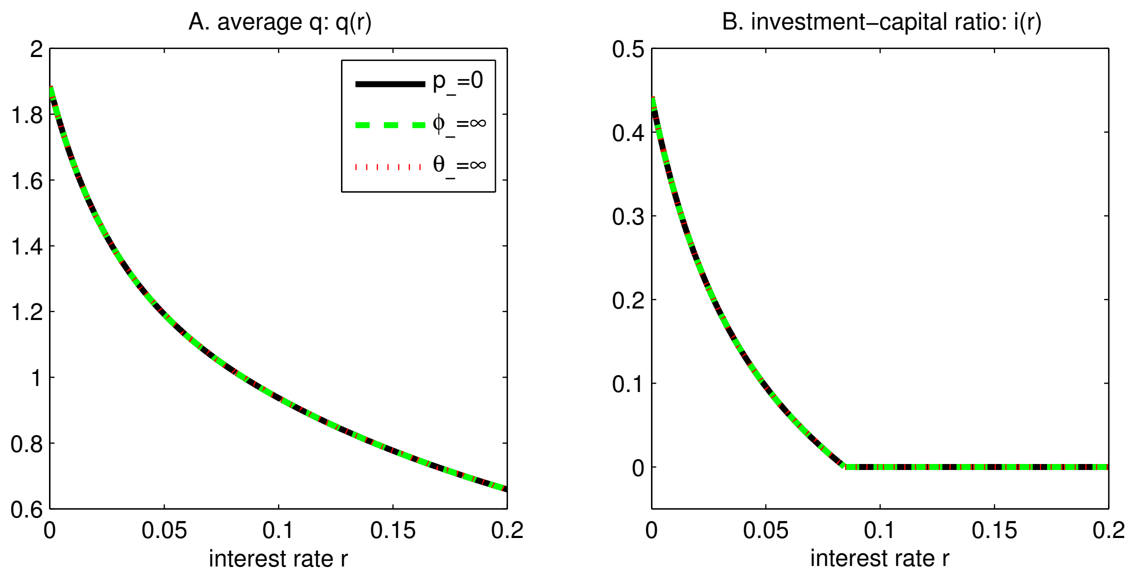

Figure 11: **Tobin's q and $i(r)$ with irreversible investment**

of installed capital to zero ($p_- = 0$), making capital completely worthless if liquidated. Alternatively, we may choose the adjustment cost for either convex or lumpy divestment to infinity, (i.e. $\theta_- = \infty$, $\phi_- = \infty$). The three cases all deliver identical solutions for both the divestment and the positive investment regions. Figure 11 plots Tobin's q and the optimal investment-capital ratio $i(r)$ under irreversibility. As in our baseline model, investment varies significantly with the level of the interest rate. Ignoring interest rate dynamics induces significant error for Tobin's q and investment.

Derivation for Proposition 4. With homogeneity property, we conjecture that there are three regions (positive, zero, and negative investment regions), separated by two endogenous cutoff interest-rate levels $\underline{r}$ and $\overline{r}$. Firm value in the three regions can be written as follows,

$$V(K,r) = \begin{cases} K \cdot q_-(r), & if \quad r > \overline{r}, \\ K \cdot q_0(r), & if \quad \underline{r} \leq r \leq \overline{r}, \\ K \cdot q_+(r), & if \quad r < \underline{r}, \end{cases} \tag{C.13}$$

Importantly, at $\underline{r}$ and $\overline{r}$, $V(K,r)$ satisfies value-matching, smooth-pasting, and super contact conditions, which imply (C.6), (C.7), and (C.8), respectively. Note that (C.5) is the natural boundary condition at $r = 0$ and (C.9) reflects that firm value vanishes as $r \to \infty$. Other details are essentially the same as those in Proposition 1.

When the fixed cost for investment ϕ_+ is sufficiently large, there is no investment region, i.e. $\underline{r} = 0$. Additionally, the condition at $r = 0$, (C.5), is replaced by the following condition,

$$\pi - \delta q_0(0) + \kappa \xi q_0'(0) = 0 \,. \tag{C.14}$$

In sum, for the case with inaction and divestment regions, the solution is given by the linked ODEs (C.3)-(C.4) subject to (C.14), the free-boundary conditions for the endogenous threshold $\overline{r}$ given as the second set of conditions in (C.6)-(C.8), and the limit condition (C.9).

Similarly, if the cost of divestment ϕ_- is sufficiently high, the firm has no divestment region, i.e. $\overline{r} = \infty$. The model solution is given by the linked ODEs (C.2)-(C.3) subject to (C.5), the free-boundary conditions for $\underline{r}$ given as the first set of conditions, and $\lim_{r\to\infty} q_0(r) = 0$.

D Serially correlated productivity shocks

We now extend our baseline convex model to allow for serially correlated productivity shocks. Let s_t denote the state (regime) at time t. The expected productivity in state s at any time t, $\pi(s_t)$, can only take on one of the two possible values, i.e. $\pi(s_t) \in \{\pi_L, \pi_H\}$ where $\pi_L > 0$ and $\pi_H > \pi_L$ are constant. Let $s \in \{H, L\}$ denote the current state and $s-$ refer to the other state. Over the time period $(t, t + \Delta t)$, under the risk-neutral measure, the firm's expected productivity changes from π_s to π_{s-} with probability $\zeta_s \Delta t$, and stays unchanged at π_s with the remaining probability $1 - \zeta_s \Delta t$. The change of the regime may be recurrent. That is, the transition intensities from either state, ζ_H and ζ_L, are strictly positive. The incremental productivity shock dX after risk adjustments (under the risk neutral measure) is given by

$$dX_t = \pi(s_{t-})dt + \epsilon(s_{t-})d\mathbb{Z}_t \,, \quad t \geq 0 \,. \tag{D.1}$$

The firm's operating profit dY_t over the same period $(t, t + dt)$ is also given by (6) as in the baseline model. The homogeneity property continues to hold. Again, for illustrative simplicity, we set the liquidation value to zero, i.e., $\ell = 0$. The following theorem summarizes the main results.

Theorem 1 *Tobin's q in two regimes, $q_H(r)$ and $q_L(r)$, solves the following linked ODEs:*

$$rq_s(r) = \pi_s - c(i_s(r)) + (i_s(r) - \delta)q_s(r) + \mu(r)q_s'(r) + \frac{\sigma^2(r)}{2}q_s''(r) + \zeta_s(q_{s-}(r) - q_s(r)), \ s = H, L, \tag{D.2}$$

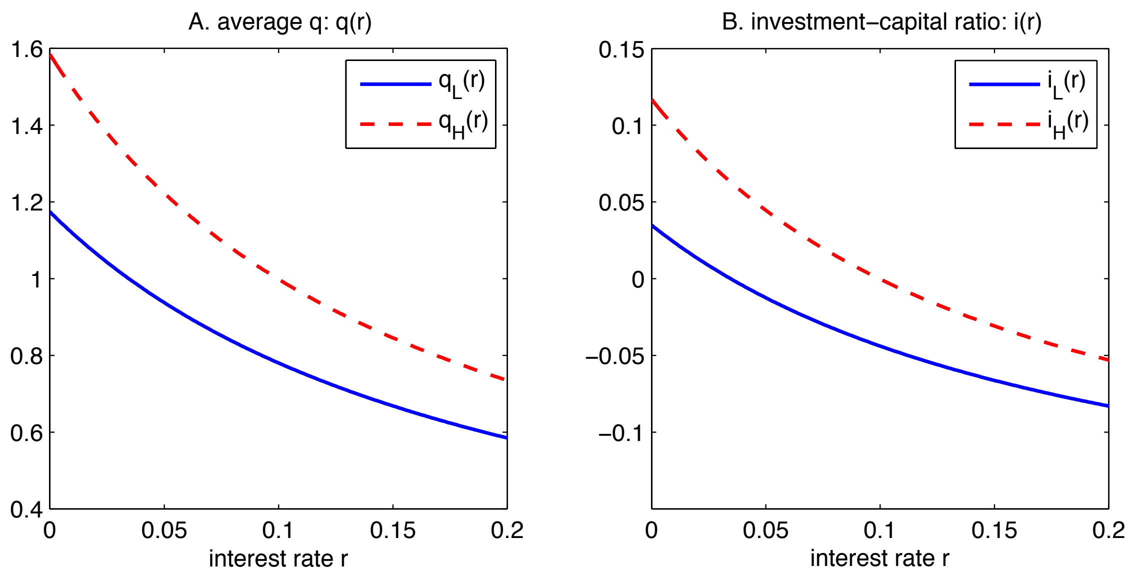

Figure 12: **Tobin's q and $i(r)$ with serially correlated productivity shocks**

subject to the following boundary conditions,

$$\pi_s - c(i_s(0)) + (i_s(0) - \delta)q_s(0) + \kappa\xi q_s'(0) + \zeta_s(q_{s-}(0) - q_s(0)) \quad = \quad 0\,, \tag{D.3}$$

$$\lim_{r\to\infty} q_s(r) \quad = \quad 0\,. \tag{D.4}$$

The optimal investment-capital ratios in two regimes $i_H(r)$ and $i_L(r)$ are given by

$$i_s(r) = \frac{q_s(r) - 1}{\theta}\,, \quad s = H,\, L. \tag{D.5}$$

Figure 12 plots Tobin's average q and the investment-capital ratio $i(r)$ for both the high- and the low-productivity regimes. We choose the expected (risk-neutral) productivity, $\pi_H = 0.2$ and $\pi_L = 0.14$, set the (risk-neutral) transition intensities at $\zeta_L = \zeta_H = 0.03$. The expected productivity has first-order effects on firm value and investment; both $q_H(r)$ and $i_H(r)$ are significantly larger than $q_L(r)$ and $i_L(r)$, respectively. Additionally, both $q_H(r)$ and $q_L(r)$ are decreasing and convex as in the baseline model. Our model with serially correlated productivity shocks can be extended to allow for richer adjustment cost frictions such as the price wedge and fixed costs as we have done in the previous section, and multiple-state Markov chain processes for productivity shocks.

E Data Construction

Aggregate Data. We use data for aggregate investment I and capital stock K from the national account and fixed asset tables available from the Bureau of Economic Analysis (BEA). We construct annual series of the aggregate investment rate, denoted by IK, as $IK_t = \frac{I_t}{0.5\times(K_{t-1}+K_t)}$, where investment I is gross private nonresidential fixed investment from NIPA Table 1.1.5 and capital K is nonresidential fixed asset from NIPA fixed asset Table 1.1. Note that investment I and capital K are scaled by the annual implicit price deflator for the gross private nonresidential fixed investment, reported in NIPA Table 1.1.9.

We obtain financial information from Compustat for US publicly held companies with information for two or more consecutive years. We use fiscal-year annual company data from balance sheets, income statements, and cash flow statements, and omit observations with negative total assets (AT) or current assets (ACT). Utilities and financial firms are excluded from the sample. Specifically, We omit firms whose primary standard industry classification (SIC) code is between 4900 and 4999 (utility firms) or between 6000 and 6999 (financial firms).

Firm-level Data. The firm-level investment rate (i.e., the change in gross capital stock) is defined as $IK_{i,t} = \frac{CAPX_{i,t}}{0.5\times(K_{i,t-1}+K_{i,t})}$ where K is the firm's net property, plant and equipment ($PPENT$) and $CAPX$ is its capital expenditure. Both K and $CAPX$ are scaled by the annual implicit price deflator of gross private nonresidential fixed investment, reported in NIPA Table 1.1.9. Our firm-level control variables are the following variables. Book leverage is defined as $BLev = (DLC + DLTT)/(DLC + DLTT + CEQ)$, where DLC is debt in current liabilities, $DLTT$ is long-term debt, and CEQ is the Compustat common book equity. Return on assets (ROA) is calculated as $ROA = Earnings/AT$, where $Earnings$ is defined as the sum of income before extraordinary items (IB), interest expense ($XINT$), and income statement deferred taxes ($TXDI$). $Tang = PPEGT/AT$, where $PPEGT$ is gross property, plant, and equipment. We further control for firm size, proxied via the logarithm of sales. In the unreported robustness checks, we also control for the Tobin's Q, which is computed as MV/AT, where MV is the market value of assets and is given by $MV = AT + ME - CEQ - TXDB$, ME is the CRSP market value of equity (calculated as the December stock price times shares outstanding), and $TXDB$ denotes the balance sheet deferred taxes. All the variables are winsorized at the 0.5 and 99.5 percentiles.

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Table 1

Descriptive statistics

This table reports the annual average value (Mean) and standard deviation (Std) of the variables of interests from 1963 to 2014. Panel A reports the aggregate statistics. Investment rate (*IK*) is the real gross private nonresidential fixed investment scaled by real private nonresidential fixed asset. *Baa-Tb10y* is the Moody's Baa corporate bond yield in excess of the 10-year Treasury rate. *Baa-Aaa* is the Moody's Baa bond yield in excess of Moody's Aaa yield. Relative bond price (*Rela BP*) is defined as $\frac{0.1+\text{10-year Treasury rate}}{0.1+Baa\text{ bond yield}}$ following Philippon (2009). Idiosyncratic volatility (*IdioV*) is the cross-sectional return volatility based on CRSP. Book leverage (*BLev*) is the total liabilities of the nonfinancial corporate business sector from the Flow of Funds scaled by the stock of capital from NIPA. The price-to-dividend ratio (*PD*) is the ratio between S&P composite stock price index and the sum of all dividends accruing to stocks in the index from Robert Shiller's website. Panel B reports firm-level statistics. Investment rate (*IK*) is capital expenditure over net property plant and equipment. Book leverage (*BLev*) is total liabilities scaled by the sum of total liabilities and the book value of common equity. Return on assets (*ROA*) is earnings over total assets. Tangibility (*Tang*) is gross property, plant, and equipment scaled by total assets. Firm size is measured by logarithm of sales (*Sales*).

A. Aggregate			B. Firm-level		
	Mean	Std		Mean	Std
IK	0.11	0.01	IK	0.29	0.25
Baa-Tb10y	2.03%	0.74%	BLev	0.29	0.24
Baa-Aaa	1.03%	0.42%	ROA	0.09	0.05
Rela BP	0.89	0.04	Tang	0.54	0.37
IdioV	0.56	0.15	Sales	0.49	2.31
BLev	0.48	0.06			
PD	37.28	16.80			

Table 2

Predicting firm-level investments

This table reports the predictive regression results of firm-level investments. Relative bond price (*Rela BP*) is defined as $\frac{0.1+\text{10-year Treasury rate}}{0.1+Baa\text{ bond yield}}$ following Philippon (2009). *Baa-Tb10y* is the Moody's Baa corporate bond yield in excess of the 10-year Treasury rate. *Baa-Aaa* is the Moody's Baa bond yield in excess of Moody's Aaa yield. Investment rate (*IK*) is capital expenditure over net property plant and equipment. Book leverage (*BLev*) is total liabilities scaled by the sum of total liabilities and the book value of common equity. Return on assets (*ROA*) is earnings over total assets. Tangibility (*Tang*) is gross property, plant, and equipment scaled by total assets. Firm size is measured by logarithm of sales (*Sales*). All the regressions include the firm fixed effect. Standard errors are clustered by firm and by time. Sample is from 1963 to 2014.

	[1]	[2]	[3]	[4]	[5]	[6]
Rela BP	0.906			0.390		
[t]	9.310			5.229		
Baa-Tb10y		-4.326			-2.229	
[t]		-6.710			-5.627	
Baa-Aaa			-2.627			-2.865
[t]			-1.891			-4.253
BLev				-0.158	-0.157	-0.156
[t]				-14.523	-14.541	-14.698
ROA				0.649	0.660	0.690
[t]				22.434	22.524	23.139
Tang				-0.238	-0.240	-0.244
[t]				-19.917	-19.903	-20.047
Sales				-0.035	-0.037	-0.041
[t]				-12.517	-13.461	-16.817
No. obs	166,293	166,293	166,293	127,528	127,528	127,528
R^2	0.02	0.02	0.00	0.14	0.14	0.14

Table 3

Predicting aggregate investments

This table reports the predictive regression results of aggregate investment. Investment rate (*IK*) is the real gross private nonresidential fixed investment scaled by real private nonresidential fixed asset. Relative bond price (*Rela BP*) is defined as $\frac{0.1+\text{10-year Treasury rate}}{0.1+Baa\text{ bond yield}}$ following Philippon (2009). *Baa-Tb10y* is the Moody's Baa corporate bond yield in excess of the 10-year Treasury rate. *Baa-Aaa* is the Moody's Baa bond yield in excess of Moody's Aaa yield. *Tb3m* is the real 3-month Treasury rate. Idiosyncratic volatility (*IdioV*) is the cross-sectional return volatility based on CRSP. Book leverage (*BLev*) is the total liabilities of the nonfinancial corporate business sector from the Flow of Funds scaled by the stock of capital from NIPA. The price-to-dividend ratio (*PD*) is the ratio between S&P composite stock price index and the sum of all dividends accruing to stocks in the index from Robert Shiller's website. The slopes of *PD* are multiplied by 100. [*t*]s are heteroskedasticity and autocorrelation consistent *t*-statistics (Newey-West.) Sample is from 1963 to 2014.

	[1]	[2]	[3]	[4]	[5]	[6]
Rela BP	0.107			0.102		
[*t*]	4.373			2.554		
Baa-Tb10y		-0.678			-0.598	
[*t*]		-4.825			-3.335	
Baa-Aaa			-1.197			-0.984
[*t*]			-2.693			-4.453
Tb3m				0.152	0.176	0.231
[*t*]				2.107	3.334	5.229
IdioV				-0.001	0.006	0.004
[*t*]				-0.113	0.552	0.461
BLev				-0.042	-0.053	-0.076
[*t*]				-1.728	-2.642	-4.203
PD				0.040	0.030	0.030
[*t*]				4.916	4.202	3.066
R^2	0.271	0.305	0.294	0.620	0.653	0.682