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A "Position Paradox" in Sponsored Search Auctions

Kinshuk Jerath, Liye Ma
Tepper School of Business, Carnegie Mellon University, Pittsburgh, Pennsylvania 15206
{kinshuk@cmu.edu, liyem@andrew.cmu.edu}

Young-Hoon Park
Johnson Graduate School of Management, Cornell University, Ithaca, New York 14853, yp34@cornell.edu

Kannan Srinivasan
Tepper School of Business, Carnegie Mellon University, Pittsburgh, Pennsylvania 15206, kannans@cmu.edu

We study the bidding strategies of vertically differentiated firms that bid for sponsored search advertisement positions for a keyword at a search engine. We explicitly model how consumers navigate and click on sponsored links based on their knowledge and beliefs about firm qualities. Our model yields several interesting insights; a main counterintuitive result we focus on is the "position paradox." The paradox is that a superior firm may bid lower than an inferior firm and obtain a position below it, yet it still obtains more clicks than the inferior firm. Under a pay-per-impression mechanism, the inferior firm wants to be at the top where more consumers click on its link, whereas the superior firm is better off by placing its link at a lower position because it pays a smaller advertising fee, but some consumers will still reach it in search of the higher-quality firm. Under a pay-per-click mechanism, the inferior firm has an even stronger incentive to be at the top because now it only has to pay for the consumers who do not know the firms' reputations and, therefore, can bid more aggressively. Interestingly, as the quality premium for the superior firm increases, and/or if more consumers know the identity of the superior firm, the incentive for the inferior firm to be at the top may increase. Contrary to conventional belief, we find that the search engine may have the incentive to overweight the inferior firm's bid and strategically create the position paradox to increase overall clicks by consumers. To validate our model, we analyze a data set from a popular Korean search engine firm and find that (i) a large proportion of auction outcomes in the data show the position paradox, and (ii) sharp predictions from our model are validated in the data.

Key words: sponsored search advertising; search cost; vertical differentiation; bidding strategy; pay per impression; pay per click
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1. Introduction

Sponsored search advertising has grown into one of the major forms of online advertising in the past decade and is predicted to grow at an annual compound rate of more than 12% in the near future (Jupiter Research 2007). Firms—global and local, big and small—now actively advertise in the "sponsored links" sections of popular search engines such as Google, Yahoo!, and Bing in many countries; Yandex in Russia; Baidu in China; and Daum and Naver in Korea. When a consumer searches for a specific keyword on the search engine, she is presented with two lists of clickable links: one is a list of organic search results, and the other is a list of sponsored links. The list of sponsored links is determined by auctioning the positions to firms that want to advertise in response to the searched keyword. Advertised links are typically ordered in decreasing order of the firms' bids, and some search engines, such as Google, also augment bids by firm-specific weights.

The spectacular commercial success of sponsored search advertising has motivated several recent academic studies on it, which have significantly increased our understanding of the various phenomena operating in this advertising market. A main takeaway from the empirical work in this area is that advertisements at higher positions attract more clicks from consumers (e.g., Agarwal et al. 2009, Feng et al. 2007, Ghose and Yang 2009). Most theoretical papers focus on optimal bidding strategies of advertisers and, in accordance with the empirical observation above, assume that higher positions obtain more clicks (e.g., Edelman et al. 2007, Katona and Sarvary 2010, Varian 2007) and sometimes augment position-specific click-through rates with firm-specific quality scores. Consumers' click-through rates, however, are mostly assumed exogenously.

In this paper, we take a step forward by explicitly modeling how consumers navigate search results based on their knowledge and beliefs about the

qualities of vertically differentiated firms and their positions in the sponsored list. The links that a consumer clicks on are an outcome of this process, which in turn determines the bidding strategies of the firms. Our assumptions are in accordance with recent empirical work that finds that consumer navigation and click behavior depends significantly on the quality and order of competing firms and the keyword under consideration (Ghose and Yang 2009, Park and Park 2010, Rutz and Bucklin 2011, Rutz et al. 2011).

More specifically, we assume that there is a "superior" and an "inferior" firm (where the superior firm provides higher net valuation to consumers). A fraction of the consumers know the identity of the superior firm (the "informed" consumers); the rest do not (the "uninformed" consumers). The different types of consumers navigate and click on search results differently. In this framework, we obtain several results on the optimal bidding strategies of firms and the resulting orderings. Under certain conditions of consumer knowledge about firm qualities and consumer search cost (or "click cost"), we obtain novel results on the outcomes of the position auction, whereas under other conditions, our results are similar to those in the extant literature.

We first study the pay-per-impression mechanism (where all firms are charged whenever a consumer searches the keyword and their links are displayed). An intriguing finding revealed in our study is the "position paradox"—under certain conditions, a superior firm may bid lower than an inferior firm and obtain a position below it, but it still obtains more clicks than the inferior firm. Surprisingly, as the quality premium for the superior firm increases, the inferior firm is more likely to be at the top. Intuitively speaking, a superior firm may prefer a lower-ranked position over a higher-ranked one if it can receive only slightly fewer clicks at the lower-ranked position but can substantially reduce its cost there. An inferior firm, on the other hand, may want to take the higher-ranked position, even if it has to pay more while still receiving fewer clicks than the superior firm, because it would receive substantially fewer clicks at the lower-ranked positions. We show that these trade-offs can be reduced to an interaction among three basic effects—"residual demand," "incremental value," and "differential cost"—which we explain later.

Next, we examine the pay-per-click mechanism (where a firm is charged only when a consumer clicks on its link). Under this mechanism, the position paradox is strengthened; i.e., the inferior firm is even more likely to be at the top. Surprisingly, as the number of informed consumers increases, the position paradox gets strengthened further. We also find that as the subjective element of consumers' preferences becomes less important, the position paradox becomes more likely. Furthermore, we use a data set on sponsored search auctions from a popular Korean search engine firm and provide indirect empirical support for our theory by showing that (i) a large proportion of auction outcomes in the data show the position paradox, and (ii) sharp predictions from our analytical model regarding observable data patterns are validated.

Our base model assumes that consumers have homogeneous search cost and that uninformed consumers are boundedly rational. We later extend the base model to show that our key results hold in the case of heterogeneous search costs across consumers, as well as in the case of fully rational uninformed consumers. To further ensure robustness, we also show that the results hold under a "bid-weighting" mechanism used by some popular search engines. Finally, we discuss a potential alternative explanation for the position paradox arising because of the presence of the organic search results and show that our theory provides a more plausible explanation.

Prior theoretical work has largely ignored the position paradox or has exogenously assumed some situations similar to it in minor extensions, as in Edelman et al. (2007), Katona and Sarvary (2010), and Varian (2007). In contrast, we carefully study this important phenomenon by explicitly modeling consumer navigation behavior as to how consumers respond to the qualities and positions of advertisers in the sponsored search list. In other words, one of our main contributions is that rather than assuming consumer click behavior in a "reduced-form" way, we detail its "structural" specification and endogenously derive the equilibrium implications, one of which is the position paradox. Certain other papers also model the consumer side in position auctions. For instance, Athey and Ellison (2011) and Chen and He (2006) model consumer navigation but provide the conventional result that firms get rank ordered by decreasing qualities. After we develop our model and present our insights, we make a sharper comparison with these papers. Shin (2009) and Xu et al. (2009) model other aspects of consumer response such as reaction to price and behavioral aspects such as assimilation and contrast effects within the list of results.

A number of popular search engines use a bid-weighting mechanism by multiplying the participating firms' bids by firm-specific weights. Although search engines do not release details about how they construct their bid weights, the conventional belief is that these weights are positively correlated with firm qualities (Liu et al. 2010, Weber and Zheng 2007). We show that our main results hold if the search engine uses such a bid-weighting scheme. More interestingly, we show that the search engine might have the reverse incentive, i.e., attach a larger weight to

the inferior firm's bid to create the position paradox, which can generate overall greater clicks from consumers.

The nascent literature on sponsored search also covers various other aspects of the phenomenon. Chan and Park (2009), Goldfarb and Tucker (2011), and Yao and Mela (2011) empirically study bidding strategies of firms. Abhishek et al. (2009) show how to account for aggregation biases in data recording while empirically analyzing bidding strategies of firms. Zhu and Wilbur (2011) analyze hybrid auctions in which advertisers decide to bid on a pay-per-impression or pay-per-click basis.

The rest of this paper is organized as follows. In §2, we describe our analytical model, and in §3, we analyze it. In §4, we offer empirical support for our theory. In §5, we consider various extensions of our basic model and show its robustness. In §6, we compare our work to other closely related papers. In §7, we conclude with a discussion.

2. Model

We model two firms, S and I, competing for sponsored search advertisement positions for a specific keyword at a search engine firm. Firm S, the superior firm, has a higher-quality product than firm I, the inferior firm. The inferior firm provides a consumer a net product utility (i.e., the utility of the product minus its price) of $V > 0$, whereas the superior firm provides a consumer a net utility of $V + Q$. Here, $Q > 0$ represents the quality premium of firm S over firm I. The per-unit margin of firm i is denoted by m_i, $i \in \{S, I\}$, and the superior firm has a higher per-unit margin; i.e., $m_S > m_I > 0$. This assumption is only made to highlight the position paradox by making it more difficult to obtain, as we will elaborate later. We use the term "product" throughout the paper, but this could also be a "service." For example, a retailer could provide better product descriptions, a more secure online payment mechanism, better customer service, etc., than another retailer, even if both are selling products produced by other manufacturers.

A group of consumers, with mass normalized to 1, search the keyword in question at the search engine. In response, the search engine returns an ordered list of sponsored links. All consumers know that there are two firms in the market, one offering quality $V + Q$ and another offering quality V. However, all consumers do not know which firm offers which product. Specifically, there are two types of consumers. The first type is the informed consumers, who can tell whether a firm is the superior firm (offering quality $V + Q$) or the inferior firm (offering quality V) upon viewing its name or URL on its advertisement link in the search results. The second type is the uninformed consumers, who cannot tell a firm's quality from the advertisement link itself. These consumers have to search for this information by clicking on a firm's link and obtaining information about the product, e.g., find out the product specifications and price, read consumer reviews, etc. After this exercise, the uninformed consumers can also determine the quality of this firm. We assume that the size of the informed consumers is $\phi \in (0, 1)$, so the size of the uninformed consumers is $1 - \phi$. The parameter ϕ can be interpreted as a measure of how widespread the reputation of the superior firm is in the market. The assumption that some consumers know that the two firms offering different quality levels exist even if they do not know which is which can be better understood through an example. Consider a consumer who is shopping online. Through general research or communicating with friends, she may know that some stores charge \$3.95 for shipping, whereas other stores offer free shipping and second-day delivery. However, when searching online, she may not know which store offers which shipping feature, even though she knows that both types of stores exist.

However, knowing the qualities of firms alone is not sufficient for a consumer to make the purchase decision—she also has to assess her subjective "match" with a product. This match can only be assessed after clicking on a firm's link and obtaining information about the product. Hence, before purchasing a product, both informed and uninformed consumers have to click on a firm's link to assess their match with the product being offered. In any consumer's purchase decision, product quality can be interpreted as an objective dimension and match can be interpreted as a subjective dimension.

We assume that the informed consumers always start their search with the superior firm irrespective of its position and may go to the inferior firm if they do not obtain a match with the superior firm. For the uninformed consumers, we consider two settings. In the focal setting, we assume that they are boundedly rational (Simon 1955) and start their search with the firm at the top, which could be the superior or the inferior firm; based on the quality and the match, they may then choose to search further or stop.[1] Given that the population of consumers using the Internet is very diverse and heterogeneous, and considering the

[1] The bounded rationality of uninformed consumers can be interpreted in the following manner. They are trading off between two "costs": (1) the cost of thinking through the bidding strategies of the firms to figure out how they will be ordered in equilibrium, and (2) the expected cost of the search effort they have to expend in searching through the links of the firms using the heuristic of starting from the top and going downward. Given their bounded rationality, they find the first option to be more costly than the second option. Therefore, they choose to search rather than compute the equilibrium.

evidence from behavioral economics that shows that consumers often do not have perfect strategic foresight (Ho et al. 2006), the assumption that some consumers are boundedly rational is a reasonable one. Similar assumptions on bounded rationality of a fraction of consumers have been used in other analytical studies (e.g., Gabaix and Laibson 2006). The assumption that these consumers start searching from the top position is in accordance with the findings in the literature on how online consumers process ordered lists (e.g., Granka et al. 2004, Hoque and Lohse 1999).

The focal setting with bounded rationality is a simple one that helps us obtain our results in a parsimonious way. However, our main results *do not* depend on the assumption of bounded rationality. We illustrate this in §5.2 by considering a second setting in which uninformed consumers are fully rational but have incomplete information (specifically, they have beliefs on the type of the superior firm as being a high-margin or a low-margin type).

Consistent with the existing literature on search, we assume that the first search is free (this assumption does not qualitatively affect our results), and a search cost of $s > 0$ applies for subsequent searches. As several studies have shown, this search cost can be substantial and has a significant impact on how much an online consumer searches (Brynjolfsson et al. 2009, Johnson et al. 2004). We also assume that once a consumer has invested this cost for reading about a firm's offering, she can go back to this firm without incurring any further cost.[2] For simplicity, we assume that the match probability is the same for both firms and is equal to p. Assuming this to be different for different firms does not alter our results qualitatively. We assume that every consumer makes her purchase or subsequent search decisions to maximize her expected utility.

The search engine can auction the positions either through a pay-per-impression mechanism (in which both firms pay their respective fees whenever a consumer searches the keyword) or a pay-per-click mechanism (in which a firm pays only when its link is actually clicked following a keyword search). In both mechanisms, we assume that both firms submit their bids simultaneously. The firm that bids higher is placed on top, and the other is placed at the bottom. (Ties are broken randomly with equal probability.) The winning firm pays the amount of the losing firm's bid, and the losing firm pays the minimum bid $\underline{b} > 0$. Note that the assumption of one-shot simultaneous bidding is a deviation from the possibility of continuous asynchronous bidding allowed in auctions run by some popular search engines. As in previous theoretical work on position auctions, we make this assumption for simplicity. Later, we discuss that changing this assumption is not expected to change our basic insights.

We model the game in two stages. In the first stage, both firms submit bids and are ranked by the search engine. In the second stage, each consumer conducts her search. We use the concept of subgame perfect Nash equilibrium, and obtain a locally envy-free equilibrium (Edelman et al. 2007, Varian 2007) in which neither firm wants to exchange its position with the other firm.

Before we proceed to the analysis, we note that we do not model price competition (and V and $V + Q$ denote utilities of the products minus their respective prices). Advertising expenditure is only one of the many factors affecting price, and sponsored search advertising is only one of the many forms of advertising typically used by a firm—in this paper, we assume that a firm's sponsored search bidding strategy does not affect its pricing decision. This assumption is commonly invoked in the literature on position auctions. We also note that the insights we obtain from the model can also be obtained if there are more than two firms. To confirm this, we have conducted the analysis with three firms. The details of this analysis are available from the authors upon request.

3. Analysis

3.1. Pay-per-Impression Auction

In this section, we study the pay-per-impression case, in which a firm pays the advertisement fee whenever a consumer searches the keyword and the link is displayed. Depending on the values of the parameters s, p, m_S, m_I, V, and Q, multiple scenarios exist, each with different optimal search behavior by consumers and, therefore, different optimal bidding behavior by firms. Behind all these scenarios, however, are two key factors that drive the bidding behavior of firms.

The first factor is that of "residual demand," which intuitively means that the firm placed on top may not get all the sales. This can happen for two reasons. First, this arises from the uncertainty in finding a matched product because of the subjective component of consumer preferences. When a consumer goes to a firm's website, she finds a matched product only

[2] Alternatively, we could incorporate a fatigue-based second-visit cost $f > 0$ in the model. The second visit to a firm's website is expected to be of much smaller cost than the first visit because the consumer already has most of the information; i.e., f is expected to be significantly smaller than s. In this case, our insights remain unchanged. Moreover, the position paradox emerges even if f is large. In fact, our analysis (not presented here) shows that the position paradox emerges even if we assume that $f = s$ (i.e., the cost of the second visit to a firm's website is the same as the cost of the first visit to its website).

with probability p. If a match is not found, the consumer may search on. Second, a consumer may decide to search on if the first firm she visits is the inferior one, even if she already finds a match there, provided the quality premium, Q, is sufficiently high to outweigh her search cost. This factor makes it possible for a losing bidder to still make sales, which makes a sponsored search auction qualitatively different from a standard winner-take-all auction. The second factor, partly arising from the first, is that of "incremental value." Because both the winning and losing bidders may make positive sales, it is the difference in revenue between winning and losing that decides a firm's bid (not the absolute revenue from winning). Hence, a firm decides its bid based on the additional profit of being at the top position.

To start the analysis, consider a scenario in which $p\max\{Q, V\} < s \leq p(V + Q)$; i.e., the search cost is higher than both the expected quality premium and the expected utility of visiting the inferior firm, but it is lower than the expected utility of visiting the superior firm. In this case, if a consumer finds a match at the first firm she visits, she will buy the product and stop. If she does not find a match there, then if the other firm is the superior firm, she will search on, but if the other firm is the inferior one, she will stop.

Consider an uninformed consumer. Being uninformed, she will still start from the top link and, after clicking on it, will be able to identify it as the superior or the inferior firm, and determine whether or not she finds a match. If the superior firm has the top link and the consumer finds a match there, she will buy and stop. Moreover, she will stop even if she does not find a match, as explained above. If the top link belongs to the inferior firm, the consumer will stop searching if she finds a match but will continue searching if she does not find a match. An informed consumer, in contrast, will always start from the superior firm. If she finds a match, she will purchase and stop, and even if she does not find a match, she will still stop.

We denote the profit of firm $i \in \{S, I\}$ when it is placed at position $j \in \{1, 2\}$ by $\Pi_{i,j}$.[3] Given the expected search behavior of consumers, if the superior firm is placed on top and pays b_S per search, the expected profits for the two firms are given by

$$E[\Pi_{S,1}] = pm_S - b_S \quad \text{and} \quad E[\Pi_{I,2}] = 0 - \underline{b}.^{4}$$

If the inferior firm is placed on top and pays b_I per search, the expected profits for the two firms are given by

$$E[\Pi_{S,2}] = \phi pm_S + (1-\phi)(1-p)pm_S - \underline{b} \quad \text{and}$$
$$E[\Pi_{I,1}] = (1-\phi)pm_I - b_I.$$

Note that in these expressions, b_S and b_I denote the payment per search for firm S and firm I, respectively, and *not* their equilibrium bids. These equilibrium bids, denoted by b_S^* and b_I^*, will be derived subsequently.

We can observe from the above that the superior firm obtains some clicks even if it is at the second position. This is a direct outcome of the effect of residual demand—the search cost is such that a consumer who does not find a match at the inferior firm will continue searching for the superior firm. However, the search cost is high enough that the inferior firm obtains no clicks if it is in the second position. Consequently, the effect of incremental value suggests that the inferior firm will compete more for the top position under certain conditions. We see from the derivations below for the firms' bids that this is indeed the case.

We can derive the equilibrium bids for each firm for position 1 by noting that a firm in position 2 pays $\underline{b}$ and will bid an amount such that if it indeed gets the top position and has to pay this amount, its profit should be equal to its profit at the bottom position. Intuitively, a firm will bid an amount equal to the additional benefit of the top position over the bottom one. These bids will be Nash equilibrium bids and also characterize exactly the locally envy-free equilibrium defined in Edelman et al. (2007). We also note that this is a weakly dominant strategy for both firms.[5]

In accordance with this, firm S will be willing to pay up to b_S^* for position 1, where b_S^* equates its profit from positions 1 and 2; i.e.,

$$pm_S - b_S^* = \phi pm_S + (1-\phi)(1-p)pm_S - \underline{b}$$
$$\Longrightarrow \quad b_S^* = (1-\phi)p^2 m_S + \underline{b}.$$

[3] Throughout the paper, we use "position 1," "above," and "top" interchangeably with each other and "position 2," "below," and "bottom" interchangeably with each other.

[4] Note that this profit is negative. However, we have assumed that firms do not exit the auction even if they make negative profit. This is a purely technical assumption that keeps the analysis cleaner; it has no effect on the insights from the model.

[5] As discussed in Edelman et al. (2007) and Varian (2007), a range of bids can sustain an equilibrium in a position auction. From this range, we consider bids that conform to the following rule: every firm sets its bid at the highest level such that if this bid helps it move up in the ranking, it will make a profit. This is the rule that both these papers also prefer. Our insights hold under other rules for bids (e.g., every firm sets its bid at the highest level such that the firm above it is not induced to move down by this bid; i.e., the equilibrium is not disturbed). Furthermore, even with multiple players, there is no ambiguity in computing the equilibrium once we decide which rule is used to calculate the equilibrium bids.

Similarly, firm I will be willing to pay up to b_I^* for position 1, where b_I^* equates its profit from positions 1 and 2; i.e.,

$$(1-\phi)pm_I - b_I^* = 0 - \underline{b} \implies b_I^* = (1-\phi)pm_I + \underline{b}.$$

In this scenario, both firms value the top position more than they value the bottom one. If the margin of the superior firm is significantly higher than that of the inferior firm, i.e., $m_S > m_I/p$, then the superior firm will bid higher and be placed on top. Otherwise, the inferior firm will be placed on top. Furthermore, if the inferior firm is on top, it will obtain more clicks only if $\phi < p/(1+p)$; otherwise, the superior firm will obtain more clicks. Therefore, in this scenario, if $m_S \le m_I/p$ and $\phi \ge p/(1+p)$, then we obtain the position paradox.

The position paradox can occur for other parameter ranges as well. For completeness, we conduct the analysis for the full parameter space and summarize the results in Table 1. Note that we have analyzed in detail above Scenario IV from Table 1. The detailed analysis of the other scenarios is available in §A.1 of the appendix. These scenarios can be summarized in the following way. In Scenario I, both firms bid the same amount, and the firms are ranked randomly with equal probability by the search engine. If the superior firm is placed below, all consumers click on its link, but only a $(1-p)$ fraction of consumers click on the inferior firm's link. In Scenario II, the superior firm always values both positions equally, whereas the inferior firm always values the top position more. In equilibrium, the inferior firm will bid higher to be placed on top but obtain fewer clicks. Therefore, we can obtain the position paradox in Scenarios I and II as well. Note that even if there are no informed consumers, the superior firm may still want to be at the bottom position, although this holds in a smaller parameter region because it will receive fewer clicks. In Scenarios III and V, the superior firm always outbids the inferior firm and is placed at the top (i.e., we do not see the position paradox). The above results give us the following proposition.

Table 1 Equilibrium Bids Under the Pay-per-Impression Mechanism

Scenario	Firm S bid Firm I bid
I: $s \le p\min\{Q, V\}$	$\underline{b}$ $\underline{b}$
II: $p\min\{Q, V\} < s \le p\max\{Q, V\}$ and $Q > V$	$\underline{b}$ $(1-\phi)(1-p)pm_I + \underline{b}$
III: $p\min\{Q, V\} < s \le p\max\{Q, V\}$ and $Q < V$	$(1-\phi)p^2m_S + \underline{b}$ $(1-\phi)p^2m_I + \underline{b}$
IV: $p\max\{Q, V\} < s \le p(V+Q)$	$(1-\phi)p^2m_S + \underline{b}$ $(1-\phi)pm_I + \underline{b}$
V: $s > p(V+Q)$	$(1-\phi)pm_S + \underline{b}$ $(1-\phi)pm_I + \underline{b}$

Proposition 1. *In the pay-per-impression auction, we observe the position paradox if*

- $s \le p\min\{Q, V\}$ (*with probability* 1/2), *or*
- $pV < s \le pQ$, *or*
- $p\max\{Q, V\} < s \le p(V+Q)$, $m_S \le m_I/p$, *and* $\phi \ge p/(1+p)$.

Two more points are worth noting. First, as we can see from the equilibrium bids in each of the five scenarios, the size of the fraction of informed consumers has no effect on which firm will be the winner (although it does affect the bids). This may not be obvious at first, as informed consumers will always click on the superior firm first; so the more such consumers, the better it is for the superior firm. However, recall that one key intuition is the incremental demand. Because the informed consumers do not change their click behavior in response to different link positioning, it is only the uninformed consumers that the two firms are competing for. Furthermore, for pay-per-impression, both firms will pay for both types of consumers anyway, regardless of whether and which links the consumers click. Therefore, although the size of the informed consumers changes the expected values of the top position to both firms, it changes them in a proportional manner that has no bearing on which firm wins the auction.

The second point is that a larger quality premium does not make the superior firm more likely to be the winning bidder. Quite the contrary, comparing Scenarios II and III shows that a larger quality premium may, in fact, make the inferior firm more likely to win the auction. Although surprising at first look, this can again be understood from the key intuitions of incremental value and residual demand—the larger the quality premium, the more likely that the superior firm will be searched even if it is placed at the bottom. This reduces the equilibrium bid of the superior firm, therefore making the inferior firm more competitive. We state this result in the following proposition.

Proposition 2. *In the pay-per-impression auction, under certain conditions, a larger quality premium for the superior firm makes the inferior firm more likely to win the auction.*

This analysis shows that in Scenarios III, IV, and V, the per-unit margins of the firms influence which firm wins the auction. The reader may recall that we have made an exogenous assumption that $m_S > m_I$. If we relax this assumption, it will only become more likely for the inferior firm to be placed on top. (Intuitively, if $m_S < m_I$, then the inferior firm will have the characteristic of the high-valuation bidder, given its higher per-unit profit.) Therefore, the assumption $m_S > m_I$ serves to *raise* the bar for the inferior firm to win the auction.

This highlights the effects of consumer navigation and click behavior discussed previously by showing that they can bring about the position paradox even in the case where other factors (such as margin) work in the opposite direction.

3.2. Pay-per-Click Auction

We now analyze the case of the pay-per-click auction mechanism, where a firm pays the advertisement fee only when its link is clicked. We consider the setting with boundedly rational uninformed consumers. The two key factors discussed in the pay-per-impression case—residual demand and incremental value—continue to apply here. In addition, there is another key factor that changes the outcome of the auction compared to the pay-per-impression case and, in general, makes the superior firm even more likely to end up at the bottom position.

We call this effect the "differential cost" effect. Both firms know that some consumers are informed, and these consumers will start with the superior firm no matter how links are positioned. In the pay-per-click case, the inferior firm will pay for these consumers only if they actually click on its link; i.e., it will pay for them selectively. This is different from the pay-per-impression case, in which both firms pay as long as the links are displayed; i.e., both firms pay for searches by every consumer. In other words, under pay per click, the inferior firm will not unnecessarily pay for the informed consumers who never click on it. This reduces the expected cost per search of the inferior firm, thereby increasing its bidding capacity. In contrast, the superior firm will see the added fee (the additional amount needed to be at the top) paid for these consumers as pure waste, because these consumers will click on the firm's link anyway. Hence, this effect, on the margin, can increase the relative bidding power of the inferior firm.

Similar to the pay-per-impression case, there are five scenarios depending on the values of parameters s, p, m_S, m_I, V, and Q, each resulting in different optimal bids and link positions. In each scenario, the search behavior of consumers and the expected revenues of firms when either is placed on top are exactly the same as they are in the corresponding scenario in the pay-per-impression case. The only difference is on the cost side—in pay per click, a firm pays its bid weighted by the probability of click (which is ≤ 1), whereas in pay per impression, the firm pays its bid for the full mass of all consumers who search (which is equal to 1). As mentioned above, the inferior firm can bid more aggressively in a pay-per-click auction. The result is that the position paradox occurs for a wider range of the parameter values. In the following, we highlight the basic insights by discussing Scenario IV in detail and summarize the results for the other scenarios. We provide the full analysis in §A.2 of the appendix.

Table 2 Equilibrium Bids Under the Pay-per-Click Mechanism

Scenario	Firm S bid Firm I bid
I: $s \leq p\min\{Q, V\}$	$\underline{b}$ $\underline{b}$
II: $p\min\{Q, V\} < s \leq p\max\{Q, V\}$ and $Q > V$	$\underline{b}$ $(1-p)pm_I$
III: $p\min\{Q, V\} < s \leq p\max\{Q, V\}$ and $Q < V$	$(1-\phi)p^2m_S + (1-(1-\phi)p)\,\underline{b}$ $((1-\phi)p^2m_I + (1-p)\,\underline{b})/(1-\phi p)$
IV: $p\max\{Q, V\} < s \leq p(V+Q)$	$(1-\phi)p^2m_S + (1-(1-\phi)p)\,\underline{b}$ pm_I
V: $s > p(V+Q)$	$(1-\phi)pm_S + \phi\,\underline{b}$ pm_I

Given the position, the expected revenues in the pay-per-click auction are as in the pay-per-impression auction, so we now look at the expected cost. As explained in §3.1, if the superior firm is placed on top, the probability that it will be clicked is 1; if it is placed at the bottom, this probability is $\phi + (1-\phi)(1-p)$. If the inferior firm is placed on top, its probability of being clicked is $1-\phi$; if it is placed at the bottom, this probability is 0.

If the superior firm pays b_S per click and is placed on top (so the inferior firm pays $\underline{b}$ per click), then the expected profits of the firms are $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0$. If the inferior firm pays b_I per click and is placed on top (so the superior firm pays $\underline{b}$ per click), then the expected profits of the firms are $E[\Pi_{S,2}] = (\phi + (1-\phi)(1-p))(pm_S - \underline{b})$ and $E[\Pi_{I,1}] = (1-\phi)(pm_I - b_I)$. The equilibrium bids can be obtained as follows:

$$pm_S - b_S^* = (\phi + (1-\phi)(1-p))(pm_S - \underline{b})$$
$$\Longrightarrow \quad b_S^* = (1-\phi)p^2m_S + (1-(1-\phi)p)\,\underline{b} \quad \text{and}$$
$$(1-\phi)(pm_I - b_I) = 0 \quad \Longrightarrow \quad b_I^* = pm_I.$$

If $(1-\phi)p^2m_S + (1-(1-\phi)p)\,\underline{b} > pm_I$, then the superior firm wins the auction; otherwise, the inferior firm wins the auction. If the inferior firm wins, it will obtain more clicks than the superior firm only if $\phi < p/(1+p)$; otherwise, the superior firm will obtain more clicks even though it is at the bottom position, and we obtain the position paradox.

We summarize the results for the other scenarios in Table 2. (The details are available in §A.2 of the appendix.) Interestingly, because of the new differential cost effect in the pay-per-click auction, the position paradox can occur in *all five* scenarios. These results give us the following proposition.

Proposition 3. *In the pay-per-click auction, we observe the position paradox if*

- $s \leq p\min\{Q, V\}$ (*with probability* 1/2), *or*
- $pV < s \leq pQ$ *and* $p(1-p)m_I \geq \underline{b}$, *or*

- $pQ < s \le pV$, $(1-\phi)p(m_I - (1-\phi p)m_S) \ge \phi \underline{b}$, *and* $\phi \ge 1/2$, *or*
- $p\max\{Q, V\} < s \le p(V+Q)$, $pm_I \ge (1-\phi)p^2 m_S + (1-(1-\phi)p)\underline{b}$, *and* $\phi \ge p/(1+p)$, *or*
- $s > p(V+Q)$, $pm_I \ge (1-\phi)pm_S + \phi\underline{b}$, *and* $\phi \ge 1/2$.

Furthermore, as we have already discussed above, as the number of informed consumers increases, the inferior firm bids higher and is more likely to win the position auction. This gives us a surprising result—the more widespread the reputation of the superior firm, the more likely it is that the inferior firm will win the position auction. We state this result in the following proposition.

Proposition 4. *In the pay-per-click auction, as the fraction of informed consumers in the population increases, the inferior firm is more likely to win the auction.*

Furthermore, the subjective match probability in our model, p, can be considered as representing how "picky" consumers are for the product category. A smaller p implies that there is a smaller chance of finding a subjective match; i.e., consumers are pickier. For instance, this subjective element may be more important when purchasing a pair of sunglasses than when purchasing a webcam, so a keyword related to sunglasses will have a smaller match probability p than a keyword related to webcams. Viewed from this angle, a higher p (less picky) implies higher expected utility from continued search, which in turn implies greater consumer search. Therefore, broadly speaking, if the subjective match element in a category is less important and consumers are less picky, the position paradox can be more likely in both auction mechanisms.

4. Empirical Support

To empirically validate our model's predictions, we obtained a database of sponsored search advertisements from a leading search engine firm in Korea. For a given keyword, the search engine uses a pay-per-impression position auction to sell up to five different advertising positions in the sponsored list to potential advertisers. For a given keyword, the data consist of the daily positions of the advertisers and the corresponding daily impressions (i.e., the number of times the keyword was searched) and click counts at each position over a 15-day period in July 2008. Our data set is unique in that we have click counts at all positions for each keyword, whereas most previous empirical studies have click counts only for one advertiser.

We have the exact URL of each advertiser when the keyword is searched. However, we do not have data on quality scores of advertisers. Therefore, we decided to use a firm's reputation to impute the quality of a given Web link. We hired three independent annotators from the United States and Korea to assess, for each keyword, whether each advertiser is a high-quality or a low-quality firm. Among the 246 keywords provided for this research, we excluded keywords if annotators could not recognize the firms by their names, and we only considered keywords for which annotators could confidently classify the qualities of the firms. We also excluded keywords when only one advertiser was displayed during the data period. We then computed the proportion of agreements on the quality of the firms across keywords between the annotators. The interrater reliability score ranged from 0.90 to 0.94, indicating a very high level of reliability. From the high-quality firms in a given keyword search, we selected the best-ranked firm among them as the superior firm. Likewise, from the low-quality firms, we selected the best-ranked firm among the low-quality firms as the inferior firm. (Selecting the worst-ranked firm among the low-quality firms as the inferior firm gives qualitatively similar results.) Finally, we ended up with a total of 102 keywords and categorized each into one of the following three different configurations based on the average number of clicks over the data period:

Configuration 1 (C1). The superior firm is above the inferior firm, and the superior firm obtains more clicks.

Configuration 2 (C2). The inferior firm is above the superior firm, and the inferior firm obtains more clicks.

Configuration 3 (C3). The inferior firm is above the superior firm, but the superior firm obtains more clicks.

We find that all three of these configurations have significant representation in our data set. Of the 102 keywords, we find 48, 25, and 27 keywords in C1, C2, and C3, respectively.[6, 7] Table 3 shows the daily average number of clicks corresponding to the three different configurations. The extant literature focuses on C1 as an equilibrium configuration, whereas our theoretical model predicts that C2 and C3 can also arise in equilibrium. We observe that C2 and C3 occur in 25% and 27% of all cases, respectively, and this offers direct confirmation of our results. Note that the data from

[6] We note two points here: (1) Our database also contained the case in which the superior firm is above the inferior firm and obtains fewer clicks, but this was only the case for two keywords. (2) The classification of each keyword into one of the different configurations is very stable, primarily because advertisers and their ranks do not change significantly during the 15-day data period.

[7] Animesh et al. (2010) empirically document situations in which low-quality firms are placed above high-quality firms in sponsored search results and implicitly assume in their study that firms at the top must be obtaining more clicks.

Table 3 Daily Average Number of Clicks for the Different Configurations

	C1	C2	C3	Total
No. of keywords	48	25	27	100
Average no. of clicks				
High-quality firm	16.80	4.18	13.34	12.68
Low-quality firm	6.37	11.58	5.42	7.42

a pay-per-impression auction provide a conservative test of the position paradox because, as shown by our analytical model, it is less likely to occur under the pay-per-impression auction compared with the pay-per-click auction.

After convincingly establishing the existence of the position paradox outcome, we now use our analytical model to derive two sharp predictions related to click-through rates (CTRs) in configurations C1, C2, and C3, and then we test them on our data. Because our data are from a pay-per-impression auction, we draw on our results in §3.1. Our theory based on consumer search cost provides specific predictions regarding which configuration (C1, C2, and/or C3) can occur under which search cost scenario (Scenarios I–V). Note that the search cost s increases from Scenario I to Scenario V. As Table 4, panel a, shows, C1 can occur under Scenarios I, III, IV, and V; C2, only under Scenario IV; and C3, under Scenarios I, II, and IV. Furthermore, each keyword in our data is categorized into one of the three configurations C1, C2, or C3. Assuming that different keywords (corresponding to different products/categories) have different search costs for consumers, and given that higher search cost will lead to fewer clicks on the sponsored list, Table 4, panel a, provides the following two predictions:[8]

Prediction 1 (P1). Average CTRs across all keywords categorized into C2 should be smaller than the average CTRs across all keywords categorized into C1 and C3.

Prediction 2 (P2). The dispersion in CTRs across all keywords categorized into C2 should be smaller than the dispersion in CTRs across all keywords categorized into C1 and C3.

The reasoning behind P1 is that C2 can occur only in a high search cost scenario, whereas C1 and C3 can occur in low search cost scenarios as well. The reasoning behind P2 is that C2 can occur in only one search cost scenario, whereas C1 and C3 can occur in multiple search cost scenarios.

To test these predictions, we calculate the average CTRs and the standard deviation (dispersion) in CTRs across all keywords categorized into each configuration. The values are reported in Table 4, panel b. From

[8] We thank an anonymous reviewer for suggesting this approach of developing testable predictions based on search cost.

Table 4 Development and Testing of Predictions

	C1	C2	C3
(a) Search cost scenarios under which each configuration can occur under the pay-per-impression mechanism			
Scenario I	✓		✓
Scenario II			✓
Scenario III	✓		
Scenario IV	✓	✓	✓
Scenario V	✓		
(b) Statistics across all keywords classified into each configuration			
Avg. CTRs	0.10	0.05	0.09
Std. dev. of CTRs	0.13	0.07	0.13

the first row, we can see that P1 regarding C2 having the lowest average CTR holds. From the second row, we can see that P2 regarding C2 having the least dispersion in CTRs holds. Furthermore, from Table 4, panel a, we can also conjecture that the mean and standard deviation numbers for C1 and C3 should be similar, which, as shown in Table 4, panel b, is indeed the case.

To summarize, our empirical analysis offers support for our theory by showing that (i) a large proportion of auction outcomes in the data show the position paradox, and (ii) two sharp predictions from our analytical model are validated in the data. We note that this empirical support is indirect in nature, because it is based on observing data patterns implied by the model rather than testing direct predictions for different keywords (e.g., whether the position paradox will occur for a keyword or not). To test the theory directly, we need to know the search cost for each keyword as well as the quality levels of each firm. Quantifying search cost and quality using observed click data, however, is an interesting and substantial empirical question in its own right, which we leave for future work.

5. Model Extensions and Robustness

5.1. Heterogeneous Search Costs

In the basic model, we assumed that all consumers have the same search cost s. However, our insights also hold if consumers are differentiated in their search costs, and firms bid to maximize their expected profits across all consumers. We consider such a model in §TA1 in the electronic companion, available as part of the online version that can be found at http://mktsci.pubs.informs.org/, and we briefly state the results here. In a pay-per-impression auction, we find that the position paradox can arise, and as before, it is more likely as the quality premium Q offered by the superior firm increases. In a pay-per-click auction, we again find that the position paradox can arise, and as before, it

is more likely as the number of informed consumers in the market increases. A new result we find is that, in both auctions, the position paradox is more likely as the base level of quality V in the market increases. Intuitively, higher V motivates a larger fraction of consumers to continue searching, which reduces the incentive of the superior firm to bid high to be placed on top.

5.2. Fully Rational Uninformed Consumers

In the basic model, we assumed that uninformed consumers are boundedly rational and always start from the first link. While we believe it is a reasonable assumption given empirical observations, and it helps highlight our insights in a parsimonious way, this assumption is not required for our results. In this section, we show by using a simple extension of the model that the position paradox can occur even if uninformed consumers are fully rational (and, therefore, can correctly figure out how the two firms will bid and how they will be ranked).

We extend the model to a game of incomplete information by allowing the superior firm to come from one of two possible "types." The first type, which we call the "high-margin" type of the superior firm, has a margin of m_{SH}, whereas the second type, the "low-margin" type, has a margin of m_{SL}. We assume $m_{SL} < m_I/p < m_{SH}$. We assume that the superior firm knows its true type and the inferior firm also knows the true type of the superior firm before bidding, but the consumers do not. Instead, consumers have a probabilistic prior belief on the superior firm's type, where they assign a prior probability $\gamma \in (0, 1)$ that the superior firm is of the high-margin type.

We illustrate how the position paradox can arise by considering a pay-per-impression auction for Scenario IV; i.e., $p\max\{Q, V\} < s \le p(V+Q)$. If we fix the type of the superior firm, then as we have proven previously, the high-margin superior firm will bid $b^*_{SH} = (1-\phi)p^2 m_{SH} + \underline{b}$, the low-margin superior firm will bid $b^*_{SL} = (1-\phi)p^2 m_{SL} + \underline{b}$, and the inferior firm will bid $b^*_I = (1-\phi)pm_I + \underline{b}$ regardless of which type of superior firm it is competing against. The result is that the inferior firm will be on top if it is competing against a low-margin superior firm, and it will be at the bottom if it is competing against the high-margin superior firm.

When this margin information is unknown to the consumers, however, the prior belief of the uninformed consumers becomes important (the prior belief of informed consumers does not matter, as they recognize the superior firm and always start searching from the superior firm). Intuitively, if these consumers believe that the superior firm is more likely of the high-margin type, i.e., $\gamma > 0.5$, then they would believe that the superior firm is more likely to be on top and would start searching from the top link.

More formally, in the game setup as above, if $\gamma > 0.5$, then the following strategy profile constitutes a perfect Bayesian Nash equilibrium:

- The informed consumers always start from the superior firm and stop whether or not a match is found. The uninformed consumers always start from the top link. If it is the superior firm, the consumers stop whether or not a match is found. If the firm at the top is the inferior firm, then the consumers stop if a match is found and proceed to the other firm if a match is not found.
- The high-margin-type superior firm bids $b^*_{SH} = (1-\phi)p^2 m_{SH} + \underline{b}$. The low-margin-type superior firm bids $b^*_{SL} = (1-\phi)p^2 m_{SL} + \underline{b}$. The inferior firm bids $b^*_I = (1-\phi)pm_I + \underline{b}$.

The proof is straightforward. We have proven earlier that both (b^*_{SH}, b^*_I) and (b^*_{SL}, b^*_I) are equilibrium strategies in their respective compelete information cases when the uninformed consumers start from the top. The strategy of the uninformed consumers to start from the top is therefore optimal given their prior $\gamma > 0.5$. This implies that the superior firm will obtain $\phi + (1-\phi)(1-p)$ clicks, whereas the inferior firm will obtain $(1-\phi)$ clicks.

This result shows that even when uninformed consumers are fully rational but are uninformed about the identity and type of the superior firm, the position paradox can occur. This is because the low-margin-type firm bids lower than the inferior firm, resulting in the configuration in which the inferior firm is placed on top. Furthermore, if $\phi \ge p/(1+p)$, the superior firm obtains more clicks as well. This is not surprising, as the underlying drivers of the phenomenon, residual demand and incremental value, remain the same whether consumers are boundedly or fully rational.

5.3. Bid Weighting

To improve revenue yield, popular search engines such as Google and Yahoo! augment each firm's bid by a certain firm-specific weight and rank the firms based on the weighted bid. Although in our model we have not treated the search engine as strategic, in this section we briefly consider the implications of strategic bid weighting.

A major hindrance in modeling bid weighting realistically is that search engines do not publicly declare how they determine firm-specific weights. However, the conventional belief is that firm-specific weights must be positively correlated with firm qualities, which would allow a high-quality firm to be ranked higher even with a slightly lower bid than a low-quality firm. The intuition is that a higher-quality firm can generate more clicks and should be favored for better positions. It is straightforward to see that in our model, the position paradox can still occur under such

weighting, as long as the relative weight attached to the superior firm's bid is not too large. We offer a more formal argument in §TA2 of the electronic companion.

Surprisingly, our model also shows that, depending on the situation, a search engine may want to give higher weight to the *inferior* firm's bid. Because a consumer may continue to search if she does not find a matching product at the higher-ranked firm, and the superior firm's product makes it more worthwhile to continue searching, the search engine may benefit from intentionally placing the inferior firm above the superior one, which could increase the overall number of clicks.[9]

We illustrate this by using a numerical example of a pay-per-click auction. First, we update the ranking and payment rule to accommodate bid weighting: an inferior firm's bid will be multiplied with a weight w, which can be either larger or smaller than 1, and the winning firm will pay the minimum amount required to stay in that position (which is the payment rule followed by popular search engines that practice bid weighting). Therefore, if the superior firm bids b_S and the inferior firm bids b_I, then if $b_S > wb_I$, the superior firm is placed on top and pays wb_I per click, but if $b_S < wb_I$, then the inferior firm is placed on top and pays b_S/w per click. This bid weighting does not change the impact of the three factors of resudial demand, incremental value, and differential cost, so the equilibrium bids remain the same. Next, we assume that $\phi = 0.5$, $p = 0.5$, $\underline{b} = 1$, $m_I = 2$, and $m_S = 4$, and we assume that the values of V, Q, and s put the situation into Scenario IV; i.e., $p\max\{Q, V\} < s \leq p(V+Q)$. In this case, the superior firm will bid $b_S^* = (1-\phi)p^2 m_S + (\phi + (1-\phi)(1-p))\underline{b} = 1.25$, and the inferior firm will bid $b_I^* = pm_I = 1$.

Without bid weighting, the superior firm wins the auction and is placed at the top. Both types of consumers will start from the top and, regardless of the result, will not move on to the inferior firm, as the search cost outweighs the expected utility of the inferior firm. Therefore, the revenue of the search engine is $R = 1 \cdot 1 = 1$, which is the probability of the superior firm being clicked times the per-click revenue of 1 (the bid of the inferior firm).

However, if the search engine adopts bid weighting and sets $w = 1.3$, then the inferior firm will win and be placed on top. For every click, the inferior firm will have to pay 1.25/1.3 to maintain its position, whereas the superior firm will pay the minimum bid of 1. An uninformed consumer will start from the inferior firm and will proceed to the superior firm if a match is not found. This brings revenue $1.25/1.3 + 0.5 \cdot 1$ per uninformed consumer to the search engine. An informed consumer will click directly on the superior firm, bringing a revenue of 1 to the search engine. Since $\phi = 0.5$, the total revenue of the search engine is $R = 0.5(1.25/1.3 + 0.5 \cdot 1) + 0.5 \cdot 1 = 1.23$, which is higher than the revenue without bid weighting. (Actually, setting w arbitrarily close to 1.25 from above will increase revenue further in this case, but we just use this for illustration instead of performing rigorous optimization.) In this example, the key insight is that placing the inferior firm on top will generate an expected 1.5 clicks per uninformed consumer, more than the one click if the superior firm is placed on top. Interestingly, this shows that sometimes it may be in the search engine's interest to use bid weighting to intentionally create a position paradox.

5.4. Impact of the Organic Listing

In response to a keyword search, a search engine returns two sets of results: the organic list and the sponsored list (if there are any bidders). This can provide an alternative explanation for the position paradox, which is as follows. A superior firm is more likely to be ranked higher than an inferior firm in the organic list by virtue of its higher quality. Because consumers are likely to view and navigate the two lists jointly (Yang and Ghose 2010), a superior firm should be able to obtain more clicks by consumers through its higher position on the organic list. Therefore, it may not have the incentive to pay a large amount to be placed above the inferior firm on the sponsored list, which explains the position paradox.[10] Theoretically, the above explanation is a valid one. However, it is based on the key assumption that there is significant overlap between the organic and sponsored results at a typical search engine. We investigate this issue in detail and find weak support for this assumption, which undermines the plausibility of this alternative explanation.

We have already shown that the position paradox outcome is prominent in the Korean data. We consulted our data provider, and they informed us that their organic listings are generated from a proprietary data set consisting of data from blogs, cafés (i.e., online communities run by the portal associated with the search engine), and a knowledge database where online users post questions and other users provide answers. Importantly, while collecting this data set for organic search results, they explicitly avoid the websites of commercial sellers (i.e., advertisers for sponsored results). Therefore, they can guarantee "close-to-zero" overlap between the organic and the sponsored listings (for which commercial sellers bid). In other words, for each of the 100 keywords in our Korean data set, we can be assured that

[9] We thank the associate editor for suggesting this intuition.

[10] We thank the editor and the associate editor for raising this key criticism.

there is almost no overlap between the two sets of search results. Therefore, the alternative theory cannot explain the position paradox occuring in these data.

Next, we tested for overlap between organic and sponsored results at the popular search engine Google. To do this, we collected category names for 402 product categories on the website http://www.consumersearch.com and used these category names as search phrases on Google. These are commercial category names, and each has many sellers bidding for the sponsored listing. For example, some of these keywords are "Inkjet printers," "Best wine under $12," and "Auto GPS." We crawled both the sponsored results and the organic results for these 402 keywords and calculated the overlap in sellers between these lists. If two different links led to different pages in the same Web domain, we still considered it an overlap.[11]

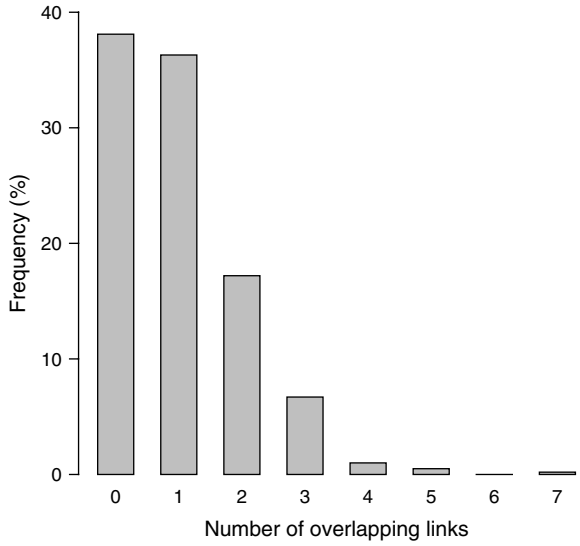
Interestingly, we found that this overlap is minimal—if we consider the first page of search results, approximately 40% of these keywords have no overlap at all between the two lists, and approximately 40% have only one common link. The histogram of the overlapping links is shown in Figure 1. (The maximum possible overlap can be about 10 links on average.) The histogram remains similar if we consider results beyond the first page, if we crawl the results on a different day, or if we use a different search engine. We also note that the nonoverlapping organic links primarily refer to information websites such as Wikipedia.com, expert and consumer review websites, blogs, etc., instead of sellers, much like in the Korean data.[12]

This analysis offers strong indication that the alternative explanation—that the position paradox might arise because the superior firm can rely on the free organic results for clicks—has weak support. On the other hand, we show in §4 that two sharp predictions from our analytical model are validated in the Korean data, which supports our theory. Furthermore, the observation that organic results primarily link to information provision websites suggests that, for some keywords, uninformed consumers may get an indication of firm qualities from the organic results. This might lead to an increase in the proportion of informed consumers, which, as per Proposition 4, could even strengthen the position paradox.

[11] The full list of keywords with overlap statistics is provided in §TA3 of the electronic companion. Note that we use all category names listed on the website, and there is no bias in keyword selection.

[12] We also used English translations of the 100 keywords in our Korean data and crawled the search results on Google. As above, we found a very small overlap between the sponsored and organic results.

Figure 1 **Histogram of Overlap Between Organic and Sponsored Results**



6. Comparison with Related Papers

Previous theoretical work has significantly enhanced our understanding of sponsored search advertising. Our results differ from the assumptions and results in some closely related papers in subtle but important ways. Edelman et al. (2007), Katona and Sarvary (2010), and Varian (2007) make exogenous assumptions about CTRs without modeling consumer search. These papers use "quality score" or "inherent attractiveness" parameters associated with firms to increase their CTRs, which implies exogenously assuming that a lower link can obtain more clicks than an upper link, not endogenously deriving or predicting this phenomenon. In other words, the assumptions in these papers do not endogenize the CTRs because the effect of firm attractiveness on clicks remains the same irrespective of its position in the list.

In contrast, we explicitly model how consumers navigate and click on search results based on their knowledge and beliefs about firm qualities. The fundamental insight that our analysis brings out, which cannot be obtained if CTRs are exogenously assumed, is that the value of a position to a firm cannot be determined in isolation. We argue that the effect of firm quality or attractiveness on a firm's CTR should depend on its "competitive environment," which is determined endogenously through the firms' strategic interactions. For example, the value of a lower link to a firm should very well depend on which firm is placed above it. If the firm placed above offers limited products at unattractive prices, the firm at the link below will obtain more clicks compared with the case in which the first firm offers a great product variety at a cheap price. Therefore, although the quality differential between firms is exogenous in our model,

its impact on CTRs depends on the configuration of the list.

Second, the notion of the superior firm is very different between the above-mentioned papers and our paper. In the basic model of Edelman et al. (2007), each firm is described by a "value per click" (VPC), and firms order themselves by decreasing VPC. In their extended model, each firm is described using two dimensions: a "quality score" (QS) and a VPC. In this case, firms order themselves by decreasing QS $\times$ VPC. This means that if a firm is placed below another firm, then either its QS or its VPC or both must be lower. Varian (2007) has almost exactly the same result. In our case, however, the superior firm that is placed lower has *nothing* worse than the inferior firm a priori—it offers higher net valuation to consumers (which is analogous to the QS), it has a higher margin (which is analogous to the VPC), and it has the same match probability (which we did not make higher to avoid confusion, but this can be done and the result still holds). The resulting CTR of the superior firm is also higher. In the final outcome, however, it is placed below the inferior firm. In other words, when multiple dimensions of quality are considered in the above-mentioned papers, firms still order themselves by a composite quality score (i.e., QS $\times$ VPC). In our paper, there is no room for ambiguity on which firm is the superior one, and we still show that it can be placed lower in equilibrium.

There are some other working papers that also model search by consumers (Athey and Ellison 2011, Chen and He 2006). However, they do not generate the position paradox and many of our other results. The key difference is that in these models, all consumers rationally believe that firms are ordered by decreasing quality, and firms indeed order themselves in that way in equilibrium. In our model, uninformed consumers who are boundedly rational always start searching from the top even if the lower-quality firm is at the top, which under some conditions leads to firms endogenously ordering themselves by increasing quality. The empirical evidence we provide offers strong support for our theory. We also show that even if the uninformed consumers are rational, then under some conditions they will still always start searching from the top because of asymmetric information about firm types. Furthermore, we obtain other counterintuitive results, such as the result that bid weights can be negatively correlated with firm quality, which these papers do not generate.

To summarize, previous papers either do not model consumer search, or if they model consumer search, then they do not derive the position paradox. We model consumer search and find that under certain conditions firms order themselves in descending order of quality (in agreement with previous research), whereas under other conditions the reverse ordering may be observed.

7. Conclusions and Discussion

Surfers on the World Wide Web rely on search engines to direct them to websites that contain content of immediate interest to them. In sponsored search advertising, firms bid for links to their websites to be displayed in response to a keyword that consumers search. This provides a highly targeted advertising medium, and therefore, sponsored search advertising has become very popular with advertisers in a very short time. In this paper, we study the bidding strategies of vertically differentiated firms that bid for sponsored search advertisement positions for a keyword at a search engine. We explicitly model consumer navigation and clicking of search results based on the firms' positions and the consumers' own knowledge and beliefs about firm qualities. We derive several results on how firms will bid in position auctions.

Interestingly, we find the existence of an important paradoxical outcome in which a high-quality firm bids less than a low-quality firm to be placed below it, yet it still obtains more clicks than the low-quality firm. In a pay-per-impression auction, the position paradox can occur because a superior firm may be confident that even if it is placed lower, it will get a sufficient mass of residual consumers because of its superior quality, whereas an inferior firm will obtain a sufficient click-through rate only if it is placed above the superior firm. In a pay-per-click auction, the position paradox becomes even more likely because the inferior firm only has to pay for the uninformed consumers and can, therefore, bid more aggressively and win more often. We decompose these trade-offs into three effects—residual demand, incremental value, and differential cost.

We show that the position paradox is more likely to occur when the quality differential between the two firms is larger and when the high-quality firm has a better reputation (i.e., more consumers know its identity). Using a data set obtained from a leading Korean search engine firm, we show empirical evidence to support our theory. In an extension to the basic model, we show that the position paradox persists under quality-based bid weighting by the search engine. In fact, contrary to conventional belief, it may be in the search engine's interest to attach higher weights to the bids of lower-quality firms to increase their rank in the list. In other words, the search engine may have the incentive to use bid weighting to intentionally create a position paradox.

At a fundamental level, our analysis adds to the existing literature on sponsored search auctions by highlighting the insight that the value of a position

to a firm depends on its competitive environment. Therefore, metrics that are often invoked in studies on sponsored search auctions, such as click-through rate and value per click, may not always be exogenously assumed. Rather, these metrics should be determined endogenously by understanding the strategic interactions among firms, which in turn are influenced by how consumers respond (based on the configuration of the list of sponsored links, their own information sets, as well as their cognitive abilities).

We now discuss some avenues for future work. First, mechanism design for position auctions is not in the scope of our paper because we do not consider the search engine as a strategic player. Nevertheless, our counterintuitive result that larger weights may be attached to bids from lower-quality firms indicates that a rigorous study of mechanism design incorporating firm reputation effects, heterogeneous consumer knowledge of this reputation, and consumer search can be a fruitful area for future work.

Second, we have simplified the auction from a continuous asynchronous bidding auction to a one-shot simultaneous bidding auction, as do almost all other theoretical papers on this subject. This allows us to focus on our key insights while keeping the analysis tractable. Moreover, even though sponsored search auctions are run by the search engine on a continuous basis, the bidding activity is usually far from continuous. As pointed out by Athey and Nekipelov (2010), it usually does not make much sense for bidders to change bids continuously because the effort associated with continuous bidding may be too high, and the feedback bidders get from the search engine is usually much slower and much more aggregated (typically at the daily level). Moreover, not all search engines allow continuous bidding. For example, our Korean search engine allows bidders to bid only once a day, and the outcome then remains fixed for that day. Interestingly, we observe in our data that bidders typically do not change their bids for many weeks at a stretch. This stability in bids indicates that the bidders have settled into some sort of equilibrium. Therefore, we believe that a simplification of the bidding game from continuous asynchronous bidding to one-time simultaneous bidding should not change our basic insights. Modeling a repeated position auction presents an interesting, but challenging, opportunity for future research.

Third, in our model, consumers know the quality levels in the market and search to find out which firm offers which quality level. As long as consumers' beliefs about quality levels in the market do not deviate significantly from true quality levels, our results will hold. Because the position paradox is generated as an outcome of consumer search, we expect it to be more prominent if there is additional uncertainty about quality levels, because consumers will search more. Building quality uncertainty explicitly into the model is an interesting direction for future work.

Finally, our empirical exploration in §5.4 indicates that the apparently common belief that there is a large overlap between organic and sponsored results at major search engines might actually be a misconception. We find that not only is this overlap very small for most keywords, but the nature of the results is also very different (information-based links in organic results versus commercial links in sponsored results). Future studies can explore this interesting observation in more detail, which can aid in the understanding of how consumers process organic and sponsored results jointly.

8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

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Appendix

A.1. Analysis for Pay-per-Impression Auction

Scenario I: $s \leq p\min\{Q, V\}$. This is a low-search-cost scenario. If the top link is of the superior firm, an uninformed consumer will buy the product and stop if she finds a match. If she does not find a match, she will click on the bottom link of the inferior firm. If the top link is of the inferior firm, an uninformed consumer will click on the bottom link, regardless of whether she finds a match at the top link. If she finds a match at the superior firm, she will buy the product and stop. Otherwise, she will either buy the product from the inferior firm if a match had been found when she first searched there or stop if not. An informed consumer always starts from the superior firm, buys from it if a match is found, and searches the inferior firm in case there is no match. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = (1-p)pm_I - \underline{b}$, and $E[\Pi_{S,2}] = pm_S - \underline{b}$ and $E[\Pi_{I,1}] = (1-p)pm_I - b_I$. The equilibrium bids are $b_S^* = \underline{b}$ and $b_I^* = \underline{b}$; i.e., both firms will bid the minimum required amount because neither has an extra advantage from the top position. We assume that the tie is broken randomly with equal probability. If the superior firm

is placed second, all consumers click on its link, but only a $(1-p)$ fraction of consumers click on the inferior firm's link. Therefore, we can obtain the position paradox.

Scenario II: $p\min\{Q,V\} < s \le p\max\{Q,V\}$ **and** $Q > V$ **(i.e.,** $pV < s \le pQ$**).** In this scenario, the search cost is higher than the expected utility of visiting the inferior firm but lower than the expected quality premium from the superior firm. If a consumer visits the superior firm first, she will not visit the inferior firm irrespective of whether she finds a match at the superior firm or not. If she visits the inferior firm first, she visits the superior firm irrespective of whether she finds a match at the inferior firm or not. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0 - \underline{b}$, and $E[\Pi_{S,2}] = pm_S - \underline{b}$ and $E[\Pi_{I,1}] = (1-\phi)(1-p)pm_I - b_I$. Thus, position does not matter to the superior firm, whereas the inferior firm strictly prefers the top position. The equilibrium bids are $b_S^* = \underline{b}$ and $b_I^* = (1-\phi)(1-p)pm_I + \underline{b}$. In equilibrium, the inferior firm is always placed on top, but it obtains fewer clicks; i.e., the position paradox arises.

Scenario III: $p\min\{Q,V\} < s \le p\max\{Q,V\}$ **and** $Q < V$ **(i.e.,** $pQ < s \le pV$**).** In this scenario, if a consumer first visits the superior firm and finds a match, she will still buy and stop. If she first visits the inferior firm and finds a match, she will also buy and stop, because the expected quality premium does not warrant an additional search. If a consumer does not find a match in the first firm she visits, she will continue to search the other firm. As in the first two scenarios, an uninformed consumer will first search whichever firm that is on top, whereas an informed one will search the superior firm first. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = (1-p)pm_I - \underline{b}$, and $E[\Pi_{S,2}] = \phi pm_S + (1-\phi)(1-p)pm_S - \underline{b}$ and $E[\Pi_{I,1}] = \phi(1-p)pm_I + (1-\phi)pm_I - b_I$. The equilibrium bids are $b_S^* = (1-\phi)\cdot p^2 m_S + \underline{b}$ and $b_I^* = (1-\phi)p^2 m_I + \underline{b}$. In this scenario, both firms will generate higher revenue when placed on top than when placed on bottom. However, since $m_S > m_I$, the superior firm has "more to lose" at the bottom position and will outbid the inferior firm.

Scenario IV: $p\max\{Q,V\} < s \le p(V+Q)$**.** Please see §3.1.

Scenario V: $s > p(V+Q)$**.** In this scenario, the search cost is higher than even the expected utility of visiting the superior firm. In this case, a consumer will stop after conducting the first search, regardless of which firm is visited or whether a match is found. An uninformed consumer starts from the top, whereas an informed consumer starts from the superior firm. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0 - \underline{b}$, and $E[\Pi_{S,2}] = \phi pm_S - \underline{b}$ and $E[\Pi_{I,1}] = (1-\phi)pm_I - b_I$. The equilibrium bids are $b_S^* = (1-\phi)pm_S + \underline{b}$ and $b_I^* = (1-\phi)pm_I + \underline{b}$. In this scenario, the superior firm has more to lose if it does not win the top position than does the inferior firm. Hence, it will bid to be placed on top.

A.2. Analysis for Pay-per-Click Auction

In the pay-per-click auction, the revenues for each firm stay the same but the payments change because they depend on the actual clicks (and not just the impressions). We use the revenue expressions mentioned above and only discuss the payments based on clicks.

Scenario I: $s \le p\min\{Q,V\}$**.** The analysis of this low-search-cost scenario is the same as in the pay-per-impression case because payments remain the same.

Scenario II: $p\min\{Q,V\} < s \le p\max\{Q,V\}$ **and** $Q > V$ **(i.e.,** $pV < s \le pQ$**).** In this scenario, when the inferior firm is placed on top, the probability that it is clicked is $1-\phi$ (the portion of uninformed consumers); when it is placed on bottom, no consumer clicks on it. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0$, and $E[\Pi_{S,2}] = pm_S - \underline{b}$ and $E[\Pi_{I,1}] = (1-\phi)(1-p)pm_I - (1-\phi)b_I$. The equilibrium bids are $b_S^* = \underline{b}$ and $b_I^* = (1-p)pm_I$. The insight is that because the inferior firm pays only when its link is clicked, it can bid the expected revenue conditional on the link being clicked, instead of the unconditional expected revenue as in the pay-per-impression case. Hence, we again have a case in which the inferior firm emerges on top but obtains fewer clicks.

Scenario III: $p\min\{Q,V\} < s \le p\max\{Q,V\}$ **and** $Q < V$ **(i.e.,** $pQ < s \le pV$**).** In this scenario, if the superior firm is placed on top, the probability that it will be clicked is 1. If it is placed at the bottom, this probability is $\phi + (1-\phi)(1-p)$. If the inferior firm is placed on top, its probability of being clicked is $\phi(1-p) + (1-\phi)$, whereas if it is placed at the bottom, the probability of it being clicked is $1-p$. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = (1-p)\cdot(pm_I - \underline{b})$, and $E[\Pi_{S,2}] = (\phi + (1-\phi)(1-p))(pm_S - \underline{b})$ and $E[\Pi_{I,1}] = (\phi(1-p) + (1-\phi))(pm_I - b_I)$. The equilibrium bids are $b_S^* = (1-\phi)p^2 m_S + (1-(1-\phi)p)\underline{b}$ and $b_I^* = ((1-\phi)p^2 m_I + (1-p)\underline{b})/(1-\phi p)$. Depending on the values of the parameters, either firm may end up at the top position. This is different from the pay-per-impression case, in which the superior firm always wins in this scenario. Note that the differential cost effect is driving this difference. If the inferior firm wins in this scenario, then it will still obtain fewer clicks if $\phi \ge 1/2$.

Scenario IV: $s > p\max\{Q,V\}$ **and** $s \le p(V+Q)$**.** Please see §3.2.

Scenario V: $s > p(V+Q)$**.** In this scenario, if the superior firm is placed on top, the probability that it will be clicked is 1. If it is placed at the bottom, this probability is ϕ. If the inferior firm is placed on top, the probability that it will be clicked is $1-\phi$, whereas if it is at the bottom, this probability is 0. Therefore, $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0$, and $E[\Pi_{S,2}] = \phi(pm_S - \underline{b})$ and $E[\Pi_{I,1}] = (1-\phi)(pm_I - b_I)$. The equilibrium bids are $b_S^* = (1-\phi)pm_S + \phi\underline{b}$ and $b_I^* = pm_I$. Either firm may end up at the top position, and as ϕ increases, the inferior firm bids higher and will win the auction. Furthermore, if the inferior firm wins, then it will still obtain fewer clicks if $\phi \ge 1/2$.

An assumption made in the above analysis is that in all equilibria, both firms' bids as specified in the equations are larger than or equal to the minimum bid ($\underline{b}$). The assumption will hold as long as both firms' margins are considerably larger than the minimum bid and when the match probability is not too close to 0 or 1.

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