# A NOTE ON THE RECOVERABILITY AND UNIQUENESS OF CHANGING TASTES

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This paper demonstrates that it will be impossible, by observing an agent's demand behavior, to either refute or confirm the general taste change hypothesis without substantially restricting the class of eligible preferences.

## 1. Introduction

For the past decade, the phenomenon of changing tastes in the context of intertemporal allocation problems has received wide attention in the literature [see, for example, Hammond (1976), Peleg and Yaari (1973), Pollak (1968), and Yaari (1977)]. Much of this effort has involved a consideration of various solution concepts (strategies) by which taste change may be meaningfully taken into account.

The concepts most widely considered [see, for example, Hammond (1976), Strotz (1956), or Pollak (1968)] are generally referred to as naive and sophisticated choice. Under naive choice, the agent in each time period allocates his wealth over current and future consumption in a manner optimal with respect to his preferences in force in that period. With changing tastes, these optimal consumption plans will thus continually be revised. Alternatively, sophisticated choice requires that the agent chooses, in each time period, an optimal plan from the subset of those he will actually subsequently follow.

It is then natural to ask if there are observable properties of an agent's

intertemporal demand functions which would uniquely identify which strategy concept (naive or sophisticated) he was applying. Unfortunately, such properties cannot in general be found. As first noted by Pollak (1968), certain (changing) preference structures can produce identical demand behavior under either solution concept. In this note such phenomenon is shown to occur for a large class of examples which includes those of Pollak (1968). But, more strikingly, this common demand behavior can be shown, in certain cases, to be rationalizable by a single multiperiod utility function which exhibits no change of taste whatsoever. Lastly, we demonstrate that this latter result may be understood as illustrating a well known demand aggregation theorem of Chipman (1974).

Our work thus has two significant consequences: (i) in general, it will be impossible, by observing his demand behavior, to infer what solution concept (if any) an agent whose tastes are changing may have chosen to adopt, and (ii) it will, in general, be impossible, empirically, to either refute or confirm the general taste change hypothesis without substantially restricting the class of possible preferences. This latter conclusion is valid since, for certain specifications, the same intertemporal demand behavior can be shown to result from the sequential choice of an agent whose tastes are changing as from an agent whose tastes are not changing at all. [This follows from the fact that certain (taste change induced) intertemporal demand behavior can be rationalized by a well behaved multiperiod utility function no exhibiting any change in tastes.]

Section 2 of the paper presents a suggestive example, and relates it to the work of Chipman (1974), while section 3 contains a brief literature review.

## 2. Example and basic theory

For the greatest simplicity of presentation, the discussion is cast in the context of a certainty, single consumption good world where the agent seeks to allocate his income over three periods. In what is to follow, let $y_1$ denote initial endowment wealth, and $c_t (t = 1, 2, 3)$ denote, respectively, consumption in periods $t = 1, 2, 3$; correspondingly, let $p_t (t = 1, 2, 3)$ denote the (present value) prices in each of the indicated time periods. Agents' time preferences for periods one and two are given the following general forms: $U_1(c_1,c_2,c_3)$: $C_1 \times C_2 \times C_3 \rightarrow R_+$, and $U_2(c_2,c_3|c_1)$: $C_2 \times C_3 \rightarrow R_+$, where, $\forall \, t$, $C_t$ denotes the range of possible consumption

values in period $t$. Both $U_1(\cdot)$ and $U_2(\cdot)$ will be assumed continuous, monotone increasing, and strictly convex to the origin. If there exists no monotone increasing transformation $T$ for which $U_2(c_2,c_3|\bar{c}_1) = TU_1(\bar{c}_1,c_2,c_3)$, we say that an agent's tastes have changed. Although our results hold for more elaborate settings, such generality is unnecessary to the basic point.

We now illustrate a situation in which, although agents' tastes have changed, the demand behavior resulting from the application of either the native or sophisticated solution perspectives is identical. Furthermore, it is shown that this common demand behavior may be rationalized by a single intertemporal utility function which can actually be exhibited.

*Example 1.* Suppose that $U_1(c_1,c_2,c_3) = c_1^{\alpha_1}c_2^{\alpha_2}c_3^{\alpha_3}$, and $U_2(c_2,c_3) = c_2^{\beta_2}c_3^{\beta_3}$, where $\sum_{t=1}^{3}\alpha_t = \sum_{t=2}^{3}\beta_t = 1$; $\alpha_t \neq \beta_t$ for $t = 2, 3$. Under the naive perspective, which is first employed for illustrative purposes, the agent initially solves:

$$\max c_1^{\alpha_1}c_2^{\alpha_2}c_3^{\alpha_3} \quad \text{subject to} \quad p_1c_1 + p_2c_2 + p_3c_3 \leqslant y_1.$$

A simple and straightforward calculation yields that $c_1 = \alpha_1 y_1/p_1$ and that $c_2 = \alpha_2 y_1/p_2$ and $c_3 = \alpha_3 y_1/p_3$. Then, in the second period, the agent revises these plans, due to his change of taste, by solving

$$\max c_2^{\beta_2}c_3^{\beta_3} \quad \text{subject to} \quad p_2c_2 + p_3c_3 \leqslant (1 - \alpha_1)y_1 = (\alpha_2 + \alpha_3)y_1,$$

to obtain $c_2 = \beta_2(\alpha_2 + \alpha_3)y_1/p_2$ and $c_3 = \beta_3(\alpha_2 + \alpha_3)y_1/p_3$. The demand pattern actually observed is thus summarized by $c_1 = \alpha_1 y_1/p_1$, $c_2 = \beta_2(\alpha_2 + \alpha_3)y_1/p_2$, and $c_3 = \beta_3(\alpha_2 + \alpha_3)y_1/p_3$. An application of the sophisticated choice procedure can be shown to yield identical results. [1] Clearly, the agent's optimal plan is inconsistent [in the Strotz (1956) sense] as period one planned consumption for period two is revised from $\alpha_2 y_1/p_2$ to $\beta_2(\alpha_2 + \alpha_3)y_1/p_2$. Furthermore, a straightforward calculation confirms that this common demand behavior can be rationalized by the following three period representation:

$$\hat{U}_1(c_1,c_2,c_3) = c_1^{\alpha_1}c_2^{\beta_2(\alpha_2 + \alpha_3)}c_3^{\beta_3(\alpha_2 + \alpha_3)}.$$

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[1] This is not surprising as our functional forms are adaptations of those analyzed by Pollak (1968), who derives a similar result in a somewhat different context.

How is it possible that this common naive/sophisticated demand solution can be rationalized? To answer this question, first notice that one may, formally, consider the successive preference functions of a single agent whose tastes are changing as equivalently representing the preference orderings of distinct individuals who act sequentially. Notice also that the distribution of income between period 1 and periods 2 and 3 is price $(p_1, p_2, p_3)$ and aggregate income $(y_1)$ independent (hereafter *PAII*). Regarding the successive preference orderings as representing different individuals, this is to say, equivalently, that the distribution of income across individuals is *PAII*. These facts, along with the homotheticity of the successive preference orderings, casts our problem in the context of a demand aggregation result of Chipman (1974, Theorem 4, p. 32). This is so as rationalizing the common naive/sophisticated demand behavior is formally equivalent to rationalizing the aggregate demand behavior which our naive/sophisticated solution may also be interpreted to represent.

The following Lemma offers conditions sufficient to ensure that the naive and sophisticated solutions exhibit a distribution of income among periods which is *PAII*: [2]

*Lemma 1.   Consider an N-period allocation problem where the set of all feasible allocations is described by* $C = \{(c_1, c_2, \ldots, c_N): \Sigma_{t=1}^N p_t c_t \leqslant y_1\}$. *Consider also a family of preference orderings (which represent an agent's sequentially changing tastes)* $\{U_i(\cdot)\}_{i=1}^{l \leqslant N}$, *where* $U_i(c_i, c_{i+1}, \ldots, c_N): C_i \times \ldots \times C_N \to R_+$. *For all i, the utilities* $U_i$ *are strictly convex to the origin, continuous, increasing, homothetic, and* $\forall\ i,\ i < l$, $U_i$ *has the property that its optimal allocation of income across consumption possibilities* $(C_i, C_{i+1}, \ldots, C_N)$ *is PAII. Then, under either the naive or sophisticated perspective, the proportion of income spent in each time period s, s = 1, 2, \ldots, l - 1, and collectively in time periods s = l, l + 1, \ldots, N is independent of prices* $(p_1, p_2, \ldots, p_N)$ *and total income,* $y_1$.

Proof.   See the appendix.

*This Lemma, in turn, allows the principal result.*

*Theorem 1.   Consider a collection of feasible allocations C and a family of preference orderings* $\{U_1(\cdot)\}_{i=1}^{l \geqslant N}$ *(which may be interpreted as representing an agent's sequentially changing tastes) satisfying the conditions of Lemma*

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[2] The naive/sophisticated solutions need not agree, however, for each to exhibit this property.

1. *Suppose, in addition, that $U_i(\cdot)$ is continuously differentiable and that $\partial U_i(\cdot)/\partial c_{l+i}|_{c_{l+i}=0} = +\infty, \forall\ i,\ 0 \leqslant i \leqslant N - l$. Then there exists a utility function $U^N\colon C \to R$ which generates the naive demand system and which is strictly convex to the origin, positively homogeneous of degree one, and continuous. Similarly, there exists a utility function $U^s\colon C \to R$ which generates the sophisticated demand system, and which is also strictly convex to the origin, positively homogeneous of degree one, and continuous.*

*Proof.* As before, we consider the case of naive choice; sophisticated choice is similar, though more complex notationally.

Denote the total expenditures under naive choice in periods $\{1, 2,\ldots,l-1\}$ by, respectively $\{E_1, E_2,\ldots,E_{l-1}\}$, and denote the collective expenditure in periods $\{l,\ l+1,\ldots,N\}$ by $E_{l,N}$; $\Sigma_{s=1}^{l-1}E_s + E_{l,N} = y_1$. By Lemma 1 we know that these proportions are price independent, and that if initial wealth $y_1$ changes to $\lambda y_1$, total expenditures for periods $\{1, 2,\ldots,l-1\}$ and periods $\{l,\ l+1,\ldots,N\}$ will become, respectively, $\{\lambda E_1, \lambda E_2,\ldots,\lambda E_{l-1}\}$ and $\lambda E_{l,N}$.

Define another family of preference orderings $\{\hat{U}_i(\cdot)\}_{i=1}^{l}$ by

$$\hat{U}_i(c_i,\ldots,c_N) = c_i^\delta, \qquad 1 \leqslant i < l-1, \quad Q < \delta < 1,$$

$$\hat{U}_i(c_i,\ldots,c_N) = U_i(c_i,\ldots,c_N), \qquad i = l.$$

If ordering $\hat{U}_i$ is regarded as receiving income share $\rho_i y_i$, where

$$\rho_i = E_i/y_1, \qquad 1 \leqslant i < l-1,$$
$$\phantom{\rho_i} = E_{l,N}/y_1, \qquad i = l,$$

then the distribution of income to orderings $\{\hat{U}_i\}_{i=1}^{l}$, as determined by the application of naive choice is *PAII*. Furthermore, the demand functions corresponding to these utilities are real valued, differentiable [by the properties of the $U_i(\cdot)$ functions and the continuity of the budget constraints] and exhaust all income. Lastly, each element of the collection $\{\hat{U}_i\}_{i=1}^{l}$ is homothetic and, for the income distribution determined by the exercise of naive choice, the consequent consumption demands coincide with those of naive choice. (In this way a time dependent allocation problem has essentially been transformed into a time independent one.)

Thus by Chipman (1974, Theorem 4) there exists a utility function $U^N(c_1, c_2,\ldots,c_N)$, with the asserted properties, which gives the same demand behavior, for any prices and aggregate income $y_1$, as would result from the actions of the individuals $\{\hat{U}\}_{i=1}^{l}$. But these demands coincide

with those of naive choice where we identify demand $c_i$ of $U^N(c_1,\ldots,c_N)$ as occurring at time $i$. But then $U^N(c_1,c_2,\ldots,c_N)$ rationalizes the naive choice behavior.    Q.E.D.

As we conclude this section, several clarifying remarks are in order:

*Remark 1.*    A reading of the proof of the Lemma makes clear the fact that the last utility $U_l$ (which determines allocations in periods $l$, $l+1,\ldots,N$) need only be homothetic and need not assign a distribution of income to periods $l$, $l+1,\ldots,N$ which is independent of prices. Since the income assigned to 'agent' $U_l$ is a residual remaining from agents $\{U_i\}_{i=1}^{l-1}$, a fixed distribution of income to agents $\{U_i\}_{i=1}^{l-1}$ (guaranteed by the price independence assumption) is sufficient to guarantee a fixed distribution of income to all agents, as Theorem 1 requires.

*Remark 2.*    Pollak (1971, p. 402) shows that the class of utility functions which are separable, homothetic, and exhibit the weaker property of 'expenditure proportionality' is the Bergson family. Of this family, the subclass satisfying the *PAII* condition are those of the form

$$U(c_1,c_2,\ldots,c_T) = \sum_{k=1}^{T} a_k \log c_k.$$

Although the results of this note nowhere require separability, it is difficult to construct examples of utility functions which are homothetic and exhibit expenditure proportionality yet which are not separable. Thus, for practical purposes, the eligible utility functions for orderings $\{U_i\}_{i=1}^{l-1}$ must be of the forms listed above.

*Remark 3.*    although the example illustrates a case in which the naive and sophisticated solutions agree, neither Lemma 1 nor Theorem 1 assert that this will be true generally.

The claims made in the introduction have been verified. Our example suggests the impossibility of inferring naive or sophisticated strategies from observed demand behavior while our Theorem confirms the difficulty in asserting, categorically, taste change at all.

## 3. Concluding comments

Other authors have analyzed problems resembling ours. We should first mention Pollak (1968). In a more general setting, using functional

forms similar to those of our example, Pollak (1968) similarly demon-
strates that the naive and sophisticated solutions can agree. He does not,
however, attempt to rationalize the common naive/sophisticated demand
behavior. Phlips and Spinnewyn (1979) similarly show the observational
equivalence of certain naive and sophisticated demand systems in a taste
change context. Their perspective is somewhat different, however, as they
derive this observational equivalence as justification for the use of certain
well known naive models as proxies for ones which would more fully
consider changing tastes. The Phlips–Spinnewyn (1979) models consider,
furthermore, the case of taste changes which are endogenous, in the sense
that tastes in a given period depend upon the levels of past consumption.
This differs from our analysis where the taste change mechanism can
evidently be exogenous.

## Appendix

*Proof of Lemma 1.* We illustrate with the naive case; sophisticated is
similar. Thus, at time 1, the agent solves

$$\max U_1(c_1, c_2, \ldots, c_N) \quad \text{subject to} \quad \sum_{i=1}^{N} p_i c_i \leqslant y_1.$$

By the strict convexity to the origin and continuity of $U_1(\cdot)$, and the
compactness of the budget set, a unique solution must exist, which we
denote by $(c_1^1, c_2^1, \ldots, c_N^1)$. By homotheticity, if income changes from $y_1$ to
$\lambda y_1$, the optimal consumption plan changes from $(c_1^1, c_2^1, \ldots, c_N^1)$ to
$(\lambda c_1^1, \lambda c_2^1, \ldots, \lambda c_N^1)$. Furthermore, by assumption, if prices change from
$(p_1, p_2, \ldots, p_N)$ to $(\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_N)$, the new optimal consumption vector
$(\hat{c}_1^1, \hat{c}_2^1, \ldots, \hat{c}_N^1)$ satisfies, $\forall i = 1, 2, \ldots, N$, $p_i c_i^1 = \hat{p}_i \hat{c}_i^1$. Thus, for any price
vector, and any income level, the proportion of income $y_1$ spent in period
one is constant.

This process continues by induction. For period $s < l$, the agent solves

$$\max U_s(c_s, c_{s+1}, \ldots, c_N) \quad \text{subject to} \quad \sum_{i=s}^{N} p_i c_i \leqslant y_1 - \sum_{i=1}^{s-1} p_i c_i^i,$$

where $c_i^i$ denotes optimal period $i$ consumption. By the same reasoning,
this has a unique solution $(c_s^s, c_{s+1}^s, \ldots, c_N^s)$. If income $y_1$ changes to $\lambda y_1$,
the agent solves (since demands in periods $i = 1, 2, \ldots, s - 1$ change from

$c_i^i$ to $\lambda c_i^i$ by the induction hypothesis)

$$\max U_s(c_s, c_{s+1}, \ldots, c_N) \quad \text{subject to} \quad \sum_{i=s}^{N} p_i c_i \leqslant \lambda y_1 - \lambda \sum_{i=1}^{s-1} p_i c_i^i,$$

which, by homotheticity, has the solution $(\lambda c_s^s, \lambda c_{s+1}^s, \ldots, \lambda c_N^s)$. If prices were to change to $(\hat{p}_1, \ldots, \hat{p}_N)$, the demands under naive choice for periods 1, 2,..., $s-1$, would change to $(\hat{c}_1^1, \hat{c}_2^2, \ldots, \hat{c}_{s-1}^{s-1})$, but nevertheless satisfy, $p_i c_i^i = \hat{p}_i \hat{c}_i^i$, $i = 1, 2, \ldots, s-1$. Thus, in period $s$, the agent would face the allocation problem

$$\max U_s(c_s, c_{s+1}, \ldots, c_N) \quad \text{subject to}$$

$$\sum_{i=s}^{N} \hat{p}_i c_i \leqslant y_1 - \sum_{i=1}^{s-1} \hat{p}_i \hat{c}_i^i = y_1 - \sum_{i=1}^{s-1} p_i c_i^i.$$

Again, by assumption, the optimal allocation for period, $s$, $\hat{c}_s^s$, would satisfy $p_s c_s^s = \hat{p}_s \hat{c}_s^s$. For arbitrary time periods $s < l$, therefore, the proportion of income spent in that time period is independent of all prices and of total aggregate income $y_1$. Since the proportion of the income available collectively for expenditure in periods $l, l+1, \ldots, N$ is a residual, it too is independent of prices $(p_1, p_2, \ldots, p_N)$ and total income.    Q.E.D.

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