

# A Theory of Voluntary Disclosure and Cost of Capital

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## Abstract

This paper explores the links between firms' voluntary disclosures and their cost of capital. Existing studies investigate the relation between mandatory disclosures and cost of capital, and find no cross-sectional effect but a negative association in time-series. In this paper, I find that when disclosure is voluntary firms that disclose their information have a lower cost of capital than firms that do not disclose, but the association between voluntary disclosure and cost of capital for disclosing and non-disclosing firms is positive in aggregate. I further examine whether reductions in cost of capital indicate improved risk-sharing or investment efficiency. I also find that high (low) disclosure frictions lead to overinvestment (underinvestment) relative to first-best. As average cost of capital proxies for risk-sharing but not investment efficiency, the relation between cost of capital and ex-ante efficiency may be ambiguous and often irrelevant.

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# 1 Introduction

In this paper, I study the association between voluntary disclosure and cost of capital and examine whether cost of capital captures some of the real effects of voluntary disclosure (i.e., investment and risk-sharing efficiency). I address two main questions: first, at the individual firm level, do firms that voluntarily disclose more information experience a lower cost of capital relative to firms that do not disclose? Second, at the economy-wide level, do voluntary firm disclosures affect average cost of capital and what are the consequences of such endogenous disclosures on ex-ante economic efficiency? Answering the first question allows us to better understand the economic forces underlying firms' disclosures, their effects on individual firms' cost of capital, and the cross-sectional differences in costs of capital between disclosing and non-disclosing firms. Providing an answer to the second question can help us tie the average cost of capital with firms' voluntary disclosures and the level of investment across economies at different stages in their life cycles.

Although the evidence is still relatively recent, a number of empirical studies document a negative cross-sectional association between disclosure quality and cost of capital (Botosan (1997), Sengupta (1998), Botosan and Plumlee (2002), Ecker et al. (2006) and Francis et al. (2008)). In this literature, most of the existing research uses a cross-sectional research design, comparing differences in cost of capital between firms with different disclosure quality *at a given point in time*. My first contribution is to demonstrate, within a formal model, that disclosure quality may be an explanatory factor for the cross-section of expected returns. This prediction differs from the prior theoretical literature in the area (Easley and O'Hara (2004), Hughes, Liu, and Liu (2007), Lambert, Leuz and Verrecchia (2007) and Christensen, de la Rosa, and Feltham (2010)) in which the main focus is the time-series change in the economy-wide risk premium between two different time periods, i.e. pre-disclosure versus post-disclosure. In these models, since the economy-wide effect of disclosure affects all firms in the period, there is not necessarily a cross-sectional difference between the cost of capital of disclosing versus non-disclosing firms.<sup>1</sup> I find that, if the disclosure friction is high, firms making more voluntary disclosures have a lower cost of capital. The rationale for this result is tied to the relation between voluntary disclosures and investors' updated estimate of the firms' systematic risk per dollar

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<sup>1</sup>This can be easily illustrated with a brief example (see Lambert, Leuz, and Verrecchia (2007) for more details). Suppose that the economy features only two firms, *A* and *B*. If *A* discloses, some uncertainty relating the systematic shock will be realized, possibly leading (on average) to a reduction in the risk premium for *both A* and *B*. This does not imply (as is usually tested empirically) that the risk premium of *A* will be lower than that of *B*.

of expected cash flows. Conditional on a voluntary disclosure, investors expect higher cash flows which dilute the firms' sensitivity to systematic risk, in turn decreasing cost of capital and increasing market value.

My second main contribution is to explain the effect of managerial reporting discretion on cost of capital. When disclosure itself is a choice, the interpretation of empirical results must take into account the endogeneity of the disclosure decision (Skaife, Collins, and LaFond (2004), Nikolaev and Van Lent (2005), and Cohen (2008)). I show that the cross-sectional association between disclosure quality and cost of capital is closely tied to the nature of managerial discretion. In particular, I establish a relationship between, on the one hand, cost of capital, and, on the other hand, the disclosure friction and the total amount of voluntary disclosure in the economy. By contrast, most of the prior theoretical literature on cost of capital focuses on an exogenous signal that is publicly reported and thus does not discuss disclosure as an endogenous response in the economic environment. I find that when there is more voluntary disclosure (lower disclosure friction), the cost of capital of all disclosing firms is increasing as well as the cost of capital of non-disclosing firms. More voluntary disclosure means that more firms with lower cash flows and a lower market price disclose while investors perceive non-disclosing firms as having even lower cash flows.

Lastly, I examine the relationship between measures of cost of capital and the real effects of disclosure. At a conceptual level, cost of capital is only a price variable and, as such, it is only relevant for policy-making to the extent that it may proxy for real effects, i.e. related to investors' final consumption. In this model, I incorporate two real effects: the market's ability to properly diversify away firm-specific risks (risk-sharing efficiency) and the efficiency of liquidation decisions (investment efficiency) tied to the asymmetric information about firms that did not disclose. In particular, I provide a linkage between cost of capital measures and the existing literature on the real effects of disclosure (Pae (1999), Hughes and Pae (2004), Liang and Wen (2007), Hughes and Pae (2010) and Li, Liang, and Wen (2011)). The third main result of this paper is that the average cost of capital is an appropriate proxy for overall ex-ante efficiency only when the voluntary disclosure is low and investors are not fully diversified ex-ante. The average cost of capital is irrelevant as a measure of economic inefficiency when ex-ante diversification is available. When most firms are uninformed (i.e., the disclosure friction is high), firms that do not disclose are more likely to be uninformed and thus are financed, leading to overinvestment. Because more voluntary disclosure increases the dispersion in market prices, it also increases average cost of capital and decreases the market's risk-sharing efficiency.<sup>2</sup>

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<sup>2</sup>In resolving uncertainty, information also erodes risk-sharing opportunities when it is publicly revealed

Risk-sharing is impaired due to endogenous changes in equilibrium prices arising from the change in information. Since firms are financed regardless of their disclosure, more voluntary disclosure does not affect investment efficiency. Hence, under overinvestment, a decrease in the disclosure friction corresponds to greater average cost of capital and lower ex-ante efficiency. By contrast, when most firms are informed (i.e., the disclosure friction is low), firms that do not disclose are not financed, leading to underinvestment as some uninformed high-value firms are liquidated. I show that more voluntary disclosure implies lower underinvestment and increases total wealth in the economy. Hence, more disclosure increases ex-ante efficiency but does not change the average cost of capital.

I distinguish in my study the average cost of capital (defined as the equally-weighted average return by all firms) from the risk premium or equivalently the return on the market portfolio (defined as the value-weighted average return). The return on the market portfolio may not vary with disclosure, in a given equilibrium, because, in response to shocks to their wealth, investors rebalance their holdings of risky assets. However the return of the market portfolio is higher under overinvestment than under underinvestment. There are few comparable results in the literature on cost of capital and economic efficiency. Gao (2010) discusses the effect of information on investor welfare but in the different context of intergenerational risk-sharing and mandatory disclosure.<sup>3</sup>

The model in this paper extends the Dye-Jung-Kwon voluntary disclosure model (Dye (1985), Jung and Kwon (1988)), hereafter DJK, by incorporating it into the general equilibrium capital asset pricing model. In the economy, each of a large number of risk-averse investors owns a firm's new project whose terminal cash flow, if financed, contains a firm-specific and an economy-wide cash flow component. As in DJK, there is a disclosure friction: each firm may or may not privately observe information about the firm-specific component and, when endowed with information, strategically chooses whether to publicly disclose. Investors observe public disclosures and non-disclosures, and rationally price each firm. The friction affects the proportion of firms voluntarily disclosing and both the fraction of firms liquidated and the risk premium demanded by investors. I show how the friction may reduce cost of capital, both at the firm level (if a particular firm discloses more relative to its peers) and at the aggregate market level (if all firms disclose more overall).

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before trading. "Public information . . . in advance of trading adds a significant distributive risk" (Hirshleifer 1971, p. 568). However, my result differs from Hirshleifer (1971) in that I show how changes to voluntary disclosure may lead to greater price dispersion and study aggregate cost of capital, while Hirshleifer focuses on efficiency after price dispersion has increased.

<sup>3</sup>Another important difference between Gao (2010) and my paper is that his model is one with a single firm and a single agent per generation. To my knowledge, an extension of his results to an economy with multiple firms and agents is non-trivial.

## Related Literature

My paper is related to three strands of literature: voluntary disclosure, accounting quality and cost of capital, and the real effects of disclosure on cost of capital.

The voluntary disclosure literature studies firms' endogenous disclosure decisions and their consequences on the type of information disclosed (Verrecchia (1983), Dye (1985)). These models do not incorporate systematic risk. Therefore, disclosing and non-disclosing firms will receive the same cost of capital, namely the risk-free rate.<sup>4</sup> To explain the cross-section of expected returns, my paper combines voluntary disclosure with asset pricing in the presence of systematic risk. To my knowledge, the only studies that focus on voluntary disclosures and systematic risk are those of Kirschenheiter and Jorgensen (2003, 2007). They focus on disclosures about risk, more applicable to financial products, such as value-at-risk, new ventures or exposure to interest rates. As is common in the voluntary disclosure literature, I consider disclosures about expected or projected cash flows, such as asset values, earnings' forecasts, sales projections, expense reductions or asset acquisitions. There are also several other important differences between their research design and mine, such as the nature of the disclosure process, the size of the economy, investors' preferences or productive decisions. While Kirschenheiter and Jorgensen do not measure the average cost of capital (average return by all firms), they find that the equity risk premium (expected return of the market portfolio) is increasing in information availability.

My paper also contributes to the literature on the relation between accounting quality and cost of capital. This literature has explored the effects of exogenous information on risk premia. I focus on the endogenous disclosure caused by the information asymmetry between firms and investors, which is different from the information asymmetry among investors in Easley and O'Hara (2004) and Hughes, Liu, and Liu (2007). Easley and O'Hara (2004) find a higher cost of capital if there is more private information and less precise information in a finite economy. In their model, a proportion of investors receive information while the remaining fraction of investors do not receive information. They explain that the uninformed investors will demand a higher risk premium for trading securities on which they face information risk. Hughes, Liu, and Liu (2007) prove that this result does not hold when the economy becomes large, as more information may only af-

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<sup>4</sup>Other studies investigate the interactions between voluntary disclosures and the economic and informational environment. Bertomeu, Beyer, and Dye (2011) study whether firms' voluntary disclosures can reduce asymmetric information in financial markets and lead to cheaper financing. Managers might also know several pieces of information and the decision to voluntarily disclose their information depends on the correlation and precision about the signals (Kirschenheiter (1997)) and the mandatory disclosure environment (Einhorn (2005)).

fect the (aggregate) market premium but not a firm’s cost of capital directly: specifically, information about the systematic factor is the only information priced by the market. As noted by Christensen, de la Rosa, and Feltham (2010), disclosure should only affect the timing of resolution of uncertainty, and thus a commitment to disclosure does not increase welfare of the manager disclosing or, even, that of investors. In comparison to this literature, I show that firms that choose to disclose have an unambiguously lower cost of capital after disclosure has occurred than firms that choose not to disclose.

The rest of the paper proceeds as follows. Section 2 presents the model. Sections 3 and 4 determine the characteristics of the two types of equilibria of the model. Section 5 examines the impacts of an exogenous disclosure friction on cost of capital both at the firm level and the macroeconomic level. Section 6 focuses on the efficiency implications of more information availability. Section 7 concludes. All omitted proofs are given in Appendix.

## 2 The Model

### 2.1 Timeline

The economy is populated by a large number of investors and firms. I briefly describe the main sequence of events hereafter and summarize them in Figure 1.

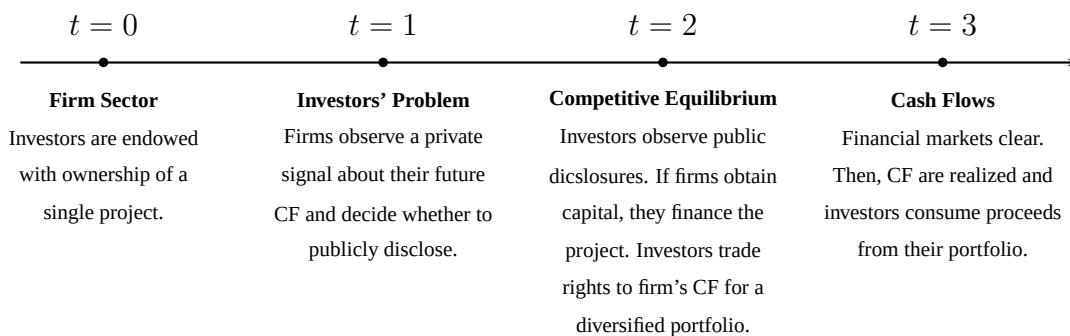


Figure 1: Timeline

At date 0, each investor is endowed with the ownership of a single project, which entitles the owner to the future cash flows of the project if the firm is eventually financed. I later refer to this project as “the firm.” In this paper the investor is not the manager of the firm.<sup>5</sup>

<sup>5</sup>Leland and Pyle-type signalling considerations are beyond the scope of my analysis. This assumption is similar to the voluntary disclosure literature, which considers the manager as a person distinct from the investors. Informed trading by managers (unlike disclosure management) is explicitly prohibited by the

At date 1, each firm receives private information about the future cash flow (CF) of the project with probability  $1 - \eta$ . The information may be publicly disclosed. The disclosure problem is described in more details in Subsection 2.2 “Firm Sector.”

At date 2, all investors observe all public disclosures (if any). Firms’ projects valued at a positive price are financed. Investors trade the rights to their firms’ cash flow for a diversified portfolio. Their portfolio choice decision is described in Subsection 2.3 “Investors’ Problem.”

At date 3, financial markets clear; the market-clearing prices are determined in Subsection 2.4 “Competitive Equilibrium.” Then, uncertainty about the firm’s cash flows is realized, and investors consume the cash flows received from their portfolio.

## 2.2 Firm Sector

I discuss here the characteristics of the firms and describe the events occurring at date 1.

### Firms’ Cash Flows

There is a continuum of firms, and each firm can generate a stochastic cash flow  $\pi$  if the firm is financed, net of the required investment, and zero otherwise. For tractability and given the focus on a multi-firm economy, I restrict attention to a setting in which the project is financed or not, and do not consider the scale of investment. I assume that  $\pi = \epsilon + y$ , where  $\epsilon$  is a firm-specific i.i.d. random variable (the indexation on each firm is omitted to save space) and  $y$  is a systematic risk factor (common for all firms). This factor model approach is similar to Jorgensen and Kirschenheiter (2003, 2007) and Hughes, Liu, and Liu (2007).<sup>6</sup> The firm-specific shock  $\epsilon$  captures the firm’s idiosyncratic (diversifiable) risk, has a distribution  $H(\cdot)$ , density  $h(\cdot)$  with mean  $\mathbb{E}(\epsilon) = \theta$ , full support over  $\mathbb{R}$  and is independent of  $y$ . The common shock  $y$  is assumed to have a density  $f(\cdot)$  and full support over  $[\underline{y}, +\infty)$  (where  $\underline{y} > -\theta$ ) and  $\mathbb{E}(y) = 0$ .<sup>7</sup> I normalize the mass of all firms in the economy to one. Therefore, defining  $CF_m(y)$  as the payoff in unit of consumption of all firms (hereafter, the “market portfolio”),  $CF_m(y)$  must be equal to  $\text{Prob}(Inv)(\mathbb{E}(\epsilon|Inv) + y)$ , where  $Inv$  represents the event that the firm is financed and

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SEC; further, in practice, managers’ trades constitute a very small portion of the total volume traded and would only marginally affect asset prices.

<sup>6</sup>Under the assumptions of the capital asset pricing model (e.g., Mossin (1966)), firms’ cash flows can always be decomposed into an idiosyncratic and a (suitably constructed) systematic factor and thus, such a decomposition is without loss of generality.

<sup>7</sup>The restriction to  $\mathbb{E}(y) = 0$  is without loss of generality; if  $\mathbb{E}(y) \neq 0$ , one could relabel  $y' = y - \mathbb{E}(y)$ , with mean zero, and  $\epsilon' = \epsilon + \mathbb{E}(y)$ , with no change to the results or analysis. In other words, a revision of the economy’s growth would be captured in this model by the common mean of the firm-specific factor  $\theta$ .

$Prob(Inv)$  is the probability of a firm being financed. Following this observation, I will denote a realization of  $y$  as a state of the world.

### Disclosure Decisions

Firms observe a perfect signal  $s$  on their idiosyncratic cash flow  $\epsilon$  with probability  $1 - \eta$ .<sup>8</sup> Given that this is already the main object of the prior literature (Lambert et al. (2007), Christensen, de la Rosa, and Feltham (2010)), I assume in my model that firms do not disclose information about the state of the world  $y$ . Firms decide whether to release their private information upon receipt. As in DJK, disclosures are truthful but the firm cannot credibly communicate an absence of information endowment. Firms take the set of possible prices as given when they disclose. I define  $P_\epsilon$  (to be endogenously determined) as the market price if the firm's signal  $s = \epsilon$  is disclosed and observed. The price  $P_\emptyset$  is offered if no additional signal is revealed. This price is equal to the trading value of a non-disclosing firms or zero if this trading value is negative (in which case the firm is not financed). Firms maximize the value of their current owner and disclose if they learn their information if and only if  $P_\epsilon \geq P_\emptyset$ . I denote the optimal disclosure threshold  $\epsilon^*$ , above which all firms decide to voluntarily disclose.

## 2.3 Investors' Problem

Now I discuss the characteristics of investors, and describe the events occurring at date 2.

### Preferences

Investors are each initially endowed with one firm. They have a constant relative risk-aversion (CRRA) utility function  $u(x) = x^{1-\alpha}/(1-\alpha)$ , where  $x$  is final consumption and  $\alpha > 0$  is an investor's Arrow-Pratt relative risk-aversion coefficient.<sup>9</sup> Each investor is initially undiversified; thus, it is optimal for him to sell the project and invest in a diversified portfolio.

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<sup>8</sup>The results, and proofs, are unchanged if one assumes instead that firms receive a noisy signal with probability  $1 - \eta$ , say  $s'$ , on  $\epsilon$ . Given that the estimation risk on  $\epsilon$  is purely idiosyncratic, it would not be priced, and thus one could relabel the model by replacing  $\epsilon$  by  $\epsilon' = \mathbb{E}(\epsilon|s')$ .

<sup>9</sup>If  $\alpha = 1$ ,  $u(\cdot)$  is set to  $u(x) = \ln(x)$ . As the CRRA utility is not defined for negative values, I assume that if  $x < 0$  then  $u(x)$  goes to  $-\infty$ . All the results of the model carry over for: (i) other types of ownership (certain investors own multiple projects or share ownership), (ii) if some investors do not own a project, (iii) if all investors also have an i.i.d. personal wealth, in addition to their project. The only required assumption is that not all investors are perfectly diversified ex-ante. The result on the difference in returns between disclosing or non-disclosing firms is robust to any strictly concave utility function but the CRRA assumption is required for the comparative statics and efficiency comparisons.



## Portfolio Choice

At date 2, investors observe the information disclosed by all firms, sell their asset in a competitive market, and optimally diversify away their idiosyncratic risk. Post disclosure, investors differ in the disclosure of their firm. If the firm did not disclose, the investor can sell his firm for a price  $P_\emptyset$ . If the informed firm discloses the information, there is also a continuum of investors who can sell their firm for a price  $P_\epsilon$  where  $\epsilon \in \mathbb{R}$ . In short-hand, I denote the value of a firm  $P_\delta$ , where  $\delta \in \{\emptyset\} \cup \mathbb{R}$  is the firm's disclosure. As the form of my economy is very similar to that in Stiglitz and Cass (1970), two-fund separation holds, i.e., investors trading in a complete financial market will always choose to hold a combination of the market portfolio and the risk-free asset.<sup>10</sup>

Let  $\gamma$  be the proportion of each investor's wealth invested in the risk-free asset. The risk-free asset is in zero net supply. The remaining proportion  $1 - \gamma$  is thus invested in the market portfolio. Without loss of generality, I normalize the price of the risk-free asset to 1 (so that a risky asset with price  $x$  means that it can be exchanged for a certain consumption of  $x$ ) and denote  $P_m$  the price of the market portfolio. The portfolio choice problem of an investor can then be written as follows:

$$(\Gamma_\delta) \quad \max_{\gamma} \int f(y) U \left( P_\delta \gamma + \frac{(1 - \gamma) P_\delta}{P_m} C F_m(y) \right) dy$$

In summary, the investor has wealth  $P_\delta$ , the market value of the firm owned, and chooses  $\gamma$ , the proportion of that wealth to be invested in the risk-free asset, which yields  $P_\delta \gamma$  units of consumption at the end of the period. The rest of the wealth  $(1 - \gamma) P_\delta$  is invested in the market portfolio, which is used to buy a proportion  $(1 - \gamma) P_\delta / P_m$  of the market. This yields  $((1 - \gamma) P_\delta / P_m) C F_m(y)$  units of consumption at the end of the period. For purposes of interpretation, it is convenient to work directly with the expected return on the market portfolio,  $\mathbb{E}(R_m) \equiv \mathbb{E}(C F_m(y)) / P_m$ . I endogenize in the next section  $P_m$ , or equivalently  $\mathbb{E}(R_m)$ .

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<sup>10</sup>A formal proof is available upon request. The two-fund separation does not require CRRA, but would work for any HARA class utility. However, the model of large economy would have to be "adjusted" if another utility is used instead. With the CARA preference, a continuum of investors for a fixed endowment would lead to an arbitrarily low risk per investor and, thus, would bring risk premia to the risk-free rate (this does not happen with CRRA because as there are more investors, the wealth of each investor decreases which increases risk-aversion, so that both effects cancel and the risk premium no longer depends on the number of investors or assets, it only depends on total endowment). Thus, if one were to write a large economy with CARA, one would have to state a finite economy with one asset per investor and then let both the number of investors and the number of assets (i.e., the total wealth in the economy) become large. This makes the CARA model more cumbersome for the purpose of defining a large economy.

## 2.4 Competitive Equilibrium

I discuss here the sequence of events occurring at date 3; specifically, I state the definition of a competitive equilibrium and derive the equilibrium market prices.

### Market Pricing of Disclosing and Non-Disclosing Firms

I examine here the market pricing of a disclosing and non-disclosing firms, as a function of the market portfolio and the risk-free asset. Absent any arbitrage opportunities, the price of a firm should be equal to that of a basket of these securities that yields the same terminal cash flows in every state. To construct such a replicating portfolio, I rewrite the terminal cash flow of a firm in terms of the market portfolio and a risk-free component.<sup>11</sup> Consider first a firm that discloses  $\epsilon$ , implying a future cash flow  $\epsilon + y$ . This future cash flow can be decomposed as follows:

$$\epsilon + y = \epsilon - \mathbb{E}(\epsilon|Inv) + \frac{1}{\text{Prob}(Inv)} \underbrace{\text{Prob}(Inv) (\mathbb{E}(\epsilon|Inv) + y)}_{CF_m(y)} \quad (1)$$

It follows that the firm's future cash flow is the same as the cash flow obtained from a portfolio with (a)  $\epsilon - \mathbb{E}(\epsilon|Inv)$  units of the risk-free asset, and (b)  $1/\text{Prob}(Inv)$  units of the market portfolio. Therefore, the firm must have a value equal to the value of the latter portfolio.

$$P_\epsilon = \epsilon - \mathbb{E}(\epsilon|Inv) + \frac{1}{\text{Prob}(Inv)} \underbrace{\frac{\mathbb{E}(CF_m)}{\mathbb{E}(R_m)}}_{P_m} \quad (2)$$

Similarly, consider the case of a non-disclosing firm. The firm's cash flow is  $\epsilon + y$ , where  $\epsilon$  is unknown to investors but can be perfectly diversified by holding a portfolio of all non-disclosing firms. As a result the trading price  $\rho$  of this firm is that of a firm paying  $\mathbb{E}(\epsilon|ND) + y$ , where ND represents the event that the firm did not disclose. Using the same logic, the trading price of a non-disclosing firm that is financed is as follows:

$$\rho = \mathbb{E}(\epsilon|ND) - \mathbb{E}(\epsilon|Inv) + \frac{1}{\text{Prob}(Inv)} \underbrace{\frac{\mathbb{E}(CF_m)}{\mathbb{E}(R_m)}}_{P_m} \quad (3)$$

Given that a firm is financed if and only if it is traded for a positive price, the value of a non-disclosing firm is given by  $P_\emptyset = \rho$  if  $\rho \geq 0$  and  $P_\emptyset = 0$  otherwise.

<sup>11</sup>While each individual asset is replicated with either negative or positive quantities of the risk-free asset, the total net supply of the risk-free asset will always be zero.

## Market-Clearing and Risk Premium

I close the model by stating the general equilibrium Equations that determine the expected market return  $\mathbb{E}(R_m)$ . To avoid situations with multiple equivalent equilibria, I assume that a firm that does not expect to be financed conditional on its disclosure will not disclose (e.g., if there is some small cost of disclosure).<sup>12</sup> It follows that only firms that did not disclose may not be financed. Therefore, there are two possible equilibrium candidates: (1) overinvestment equilibria, in which all firms invest and receive a positive price even if they do not disclose, (2) underinvestment equilibria, in which firms that do not disclose - whether voluntarily or involuntarily - are not financed.

**Definition 1** An “overinvestment” (resp. “underinvestment”) equilibrium is a set of optimal portfolio choice  $\gamma_\delta$ , expected market portfolio return  $\mathbb{E}(R_m)$  and disclosure threshold  $\epsilon^{**}$ , where  $\epsilon^{**}$  is denoted  $\epsilon^{over}$  (resp.  $\epsilon^{under}$ ) in the case of overinvestment (resp. underinvestment) such that:

(i)  $\gamma_\delta$  solves the maximization problem  $(\Gamma_\delta)$ .

(ii) The risk-free asset is in zero net supply, i.e.

$$\forall y, \underbrace{0}_{\text{net supply}} = \underbrace{(1 - (1 - \eta)(1 - H(\epsilon^{**})))\gamma_\emptyset P_\emptyset}_{\text{non-disclosing firms' total demand}} + \underbrace{(1 - \eta) \int_{\epsilon^{**}}^{+\infty} \gamma_\epsilon P(\epsilon) h(\epsilon) d\epsilon}_{\text{disclosing firms' total demand}}$$

(iii)  $P_{\epsilon^{**}} = P_\emptyset = \rho > 0$  (resp.,  $P_{\epsilon^{**}} = P_\emptyset = 0 \geq \rho$ ).

Condition (i) and (ii) are standard in the general equilibrium literature; the first condition guarantees that all investors invest optimally and the second condition implies that the asset market for the risk-free asset clears. Condition (iii) captures the optimal disclosure and investment decisions. In an overinvestment equilibrium, all firms that do not disclose achieve a positive price and are financed, i.e.  $P_\emptyset = \rho > 0$  while in an underinvestment equilibrium, all firms that do not disclose are liquidated, i.e.  $P_\emptyset = 0$  and continuation would have led to a negative market price  $\rho \leq 0$ . Lastly, the optimal disclosure strategy implies that the marginal discloser is indifferent between disclosure and non-disclosure, thus  $P_{\epsilon^{**}} = P_\emptyset$ . The payoff from the market portfolio  $CF_m(y)$  is itself a function of the nature of the competitive equilibrium and, thus, also of the voluntary disclosure decision. This is an important channel through which voluntary disclosure can affect risk premia.

<sup>12</sup>The results are unchanged if this restriction is lifted; except that there may be many economically equivalent equilibria in which some low-value firms choose to disclose but still do not receive financing.

In the case of overinvestment, all firms in the economy receive financing, and therefore  $CF_m(y) = y + \theta$ . By contrast, in the case of underinvestment, all firms that do not disclose are liquidated and thus the economy as a whole will shed both low-value firms and those high-value uninformed firms, which translates into  $CF_m(y) = (1 - \eta) \int_{\epsilon_{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon$ .

## 2.5 First-Best Benchmark

I define the first-best as the solution to the model when a planner perfectly observes all information about the idiosyncratic component  $\epsilon$  and determines both the allocation of assets across investors and the financing threshold  $\epsilon^{FB}$ . In the first-best allocation, all investors should be given the same well-diversified portfolio ex-ante and thus the first-best problem is equivalent to maximizing the ex-ante CRRA utility of a representative investor defined as  $\max_{\bar{\epsilon}} \int f(y) \left( \int_{\bar{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{1-\alpha} dy$ .<sup>13</sup>

**Proposition 1** *Firms are financed if and only if their signal about future cash flows  $\epsilon$  is weakly above  $\epsilon^{FB}$  where  $\epsilon^{FB}$  is uniquely defined as follows:*

$$\epsilon^{FB} = - \frac{\int y f(y) \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{-\alpha} dy}{\int f(y) \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{-\alpha} dy} \in (0, \theta)$$

First-best prescribes not to finance firms whose expected cash flows are too low. The fundamental tension in the first-best solution is between increasing expected aggregate consumption, and decreasing total aggregate risk by liquidating some firms. The first-best threshold lies between 0 and  $\theta$ . At one extreme, financing a firm with zero idiosyncratic value ( $\epsilon = 0$ ) would increase risk without increasing aggregate consumption. At the other extreme, a firm with the expected unconditional cash flow ( $\epsilon = \theta$ ) yields a positive non-diversifiable cash flow component  $\theta + y$ , which is always strictly greater than the payoff if the firm is not financed.

## 3 Overinvestment Equilibrium

I first solve for the overinvestment equilibrium. This equilibrium exists if the non-disclosing trading price  $\rho$  is positive. I decompose the problem in three steps. First, I take the expected market portfolio return  $\mathbb{E}(R_m)$  as given and derive the optimal disclosure threshold from condition (iii) in definition 1. Second, I solve for the expected market return  $\mathbb{E}(R_m)$

<sup>13</sup>For convenience, I focus on the “anonymous” or symmetric solution, in which the planner does not advantage certain investors over others. The solution  $\epsilon^{FB}$  is unchanged if one considers the complete set of Pareto-efficient solutions in which the planner may favor certain investors over others.

based on constraints (i) and (ii) from definition 1. Third, I collect these results and formally state the competitive equilibrium of the model.

### 3.1 Disclosure Threshold Dependent on the Disclosure Friction

I write the optimal disclosure threshold  $\epsilon^{over}$  under overinvestment, i.e. the threshold that satisfies  $P_{\epsilon^{over}} = P_\emptyset$ . The disclosure threshold is given by  $\rho = P_\emptyset$  which, using the pricing functions obtained earlier reduces to the familiar DJK disclosure threshold.

**Lemma 1** *The disclosure threshold  $\epsilon^{over}$  is given by the unique solution to:*

$$\eta(\theta - \epsilon^{over}) = (1 - \eta) \int_{-\infty}^{\epsilon^{over}} H(\epsilon) d\epsilon \quad (4)$$

This equation is natural in my setting given that the voluntary disclosure model introduced in my market environment is driven by uncertainty about the information endowment in DJK. As shown by Jung and Kwon, there is a unique disclosure threshold such that the proportion of non-disclosers increases in the disclosure friction. As is common in the disclosure literature, a higher disclosure friction increases the proportion of firms *voluntarily* withholding. This comparative static is well-understood in the disclosure literature and thus I do not pursue it further here. Firms that should not have invested in first-best do not disclose ( $\epsilon^{over} > \epsilon^{FB}$ ) and are financed. In this respect, the equilibrium is consistent with its terminology of overinvestment. The asymmetric information between firms and outside investors, combined with a high disclosure friction, leads investors to infer that non-disclosing firms, are predominantly uninformed firms and thus are likely to have favorable news. Interestingly, the disclosure threshold  $\epsilon^{over}$  does not depend on the market risk premium and thus on the risk-aversion parameter  $\alpha$ . All firms are financed and once they execute their projects, they are identically affected by the common shock  $y$ , which is additively separable from the idiosyncratic cash flow  $\epsilon$ . An empirical implication of this property is that the amount of voluntary disclosure should be insensitive to the business cycle (possibly in contrast to mandatory disclosure, e.g., Bertomeu and Magee (2011)).<sup>14</sup>

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<sup>14</sup>For example, according to the model, one should not observe much time series variation in aggregate levels of disclosure as compared to, say, cross-country or cross-industry variations. Moreover, the aggregate level of disclosure should not be related to characteristics of the overall economy, such as GDP growth or market returns.

### 3.2 Market Risk Premium

I determine next the expected market portfolio return  $\mathbb{E}(R_m^{over})$  and the competitive equilibrium of the economy. Let  $\eta^{over}$  be the friction cut-off, above which there exists an overinvestment equilibrium.

**Proposition 2** *For high levels of disclosure frictions ( $\eta \geq \eta^{over}$ ), there exists a unique overinvestment competitive equilibrium  $(\gamma_\delta^{over}, \mathbb{E}(R_m^{over}), \epsilon^{over})$ , where:*

(i)  $\gamma_\delta^{over} = 0$

(ii)  $\mathbb{E}(R_m^{over}) = \frac{\theta}{\theta + Q^{over}}$  where  $Q^{over} = \frac{\int y f(y)(\theta+y)^{-\alpha} dy}{\int f(y)(\theta+y)^{-\alpha} dy} < 0$

(iii)  $\epsilon^{over}$  defined in Equation (4)

After the disclosure stage, agents have personal wealth  $P_\delta$  (where  $\delta$  may vary across agents), the market value of their firm. Each agent, then, makes different portfolio choice decisions, choosing a different quantity of risk-free asset and market portfolio. Under the assumption of CRRA utilities, all agents invest in proportion to their wealth, and this proportion does vary with the wealth of the agent. Aggregating all such consumers yields a simple expression for the equity premium that corresponds to the market premium for a representative agent owning all the firms and having the same CRRA utility function as each individual consumer.<sup>15</sup> The expected market portfolio return  $\mathbb{E}(R_m^{over})$  can be rewritten as follows:

$$\underbrace{\mathbb{E}(R_m^{over}) - 1}_{\text{risk premium}} = \frac{-Q^{over}}{\theta + Q^{over}} = \frac{-Q^{over}}{P_m^{over}}$$

The expected market return has, as predicted by the asset pricing theory, a return higher than the risk-free rate because the market portfolio is exposed to undiversifiable systematic risk. The term  $-Q^{over}/P_m^{over}$  corresponds to the equity premium in a CAPM framework. One important aspect of the model is that the equity risk premium does *not* depend on the informational frictions and/or characteristics of the disclosure environment, within

<sup>15</sup>The result also suggests some caution in interpreting single-agent models of disclosure outside of the CRRA framework. For example, CARA utility functions are rather unusual in asset pricing given that asset pricing is all about risk-taking and, unlike CRRA, CARA predict empirically counter-factual risk-taking behavior (Rubinstein (1975), Cochrane (2005)). Under CARA utilities, a billionaire, a millionaire or a minimum-wage worker would all hold exactly the same dollar amount of risky assets (their investment would only differ in terms of how much risk-free asset they hold). If utility functions are CARA, for example, there will exist a representative agent; however, the preference of this representative agent will depend on the wealth of all agents, which in turn will depend on the disclosure threshold  $\epsilon^{**}$ ; as a result, a comparative static on the disclosure threshold would require adjusting the preferences of the representative agent - which would lead to considerable analytical difficulties.

the overinvestment equilibrium. By Proposition 2, the equity premium can be fully characterized by the behavior of the representative agent who, by construction, owns all the wealth and thus does not bear the extra risk due to disclosure. This property is in sharp contrast with single-firm economies in which the diversification of any disclosure risk is, by assumption, ruled out (Yee (2006), Gao (2010)). But the result is also somewhat in contrast with prior results in a multi-firm economy (Jorgensen and Kirschenheiter (2003), Lambert, Leuz, and Verrecchia (2007), Christensen, de la Rosa, and Feltham (2010)). The main reason for the difference is that these studies are based on a finite economy where individual firm-specific risk is, by assumption, not fully diversifiable (Hughes, Liu, and Liu (2007)). It follows that disclosures always contain information about the aggregate state, realizing a component of the aggregate state, and henceforth lowering risk premia. While this latter effect has been well-studied, it is worth noting that it is entirely driven by the systematic component of the disclosure, not the firm's idiosyncratic risk per se.

## 4 Underinvestment Equilibrium

### 4.1 Risk Premium and Optimal Disclosure Threshold

The underinvestment equilibrium shares with the previous equilibrium the existence of a representative agent: specifically, risk premia can be obtained from the solution in a one-person economy. However, one major difference in this economy is that the total consumption available in the economy (the payoff of the market portfolio) depends on how many firms are financed, which itself depends on the probability of disclosure. Therefore, the disclosure friction may now affect risk premia, through its real effects on aggregate wealth. Let  $\eta^{under}$  be the friction cut-off, below which there exists an underinvestment equilibrium.

**Proposition 3** *If the level of disclosure frictions is low ( $\eta \leq \eta^{under}$ ), there exists an underinvestment competitive equilibrium, which is given as follows:*

$$\begin{aligned}
 (i) \quad & \epsilon^{under} = \epsilon^{FB} \\
 (ii) \quad & \gamma_{\delta}^{under} = 0 \\
 (iii) \quad & \mathbb{E}(R_m^{under}) = \frac{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon + (1 - H(\epsilon^{under})) Q^{under}} \\
 & \text{where } Q^{under} = \frac{\int y f(y) (\int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon)^{-\alpha} dy}{\int f(\tilde{y}) (\int_{\epsilon^{under}}^{+\infty} (\epsilon + \tilde{y}) h(\epsilon) d\epsilon)^{-\alpha} d\tilde{y}} < 0
 \end{aligned}$$

In the underinvestment equilibrium, all firms that should not have invested in first-best do not disclose and therefore are not financed. Thus, this equilibrium prescribes efficient liquidation of all low-value firms. However, there are also high-value firms that, with probability  $\eta$ , could not disclose and are not financed, leading to underinvestment relative to first-best. Neither the optimal disclosure threshold, nor the risk premium depend on the disclosure friction  $\eta$ . Intuitively, the economy functions in a “constrained” first-best environment, in which a proportion  $\eta$  of efficient firms are simply not financed, but for the remaining proportion  $1 - \eta$  of efficient firms, investments are made according to the first-best rule. The expected market portfolio return  $\mathbb{E}(R_m^{under})$  can be written as follows:

$$\mathbb{E}(R_m^{under}) - 1 = \frac{-(1 - H(\epsilon^{under}))Q^{under}}{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon + (1 - H(\epsilon^{under}))Q^{under}} > 0$$

In the underinvestment equilibrium, some firms do not invest and this leads to a decrease in the exposure of the market portfolio to the systematic risk.

## 4.2 Intermediate Disclosure Friction

Until this point I have described two types of equilibrium, the overinvestment equilibrium occurs for a sufficiently high disclosure friction and the underinvestment equilibrium occurs for a sufficiently low disclosure friction. These observations naturally imply the existence of an intermediate region of disclosure frictions in which either both types of equilibria may exist (“indeterminacy”) or no equilibrium may exist (“nonexistence”). To answer this question, recall that  $Q^{over}$  and  $Q^{under}$  are inversely correlated with the risk premium required in each type of equilibrium. It is interesting to note that, at least at first sight, the ordering of those measures may seem a-priori ambiguous. On the one hand, if fewer firms are financed, risk-averse investors’ wealth diminishes as some firms with positive cash flows do not execute their project. In response to the negative wealth effect investors require a higher risk premium. On the other hand, reducing the number of financed firms also decreases their exposure to systematic risk. Investors require less insurance and thus a lower risk premium. In the next Proposition, I show that, indeed, the risk premium is lower in an underinvestment equilibrium.

**Proposition 4** *There is always an interior region of disclosure frictions in which neither an overinvestment nor an underinvestment equilibrium exists, i.e.  $Q^{under} > Q^{over}$  and  $\eta^{under} < \eta^{over}$ . In addition, the risk premium in an underinvestment equilibrium is always less than in the overinvestment equilibrium.*



I conclude this Section with an important, yet lesser known, property of an economy with endogenous disclosure and investments. In such an economy, a competitive equilibrium never exists for a non-empty set of interior values of the disclosure friction.<sup>16</sup> The rationale for this result is that, under constant relative risk-aversion, investors demand lower risk premium with higher wealth. An underinvestment equilibrium increases the wealth of investors who can trade by selecting higher quality projects and thus increases risk tolerance. As a result, the underinvestment equilibrium also features greater appetite for financing non-disclosing projects than the overinvestment equilibrium. But if such non-disclosing projects are indeed financed, the economy shifts to an overinvestment regime with higher risk premia, in turn leading to such projects not being financed. This inherent contradiction creates a situation in which there is no competitive equilibrium.

## 5 Disclosure and Firms' Cost of Capital

### 5.1 Expected Cash Flows and Market Sensitivity

In this Section, I relate a firm disclosure to its cost of capital. Assume that the economy is such that  $\eta \geq \eta^{over}$ . Prices of a disclosing and a non-disclosing firm are characterized by the two following components:

$$P(\epsilon) = \underbrace{\epsilon}_{\text{Idiosyncratic CF}} + \underbrace{Q^{over}}_{\text{Systematic pricing}} \quad \text{and} \quad P_{\emptyset} = P(\epsilon^{over}) = \underbrace{\epsilon^{over}}_{\text{Idiosyncratic CF}} + \underbrace{Q^{over}}_{\text{Systematic pricing}}$$

The first component in the above equation corresponds to inferences about the idiosyncratic cash flow. It is increasing in the signal and, given that firms that do not disclose have, on average, low value, it is also greater for firms disclosing than for firms not disclosing. The second component corresponds to the pricing of the firm's systematic risk, and is identical for disclosing and non-disclosing firms. While the total amount of systematic risk borne by the disclosing and non-disclosing firms is identical, the total amount of risk per unit of expected cash flow is not. Because disclosing firms have higher cash flows, the sensitivity to systematic risk is diluted. This rationalizes the empirical positive association between voluntary disclosure and earnings quality documented.

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<sup>16</sup>For  $\eta \in (\eta^{under}, \eta^{over})$  there exist mixed-strategy equilibria where non-disclosing firms are not financed with a positive probability. It can be shown that in these mixed-strategy equilibria (with details available from the author): investment, the market risk premium and investors' ex-ante welfare all increase as the disclosure friction increases.

## 5.2 Disclosure, Market Beta and Cross-sectional Cost of Capital

To convert these results into statements about firm's expected returns, I define next a firm's cost of capital as investors' expected cash flow over the market price. To set up ideas, the risk premium corresponds to the cost of capital for the market portfolio. Formally, let  $R_\delta \equiv \mathbb{E}(\epsilon|\delta)/P_\delta$  be defined as the expected return for a firm disclosing  $\delta$  (i.e., the ratio of its expected cash flow to its price), where  $\delta \in \{\emptyset\} \cup \mathbb{R}$ . Finally, let  $R_D = \mathbb{E}(R_\delta|\delta \neq \emptyset)$ , be the expected return conditional on disclosure. From asset pricing models, one knows that a firm less (more) sensitive to systematic risk has a lower (higher) expected market return. The measure of the firm's sensitivity to systematic risk is the market  $\beta$  measured by the covariance of the firm's return with the market portfolio return over the variance of the market portfolio return. I relate the market  $\beta$  to the cost of capital in this model.

**Lemma 2** Suppose  $\eta \geq \eta^{over}$ . The firm's cost of capital can be expressed as follows:

$$\begin{aligned}
 R_\emptyset &= \underbrace{1}_{\text{Risk-free}} + \underbrace{\beta_\emptyset (\mathbb{E}(R_m^{over}) - 1)}_{\text{Risk Premium}} \quad \text{and} \quad R_D = \underbrace{1}_{\text{Risk-free}} + \underbrace{\beta_D (\mathbb{E}(R_m^{over}) - 1)}_{\text{Risk Premium}} \\
 \text{where } \beta_\emptyset &= \frac{\underbrace{\frac{V(y)}{P_\emptyset P_m^{over}}}_{\text{covariance}}}{\underbrace{\frac{V(y)}{P_m^{over2}}}_{\text{market variance}}} \\
 \text{and } \beta_D &= \frac{\underbrace{\int_{\epsilon^{over}}^{+\infty} \frac{V(y)}{P(\epsilon) P_m^{over}} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon}_{\text{covariance}}}{\underbrace{\frac{V(y)}{(P_m^{over})^2}}_{\text{market variance}}}
 \end{aligned}$$

Disclosing and non-disclosing firms differ by their sensitivity to the risk premium. Although non-disclosing firms have an additional variance term due to the fact that their signal is imperfectly known, this extra variance is diversifiable and therefore it is not priced. In particular, the variance due to the estimation risk on  $\epsilon$  does not appear in  $\beta_\emptyset$ . Proposition 5 compares average expected returns of disclosing firms  $R_D$  (averaged over all firms who disclosed successfully) and non-disclosing firms  $R_\emptyset$ .

**Proposition 5** In the overinvestment equilibrium ( $\eta \geq \eta^{over}$ ),  $\beta_D < \beta_\emptyset$ . That is, disclosing firms have a lower cost of capital than non-disclosing firms, i.e.  $R_D < R_\emptyset$  and more voluntary disclosure (lower  $\eta$ ) increases  $R_D$  and  $R_\emptyset$ .

The market beta of firms disclosing is lower than the market beta of firms that do not disclose. This effect is due to the fact that disclosing firms have, on average, higher idiosyncratic cash flows than non-disclosing firms. In turn, these future gains dilute some of the sensitivity to the systematic shock, offsetting the systematic risk. In contrast, non

disclosure implies low cash flows, and thus more systematic risk per unit of cash flow and a higher beta. More voluntary disclosures (lower disclosure friction) in the economy lead to a lower cost of capital of all disclosing firms as well as non-disclosing firms as the proportion of disclosing firms includes a wider range of firms with lower cash flows, i.e. more exposed to systematic risk while the remaining non-disclosing firms are more likely to have even lower cash flows.

This result sheds light on the mixed empirical findings in the current capital market literature on cost of capital. Welker (1995) and Sengupta (1998) analyze firm disclosure rankings given by financial analysts and find that firms rated as more transparent have a lower cost of capital. Botosan (1997) and Botosan and Plumlee (2002) show that firms disclosing more information in their annual reports have lower cost of capital. Ecker et al. (2006), Chen, Berger, and Li (2006) and Francis, Nanda, and Olsson (2008) also relate firm-specific information to cost of capital and find similar results.<sup>17</sup>

Finally, the discussion focuses on the overinvestment equilibrium; if the disclosure friction is low, non-disclosing firms are not financed and therefore one cannot compare the costs of capital between disclosing and non-disclosing firms. An immediate extension is to consider that all investors are perfectly diversified ex-ante. In this case one can consider a more general form of firms' cash flows by including a constant term  $\mu > 0$  so that even if firms do not invest, they still receive a constant cash flow  $\mu > 0$ . It follows that in an underinvestment equilibrium, non-disclosing firms do not invest in risky projects and keep their initial capital invested in the risk-free rate, whereas disclosing firms invest in projects whose return (cost of capital) is higher, but exposed to the systematic risk. Hence one feature to highlight for empirical work is that the level of investment in the economy is a key determinant to study cross sectionally the relation between disclosure and cost of capital.

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<sup>17</sup>My result should be separated from other standard models of disclosure (e.g., Verrecchia (1983) or Dye (1985)) that do not incorporate systematic risk. In such models, the primary object of interest is the instantaneous response of the market price to disclosure. Such response would also exist here (the non-disclosing firm's price would decrease) but the notion of cost of capital studied here is measured as the return for the (possibly long) period post disclosure, excluding the disclosure event. The benefit of using this approach is that it predicts long-term effects of disclosure, as observed empirically, versus a short adjustment. Further, the standard model predicts that, when not disclosing, a firm's market price would decrease which would lead to negative returns and/or the counterfactual empirical implication that the cost of capital of non-disclosing firms (as proxied by their market return) would be lower than the cost of capital of disclosing firms.

## 6 Disclosure Friction, Average Cost of Capital and Economic Efficiency

### 6.1 Price Dispersion and Cost of Capital

Before the average cost of capital in the economy is formally stated, it is useful to first derive the price dispersion induced by the disclosure friction. I provide in Lemma 3 two additional technical properties of the model.

**Lemma 3** *Denote  $\Delta(\cdot; \eta)$  as the distribution of  $P_\delta$ . Let  $\eta \geq \eta'$ :*

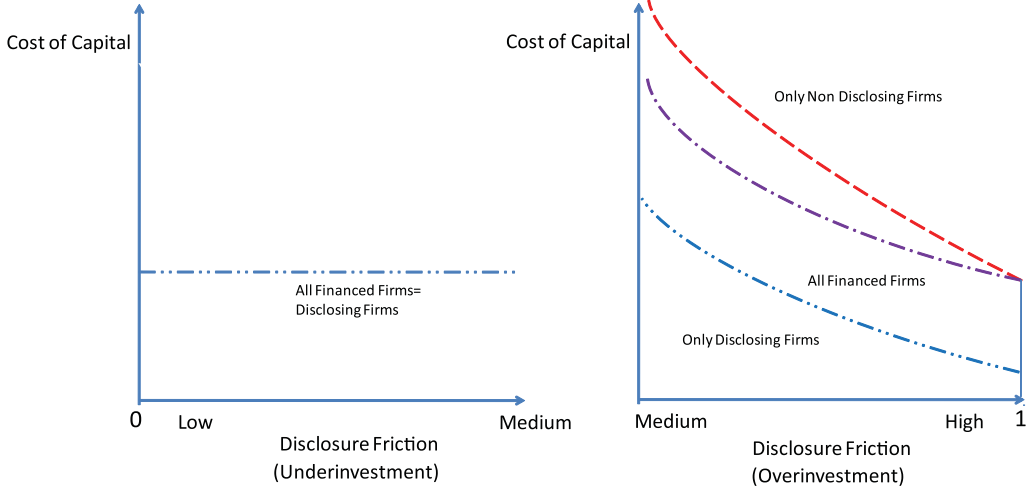
- (i) *In the overinvestment equilibrium,  $\Delta(\cdot; \eta)$  second-order stochastically dominates  $\Delta(\cdot; \eta')$ .*
- (ii) *In the underinvestment equilibrium,  $\Delta(\cdot; \eta')$  first-order stochastically dominates  $\Delta(\cdot; \eta)$ .*

The first part of lemma 3 (i) demonstrates that a lower disclosure friction increases the variability of market prices in the overinvestment equilibrium. That is, more disclosure implies a wider range of reported signals while less disclosure implies an “average” price for non-disclosing firms. It follows that a profit-maximizing but risk-averse investor would always prefer a greater disclosure friction in the overinvestment equilibrium. The second part (ii) shows that the result is reversed when the disclosure friction is sufficiently low and falls in the underinvestment equilibrium. In the underinvestment equilibrium region, decreasing the disclosure friction raises the proportion of high value firms to be informed, which implies, because the disclosure threshold coincides with first-best, that more firms choose the efficient investment.

I analyze next the average cost of capital in the economy ( $\mathcal{R} \equiv \mathbb{E}(R_\delta)$ ), which is the unconditional expected return averaging all the firm-specific returns in the economy. The average cost of capital captures the regression output in a cross-sectional equally-weighted empirical study. It is typically different from the risk premium, which is computed as the return of the market portfolio. However, a link between the two concepts is that the latter corresponds to the return of each firm, weighted by its size in the market portfolio (or value-weighted).

**Proposition 6** (i) *In the overinvestment equilibrium, disclosing and non-disclosing firms’ average cost of capital decrease in the disclosure friction. Further overall average cost of capital decreases in the disclosure friction and is greater than the risk premium.*

Figure 2: Average Costs of Capital



(ii) *In the underinvestment equilibrium, average cost of capital does not depend on the disclosure friction and is greater than the risk premium.*

Proposition 6 provides the result that maps the disclosure friction to average cost of capital. A characteristic of average cost of capital (equally-weighted basis) is to put less weight than the risk premium (value-weighted basis) on the high value firms. Thus, it stresses more the dispersion of prices, amplifying the representability of non disclosing firms with respect to their cost of capital. In the overinvestment equilibrium a lower disclosure friction increases the number of disclosures which in turn causes more cross-sectional dispersion in market prices. On a value-weighted basis, this would not affect the market risk premium. On an equally-weighted basis, firms with lower prices and higher costs of capital are over-represented (as compared to the value-weighted portfolio) implying a greater average cost of capital. As shown on Figure 2, the overinvestment equilibrium and interior disclosure friction leads to the highest possible average cost of capital. For any level of the disclosure friction such that  $\eta \leq \eta^{under}$  (underinvestment equilibrium), firms receive an average cost of capital equal to the cost of capital of all disclosing firms.<sup>18</sup> In the underinvestment equilibrium non-disclosing firms are no longer financed and aggregate cost of capital falls and remains constant for all  $\eta \leq \eta^{under}$ .

The model has implications for existing research on cost of capital. Recently, several studies have examined the consequences on firm's average cost of capital of changes on

<sup>18</sup>The average cost of capital in the economy is an unconditional expectation whereas the average cost of capital of disclosing and non-disclosing firms is a conditional expectation.

accounting standards (which may or may not improve accounting quality). Barth, Landsman, Lang, and Williams (2007) find evidence that firms applying IAS generally have higher value-relevant information than domestic standards. Leuz and Verrecchia (2000) report that a sample of firms voluntarily switching from German to international standards decreased their cost of capital. The interpretation of this empirical finding depends on whether this shift was due to: (i) firms voluntarily choosing international standards in a strategic way, (ii) the level of accounting quality (interpreted here by  $\eta$ ) in international standards and firms needed to use international standards for other non-strategic reasons. Comparing (i) and (ii), the interpretation of international standards having more accounting quality than German standards would only be confirmed in the first scenario.

## 6.2 Efficiency and Average Cost of Capital

Finally, I address the relation between the exogenous disclosure friction and economic efficiency; and whether economic efficiency can be measured by the average cost of capital. I use the term of efficiency to describe how well the accounting system maximizes investors' ex-ante expected utility (another common term is welfare).<sup>19</sup> Economic efficiency can be decomposed in two aspects: productive efficiency, i.e. whether a lower disclosure friction implements more efficient production, and risk-sharing efficiency, i.e. to what extent can financial markets help investors insure against diversifiable risk.<sup>20</sup>

**Proposition 7** *Economic efficiency is maximum at either  $\eta = 0$  or  $\eta = 1$ .*

As the disclosure friction moves away from  $\eta^{over}$  or  $\eta^{under}$ , either productive efficiency or risk-sharing improves, leading to an overall improvement in economic efficiency. Proposition 7 shows that the global efficiency optimum takes the form of a corner (or “bang-bang”) social policy with either a complete resolution of the risk-sharing with no disclosure (maximum disclosure friction), or a complete resolution of the production inefficiency with full disclosure (no disclosure friction).

**Corollary 1** *If investors are under-diversified ex-ante, an increase in average cost of capital implies a decrease in economic efficiency in the overinvestment region. Otherwise if*

<sup>19</sup>Analyzing efficiency is useful, because in theory any efficient outcomes could lead to welfare improvements in an economy with non-identical agents if a planner were to make fixed transfers (second-welfare theorem); separating efficiency from welfare considerations, in this respect, allows me to distinguish accounting from reallocate effects.

<sup>20</sup>I use the standard definition of risk-sharing, i.e. a mutually beneficial trade in which agents exchange offsetting (idiosyncratic) sources of risk. Risk-sharing is impaired when more information in advance of trading limits the risk-sharing opportunity. I contrast risk-sharing with productive efficiency, defined as operating decisions that increase the total amount of resource in the economy.

*investors are fully diversified, the average cost of capital is irrelevant to proxy for economic efficiency.*

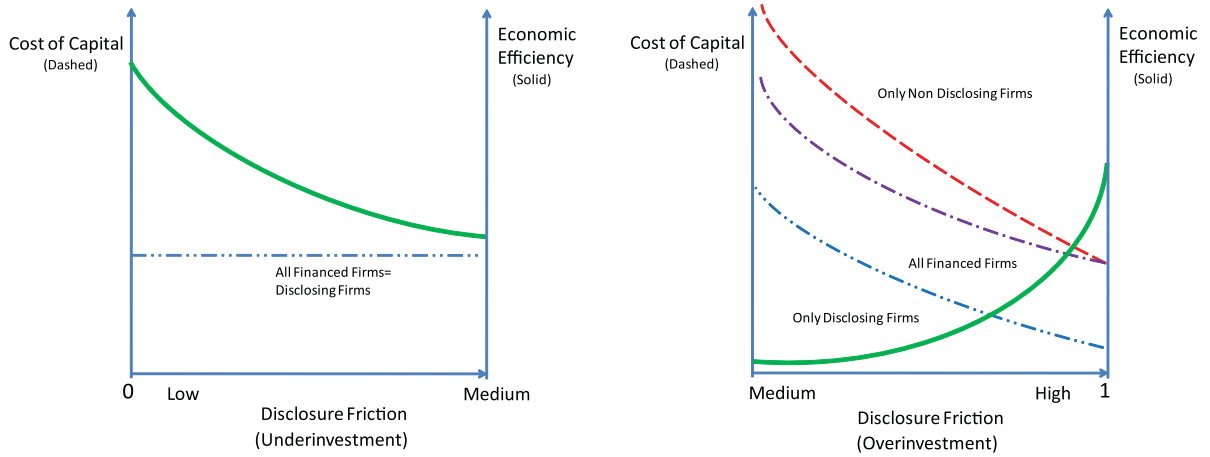
Linking cost of capital to efficiency is important, since cost of capital is a metric that can be empirically observed to evaluate a new accounting regulation. The average cost of capital can be a proxy for economic efficiency when the disclosure friction is high; specifically, in my model, the average cost of capital captures how well financial markets function at diversifying idiosyncratic risk. However if one is relaxing the assumption of underdiversification ex-ante and consider instead fully diversified investors ex-ante, the average cost of capital is irrelevant to measure economic efficiency. Alternatively when the disclosure friction is relatively low, the average cost of capital may be artificially low because high-beta non-disclosing firms are not financed but is not connected to economic efficiency in the underinvestment equilibrium.

Further, when one compares the efficiency with no friction versus the greatest disclosure friction, if efficiency is greater without the disclosure friction, then average cost of capital and efficiency are always misaligned. It is easy to verify that this condition will be verified when risk-aversion is low, so that the production efficiency concerns dominate risk-sharing concerns. The condition, however, is not sufficient to maximize efficiency, given that average cost of capital is constant when  $\eta < \eta^{under}$ ; in this case, maximizing efficiency requires also maximizing the aggregate level of investment. On the other hand, average cost of capital is aligned with efficiency when investors are not fully diversified ex-ante and risk-sharing motives dominate productive efficiency concerns (i.e., when  $\eta = 1$  is preferred to  $\eta = 0$ ). This condition will be satisfied when risk-aversion is large or few firms have signals below  $\epsilon^{FB}$ . Figure 3 represents the variation of the average cost of capital and economic efficiency as a function of the friction in the economy.

In summary, I show that the alignment between average cost of capital and economic efficiency depends on investors' risk-aversion and the distribution of firm's cash flows. From a practical perspective, in developed economies, one may possibly expect more high-value firms because such firms are filtered by existing institutions. Further, because the state may be offering a safety welfare net, one may also expect investors to be less risk-averse. One would then expect in such economies to observe lower disclosure friction and relatively high average cost of capital for disclosing firms. This prediction would be reversed in the case of emerging economies. In such economies, one would expect investors to be more risk-averse due to possible liquidity needs or more severe lemon's problems due to the lack of pre-established institutions.<sup>21</sup>

<sup>21</sup>A recent literature (Becka, Demirg-Kuntb, and Maksimovic (2008) and references therein) discusses whether developing economies may have more severe informational frictions than developed countries and

Figure 3: Cost of Capital vs Economic Efficiency (ex-ante Expected Utility)



## Conclusion

This paper provides a theory that ties together voluntary disclosure, cost of capital and economic efficiency. The model captures three main salient components: there are multiple firms and investors, voluntary disclosures are endogenous but affected by an exogenous disclosure friction, and the disclosure friction has real efficiency consequences on risk-sharing and production. I make several main observations.

- (i) If the disclosure friction is high, firms that disclose have lower cost of capital than firms that do not disclose. But more voluntary disclosure is both increasing the costs of capital of disclosing and non-disclosing firms.
- (ii) A disclosing firm's cost of capital is decreasing in its idiosyncratic expected cash flows.
- (iii) An increase from a low to a high disclosure friction implies an increase in aggregate investment.
- (iv) Economies with a high disclosure friction feature overinvestment, while those with a low disclosure friction feature underinvestment.
- (v) If investors are ex-ante undiversified and if the disclosure friction is high (low), a decrease in the disclosure friction implies an increase (no change) in average cost of capital and an decrease (increase) in economic efficiency. If diversification is available ex-ante, the average cost of capital is irrelevant to measure economic efficiency.

examines implications for growth and financing decisions.



As a path for future work, the analysis suggests several links between information availability driven by the disclosure friction and asset pricing; if one interprets information availability in my model as a proxy of accounting quality then empirical analysis should offer a more systematic methodology to use accounting quality as an asset pricing factor. A more detailed analysis is necessary to unravel how to measure changes to accounting quality that fit well the cross-section of stock returns. Finally, I focused on a one-period economy, in order to use results on aggregation with a disclosure game with multiple firms.

## Appendix: Technical Supplements

**Proof of Proposition 1:** The social planner solves the following maximization problem:

$$\max_{\tilde{\epsilon}} \int f(y) U \left( \int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \quad (\text{A-1})$$

The first order condition (FOC) of the maximization problem (A-1) yields:

$$-h(\epsilon^{FB}) \int f(y) (\epsilon^{FB} + y) U' \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy = 0$$

As  $h(\epsilon^{FB}) > 0$ , one can rewrite this FOC as  $\Phi(\epsilon^{FB}) = 0$  where:

$$\begin{aligned} \Phi(\epsilon^{FB}) &= - \int f(y) (\epsilon^{FB} + y) U' \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \\ &= -\epsilon^{FB} \int f(y) U' \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy + \int y f(y) U' \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \end{aligned}$$

Noting that  $U' > 0$ , the fact that  $\Phi(\epsilon^{FB}) = 0$  implies that  $\epsilon^{FB} = - \frac{\int y f(y) U' \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy}{\int f(y) U' \left( \int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy}$ . To prove that  $\epsilon^{FB}$  is unique and in  $(0, \theta)$ , it is sufficient to show that: (i)  $\Phi' < 0$  and, (ii)  $\Phi(0) > 0$  and  $\Phi(\theta) < 0$ .

$$\Phi'(\tilde{\epsilon}) = \int f(y) (\tilde{\epsilon} + y)^2 h(\tilde{\epsilon}) U'' \left( \int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy - \int f(y) U' \left( \int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy$$

Since  $U' > 0$  and  $U'' < 0$ ,  $\Phi' < 0$ .

$$\begin{aligned} \Phi(0) &= - \int y f(y) U' \left( \int_0^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \\ &= - \int_0^{+\infty} y f(y) U' \left( \int_0^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy - \int_{\underline{y}}^0 y f(y) U' \left( \int_0^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \\ &> - \int_0^{+\infty} y f(y) U' \left( \int_0^{+\infty} \epsilon h(\epsilon) d\epsilon \right) dy - \int_{\underline{y}}^0 y f(y) U' \left( \int_0^{+\infty} \epsilon h(\epsilon) d\epsilon \right) dy \\ &> -U' \left( \int_0^{+\infty} \epsilon h(\epsilon) d\epsilon \right) \int_{\underline{y}}^{+\infty} y f(y) dy = 0 \end{aligned}$$

where the inequality follows from the concavity of  $U$ . Moreover,  $\theta + y > 0$  implies that:

$$\Phi(\theta) = - \int f(y)(\theta + y)U' \left( \int_{\theta}^{+\infty} (\epsilon + y)h(\epsilon)d\epsilon \right) dy < 0$$

**Proof of Lemma 1:** The disclosure threshold satisfies  $P_{\epsilon^{over}} - P_{\emptyset} = 0$ . This implies, by Equations (2) and (3) that  $\epsilon^{over} - \mathbb{E}(\epsilon|ND) = 0$ . The non-disclosure conditional expectation  $\mathbb{E}(\epsilon|ND)$  can be expanded by Bayesian updating to:

$$\mathbb{E}(\epsilon|ND) = \frac{\eta\theta + (1 - \eta)H(\epsilon^{over})\mathbb{E}(\epsilon|\epsilon \leq \epsilon^{over})}{\eta + (1 - \eta)H(\epsilon^{over})}$$

The equation can be rearranged as follows,

$$\eta(\theta - \epsilon^{over}) = (1 - \eta)(H(\epsilon^{over}) - \int_{-\infty}^{\epsilon^{over}} \epsilon dH(\epsilon)) = (1 - \eta) \int_{-\infty}^{\epsilon^{over}} H(\epsilon)d\epsilon$$

This corresponds to Equation (7), p. 149 in Jung and Kwon (1988) and

$$\frac{\partial \epsilon^{over}}{\partial \eta} = \frac{\int_{\epsilon^{over}}^{+\infty} (1 - H(\epsilon^{over}))d\epsilon}{\eta + (1 - \eta)H(\epsilon^{over})} \quad (\text{A-2})$$

**Proof of Proposition 2:** I solve the maximization problem of an investor:

$$(\Gamma_{\delta}) \quad \max_{\gamma} \int f(y)(P_{\delta}\gamma + \frac{(1 - \gamma)P_{\delta}}{P_m}CF_m(y))^{1-\alpha} dy$$

The dependence on  $\delta$  is due to the price  $P_{\delta}$ . But from the program  $\Gamma_{\delta}$ ,  $P_{\delta}$  is only a constant multiplicative term of the objective function and the maximizer  $\gamma$  does not depend on  $P_{\delta}$  and so does not depend on  $\delta$ . Thus  $\gamma_{\emptyset}^{over} = \gamma_{\epsilon}^{over} = \gamma^{over}$ .

The aggregate demand of all investors in the risk-free asset AD is equal to:

$$\begin{aligned} AD &= (\eta + (1 - \eta)H(\epsilon^{over}))P_{\emptyset}\gamma_{\emptyset}^{over} + (1 - \eta) \int_{\epsilon^{over}}^{+\infty} P(\epsilon)\gamma_{\epsilon}^{over} h(\epsilon)d\epsilon \\ &= \gamma^{over} \left( (\eta + (1 - \eta)H(\epsilon^{over}))(\epsilon^{over} - \theta + \frac{\theta}{\mathbb{E}(R_m)}) \right) \\ &\quad + \gamma^{over} \left( (1 - \eta) \int_{\epsilon^{over}}^{+\infty} (\epsilon - \theta + \frac{\theta}{\mathbb{E}(R_m)})h(\epsilon)d\epsilon \right) \\ &= \gamma^{over} \left( \frac{\theta}{\mathbb{E}(R_m)} - \theta + (\eta + (1 - \eta)H(\epsilon^{over}))\epsilon^{over} \right) + \gamma^{over}(1 - \eta) \int_{\epsilon^{over}}^{+\infty} \epsilon h(\epsilon)d\epsilon \quad (\text{A-3}) \end{aligned}$$

I rewrite expression  $\int_{\epsilon^{over}}^{+\infty} \epsilon h(\epsilon)d\epsilon$  by integrating by parts as follows:

$$\int_{\epsilon^{over}}^{+\infty} \epsilon h(\epsilon)d\epsilon = \int_{\epsilon^{over}}^{+\infty} (1 - H(\epsilon))d\epsilon + \epsilon^{over}(1 - H(\epsilon^{over}))$$

Substituting this expression into Equation (A-3)

$$\begin{aligned} AD &= \gamma^{over} \left( \frac{\theta}{\mathbb{E}(R_m)} - \theta + (\eta + (1 - \eta)H(\epsilon^{over}))\epsilon^{over} \right) \\ &\quad + \gamma^{over}(1 - \eta) \left( \int_{\epsilon^{over}}^{+\infty} (1 - H(\epsilon))d\epsilon + \epsilon^{over}(1 - H(\epsilon^{over})) \right) \\ &= \gamma^{over} \left( \frac{\theta}{\mathbb{E}(R_m)} - \theta + \epsilon^{over} + \int_{\epsilon^{over}}^{+\infty} (1 - H(\epsilon))d\epsilon - \int_{-\infty}^{\epsilon^{over}} H(\epsilon)d\epsilon \right) \quad (\text{A-4}) \end{aligned}$$

The mean  $\theta$  can be rewritten as:

$$\begin{aligned}\theta &= \int_{-\infty}^{+\infty} \epsilon h(\epsilon) d\epsilon = \int_{\epsilon^{over}}^{+\infty} \epsilon h(\epsilon) d\epsilon + \int_{-\infty}^{\epsilon^{over}} \epsilon h(\epsilon) d\epsilon \\ &= \int_{\epsilon^{over}}^{+\infty} (1 - H(\epsilon)) d\epsilon + \epsilon^{over} (1 - H(\epsilon^{over})) - \int_{-\infty}^{\epsilon^{over}} H(\epsilon) d\epsilon + \epsilon^{over} H(\epsilon^{over})\end{aligned}$$

Finally expression (A-4) is equal to  $\gamma^{over}(\theta + \frac{\theta}{\mathbb{E}(R_m)} - \theta) = \gamma^{over} \frac{\theta}{\mathbb{E}(R_m)}$ . As the net supply is equal to zero, it yields  $\gamma^{over} = 0$ .

I determine next the expression of the expected market portfolio return in this economy. Simplifying the maximization problem  $(\Gamma_\delta)$ , it yields:

$$\max_{\gamma} \int f(y) \left\{ \gamma + (1 - \gamma) \frac{(\theta + y)}{P_m} \right\}^{1-\alpha} dy$$

The FOC from the investor's maximization problem with respect to  $\gamma$  is equal to:

$$\mathbb{E} \left\{ \left\{ 1 - \frac{(\theta + y)}{P_m} \right\} \left\{ \gamma + (1 - \gamma) \frac{(\theta + y)}{P_m} \right\}^{-\alpha} \right\} = 0 \quad (\text{A-5})$$

As from the market clearing condition, I showed that  $\gamma^{over} = 0$ , the FOC is reduced to:

$$\mathbb{E} \{ \{ \mathbb{E}(R_m)(\theta + y) - \theta \} (\theta + y)^{-\alpha} \} = 0 \quad (\text{A-6})$$

Simplifying,

$$\mathbb{E}(R_m^{over}) = \frac{\theta \mathbb{E}((\theta + y)^{-\alpha})}{\mathbb{E}((\theta + y)(\theta + y)^{-\alpha})} = \frac{\theta}{\theta + \underbrace{\mathbb{E}(y(\theta + y)^{-\alpha}) / \mathbb{E}((\theta + y)^{-\alpha})}_{Q^{over}}}$$

I prove next that  $Q^{over}$  is negative where  $Q^{over} = \int \frac{yf(y)U'(\theta+y)}{\int f(y)U'(\theta+y)dy} dy$ . By additivity of the integral,

$$\int_{\underline{y}}^{+\infty} yf(y)U'(\theta+y)dy = \int_0^{+\infty} yf(y)U'(\theta+y)dy + \int_{\underline{y}}^0 yf(y)U'(\theta+y)dy$$

Moreover  $\int_0^{+\infty} yf(y)U'(\theta+y)dy < \int_0^{+\infty} yf(y)U'(\theta)dy$  as  $U$  is strictly concave. Likewise  $\int_{\underline{y}}^0 yf(y)U'(\theta+y)dy < \int_{\underline{y}}^0 yf(y)U'(\theta)dy$ . By assumption  $\int_{\underline{y}}^{+\infty} yf(y)dy = E(\tilde{y}) = 0$  and it yields

$$\int_{\underline{y}}^{+\infty} yf(y)U'(\theta+y)dy < U'(\theta) \int_{\underline{y}}^{+\infty} yf(y)dy = 0$$

In the overinvestment equilibrium, the non-disclosing price  $P_\emptyset(\epsilon^{over})$  is equal to  $P(\epsilon^{over}) = \epsilon^{over} + Q^{over}$ . As  $Q^{over}$  is independent of  $\eta$ , the derivative of  $P(\epsilon^{over})$  w.r.t to  $\eta$  is equal to  $\frac{\partial \epsilon^{over}}{\partial \eta} \geq 0$ . Thus the non-disclosing price is increasing in the disclosure friction  $\eta$ . Further it implies that there exists a unique  $\eta^{over}$  such that  $P_\emptyset(\epsilon^{over}(\eta^{over})) = 0$  and  $\epsilon^{over}$  is always positive.

### Proof of Proposition 3:

If the investor has a non-disclosing firm, its wealth is equal to zero and cannot invest in the risk-free asset nor in the market portfolio. However if an investor has a disclosing firm then he solves the maximization problem  $(\Gamma_\epsilon)$ .

$$(\Gamma_\epsilon) \quad \max_{\gamma} \int f(y) \left( P_\delta \gamma + \frac{(1 - \gamma)P_\delta}{P_m} CF(y) \right)^{1-\alpha} dy$$

Simplifying,

$$(\Gamma_\epsilon) \quad \max_{\gamma} \int f(y) \left( \gamma + \frac{(1-\gamma)}{P_m} (1-\eta) \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{1-\alpha} dy$$

The maximizers of an individual investor having a disclosing firm do not depend on  $P_\epsilon$  and thus  $\epsilon$ . Therefore  $\forall \epsilon, \gamma_\epsilon^{under} = \gamma^{under}$ . The aggregate demand of all investors for the risk free asset is equal to:

$$(\eta + (1-\eta)H(\epsilon^{under}))\gamma_\emptyset^{under} P_\emptyset + (1-\eta) \int_{\epsilon^{under}}^{+\infty} \gamma_\epsilon^{under} P(\epsilon) d\epsilon = \gamma^{under} (1-\eta) \int_{\epsilon^{under}}^{+\infty} P(\epsilon) d\epsilon$$

As the net supply is equal to zero, it yields  $\gamma^{under} = 0$ .

Taking the FOC of the investor problem, it yields

$$\mathbb{E} \left\{ \left( \frac{(1-\eta) \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon}{P_m} - 1 \right) \left( \gamma + \frac{(1-\gamma)}{P_m} (1-\eta) \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} \right\} = 0$$

By the market clearing condition,  $\gamma^{under} = 0$  and the FOC is then reduced to:

$$\mathbb{E} \left\{ \left( \mathbb{E}(R_m) \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon - \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} \right\} = 0$$

$$\text{Simplifying } \mathbb{E}(R_m^{under}) = \frac{\mathbb{E} \left( \left( \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} \right)}{\mathbb{E} \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} \right)}$$

$$\text{Rearranging, } \mathbb{E}(R_m^{under}) = \frac{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon + (1-H(\epsilon^{under})) \frac{\mathbb{E} \left( y \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} \right)}{\mathbb{E} \left( \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} \right)}} \quad (\text{A-7})$$

I now turn to the determination of the disclosure threshold  $\epsilon^{under}$ .

The firm observing  $\epsilon^{under}$  has a cash flow:

$$\epsilon^{under} + y = \underbrace{\epsilon^{under} - \frac{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}{1-H(\epsilon^{under})}}_{\text{units of the risk free asset}} + \underbrace{\frac{1}{(1-\eta)(1-H(\epsilon^{under}))} CF_m}_{\text{units of the market portfolio}}$$

Its price is then  $\epsilon^{under} - \frac{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}{1-H(\epsilon^{under})} + \frac{1}{(1-\eta)(1-H(\epsilon^{under}))} P_m^{under} = 0$  as by definition this firm has a market price of zero, where

$$P_m^{under} = (1-\eta) \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon + (1-\eta)(1-H(\epsilon^{under})) Q^{under}$$

$$\text{with } Q^{under} = \int_{\underline{y}}^{+\infty} \frac{y f(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha}}{\int_{\underline{y}}^{+\infty} f(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} dy} dy$$

Replacing  $P_m^{under}$  by its expression and simplifying it yields  $\epsilon^{under} = - \frac{\int y f(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+y) h(\epsilon) d\epsilon \right)^{-\alpha} dy}{\int f(\tilde{y}) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon+\tilde{y}) h(\epsilon) d\epsilon \right)^{-\alpha} d\tilde{y}}$ .

The disclosure threshold  $\epsilon^{under}$  is independent of the disclosure friction  $\eta$ .

I now prove that  $Q^{under}$  is negative. This is similar to the proof of  $Q^{over} < 0$ .

$$Q^{under} = \int_{\underline{y}}^{+\infty} \frac{yf(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon + y)h(\epsilon)d\epsilon \right)^{-\alpha}}{\int_{\underline{y}}^{+\infty} f(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon + y)h(\epsilon)d\epsilon \right)^{-\alpha} dy} dy$$

As the utility function is strictly concave,

$$\int_0^{+\infty} yf(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon + y)h(\epsilon)d\epsilon \right)^{-\alpha} dy < \int_0^{+\infty} yf(y) \left( \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon)d\epsilon \right)^{-\alpha} dy \quad (A-8)$$

$$\text{and } \int_{\underline{y}}^0 yf(y) \left( \int_{\epsilon^{under}}^{+\infty} (\epsilon + y)h(\epsilon)d\epsilon \right)^{-\alpha} dy < \int_{\underline{y}}^0 yf(y) \left( \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon)d\epsilon \right)^{-\alpha} dy \quad (A-9)$$

By assumption  $\int_{\underline{y}}^{+\infty} yf(y)dy = \mathbb{E}(\tilde{y}) = 0$  which yields

$$Q^{under} < \left( \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon)d\epsilon \right)^{-\alpha} \int_{\underline{y}}^{+\infty} yf(y)dy = 0$$

Further in the underinvestment region, the turning point  $\eta^{under}$  is determined by equation (4) such that the price for non-disclosing firms is equal to zero. Given that  $Q^{under}$  is not equal to  $Q^{over}$ , there exists  $\eta^{under} \neq \eta^{over}$  such that  $\epsilon^{under} = -Q^{under}$ .

**Proof of Proposition 4:** Recall that  $Q^{under}$  is as follows

$$\begin{aligned} Q^{under} &= \frac{\int yf(y) \left( \int_{\epsilon^{under}}^{+\infty} h(\epsilon)(\epsilon + y)d\epsilon \right)^{-\gamma} dy}{\int f(y) \left( \int_{\epsilon^{under}}^{+\infty} h(\epsilon)(\epsilon + y)d\epsilon \right)^{-\gamma} dy} \\ &= \frac{\int yf(y) \left( \frac{\int_{\epsilon^{under}}^{+\infty} h(\epsilon)(\epsilon + y)d\epsilon}{\int_{\epsilon^{under}}^{+\infty} h(\epsilon)d\epsilon} \right)^{-\gamma} dy}{\int f(y) \left( \frac{\int_{\epsilon^{under}}^{+\infty} h(\epsilon)(\epsilon + y)d\epsilon}{\int_{\epsilon^{under}}^{+\infty} h(\epsilon)d\epsilon} \right)^{-\gamma} dy} \\ &= \frac{\int yf(y) (y + k(\epsilon^{under}))^{-\gamma} dy}{\int f(y) \int_a^{+\infty} h(\epsilon)(y + k(\epsilon^{under}))^{-\gamma} dy} \\ &= \int yz(y; k(\epsilon^{under})) dy \end{aligned}$$

$$\text{where } k(x) = \mathbb{E}(\epsilon | \epsilon \geq x) \quad \text{and} \quad z(y; a) = \frac{f(y)(y + a)^{-\gamma}}{\int f(y')(y' + a)^{-\gamma} dy'}$$

$Q^{under}$  can also be interpreted as  $Q^{over}$  in an economy with an endowment process  $y + k(\epsilon^{under})$ . The function  $z(\cdot; a)$  is a well-defined probability density function (it is commonly referred to as the risk-neutral probability measure). I shall argue next that the distribution decreases in the sense of the first-order stochastic dominance when  $a$  increases. To see this, define  $\psi(X; a)$  as the cumulative distribution function:

$$\psi(X; a) = \frac{\int_{\underline{y}}^X f(y)(y + a)^{-\gamma} dy}{\int f(y)(y + a)^{-\gamma} dy} \quad \text{and} \quad \psi_a(X; A) = \gamma \frac{K_1}{(\int f(y)(y + a)^{-\gamma} dy)^2}$$

$$\text{with } K_1 = \int f(y)(y + a)^{-\gamma-1} dy \int_{\underline{y}}^X f(y)(y + a)^{-\gamma} dy - \int_{\underline{y}}^X f(y)(y + a)^{-\gamma-1} dy \int f(y)(y + a)^{-\gamma} dy.$$

To prove first-order stochastic dominance, I need to show that  $\psi_a < 0$ , i.e.  $K_1 < 0$ .

$$\begin{aligned}\frac{\partial K_1}{\partial X} &= \int f(y)(y+a)^{-\gamma-1} dy f(X)(X+a)^{-\gamma} \\ &\quad - f(X)(X+a)^{-\gamma-1} \int f(y)(y+a)^{-\gamma} dy \\ &= f(X)(X+k(a))^{-\gamma-1} \underbrace{\int f(y)(y+k(a))^{-\gamma-1} (X-y) dy}_{K_2}\end{aligned}$$

The term  $K_2$  is strictly increasing in  $X$  and is strictly negative (positive) at  $X = \underline{y}$  ( $X = \bar{y}$ ). Therefore,  $K_1$  must therefore be first decreasing and then increasing, which in turn implies that  $K_1 < 0$  for any  $X$ . It follows that  $\psi_a(\cdot) < 0$  and thus, for two  $a' > a$ , the risk-neutral probability measure induced by the density  $z(\cdot; a')$  first-order stochastically dominates that induced by the density  $z(\cdot; a)$ . Using this and the fact that  $\theta < k(\epsilon^{under})$ ,

$$Q^{under} = \int y z(y; k(\epsilon^{under})) dy > \int y z(y; \theta) dy = Q^{over}$$

This also implies that  $\eta^{under} < \eta^{over}$ . Finally, comparing the risk premium in each equilibrium:

$$\begin{aligned}\mathbb{E}(R_m^{under}) &= \frac{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon + (1 - H(\epsilon^{under})) Q^{under}} \\ &= \frac{\mathbb{E}(\epsilon | \epsilon \geq \epsilon^{under})}{\mathbb{E}(\epsilon | \epsilon \geq \epsilon^{under}) + Q^{under}} < \frac{\theta}{\theta + Q^{over}} = \mathbb{E}(R_m^{over})\end{aligned}$$

**Proof of Lemma 2:**

The costs of capital of non-disclosing firms and disclosing firms are respectively:

$$R_\emptyset = \frac{\mathbb{E}(\epsilon | ND)}{P_\emptyset} = \frac{\epsilon^{over}}{P(\epsilon^{over})} \quad \text{and} \quad R_D = \int_{\epsilon^{over}}^{+\infty} \frac{\epsilon}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon$$

As  $P_\emptyset = P(\epsilon^{over}) = \epsilon^{over} + Q^{over}$  and  $P(\epsilon) = \epsilon + Q^{over}$ , a simple rewriting of the costs of capital yields:

$$R_\emptyset = 1 - \frac{Q^{over}}{P_\emptyset} \quad \text{and} \quad R_D = 1 - Q^{over} \int_{\epsilon^{over}}^{+\infty} \frac{h(\epsilon)}{P(\epsilon)(1 - H(\epsilon^{over}))} d\epsilon$$

Looking closely at the prices, I can express them as a CAPM formulation:

$$\begin{aligned}R_\emptyset &= \underbrace{1}_{\text{Riskfree}} + \underbrace{\frac{P_m^{over}}{P_\emptyset}}_{\beta_\emptyset} \underbrace{(\mathbb{E}(R_m^{over}) - 1)}_{\text{Risk Premium}} \\ R_D &= \underbrace{1}_{\text{Riskfree}} + \underbrace{\int_{\epsilon^{over}}^{+\infty} \frac{P_m^{over}}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon}_{\beta_D} \underbrace{(\mathbb{E}(R_m^{over}) - 1)}_{\text{Risk Premium}}\end{aligned}$$

I derive the beta  $\beta$  as an expression of the covariance between the firm's return and the market portfolio return over the variance of the market portfolio return.

$$\begin{aligned}\beta_\emptyset &= \frac{P_m^{over}}{P_\emptyset} = \frac{\underbrace{V(y)}_{\text{covariance}}}{\underbrace{P_\emptyset P_m^{over}}_{\text{market variance}}} / \frac{V(y)}{P_m^{over2}} \\ \beta_D &= \int_{\epsilon^{over}}^{+\infty} \frac{P_m^{over}}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon = \underbrace{\int_{\epsilon^{over}}^{+\infty} \frac{V(y)}{P(\epsilon) P_m^{over}} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon}_{\text{covariance}} / \underbrace{\frac{V(y)}{P_m^{over2}}}_{\text{market variance}}\end{aligned}$$

**Proof of Proposition 5:**

Further  $P_\emptyset(\epsilon^{over}) = P(\epsilon^{over})$ . As  $\forall \epsilon \geq \epsilon^{over}$ ,  $P(\epsilon^{over}) \leq P(\epsilon)$ ,  $R_\emptyset \geq R_D$ .

**Proof of Lemma 3:**

**(i) Let us prove that if  $\eta \geq \eta^{over}$ , for  $\eta < \eta'$ ,  $\Delta(\cdot; \eta')$  second-order stochastically dominates  $\Delta(\cdot; \eta)$ .** For a price  $p < P_\emptyset(\epsilon^{over}(\eta))$ ,  $\Delta(p; \eta) = 0$  otherwise for a price  $P_\emptyset(\epsilon^{over}(\eta)) \leq p \leq \bar{p}$ ,  $\Delta(p; \eta) = \{\eta + (1 - \eta)H(\epsilon^{over}(\eta))\} + (1 - \eta) \{H(p - Q^{over}) - H(\epsilon^{over}(\eta))\}$ . Let us define  $\eta < \eta'$  and  $T(\bar{p})$  as the area between the two curves  $\Delta(\cdot; \eta)$  and  $\Delta(\cdot; \eta')$  in  $[P_\emptyset(\epsilon^{over}(\eta)), \bar{p}]$ , specifically  $T(\bar{p}) = \int_{P_\emptyset(\epsilon^{over}(\eta))}^{\bar{p}} (\Delta(p; \eta) - \Delta(p; \eta')) dp$ . By additivity of the integral I rewrite  $T(\bar{p})$ :

$$T(\bar{p}) = \int_{P_\emptyset(\epsilon^{over}(\eta))}^{P_\emptyset(\epsilon^{over}(\eta'))} (\Delta(p; \eta) - 0) dp + \int_{P_\emptyset(\epsilon^{over}(\eta'))}^{\bar{p}} (\Delta(p; \eta) - \Delta(p; \eta')) dp$$

Consider  $\eta' = \eta + \mu$ , then  $T(\bar{p})$  becomes:

$$T(\bar{p}) = \int_{P_\emptyset(\epsilon^{over}(\eta))}^{P_\emptyset(\epsilon^{over}(\eta+\mu))} \Delta(p; \eta) dp + \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\bar{p}} (\Delta(p; \eta) - \Delta(p; \eta + \mu)) dp$$

Further define  $A(\mu) = \int_{P_\emptyset(\epsilon^{over}(\eta))}^{P_\emptyset(\epsilon^{over}(\eta+\mu))} \Delta(p; \eta) dp$  and  $B(\mu) = \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\bar{p}} (\Delta(p; \eta) - \Delta(p; \eta + \mu)) dp$ . I differentiate the above expressions w.r.t  $\mu$ :

$$\begin{aligned} A'(\mu) &= \frac{\partial \epsilon^{over}(\eta + \mu)}{\partial \eta} \Delta(P_\emptyset(\epsilon^{over}(\eta + \mu)); \eta) \\ B'(\mu) &= -\frac{\partial \epsilon^{over}(\eta + \mu)}{\partial \eta} \{ \Delta(P_\emptyset(\epsilon^{over}(\eta + \mu)); \eta) - \Delta(P_\emptyset(\epsilon^{over}(\eta + \mu)); \eta + \mu) \} \\ &\quad - \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\bar{p}} ((1 - H(\epsilon^{over}(\eta + \mu))) + (1 - \eta - \mu)h(\epsilon^{over}(\eta + \mu)) \frac{\partial \epsilon^{over}(\eta + \mu)}{\partial \eta} \\ &\quad - H(p - Q^{over}) + H(\epsilon^{over}(\eta + \mu)) - (1 - \eta - \mu)h(\epsilon^{over}(\eta + \mu)) \frac{\partial \epsilon^{over}(\eta + \mu)}{\partial \eta}) dp \\ &= -\frac{\partial \epsilon^{over}(\eta + \mu)}{\partial \eta} \{ \Delta(P_\emptyset(\epsilon^{over}(\eta + \mu)); \eta) - \Delta(P_\emptyset(\epsilon^{over}(\eta + \mu)); \eta + \mu) \} \\ &\quad - \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\bar{p}} (1 - H(p - Q^{over})) dp \end{aligned}$$

When  $\mu = 0$ ,  $A'$  can be simplified to  $\frac{\partial \epsilon^{over}(\eta)}{\partial \eta} \Delta(P_\emptyset(\epsilon^{over}(\eta)); \eta) \geq 0$ .

Likewise  $B'$  is equal to  $-\int_{P_\emptyset(\epsilon^{over}(\eta))}^{\bar{p}} (1 - H(p - Q^{over})) \leq 0$ .

Replacing  $\frac{\partial \epsilon^{over}(\eta)}{\partial \eta}$  by expression (A-2) and simplifying, I compute  $A'(0) + B'(0)$ :

$$\int_{\epsilon^{over}(\eta)}^{+\infty} (1 - H(\epsilon)) d\epsilon - \int_{P_\emptyset(\epsilon^{over}(\eta))}^{\bar{p}} (1 - H(p - Q^{over})) dp$$

Consider the change in variable  $\epsilon = p - Q^{over}$ , I further simplify the above expression by:

$$\int_{\epsilon^{over}(\eta)}^{+\infty} (1 - H(\epsilon)) d\epsilon - \int_{\epsilon^{over}(\eta)}^{\bar{\epsilon}} (1 - H(\epsilon)) d\epsilon \quad (\text{A-10})$$

If  $\bar{\epsilon}$  converges to  $+\infty$  then expression (A-10) is equal to zero. Thus  $\forall \bar{\epsilon}$ , expression (A-10) is positive. It also means that  $\forall \bar{p}$ ,  $T(\bar{p}) \geq 0$  i.e. the probability mass of  $\Delta(\cdot; \eta)$  is more spread out than the probability mass of  $\Delta(\cdot; \eta')$ , which implies that  $\Delta(\cdot; \eta)$  is "riskier" than  $\Delta(\cdot; \eta')$ .

**(ii) Let us prove that if  $\eta \leq \eta^{under}$ , for  $\eta' < \eta$ ,  $\Delta(\cdot; \eta')$  first-order stochastically dominates  $\Delta(\cdot; \eta)$ .**

It has been proven that  $P(\epsilon^{under})$  does not depend on  $\eta$ . Thus the range of prices is the same for a given  $\eta$

or  $\eta'$ . Let us take the difference between  $\Delta(p; \eta')$  and  $\Delta(p; \eta)$  for  $p \in [0, \bar{p}]$ .

$$\begin{aligned} & \Delta(p; \eta') - \Delta(p; \eta) \\ = & (\eta' + (1 - \eta')H(\epsilon^{under})) + (1 - \eta') (H(p - Q^{under}) - H(\epsilon^{under})) \\ & - ((\eta + (1 - \eta)H(\epsilon^{under})) + (1 - \eta) (H(p - Q^{under}) - H(\epsilon^{under}))) \end{aligned}$$

Replacing  $\eta$  by  $\eta' + \mu$  it yields

$$\begin{aligned} & \Delta(p; \eta') - \Delta(p; \eta' + \mu) \\ = & (\eta' + (1 - \eta')H(\epsilon^{under})) + (1 - \eta') (H(p - Q^{under}) - H(\epsilon^{under})) \\ & - ((\eta' + \mu + (1 - \eta' - \mu)H(\epsilon^{under})) + (1 - \eta' - \mu) (H(p - Q^{under}) - H(\epsilon^{under}))) \\ = & -\mu(1 - H(\epsilon^{under})) + \mu (H(p - Q^{under}) - H(\epsilon^{under})) \\ = & -\mu(1 - H(p - Q^{under})) \leq 0 \end{aligned}$$

Therefore  $\Delta(\cdot; \eta')$  first order stochastically dominates  $\Delta(\cdot; \eta)$ .

**Proof of Proposition 6:**

If  $\eta \geq \eta^{over}$ , I will prove that  $R_D$  is decreasing in  $\eta$ .

$$R_D = 1 - \int_{\epsilon^{over}(\eta)}^{+\infty} \frac{Q^{over}}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}(\eta)))} d\epsilon$$

Differentiating  $R_D$  w.r.t  $\eta$ , it yields

$$-\frac{1}{(1 - H(\epsilon^{over}))^2} h(\epsilon^{over}) J'(\eta) \int_{\epsilon^{over}}^{+\infty} \frac{Q^{over} h(\epsilon)}{P(\epsilon)} d\epsilon + J'(\eta) \frac{Q^{over} h(\epsilon^{over})}{(1 - H(\epsilon^{over})) P(\epsilon^{over})}$$

Simplifying it yields

$$-\frac{J'(\eta) Q^{over} h(\epsilon^{over})}{(1 - H(\epsilon^{over}))} \left( \frac{1}{(1 - H(\epsilon^{over}))} \int_{\epsilon^{over}}^{+\infty} \frac{h(\epsilon)}{P(\epsilon)} d\epsilon - \frac{1}{P(\epsilon^{over})} \right) \quad (\text{A-11})$$

As  $\forall \epsilon > \epsilon^{over}$ ,  $P(\epsilon^{over}) \leq P(\epsilon)$ , and  $\frac{\partial \epsilon^{over}(\eta)}{\partial \eta} \geq 0$  and  $Q^{over} < 0$ , the derivative of  $R_D$  with respect to  $\eta$  given by expression (A-11) is negative. Therefore  $R_D$  is decreasing in  $\eta$ .

$P_\emptyset(\epsilon^{over}) = P(\epsilon^{over}) = \epsilon^{over} + Q^{over}$ . Differentiating  $P(\epsilon^{over})$  with respect to  $\eta$  yields  $\frac{\partial \epsilon^{over}(\eta)}{\partial \eta} \geq 0$ .

Thus  $R_\emptyset$  is decreasing in  $\eta$ .

If  $\eta \leq \eta^{under}$ ,  $\epsilon^{under}$  and  $Q^{under}$  do not depend on  $\eta$ , so  $R_D$  is independent of  $\eta$ .

$$R_D = 1 - Q^{under} \int_{\epsilon^{under}}^{+\infty} \frac{h(\epsilon)}{P(\epsilon)(1 - H(\epsilon^{under}))} d\epsilon$$

I turn to the comparative statics on the average cost of capital  $\mathcal{R}$ .

If  $\eta \geq \eta^{over}$ ,  $R_\delta = 1 - \frac{Q^{over}}{P_\delta}$ . Thus ordering  $\mathcal{R}(\eta)$  and  $\mathcal{R}(\eta')$  boils down to ordering  $1/P_\delta(\eta)$  and  $1/P_\delta(\eta')$ .

I know that the function  $1/P_\delta$  is a convex and decreasing function. I further proved that for  $\eta' < \eta$ ,  $\Delta(\cdot; \eta)$  second order stochastically dominates  $\Delta(\cdot; \eta')$ . This implies that  $\mathbb{E}(1/P_\delta(\eta)) \leq \mathbb{E}(1/P_\delta(\eta'))$ .

If  $\eta \leq \eta^{under}$  then  $\mathcal{R} = R_D$  as non-disclosing firms are not financed and

$R_D = 1 - Q^{under} \int_{\epsilon^{under}}^{+\infty} \frac{h(\epsilon)}{P(\epsilon)(1 - H(\epsilon^{under}))} d\epsilon$ , independent of  $\eta$ . I now compare the average cost of capital to the risk premium. By applying Jensen's inequality,

$$\mathbb{E}(R_m^{over}) \leq \mathcal{R} \quad \text{and} \quad \mathbb{E}(R_m^{under}) \leq R_D \quad (\text{A-12})$$

**Proof of Proposition 7:**

For  $\eta \geq \eta^{over}$ , the investor's indirect utility function with a firm disclosing  $\delta$  is equal to

$$\frac{1}{1 - \alpha} P_\delta^{1 - \alpha} \int f(y)(\theta + y)^{1 - \alpha} dy$$



The indirect utility is concave in  $P_\delta$ . Further I know that for  $\eta < \eta'$ ,  $\Delta(\cdot; \eta')$  second order stochastically dominates  $\Delta(\cdot; \eta)$ , which implies that

$$\begin{aligned} & \frac{1}{1-\alpha} \int P_\delta^{1-\alpha} \int f(y)(\theta+y)^{1-\alpha} dy d\Delta(P_\delta; \eta') \\ & \geq \frac{1}{1-\alpha} \int P_\delta^{1-\alpha} \int f(y)(\theta+y)^{1-\alpha} dy d\Delta(P_\delta; \eta) \end{aligned}$$

For  $\eta \leq \eta^{under}$ , the investors' aggregate expected utility for a given  $\eta$  is equal to

$$(1-\eta) \int_{\epsilon^{under}}^{+\infty} \frac{1}{1-\alpha} P(\epsilon)^{1-\alpha} \int f(y) \left( \int_{\epsilon^{under}}^{+\infty} (1-\eta)(\epsilon+y)h(\epsilon)d\epsilon \right)^{1-\alpha} dy h(\epsilon)d\epsilon$$

consider  $\eta > \eta'$

$$\begin{aligned} & (1-\eta) \int_{\epsilon^{under}}^{+\infty} \frac{1}{1-\alpha} P(\epsilon)^{1-\alpha} \int f(y) \left( \int_{\epsilon^{under}}^{+\infty} (1-\eta)(\epsilon+y)h(\epsilon)d\epsilon \right)^{1-\alpha} dy h(\epsilon)d\epsilon \\ & \leq (1-\eta') \int_{\epsilon^{under}}^{+\infty} \frac{1}{1-\alpha} P(\epsilon)^{1-\alpha} \int f(y) \left( \int_{\epsilon^{under}}^{+\infty} (1-\eta')(\epsilon+y)h(\epsilon)d\epsilon \right)^{1-\alpha} dy h(\epsilon)d\epsilon \end{aligned}$$

#### **Proof of Corollary 1:**

By Proposition 6, it has been proven that the average cost of capital in the economy is decreasing in the disclosure friction  $\eta$ , if  $\eta \geq \eta^{over}$ . Further by Proposition 7, the investors' expected utility increases in the disclosure friction  $\eta$ , if  $\eta \geq \eta^{over}$ .

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